

Column Properties :

$$\begin{cases} L = 264 \text{ in} \\ E = 29500 \text{ Ksi} \\ I = 307 \text{ in}^4 \\ A = 11.7 \text{ in}^2 \end{cases}$$

Section: W12 × 40

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29500 \times 307}{264^2} = 1282.48 \text{ Kips}$$

$$k_s = \frac{P_{cr}}{L_c} = \frac{1282.48}{264} = 4.858 \frac{\text{Kips}}{\text{in}}$$

number of column=3  $\rightarrow k_n = \underline{2.21} k_s = \underline{2.21} \times 4.858 = 10.736 \frac{\text{Kips}}{\text{in}}$

$$\beta_{br} = \frac{EA_{br}}{L_{br}} \geq k_n \rightarrow \frac{29500 \times A_{br}}{100} \geq 10.736 \rightarrow A_{br} \geq \frac{10.736 \times 100}{29500} = 0.0364 \text{ in}^2$$

Brace 1 Properties ( $\beta = \beta_i$ ):

$$\begin{cases} L = 100 \text{ in} \\ E = 29500 \text{ Ksi} \\ I = 10 \text{ in}^4 \\ A = 0.1456 \text{ in}^2 \end{cases}$$

Brace 2 Properties ( $\beta = \beta_i$ ):

$$\begin{cases} L = 100 \text{ in} \\ E = 29500 \text{ Ksi} \\ I = 10 \text{ in}^4 \\ A = 0.0728 \text{ in}^2 \end{cases}$$

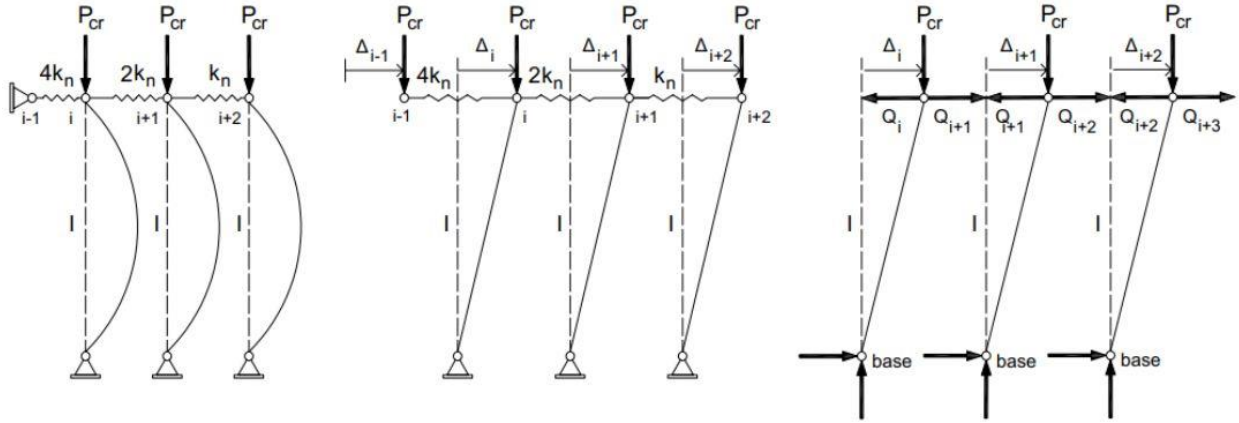
Brace 3 Properties ( $\beta = \beta_i$ ):

$$\begin{cases} L = 100 \text{ in} \\ E = 29500 \text{ Ksi} \\ I = 10 \text{ in}^4 \\ A = 0.0364 \text{ in}^2 \end{cases}$$

$$A_{brace1}(\beta = 2\beta_i) = 0.2912 \text{ in}^2$$

$$A_{brace2}(\beta = 2\beta_i) = 0.1456 \text{ in}^2$$

$$A_{brace3}(\beta = 2\beta_i) = 0.0728 \text{ in}^2$$



$$Q_i = 4k_n(\Delta_i - \Delta_{i-1}) \rightarrow Q_i + 4k_n\Delta_{i-1} - 4k_n\Delta_i = 0 \quad (1)$$

$$Q_i L - Q_{i+1} L - P_{cr}\Delta_i = 0 \rightarrow Q_i - Q_{i+1} - \frac{P_{cr}}{L}\Delta_i = 0 \quad (2)$$

$$Q_{i+1} = 2k_n(\Delta_{i+1} - \Delta_i) \rightarrow Q_{i+1} + 2k_n\Delta_i - 2k_n\Delta_{i+1} = 0 \quad (3)$$

$$Q_{i+1} L - Q_{i+2} L - P_{cr}\Delta_{i+1} = 0 \rightarrow Q_{i+1} - Q_{i+2} - \frac{P_{cr}}{L}\Delta_{i+1} = 0 \quad (4)$$

$$Q_{i+2} = k_n(\Delta_{i+2} - \Delta_{i+1}) \rightarrow Q_{i+2} + k_n\Delta_{i+1} - k_n\Delta_{i+2} = 0 \quad (5)$$

$$Q_{i+2} L - Q_{i+3} L - P_{cr}\Delta_{i+2} = 0 \rightarrow Q_{i+2} - Q_{i+3} - \frac{P_{cr}}{L}\Delta_{i+2} = 0 \quad (6)$$

$$\begin{bmatrix} 4k_n & 1 & -4k_n & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{P_{cr}}{L} & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2k_n & 1 & -2k_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{P_{cr}}{L} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_n & 1 & -k_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{P_{cr}}{L} & -1 \end{bmatrix} \begin{Bmatrix} \Delta_{i-1} \\ Q_i \\ \Delta_i \\ Q_{i+1} \\ \Delta_{i+1} \\ Q_{i+2} \\ \Delta_{i+2} \\ Q_{i+3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Delta_{i-1} = Q_{i+n} = 0$$

$$k_n = mk_s$$

$$k_s = \frac{P_{cr}}{L}$$

$$\begin{bmatrix} 1 & -4mk_s & 0 & 0 & 0 & 0 \\ 1 & -k_s & -1 & 0 & 0 & 0 \\ 0 & 2mk_s & 1 & -2mk_s & 0 & 0 \\ 0 & 0 & 1 & -k_s & -1 & 0 \\ 0 & 0 & 0 & mk_s & 1 & -mk_s \\ 0 & 0 & 0 & 0 & 1 & -k_s \end{bmatrix} \begin{Bmatrix} Q_i \\ \Delta_i \\ Q_{i+1} \\ \Delta_{i+1} \\ Q_{i+2} \\ \Delta_{i+2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \rightarrow$$

$$8m^3(k_s)^3 - 22m^2(k_s)^3 + 10m(k_s)^3 - (k_s)^3 = 0 \rightarrow$$

$$k_s^3[8m^3 - 22m^2 + 10m - 1] = 0$$

$$\begin{cases} f(m) \cdot (k_s)^3 = 0 \\ (k_s)^3[8m^3 - 22m^2 + 10m - 1] = 0 \end{cases} \Rightarrow f(m) = 8m^3 - 22m^2 + 10m - 1$$