apply hedamart on 0 inititalised state ---> or apply fourier transform --> both generate same result

Note Title 09-10-2024

Quantum Phase Estimation (QPE)

↳ To extract the energy (eigen value) of the quantum operator (U) or to obtain the period in shor's algorithm.

↳ Linear system of Equation

Ex $\quad A x = \lambda x \quad \approx \quad \boxed{U|\phi\rangle = \lambda |\phi\rangle}$

λ → Eigen value of unitary U

$$\Downarrow$$

$$\boxed{U|\phi\rangle = e^{2\pi i\theta} |\phi\rangle}$$

→ Eigen value containing the phase θ of U

$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$, $e^{i\pi/2} = i$

Ex $\quad S|1\rangle = e^{i\pi/2} |1\rangle$ } $\theta = \frac{1}{4} = 0.25$

$e^{2\pi i \frac{1}{4}} = e^{i\pi/2}$

$e^{i\pi/2} = i$

0.1
0.010

Ex $\quad \theta = 0.25 \approx 0.010$

3-qubit $|010\rangle = |2^n\theta\rangle$

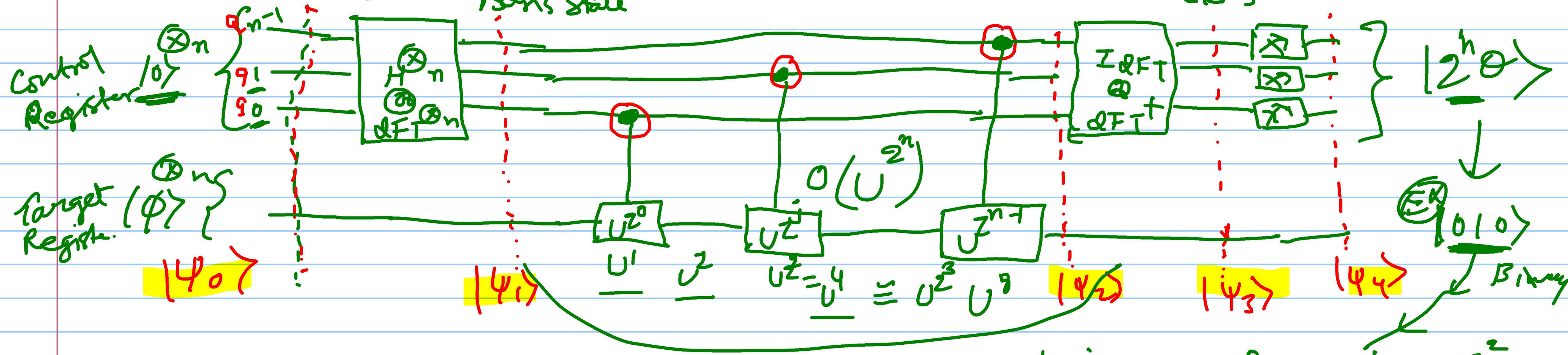
QPE Circuit

$H^{\otimes n} = QFT^{\otimes n}$ if $(|0\rangle^{\otimes n})$

$H(|0\rangle) = |+\rangle$ $\quad$ $H(|1\rangle) = |-\rangle$

Computi. Basis Sta $\xrightarrow{QFT}$ Fourier Basis State

FBS $\xrightarrow{QFT^{\dagger}}$ CBS



Using Phase Kickback mechanism the phase value θ gets encoded in the control register

$= 0 \times 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2$

$= (2)$ Decimal

Algo Step.

$$|\psi_0\rangle = |0\rangle^{\otimes n} \otimes |\phi\rangle^{\otimes n}$$

Solution $= \dfrac{2}{\text{Total no. of states syste } (n=3)} = \dfrac{2}{2^3} = \dfrac{2}{8} = \boxed{\dfrac{1}{4}}$

$$|\psi_1\rangle = (H^{\otimes n} \otimes I^{\otimes n})(|\psi_0\rangle) \cong (QFT^{\otimes n} \otimes I^{\otimes n})$$

$$= \frac{1}{\sqrt{2^n}}(|0\rangle + |1\rangle)^{\otimes n} \otimes |\phi\rangle$$

Ex

$$\left[\, |+\rangle \otimes |\phi\rangle = \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |\phi\rangle = \frac{|0\rangle|\phi\rangle + |1\rangle|\phi\rangle}{\sqrt{2}} \,\right]$$

$|\psi_2\rangle = (U^{2^j})\,|\phi\rangle$ to perform $2^j \to$ Phase Rotations

$$|\psi_1\rangle \longrightarrow U^{2^j}|\phi\rangle = e^{2\pi i 2^j \theta}|\phi\rangle$$

= when the qubit of control register is in ket $|1\rangle$, we apply controlled unitary CU^{2^j} (where $j = 0$ to $n-1$) $\quad n = 3 \quad \{0, 1, 2\}$

$$= \frac{1}{\sqrt{2^n}}\Big(\big(|0\rangle|\phi\rangle + |1\rangle \underset{I}{U^{2^0}}|\phi\rangle\big) \otimes \big(|0\rangle|\phi\rangle + |1\rangle \underset{I}{U^{2^j}}|\phi\rangle\big) \otimes \big(|0\rangle|\phi\rangle + |1\rangle U^{2^{n-1}}|\phi\rangle\big)\Big)$$

$$= \frac{1}{\sqrt{2^n}}\Big(\big(|0\rangle + |1\rangle e^{2\pi i 2^0 \theta}|\phi\rangle\big) \otimes \big(|0\rangle + |1\rangle e^{2\pi i 2^j \theta}|\phi\rangle\big) \otimes \big(|0\rangle + |1\rangle e^{2\pi i 2^{n-1}\theta}|\phi\rangle\big)$$

$$= \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} e^{2\pi i \theta j} \underbrace{|j\rangle}_{CR} \otimes \underbrace{|\phi\rangle}_{TR} \Big\} \cong QFT$$

$$|\psi_3\rangle = QFT^{\dagger}|\psi_2\rangle$$

$$= \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} e^{2\pi i \theta j} |j\rangle \otimes |\phi\rangle \xrightarrow{QFT^{\dagger}}$$

$$= \frac{1}{2^n} \sum_{x=0}^{2^n-1} \sum_{j=0}^{2^n-1} e^{\frac{-2\pi i}{2^n}(x - 2^n\theta)} |x\rangle \otimes |\phi\rangle$$

$|\psi_4\rangle$ = Apply Measurement

CR $\rightarrow$ $\underline{|2^n\theta\rangle}$ $\simeq$ Eigenvalue of Unitary operator