Flow Around a Circular Cylinder

Reading: Section 6.3 (pp. 241–253), Section 9.4 (pp. 428–433), Section 9.7 (pp. 445–459) from Pritchard, 8^{th} ed.

Objectives

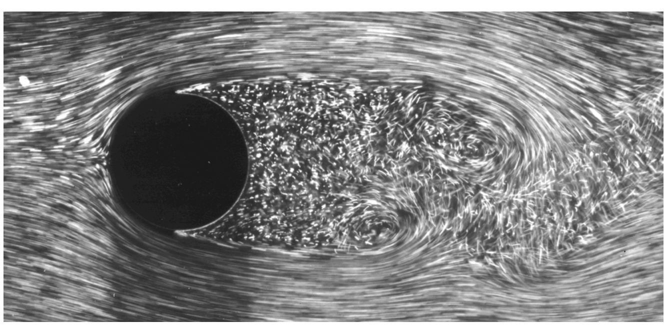
(i) measure the vortex shedding frequency from a circular cylinder as a function of Reynolds number,

(ii) calculate the drag force acting on a circular cylinder through the measurement of the wake velocity profile along with the use of the conservation of momentum equation, and

(iii) compare your experimental data to that published in the textbook.

Background

Flow fields associated with bluff (non-streamlined) bodies are of great significance in many engineering applications. Typical applications include the flow around aircraft struts, flow past cabling systems, building aerodynamics and the flow around heat transfer surfaces. The flow past a cylinder in a uniform crossflow, as shown in Figure 1, is a classical configuration that displays many of the features generic to bluff body flows. In the present lab two of the most significant of these features are explored: namely the periodic shedding of vortices from the aft of the body, and the drag force resulting from the separated wake behind the cylinder.



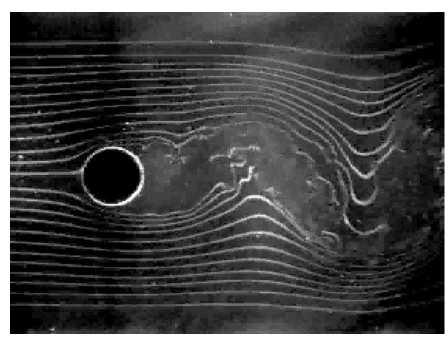


Figure 1: Example visualization images of the flow around a circular cylinder.

Vortex Shedding

The shedding phenomenon is characterized by the non-dimensional frequency or Strouhal number ($St = n\,D/U_\infty$, where n is the shedding frequency, D is the diameter, and U_∞ is the approach velocity) as a function of Reynolds number ($Re = D\,U_\infty/\nu$, where ν is the kinematic viscosity). Figure 2 shows the expected behavior (based on published data).

Note, the line thickness in the figure indicates the uncertainty in the data. It is clear that for $300 < Re < 10000$, the Strouhal number remains nearly constant.

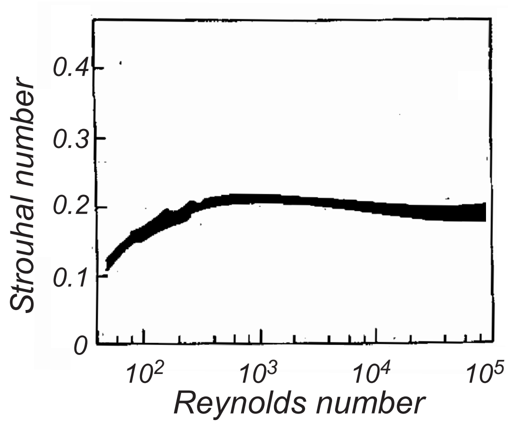


Figure 2: Strouhal number versus Reynolds number, based on the experimental work of Lienhard, 1966.

In this lab, a hot-wire probe will be used to measure the shedding frequency. A hot-wire probe consists of a very fine wire (in this case, 5μm diameter) that comprises one leg of a Wheatstone bridge. When the circuit is powered, the wire is heated to a constant temperature. As the air flows over the probe, the hot-wire is cooled. A fast response compensating circuit keeps the wire at its pre-set temperature by adjusting the current flow through the wire. This changing electrical signal can then be related (through a calibration procedure) to the flow speed. The fast response of the hot-wire makes it well suited for measuring turbulent and unsteady flows. In the present lab, the voltage output from the hot-wire is digitized using an analog-to-digital converter. The recorded signal captures the temporal change in the voltage output from the hot-wire system resulting from the periodic vortex shedding phenomenon. Here, we are just looking at the frequency content of the signal. We will NOT use the hot-wire to make actual velocity measurements. Note, in order to extract flow speed information from a hot-wire, rather than just the frequency content of the signal, the hot-wire MUST first be calibrated against known reference velocities.

Drag

The net drag on the cylinder is determined through an experimental application of the integral form of the conservation of momentum equation. A sketch of the problem including the control volume is shown in Figure 3. The x-component of the conservation of momentum equation is

$$-F_D + P_1 A - P_2 A = \int_{\mathrm{CS}} u \, \rho \, \vec{V} \cdot d\vec{A}, \tag{1}$$

where F_D denotes the force required to hold the cylinder in place (and is equivalent to the drag force acting on the cylinder), P_1 and P_2 are the gage pressures acting on control surfaces 1 and 2, and A is the cross sectional area of the wind tunnel. Remember that the conservation of mass equation also needs to be satisfied for this same control volume. Therefore, if we know that the velocity decreases in the region behind the cylinder, then the velocity has to increase near the walls of the wind tunnel to compensate. Of course, in the immediate vicinity of the wind tunnel walls, a boundary layer exists due to the no-slip condition at the walls. However in the present wind tunnel, these boundary layers are very small (< 0.25 inches thick). To determine the drag force F_D in (1), the wake profile $u(y)$ must be measured and integrated. This will be achieved in the present lab using a Pitot-static probe and pressure transducer at an x location between $7\,D$ and $10\,D$ downstream of the cylinder. This downstream distance is chosen because Pitot-static probe data cannot be trusted in the wake region directly behind the cylinder, where the flow recirculates. The wake profile $u(y)$ is obtained by traversing the Pitot-static probe in the $\pm y$ directions and using Bernoulli's equation to convert dynamic pressure to velocity.

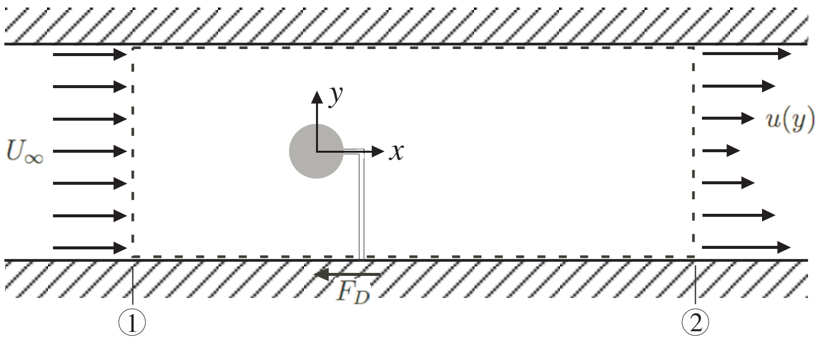


Figure 3: Control volume (dashed line) for the integral conservation of momentum analysis.

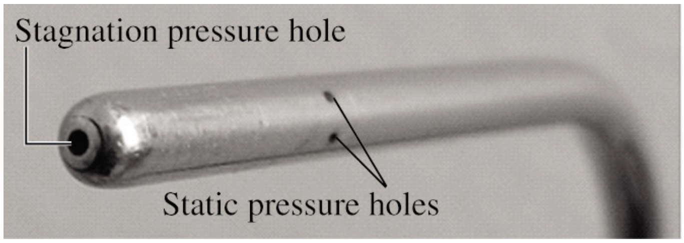
<u>Measuring Velocity using a Pitot-Static Probe</u>

Figure 4 shows a photograph of the tip of a Pitot-static probe and a schematic of how the probe can be connected to a manometer in order to measure the dynamic pressure. The dynamic pressure reading may then be used to calculate the velocity of the flow using Bernoulli's equation,

$$P_s + \frac{\rho\, u^2}{2} = P_t, \tag{2}$$

where P_s is the *static* (thermodynamic) pressure and P_t is the *total* (stagnation) pressure at the tip of the probe, ρ denotes the density of the air, and u is the velocity of the flow. The quantity $1/2\,\rho\, u^2$ represents the *dynamic* pressure. Bernoulli's equation, as written in (2) assumes that P_s and P_t are measured along an inviscid, incompressible, steady streamline. By aligning the Pitot-static probe along such a streamline, one can measure the pressure difference $(P_t - P_s)$ with a simple manometer, and therefore obtain the velocity at the tip of the probe.

Operationally, this means that the Pitot-static probe must be aligned such that the velocity vector points into the tip of the probe. Physically, this alignment causes a streamline to stop or stagnate at the tip, so that there is no actual mass flow rate through the Pitot-static probe. For this reason, the total pressure P_t is often called the stagnation pressure. On the other hand, since the static pressure represents the pressure of a static flow, it is measured perpendicular to the streamline. Therefore, the static pressure is measured through small holes drilled around the periphery of the Pitot-static probe. The difference between the total



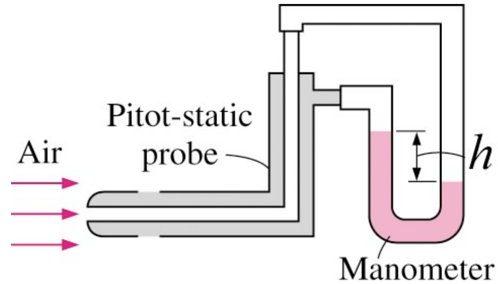


Figure 4: Photograph of a Pitot-static probe (left) and schematic of a Pitot-static probe connected to a manometer (right).

and static pressure can be measured with a liquid-filled manometer or a pressure transducer. In this lab, we will utilize a digital pressure transducer connected to a data acquisition system.

One can show (left as an exercise for the student) that the above equation can be written for the velocity in the following form

$$u = C\sqrt{\frac{h\,(T + 273.15)}{H_{\text{baro}}}}, \tag{3}$$

where

$C = 4.7538$ [$\mathrm{m \cdot s^{-1} \cdot in^{1/2} \cdot (mm\ K)^{-1/2}}$],
u is the velocity [m/s],
h is the dynamic pressure [mm of mercury],
T is the air temperature [°C], and
H_{baro} is the barometric pressure [inches of mercury]

Importantly, equation (3) is a *dimensional* equation. Therefore, you MUST make sure that the values are expressed in the correct UNITS as shown above. The wake profile data are then integrated numerically according to (1). Previous experimental work has determined that the non-dimensional drag force of a cylinder follows the behavior shown in Figure 5. The non-dimensional drag force is defined as $C_D = F_D/(1/2\,\rho U_\infty^2\, D\,\ell)$, where ℓ is the length of the cylinder, and ρ denotes the density of the air.

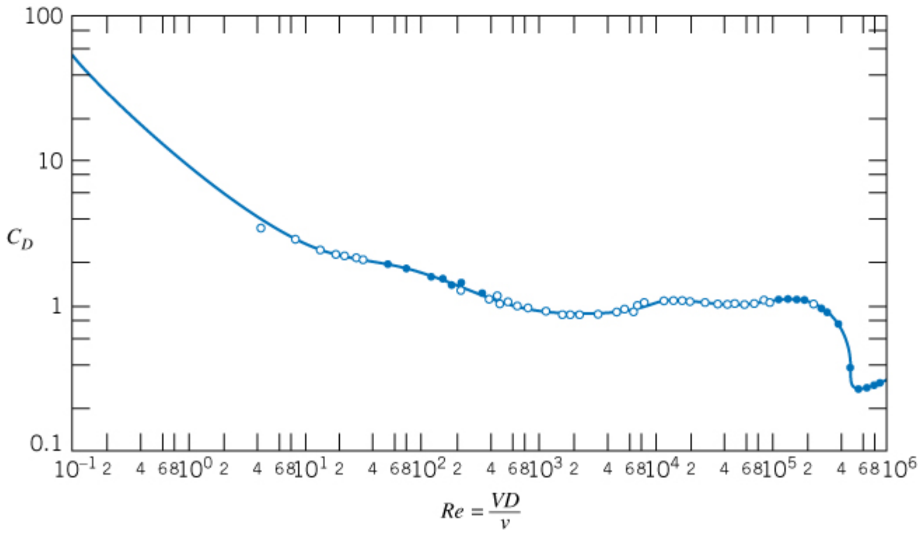


Figure 5: Drag coefficient versus Reynolds number, based on a compilation of experimental data from Schlichting, 1979.

Laboratory Tasks

Experiments are conducted in a wind tunnel with a 12 inch $\times$ 12 inch cross section. A schematic is given in Figure 6 showing the coordinate system that will be used. The circular cylinder has a diameter of $D = 0.75$ inch and is mounted horizontally across the test section of the wind tunnel. The approach flow U_∞ may be varied by controlling the power supplied to the wind tunnel fan via a variable frequency drive.

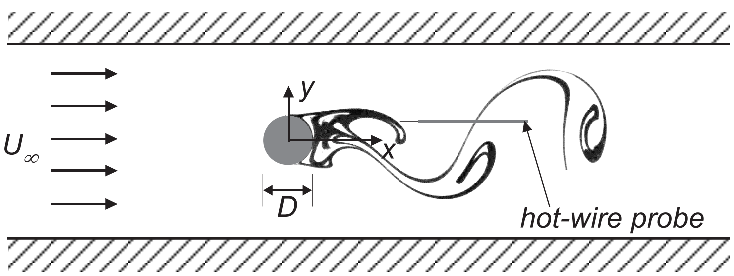


Figure 6: Schematic (side view) of the vortex shedding experiment in the wind tunnel.

The lab procedures are outlined below. Students MUST use the posted sheet to record the raw data acquired in the laboratory.

1. Determine the air density:
   (a) Measure the room temperature using the thermometer.
   (b) Measure atmospheric pressure using the mercury barometer.
   (c) Use the ideal gas law to calculate the local air density.

2. Measure the shedding frequency:

   (a) Make sure the hot-wire probe is located at about $x = 1.5\,D$, $y = 0.7\,D$.

   (b) Make sure the hot-wire probe circuit output is connected to channel ai1 on the BNC connector block. The connector block is the interface to the Data Acquisition (DAQ) system which performs the analog-to-digital conversion of the hot-wire voltage signal.

   (c) Start the wind tunnel.

   (d) Using the Lab4_HotWire.vi program on the computer, collect 5 seconds of data from the hot-wire at a rate of f_{DAQ} =1kHz. Save the file as: "Sec0##_Fan##_hotwiredata.txt", where the #'s are your lab section number and the fan frequency from the variable frequency drive, respectively.

   (e) Observe the hot-wire response on the computer monitor. The period ΔT of the oscillations indicates the vortex shedding frequency. Figure 7 gives a sample of the digitally recorded output from the hot-wire probe. The arrows indicate the locations of the troughs in the signal due to the periodicity of the vortex shedding phenomenon. The frequency is then $f = (\Delta T)^{-1}$. For the laboratory, report you can estimate ΔT by plotting the voltage signal, visually identifying the troughs, finding the number of points m between two successive troughs, and then calculating $\Delta T = m/f_{DAQ}$. To obtain a reasonable estimate of ΔT with this method, you should average over at least 10 periods. Alternatively, the troughs can be identified programmatically by identifying local minima in the signal and then calculating the number of points between successive minima.

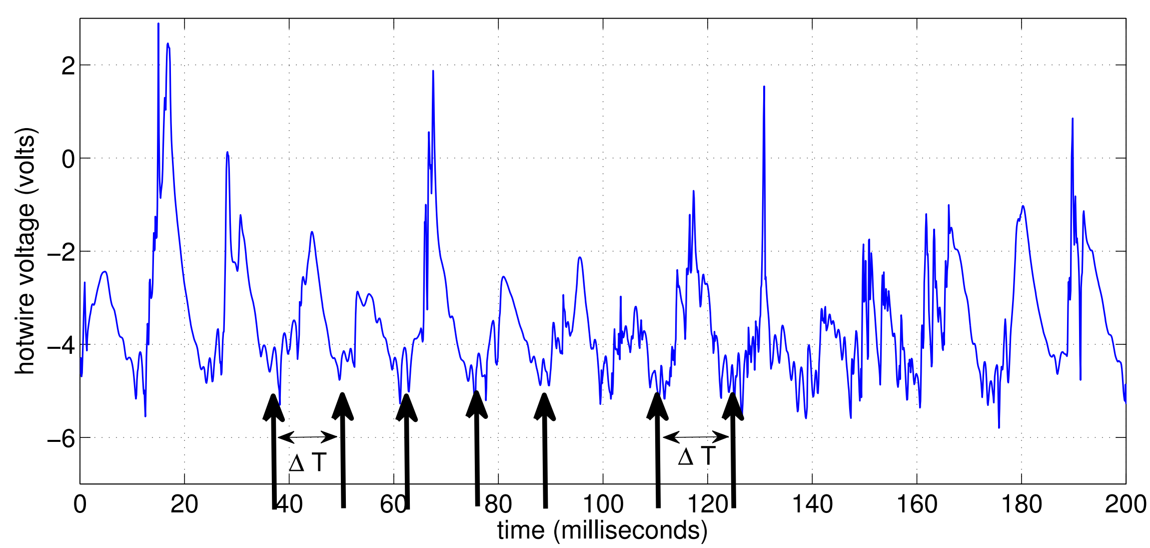


Figure 7: Sample voltage output from the hot-wire probe located downstream of the circular cylinder plotted using Matlab. In the case shown, $\Delta T \approx 12.5$ ms and $f = 80$ Hz.

   (f) Repeat for 5 different approach flow speeds by adjusting the power supplied to the wind tunnel fan.

3. Measure the freestream velocities
   (a) Move the Pitot-static tube to a location that is slightly upstream of the cylinder.
   (b) Correctly connect the tubes from the Pitot-static probe to the transducer. Make sure the transducer is connected to channel ai0 of the BNC connector block.
   (c) Measure the dynamic pressure from the Pitot-static probe with the transducer using the Lab4_transducer.vi program on the computer. Collect 30 seconds of data and save it as "Sec0##_Fan##_SheddingPd.txt", where the #'s are your lab section number and the fan frequency from the variable frequency drive, respectively.
   (d) Repeat for the same 5 different approach flows used above to determine the vortex shedding frequency by adjusting the power supplied to the wind tunnel fan.

4. Measure wake profile:
   (a) Set the wind tunnel speed so the approach flow is between U_∞ =15–20 m/s.
   (b) Determine the UNITS on the electronic display for the vertical traverse.
   (c) Move the Pitot-static probe upstream of the cylinder ($x/D \approx -7$, $y/D \approx 5$). Measure the dynamic pressure as well as the *static* pressure, P_1 at this location for 30 seconds using the transducer and Lab4_Transducer.vi. Save the files as "Sec0##_Approach_Dynamic.txt" and "Sec0##_Approach_Static.txt" respectively, where the #'s are your lab section number.
   (d) Move the Pitot-static probe downstream of the cylinder ($x/D \approx 8$, $y/D = 5$). Measure the *static* pressure, P_2 by taking a 30 second data set and saving it as "Sec0##_Wake_Static.txt".
   (e) Move the Pitot-static probe to $x/D \approx 8$, $y/D = 0$. Note the readout on the electronic display for this y position.
   (f) Determine the increments that you will move the Pitot-static probe in order to get 20–30 data points across the entire wake. Write these positions on your data sheet. Importantly, the bulk of your data points should be concentrated in the wake region, the width of which is only a few inches.
   (g) Traverse the Pitot-static tube across the wake and record the dynamic pressure at each of the points by taking a 30 second data set at each point and saving it as "Sec0##_Wake##_Dynamic.txt", where the # are your lab section number and the Pitot-static tube position number as recorded on your data sheet, respectively.

Work Due

Plots:

1. Plot of the velocity profile in the wake, i.e., u (on the abscissa) as a function of y (on the ordinate). Units on u and y should be m/s and m, respectively. Use markers, such as circles, for your data and connect the data points with a solid line.
2. Plot of the hot-wire probe voltage output versus time for one of the data sets you acquired. Indicate a pair of successive troughs using either dashed lines or other annotation features in Matlab. Be sure in the caption to state the approach velocity for this data set. Plot the data with a solid line (no markers).
3. Plot of St (on the ordinate) versus Re (on the abscissa). The Reynolds number should be plotted in logarithmic coordinates; while the Strouhal number should be plotted in linear coordinate, similar to Figure 2 of this handout. The range on St should be from 0 to 0.3. Indicate your data with markers, such as circles, and connect the data points with a solid line.
4. Plot of C_D (on the ordinate) versus Re (on the abscissa). Both C_D and Re should be plotted on logarithmic coordinates, similar to Figure 5 of this handout. Include both your data as well as the published data (download as a text file from CANVAS). Include a legend that distinguishes between your data and the data from the textbook. Your data should be plotted using markers only, such as circles; while, the textbook data should be plotted as a solid line.

Matlab Code:

Append your Matlab codes used to analyze the data and generate the required plots.

Hand Calculation:

Append a scan of your derivation of (3). This must be nearly done to receive credit. Start your derivation with a schematic of a Pitot-static tube. Draw the stagnation streamline and show how application of Bernoulli's equation leads to (2). From there, substitute in the ideal gas law to calculate air density based on measurements of the ambient temperature and barometric pressure. Assume your measurements are in °C for temperature and inches of mercury for barometric pressure. In addition, the pressure difference measured by our pressure transducer will output in mm of mercury. Show in your derivation, the appropriate unit conversion in order to obtain the velocity in m/s.

Short-Answer Questions:

1. State the percent difference in the drag coefficient C_D obtained from your data compared with the "accepted value" at the same Reynolds number. Write this as a complete sentence that includes the value of your Reynolds number.
2. Speculate on how the experiment or data analysis methods could be modified to improve accuracy. Write 3–4 sentences, including any references as necessary to support your response.

Help Constructing a MATLAB code

Write a MATLAB code to convert measured dynamic pressures to velocities using (3), read in the textbook data, numerically integrate the wake profile, and draw the required plots. IMPORTANT: You must comment your Matlab code to receive full credit. Try to write the Matlab code on your own. If you get stuck, you can consult the following outline below. Note, this is NOT a complete code. You will need to calculate some statistics on your own. For example, mean static pressure values for the wake profile component.

1. Download all the necessary data files from your section's folder from the CANVAS Lab4 Data Files, and read them into MATLAB using the load command. This includes each of the dynamic pressure files associated with the different velocities used in the shedding frequency experiment, the two static pressure files used to determine the drag force, and the many dynamic pressure files used to obtain the velocity in the wake. It is recommended that this is done inside a FOR-LOOP.
2. Define important parameters at the top of the code (use % for comments):

```
% diameter of cylinder (m)
D = 0.75*0.0254;

% width of wind tunnel (m)
w = 12*0.0254;

% barometric pressure (inches of mercury)
Hbaro = [type your value here];

% air temperature (Celsius)
T = [type your value here];

% air density - using ideal gas law (kg/m^3)
rho = [type equation here - CHECK UNITS];

% kinematic viscosity of air (m^2/s)
nu = [type your value here];

% mean dynamic pressure associated with U_infinity for wake profile
DPinfty_wake = [type your value here];

% mean static pressures measured in wake profile experiments
P1 = [type yours here];
P2 = [type yours here];

% constant for Bernoulli's equation (m*s^{-1}*in^{1/2}*(mm*K)^{-1/2})
C = 4.7538;
```

3. Read in textbook data for C_D versus Re plot (download file from WebCT):

```
% read in textbook data for CD versus Re
[Re_book,Cd_book] = textread(['CD_RE_Textbook.dat'],'%f %f');
```

4. Determine the shedding frequencies:

```
% Determine the shedding frequencies from the hotwire voltage files
% using one of the methods described in the Laboratory Tasks section
% of handout.  This will require you to either (i) make plots and
% manually count points between events or (ii) calculate the frequency
% using an automatic minima detection algorithm.

% The following outlines how to calculate the vortex shedding
% frequency by local minima detection.  Please note that the given
% section of code is very sensitive to the choices for vmin and tmin
% and that the result (DeltaT) should be spot checked against
% estimates from raw voltage plots (i.e. the manual method of plotting
% and counting points between visually detected minima).
%
% set data parameters
f=[type your value here]];    %sampling frequency (Hz)
vmin=[type your value here];  %threshold min (should be < mean value)
tmin=[type your value here];  %minimum time between events (msec)

consec=round(tmin*f/1000);  %tmin in points
V = load([type your filename here]);

%set the threshold for the signal
I=find(V < vmin);

% find the beginning and ending of troughs
% by looking for the consecutive points > tmin
II=find(I(2:length(I))-I(1:length(I)-1) > consec);

% loop through troughs and find the index of the minimum value
for i=1:length(II)-1
    [temp,minindex(i)]=min(V(I(II(i)):I(II(i+1))));
    minindex(i)=minindex(i)+I(II(i));
end

% calculate DeltaT by finding how many points are between identified
% minima and multipling by the physical time between two successive
% measurements
DeltaT=mean(minindex(2:length(minindex))- ...
minindex(1:length(minindex)-1))/f*1000;
```

5. Height of control volume will just span the total distance traversed by the Pitot-static probe (NOT the height of the wind tunnel)

```
% define the wake profile positions data was collected at
y_wake = [fill in the values from lab];
% height of control volume
H = abs(y_wake(length(y_wake)) - y_wake(1));
```

6. Convert pressures to velocities:

```
% convert dynamic pressure to velocities in m/s
v_vortex = C*sqrt(Pd_vortex*(T+273.15)/Hbaro);
v_wake = C*sqrt(Pd_wake*(T+273.15)/Hbaro);

% velocity of approach flow in wake profile experiment
vinfty = [you type it in];
```

7. Reynolds number:

```
% Reynolds number
Re_vortex = v_vortex*D/nu ;
Re_wake = vinfty*D/nu ;
```

8. Nondimensional vortex shedding frequencies

```
% nondimensional vortex shedding frequencies
f_vortex = 1./Dt_vortex;
St = f_vortex*D./v_vortex;
```

9. Mass flux through control surfaces. Numerically integrate data in wake profile using trapezoidal integration

```
% mass flux through control surfaces
MassFlux1 = rho*vinfty*w*H;
MassFlux2 = trapz(y_wake,v_wake)*rho*w;
```

10. Momentum flux through control surfaces

```
% momentum flux through surface 1 and 2
MomFlux1 = [you type here];
MomFlux2 = trapz(y_wake,v_wake.^2)*rho*w;
```

11. Supply any other necessary calculations here, including drag force calculation, drag coefficient, etc.

12. Plot out wake profile:

```
% plot out dimensional wake profile
figure;plot([type it here]);
xlabel('[type it here]');
ylabel('[type it here]');
```

13. Semilogarithmic plot of St versus Re:

```
% plot out St versus Re
figure;semilogx(Re_vortex,St);
xlabel('Re');
ylabel('St');

% set limits on axis
set(gca,'ylim',[0,0.3]);
```

14. You write in the remaining lines of code below for the remaining plots