TWO-DIMENSIONAL STEADY-STATE CONDUCTION

Introduction

Steady-state heat conduction in a two-dimensional geometry is controlled by the thermal conditions at the boundaries. Analytical solutions may be performed in some simple geometries, such as rectangular or cylindrical flat surfaces, when boundary conditions are completely defined on each surface. The boundary conditions may be combinations of boundary conditions of the first kind (constant temperature), second kind (constant heat flux), and third kind (convection condition). Unfortunately, analytical solutions are difficult to perform and only simple geometries may be used. As an alternative to analytical solutions, numerical techniques, such as the finite-difference technique or the finite element method, may be employed to approximate the temperature field in the 2-D geometry. Regardless of the numerical method, appropriate boundary conditions must still be employed in order to solve for the temperature distribution in the solid. Two-dimensional conduction may also be studied experimentally. In this case, the object must be constructed in such a way that its thermal behavior may be assumed to be two-dimensional in nature. In other words, the heat flux and significant temperature gradients occur primarily in a single plane. In this way, heat transfer in the third dimension is minimized and the temperature field in the principal plane is sufficient to understand the thermal performance.

This exercise uses a unique rectangular heated flat plate that is designed to produce heat transfer that is very nearly two-dimensional. One of the plate edges is insulated while the other three edges may be either heated or cooled. The wide variety of thermal boundary combinations makes it possible to produce a diverse set of heat flux patterns in the plate. The temperatures on the plate surface may be measured with a thermal imaging camera (TIC), producing a complete surface temperature map. The TIC also produces a color isothermal map, which helps visualize the temperature field and accompanying heat flux map. The measured temperatures on the perimeter of the flat plate will be used as inputs to a numerical model. The temperature field produced by the finite-difference numerical model will be compared to measured temperatures from the flat plate. The goal of the numerical exercise is to validate the finite-difference method as an appropriate and accurate tool for predicting thermal performance in two-dimensional structures.

Objectives

The objectives of this lab are: 1) to become familiar with two-dimensional conduction produced by heated/cooled edges of a flat plate, 2) to become familiar with the use of a thermal imaging camera for temperature measurement and imaging, 3) to computationally determine the temperature distribution using the finite-difference method, and 4) to compare the experimental results with the numerical results.

Experimental Apparatus

The schematic of the flat plate apparatus is shown in Fig. 1. The flat plate is machined aluminum with a thickness of 7.62 cm (3″) at the edges and approximately 0.635 cm (1/8″) in the center portion of the plate. The exposed plate surface is 15.24 cm by 15.24 cm (6″ square) and has been painted black. Foam insulation 3.81 cm (1.5″) thick is attached to the rear of the plate to reduce heat loss. All four edges of the plate are also insulated with 3.81 cm (1.5″) of foam insulation. The bottom edge is completely insulated and may be assumed to be adiabatic. Six small thermoelectric coolers (TEC) are attached to each of the two vertical edges and the top edge. The arrays of TECs are powered by three separate voltage supplies (one for each edge array), which include switches to control polarity (and thus set the TEC array to either heat or cool). The flat plate assembly is mounted vertically and a thermal imaging camera is set up to view the black aluminum surface.

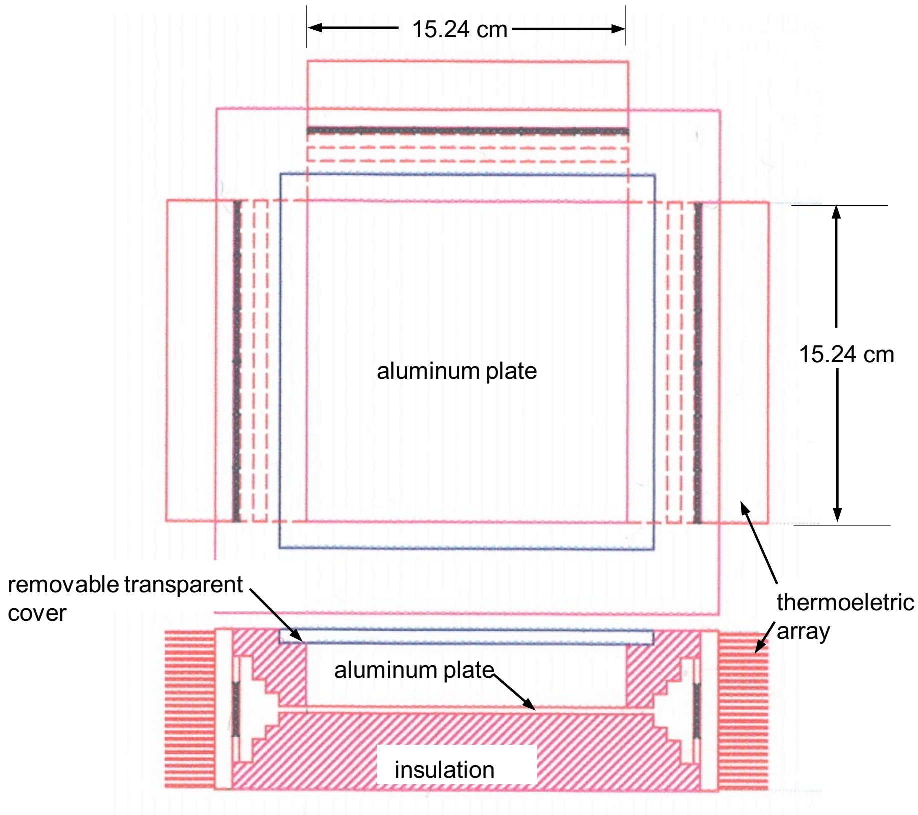

Figure 1. Schematic of flat plate with edge heating and/or cooling.

Thermoelectric coolers are solid-state heat pumps used in applications where temperature stabilization, temperature cycling, or cooling below ambient may be required. There are many commercial products using thermoelectric coolers, including CCD cameras, laser diodes, microprocessors, blood analyzers, portable picnic coolers, and the thermal imaging camera used in this experiment. Thermoelectrics are based on the Peltier Effect, in which a temperature differential occurs when a DC current is applied across two dissimilar materials. Thermoelectrics can be used to both heat and cool, depending on the direction of the current. Using a thermoelectric in the heating mode is very efficient because all the electric resistance heating (Joule heating) and the load from the cold side are pumped to the hot side. This reduces the power needed to achieve the desired heating.

The thermal imaging camera is actually an infrared camera, which is a non-contact device that detects infrared energy (heat) and converts it into an electronic signal. The signal is then processed to produce a thermal image on a video monitor and to perform temperature calculations. Heat sensed by an infrared camera can be very precisely quantified, which facilitates monitoring of thermal performance and the identification of potential heat-related problems in industrial settings. The precise temperature measurement and the visual image produced by the camera are the features of most interest in this activity. The camera sensor consists of a pixel array of 320 columns and 240 rows, with the origin in the upper left of the image.

Procedure

A. Experimental determination of the temperature field with FLIR T-420 infrared camera

1. The TA will perform initial camera installation and setup. Make sure the camera is connected to the PC via USB cable.

2. If not already in a USB port, insert the FLIR software security dongle into a free port.

3. Turn the camera on using the power button on the lower right of the back of the camera.

4. Load the FLIR ExaminIR software.

5. You should see the software startup dialog and select the T420/T440… camera. The image in the software window should now be a live image through the camera.

6. Focus the camera via software using the Cntrl-Alt-A key combo.

7. Moving the mouse into the image window, the pixel row, column, and temperature are displayed adjacent to the cursor. Place the mouse at a spot that you expect will undergo significant temperature change when the TA changes the TEC voltage settings and polarity (effectively changing the thermal boundary conditions for the 2-D conduction condition).

8. The TA should now change the power supply voltage and polarity settings to each TEC array such that each group views a unique combination of heated, cooled, and adiabatic edges for the flat plate. The temperature of the selected point should be observed over time until a new steady-state condition is achieved.

9. Once a steady-state condition is known to exist, the image should be paused and written to the hard drive as both an image and a text file of pixel temperature values.

    a. To pause the image, press the keyboard Pause/Break key (above the “end” key).

    b. To export the paused image, use this menu sequence: File>Export, then select png, bmp, or jpg file format; then complete the file dialog.

    c. To export the paused image as a comma separated variable file readable by Excel, Matlab, etc., use this menu sequence: File>Export, then select comma separated variable file format; then complete the file dialog.

10. Verify that the file exports worked correctly. If so, turn the camera off with the power button.

B. Numerical determination of the temperature field

Conduct a 2-D finite-difference analysis of the square flat plate using the same grid as that used in the experiment. The two-dimensional steady-state heat diffusion equation with no generation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

As shown in Fig. 2, when the grid spacing is equal in the two directions ($\Delta x = \Delta y$), Eq. (1) in finite-difference form may be written

$$T_{m,\,n+1} + T_{m,\,n-1} + T_{m+1,\,n} + T_{m-1,\,n} - 4T_{m,n} = 0 \tag{2}$$

The boundary conditions for the finite-difference model should be consistent with those existing in the experimental apparatus. The measured temperatures on the boundaries where the TEC arrays are positioned should be used as known boundary temperatures in the numerical model. The adiabatic boundary of the flat plate should be modeled as an adiabatic boundary in the numerical model. Singularity points (corners) may be modeled as either adiabatic boundaries or as constant temperature (use measured temperatures) boundaries. The program should be written in Matlab. Either the matrix inversion and multiplication method or an iterative technique (Gauss-Seidel) should be employed to solve the system of equations for the temperature field. If Gauss-Seidel iteration is used, a 0.002% convergence criterion should be applied to all grid point temperatures. This convergence criterion is expressed as

$$\frac{T_{\text{new}} - T_{\text{old}}}{T_{\text{old}}} \leq 0.00002 \tag{3}$$

where T_{new} is the new temperature value and T_{old} is the previous iteration temperature. Once the temperature field is known, determine and plot the corresponding isotherms in the two-dimensional region. It should be noted that since the prescribed boundary conditions for the finite-difference solution have values equal to the measured temperatures, the numerically determined temperatures should have values nearly equal to the measured temperatures at each grid point.

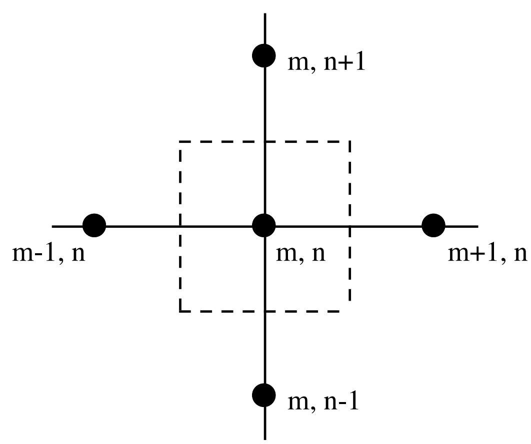

Figure 2. Grid notation for finite-difference formulation.

Required Plots

1. Display the <u>measured</u> temperatures in a filled-color contour plot. Be sure to include a colorbar. Provide labels for the horizontal and vertical axes, as well as the colorbar. All labels must include appropriate units.

2. Display the computed temperatures in a filled-color contour plot. Be sure to include a colorbar. Provide labels for the horizontal and vertical axes, as well as the colorbar. All labels must include appropriate units.

3. Display the measured temperatures in a line-contour plot showing 15-20 isotherms. Make sure each isotherm is labeled with the corresponding temperature value. Provide labels for the horizontal and vertical axes that include appropriate units.

4. Display the computed temperatures in a line-contour plot showing 15-20 isotherms. Make sure each isotherm is labeled with the corresponding temperature value. Provide labels for the horizontal and vertical axes that include appropriate units.

5. Display the difference between the measured and computed temperatures in a filled-color contour plot. Be sure to include a colorbar. Provide labels for the horizontal and vertical axes, as well as the colorbar. All labels must include appropriate units.

Discussion Items

1. Discuss the differences between the measured and computed temperatures. What is the average difference (state in terms of a percentage)? Are there regions in the domain where the differences are greater? If so, explicitly state where the difference is a maximum and state the corresponding maximum percent difference. Explain why differences are not uniform across the entire domain.

2. Discuss the boundary conditions used in the numerical model. Does the experimental temperature data at or near the boundary indicate that an adiabatic condition is appropriate for the numerical model? Discuss the data obtained on boundaries where the TECs are located. Is a uniform temperature boundary condition an appropriate assumption for the numerical model?

3. Discuss sources of error in the experiment that may lead to differences in the measured and computed data. Are there design issues associated with the flat plate that may cause the performance to deviate from the idealized case of two-dimensional conduction? Suggest design improvements that will produce a better comparison with the idealized two-dimensional conduction case.

References

Bergman, T.L., Lavine, A.S., Incropera, F.P., Dewitt, D.P., *Fundamentals of Heat and Mass Transfer*, 7th Ed., John Wiley & Sons, New York, 2011.