A pareto-optimal approach to numerical scheme design for conservation laws.

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Most discrete approximations to first and second order derivatives in governing laws are generally designed through the Taylor's series polynomials as well as the well known modified wavenumber analysis approach of Lele (J. Comp. Phys., 103(1), 16-42). The base philosophy of the latter is characterized by a minimization of the dispersive error of the scheme (in essence the error in representing spectral quantities accurately). In addition, it is also desired to obtain suitable amounts of dissipation within a scheme so that signals of derivatives remain viable for nonlinear wave propagation. This problem of maximizing dispersion while minimizing (or maximimizing) dissipation has generally been studied in a rather ad-hoc fashion by previous researchers. Throughout the vast body of computational physics literature, to our knowledge, there has been no investigation of this framework within a pareto-optimal paradigm that gives the user access to desired combinations of these two competing objectives. This study aims to obtain pareto fronts for multiple coefficients of a stencil that evaluates the first order derivative and to test their viability for a linear wave equation problem. Through our efforts we aim to construct finite difference stencils which are ideally suited to a specific dissipation and dispersion with profound implications for large eddy simulation.