TRANSIENT CONDUCTION LAB

Objectives

1. To observe the effect of convective heating on the transient thermal response for a simple three-dimensional shape.

2. To become more familiar with the analytical solutions to the heat equation in multi-dimensional transient systems.

3. To determine the heat transfer coefficient for a transient conduction problem by a direct application of the analytical solution.

4. To determine the effect of thermal conductivity on the transient thermal response of an object exposed to a convective heating.

References

1. *Fundamentals of Heat and Mass Transfer*, T.L. Bergman, A.S. Lavine, F.P. Incropera, D.P. DeWitt, 7th Ed., John Wiley & Sons, 2011.

Introduction

When an object is suddenly subjected to a change in environmental conditions, some time will elapse before a new equilibrium temperature is established. In the transient heating or cooling process between the initial and final equilibrium states, the analysis must consider the change in internal energy of the object with time, and the boundary conditions must be modified to match the physical situation. Unsteady heat transfer analysis is of practical interest due to the large number of heating and cooling processes that occur in industry.

Exact analytical solutions of the heat equation are available for many regular one-dimensional shapes, such as an infinite plane wall, an infinite cylinder, and a sphere. These solutions may be combined in an appropriate manner to determine exact solutions for objects in which two- and sometimes three-dimensional effects are significant. In this lab exercise, the transient heating of a sphere, and a short cylinder will be studied by measuring the temperature at the center of each object over time. Thus, only the solution for the transient temperature response at the shape center is required. All of the exact solutions are in the form of an infinite series; however, when the Fourier number $Fo > 0.2$, a one-term approximation provides acceptable accuracy. Details of the one-term approximate solution at the center of each object are provided below.

Sphere

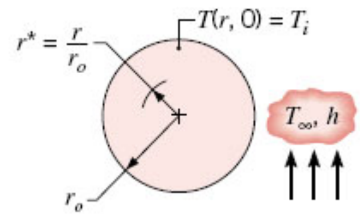


Fig. 1 Sphere with initial temperature subjected to sudden convection condition

The nondimensional center temperature θ_0^* is defined as

$$\theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} \quad (1)$$

where T_0 is the center temperature, T_∞ is the ambient temperature, T_i is the initial temperature and other system variables are shown in Fig. 1.

The one-term approximate transient solution for a sphere of radius r_0 at time t is

$$\theta_0^* = C_1 \exp\left(-\zeta_1^2\, Fo\right) \quad (2)$$

where

$$Fo = \alpha t / r_0^2\ , \quad (3)$$

$$\alpha = k/\rho c_\mathrm{p}\ , \quad (4)$$

$$C_1 = \frac{4[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)]}{2\zeta_1 - \sin(2\zeta_1)}\ , \quad (5)$$

and the eigenvalue ζ_1 is the first root of the transcendental equation

$$1 - \zeta_1 \cot \zeta_1 = Bi \quad (6)$$

with

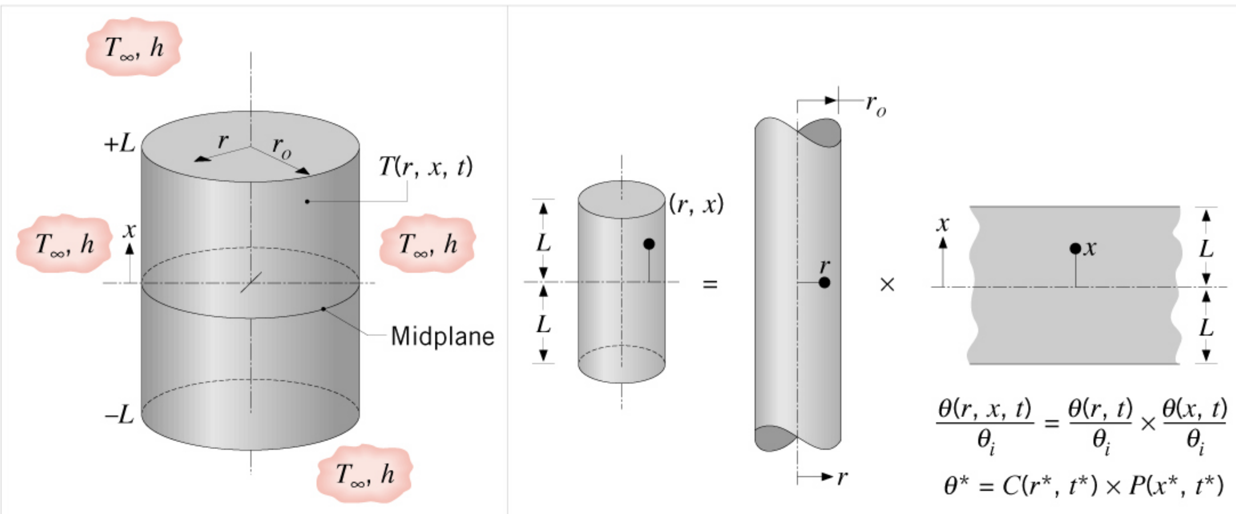
$$Bi = \frac{hr_0}{k}. \quad (7)$$

Note, the above solution in (2) is only valid for $Fo > 0.2$, i.e., it is not valid immediately at the start of the heating/cooling near t =0.

Short Cylinder

Consider a short cylinder (Fig. 2) initially at a uniform temperature T_i. It is suddenly immersed in a fluid of temperature $T_\infty \neq T_i$. Since the axial and radial dimensions are on the same order, heat transfer within the cylinder will be comparable in both the radial and axial directions. Thus, the cylinder temperature depends on x, r, and t. The heat equation in cylindrical coordinates, assuming constant properties and no generation, could be solved for the desired temperature distribution. A closed form solution can be obtained using the separation of variables technique. This result is expressed as the product of the temperature solution in the axial direction (plane wall solution) and the temperature solution in the radial direction (infinite cylinder solution). Thus, the two-dimensional solution is expressed as a product of one-dimensional solutions corresponding to those for a plane wall of thickness $2L$ and an infinite cylinder of radius r_0.

Fig. 2 Two-dimensional transient conduction in a short cylinder.



Since only the center temperature of the short cylinder is measured, the analytical solution for the center temperature is of interest here. The nondimensional center temperature θ_0^* is given by

$$\theta_0^* = \frac{T(0, 0, t) - T_\infty}{T_i - T_\infty} = C(0, t)\, P(0, t) \tag{8}$$

where C and P are the nondimensional center temperatures for the infinite cylinder and infinite plane wall, respectively. For $Fo > 0.2$, these solutions may be simplified to their one-term approximations. The one-term transient nondimensional temperature solution for the infinite plane wall centerline is

$$\theta_0^* = P(0, t) = C_1 \exp(-\zeta_1^2\, Fo)\,, \tag{9}$$

where $$C_1 = \frac{4 \sin\zeta_1}{2\zeta_1 + \sin(2\zeta_1)} \tag{10}$$

and the eigenvalue ζ_1 is the first root of the transcendental equation

$$\zeta_1 \tan \zeta_1 = Bi \quad . \tag{11}$$

For the infinite plane wall, $Bi = hL/k$ and $Fo = \alpha t/L^2$, where L is the distance from the cylinder center to one of the flat surfaces.

For the infinite cylinder, $Bi = hr_0/k$ and $Fo = \alpha t/r_0^2$, where r_0 is the cylinder radius. The one-term transient nondimensional temperature solution for the center of an infinite cylinder is

$$\theta_0^* = C(0, t) = C_1 \exp(-\zeta_1^2 \, Fo) \, , \tag{12}$$

where $$C_1 = \frac{2}{\zeta_1} \frac{J_1(\zeta_1)}{J_0^2(\zeta_1) + J_1^2(\zeta_1)} \, , \tag{13}$$

J_1 and J_0 are Bessel functions of the first kind of order 1 and 0, respectively, and the eigenvalue ζ_1 is the first root of the transcendental equation

$$\zeta_1 \frac{J_1(\zeta_1)}{J_0(\zeta_1)} = Bi \quad . \tag{14}$$

Experimental Apparatus

The primary component of the transient heat transfer apparatus, shown schematically in Fig. 3, is a 0.0454 m^3 (12 gallon) tank separated into two zones. The inner circulation chamber has water flowing in an upward direction over a submersed three-dimensional shape. The test specimens are of various materials and shapes, including a sphere, and a short cylinder. Table 1 provides transport and thermodynamic properties for the specimen materials. Each specimen has an imbedded copper/constantan Type-T thermocouple with the thermocouple junction at the center of the object. Thermocouple output (specimen and water temperature) as a function of time are acquired, stored and displayed using a *LabVIEW* program. At the completion of data acquisition, all stored data and the temperature/time plot may be printed. A constant water temperature is maintained by a thermostat and thermocouple that control the power to an electric resistance heating element. The maximum water temperature should not exceed 70°C. Approximate water temperatures may be set by the thermostat; however, an accurate water temperature measurement is obtained by a submersed thermocouple. A circulating pump provides the continuous flow of water from the outer chamber to the circulation chamber resulting in a forced convection condition on the specimen.

Table 1. Thermodynamic and Transport Properties of Specimen Materials

Material	Density (kg/m^3)	Thermal Conductivity (W/m-K)	Specific Heat (J/kg-K)
Aluminum 2024T351	2760	121.4	895.8
Brass 360	8500	116.0	382.6

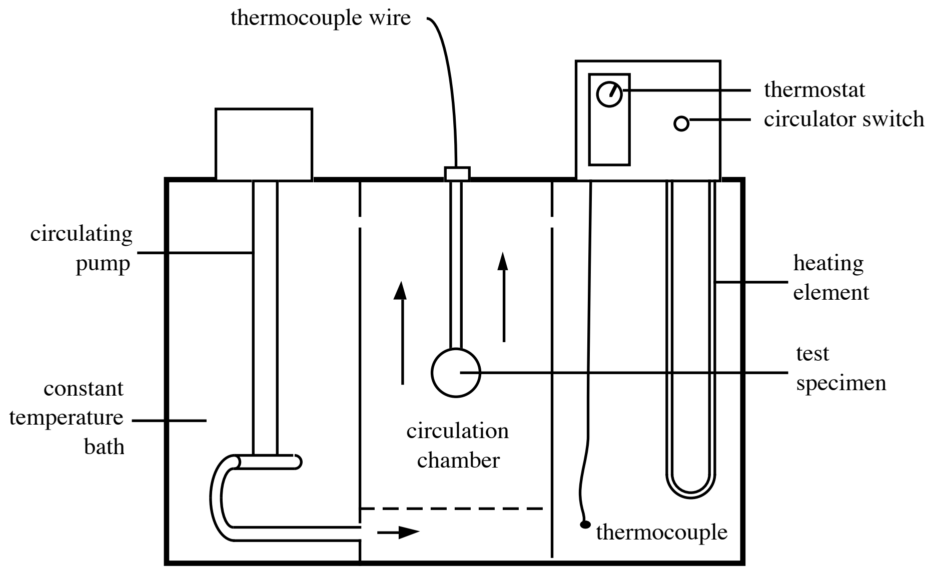


Fig. 3. Schematic of transient heat transfer apparatus

Procedure

1. The TA will assign each group a particular shape for the exercise. Two materials of the same shape are to be tested. All specimens are cooled to an initial temperature near 0°C in an ice bath.

2. Measure the important physical dimensions of each assigned specimen.

3. Determine that the water in the transient heat transfer tank is at a steady-state condition.

4. Attach the thermocouple adapter cable to the first shape you have chosen. The water bath thermocouple is already connected. Start the *transientconduct2016.vi* LabView data acquisition program using the white right arrow key on the menu bar. Now remove a specimen from the ice bath and place the specimen in the hot water bath.

5. Monitor the specimen temperature as a function of time until the center temperature is within approximately 5% of the maximum achievable temperature change (i.e., $\theta_0^* \approx 0.05$). At this point, use the Grab Data button to stop data acquisition.

6. With the second material of the same shape, repeat steps 2-5.

7. For later access to the data, both data sets should now be transferred to a portable storage device, printed, emailed to your account, or saved in the Canvas class folder.

8. Analyze the data to determine the heat transfer coefficient and thermal conductivity of the object.

Data Analysis

The collected data from your experiments can be used to calculate the convective heat transfer coefficient h. We will focus on the case of the sphere. To do this, we need to curve fit the experimental data to the analytical solution in (2). If you plot the nondimensional temperature θ_0^* versus Fourier number Fo for your data on a semi-logarithmic plot, you will notice a linear region. This means the physical behavior of the system follows an exponential function. We can use Matlab's `polyfit` command to perform the curve fit. To do this, we need to recast the model equation (2) into an equivalent form of a straight line. This is easily done by taking the natural logarithm of both sides of (2) to yield

$$y = mx + b \tag{15}$$

where $y=ln(\theta_0^*)$, $x=Fo$, $m=-\zeta_1^2$, and $b=ln(C_1)$. The derivation is left for the student to complete. We then curve fit the data (transformed into semi-logarithmic coordinates) to find the best-fit values for m and b. Note, in performing this curve, it is imperative you only use the data that exhibit an exponential trend. This means you will need to ignore the data near the beginning of the experiment. Once you know the value for m, you can use equation (6) to obtain the Biot number Bi, and then equation (7) to calculate h.

Because h depends only on the characteristics of the fluid flow driving the convection as well as the geometry of the object, it is reasonable to expect that the measured h values for both test cases should be very similar, i.e., h should NOT depend on the material properties of the object. We can exploit this fact in order to <u>measure</u> the thermal conductivity of an unknown material. To examine this idea, we will measure the thermal conductivity of both the aluminum and brass spheres and compare with the listed values in Table 1. For example, for the case of the brass sphere, we take the Bi value from the curve fit to the brass data, and calculate k using the h value estimated from the aluminum sphere:

$$k_{\text{brass}} = \frac{h_{\text{aluminum}}\, r_0}{Bi_{\text{brass}}} \tag{16}$$

A similar procedure is followed for calculating k of the aluminum sphere using the estimated h value from the experimental data with the brass sphere.

Required Plots and Tables

1. On a single figure, plot the raw temperature data (on the vertical axis) versus time (on the horizontal axis) for both the aluminum and brass spheres. Use open circles for the center temperatures and a dashed line for the bath temperatures --- color the markers & dashed line blue for aluminum and red for brass. Do NOT connect the center temperature data points with a line. Since the bath temperature changes insignificantly during the course of each experiment, plot only the average bath temperature for the experiment (do NOT plot the individual data points for the bath temperature since this will simply clutter your plot). Be sure to include a legend and appropriate units on your axes labels.

2. Create a semi-logarithmic plot of the nondimensional temperature θ_0^* (vertical axis, logarithmic scale) versus the Fourier number *Fo* (horizontal axis, linear scale) for the **aluminum** sphere. Use blue circles for the data. Overlay your curve fit of the data using a solid line. Be sure to include a legend.

3. Create a similar plot, as explained in item 2 above, for the **brass** sphere.

4. Create a table that looks like the one below containing the values from your analysis.

Material	*Bi*	*h* (W/m^2 K)	Thermal conductivity (W/m K)		
			listed	measured	% difference
Aluminum 2024T351			121.4		
Brass 360			116.0		

Short-Answer Questions

1. State the percent difference in *h* values measured between the two test cases. Is this reasonable? Briefly discuss any ideas you have for increasing the accuracy of the *h* values that can be obtained from this experiment.

2. Evaluate whether the one-term approximation for the analytical solution is appropriate in this application. To do this, quantify the "goodness" of your curve fits in Plots 2 & 3, AND state the time (in seconds), along with the corresponding *Fo* value, after which the data appear to follow an exponential trend. Your response to this question should be in the form of two or more complete sentences. [Note: your goodness-of-fit calculations should be placed in your Matlab code with the output displayed to the screen.]

3. Would it be appropriate to predict the transient temperature response of the tested object using the lumped capacitance method? Provide a response in the form of two or more complete sentences that reference the magnitude of the Biot numbers for both test cases examined. [Hint: recall that if the Biot number is small (i.e., $Bi < 0.1$), then the temperatures inside the body will not vary significantly in space. In this case, it is usually sufficient to consider the body to be at a uniform temperature.]