CONVECTION FROM A FLAT PLATE WITH AN UNHEATED STARTING LENGTH

Objectives

The objective of this experiment is to determine the local heat transfer coefficient and Nusselt number for a heated flat plate with an unheated starting length. In addition, the average heat transfer coefficient and Nusselt number are to be computed from the experimental data. All data are to be compared to theoretical predictions.

Introduction

One of the fundamental problems of interest in convection heat transfer considers flow over a heated flat plate. While the geometry could not be any more basic, parallel flow over a flat plate, as shown in Figure 1, occurs in a number of engineering applications. In addition, this geometry may be a good first approximation for flow over surfaces that are slightly contoured, such as airfoils or turbine blades. In this type of external flow, the boundary layers develop freely, without any constraints imposed by adjacent surfaces. Consequently, there will always be a region of the flow outside the boundary layer in which velocity and temperature gradients are negligible.

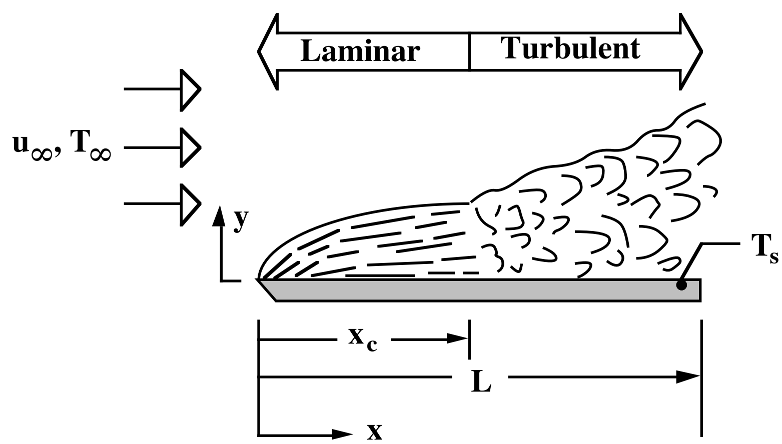


Figure 1. Parallel Flow over a Flat Plate

By nondimensionalizing the boundary layer conservation equations, it can be shown that the local and average convection coefficients may be correlated by equations of the form

$$Nu_x = f_1\left(x^*, Re_x, Pr\right) \tag{1}$$

$$\overline{Nu}_x = f_2\left(Re_x, Pr\right) \tag{2}$$

where Nu_x = local Nusselt number
Re_x = local Reynolds number
Pr = Prandtl number

x^* = dimensionless axial location, (x/L)

Definitions for the local Nusselt number and Reynolds number are

$$Nu_x = \frac{h_x x}{k_f} \tag{3}$$

$$Re_x = \frac{u_\infty x}{\nu} \tag{4}$$

where h_x = local heat transfer coefficient
k_f = thermal conductivity of the fluid
u_∞ = free stream velocity
ν = kinematic viscosity

Note that the properties of the fluid are based on the film temperature and that the subscript x has been appended to the Nusselt number to emphasize that the interest here is in local conditions on the surface. The overbar indicates an average value of the Nusselt number from $x^* = 0$ to the point of interest. One of the primary objectives of convection heat transfer is to find the functions f_1 and f_2. Either theoretical or experimental approaches can be used for this purpose.

Two boundary conditions frequently used in heat transfer analysis are 1) uniform surface temperature (isothermal) and 2) uniform surface heat flux (isoflux). For a uniform heat flux boundary condition on a flat plate exposed to forced convection flow, integral methods [1] may be used to determine the local Nusselt number for laminar flow. The correlation that results (for the case with a fully heated flat plate, i.e., no unheated starting length) is

$$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}, \qquad Pr \geq 0.6 \tag{5}$$

In a similar fashion, it can be shown that for turbulent flow

$$Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3}, \qquad 0.6 \leq Pr \leq 60 \tag{6}$$

When compared to the results for a uniform surface temperature, these solutions give values of Nu_x that are 36% and 4% larger for laminar and turbulent flow, respectively.

Although relatively simple, a convective heat transfer process for parallel flow over a flat surface is a fairly common engineering occurrence. On some occasions, such as in electronic chip cooling, the heat source is located some distance from the leading edge, resulting in thermal boundary layer growth lagging that of the hydrodynamic boundary layer. In this case, the surface temperature of the unheated section equals that of the fluid ($T_s = T_\infty$). As shown in Fig. 2, the velocity boundary layer begins to develop at the leading edge ($x = 0$), while the thermal boundary layer development starts at $x = \xi$. An integral boundary layer solution can be used [1] to develop an expression for the local Nusselt number that accounts for the unheated length. The equation for *laminar* flow is

$$Nu_x = \frac{Nu_x|_{\text{fully heated}}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} = \frac{0.453\,Re_x^{1/2}\,Pr^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} \tag{7}$$

where ξ denotes the length of the unheated region as illustrated in Figure 2. Similarly, the unheated starting length effect on heat transfer for a flat plate experiencing *turbulent* flow can be expressed as

$$Nu_x = \frac{Nu_x|_{\text{fully heated}}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}} = \frac{0.0308\,Re_x^{4/5}\,Pr^{1/3}}{\left[1-(\xi/x)^{9/10}\right]^{1/9}} \tag{8}$$

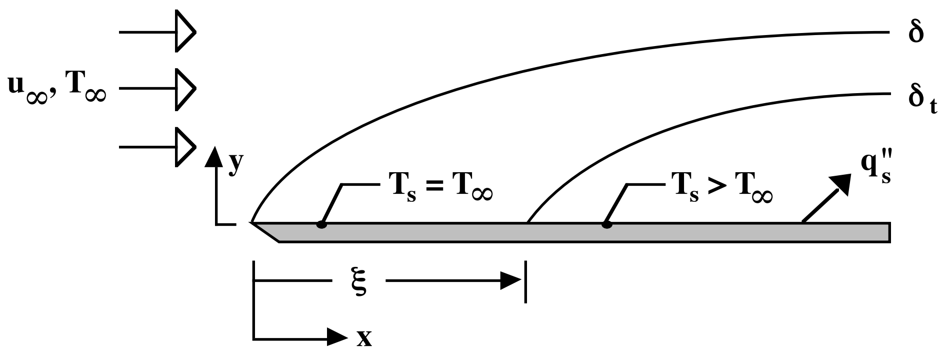


Figure 2. Flat Plate in Parallel Flow with an Unheated Starting Length

The average heat transfer coefficient $\bar{h}_L$ for the heated portion of the plate is given by

$$\bar{h}_L = \frac{1}{L-\xi}\int_{\xi}^{L} h(x)dx \tag{9}$$

The average Nusselt number for the heated portion of the plate for both laminar and turbulent flow and for isothermal and isoflux boundary conditions may also be determined analytically [3]. For laminar flow with an isoflux boundary condition, the result is

$$\overline{Nu}_L = 2\left(Nu_L|_{\text{fully heated}}\right)\left[1-(\xi/L)^{3/4}\right]^{2/3} = 2\left(0.453\,Re_L^{1/2}\,Pr^{1/3}\right)\left[1-(\xi/L)^{3/4}\right]^{2/3} \tag{10}$$

The bracketed term along with its exponent in eq. (10) represent the unheated starting length effect on $\overline{Nu}_L$. In addition, the coefficient 2 in eq. (10) is associated with the ratio of the average heat transfer coefficient to the local value for the fully heated plate. Finally, recall that L represents the distance from the leading edge of the plate to the end of the heated section, i.e., L=77+153 mm (based on the schematic given in Figure 3).

As always, the local plate temperature $T_s(x)$, heat transfer coefficient, and heat flux q_s'' are related through Newton's Law of Cooling

$$q_s'' = h_x\left[T_s(x) - T_\infty\right] \tag{11}$$

Experimental Apparatus

A small heated flat plate has been designed and constructed to provide experimental conditions that closely mimic those assumed in the theoretical solutions. The plate is installed in the "instructional" wind tunnel (the test section has a 30.48 cm x 30.48 cm cross section) so that controlled airflow conditions can be produced. In order to eliminate difficulties associated with insulating one of the surfaces (so that heat is transferred from only one surface), the flat plate has been designed to produce symmetric thermal boundary conditions (i.e., heat transfer occurs from both the top and bottom surfaces).

Each flat plate (there are two identical surfaces, on the top and bottom of the plate apparatus) consists of a smooth rectangular surface, 297 mm long by 144 mm wide. The heated section (153 mm x 68 mm) is centered in the smooth surface and is surrounded by Teflon insulating material. Heat is provided by six (6) flat electric-resistance strip heaters, each with dimensions of 76.2 mm by 25.4 mm, that are mounted side-by-side so that the air flows across the heater width. The heater arrangement provides a nearly isoflux boundary condition on the plate surfaces. The strip heaters have been wired in parallel and power is supplied to the heaters through an AC power supply. The total power to the heaters is determined from measurements of the total voltage V supplied to the heaters and the resistance of the heater circuit R.

$$q_s = \frac{V^2}{R} \tag{12}$$

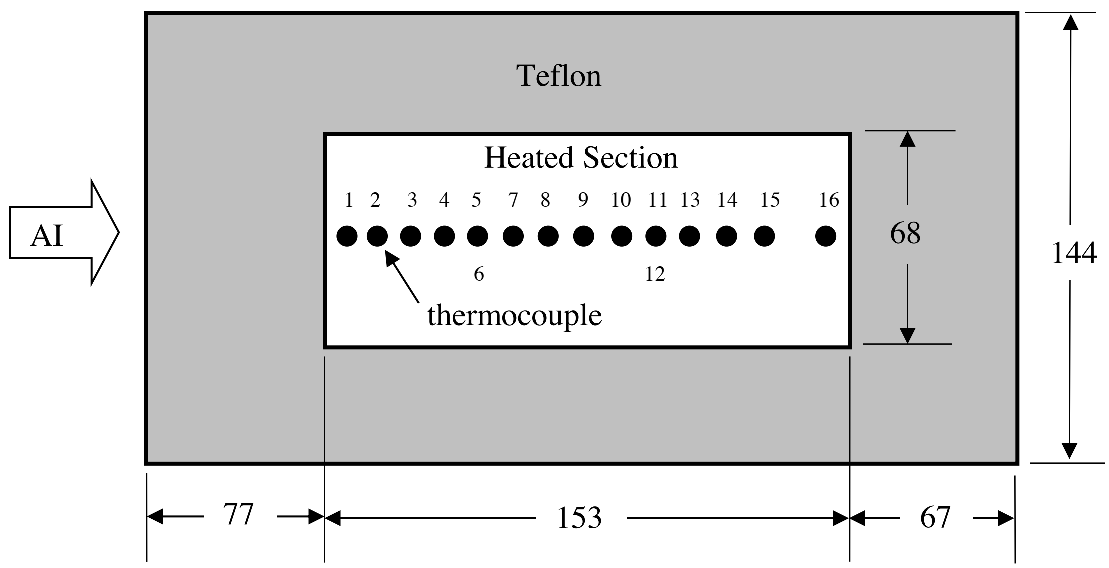
The heat flux is computed by noting that the heat is dissipated from two heated plate surfaces.

$$q_s'' = \frac{q_s}{2A_{heated}} \tag{13}$$

The heated surface is made of a composite material with the layers oriented in a vertical direction such that the thermal conductivity in the vertical direction is approximately 10 times that in the horizontal direction. This design feature reduces axial conduction along the heated surface. Teflon insulating material surrounds the plate/heater assembly. Only the heated plate surfaces are exposed to the airflow and the Teflon material minimizes heat loss in the horizontal direction to the surroundings. An aerodynamic leading edge is attached to the front of the heated plate and it is designed to prevent boundary layer separation such that a classical hydrodynamic boundary layer will be established.

Thermocouples are located at various spots beneath the two plate surfaces. The thermocouple beads are inserted into small holes drilled from the plate backside to a depth nearly equal to the plate thickness. A high thermal conductivity epoxy secures the thermocouples within the holes.

Thermocouples are connected to a LabVIEW system for readout. A dimensioned drawing of the flat plate apparatus, including the thermocouple locations, is shown in Figure 3. Thermocouple locations are referenced from the leading edge. Thermocouples 6 and 12 are located on the bottom plate surface and are at the same distance from the leading edge as thermocouples 5 and 11, respectively. All dimensions in millimeters. The flat plate has a total thickness of 13.9 mm.



Thermocouple	Location (mm)	Thermocouple	Location (mm)
1	85	9	153
2	92	10	162
3	102	11	173
4	112	12	173
5	123	13	186
6	123	14	196
7	134	15	209
8	143	16	219

Figure 3. Heated Flat Plate Dimensions and Thermocouple Locations

Procedure

A LabVIEW data acquisition system for thermocouples and a pre-written program, along with a handheld multimeter, will be used to obtain required data.

1. Measure and record the local ambient pressure using the barometer in the lab. Don't forget to correct the pressure value for elevation as indicated in the note adjacent to the barometer. Also record the room temperature. Ambient pressure and local film temperature will be used with the ideal gas equation of state to determine local values of density.

2. With the Variac variable transformer power supply switched "off," measure and record the resistance of the parallel heater circuit using the digital multimeter.
   a) Unplug the white power cable plug from the power outlet of the Variac.
   b) Connect the multimeter to the BNC connector on the top of the silver junction box. The resistance reading should be around 150 Ω. If the resistance is greater than approximately 200 Ω, one or more of the heaters is disconnected.

3. The TA will assign a voltage value that you will apply to the resistive heaters. The value will be approximately that shown on the Variac dial. A more accurate measure of the voltage should be made using the multimeter as follows:
   a) First make sure the toggle switch on the Variac is in the "off" position.
   b) Plug the white plug power cable into the outlet on the Variac.
   c) Disconnect the resistive heaters from the junction box by pulling apart the four black connectors on the wires between the resistive heaters and the junction box.
   d) Set the Variac power level to the value assigned by your TA. Make sure a cable still connects the junction box to the multimeter.
   e) Switch "on" the Variac power supply using the toggle switch on the Variac.
   f) Switch the multimeter to voltage measurements.
   g) Measure and record the voltage indicated by the multimeter.

4. Now you will reconnect the circuits for powering the heaters:
   a) Turn "off" the Variac power switch.
   b) Disconnect the coaxial cable from the junction box.
   c) Reconnect the power supply to the resistive heaters using the four black connector pairs.
   d) Connect heater cables of the same color together, black to black and white to white.
   e) Turn off the multimeter.

5. On the PC, load the LabView program (WindTunnelConvectionME3650-2013.vi) created for this exercise and select the run button in the upper left corner of the LabVIEW workspace.

6. Turn the Variac "on" using the toggle switch. View the thermocouple measurements on the LabVIEW screen as the plate warms up. If all thermocouples indicate an increasing temperature with time, then the heaters are working properly. Do not allow plate temperatures to exceed 40°C at this time.

7. Start the wind tunnel (procedure given below) to supply airflow over the heated flat plate.

Laminar flow over the plate is the desired condition for the experiment and this condition should be present at free stream velocities less than approximately 15 m/s. The flow speed should be selected such that transition will not occur on the plate. The plate should not be allowed to reach a maximum local temperature of more than 65 to 70°C. Temperatures greater than that will likely damage the components. Maximum temperatures will be found at the last six thermocouple locations. The following airspeed to voltage correlation should be followed to prevent overheating the plate. Note, for freestream velocities greater than 15 m/s, the boundary layer flow along the flat plate will be turbulent.

Free Stream Velocity (m/s)	Fan Frequency (Hz)	Voltage (V)
5 – 10	9 – 16.2	30 – 40
10 – 15	16.2 – 23.3	40 – 50
> 15	> 23.3	50 – 60

The fan frequency ν (Hz) and free stream velocity u_∞ (m/s) can be correlated according to the following equation.

$$u_\infty = 0.704\nu - 1.373 \tag{14}$$

8. It should take 10-15 minutes for the plate to reach a steady state condition. Verify the steady state condition by examining the thermocouple readings, which should not change by more than ± 0.2°C over a specified time interval.

9. At steady state, record the following:
   a) all plate temperatures (by printing the screen view or requesting data to be saved to a file),
   b) air speed (or fan frequency),
   c) room temperature.

10. If more than one data set is needed, it is easiest to start with a lower voltage and flow velocity and then increase the settings to the next data point.

11. Once the experiment is complete, lower the voltage setting on the Variac and turn the switch to off. <u>Continue to run the wind tunnel for 5 to 10 minutes to allow the airflow to cool the plate</u>.

Operating the Wind Tunnel

1. Turn the black handle on the power supply box (at the far right) to the ON position.

2. If the "PANEL CONTROL" indicator light on the control panel is not lit, press "CTRL" until it illuminates.

3. Using the up and down arrow on the keypad, enter the desired fan frequency (Hz), then press "RUN."

4. Simply push STOP to stop the fan.

5. While running, the speed may be changed by changing the frequency using the up/down arrows located on the display.

Data Reduction

1. Use the experimental data along with eqs. (3), (11), (12), and (13) to compute the heat flux q''_{exp}, as well as local values of the heat transfer coefficient $h_{x,exp}$, and Nusselt number $Nu_{x,exp}$. All air properties should be evaluated at the local film temperature, $T_f(x)$, defined as the average temperature between the surface and freestream flow,

$$T_f(x) = \frac{T_s(x) + T_\infty}{2}$$

2. Use the theoretical model for laminar convection with an isoflux boundary condition to compute the predicted local values of $Nu_{x,th}$, $h_{x,th}$, and $T_{s,th}(x)$. Since the flow is expected to be laminar, use eqs. (5) and (7) to determine $Nu_{x,th}$. The convection coefficient, $h_{x,th}$, is then calculated using $Nu_{x,th}$ and eq. (3). The predicted surface temperature, $T_{s,th}(x)$, is calculated from eq. (11) using q''_{exp}, T_∞, and $h_{x,th}$. All properties should be based on local film temperature.

3. Compute experimental and theoretical values of both the average heat transfer coefficient and average Nusselt number for the heated plate. The experimental heat transfer coefficient $\bar{h}_{L,exp}$ is determined from eq. (9) using a numerical integration scheme, such as the trapezoidal rule or Simpson's 1/3 rule. $\overline{Nu}_{L,exp}$ is then be calculated from eq. (3). The theoretical average Nusselt number $\overline{Nu}_{L,th}$ is computed from eqs. (5) and (10) and the theoretical average heat transfer coefficient $\bar{h}_{L,th}$ is then computed from eq. (3).

4. Estimate the heat loss from the plate due to radiation to the surroundings,

$$q''_{\text{rad}} = \epsilon\,\sigma\left(T_{s,x}^4 - T_\infty^4\right)$$

   where ϵ is the emissivity of the plate and $\sigma=5.6703\times10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant. Note, the heated section of the flat plate is comprised of a composite material. The exact emissivity of this material is not known, but most carbon fiber composite materials have a relatively high emissivity. Therefore, we will take an approximate value of 0.7 for the emissivity here.

Required Plots and Tables

1-3. Create one plot each of Nu, h, and T_s versus x. In each plot, compare the experimentally determined parameters, $Nu_{x,exp}$, $h_{x,exp}$, and $T_{s,exp}(x)$, with the theoretical predictions, $Nu_{x,th}$, $h_{x,th}$, and $T_{s,th}(x)$. Use open circles for the experimental values and a solid line for the theoretical values. Include appropriate axis labels with units and a legend for each plot.

4. Create a table as shown below with the values from your calculations for the average heat transfer coefficient and average Nusslet number.

	$\overline{Nu}_L$	$\overline{h}_L$
Experiment		
Theory		

Discussion Items

1. State the percent difference between the theoretical and experimental calculations of Nu, h, and T_s as shown in your plots. Note any regions (in terms of x) where the agreement between theory and experiment are more/less favorable. Provide a plausible explanation for any discrepancies observed.

2. State the percent difference between the experimental and theoretical values for the average heat transfer coefficient and average Nusselt number.

3. State the estimated value of heat loss due to radiation and compare this with the measured heat flux q''_{exp}. Does the radiation heat transfer help to explain any discrepancies that are observed between the experimental and theoretical data?

4. Is the boundary layer expected to be laminar over the entire heated surface? Provide a numerical justification for your answer. [Hint: calculate the Reynolds number at the end of the heated plate and compare it with the critical Reynolds number for a flat plate]

References

1. Kays, W.M., and Crawford, M.E., Convective Heat and Mass Transfer, 3rd Ed., McGraw-Hill, Inc., New York, 1993.

2. Incropera, F.P. and Dewitt, D.P., Fundamentals of Heat and Mass Transfer, 4th Ed., John Wiley & Sons, New York, 1996.

3. Ameel, T.A., Average Effects of Forced Convection Over a Flat Plate with an Unheated Starting Length, *International Communications in Heat and Mass Transfer,* Vol. 24, No. 8, pp. 1113-1120, 1997.