Amt Assignment 3 Name: Roomi Uboueja
Student Number: 13939 6969

Ques 1 $(\sqrt[5]{x})y^7\,dy - 3(\sqrt[5]{x})\,dy = -2y^3\,dx$

Taking $\sqrt[5]{x}\,y^3$ and dividing it on both sides

$$\frac{(\sqrt[5]{x})y^{\cancel{7}\,4}\,dy}{(\sqrt[5]{x})y^3} - \frac{3\sqrt[5]{x}\,dy}{(\sqrt[5]{x})y^3} = \frac{-2y^3\,dx}{(\sqrt[5]{x})y^3}$$

$$y^4\,dy - \frac{3}{y^3}\,dy = \frac{-2}{\sqrt[5]{x}}\,dx$$

Integrating both sides.

$$\int y^4\,dy - \int \frac{3}{y^3}\,dy = \int \frac{-2}{\sqrt[5]{x}}\,dx$$

$$\frac{y^{4+1}}{4+1} - \frac{3.y^{-3+1}}{-3+1} = \frac{-2.x^{-1/5+1}}{-1/5+1} + C$$

$$\frac{y^5}{5} + \frac{3y^{-2}}{+2} = \frac{-2x^{4/5}}{-4/5} + C$$

$$\frac{1}{5}y^5 + \frac{3}{2y^2} = \frac{5}{2}x^{4/5} + C$$

$$\boxed{\Rightarrow \quad \frac{1}{5}y^5 = -\frac{3}{2}y^{-2} + \frac{5}{2}x^{4/5} + C}$$

$$\mathcal{L}^{-1}\left(\frac{5s}{s^2+2^2}\right) = \frac{d}{dt}\left(\mathcal{L}^{-1}\left(\frac{5}{s^2+4}\right)\right)$$

$$+ \mathcal{L}^{-1}\left(\frac{5}{s^2+4}\right)(0)$$

$$= \frac{d}{dt}\left(\mathcal{L}^{-1}\left(\frac{5}{2}\cdot\frac{2}{s^2+4}\right)\right) + 0$$

$$= \frac{d}{dt}\left(\frac{5}{2}\sin(2t)\right)$$

$$= \frac{5}{2}\cos(2t)$$

Now,

$$y = \frac{7t}{12} - \frac{7}{24}\sin 2t + \frac{1}{12}\sin 2t + \frac{5}{2}\cos 2t$$

$$\boxed{y = \frac{7t}{12} + 5\cos 2t + \frac{5}{24}\sin 2t}$$

Ques 2 $(6y^5+x)y' = y$ $\quad x=3$ when $y=2$

$$(6y^5+x)dy = ydx$$

$$6y^5dy + xdy = ydx$$

$$6y^5 dy = ydx - xdy$$

$$6y^3dy = \frac{ydx - xdy}{y^2}$$

$$6y^3dy = d\left(\frac{x}{y}\right)$$ Integrating both sides

$$\int 6y^3 dy = \int d\frac{x}{y}$$

$$\frac{6y^4}{4} = \frac{x}{y} + C$$

using $x=3$, $y=2$ to find C.

$$\frac{3}{2}y^5 = x + Cy$$

$$\frac{3}{2}(2^5) = 3 + C(2)$$

$$3\times 16 = 3 + 2C$$

$$C = \frac{45}{2}$$

$$\frac{3}{2}y^5 = x + \frac{45}{2}y$$

$$\frac{3}{2}y^5 - \frac{45y}{2} = x$$

$$\boxed{= \frac{1}{2}y(y^4 - 45) = x}$$

3) $3y'' + 12y = 7t \qquad y(0) = 5 \qquad y'(0) = 1$

Take LT on both Sides.

$$3\mathcal{L}(y'') + 12\mathcal{L}(y) = 7\mathcal{L}(t)$$

$$3\left[s^2\mathcal{L}(y) - sy(0) - y'(0)\right] + 12\mathcal{L}(y) = 7\frac{1}{s^2}$$

using $y(0) = 5$ & $y'(0) = 1$

$$3\left[s^2\mathcal{L}(y) - s(5) - 1\right] + 12\mathcal{L}(y) = \frac{7}{s^2}$$

$$3\left[s^2\mathcal{L}(y) - 5s - 1\right] + 12\mathcal{L}(y) = \frac{7}{s^2}$$

$$3s^2\mathcal{L}(y) - 15s - 3 + 12\mathcal{L}(y) = \frac{7}{s^2}$$

$$\mathcal{L}(y)\,(3s^2 + 12) - 15s - 3 = \frac{7}{s^2}$$

$$\mathcal{L}(y)\,(3s^2 + 12) = \frac{7}{s^2} + 3 + 15s$$

$$\mathcal{L}(y) = \frac{7}{s^2(3s^2+12)} + \frac{3}{3s^2+12} + \frac{15s}{(3s^2+12)}$$

$$\mathcal{L}(y) = \frac{7}{3}\,\frac{1}{s^2(s^2+4)} + \frac{\cancel{3}}{\cancel{3}}\,\frac{1}{s^2+4} + \frac{\cancel{15}^{5}}{\cancel{3}}\,\frac{s}{s^2+4}$$

$$\mathcal{L}(y) = \frac{7}{3}\frac{1}{s^2(s+2)^2} + \frac{1}{s^2+2^2} + \frac{5s}{s+2^2}$$

Taking L.T Inverse on both side

$$\mathcal{L}^{-1}(\mathcal{L}y) = \mathcal{L}^{-1}\left(\frac{7}{3s^2(s^2+2^2)}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^2+2^2}\right)$$

$$+ \mathcal{L}^{-1}\left(\frac{5s}{s^2+2^2}\right)$$

$$\mathcal{L}^{-1}\left(\frac{7}{3s^2(s^2+2^2)}\right) = \mathcal{L}^{-1}\left(\frac{7}{12s^2} - \frac{7}{12(s^2+4)}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{7}{12s^2}\right) - \mathcal{L}^{-1}\left(\frac{7}{12(s^2+4)}\right)$$

$$= \frac{7t}{12} - \frac{7}{24}\sin(2t)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+2^2}\right) = \mathcal{L}^{-1}\left(\frac{1}{2}\cdot\frac{2}{s^2+2^2}\right)$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^2+2^2}\right)$$

$$\boxed{= \frac{1}{2}\sin(2t)}$$

$$\mathcal{L}^{-1}\left(\frac{5s}{s^2+2^2}\right) = \frac{d}{dt}\left(\mathcal{L}^{-1}\left(\frac{5}{s^2+4}\right)\right)$$

$$+ \mathcal{L}^{-1}\left(\frac{5}{s^2+4}\right)(0)$$

$$= \frac{d}{dt}\left(\mathcal{L}^{-1}\left(5/2 \cdot \frac{2}{s^2+4}\right) + 0\right.$$

$$= \frac{d}{dt}\left(\frac{5}{2}\sin(2t)\right)$$

$$= \frac{5}{2}\cos(2t)$$

Now,

$$y = \frac{7t}{12} - \frac{7}{24}\sin 2t + \frac{1}{12}\sin 2t + \frac{5}{2}\cos 2t$$

$$\boxed{y = \frac{7}{12}t + 5\cos 2t + \frac{5}{24}\sin 2t}$$

Ques 4 $y'' + 4y' + 4y = 9te^{-2t}$ $y(0)=5$
$y'(0)=3$

Take $\mathcal{L}t$ on both sides.

$$\mathcal{L}(y'') + 4\mathcal{L}(y') + 4\mathcal{L}(y) = 9\,\mathcal{L}(te^{-2t})$$

$$s^2\mathcal{L}(y) - s\,y(0) - y'(0) + 4(s\mathcal{L}(y) - y(0)) + 4\mathcal{L}(y) = \frac{9}{(s+2)^2}$$

$$s^2\mathcal{L}(y) - 5\,y(0) - y'(0) + 4s\mathcal{L}(y) - 4(y(0)) + 4\mathcal{L}(y) = \frac{9}{(s+2)^2}$$

using $y(0)=5$, $y'(0)=3$

$$s^2\mathcal{L}(y) - 5\cdot 5 - 3 + 4s\mathcal{L}(y) - 4\cdot 5 + 4\mathcal{L}(y) = \frac{9}{(s+2)^2}$$

$$\mathcal{L}(y)(s^2+4s+4) - 5s - 3 - 20 = \frac{9}{(s+2)^2}$$

$$\mathcal{L}(y)[s^2+4s+4] = \frac{9}{(s+2)^2} + 5s + 23$$

$$\mathcal{L}(y) = \frac{9}{(s+2)^2} + \frac{5s}{(s+2)^2} + \frac{23}{(s+2)^2}$$

Takeng inverse Laplace on both sides.

$$\mathcal{L}^{-1}(\mathcal{L}(y_1)) = \mathcal{L}^{-1}\left(\frac{9}{(s+2)^4}\right) + \mathcal{L}^{-1}\left(\frac{5s}{(s+2)^2}\right) + \mathcal{L}^{-1}\left(\frac{23}{(s+2)^2}\right)$$

$$\mathcal{L}^{-1}\left(\frac{9}{(s+2)^4}\right) = e^{-2t}\,\mathcal{L}^{-1}\left(\frac{9}{s^4}\right)$$

$$= e^{-2t} \cdot \frac{3t^3}{2}$$

$$= \frac{3e^{-2t}t^3}{2}$$

$$\mathcal{L}^{-1}\left(\frac{5s}{(s+2)^2}\right) = \mathcal{L}^{-1}\left(\frac{5}{s+2}\right) - \mathcal{L}^{-1}\left(\frac{10}{(s+2)^2}\right)$$

$$= 5e^{-2t} - e^{-2t}.10t.$$

$$\mathcal{L}^{-1}\left(\frac{23}{(s+2)^2}\right) = e^{-2t}\,\mathcal{L}^{-1}\left(\frac{23}{s^2}\right)$$

$$= e^{-2t}.\,23t$$

Now $y = \dfrac{3e^{-2t}t^3}{2} + 5e^{-2t} - e^{-2t}.10t + e^{-2t}.23t$

$$\boxed{y = \frac{3}{2}t^3 e^{-2t} + 5e^{-2t} + 13te^{-2t}}$$

Ques $y'' + 9y = \sin 3t \qquad y(0) = 4 \quad y'(0) = 2$

Taking L.T on both sides.

$$\mathcal{L}(y'') + 9\mathcal{L}(y) = \mathcal{L}(\sin 3t)$$

$$s^2(\mathcal{L}y) - s\,y(0) - y'(0) + 9\mathcal{L}(y) = \frac{3}{s^2+9}$$

using $y(0) = 4 \quad y'(0) = 2$

$$s^2\mathcal{L}(y) - s\cdot 4 - 2 + 9\mathcal{L}(y) = \frac{3}{s^2+9}$$

$$s^2\mathcal{L}(y) - 4s - 2 + 9\mathcal{L}(y) = \frac{3}{s^2+9}$$

$$\mathcal{L}(y)\,[s^2+9] - 4s - 2 = \frac{3}{s^2+9}$$

$$\mathcal{L}(y)\,[s^2+9] = \frac{3}{s^2+9} + 4s + 2$$

$$\mathcal{L}(y) = \frac{3}{(s^2+9)^2} + \frac{4s}{(s^2+9)} + \frac{2}{s^2+9}$$

Taking Inverse Laplace on both sides.

$$\mathcal{L}^{-1}(2y) = \mathcal{L}^{-1}\left(\frac{3}{(s^2+9)^2}\right) + \mathcal{L}^{-1}\left(\frac{4s}{s^2+9}\right) + \mathcal{L}^{-1}\left(\frac{2}{s^2+9}\right)$$

$$\mathcal{L}^{-1}\left(\frac{3}{(s^2+9)^2}\right) = \frac{1}{18}\mathcal{L}^{-1}\left(\frac{2\cdot 3}{s^2+3^2}\right)^2$$

$$= \frac{1}{18}(\sin 3t - 3t\cos 3t)$$

$$\mathcal{L}^{-1}\left(\frac{4s}{s^2+9}\right) = \frac{d}{dt}\left(\mathcal{L}^{-1}\left(\frac{4}{s^2+9}\right)\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2+9}\right)(0)$$

$$= \frac{d}{dt}\left(\frac{4}{3}\sin 3t\right)$$

$$= 4\cos 3t.$$

$$\mathcal{L}^{-1}\left(\frac{2}{s^2+9}\right) = \frac{2}{3}\mathcal{L}^{-1}\left(\frac{3}{s^2+3^2}\right)$$

$$= \frac{2}{3}\sin 3t$$

Now $y = \frac{1}{18}(\sin 3t - 3t\cos t) + 4\cos 3t + \frac{2}{3}\sin 3t$