Romerico David, Haley Elliott, Julian Halsey, Nkechiyem Molokwu
COSC 336
02/22/2024

Assignment 2

Exercise 1:

a. $\Theta((4n+1)(4^{log_2(n)}))$
$= \Theta(4n^3 + n^2)$
$= \Theta(n^3)$

b. $t_1(n) = n^2 + n$
$t_2(n) = n^2$

c. $t_3(n) = n^2$ since $t_3(2n) = (2n)^2 = 4n^2 = \theta(n^2)$

d. $t_4(n) = 2^n$ since $t_4(2n) = 2^{2n}$

Exercise 2:

	A	B	O	o	Ω	ω	Θ
a.	$\log^k n$	n^ϵ	Yes	Yes	No	No	No
b.	n^k	c^n	Yes	Yes	No	No	No
c.	$\sqrt{n}$	$n^{\sin n}$	No	No	No	No	No
d.	2^n	$2^{n/2}$	No	No	Yes	Yes	No
e.	$n^{\log c}$	$c^{\log n}$	Yes	No	Yes	No	Yes
f.	$\log(n!)$	$\log(n^n)$	Yes	No	Yes	No	Yes

Exercise 3:

a. $(n \cdot n)(\frac{n}{2}) = \frac{n^3}{2}$
$=> \Theta(\frac{n^3}{2})$
$= \Theta(n^3)$

b. $n + \frac{n}{2}$
$=> \Theta(n + \frac{n}{2})$

$= \Theta(n)$

c. $(n \cdot n)(n \cdot n) = n^4$
$=> \Theta(n^4)$

d. $\Theta(\log_2 n)$

e. $(n)(log_2(n \cdot n))$
$=> (n)(\log_2(n^2))$
$=> (n)(2\log_2 n)$
$= \Theta(n \log_2 n)$

Exercise 4:

a. $S_1 = 500 + 501 + 502 + ... + 999$
$n = 999 - 500 + 1 = 500, \;\; a = 500, \;\; d = 501 - 500 = 1$

$S_1 = (500)(500) + (\frac{1(500(500-1))}{2})$
$= 250000 + (\frac{249500}{2})$
$= 250000 + 124750$
$= 374750$

b. $S_2 = 1 + 3 + 5 + ... + 999$
$n = \frac{999-1}{2} + 1 = 500, \; a = 1, \; d = 2$

$S_2 = (500)(1) = (\frac{2(500(500-1))}{2}$
$= 500 + (500(500-1))$
$= 500 + (500(499)$
$= 500 + 249500$
$= 250000$

c. $n = 30, \; k = 4$
Number of possible committees $= \binom{n}{k} = \binom{30}{4}$
$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
$=> \binom{30}{4} = \frac{30!}{4!(30-4)!} = \frac{30!}{4!26!}$

$= \frac{30 \cdot 29 \cdot 28 \cdot 7 \cdot 26!}{4!26!}$

$= \frac{30 \cdot 29 \cdot 28 \cdot 27}{24}$

$= 27405$

There are 27405 possible combinations.

d. $C_n = \binom{n}{4}$

$\binom{n}{4} = \frac{n!}{4!(n-4)!}$

$= \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)...}{4! \cdot (n-4)(n-5)...}$

$= \frac{n(n-1)(n-2)(n-3)}{4!}$

$= \frac{n^4 - 6n^3 + 11n^2 - 6n}{24}$

$= \Theta(n^4)$

$C_n = \Theta(n^4) \;\; \therefore \;\; C_n \neq o(n^3)$

Exercise 5:

Lower Bound:

$S_n \geq \int_0^n x^2 \sqrt{x} dx$

$= \int_0^n x^{5/2}$

$= \frac{2}{7} x^{7/2} \Big|_0^n$

$= \frac{2}{7} n^{7/2}$

$= n^{7/2}$

Upper Bound:

$S_n \leq \int_1^{n+1} x^2 \sqrt{x} dx$

$= \int_1^{n+1} x^{5/2}$

$= \frac{2}{7} x^{7/2} \Big|_1^{n+1}$

$= \frac{2}{7}(n+1)^{7/2} - \frac{2}{7}$

$= \frac{2}{7}(n+1)^{7/2}$

$= (n+1)^{7/2}$

$= n^{7/2}$

$\therefore \Theta(n^{7/2})$