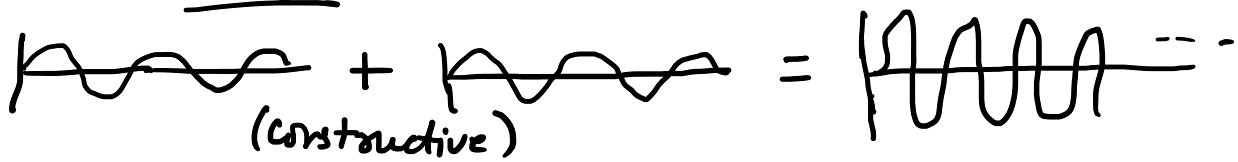
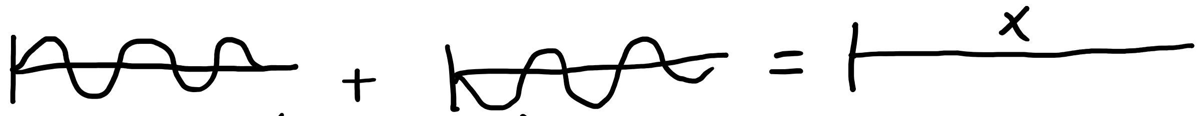
- QAA is a design strategy that enhances / amplifies the amplitudes associated to the desired item.
- QAA based on the interference mechanism (constructive / destructive)

Ex



(Constructive)

Ex



(Destructive)

- Grover's search Algorithm is the specialized case of QAA.
- Applicable over unstructured list of items, for which the classical solution is linear search $(O(N))$, where N = No. of elements.
  - $\Rightarrow (N=2^{32})\ N \rightarrow$ Exponentially large (Problem Complexity)
  - $\Rightarrow$ Classically solution is polynomially solvable.
  - $\rightarrow$ Problem can be efficiently solvable using quantum algo.

$$O(\sqrt{N})$$

- Grover's Algo works in two parts —

$\frac{\pi}{4}(\sqrt{N})$ Times

1. Grover's Oracle (U_f) — Black-Box encode the mark item. (Phase - Inversion)
2. Grover's Diffusion (U_ψ) — Amplitude Amplification (Inversion about the mean / average) (complex Prob. Amplitude are enhanced such that, the probability of measuring the desired item would be more.)

Solution state

- Let $|\bar{x}\rangle$ is the marked item state and we have in superposition of state.

$N = 2^n$ n = no. of qubits

$8 = 2^3$

- Initial state $|0\rangle^{\otimes n}$
- $|\psi\rangle = H^{\otimes n}(|0\rangle^{\otimes n}) = \frac{1}{\ldots} \sum_{x \in \{0,1\}^n} |x\rangle$

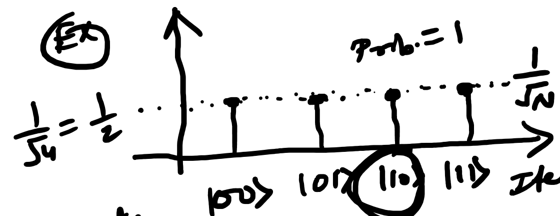
- Initial state $|0\rangle$
- $|\psi\rangle = H^{\otimes n}(|0\rangle^{\otimes n}) = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle$

- Considering the state without / other than $|\bar{x}\rangle$

Non-solution state $\leftarrow$ $|\bar{\Psi}\rangle = \frac{1}{\sqrt{2^n - 1}} \sum_{x \in \{0,1\}^n \text{ and } x \neq \bar{x}} |x\rangle$

Amplitude (4) $N = 2^n = (2)$

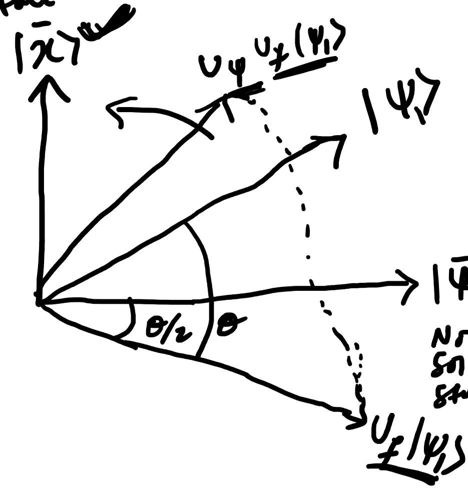
Ex



Algorithmic steps :-

(1) $|\psi_0\rangle = |0\rangle^{\otimes n}$ } Initialization

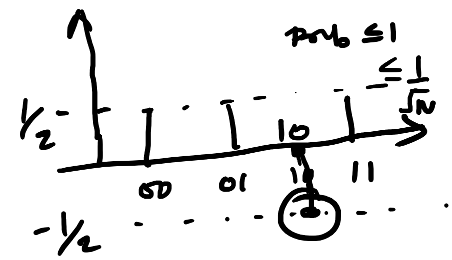
(2) $|\psi_1\rangle$ = Create the superposition
$= H^{\otimes n}(|\psi_0\rangle)$
$= \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle$



(3) Grover's Oracle $U_f|\psi_1\rangle$

$$U_f = I - 2|\bar{x}\rangle\langle\bar{x}|$$

$$U_f(|x\rangle) = \begin{cases} |x\rangle & \text{if } x \neq \bar{x} \\ -|\bar{x}\rangle & x = \bar{x} \end{cases}$$



(4) Grover's Diffusion Operator (U_ψ)

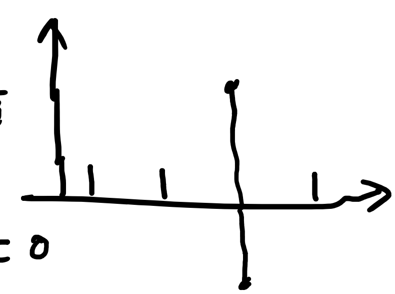
$$\boxed{U_\psi = 2|\psi\rangle\langle\psi| - I}$$

$\frac{\pi}{4}\sqrt{4} = \frac{\pi \cdot 2}{4 \cdot 2}$

$\simeq O(1)$ ✓

$$\left.\begin{aligned} &(i)\ \bar{a} = \frac{1}{N}\sum_{x=0}^{N-1} \alpha_x \\ &(ii)\ \alpha'_x = 2\bar{a} - \alpha_x \end{aligned}\right\}$$

$\frac{\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2}}{4} = \left(\frac{1}{4}\right) = \bar{a}$



$\alpha'_{00} = 2 \cdot \left(\frac{1}{4}\right) - \frac{1}{2} = 0$

$\alpha'_{01} = 0$

$\alpha'_{10} = 2 \cdot \frac{1}{4} - \left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} = 1$

$\alpha'_{11} = 0$

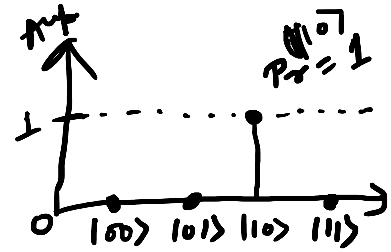
Matrix Representation of Operators

$|10\rangle$ Pr = 1

Matrix Representation of Operators

① $U_f|x\rangle = (-1)^{f(x)}|x\rangle$

$$f(x) = \begin{cases} 1 & , |x\rangle \to -|x\rangle \\ 0 & , |x\rangle \to |x\rangle \end{cases}$$

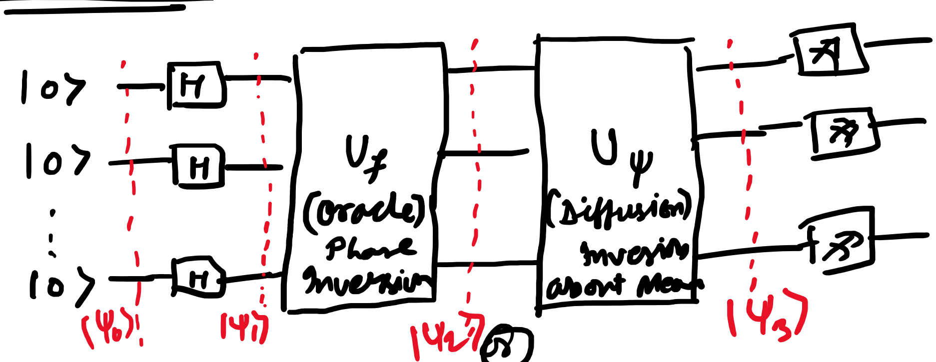


$$\downarrow$$

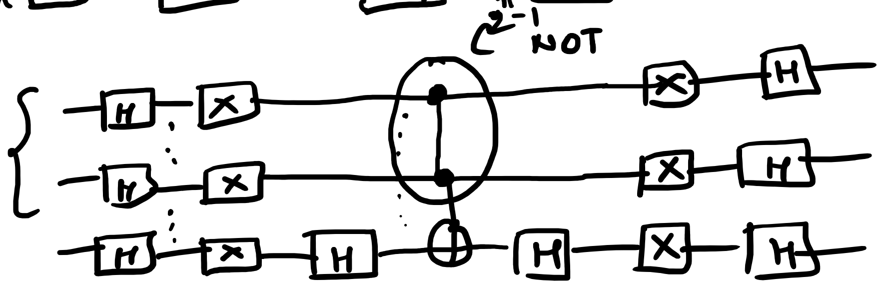
$$U_f = \begin{pmatrix} (-1)^{f(0)} & 0 & 0 & \cdots & 0 \\ 0 & (-1)^{f(1)} & & & \\ \vdots & & (-1)^{f(2)} & & \\ & & & \ddots & \\ 0 & \cdots & & & (-1)^{f(2^n-1)} \end{pmatrix}$$

② $$U_\psi = \begin{pmatrix} \frac{2}{N}-1 & \cdots & \cdots & \frac{2}{N} \\ \frac{2}{N} & \frac{2}{N}-1 & & \vdots \\ \vdots & & \ddots & \\ \frac{2}{N} & \cdots & \cdots & \frac{2}{N}-1 \end{pmatrix}$$

Grover's Circuit



Design (U_ψ)



$HXH = Z$

$HZH = X$