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Design of Experiments

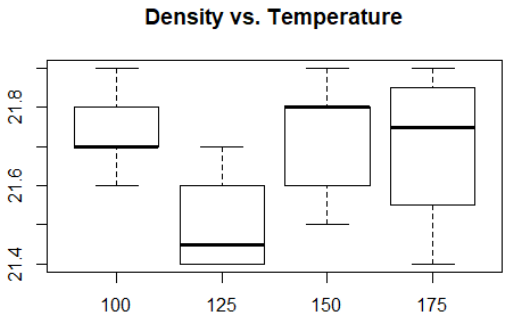
ANOVA and Multiple Comparisons

Part 1. Montgomery 5th Edition Problems:

3-4:

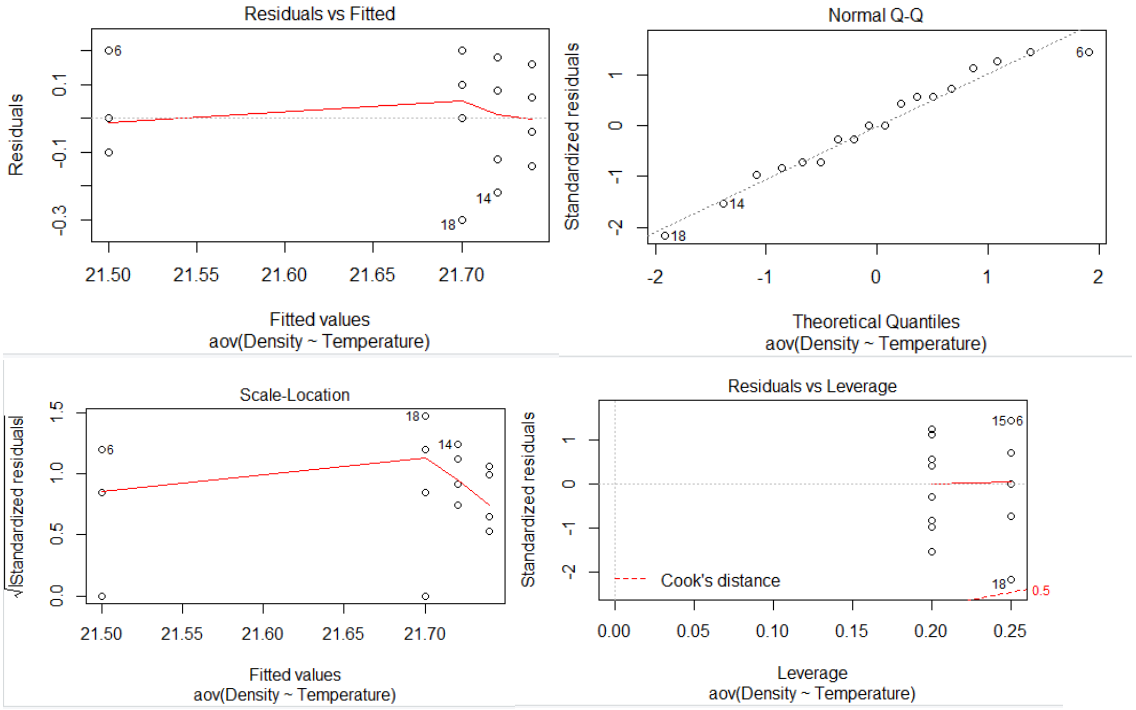
a) The results below indicate that there is not a significant correlation between density and temperature (p = 0.157)

```
> summary(DT_ANOVA_model)
            Df Sum Sq Mean Sq F value Pr(>F)
Temperature  3 0.1561 0.05204   2.024  0.157
Residuals   14 0.3600 0.02571
```

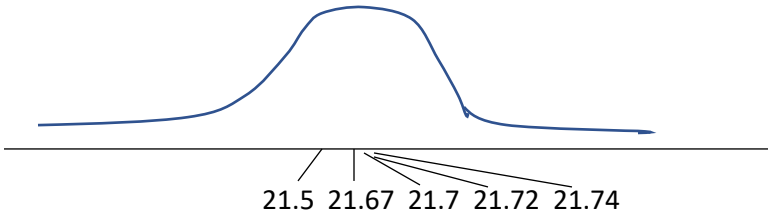


b) It's not appropriate to compare means of pairs in this case because the ANOVA already determined no significant differences. Plus, the only odd temperature in the box plot is 125, which appears to yield lower densities. The jump back up to 150 seems abnormal for a temperature-dependent function.

c) It can be seen in the top left plot of the four residual plots below that the variance is fairly uniform, although the large gap in fitted values is slightly deceiving.



d) standard deviation = 0.1742 → 3 sd = 0.55, 21.67 ± 0.55 = 21.12, 22.22 are good outer curve regions

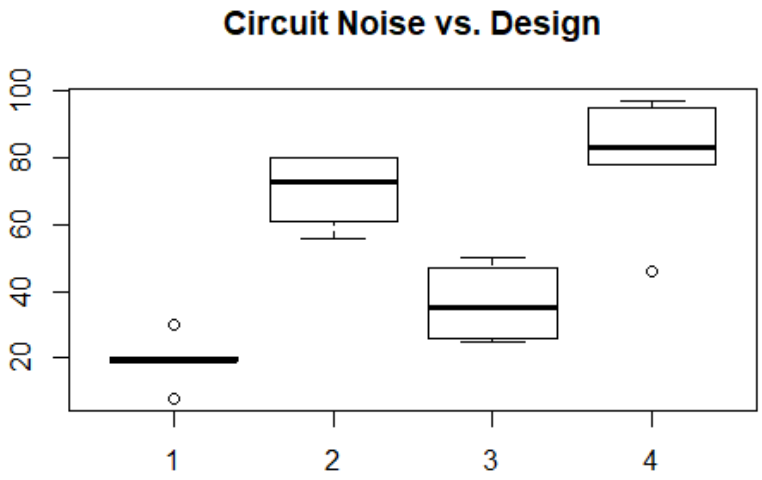


All of the means are clearly within a reasonable t-distribution

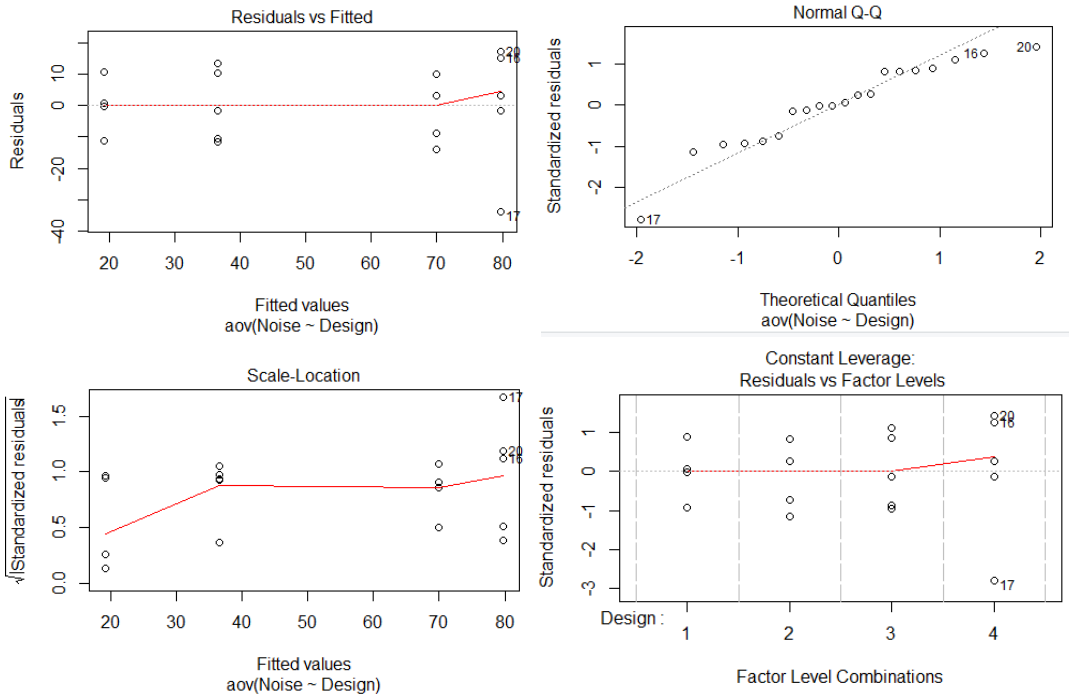
3-12:

a) The ANOVA test shown below concludes that the amount of noise is significantly not the same for all four designs. Outliers are included in this model because of the tiny sample size per circuit design.

```
> summary(CD_ANOVA_model)
            Df Sum Sq Mean Sq F value  Pr(>F)
Design       3  12042    4014   21.78 6.8e-06 ***
Residuals   16   2949     184
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```



b) The variance looks fairly uniform, and maybe only the slightest trending positively linear with fit values. I think it satisfies the assumptions well enough.

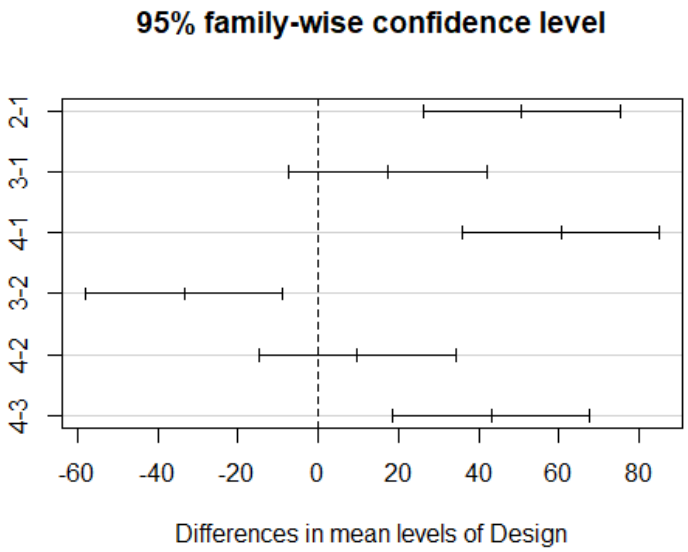


c) Just to make sure, I ran a pair-wise Tukey HSD test on the circuits, and the results were not completely conclusive (shown below). Circuits 3 and 1 are not significantly different, and 2 and 4 are also not significantly different, but those pairs are. However, I would choose circuit 1, purely based on the lower mean noise.

```
> CD_TukeyTest
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Noise ~ Design)

$Design
     diff        lwr       upr     p adj
2-1  50.8  26.235183 75.364817 0.0001159
3-1  17.4  -7.164817 41.964817 0.2194816
4-1  60.6  36.035183 85.164817 0.0000147
3-2 -33.4 -57.964817 -8.835183 0.0064088
4-2   9.8 -14.764817 34.364817 0.6703350
4-3  43.2  18.635183 67.764817 0.0006406
```



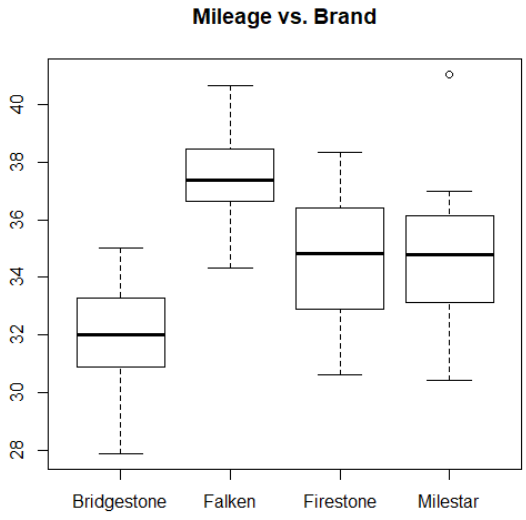
Part 2. ANOVA and Pairwise Tests:

ANOVA:

Assumptions:

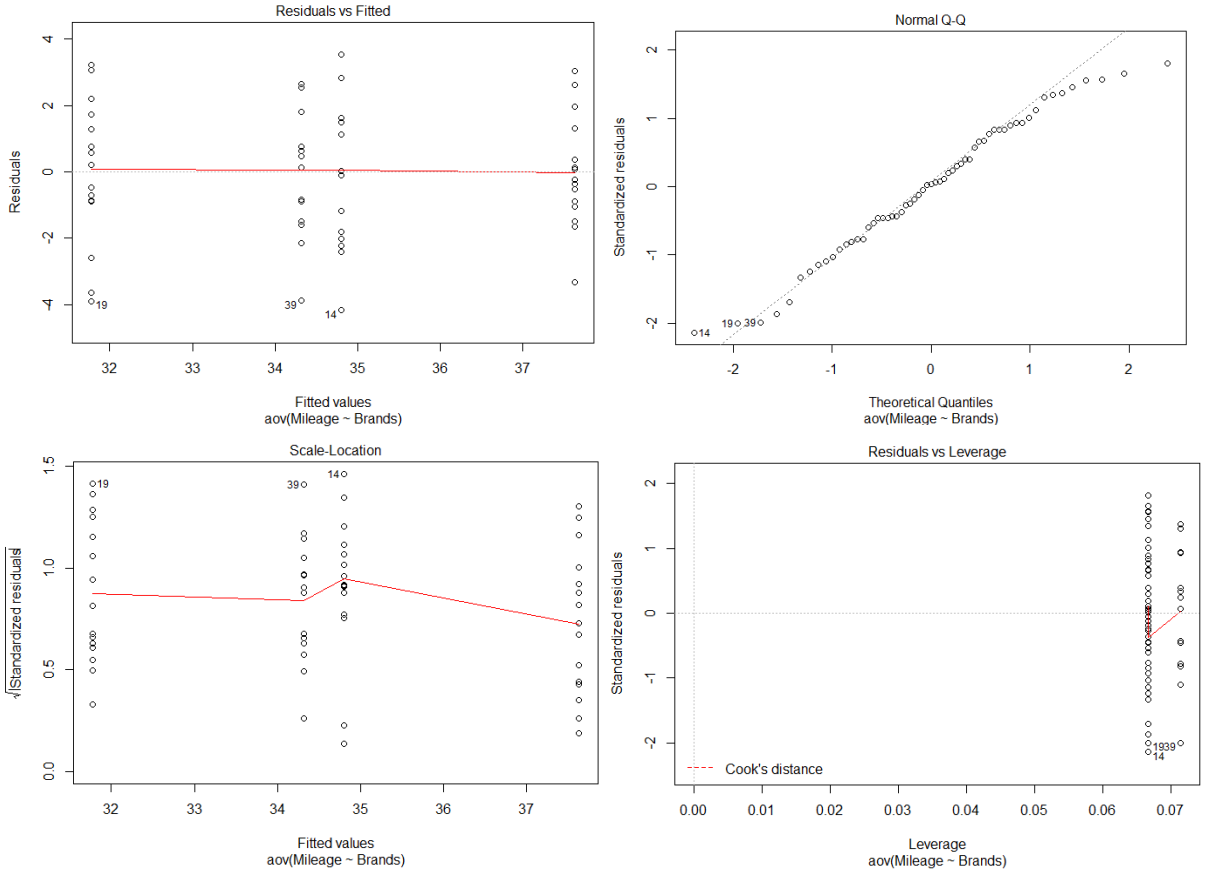
- Each brand-mileage case is independent.
- The residuals of the model are normal and small compared to the values
- The variances of each brand are similar (homoscedasticity)
- Data has no outliers (manually removed)

Visualized data:



Test results:

```
> summary(ANOVA_model)
             Df Sum Sq Mean Sq F value   Pr(>F)
Brands        3  258.2   86.06   21.11 3.16e-09 ***
Residuals    55  224.3    4.08
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```



These residual plots show that the variances are similar (top-left, can also see roughly on box-plot), the residuals are normal (top-right), the residuals follow no trend (bottom-left), and the residuals have no outliers (bottom-right, outliers previously manually removed)

Interpretation:
With a p-value of $3.16 \cdot 10^{-9}$, this test shows that there is a ***very significant*** difference in tire mileages between brands.

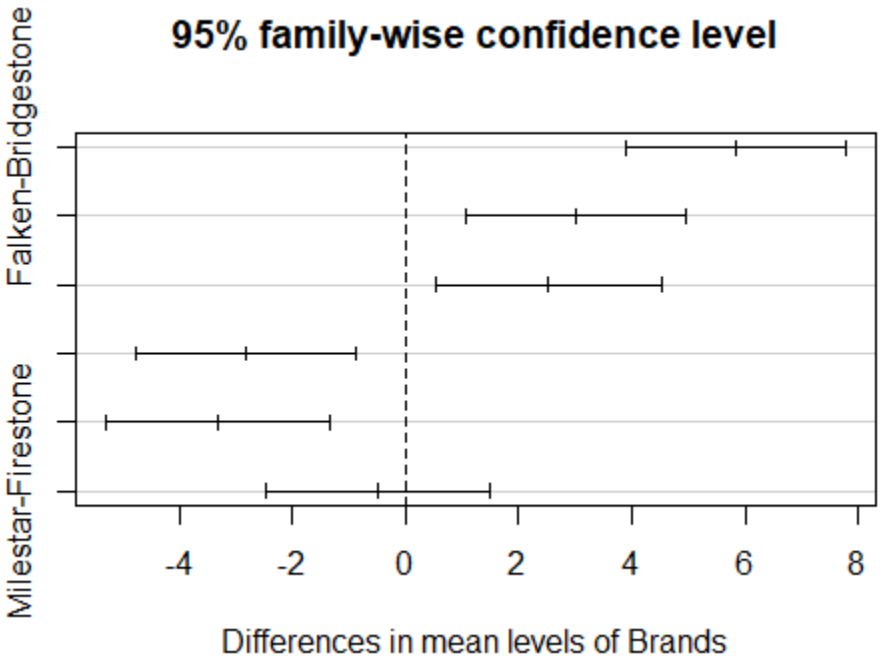
<u>Pairwise tests:</u>
Assumptions are the same as the ANOVA test. I used a Tukey HSD multiple comparisons test.

Results:

```
> TukeyTest
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Mileage ~ Brands)

$`Brands`
                           diff        lwr        upr     p adj
Falken-Bridgestone     5.8445333  3.8910084  7.7980583 0.0000000
Firestone-Bridgestone  3.0190000  1.0654750  4.9725250 0.0007871
Milestar-Bridgestone   2.5318739  0.5437705  4.5199772 0.0072470
Firestone-Falken      -2.8255333 -4.7790583 -0.8720084 0.0018185
Milestar-Falken       -3.3126595 -5.3007628 -1.3245561 0.0002729
Milestar-Firestone    -0.4871261 -2.4752295  1.5009772 0.9154050
```



Interpretation:

The image above shows the difference in sample means and 95% confidence intervals on the difference in means (lwr < $\mu_1 - \mu_2$ < upr), as well as the p-value for significance. Clearly, Falken tires are significantly better for mileage than Bridgestone, Milestar and Firestone tires (p values < 0.002). Also, Bridgestone tires are significantly worse for mileage than Falken, Milestar, and Firestone tires (p values < 0.01). There is no significant mileage difference between Milestar and Firestone tires (p = 0.915).

<u>Conclusion</u>

In conclusion, the tire brands ***do not*** all have the same mileage. Milestar and Firestone tires have very similar mileage. Falken tires have significantly higher mileage than the other brands, and Bridgestone tires have significantly lower mileage than the other brands.