National Institute of Technology Raipur
राष्ट्रीय प्रौद्योगिकी संस्थान रायपुर

DEPARTMENT OF INFORMATION TECHNOLOGY

One Week Hybrid Workshop

on

Quantum Computing and Algorithms (QCA) -2024

Last Day: Miscellaneous Topics of Workshop

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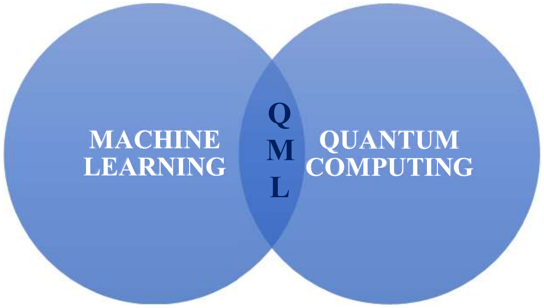
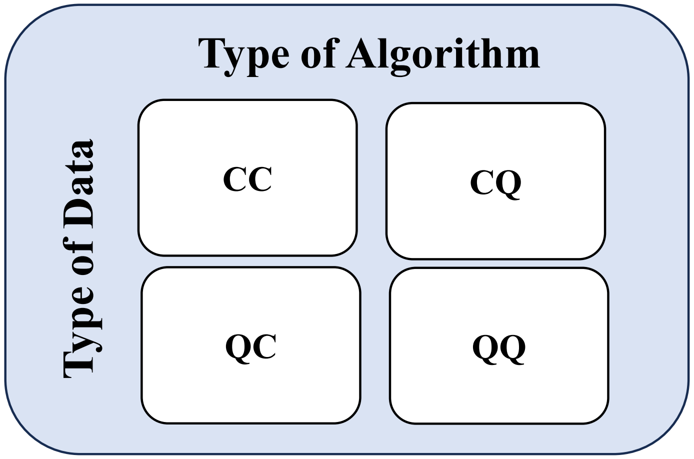
Content

1. Quantum Machine Learning Basics

Quantum Machine Guarantees:

- High Speed Computation
- Compute Intensive / Expensive
- Time Complexity Reduction
- Obtains Exponential Speedups
- High Dimensional Data Handle

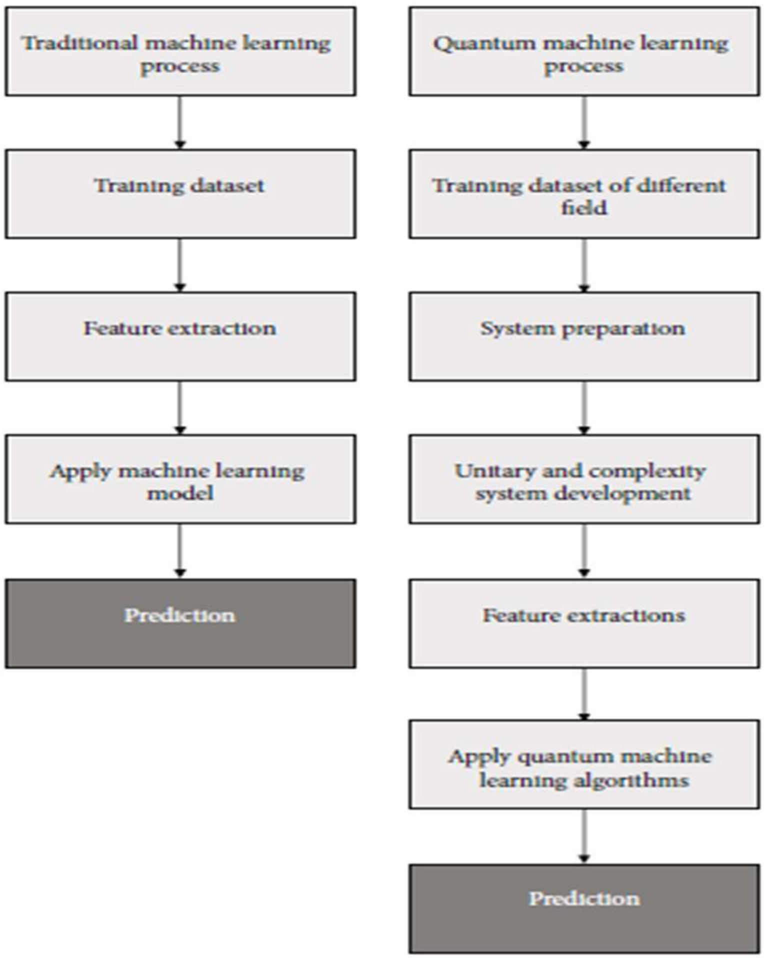


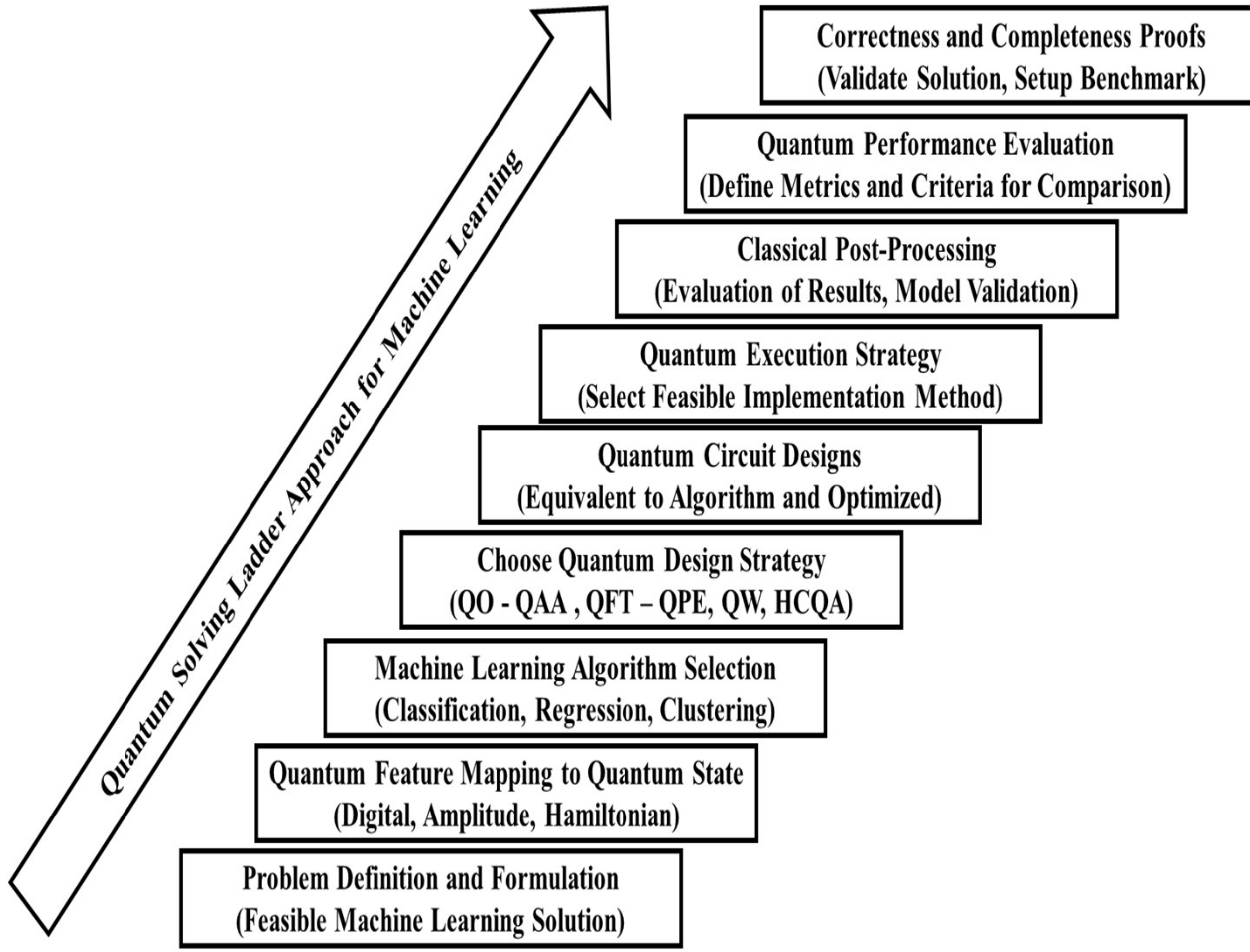


Machine Learning Viewpoint:

- Scalable, Efficient Algorithms
- Enhanced Feature Extraction
- Enhance Prediction Accuracy
- Enhances Data Visualization
- Space Increases as Dimensions

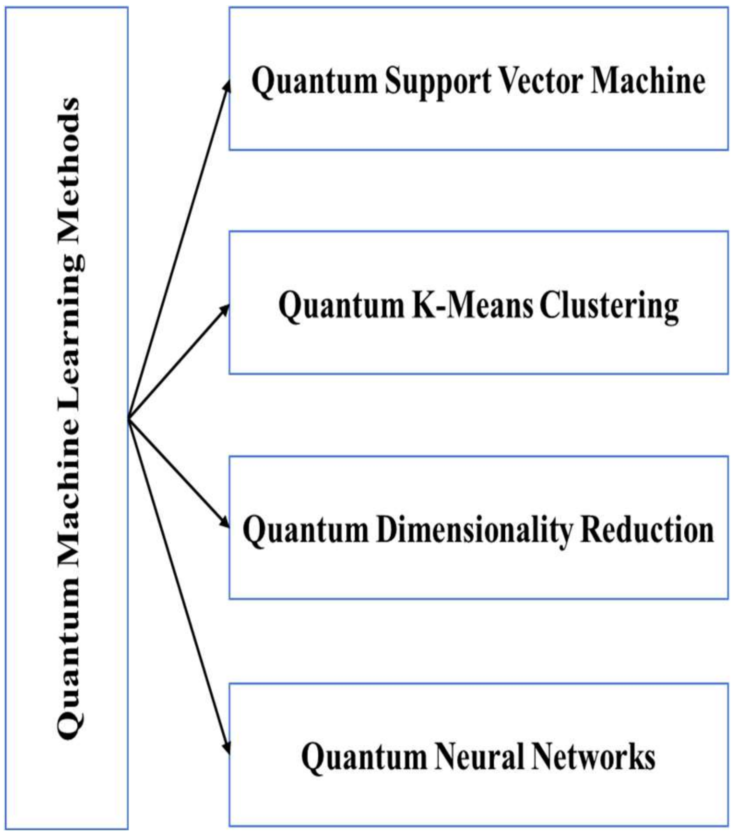
1. Quantum Machine Learning Basics





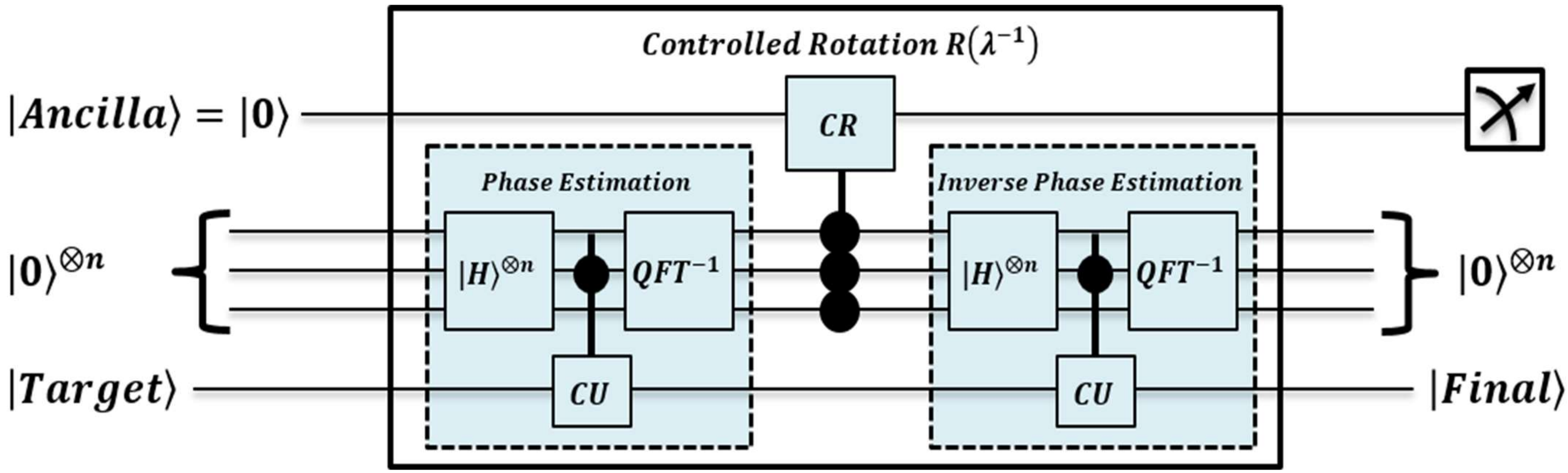
1. Quantum Machine Learning Basics

Well Known and Developed Quantum Machine Learning Algorithms and Algorithmic Advantage



Algorithm	Reference	Grover	Speedup
K-medians	Aïmeur et al. (2013)	Yes	Quadratic
Hierarchical clustering	Aïmeur et al. (2013)	Yes	Quadratic
K-means	Lloyd et al. (2013a)	Optional	Exponential
Principal components	Lloyd et al. (2013b)	No	Exponential
Associative memory	Ventura and Martinez (2000)	Yes	
	Trugenberger (2001)	No	
Neural networks	Narayanan and Menneer (2000)	Yes	
Support vector machines	Anguita et al. (2003)	Yes	Quadratic
	Rebentrost et al. (2013)	No	Exponential
Nearest neighbors	Wiebe et al. (2014)	Yes	Quadratic
Regression	Bisio et al. (2010)	No	
Boosting	Neven et al. (2009)	No	Quadratic

2. Harrow-Hassidim-Lloyd Algorithm



- **HHL realizes QPE to solve $A\,|X\rangle = |B\rangle$ as by multiplying $A^{-1}A\,|X\rangle = A^{-1}|B\rangle$ & obtains $|X\rangle = A^{-1}\,|B\rangle \equiv |Final\rangle = A^{-1}\,|Target\rangle$.**
- **In actual, logic of finding the Eigen vectors is based on computing the Eigen values of conjugate transpose of DM as A.**
- **This solution need constant execution of CR along with $2 \times log_2 D$ operations of QPE, thus time required is still $O(log_2 D)$.**

2. Harrow-Hassidim-Lloyd Algorithm

HHL Procedure

Input : Hermitian Matrix $\boldsymbol{A}$, Unitary $(\boldsymbol{CU} = \boldsymbol{e^{iAn}})$, $|\boldsymbol{Ancilla}\rangle$, A Control Register $\boldsymbol{n}$ qubits and encoded vectors $|\boldsymbol{Target}\rangle$

Output : Constructs quantum state $|\boldsymbol{Final}\rangle$ for encoded $|\boldsymbol{Target}\rangle$ register through estimating eigen values $|\boldsymbol{\lambda}\rangle$ of $\boldsymbol{A}$ based on $\boldsymbol{A^{-1}}\,|\boldsymbol{Target}\rangle$

Begin

1 Initialize control register in zero state $|\boldsymbol{0}\rangle^{\otimes \boldsymbol{n}}$ and prepare encoding of eigen vectors in $|\boldsymbol{Target}\rangle$ register as $|\boldsymbol{\phi}\rangle$

2 Apply $\boldsymbol{H}^{\otimes \boldsymbol{n}}$ on control register to create superposition state $|\boldsymbol{\psi}\rangle$

3 Perform $\boldsymbol{PE}$ procedure using $\boldsymbol{CU} = \boldsymbol{e^{iAn}}$ on encoded $|\boldsymbol{Target}\rangle$ to transform it into $|\boldsymbol{\phi_i}\rangle$ of $\boldsymbol{A}$ and stores $|\boldsymbol{\lambda_i}\rangle$ of $\boldsymbol{A}$ in control register

4 For each control qubit $|\boldsymbol{j}\rangle \in \{\boldsymbol{0}\ \boldsymbol{to}\ \boldsymbol{n-1}\}$ in control register

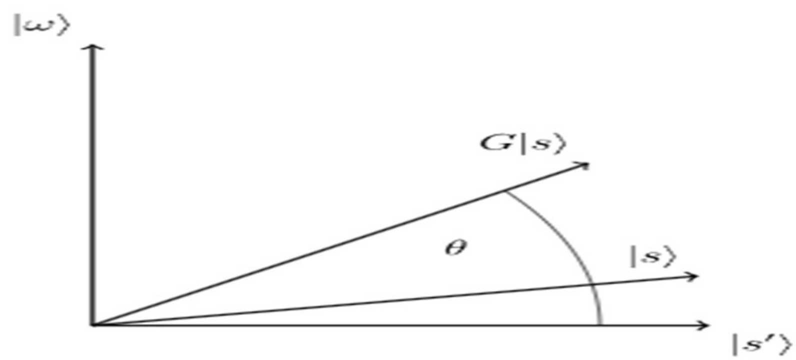
Apply a controlled rotation $(\boldsymbol{CR})$ on $|\boldsymbol{Ancilla}\rangle$ to map $|\boldsymbol{\lambda_j}^{-1}\rangle$ and to encode them in $|\boldsymbol{Ancilla}\rangle$ with the constant amplitudes

5 Perform inverse $\boldsymbol{PE}$ to make control register disentangled and turn it into $|\boldsymbol{0}\rangle^{\otimes \boldsymbol{n}}$ to obtain eigen vectors with $|\boldsymbol{\lambda_j}^{-1}\rangle$ approximations

6 Measure $|\boldsymbol{Ancilla}\rangle$ qubit, if the resulting measurement is $|\boldsymbol{1}\rangle$ then eigen value approximation $|\boldsymbol{\lambda_j}^{-1}\rangle$ is assumed as implicit applied to $|\boldsymbol{Target}\rangle$ for finding state $|\boldsymbol{Final}\rangle = \boldsymbol{A^{-1}}\,|\boldsymbol{Target}\rangle$

3. Quantum Counting

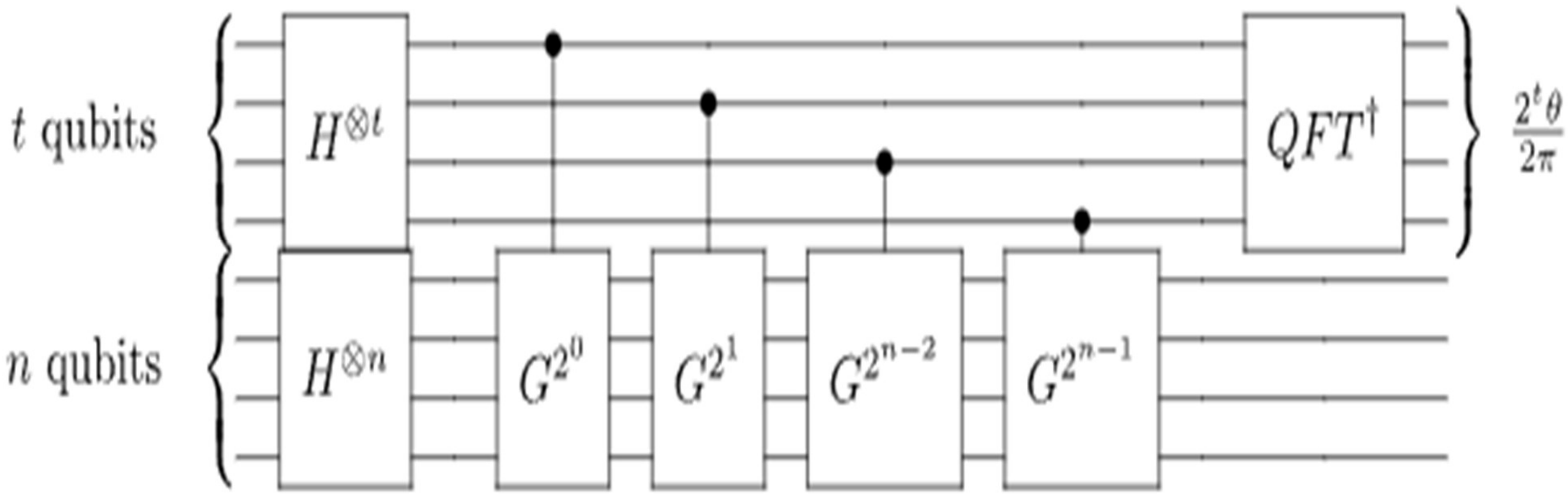
- This algorithm is interesting as it combines both quantum search and quantum phase estimation.
- Grover's algorithm attempts to find a solution to the Oracle, the quantum counting algorithm tells us how many of these solutions are present.
- In quantum counting, we simply use the quantum phase estimation algorithm to find an eigenvalue of a Grover search iteration.
- You will remember that an iteration of Grover's algorithm, G, rotates the state vector by θ in the $|\omega\rangle$, $|s'\rangle$ basis:



3. Quantum Counting

- **The percentage number of solutions in our search space affects the difference between $|s\rangle$ and $|s'\rangle$.**
- **For example, if there are not many solutions, $|s\rangle$ will be very close to $|s'\rangle$ and θ will be very small.**
- **It turns out that the eigenvalues of the Grover iterator are $e^{\pm i\theta}$, and we can extract this using quantum phase estimation (QPE) to estimate the number of solutions (M).**
- **In the $|\omega\rangle$, $|s'\rangle$ basis we can write the Grover iterator as the matrix** $G = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
- **The matrix G has eigenvectors:** $\begin{pmatrix} -i \\ 1 \end{pmatrix}, \begin{pmatrix} i \\ 1 \end{pmatrix}$
- **The state $|s\rangle$ is in the space spanned by $|\omega\rangle$, $|s'\rangle$, and thus is a superposition of the two vectors.** $|s\rangle = \alpha|\omega\rangle + \beta|s'\rangle$

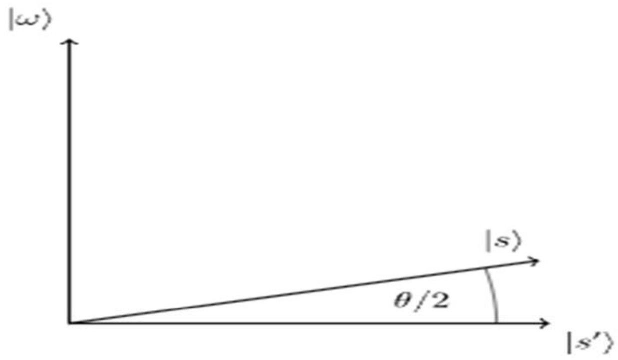
3. Quantum Counting



Finding the Number of Solutions (M):

- You may remember that we can get the angle θ/2 can from the inner product of |s⟩ and |s′⟩:

3. Quantum Counting



$$\langle s'|s\rangle = \cos\tfrac{\theta}{2}$$

And that $|s\rangle$ (a uniform superposition of computational basis states) can be written in terms of $|\omega\rangle$ and $|s'\rangle$ as:

$$|s\rangle = \sqrt{\tfrac{M}{N}}|\omega\rangle + \sqrt{\tfrac{N-M}{N}}|s'\rangle$$

The inner product of $|s\rangle$ and $|s'\rangle$ is:

$$\langle s'|s\rangle = \sqrt{\frac{N-M}{N}} = \cos\tfrac{\theta}{2}$$

From this, we can use some trigonometry and algebra to show:

$$N\sin^2\frac{\theta}{2} = M$$

5. Hybrid Quantum-Classical Algorithms

- **Introduction to Hybrid Quantum-Classical Algorithms**
  - Definition:
    - Hybrid algorithms combine quantum and classical resources to solve complex computational problems.
    - These algorithms leverage quantum computing for its speedup and classical systems for optimization and learning.
  - Purpose:
    - To address current quantum hardware limitations.
    - To solve practical problems that classical methods struggle with, like optimization, chemistry, and physics simulations.

5. Hybrid Quantum-Classical Algorithms

- **Why Hybrid Algorithms?**
  - Advantages of Hybrid Systems:
    - **Noisy Intermediate-Scale Quantum (NISQ) Era**: Quantum hardware is still noisy and imperfect, so full-scale quantum algorithms are impractical.
    - **Classical Optimization:** Classical systems can optimize quantum circuits effectively, adapting results from quantum computation to refine parameters.
    - **Applications**: Useful in optimization problems, machine learning, and molecular simulations.

5. Hybrid Quantum-Classical Algorithms

- **Overview of Variational Quantum Algorithms (VQAs)**
  - **Key Concept**:
    - VQAs utilize quantum circuits to evaluate a cost function and a classical optimizer to minimize it.
    - The quantum part of the algorithm explores a solution space, and classical methods refine the quantum output.
  - **Structure**:
    1. **Quantum Part**: Prepare a quantum state based on parameters.
    2. **Classical Part**: Measure and update the parameters to minimize a cost function.
    - **Examples**: Quantum Approximate Optimization Algorithm (QAOA), Variational Quantum Eigensolver (VQE).

5. Hybrid Quantum-Classical Algorithms

- **Quantum Approximate Optimization Algorithm (QAOA)**

  - **Purpose**: Solves combinatorial optimization problems.
  - **How It Works**:
    1. **Objective**: Minimizing a cost function represented by a Hamiltonian.
    2. **Steps**:
       - Initialize a trial wave function.
       - Apply a parameterized unitary operator composed of alternating layers of problem Hamiltonian and mixing Hamiltonian.
       - Measure the output and update parameters using a classical optimizer.
    3. **Application**: QAOA is widely applied in problems like the Max-Cut problem in graph theory and optimization of scheduling problems.
  - **Benefits**:
    - Can produce good approximate solutions to NP-hard problems.
    - Resource-efficient for early quantum hardware.

5. Hybrid Quantum-Classical Algorithms

- **Variational Quantum Eigensolver (VQE)**

  - **Purpose**: Finds the ground-state energy of a Hamiltonian, useful in quantum chemistry and materials science.
  - **How It Works**:
    1. **Objective**: Minimize the expectation value of a Hamiltonian.
    2. **Steps**:
       - Prepare a quantum state with adjustable parameters.
       - Measure the expectation value of the energy.
       - Use classical optimization techniques to minimize this energy.
    3. **Application**: VQE is often used in molecular energy simulations, helping calculate chemical reaction properties.
  - **Benefits**:
    - One of the most popular algorithms for chemistry and materials science.
    - Suits current quantum hardware with noise and limited qubits.

5. Hybrid Quantum-Classical Algorithms

- **Conclusion**

  - Hybrid Quantum-Classical Algorithms offer a promising approach in the NISQ era, combining the best of both quantum and classical worlds.
  - Algorithms like QAOA and VQE are pioneers in leveraging quantum systems for practical, real-world applications.

ANY QUESTIONS?

MANY THANKS FOR YOUR TIME