Упражнение 4.3

Уравнение Гамильтона-Якоби в сферических координатах:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2\right) - \Gamma(r) = 0$$

В ДСК:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\sum_{k=1}^{3}\left(\frac{\partial S}{\partial q_k}\right)^2\right) - \Gamma\left(\sqrt{q_1^2+q_2^2+q_3^2}\right) = 0, \quad \Gamma(r) = \delta \cdot \int \Phi(r)\,dr$$

Полный интеграл уравнений движения в сферических координатах:

$$S = a_1 t + a_2\varphi + e_1\int\sqrt{2m(\Gamma(r)-a_1) - \frac{a_3^2}{r^2}}\,dr + e_2\int\sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}\,d\theta$$

т. Якоби:

$$\det\left(\frac{\partial^2 S}{\partial q_i\,\partial a_k}\right)\Bigg|_{i,k=1}^{n} \neq 0, \quad (t,q,a)\in D$$

$$\frac{\partial S}{\partial r} = e_1\cdot\int\frac{2m\cdot\frac{d\Gamma(r)}{dr} + \frac{2a_3^2}{r^3}}{2\sqrt{2m(\Gamma(r)-a_1) - \frac{a_3^2}{r^2}}}\,dr = e_1\int\frac{m\frac{d\Gamma(r)}{dr} + \frac{a_3^2}{r^3}}{\sqrt{2m(\Gamma(r)-a_1) - \frac{a_3^2}{r^2}}}\,dr$$

$$\frac{\partial S}{\partial \varphi} = a_2$$

$$\frac{\partial S}{\partial \theta} = e_2\int\frac{-\frac{a_2^2(-2)\cdot\cos\theta}{\sin^3\theta}}{2\sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\,d\theta = e_2\int\frac{a_2^2\cos\theta}{\sin^3\theta\sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\,d\theta$$

$$\frac{\partial S}{\partial r\,\partial a_1} = e_1\cdot m\int\frac{m\frac{\partial\Gamma(r)}{\partial r} + \frac{a_3^2}{r^3}}{\sqrt[3]{2m(\Gamma(r)-a_1) - \frac{a_3^2}{r^2}}}\,dr$$

$$\frac{\partial S}{\partial r\,\partial a_2} = 0$$

$$\frac{\partial S}{\partial r\,\partial a_3} = e_1\int\frac{-a_3\left(2m\left(4r^3(\Gamma(r)-a_1) + \frac{\partial\Gamma(r)}{\partial r}\cdot r^4\right) - 2ra_3^2\right)}{\left(2m(\Gamma(r)-a_1)r^4 - r^2a_3^2\right)^{\frac{3}{2}}}$$

$$\frac{\partial S}{\partial \varphi\,\partial a_1} = 0$$

$$\frac{\partial S}{\partial \varphi\,\partial a_2} = 1$$

$$\frac{\partial S}{\partial \varphi\,\partial a_3} = 0$$

$$\frac{\partial S}{\partial \theta\,\partial a_1} = 0$$

$$\frac{\partial S}{\partial \theta\,\partial a_2} = e_2\int\frac{\cos\theta}{\sin^3\theta}\cdot\frac{-a_2}{\sin^2\theta\sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\,d\theta$$

$$\frac{\partial S}{\partial \theta\,\partial a_3} = e_2\int\frac{\cos\theta}{\sin^3\theta}\cdot\frac{a_3}{\sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\,d\theta$$

$\det(\ast) \neq 0 \Rightarrow \exists$ 6 незав. инт. кан. ур.

$$p_1 = \frac{\partial S}{\partial r} = e_1 \int \frac{m \frac{d\Gamma(r)}{dr} + \frac{a_3^2}{r^3}}{\sqrt{2m(\Gamma(r) - a_1) - \frac{a_3^2}{r^2}}}\, dr$$

$$p_2 = \frac{\partial S}{\partial \varphi} = a_2$$

$$p_3 = \frac{\partial S}{\partial \theta} = e_2 \int \frac{\cos\theta \cdot a_2^2}{\sin^3\theta \sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\, d\theta$$

$$b_1 = \frac{\partial S}{\partial a_1} = t + e_1 \cdot \int \frac{-m}{\sqrt{2m(\Gamma(r) - a_1) - \frac{a_3^2}{r^2}}}\, dr$$

$$b_2 = \frac{\partial S}{\partial a_2} = \varphi + e_2 \int \frac{-a_2}{\sin^2\theta \sqrt{a_3^2 - \frac{a_2^2}{\sin^2\theta}}}\, d\theta$$

$$b_3 = \frac{\partial S}{\partial a_3} = e_1 \int \frac{-a_3}{r^2 \sqrt{2m(\Gamma(r) - a_1) - \frac{a_3^2}{r^2}}}\, dr + e_2 \int \frac{a_3}{\sqrt{a_3^2 - \frac{a_2^2}{\sin\theta}}}\, d\theta$$