National Institute of Technology Raipur
राष्ट्रीय प्रौद्योगिकी संस्थान रायपुर

DEPARTMENT OF INFORMATION TECHNOLOGY

One Week Hybrid Workshop

on

Quantum Computing and Algorithms (QCA) -2024

Quantum Basics & Quantum Gates

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Content

1. Quantum Computing Significance

Same

$n \times 2^n$

$2^n \times m$

qubit

1010

$O(2^n)$

$O(1)$

QM - Probabilistic

CM - Deterministic

$(10) \leftarrow 1024 \rightarrow 2^{10}$

Quantum Computer Development and Quantum Computational Approaches

Quantum Mechanical Principles
- Superposition
- Entanglement
- Parallelism
- Measurement
- No Cloning

Quantum Information (Qu-Bit)
- Processing Proved to be Inherently Parallel
- Still Under Proofs for the Information Storage
- Creating Clones are not Possible (Copy Not Allow)
- Information Extraction needs "Measurement"

Quantum Computation
- Exponentially Faster, Achieves Speedup by Fast, Low Cost, Energy Efficient, Quantum Logic Gates & Circuits

EX-OR

A B	o/p
00	0
01	1
10	1
11	0

Quantum Circuit
- Mathematically represented as Unitary Matrices
- No. of Input and Output lines are same
- Reversible or Follows Bijection

Quantum Computing
- Purpose is to find Classical Equivalent Quantum Domains
- Objective is to propose Classical Equivalent Quantum Efficient Algorithms

Quantum Algorithm
- Equivalent to Quantum Circuits: Time, Space Efficient, Query, and Energy Efficient
- Mainly Two Design Strategies: Quantum Amplitude Amplification (QAA) and Quantum Fourier Transform (QFT)

Quantum Computer Realization
- Quantum Simulation (Software Approach)
- Quantum Emulation (Hardware Approach)

Quantum Historical Benchmarks
- Basic Algorithms: Deutsch, Jozsa, Simons
- Two Well Known Benchmark Algorithms: Grover's Quantum Database Search Algorithm and Shor's Prime Integer Factorization Algorithm

Quantum Technology Development
- Real Quantum Computer Technology Development
- Realistic Technology Development (Simulation / Emulation)
- Quantum Algorithm Design and Development (Quantum Cryptography (QC) and Quantum Machine Learning (QML))

2. Quantum Computational Aspects

- **Quantum-Bit (Qubit):** Basic unit of quantum information, represented using quantum states as Ket-vector.
- **Superposition:** Linear combination of quantum states, or, concurrent realization of multiple quantum states.
- **Entanglement:** Extremely strong correlation between the qubits, no matter even if separated by long distance.
- **Quantum Parallelism:** Enables simultaneous processing to perform exponential no. of operations in parallel.
- **Quantum Interference:** Biased quantum system (constructive/destructive) reinforces to desired measurement.

- **Hilbert Space:** A complex vector space with inner product, and it allows length and angle to be measured.

- **Quantum Decoherence:** Coherence refers to integrity of quantum states. Decoherence is loss of coherence.

- **Quantum Measurement:** Collapse of the quantum state into one of the desired, or possible, classical states.

- **Quantum Unitary:** Quantum gate operations that transform pure state (vector) to mixed state (density matrix)

- **Quantum Circuits:** Composed of the quantum gates that manipulate qubits to execute quantum algorithms.

3. Basic Mathematical Concepts

- A classical bit remains present in mutually exclusive manner (0 or 1), represents a classical value.

- A single qubit remains present in superposition of quantum states, and represented as Bra-Ket.

- Symbolically, the Bra-Ket is $\langle\psi|\psi\rangle$, where $|\psi\rangle$ is Ket-vector, and its dual is Bra-vector $\langle\psi|$.

  $\langle\psi| \quad |\psi\rangle$

- Qubit Ket-Notation: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ and Bra-Notation: $\langle 0| = (1 \quad 0)$ and $\langle 1| = (0 \quad 1)$

  2×1 $\quad 2\times1$ $\quad 1\times2$ $\quad 1\times2$

  qubit $= 1 \rightarrow$ $|0\rangle \quad |1\rangle$

  $\alpha_0\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\alpha_0\\0\end{pmatrix} + \alpha_1\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\\alpha_1\end{pmatrix} = \begin{pmatrix}\alpha_0\\\alpha_1\end{pmatrix}$ 2×1

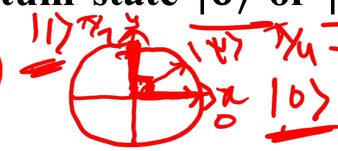
- Quantum superposition of qubit as is $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ with α_0 & α_1 are the complex coefficients or complex probability amplitudes, or square root of probabilities to measure. So, equation $|\psi\rangle = \begin{pmatrix}\alpha_0\\\alpha_1\end{pmatrix}$.

- As α_0 & α_1 are complex numbers, such that $\alpha_i = \{a + bi \mid a, b \in R \text{ and } i \in \sqrt{-1}\}$, such that $i^2 = -1$.

$|\psi\rangle = \begin{pmatrix}\alpha_0 \\ \alpha_1\end{pmatrix}$

- A measurement is needed to obtain quantum state $|0\rangle$ or $|1\rangle$ with their state probabilities $|\alpha_0|^2$ and $|\alpha_1|^2$, such that $|\alpha_0|^2 + |\alpha_1|^2 = 1$.

Ex



$\rightarrow |\psi\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{pmatrix} \begin{matrix}\leftarrow \alpha_0 \\ \leftarrow \alpha_1\end{matrix}$

$a = \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$ $\boxed{|0\rangle = \begin{pmatrix}1 \\ 0\end{pmatrix}}$ $\boxed{|1\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix}}$

$\theta = 0$ $\begin{pmatrix}1 \\ 0\end{pmatrix}$

$\theta = \pi/2$ $\begin{pmatrix}0 \\ 1\end{pmatrix}$

- So, using complex numbers, the squared magnitude can be computed as $\boxed{|a + bi|^2 = a^2 + b^2}$ due to $(a + bi)(a + bi) = (a)^2 - (bi)^2 = a^2 - b^2\, i^2 = a^2 + b^2$.

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$
$= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

- However, Bra-vector $\langle\psi|$ is a complex conjugate transpose (or, Hermitian Transpose / Adjoint) of Ket-vector $|\psi\rangle$. So, we may write $\langle\psi| = (\alpha_0{}^* \quad \alpha_1{}^*)$.

- Transpose of matrix A is $(A)^T$, however, complex conjugate of element is $(a+bi)^* = (a-bi)$. So, complex conjugate of matrix $A = (A)^*$. Finally, adjoint of matrix $A^\dagger = \left(A^T\right)^* = (A^*)^T = \langle A|$.

$$\text{Example: A} = \begin{pmatrix} 1 & i \\ 0 & 2 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 1 & 0 \\ i & 2 \end{pmatrix}^T \rightarrow \left(A^T\right)^* = \begin{pmatrix} 1 & 0 \\ -i & 2 \end{pmatrix}^* = A^\dagger \text{ (Transpose then Conjugate)}$$

$$\text{Example: A} = \begin{pmatrix} 1 & i \\ 0 & 2 \end{pmatrix} \rightarrow A^* = \begin{pmatrix} 1 & -i \\ 0 & 2 \end{pmatrix}^T \rightarrow (A^*)^T = \begin{pmatrix} 1 & 0 \\ -i & 2 \end{pmatrix}^* = A^\dagger \text{ (Conjugate then Transpose)}$$

- For unitary U the conjugate transpose is $U^\dagger$ and it performs the reverse quantum operation, such that $UU^\dagger = I$ holds true.

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- A single qubit is basically 2-dimensional quantum system explored over a Hilbert space. A qutrit is 3-dimensional $|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, and $|2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Similarly, qudit would be d-dimensional system.

4. Quantum Operations / Products

1. **Inner Product / Scalar Product $\langle\psi|\psi\rangle$** Bra-Ket

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\langle 0| = \begin{pmatrix}1 & 0\end{pmatrix}$

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\langle 1| = \begin{pmatrix}0 & 1\end{pmatrix}$

$\langle 0|0\rangle = \begin{pmatrix}1 & 0\end{pmatrix}_{1\times 2^1}\begin{pmatrix}1\\0\end{pmatrix}_{2^1\times 1} = \begin{pmatrix}1\end{pmatrix}_{1\times 1}$

$\langle 1|1\rangle = \begin{pmatrix}1\end{pmatrix}$

$\langle 0|1\rangle = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\end{pmatrix}$

$\langle 1|0\rangle = \begin{pmatrix}0\end{pmatrix}$

$$\langle\psi|\psi\rangle = 1 \text{ and } \langle\psi|\varphi\rangle = 0$$

2. **Tensor Product / Dirac's Product $(|\psi\rangle \otimes |\psi\rangle)$**

$|0\rangle \otimes |0\rangle = |00\rangle$

$|01\rangle, |10\rangle, |11\rangle$

$\boxed{\begin{pmatrix}1\\0\end{pmatrix}_{2\times1} \otimes \begin{pmatrix}1\\0\end{pmatrix}_{2\times1}} \rightarrow \begin{pmatrix}1\\0\\0\\0\end{pmatrix}_{2^2\times 1 = 4\times 1} = |00\rangle$

$\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$$|\psi\rangle \otimes |\psi\rangle = |\psi\psi\rangle$$

3. **Outer Product / Projectors $(|\psi\rangle\langle\psi|)$** Ket-Bra

$\begin{pmatrix} \end{pmatrix}_{2^n\times 1} \times \begin{pmatrix} \end{pmatrix}_{1\times 2^n}$

$\begin{pmatrix} \end{pmatrix}_{2^n\times 2^n}$

$$|0\rangle\langle 0| + |1\rangle\langle 1| \qquad |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$|0\rangle\langle 0| - |1\rangle\langle 1| \qquad \mathrm{i}\,|0\rangle\langle 1| + \boldsymbol{i}\,|1\rangle\langle 0|$$

Hilbert Space

2-Reg $|000\rangle^{\otimes 3} \rightarrow \left.\begin{array}{l} |000\rangle \\ |001\rangle \\ |010\rangle \\ \vdots \\ |111\rangle \end{array}\right\}$

Ex $|0\rangle\langle 0| + |1\rangle\langle 1|$

$\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{2\times 1}\underbrace{\begin{pmatrix}1 & 0\end{pmatrix}}_{1\times 2} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix}$

$= \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$

$= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = I$

$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $|01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

1. Quantum State as Vector
2. Superposition (1-Qubit) $|\psi\rangle^{\otimes 1} = \alpha_0|0\rangle + \alpha_1|1\rangle$

   (2-Qubit) $|\psi\rangle^{\otimes 2} = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

3. Dimension

   1-Qubit $\rightarrow 2^1 D$
   2 -"- $\rightarrow 2^2 D$ $\Big\} \rightarrow$ n-Qubit $\rightarrow 2^n D$

   $N = 2^n \rightarrow$ Qubits

   $$|\psi\rangle^{\otimes n} = \sum_{i=0}^{2^n \to N-1} \alpha_i |i\rangle$$

   $N=2$

   $$\sum_{i=0}^{2-1=1} \alpha_i |i\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

4. Inner Product $\langle\psi|\psi\rangle = 1$ $\langle\psi|\phi\rangle = 0$
5. Normalization $|\alpha_0|^2 + |\alpha_1|^2 = 1$
6. Operators $\rightarrow$ Hadamard (H), Pauli-Matrices (X, Y, Z)

   Identity (I)

7. Tensor Product $|\psi\phi\rangle$

Outer Product (Ket - Bra)

— $\boxed{I}$ —

⓪ $|0\rangle\langle 0| + |1\rangle\langle 1| = I \quad \begin{pmatrix}1&0\\0&1\end{pmatrix}_{2\times2}$

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}_{2\times1}$

$I(|0\rangle)$

$\underbrace{\begin{pmatrix}1&0\\0&1\end{pmatrix}}_{2\times2}\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{2\times1} = \begin{pmatrix}1\\0\end{pmatrix}_{2\times1}$

Pauli-Z (Phase Flip)

$\cancel{A}(|0\rangle)$

$\uparrow$

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ -

① $|0\rangle\langle 1| + |1\rangle\langle 0|$

$\begin{pmatrix}1\\0\end{pmatrix}_{2\times1}\begin{pmatrix}0&1\end{pmatrix}_{1\times2} + \begin{pmatrix}0\\1\end{pmatrix}_{2\times1}\begin{pmatrix}1&0\end{pmatrix}_{1\times2}$

$\begin{pmatrix}0&1\\0&0\end{pmatrix}_{2\times2} + \begin{pmatrix}0&0\\1&0\end{pmatrix}_{2\times2}$

$\begin{pmatrix}0&1\\1&0\end{pmatrix} = X$

(Bit Flip Operation)

— $\boxed{X}$ —

② $|0\rangle\langle 0| - |1\rangle\langle 1|$

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \qquad |0\rangle \to |0\rangle$

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix} \qquad |1\rangle \to -|1\rangle$

③ $\boxed{-i\,|0\rangle\langle 1| + i\,|1\rangle\langle 0|}$

$\begin{pmatrix}0&-i\\i&0\end{pmatrix}$

Pauli-Y (Phase + Bit Flip)

$\begin{pmatrix}0\\1\end{pmatrix}^{|1\rangle} \to \begin{pmatrix}-i\\0\end{pmatrix} = -i\begin{pmatrix}1\\0\end{pmatrix}$

$\begin{pmatrix}1\\0\end{pmatrix}^{|0\rangle} \to \begin{pmatrix}0\\i\end{pmatrix} = i\begin{pmatrix}0\\1\end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \cancel{?}\ \cancel{A}$

Pauli $\left\{ \sigma_x \quad X \right.$

$A(|0\rangle) \longrightarrow |1\rangle$

$\underset{2\times 2}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \underset{2\times 1}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}$ $|1\rangle$

$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$$\boxed{H = \frac{1}{\sqrt{2}} \sum_{x,y \in \{0,1\}} (-1)^{x \cdot y} |x\rangle\langle y|}$$

0 0
0 1
1 0
1 1

$|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| \cancel{+} - |1\rangle\langle 1| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = H$

$H(|0\rangle) = \frac{1}{\sqrt{2}} \underset{2\times 2}{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \frac{1}{\sqrt{2}} \underset{2\times 1}{\begin{pmatrix} \to 1 \\ \to 1 \end{pmatrix}}$

$\begin{pmatrix} \to \\ \to -1 \end{pmatrix}$

5. Quantum Gates & Properties

1. **Unitary**: All quantum gates are represented by **unitary matrices**. A matrix U is unitary if:

   $$U^\dagger U = UU^\dagger = I$$

   where $U^\dagger$ is the conjugate transpose of U, and I is the identity matrix. This property ensures that quantum operations are reversible and preserve the norm of the quantum state.

2. **Reversibility**: Every single-qubit gate is reversible, meaning there is an inverse gate U^{-1} such that:

   $$UU^{-1} = I$$

   This is a fundamental property of quantum operations, as quantum computations cannot erase information.

3. **Linear Operation**: Quantum gates apply linear transformations to quantum states, mapping qubits between different superpositions:

   $$U|\psi\rangle = \sum c_i U|i\rangle$$

6. Single-Qubit Quantum Gates

1-Qubit Gate $\rightarrow$ Quantum operator Matrix $(\quad)_{2^1 \times 2^1}$

2-Qubit $(\quad)_{2^2 \times 2^2} = (\quad)_{4 \times 4}$

Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}_{2\times 2}$
Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$
Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}_{2\times 2}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{2\times 2}$

n-Qubit $(\quad)_{2^n \times 2^n}$

No-Cloning Proof (Proof of Contradiction)

$$U\,(\,|\psi\rangle\,|0\rangle\,) = |\psi\rangle\,|\psi\rangle$$

Q_1 Q_2

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

Cloning of Information is not allowed

$$= \frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$$|0\rangle \otimes |0\rangle = |00\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right) \otimes |0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

$$\neq$$

Measurement

ψ → vector representation of states in a system

A → Vector Representation of measurement Apparatus

$$|\langle A|\psi\rangle|^2 = |\langle A| \times |\psi\rangle|^2 = |A^{+}\psi|^2$$

$$\Downarrow$$

$$|\langle\psi|A\rangle|^2 = |\langle\psi| \times |A\rangle|^2 = |\psi^{*}A|^2$$

7. Multi-Qubit Quantum Gates

1. **Entanglement**: Multiple qubit gates can **entangle** qubits, meaning the quantum state of one qubit depends on the state of the other qubit(s). For example, the CNOT gate can create entangled states such as the Bell state.

2. **Controlled Gates**: A controlled gate applies a transformation to the **target qubit** only if the **control qubit** is in a specific state (usually $|1\rangle$).

   - CNOT Gate (Controlled-NOT):

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

This flips the target qubit if the control qubit is $|1\rangle$.

Ex $|1\rangle$

$|\psi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $A = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow |1\rangle$

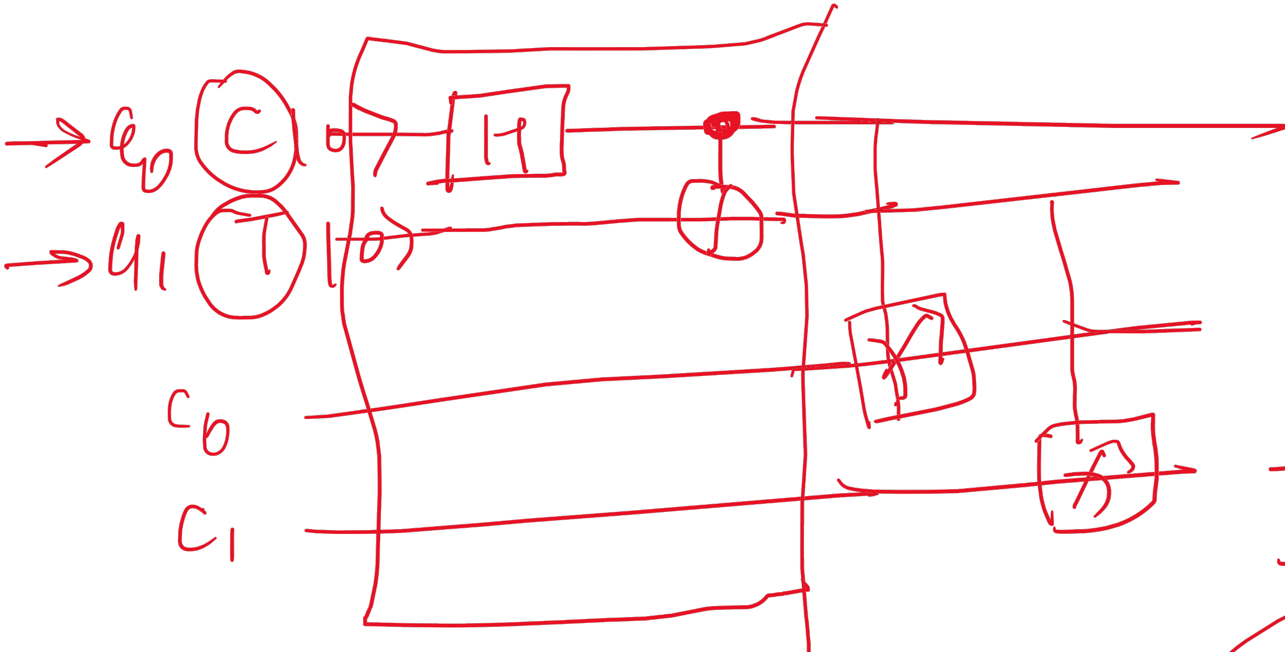
$|\langle\psi|A\rangle|^2 = \left|\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2 = (1)^2 = 1$

Ex $|\psi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ $A = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow |1\rangle$

$\left|\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$|\psi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow |1\rangle$ $A = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow |0\rangle$

$= 0$ prob



$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes |0\rangle$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

$\frac{1}{2}$

2-qubit

$|00\rangle$ $|01\rangle$ $|10\rangle$ $|11\rangle$ $\frac{1}{4}$

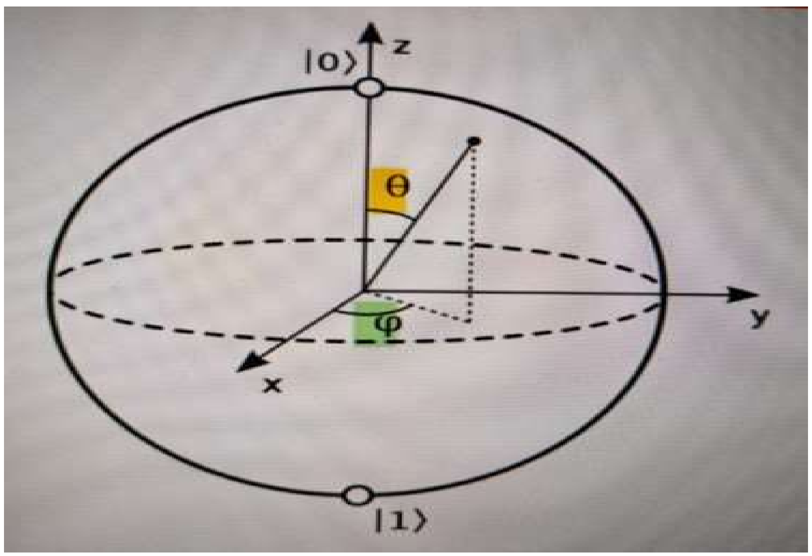
$\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^2$

8. Qubit Mapping on Bloch Sphere

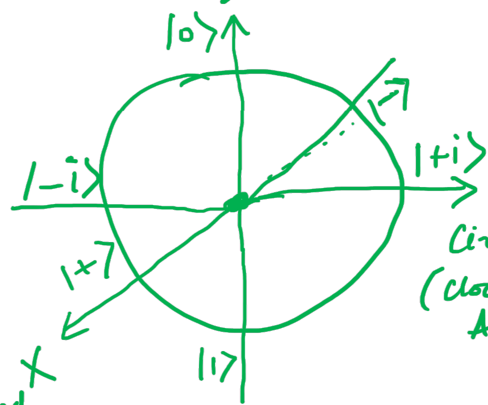
$q_0 \to |0\rangle \to H^{\otimes 1}(|0\rangle) \to \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \to |+\rangle$ ✓

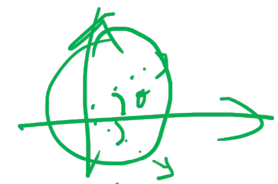
$H^{\otimes 1}(|1\rangle) \to \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \to |-\rangle$ ✓

1-qubit ( ) $2^1 \times 2^1$



Computational Basis (or) Standard Basis





$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Circular Rotation Basis (clockwise & Anticlockwise Basis)

Diagonal Basis (or) Hadamard Basis

For other detail, Please refer to the previous lecture notes.

9. Quantum Design of Classical Gates

- **Please refer to the next lecture notes.**

10. Summary on Quantum Gates

- **Please refer to the next lecture notes.**

ANY QUESTIONS?

MANY THANKS FOR YOUR TIME