Data Structures and Algorithms

COSC 336 Assignment 2

Instructions.

1. Submit by the date and time indicated on Blackboard.
2. This is a team assignment. Work in teams as in the previous assignments. Submit on Blackboard one assignment per team, with the names of all students making the team.
3. For editing your homework. I recommend that you use Latex and Overleaf, see the template files posted on the Blackboard: assignment-template.tex and assignment-template.pdf.
4. If a problem has more questions, write down your answers in the same order as the order of questions. In principle, this should help you.

Exercise 1.

a Find a Θ evaluation for the function $(4n+1)4^{\log(n)}$. (Hint: $4^{\log(n)}$ can be written in a simpler way.)

b Give an example of two functions $t_1(n)$ and $t_2(n)$ that satisfy the relations: $t_1(n) = \Theta(n^2)$, $t_2(n) = \Theta(n^2)$ and $t_1(n) - t_2(n) = o(n^2)$.

c Give an example of a function $t_3(n)$ such that $t_3(n) = \Theta(t_3(2n))$.

d Give an example of a function $t_4(n)$ such that $t_4(n) = o(t_4(2n))$.

(Note: For (b), (c), (d), the functions t_1, t_2, t_3, t_4 you pick must be selected from the common functions we have discussed, namely polynomials, logarithms, exponentials, factorial.)

Exercise 2. Fill the table from Exercise 3-2, page 61 (3-rd edition) in the textbook (also attached below), except row c, as asked in the exercise. For example the entry on the first cell in the top row is "yes" because $\log^k n = O(n^\epsilon)$. (Note: in row c all the entries are "no", because $n^{\sin n}$ oscillates.)

3-2 *Relative asymptotic growths*

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

Exercise 3. For each of the following program fragments give a $\Theta(\cdot)$ estimation of the running time as a function of n.

(a)
```
sum = 0;
for (int i = 0; i< n * n; i++) {
      for(int j =0;  j < n/2; j++)
        sum++;
}
```

(b)
```
 sum = 0;
 for (int i = 0; i< n; i++) {
sum++;}

for(int j = 0;  j < n/2; j++){
      sum++;}
```

(c)
```
 sum = 0;
 for (int i = 0; i< n * n; i++) {
      for(int j = 0;  j < n * n; j++)
           sum++
}
```

(d)
```
 sum = 0;
 for (int i = 1; i< n; i = 2*i)
           sum++
```

(e)
```
 sum = 0;
 for (int i = 0; i< n; i++) {
      for(int j = 1;  j < n * n; j = 2*j)
           sum++
}
```

Exercise 4. (a) Compute the sum $S_1 = 500 + 501 + 502 + 503 + \ldots + 999$ (the sum of all integers from 500 to 999). Do not use a program.

(b) Compute the sum $S_2 = 1 + 3 + 5 + \ldots + 999$ (the sum of all odd integers from 1 to 999). Do not use a program.

(c) A group of 30 persons need to form a committee of 4 persons. How many such committees are possible?

(d) Let C_n be the number of committees of 4 persons selected from a group of n persons. Is the estimation $C_n = o(n^3)$ correct? Justify your answer. (Hint: using the formula $\binom{n}{k}$, you can express the number of committees as a function of n.)

Exercise 5. Find a $\Theta(\cdot)$ evaluation for the sum

$$S = 1^2\sqrt{1} + 2^2\sqrt{2} + 3^2\sqrt{3} + \ldots + n^2\sqrt{n}.$$

In other words, find a function f such that $S = \Theta(f(n))$.

Show the work for both the upper bound and the lower bound. You can use the technique with integrals, or the method with bounding the terms of the sum.