# Fully Developed Pipe Flow

Reading: pp. 326–342 (Fox, Pritchard, McDonald)

### Objectives

(i) measure pressure drops and Reynolds numbers in long sections of straight pipe,

(ii) understand the effects of Reynolds number and length-to-diameter ratios on the friction factor of a long straight pipe,

(iii) estimate pipe roughnesses using the Colebrook equation along with pressure and flow rate measurements, and

(iv) estimate the minor loss coefficients of couplers.

### Background

Flow through a circular cross-section pipe has application in a large number of engineering systems. Because of this, all mechanical engineers must, at a minimum, know the basic procedures for determining flow rates, friction factors, flow losses, and pump work requirements associated with pipe flow. The present experiment considers the flow in long straight pipes with one or more couplers.

Figure 1 shows the change in the velocity profile as fluid enters a circular pipe and flows downstream. The change in the velocity profile is due to the fact that boundary layers form along the inside walls and grow inward toward the pipe centerline as the flow proceeds down the pipe. Eventually the boundary layers meet in the pipe centerline; at which point, the flow is termed *fully developed*. In the fully developed region, the velocity profile does not change with $x$. Thus, fully developed pipe flow may be considered as one-dimensional.

The Reynolds number for circular pipes is defined as

$$Re = \frac{\overline{V}\, D}{\nu}, \tag{1}$$

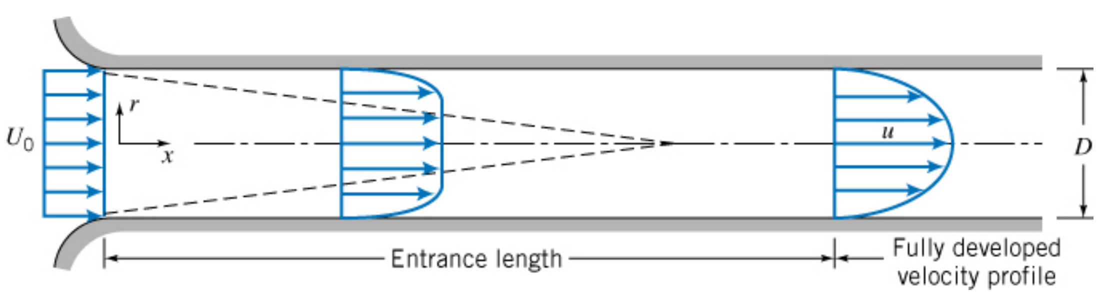


Figure 1: Illustration of the velocity profile as the flow in a circular pipe reaches the fully-developed state.

where $\overline{V}$ is the average velocity at any given cross-section, $D$ is the diameter, and $\nu$ denotes the kinematic viscosity. Depending on the Reynolds number, the flow through the pipe may be laminar or turbulent, with the general rule-of-thumb being $Re > 2300$ for turbulent flow. Note, for the laminar case, the flow becomes fully developed at $L/D \approx 100$; while, in the turbulent case, the flow becomes fully developed at $L/D \approx 20$. If the pipe is long enough, the entrance region represents a very small fraction of the overall length of the pipe, especially so in the turbulent case. Therefore, we can treat the flow through the entire length of the pipe as being fully developed without incurring too much error in the calculation.

Head Loss

For fully developed, steady flow in a horizontal pipe, the energy equation reduces to

$$\frac{\Delta P}{\rho} = h_{l,T}, \tag{2}$$

where $\Delta P$ is the pressure drop across a given length $L$ of pipe and $h_{l,T}$ is the corresponding head loss, which represents the irreversible conversion of mechanical energy to thermal energy per unit mass of fluid flowing through the pipe. To determine the major losses, we use the relation

$$h_l = f\,\frac{L}{D}\,\frac{\overline{V}^2}{2}, \tag{3}$$

where $f$ denotes the friction factor. The minor loss coefficient, $K$, for the coupler may be determined from the relation

$$h_{l,m} = K\,\frac{\overline{V}^2}{2}. \tag{4}$$

Alternatively, the minor loss due to the coupler may be expressed as an equivalent length of pipe, i.e.,

$$h_{l,m} = f\,\frac{L_e}{D}\,\frac{\overline{V}^2}{2}. \tag{5}$$

Average Velocity

Because the average velocity $\overline{V}$ appears in (1), (3), (4), and (5), we need an estimate of the average velocity. The velocity profile inside of a pipe is NOT uniform, and therefore, it is difficult to measure $\overline{V}$ directly.

For incompressible flow through a fixed control volume (C.V.), the conservation of mass reduces to

$$\int_{C.S.} \vec{V}\cdot d\vec{A} = 0. \tag{6}$$

Note, this does not require the flow to be steady, it only requires that we have a well defined volume of incompressible fluid. This form of the conservation of mass gives us an alternative way to find the average velocity ($\overline{V}$) that does not require the direct measurement of the velocity profile within a pipe. The integral in (6) at any given cross-section in the C.V. is typically defined as the volume flow rate

$$Q = \int_A \vec{V}\cdot d\vec{A}.$$

We can use the volume flow rate $Q$ to define the area averaged velocity $\overline{V}$,

$$\overline{V} = \frac{1}{A} \int_A \vec{V} \cdot d\vec{A} = \frac{Q}{A}. \tag{7}$$

From (6), we can see that for an incompressible fluid $Q$ will be constant throughout the C.V. Therefore, we can measure $Q$ at any point in the pipe network (note this only applies to single-path systems, multi-path systems require partitioning the flow between the different paths) and then use the local cross-sectional area ($A$) to calculate $\overline{V}$ at any other point.

Data Analysis

In order to estimate the friction factor for a given section of pipeline, we need to equate the left hand side of (2) to the right hand side of (3). Upon rearranging, we find

$$f = \frac{2\,\Delta P}{\rho \overline{V}^2} \frac{D}{L}, \tag{8}$$

where $\Delta P$ is the pressure drop measured over a distance $L$ of the pipe that contains no minor losses. Therefore, if we measure $\Delta P$ and $\overline{V}$, we can use (8) to estimate the friction factor. We can use this information to determine the roughness of the pipe by plotting $f$ obtained from (8) versus the Reynolds number obtained from (1) and comparing with the Moody diagram, or equivalently the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0 \, \log_{10} \left( \frac{e/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right). \tag{9}$$

Similarly, we can estimate the minor loss coefficient for the coupler using (2), (8), and (4),

$$\frac{\Delta P}{\rho} = \left( f \, \frac{L}{D} + K_t \right) \frac{\overline{V}^2}{2}, \tag{10}$$

where $\Delta P$ represents the pressure drop across the length $L$ and $K_t$ represents the total minor losses. Rearranging (10), we find

$$K_t = \frac{2\,\Delta P}{\rho \overline{V}^2} - f \, \frac{L}{D}, \tag{11}$$

where $f$ is the same value obtained from (8) for a straight section of the pipe without any couplers. Finally, we can estimate the minor loss expressed in terms of an equivalent length of pipe $L_e$ in a similar manner to obtain

$$\frac{L_e}{D} = \frac{2\,\Delta P}{f \, \rho \overline{V}^2} - \frac{L}{D}. \tag{12}$$

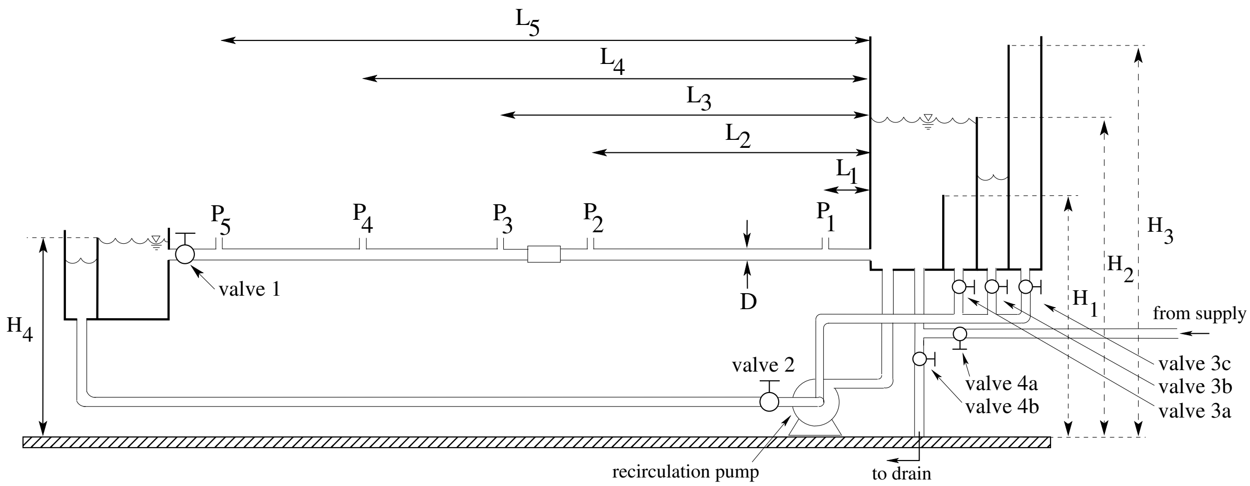


Figure 2: Schematic of the pipes, showing the relative locations of the five pressure taps, the pipe coupler and the flow control valves.

Table 1: Dimensions of the pipe diameters (units in inches).

| **Large PVC** | **Small PVC** | **Small steel** |
|---|---|---|
| (1" nominal, $D = 1.033$) | (3/4" nominal, $D = 0.81$) | (3/4" nominal, $D = 0.824$) |

**Laboratory Tasks**

Experiments are conducted using three long pipes: (i) large diameter polyvinyl chloride (PVC), (ii) small diameter PVC, and (iii) small diameter galvanized steel pipe. At the upstream end, the pipes are connected to a large supply tank and at the downstream end they empty into a smaller receiving tank. The water levels in the two tanks are both maintained at constant levels resulting in the supply of a constant head (i.e. a constant total pressure drop across the system) to the pipes. Each tank is partitioned into multiple separate volumes. The supply tank has three partitions of different heights that the water level can be filled too to maintain a constant level creating a constant head while excess water spills over the partition into the adjacent volume. The three different heights allow for three different head pressures to be created and maintained in the same system. The receiving tank has one partition creating a larger, main volume with a constant level (creating the constant head) and a smaller, spill-over volume. Water from the spill-over volumes of both the supply and receiving tanks is recirculated back to the main volume of the supply tank by a small recirculation pump. Figure 2 shows a schematic of the setup for the three pipes. A series of pressure taps is located along the length of the pipe. These taps are used to measure the *static* pressure as a function of $L$. Dimensions for the diameters of the three pipes are listed in Table 1, where all units are expressed in inches.

The lab procedures are outlined below. Students MUST use the posted sheet to record the raw data acquired in the laboratory.

1. Determine the water properties:

   (a) Measure the water temperature using the thermometer in the receiving tank.

   (b) Use Tables in Appendix A-4 to determine the density and viscosity of the water.

2. Assure that the water level in the supply tank is at the lowest of the three available heights ($H_1$). Add or remove water as necessary using valves 4a and 4b. The spill-over volume should be as full as possible while still allowing for water to spill over the partition.

3. Measure the relevant pipe and tank dimensions for the experimental system.

   (a) Measure the water level heights ($H_1$ and $H_4$) for each tank.

   (b) Measure the distances from the supply tank to each pressure tap $L_1$–$L_5$ (see figure 2) for each of the three pipes. Note that the pipes each protrude into the supply tank approximately 1 inch.

4. Obtain measurements using the large PVC pipe:

   (a) Assure that valve 3a is open to draw water from the spill-over being used.

   (b) Check to be sure the pipes are clear of air pockets by running the pump to circulate water through the pipes.

   (c) Measure the volumetric flow rate ($Q$):

      i. If open, close valve 1 (see diagram in Figure 2). Make sure both the supply and receiving tanks are still filled up to the spill over level.

      ii. Using valve 2, draw down the water level in the spill over section of the receiving tank until it is just above the level of the return hose. It is important to make sure the return hose remains full of water. Close valve 2 (valves 1 and 2 should both be closed at this point) and leave the pump on. The pump should now be recirculating water from the supply tanks' spill-over volume back to the main volume.

      iii. Open valve 1. Using the provided stop watch, time how long it takes for the spill over tank to fill up a predetermined height (say 5 inches). Close valve 1. Someone should stand ready to open valve 2 before the receiving tank over flows (and possibly close valve 3 to ensure that the pump is using it full capacity to draw water from the receiving tank). Use valve 2 to draw down the spill-over volume of the receiving tank to its initial level.

      iv. Using a tapemeasure, measure the horizontal cross-sectional area of the small spill over volume. Multiply the cross sectional area by the height of water that you measured the fill time for and divide by the time. The result will be the volume flow rate for the pipe you are testing.

      v. Repeat steps i-iv at least two more times and use the average as your volume flow rate.

   (d) Measure the drop in *static* pressure along the length of the pipe:

i. Five clear PVC tubes are provided with the laboratory setup. Each of the clear tubes has a quick connect coupler attached to one side. Make sure the tubes are clear of debris and then attach them to each pressure tap along the length of the large PVC pipe.

ii. Open valve 1. Open valve 2 as much as is necessary for the spill-over volume of the receiving tank to be held at a constant level so that the entire system is circulating water but maintaining a steady state.

iii. Using a tapemeasure, measure the height from the top of the pipe to the center of the meniscus in the clear tube for pressure tap 1. Record the static pressure on your data sheet. Do NOT forget to write down the appropriate dimensions.

iv. Repeat the above procedure for $P_2$–$P_5$ making sure to always use the same reference height in your measurements (e.g., the top of the large pipe).

v. Visually examine the trend from pressure tap 1 to pressure tap 5. The static pressure should decrease from tap 1 to tap 5. Your recorded measurements should also reflect this. Before proceeding make sure this is the case.

5. Repeat step 4 for both the small diameter PVC pipe and the galvanized steel pipe.

6. Repeat the above experiment (steps 2 through 5) using the other two supply tank water heights ($H_2$ and $H_3$) by adding water to the system using valve 4a. Use valves 3b and 3c as necessary for recirculation from the correct volume, making sure that at least one of the valve 3's is open at all times.

7. Data analysis:

   (a) Use (7) to obtain the average velocity, $\overline{V}$, from the measured volume flow rate obtained using the 'bucket filling' method.

   (b) Use pressure measurements across a straight section of the pipe without any couplers, in conjunction with (8) in order to obtain an estimate the friction factor $f$. For example, we could take $\Delta P_{3-5} = P_3 - P_5$ and $L_{3-5} = L_3 - L_5$ or take $\Delta P_{1-2} = P_1 - P_2$ and $L_{1-2} = L_1 - L_2$. Or the average of the two results could be used.

   (c) Use the pressure measurements across the coupler, in conjunction with (11) and (12) to obtain estimates of $K$ and $Le/D$ for the couplers. For example, we would use $\Delta P_{2-5} = P_2 - P_5$ and $L_{2-5} = L_2 - L_5$. We would NOT use $P_1$, because of the possibility (although small) that the flow may not be fully developed yet. Alternatively, we can also use the differential pressure between $P_2$ and $P_3$ ($\Delta P_{2-3}$) in which case the $f\frac{L}{D}$ term in (11) will be negligible (or just the $L/D$ term in 12).

   (d) Use the friction factor ($f$) and corresponding Reynolds number ($Re$) in conjunction with (9) to find the relative roughness ($e/D$) of each pipe.

**Work Due (Technical Memo)**

Your memo should consist of the following parts (see grade sheet for the points breakdown):

1. *Attachment 1: Figures and Tables.* Four figures and one table are required as explained in detail below.
   (a) Plot of $P_s$ (y-axis) versus $L/D$ (x-axis) for each of the nine tests conducted. Both axes should be plotted in linear coordinates (NOT logarithmic). You should have nine lines on the figure — three different Reynolds numbers for each of the three different pipes. Use markers to highlight the actual data points. Include a legend that indicates the pipe and Reynolds number corresponding to each line.
   (b) A plot of $f$ (y-axis) versus $Re$ (x-axis). Both $f$ and $Re$ should be plotted on logarithmic coordinates similar to the Moody diagram (Figure 8.12 in the textbook). On the plot, there should be (i) three lines based on the Colebrook equation — one line of constant $e/D$ for each of the three pipes, (ii) nine data points — one data point for each pipe at each flow rate. Data should be plotted using three different markers: ○, □, ◇, where each marker represents a different pipe. A legend should be included that indicates the pipe (for the case of the data markers) and the $e/D$ values (for the case of the Colebrook lines).
   (c) A table of the relative roughnesses ($e/D$) and minor loss coefficients associated with the couplers ($K$, $L_e/D$) for the three different pipes. The table should have the following columns: pipe, $e/D$, $K$, $L_e/D$.

2. *Attachment 2: Matlab Code(s).* Attach all of your Matlab codes used to analyze the data and generate the required plots.

3. *Summary.* In this section, provide a short summary (about 3–5 sentences) of the laboratory exercise that includes the purpose of the lab, the main results obtained, and the conclusion. The conclusion, in this case, should involve a statement of how well the data agreed with the Moody diagram. The Summary is generally the last section to be written, but appears as the first section in the completed Memo.

4. *Results.* Write one short descriptive sentence for each table/figure in Attachment 1. The table/figure number must be stated in the sentence. For example, "Figure 1 shows a plot of the friction factor versus the Reynolds number for the three pipes tested".

5. *Discussion.* Write 1–2 paragraphs that comment on the following.
   - State whether the flow is laminar or turbulent based on your measured Reynolds numbers. Discuss how this affects the measured pressure drop.
   - State the percent contribution of the major and minor losses to the total pressure drop ($\Delta P/\rho$) for the 6 experiments. Comment on whether there are noticeable trends. For example, does the major loss tend to contribute more to friction than the minor loss as the Reynolds number increases? What about as the pipe diameter or roughness length increases?
   - State the calculated roughness lengths based on your data analysis, and compare with those values listed in the textbook for similar pipe materials.

- State the calculated minor loss coefficients for the unions based on your data analysis, and compare to the standard values listed in the textbook.
- State the power (in units of Watts) required to drive the flow through the three different pipes at the highest measured flow rate. Discuss which pipe requires the most power and explain why.

**Plotting the Moody Diagram in MATLAB:**

The following code may be used to plot the Colebrook equation, for purposes of comparing with your own data. Recall, the Colebrook equation is a curve fit to the data shown in the Moody diagram.

1. First create a MATLAB function by placing the following lines of code into a file called "Colebrook_Fun.m":

```
function z = Colebrook_Fun(f,Re,eD)
z = 1/sqrt(f) + 2*log10(eD/3.7 + 2.51/(Re*sqrt(f)));
```

2. Second, create a Matlab script by typing the remaining lines of code into a separate file called "PipeLab.m". Based on the Moody diagram, select a value for $e/D$ that appears to match your data:

```
eD = [type your selected value here];
```

3. Define a vector of Reynolds numbers for use in plotting the Colebrook equation:

```
Re = [1e4:1e4:9e4,1e5:1e5:1e6]';
```

4. In order to produce the data for the Moody diagram lines, loop through the Reynolds numbers and use the "fzero" command to invert the Colebrook equation, thereby obtaining the $f$ values corresponding to each $Re$. NOTE, the syntax given for the "fzero" command requires MATLAB version 7 or higher:

```
for (k=1:length(Re))
  f(k,1) = fzero(@(f) Colebrook_Fun(f,Re(k),eD),0.02);
end
```

5. Plot the results on a log-log plot. This should look very similar to the Moody diagram in the textbook:

```
loglog(Re,f,'b-');
```

6. If your data do not fall on this line, try a different $e/D$ value and repeat the procedure. Or, instead of using a trial-and-error method, you can write an optimization routine (for extra credit) to find the best value of $e/D$ that fits your data to the Colebrook equation.