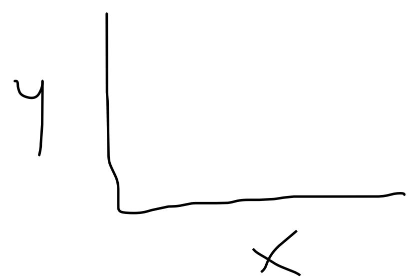
Image Representation for a 2x2 binary image



$$
y \quad \begin{array}{c|cc|} & 0 & 1 \\ \hline 0 & 00 & 01 \\ 1 & 10 & 11 \\ \hline \end{array} \text{ positions}
$$

x

$$
\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \text{ intensity}
$$

Intensity

0 black

1 white

classical image $\longrightarrow$ quantum state.

NEQR — 2006

Intensity — 0/1 ——— 1 quantum reg

position — 00/01/10/11 ——— 2 quantum reg

3 = 3 classical

quantam reg.

Representation of a qubit

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

α — prob of being in state 0

β — " state 1

$|\psi\rangle$ → ket vector → column vector

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1 \longrightarrow \text{Born rule}$$

$$P_{zero} = \frac{1}{2} = \alpha^2 \quad \alpha = \frac{1}{\sqrt{2}}$$

$$P_{one} = \frac{1}{2} = \beta^2 = \beta = \frac{1}{\sqrt{2}}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \rightarrow$$ Superposition state

$\boxed{H} \rightarrow$ Hadamard $\longrightarrow$ superposition

$$|0\rangle \rightarrow \boxed{H} \longrightarrow \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

$\otimes$

$$|1\rangle \longrightarrow \boxed{H} \longrightarrow \frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$$\otimes \ \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

$$\frac{1}{\sqrt{2}}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25 \rightarrow H$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix}\right]$$

$$\frac{1}{\sqrt{2}}\left[\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1 & -1\end{bmatrix}\right]$$

$$=\frac{1}{\sqrt{2}}\left[\begin{matrix}1 & 1\\ 0 & 0\end{matrix}+\begin{matrix}0 & 0\\ 1 & -1\end{matrix}\right]$$

$$H=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

Matrix of H gate

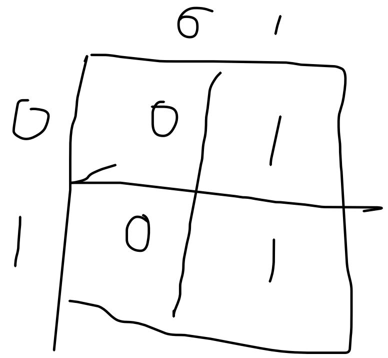
$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

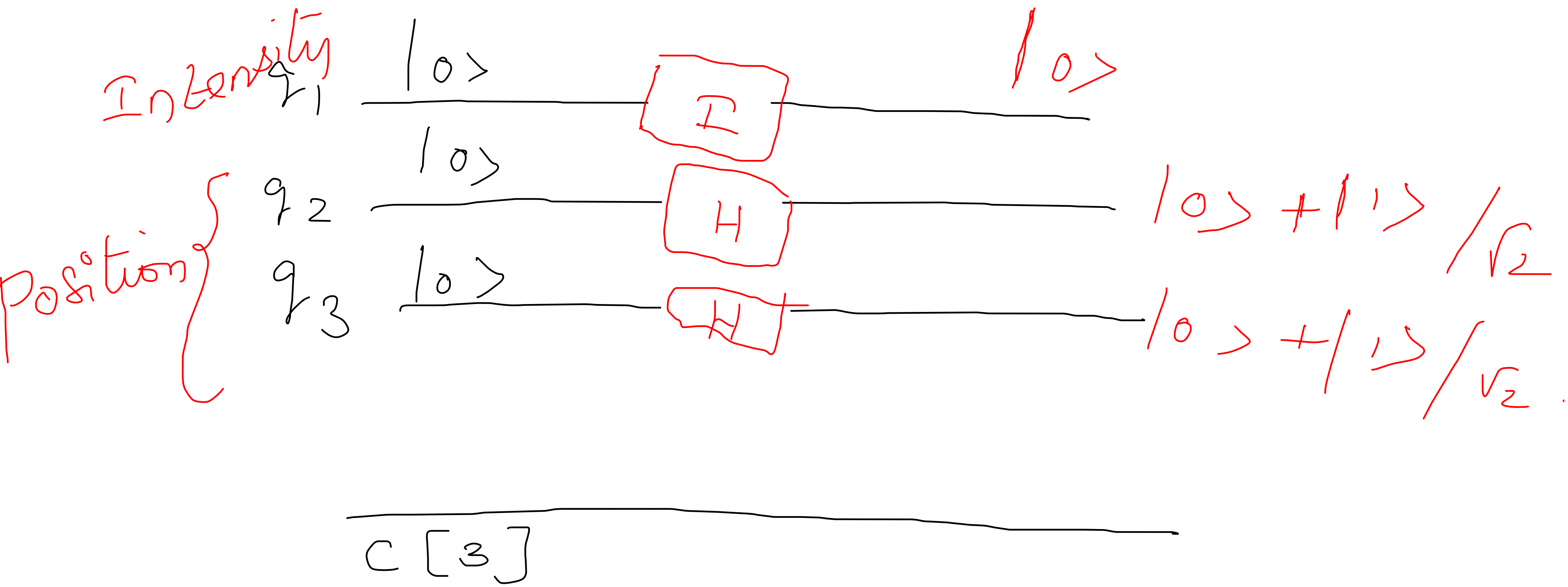
$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \begin{matrix} 1 \\ 0 \end{matrix} + \begin{matrix} 0 \\ 1 \end{matrix}$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right]$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \begin{matrix} 1 \\ 0 \end{matrix} - \begin{matrix} 0 \\ 1 \end{matrix}$$

$$= \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right)$$


0
1
0
0
1
1
0
1


Intensity
q_1
$|0\rangle$
I
$|0\rangle$
Position
q_2
$|0\rangle$
H
$|0\rangle + |1\rangle / \sqrt{2}$
q_3
$|0\rangle$
H
$|0\rangle + |1\rangle / \sqrt{2}$
c[3]

$$\frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right] \otimes \left[|0\rangle + |1\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]$$

$$\frac{1}{4} = 0.25$$

I gate → bell state eqn

$$\frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]|0\rangle$$

pixel intensity

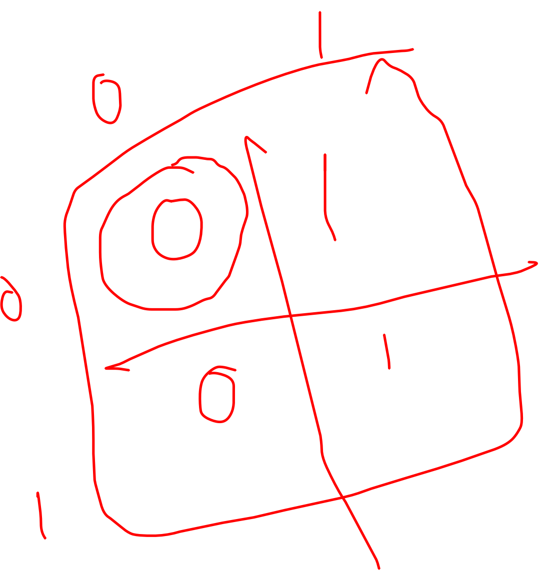
$$= \frac{1}{2}\Big[|\underline{00}0\rangle + |\underline{01}0\rangle + |\underline{10}0\rangle +$$

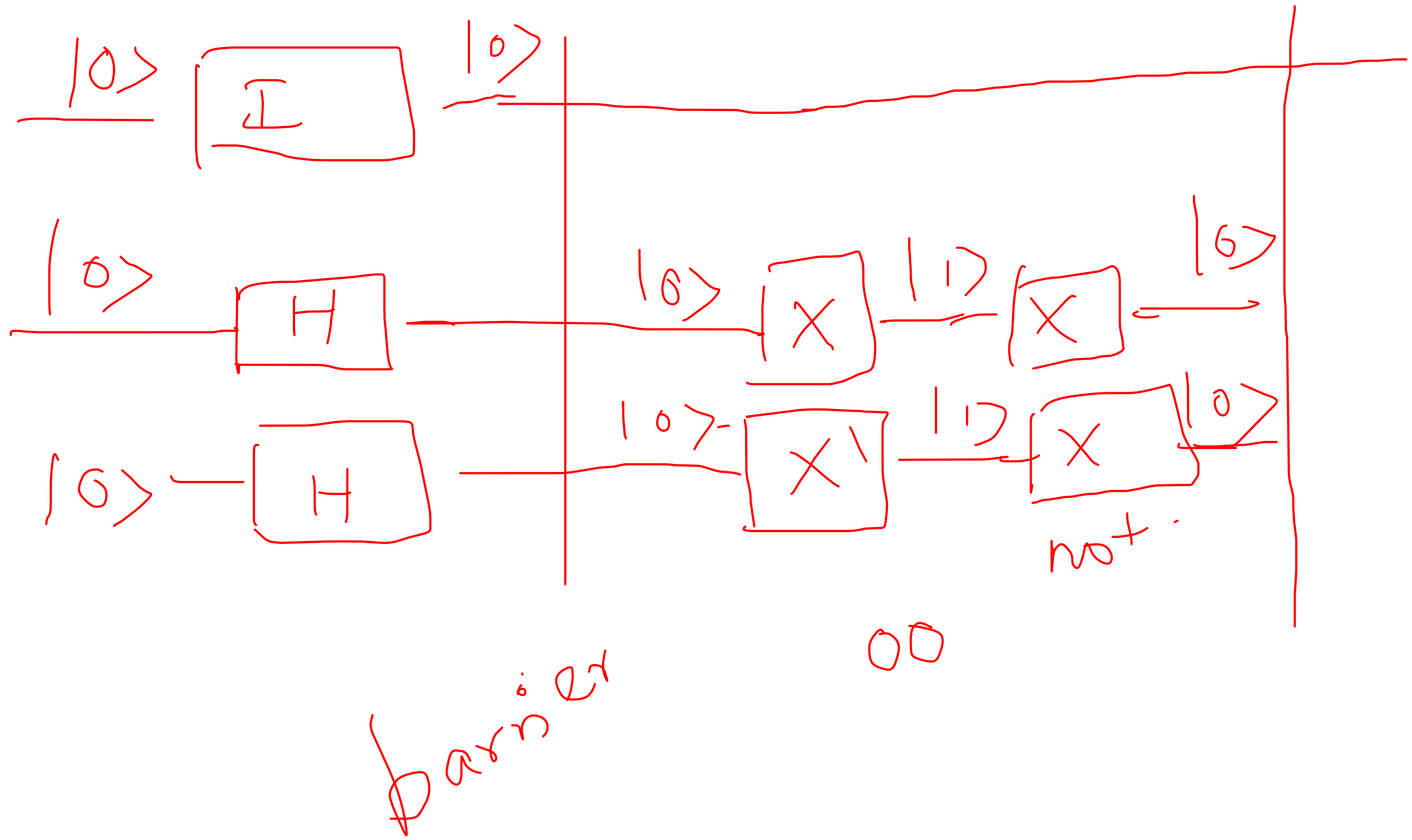
$$|\underline{11}0\rangle\Big]$$

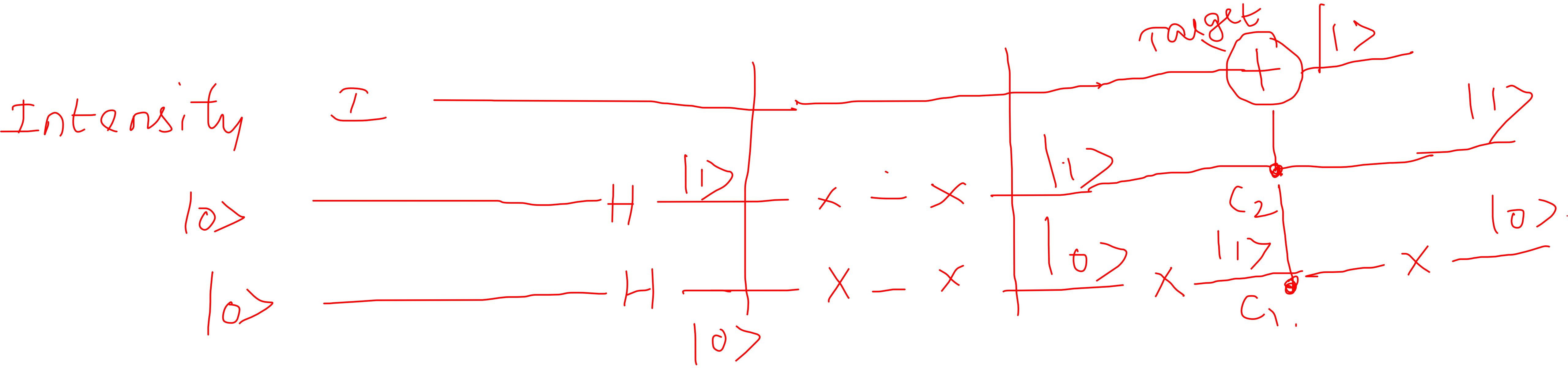
pixel position

Intensity

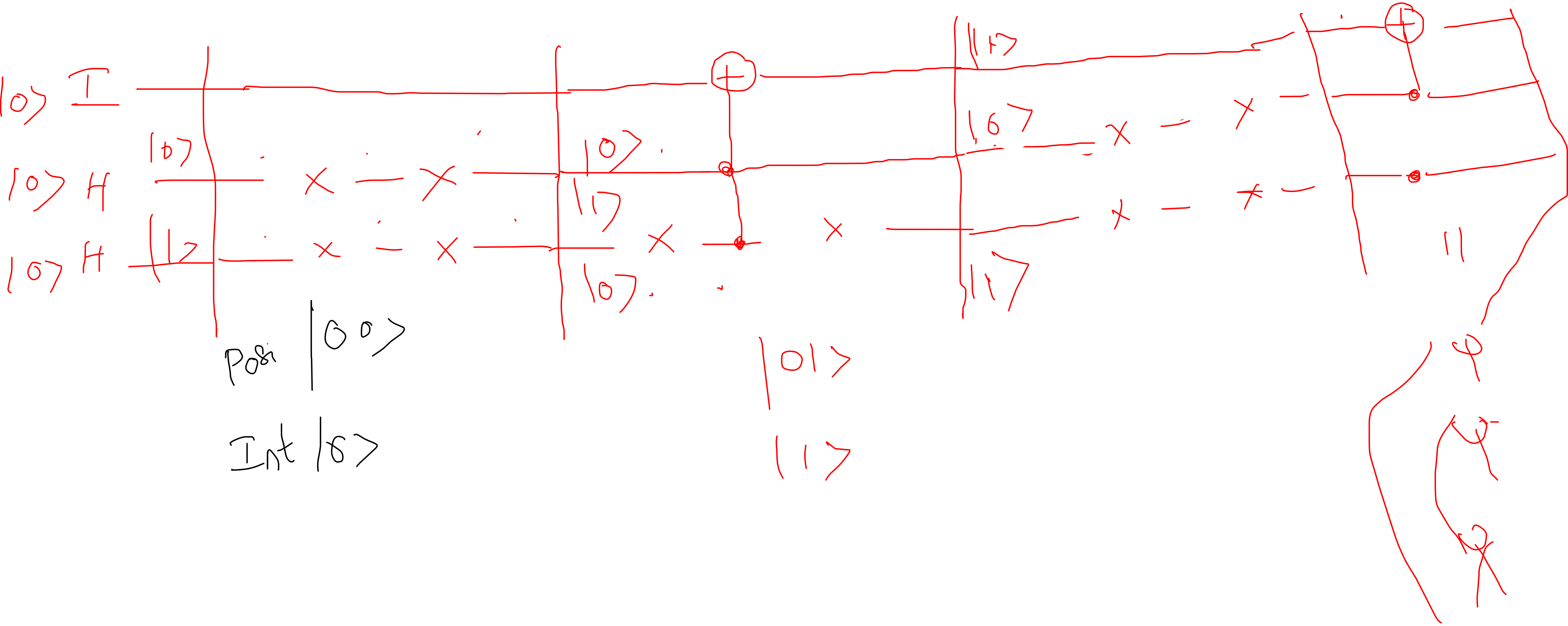
Position







$c_1 = c_2 = 1$.



|0> I
|0> H
|0> H
|+>
|1>
|0>
|1>
|0>
|1>
|0>
|1>
Pos |00>
Int |0>
|01>
|1>

Identity gate – I – buffer
do nothing.

C_1	C_2	T_1
0	0	remain the same
0	1	
1	0	

H → Superposition

X – not gate.

C = 11 → flip the target bit

CCNOT / TOffolli gate → 2 control bits
1 target bits