Flow Around a Circular Cylinder — Data Analysis Notes

Background

Any solid object moving with a velocity $\vec{V}$ through a fluid will experience a drag force F_D. This drag force is caused by the no-slip condition between the surface of the object and the fluid in direct contact with it. Importantly, the same drag force would be experienced if the object were stationary, with the fluid flowing around it at a velocity $\vec{V}$, as measured far upstream of the object.

By definition, the drag force acting on the object is the net component of the force in the direction of the mean flow. For the case of the flow around a circular cylinder, as shown in Figure 1, the flow approaches the cylinder along the horizontal x-direction. Therefore, the drag force F_D is the net x-component of the force on the cylinder, as determined by integrating the x-component of the pressure and shear stress around the entire surface of the cylinder. Note, F_D also represents the force necessary to hold the cylinder stationary in this flow, i.e., it represents the net horizontal reaction force at the mounting points.

The conservation equation for the x-component of momentum applied to the control volume shown in Figure 1 can be written as

$$-F_D + P_1\,A - P_2\,A - 4\int_{A_{\mathrm{wall}}} \tau_w\,dA = \int_{A_1+A_2} u\,\rho\,\vec{V}\cdot d\hat{A}, \tag{1}$$

where u denotes the x-component of the velocity, P_1 and P_2 denote the static pressure acting along the upstream and downstream surfaces of the control volume, respectively, and τ_w denotes the surface shear stress acting along the wind tunnel side walls. Note, there are four side walls bounding the control volume, hence the factor of 4 multiplied by the shear stress term in (1). Furthermore, in writing (1), we have assumed that the static pressure is uniform across surfaces 1 and 2. This is a very good assumption as long as the streamlines do not exhibit too much curvature (over a sufficiently long time average). You should have verified this in the lab.

In order to simplify (1), we will use the following assumptions and relations:

- → flow is two-dimensional so that the z-component of velocity (out of the page) is zero;
- → since positive $d\hat{A}$ points outward from the control volume along the x direction, $\vec{V}\cdot d\hat{A}$ is equal to $-u\,dy$ for surface 1 and $+u\,dy$ for surface 2;

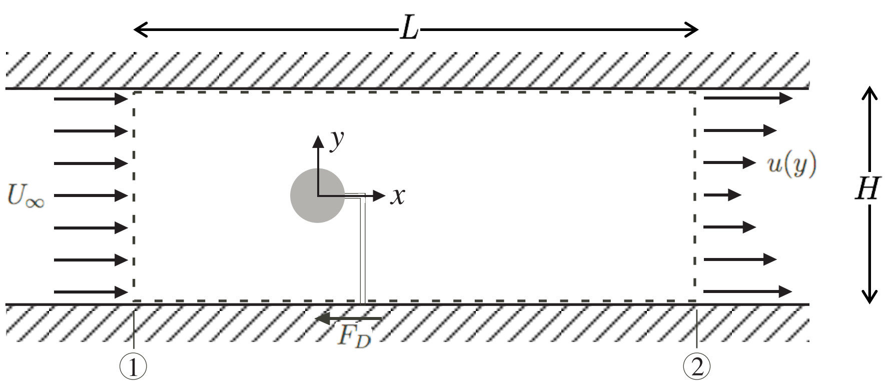

Figure 1: Schematic of the flow around a circular cylinder with the control volume indicated by the dashed line.

→ the upstream velocity along surface 1 is uniform with a magnitude of U_∞;

→ since the cross-sectional area of the wind tunnel test section is square with a side length of H, the surface areas A_1 and A_2 are equal to H^2;

→ the surface area of the side walls A_{wall} is equal to $H\,L$.

With the above information, (1) can be simplified to obtain an equation for the drag force acting on the cylinder,

$$F_D = H^2\,(P_1 - P_2) - \underbrace{4\,H \int_0^L \tau_w(x)\,dx}_{\text{friction drag on walls}} + \rho\,H^2\,U_\infty^2 - \rho\,H \int_{-H/2}^{H/2} u^2(y)\,dy\,. \tag{2}$$

In the handout for this lab, the second term on the right hand side, representing the friction drag along the side walls, was omitted. This was done, because it can generally be considered as a small correction as long as the distance L is not too large. Your task is to estimate the magnitude of this term to assess whether it is good engineering judgment to omit this term or not in the analysis.

Friction Drag Along Wind Tunnel Side Walls

We will use published empirical relations to estimate the friction drag along the wind tunnel walls. The drag coefficient due to friction along a surface is given generally by

$$C_D = \frac{\int_{A_{\text{wall}}} \tau_w\,dA}{\frac{1}{2}\,\rho\,U_\infty^2\,A}. \tag{3}$$

Note, for flow over a flat plate parallel to the flow, the surface shear stress, τ_w, will vary with distance x downstream from the leading edge of the plate. For this reason, we cannot assume that τ_w is constant and simply pull it out of the integral. Fortunately, the textbook[†] has already performed the integration for different Reynolds number regimes. The results are displayed in Figure 2. Here, the Reynolds number is defined based on the distance L from the leading edge of the plate,

$$Re_L = \frac{\rho\,U_\infty\,L}{\mu}\,. \tag{4}$$

Different empirical relations have been suggested for the three regimes: laminar, transition, and turbulence. Note, the laminar regime for flow over a flat plate parallel to the direction of the flow is generally accepted to be given by $Re_L < 5 \times 10^5$. However, there can be some variability in the actual Re value at which any real flow starts to transition to turbulence. The empirical relations referenced in Figure 2 are as follows:

$$C_D = \frac{1.33}{\sqrt{Re_L}}, \qquad Re_L < 5 \times 10^5 \text{ (laminar)}, \tag{5}$$

$$C_D = \frac{0.0742}{Re_L^{1/5}} - \frac{1740}{Re_L}, \qquad 5 \times 10^5 < Re_L < 10^7 \text{ (transition)} \tag{6}$$

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}}, \qquad Re_L < 10^9 \text{ (turbulence)}. \tag{7}$$

[†]Pritchard, P., *Introduction to Fluid Mechanics, 8th ed.*, John Wiley & Sons, Inc., 2011, pg. 446–48.

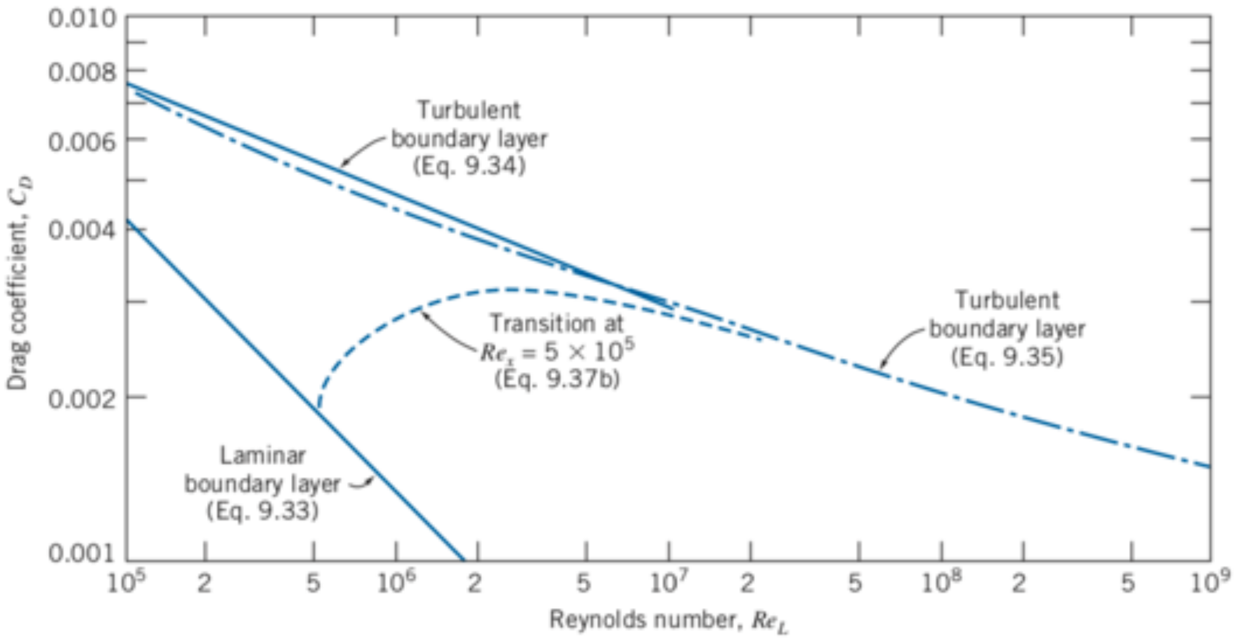

Figure 2: Skin friction drag coefficient as a function of Reynolds number for flow over a flat plate parallel to the flow.

One challenge is estimating L for the case of the experiment in the wind tunnel. Since there is no sharp leading edge for the wind tunnel side walls, we have to make an assumption about where the boundary layers start on these walls. One can argue that the best choice is to assume the boundary layers start at the exit of the contraction and entrance to the test section as illustrated in Figure 3.

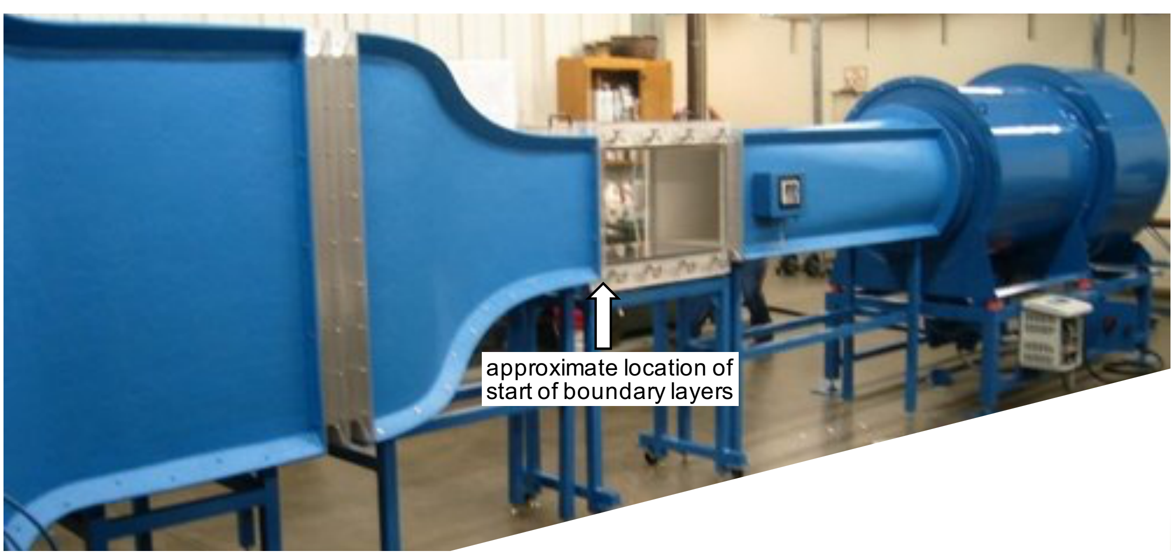

Figure 3: Photograph of the wind tunnel showing the approximate location of the start of the boundary layer growth along the tunnel side walls.