#### Programmazione concorrente

Laurea Magistrale in Ingegneria Informatica Università Tor Vergata

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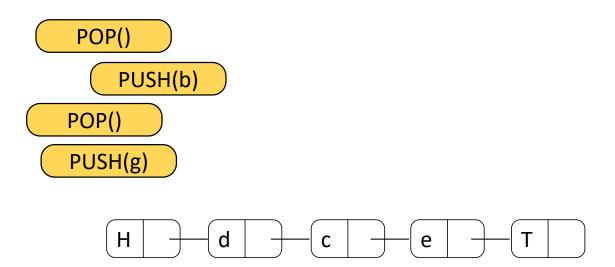
#### **Concurrent data structures**

- 1. Stack
- 2. Set
- 3. Priority queues
- 4. FIFO queues

# Concurrent Data Structures: Stacks

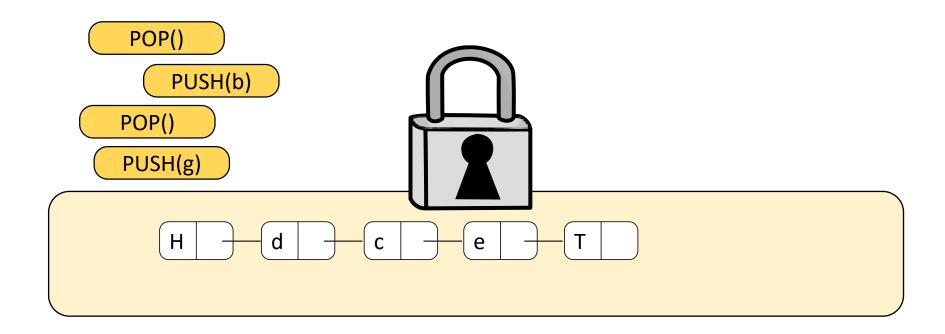
#### Stack implementation

- Stack methods:
  - push(v)
  - pop()
- Implemented as a linked list



# **Concurrent stack implementations**

Resort to a global lock



#### **Read-Modify-Write**

 RMW instructions allow to read memory and modify its content in an apparently instantaneous fashion.

```
1.RMW(MRegister *r, Function f){
2. atomic{
3. old = r;
4. *r = f(r);
5. return old;
6. }
7.}
```

 Even conventional atomic Load and Store can be seen as RMW operations

#### Compare-And-Swap

- Compare-and-Swap (CAS) is an atomic instruction used in multithreading to achieve synchronization
  - It compares the contents of a memory area with a supplied value
  - If and only if they are the same
  - The contents of the memory area are updated with the new provided value
- Atomicity guarantees that the new value is computed based on up-to-date information
- If, in the meanwhile, the value has been updated by another thread, the update fails
- This instruction has been introduced in 1970 in the IBM 370 trying to limit as much as possible the use of spinlocks

#### Compare-And-Swap

 RMW instructions allow to read memory and modify its content in an apparently instantaneous fashion.

```
1. CAS(Mregister *r, Value old_value, Value new_value f){
2. atomic{
3.  Value res = *r;
4.  if(*r == old_value) *r = new_value;
5.  return res;
6. }
7. }
```

- CAS is implemented by x86 architectures (see CMPXCHG)
- gcc offers the \_\_sync\_val\_compare\_and\_swap builtin

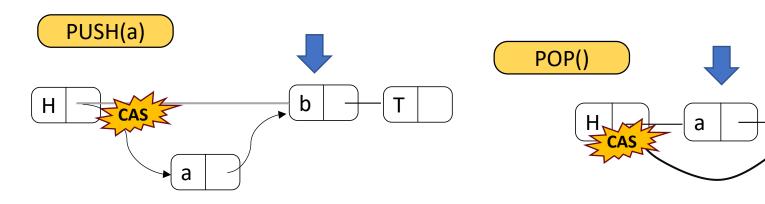
#### **Attempt 1**

#### Push:

- Get head next
- 2. Insert the new item with a CAS
- 3. If CAS fails, restart

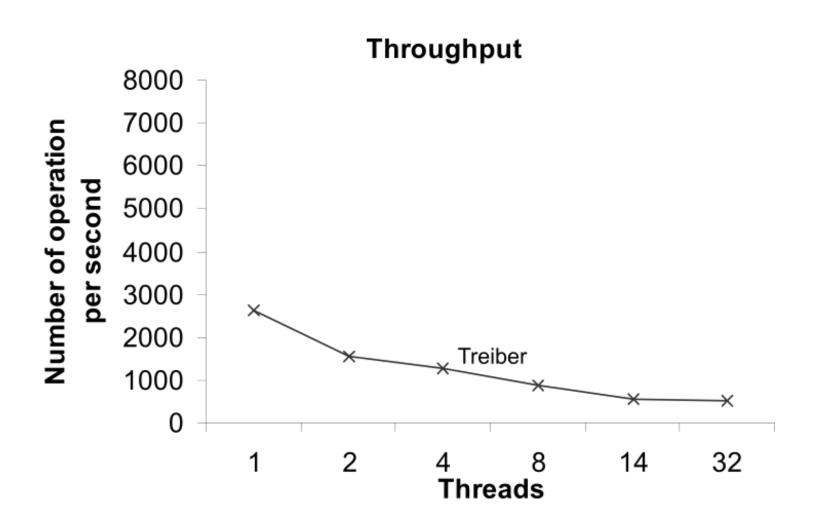
#### Delete:

- Get head next
- Disconnect the item with a CAS
- 3. If CAS fails, restart



• Is it scalable?

#### Non-blocking stack – Attempt 2 [Treiber+BO]



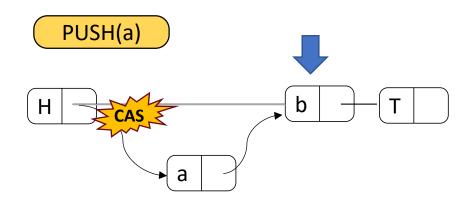
#### Non-blocking stack – Attempt 2 [Treiber+BO]

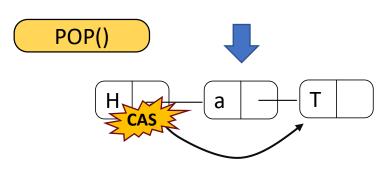
#### Push:

- Get head next
- Insert the new item with a CAS
- 3. If CAS fails, restart backoff and restart

#### Delete:

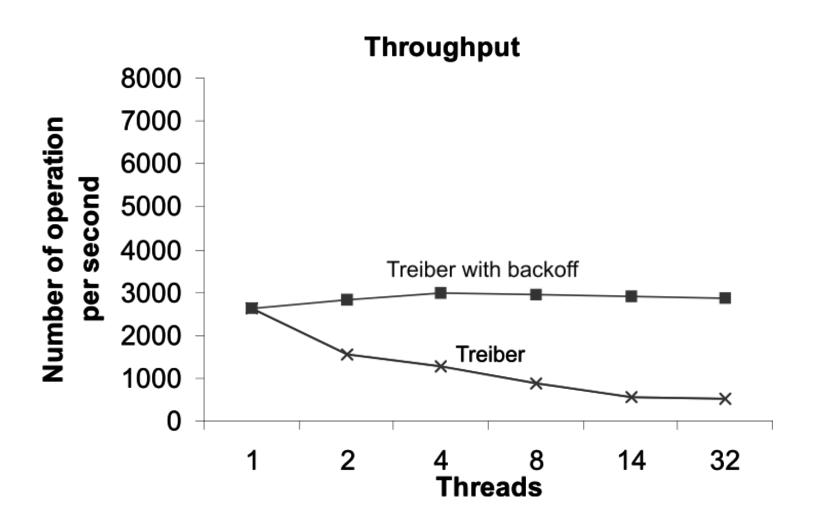
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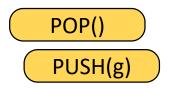
• Is it scalable?

#### Non-blocking stack – Attempt 2 [Treiber+BO]



#### Concurrent stack implementations

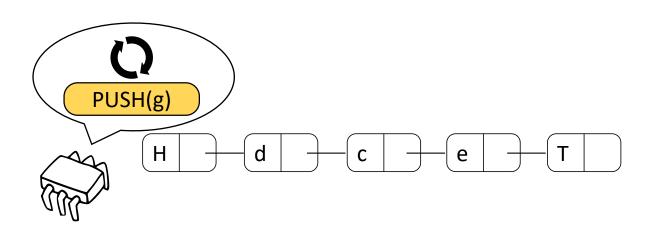
- Resort to a global lock
  - Do not scale
- Resort to a naïve non-blocking approach
  - Do not scale
- Resort to a naïve non-blocking approach + Back off
  - Do not scale, but conflict resilient
- How achieve scalability? Make back-off times useful





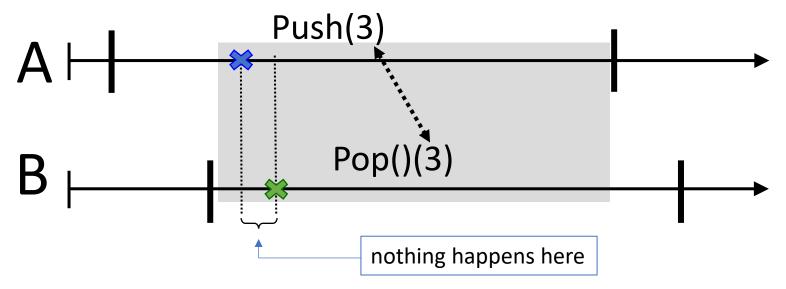
### Non-blocking stack – Attempt 3

How to take advantage of back-off times?



#### **Observation**

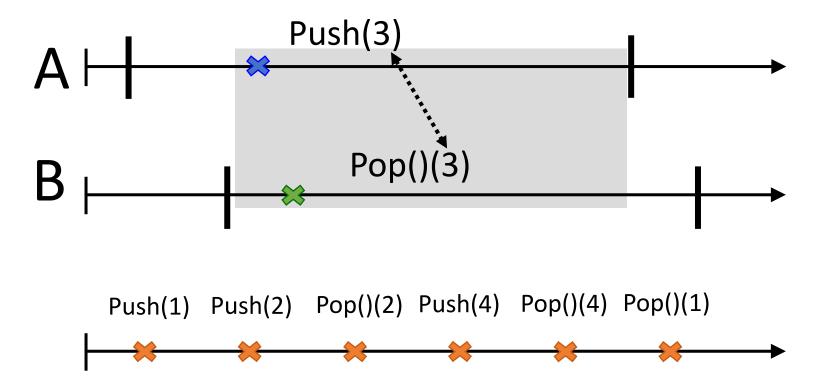
Concurrent matching push/pop pairs are always linearizable



- A push A and a pop B are:
  - concurrent to each other
  - B returns the item inserted by A
- ⇒ we can always take two points such that:
  - A is the last one to insert an item before A linearizes
  - B appears to extract the last item inserted (by A)

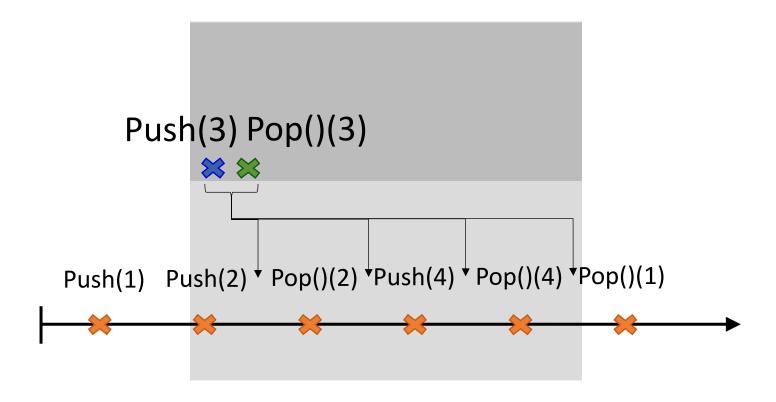
#### **Observation**

Concurrent matching push/pop pairs are always linearizable



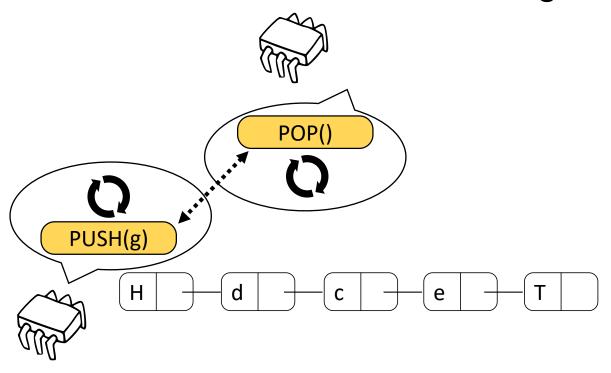
#### **Observation**

Concurrent matching push/pop pairs are always linearizable



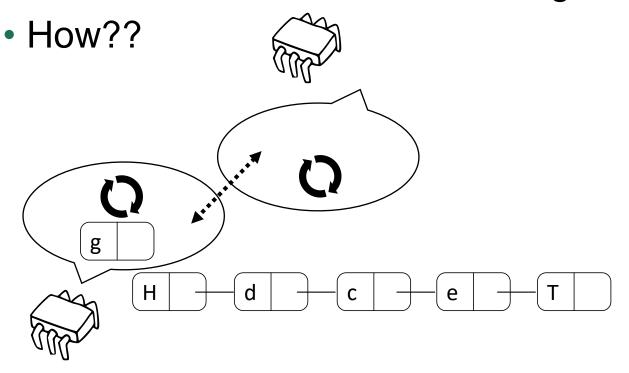
#### Non-blocking stack – Attempt 3

- How to take advantage of back-off times?
- Hope that an opposite operation arrives while waiting
- Match the two without interacting with the stack



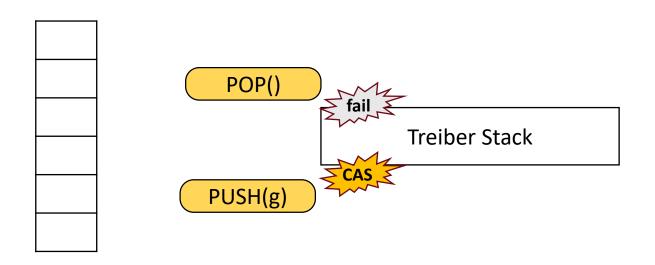
### Non-blocking stack – Attempt 3

- How to take advantage of back-off times?
- Hope that an opposite operation arrives while waiting
- Match the two without interacting with the stack



#### Non-blocking stack – Elimination stack

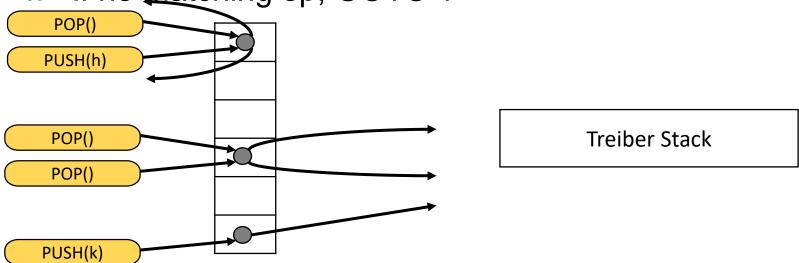
- Pair the Treiber stack with an array
- Algorithm:
  - 1. Update the original stack via CAS
  - If CAS fails, publish the operation in a random cell of the array



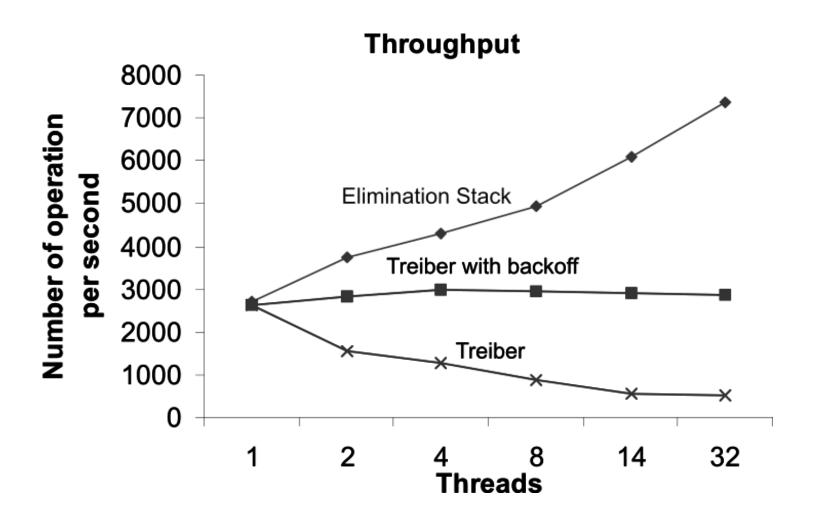
#### Non-blocking stack – Elimination stack

- Pair the Treiber stack with an array
- Algorithm:
  - 1. Update the original stack via CAS
  - If CAS fails, publish the operation in a random cell of the array
  - 3. Wait for a matching operation

4. If no matching op, GOTO 1



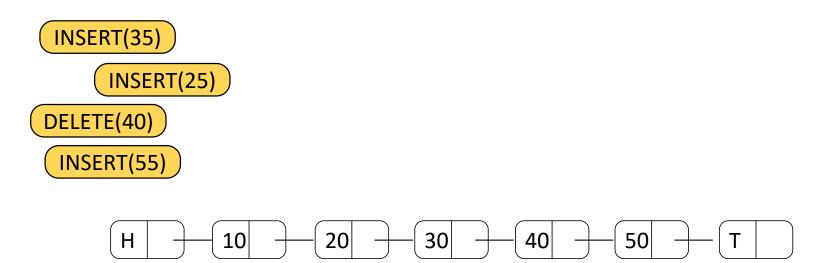
#### Non-blocking stack – Attempt 3



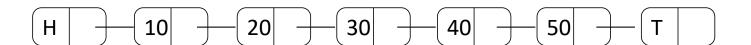
# Concurrent Data Structures: Sets

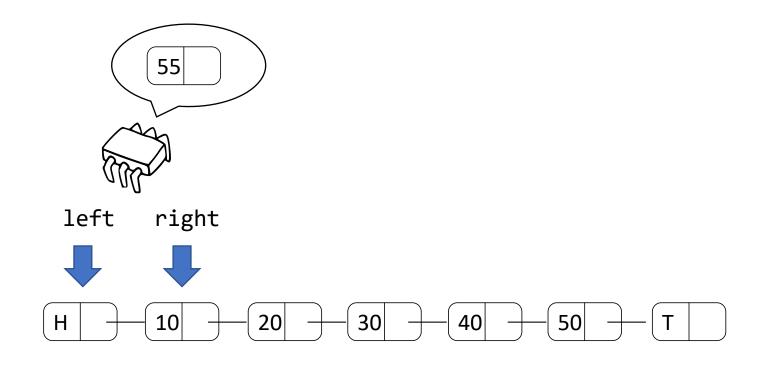
#### **Set implementations**

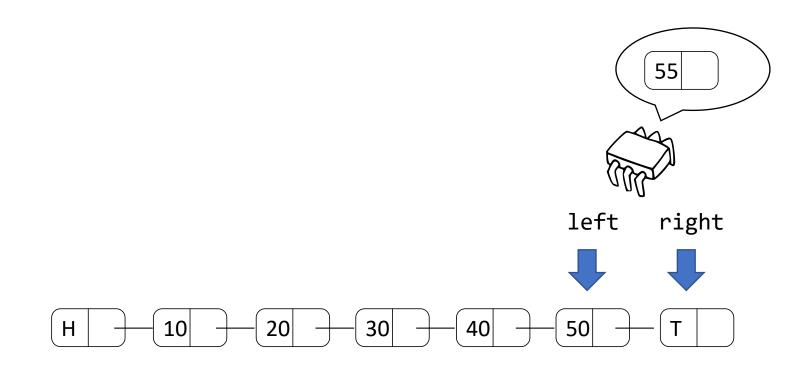
- Set methods:
  - insert(k)
  - delete(k)
  - find(k)
- Implemented as an ordered linked list

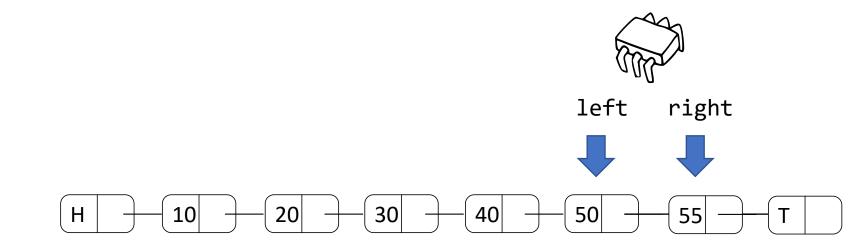


INSERT(55)



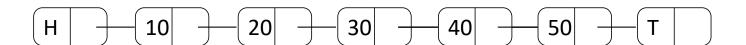




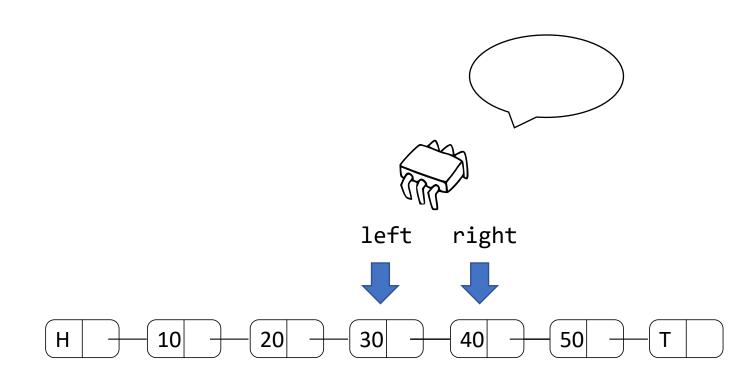


## **Delete algorithm**

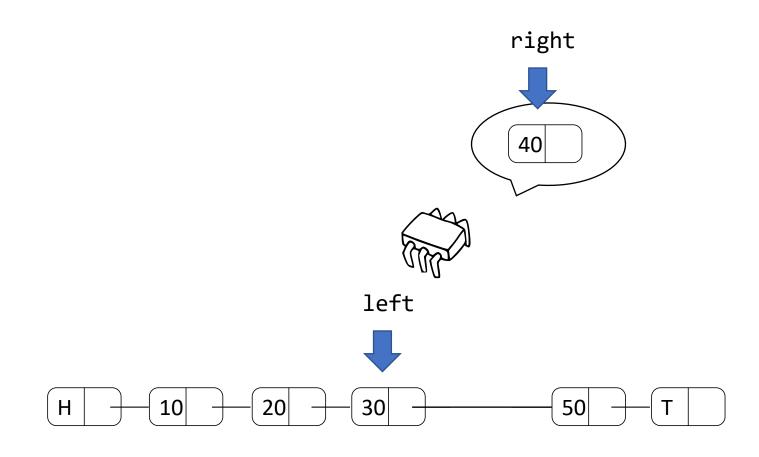
DELETE(40)



# **Delete algorithm**



# **Delete algorithm**

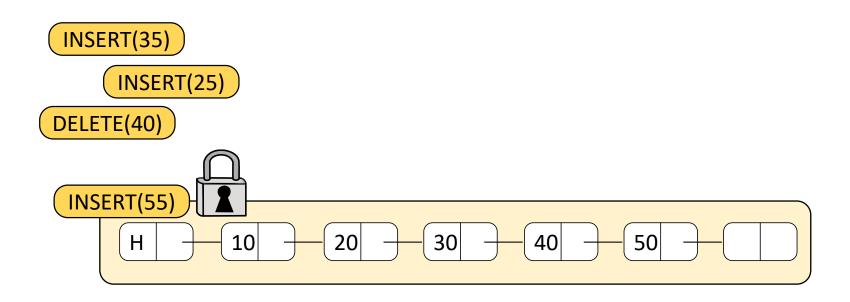


#### Sequential set implementation

```
bool do_operation(int k, int op_type){
2.
     bool res = true;
3.
     node *1,*r;
4.
5.
     1 = search(k, &r);
     switch(op_type){
6.
       case(INSERT):
7.
         if(r->key == k)
8.
           res = false;
9.
10.
       else
11.
           1->next = new node(k,r);
12.
         break;
13.
       case(DELETE):
14.
         if(r->key == k)
15.
           1-next = r-next;
16.
      else
17.
         res = false;
18.
         break;
19.
20.
21.
22.
     return res;
23.}
```

```
1. node* search(int k, node **r){
     node *1, *r_next;
2.
3. 1 = set \rightarrow head;
4.
   *r = 1->next;
5.
6.
7. r next = (*r)->next;
     while((*r)->key < k){
8.
9.
10. 1 = *r;
11.
       *r = r next;
12.
13.
      r next = (*r)->next;
14. }
15.}
```

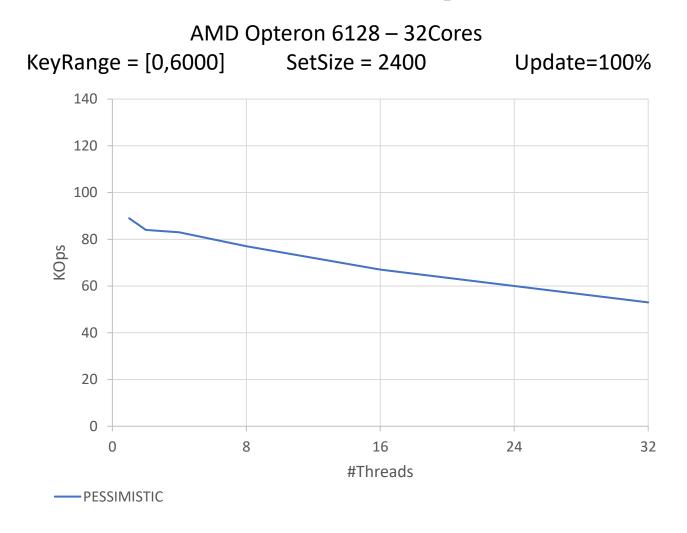
- PESSIMISTIC approach
- Synchronize via global lock

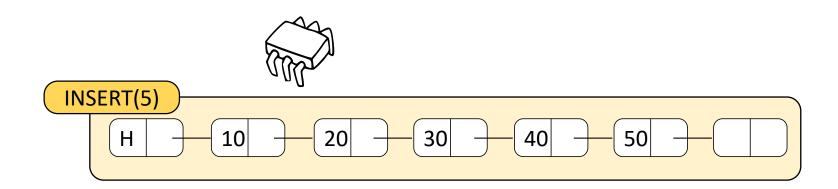


#### Concurrent set – Attempt 1 (SRC)

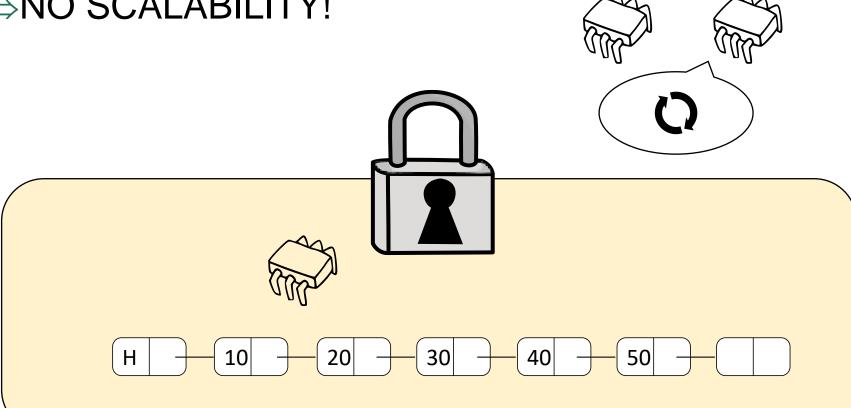
```
1. bool do_operation(int k, int op_type){
2.
     bool res = true;
3.
    node *1,*r;
    LOCK(&glock);
4.
5.
    1 = search(k, &r);
    switch(op type){
6.
      case(INSERT):
7.
        if(r->key == k)
8.
        res = false;
9.
10.
      else
11.
          1->next = new node(k,r);
12.
        break;
13.
      case(DELETE):
14.
        if(r->key == k)
      1->next = r->next;
15.
16.
     else
17.
        res = false;
18.
        break;
19.
    UNLOCK(&glock);
20.
21.
22.
     return res;
23.}
```

```
1. node* search(int k, node **r){
     node *1, *r next;
2.
3. 1 = set \rightarrow head;
4.
   *r = 1->next;
5.
6.
7. r next = (*r)-next;
     while((*r)->key < k){
8.
9.
10. l = *r;
11.
       *r = r next;
12.
13. r next = (*r) - next;
14. }
15.}
```





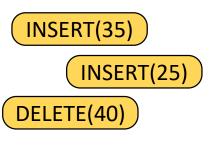
- PESSIMISTIC approach
- Synchronize via global lock
- ⇒NO SCALABILITY!

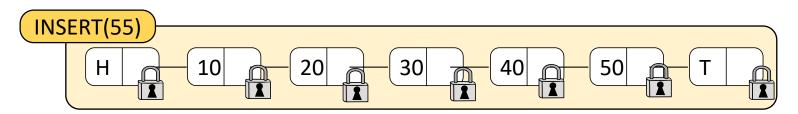


...zZz...

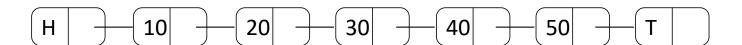
## Concurrent set – Attempt 2

- Fine-grain approach
- Each node has its own lock
- Keep two locks at a time (lock coupling):
  - One on the current node
  - One on its predecessor

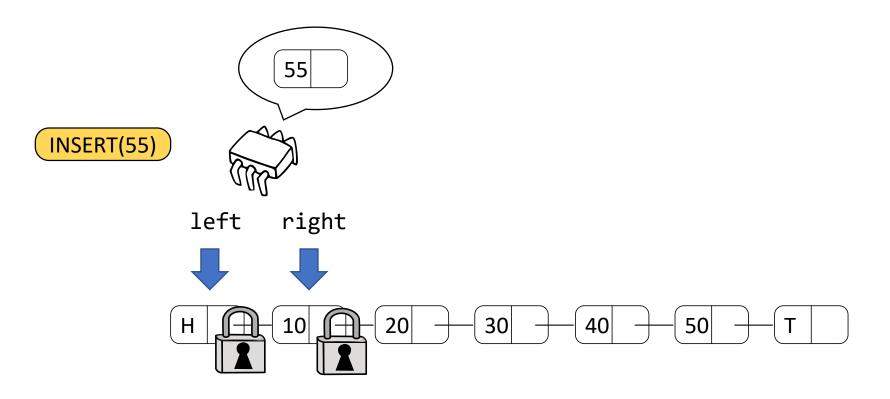




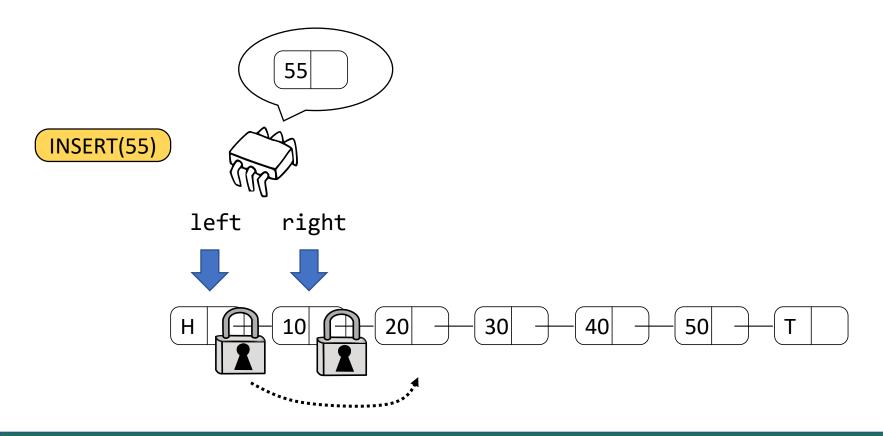
INSERT(55)



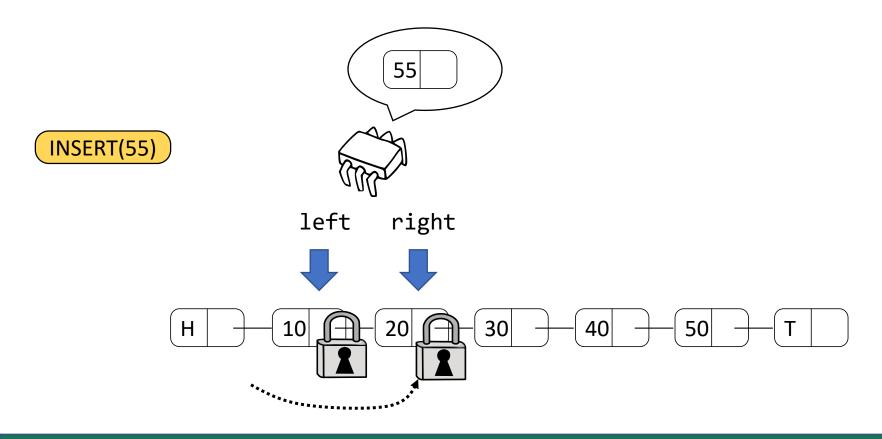
- Keep two locks at a time (lock coupling):
  - One on the current node
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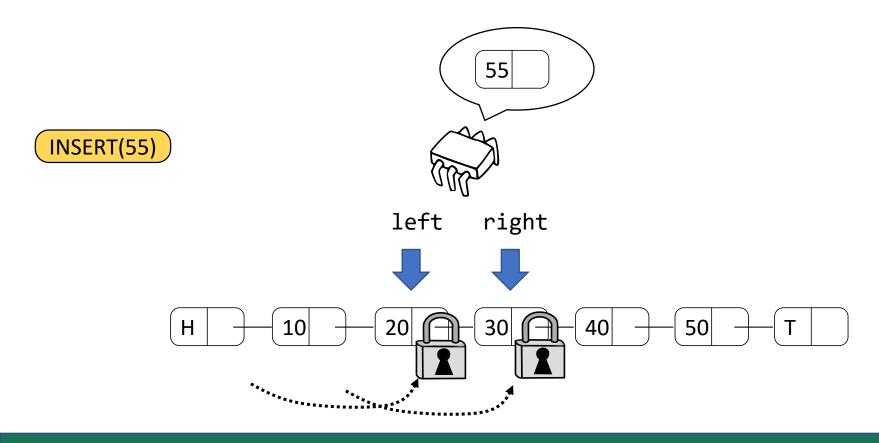
- Keep two locks at a time (lock coupling):
  - One on the current node
  - One on its predecessor



- Keep two locks at a time (lock coupling):
  - One on the current node
  - One on its predecessor

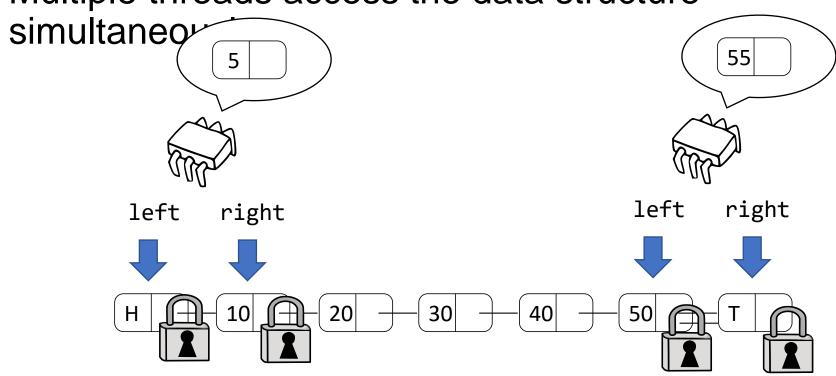


- Keep two locks at a time (lock coupling):
  - One on the current node
  - One on its predecessor



- Keep two locks at a time (lock coupling):
  - One on the current node
  - One on its predecessor

Multiple threads access the data structure

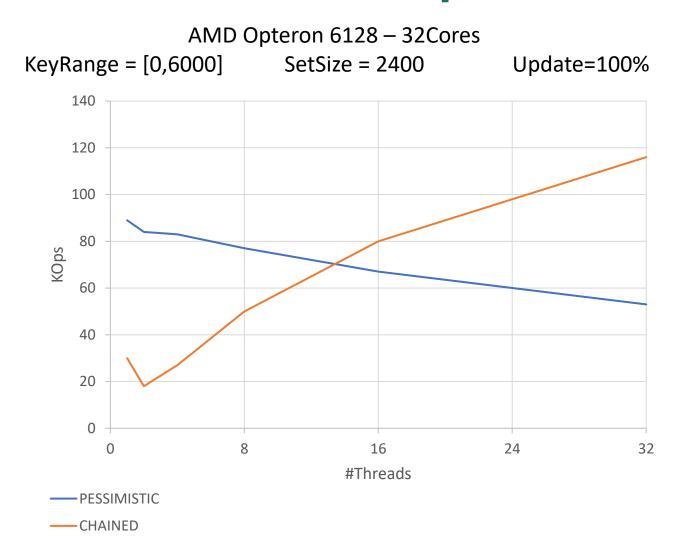


## Concurrent set – Attempt 2 (SRC)

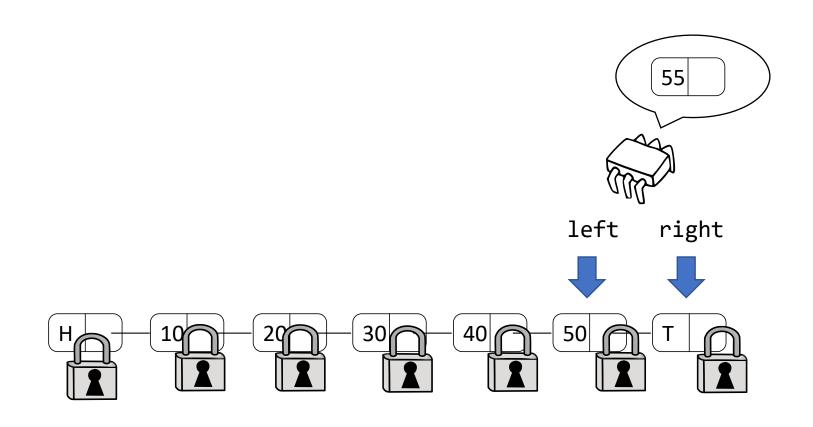
```
1. bool do_operation(int k, int op_type){
2.
     bool res = true;
3.
     node *1,*r;
    LOCK(&glock);
5.
     1 = search(k, \&r);
     switch(op type){
6.
       case(INSERT):
7.
8.
         if(r->key == k)
           res = false;
9.
10.
         else
11.
           1->next = new node(k,r);
12.
         break;
13.
       case(DELETE):
14.
         if(r->key == k)
15.
           1-next = r-next;
16.
       else
17.
         res = false;
18.
         break;
19.
    UNLOCK(&clock)
20.
    UNLOCK(&1->lock);
21.
    UNLOCK(&r->lock);
22.
23. return res;
```

```
1. node* search(int k, node **r){
     node *1, *r next;
2.
3.
    1 = set->head;
4. LOCK(\&1->lock);
   *r = 1->next;
5.
    LOCK(&(*r)->lock);
6.
    r next = (*r)->next;
7.
8.
    while((*r)->key < k){
9.
      UNLOCK(&1->lock);
10.
      1 = *r:
11.
       *r = r next;
      LOCK(&(*r)->lock);
12.
13.
      r next = (*r)->next;
14. }
15. }
```

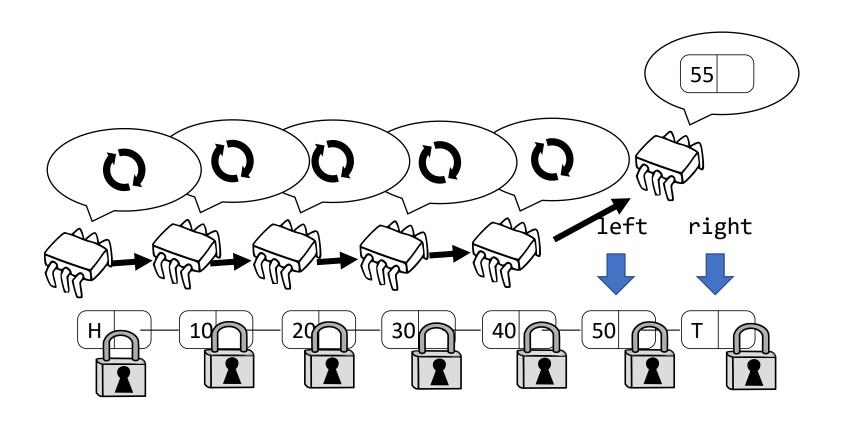
## Concurrent set – Attempt 2



Allows an increased parallelism but...

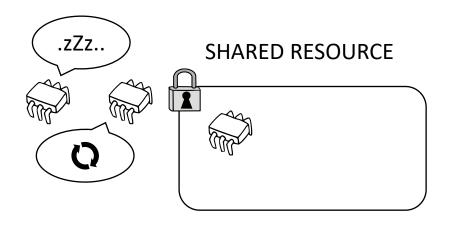


- Allows an increased parallelism but...
- High costs for lock handover

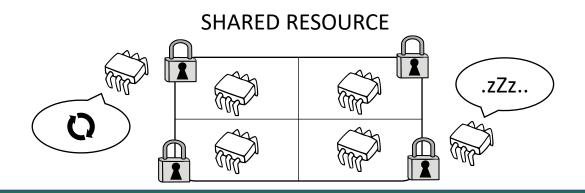


### Recap

- Explored two <u>blocking</u> strategies:
- 1. Global (coarse-grain) lock

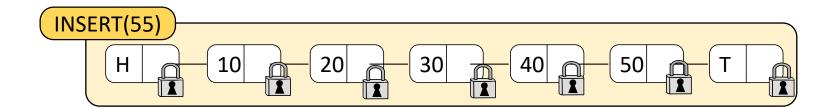


2. (Fine-grain) Lock coupling



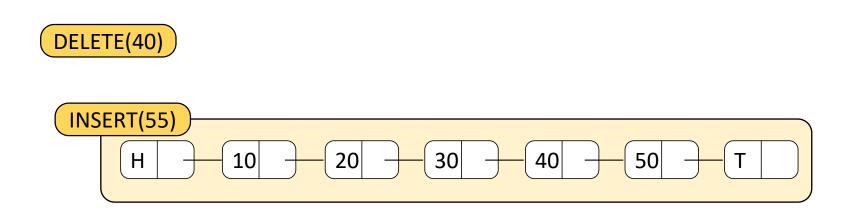
## Concurrent set – Attempt 3

DELETE(40)



## Concurrent set – Attempt 3

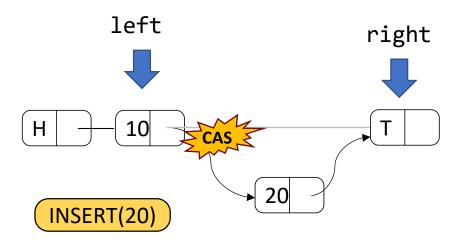
- NON-BLOCKING approach [Harris linked list]
- Search without acquiring any lock
- Apply updates with individual atomic instructions



## Non-blocking insert & delete algorithms

#### Insert:

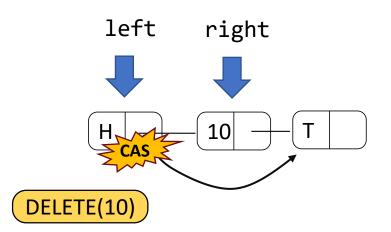
- Search left and right nodes
- Insert the new item with a CAS
- If CAS fails restart from 1

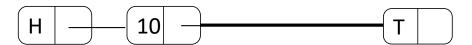


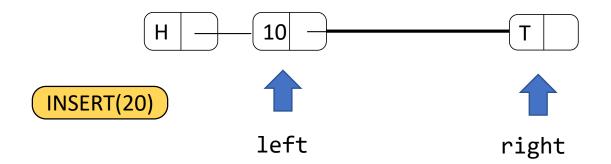
• Is it correct?

#### Delete:

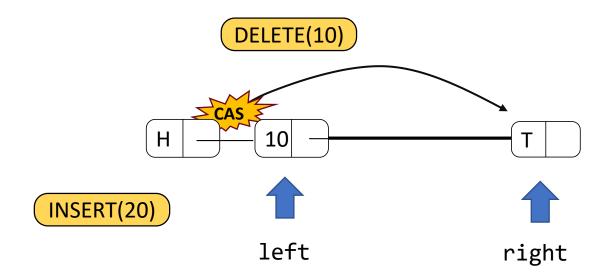
- 1. Search left and right nodes
- Disconnect the item with a CAS
- 3. If CAS fails restart from 1



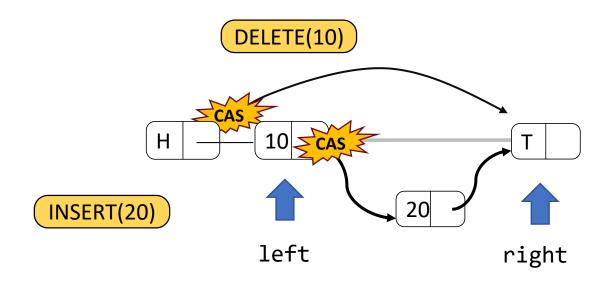




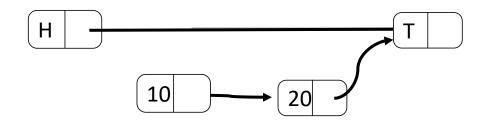
- 1. Thread A gets left and right node and go to sleep
- 2. Thread B disconnects the node containing 10
- 3. Thread A wakes up and add 20 after 10
- The new item is lost



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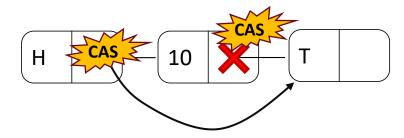
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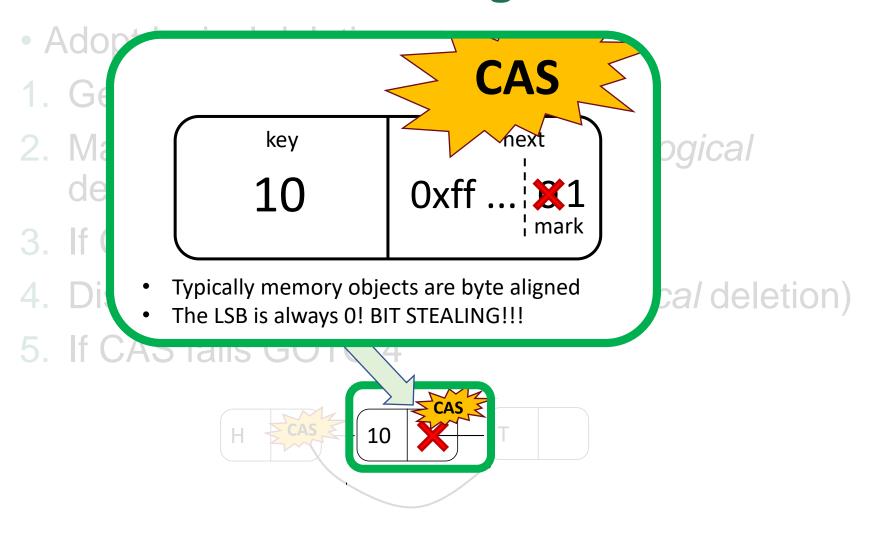
- 1. Thread A gets left and right node and go to sleep
- 2. Thread B disconnects the node containing 10
- 3. Thread A wakes up and add 20 after 10
- 4. The new item is lost

## The correct delete algorithm

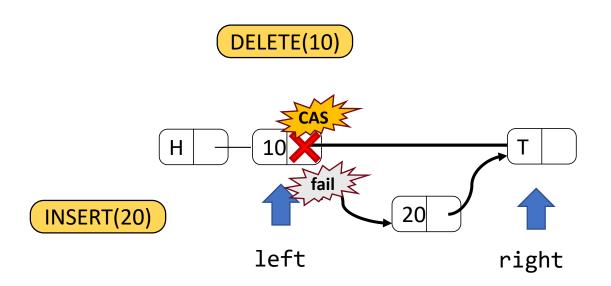
- Adopt logical deletion:
- 1. Get left and right node
- Mark the item as deleted via CAS (logical deletion)
- 3. If CAS fails GOTO 1
- 4. Disconnect the item via CAS (physical deletion)
- 5. If CAS fails GOTO 4



## The correct delete algorithm



### The correct delete algorithm

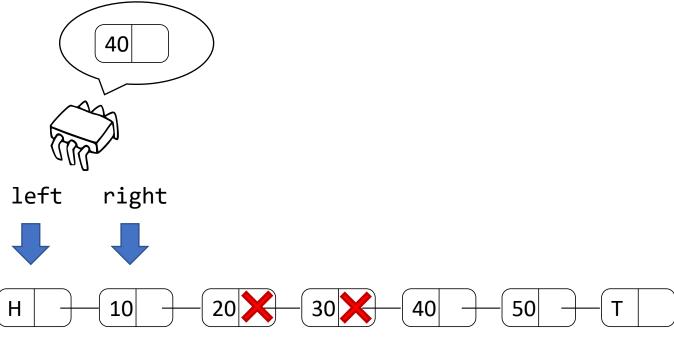


- Updates of the "next" field by two opposite concurrent operations cannot both succeed
- What to do upon conflict (failed CAS)? RESTART FROM SCRATCH!!

- The search returns two adjacent <u>non-marked</u> (left and right) nodes
- Housekeeping: disconnect logically delete items during searches



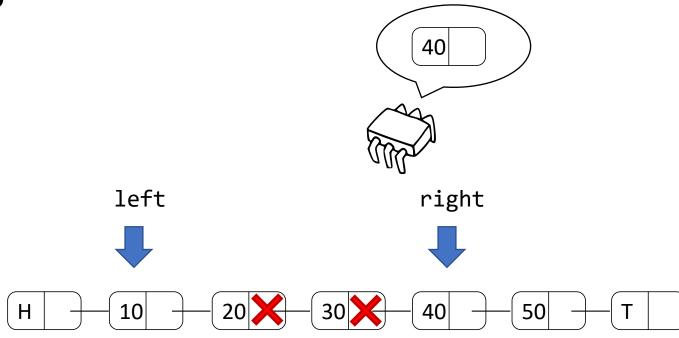
- The search returns two adjacent <u>non-marked</u> (left and right) nodes
- Housekeeping: disconnect logically delete items during searches



The search returns two adjacent <u>non-marked</u> (left and right) nodes

Housekeeping: disconnect logically delete items

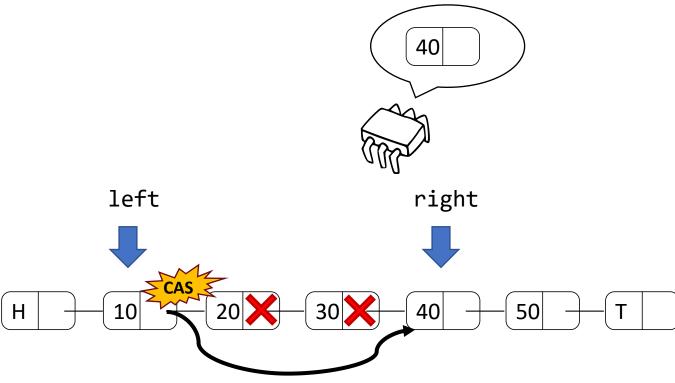
during searches



 The search returns two adjacent <u>non-marked</u> (left and right) nodes

Housekeeping: disconnect logically delete items

during searches



## Concurrent set – Attempt 3 (SRC)

```
1. bool do operation(int k, int op type){
2.
    node *1,*r, *n = new node(k);
3.
    1 = search(k, &r);
                                      /* get left and right node */
4.
    switch(op_type){
5.
      case(INSERT):
        if(r->key == k) return false; /* key present in the set */
6.
7.
  n->next = r;
8.
                                      /* insert the item
    1->next = n;
9.
10.
11.
        break;
12.
      case(DELETE):
13.
        if(r->key != k) return false; /* key not present
                                                                */
14.
        1- next = r- next; /* remove the key
                                                                */
15.
16.
17.
18.
        break;
19.
    }
20. return true;
```

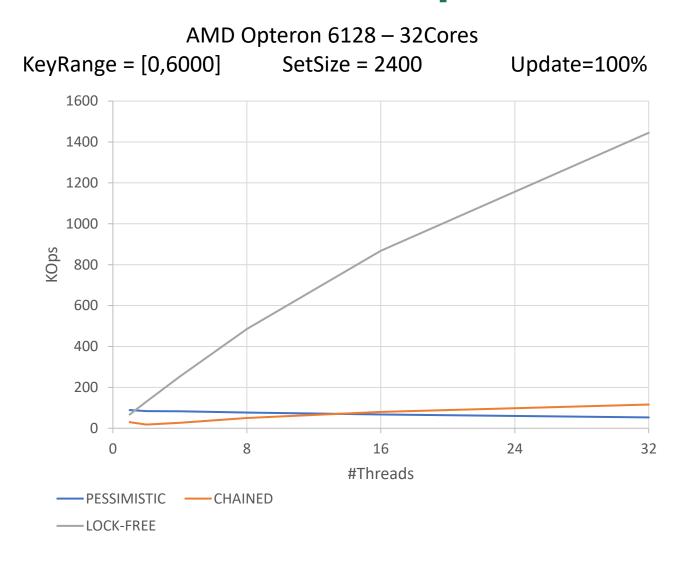
## Concurrent set – Attempt 3 (SRC)

```
1. bool do operation(int k, int op type){
2.
    node *1,*r, *n = new node(k);
3.
    1 = search(k, &r);
                                       /* get left and right node */
    switch(op_type){
4.
5.
      case(INSERT):
        if(r->key == k) return false; /* key present in the set */
6.
7.
        n-next = r;
                                       /* insert the item
8.
       1 >next - n;
        if(!CAS(&l->next, r, n))
9.
10.
           goto 3; /* insertion failed the item -> restart */
11.
         break;
12.
       case(DELETE):
13.
         if(r->key != k) return false; /* key not present
                                                                 */
14.
         1 >noxt - n >noxt:
                           /* remove the key
                                                                 */
        if(is_marked_ref((l=r->next)) | !CAS(&r->next, 1, mark(1)))
15.
16.
           goto 3; /* insertion failed the item -> restart */
        search(k,&r);
17.
                                       /* repeat search
18.
         break;
19.
     }
20.
    return true;
```

## Concurrent set – Attempt 3 (SRC)

```
1. node* search(int k, node **r){
2.
     node *1, *t, *t next, *1 next;
   *t = set->head;
3.
   t next = t->head->next;
4.
   while(1){
                                      /* FIND LEFT AND RIGHT NODE */
6.
         if(!is_marked(t_next)){
7.
            1 = t:
8.
            1 next = t next;
9.
        }
10. t = get_unmarked_ref((t_next);
11. t next = t->next;
12.
        if(!is marked ref(t next) && t->key >= k) break;
13.
14. *r = t:
15. /* DEL MARKED NODES */
16.
     if(1 next != *r && !CAS(&1->next, 1 next, *r) goto 3;
17.
     return 1;
18.}
```

## Concurrent set – Attempt 3



## Safety and liveness guarantees

- The algorithm is NON-BLOCKING (LOCK-FREE):
  - If a thread A is stuck in a retry loop => a CAS fails each time
  - If a CAS fail, it is because of another CAS that was successfully executed by a thread B
  - Thread B is making progress
- The algorithm is LINEARIZABLE:
  - Each method execution take effect in an atomic point (a successful CAS) contained between its invocation and reply
  - The order obtained by using these points has been proved to be compliant with the Set semantic

## Progress (Lock freedom)

- Each method update method has two main steps
  - A search, which might end with a CAS
  - A CAS to insert delete a node
- 1. Suppose an update method is stuck in a search:
  - The key range is finite, so the number of node is finite
  - It continuously fails to disconnect marked nodes
  - It means that new nodes have been both inserted and marked!
    - Other threads have completed update methods
- 2. Suppose an updated method always fails its last step (insertion or marking)
  - Other threads have modified the target next pointer
  - If it is due to the disconnection of marked nodes, see point 1
  - If it is due to the updated step other methods have completed

## Safety (Linearizability)

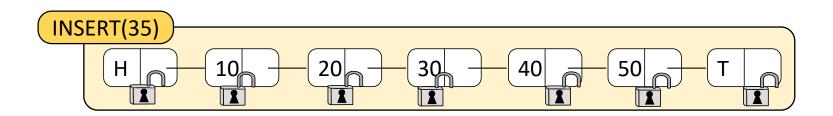
- 1. The search returns 2 adjacent nodes in an atomic point
  - 1. The read of next field of the left node
  - 2. The CAS that make left and right adjacent
- It is like that the search made a snapshot of interested key interval
- 2. Find, unsuccessful delete and unsuccessful insert linearize with the search (1.1 or 1.2)
- 3. Insert linearizes with the successful CAS to connect a new node
- 4. Delete linearizes with the successful CAS to mark a node

#### **Problems**

- It is not possible to flip a bit of a reference on memory-managed languages (e.g. JAVA)
- How to solve?

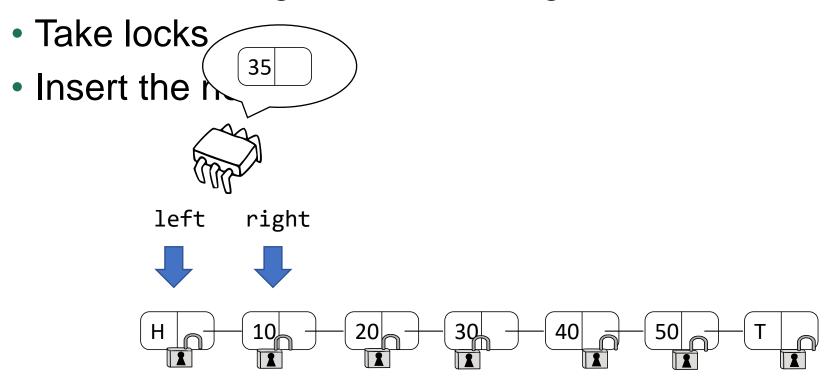
## Locks + Optimism

- Use one lock per node
- Move "marked" to a dedicated field

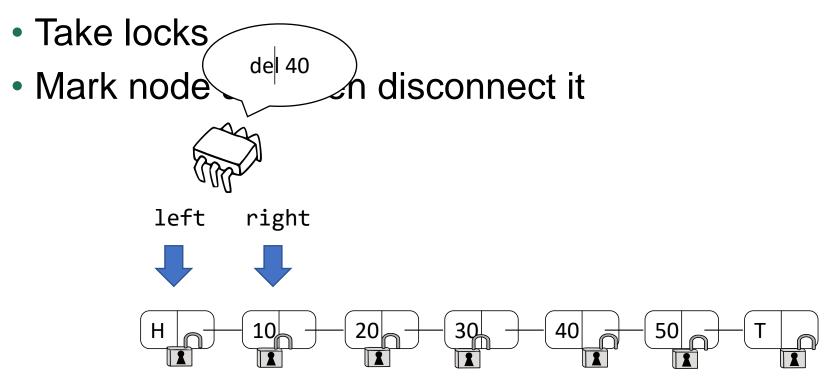


#### Locks + Optimism (insert)

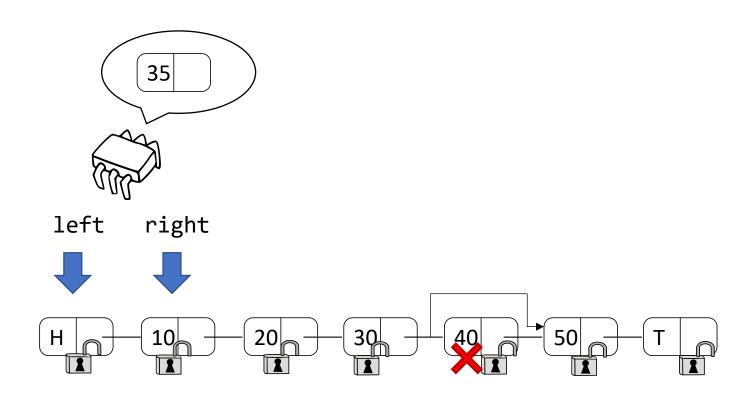
- Use one lock per node
- Move "marked" to a dedicated field
- Find left and right without taking locks!



- Use one lock per node
- Move "marked" to a dedicated field
- Find left and right without taking locks!



- Why "optimistic"? Do work (search) and hope nothing wrong happens!
- What could go wrong?



- Why "optimistic"? Do work (search) and hope nothing wrong happens!
- What could go wrong?
  - Left and/or right being marked
  - Left and right not adjacent
- How to solve?
- Validation of search results:
  - Left unmarked
  - Right unmarked
  - Left.next = right

- Why "optimistic"? Do work (search) and hope nothing wrong happens!
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#### Locks + Optimism = Lazy List

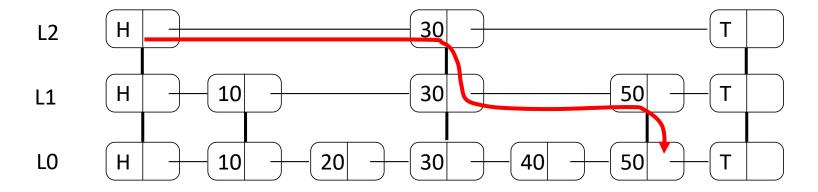
- What about correctness?
- What about progress?

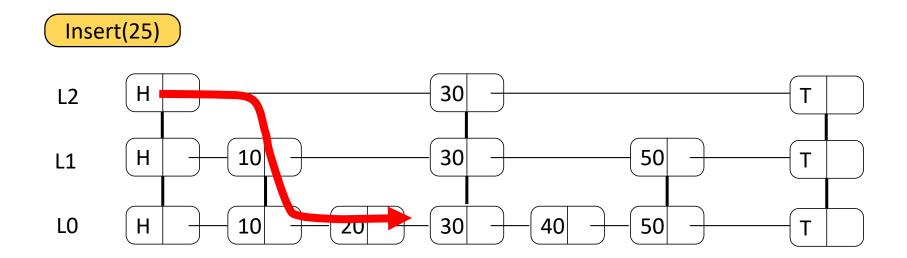
#### Can we do better?

- Costs: O(n)
- Starting from scalable "simple" set implementation we can build faster set implementations
  - Hash table: O(1)
    - Array of buckets
    - Buckets are concurrent ordered-list based sets
- We know that a search in an ordered set could be more efficient O(log(n))
- How?

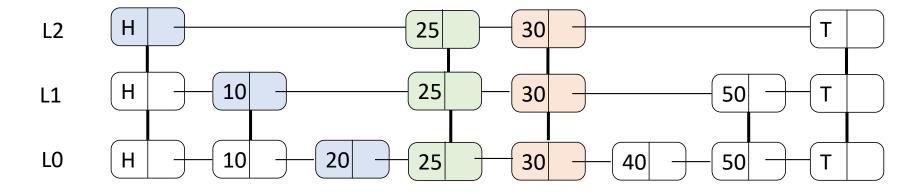
- Generalization of sorted linked lists
- Randomized data structure
- Costs: O(log(n))
- Idea:
  - 1. Maintain a core sorted linked list L0
  - 2. Use additional sorted linked lists Li such that:
    - 1. Li ⊂ Li-1
    - 2.  $|Li| \approx |Li-1|/2$
  - 3. Searches use lists in decreasing order

#### Search(50)





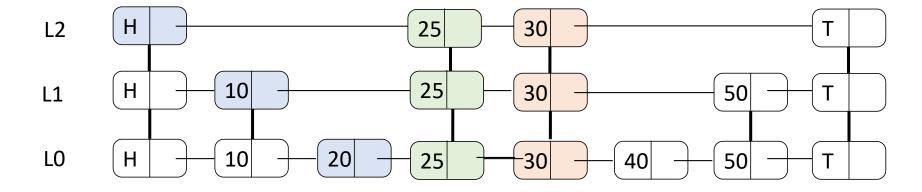
#### Insert(25)



Should I insert 25 at L1? Flip a coin!

Should I insert 25 at L2? Flip a coin!

#### Delete(25)



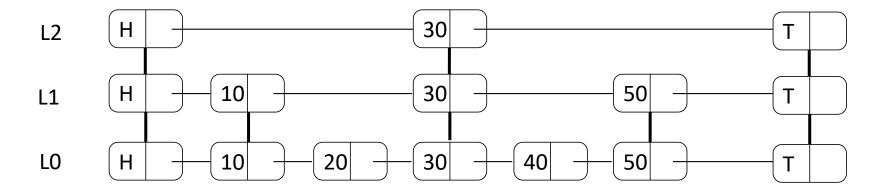
- How many (expected) keys for each level?
- L0 = N
- L1 = N/2
- L2 = N/4

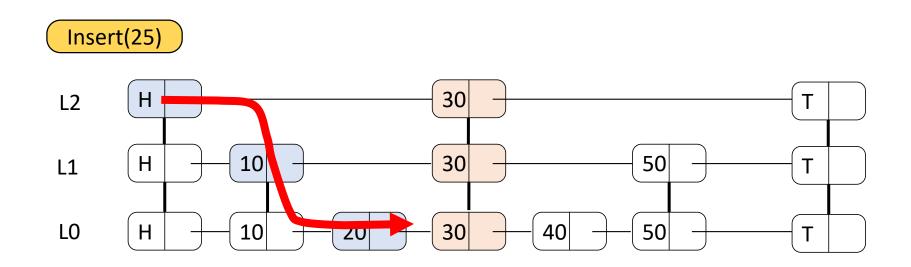
. . .

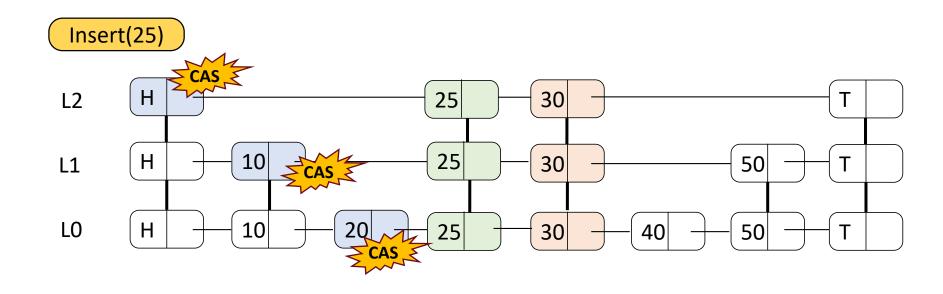
• L(logN) = 1

How many steps per level?

#### Insert(25)







Delete(25)

Η

LO

10

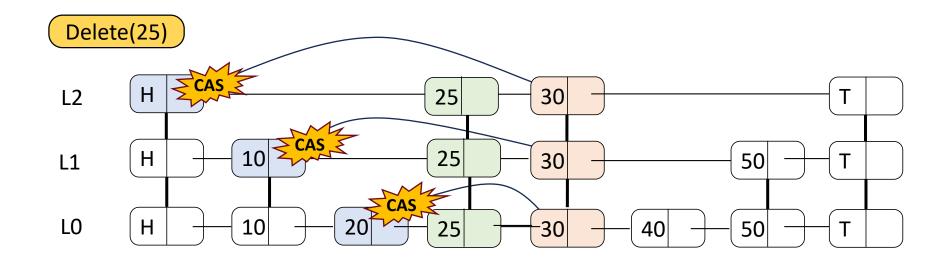
20

#### 

-30

40

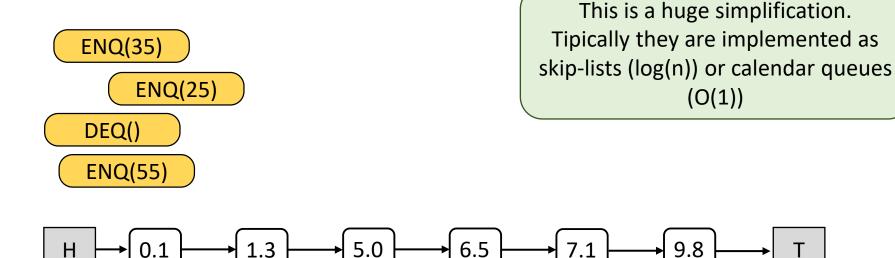
50



# Concurrent Data Structures: Priority queues

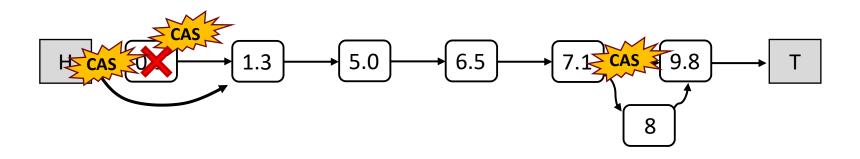
#### **Priority queue implementations**

- Priority Queue methods:
  - enqueue(k): adds a new item
  - dequeue(): returns and remove the highest priority item
- Implemented as an ordered linked list ◄

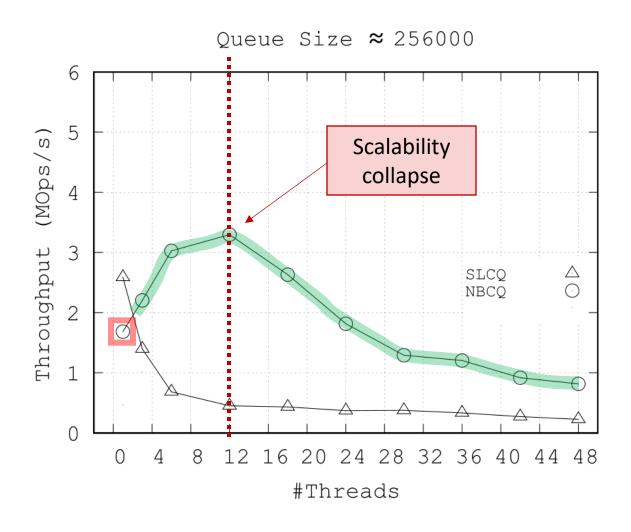


#### Priority queue – Attempt 1

- Enqueue: works as insertions in the non-blocking Set
  - Connect via CAS
- Dequeues: work as deletions in the non-blocking Set
  - Mark as logically deleted, but
  - DISCONNECT IMMEDIATELY
- Is it scalable?

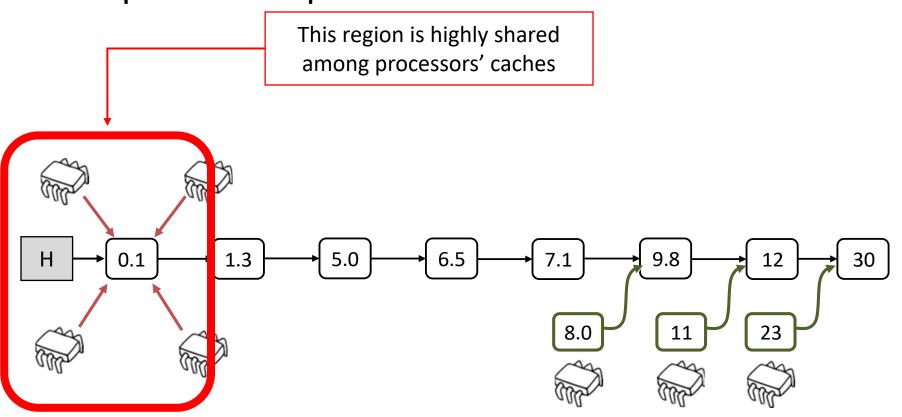


#### Priority queue – Attempt 1



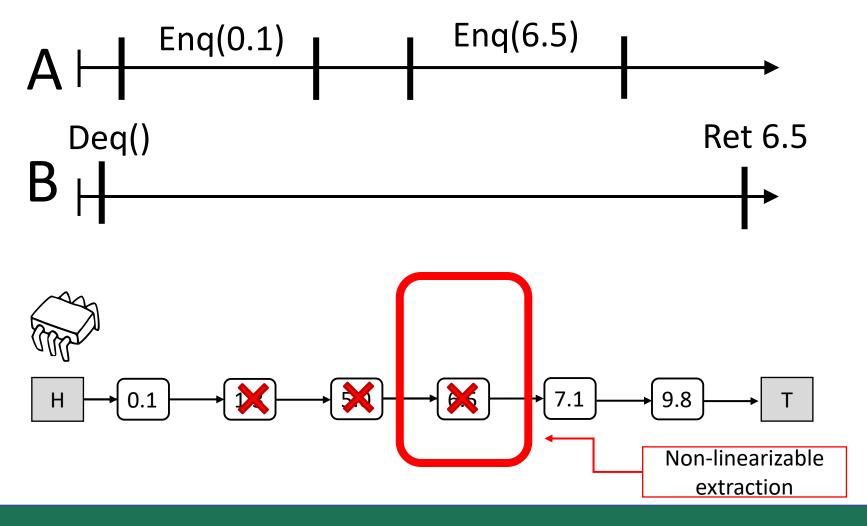
#### Priority queues: an inherently "sequential" semantic

- Enqueue offers a high level of disjoint access parallelism
- Dequeues are prone to conflicts



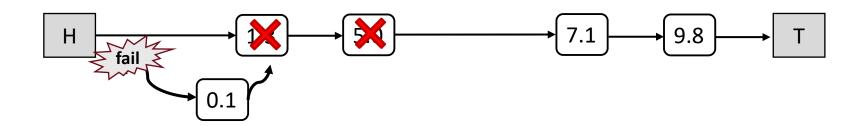
#### Lazy deletion within priority queues

 If we use lazy deletion "as is", we might obtain nonlinearizable extractions



#### Correct lazy deletion within priority queues

- To implement correct extractions with lazy deletions there are two main approaches
- Move the logical mark of a node in the field "next" of its predecessor

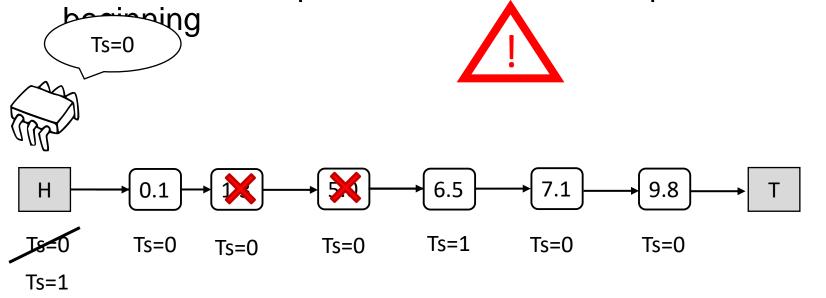


#### Correct lazy deletion within priority queues

 To implement correct extractions with lazy deletions there are two main approaches

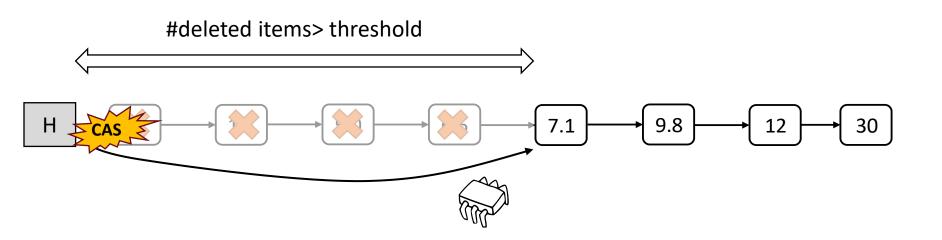
#### 2. Use logical timestamps:

- incremented each time a new minimum has been inserted
- extract item compatible with the timestamp read at the



#### PQ – Attempt 2 - Introducing Conflict Resiliency

- Lazy deletion
- Skip logically deleted items ⇒ IT INCREASES THE NUMBER OF STEPS ⇒ EXPENSIVE IN TERMS OF IMPACT ON CACHE
- Periodic Housekeeping



#### Priority queue – Attempt 2



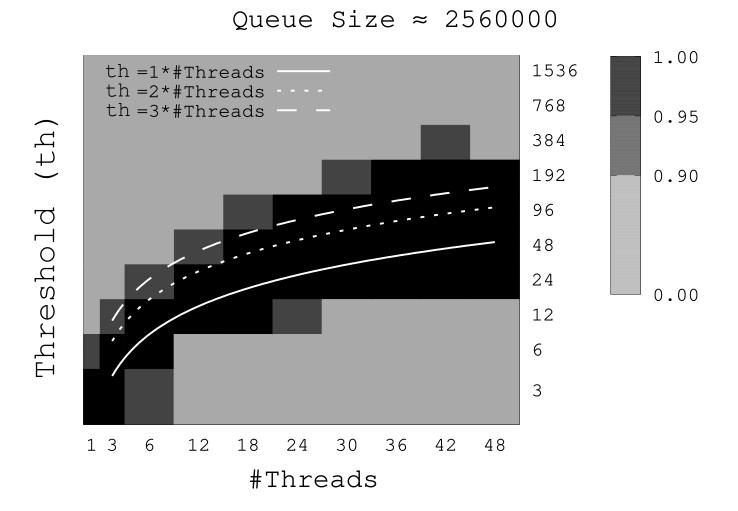
## On the conflict resiliency trade off

 The number of steps per dequeue and costs of housekeeping are <u>dependent</u>:

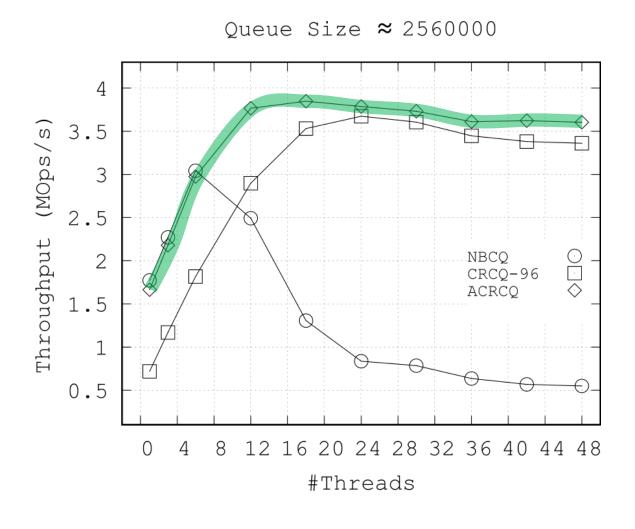




#### Conflict resiliency trade offs



#### Priority queues – Attempt 3



#### Open challenges

How to achieve scalability for priority queues?

- NO ANSWER for correct priority queue
- The research moved on looking for RELAXED SEMANTICS for priority queues
  - Enable scalability for extractions by removing an item which is "near" the minimum
- Explore orthogonal approaches by guaranteeing RELAXED CORRECTNESS CONDITIONS
  - K-linearizability
  - Quasi-linearizabilty
  - Quiescent consistency
  - Sequential consistency?
- Explore new hardware capabilities (e.g. HTM)

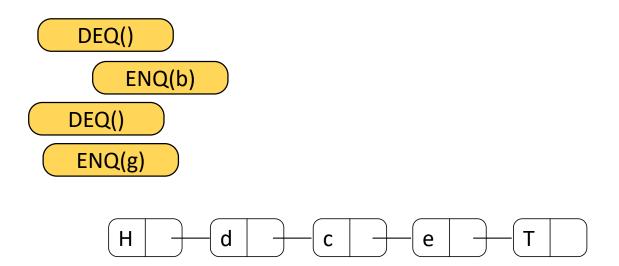
#### Why linearizable non-blocking algorithms?

- Performance is a good reason, but not the unique one
- The composition of linearizable algorithm is still linearizable
- Blocking algorithms (and their composition) might suffer from deadlocks, priority inversions and convoying
- The composition of non-blocking algorithms is nonblocking as a whole (progress property of individual algorithm might be hampered)
- Libraries should implement their algorithms in a nonblocking linearizable fashion
  - E.g. Java implements non-blocking concurrent data structure

# Concurrent Data Structures: FIFO queues

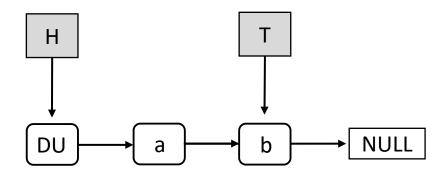
### FIFO queue implementation

- Queue methods:
  - enqueue(v)
  - dequeue()
- Implemented as a linked list



#### FIFO queue implementation

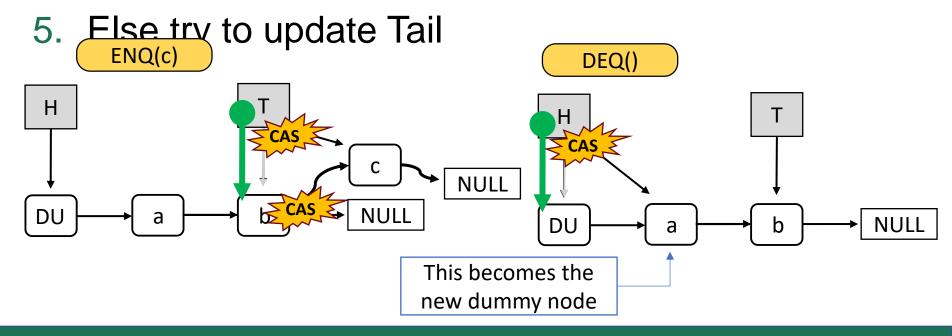
- Slightly different
- One dummy node, two pointers to access the data structure:
  - Head: points ALWAYS to a DUMMY node item
  - Tail: SHOULD point to the youngest item



#### FIFO queue implementation

- Insert:
- 1. Get node pointed by tail
- 2. Scan until next is NULL
- 3. Try to insert with CAS
- 4. If KO goto 1

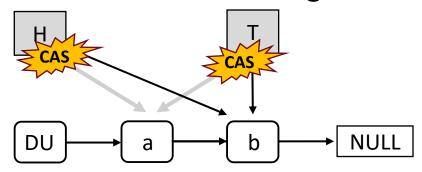
- Dequeue:
- 1. Get node pointed by head
- 2. Try to update head with its next
- 3. If KO goto 1



# The whole story

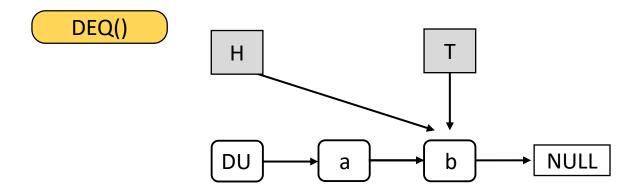
- Since the insertion of a new item and the tail update are two separate RMW they might be inconsistent
- Also dequeuers might need to update tail before updating head

• This ensures that TAIL cannot go behind HEAD

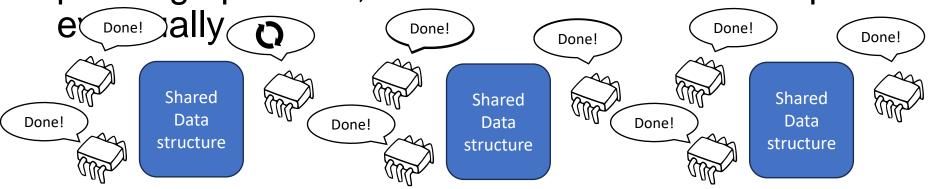


#### **Emptiness condition**

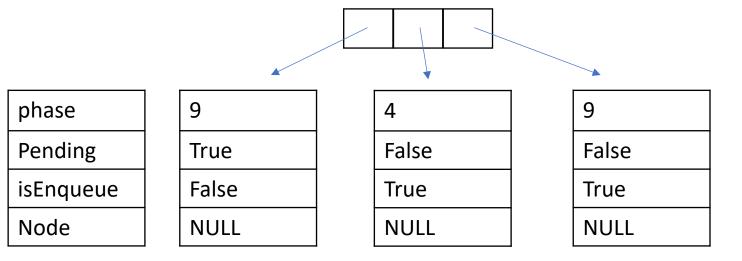
 There is a NULL node after the one pointed by HEAD



- What about a wait-free queue?
- Wait-free means that all method invocations are guaranteed to complete
- Can we modify the lock-free FIFO queue to achieve this?
- Lock-free means that some thread might starve
- If before starting any new operation we complete a pending operation, all method invocation complete



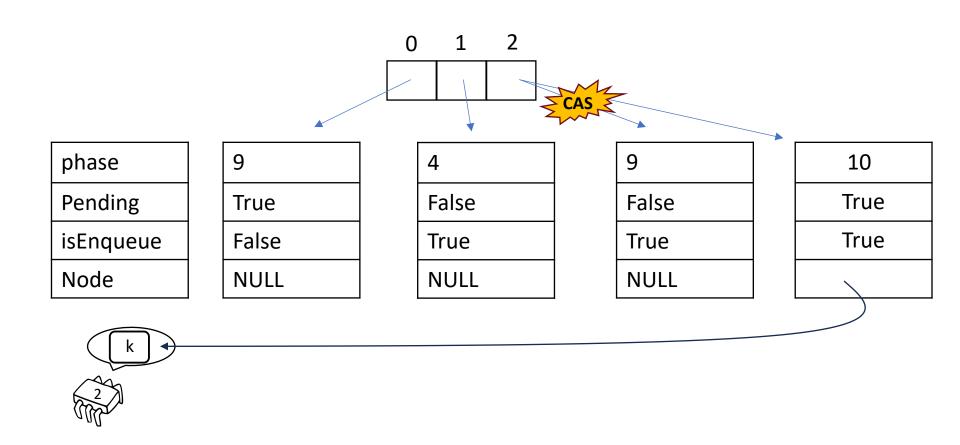
We need to be aware of pending calls



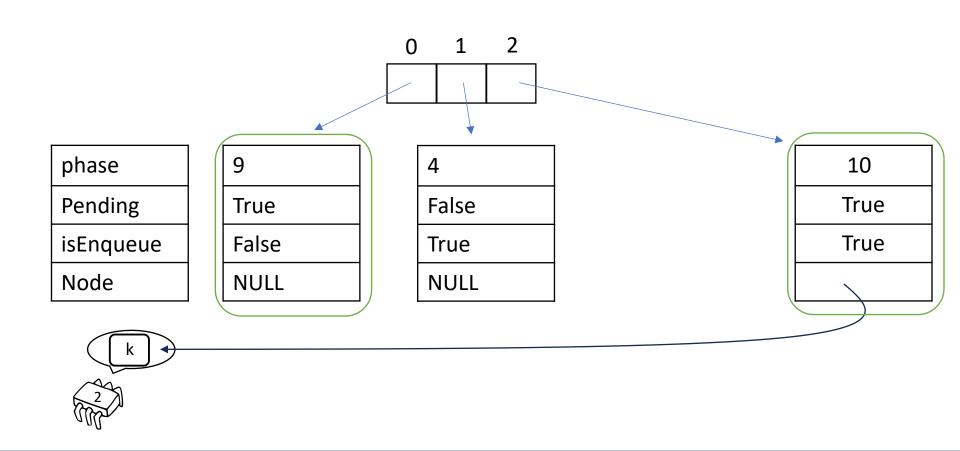
- Split operations on the linked list into 2 steps:
  - 1. Modify nodes for enqueue/dequeue (main step)
  - Modify head/tail pointers (finishing step)

- Enqueue/Dequeue structure
  - 1. Publish op record
  - 2. Get the set **S** of all pending ops whose record has been previously or concurrently published
  - 3. Help any operation in S
  - 4. Do a finishing step

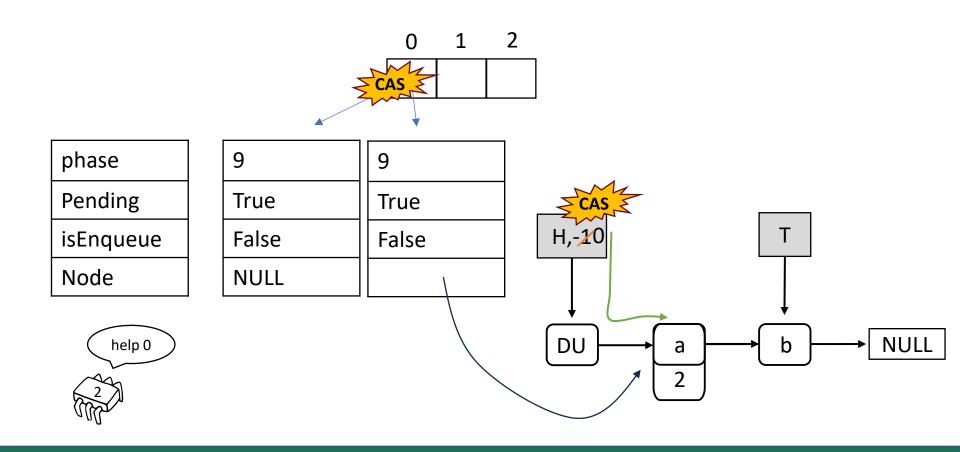
- Enqueue/Dequeue structure
  - 1. Publish op record



- Enqueue/Dequeue structure
  - 2. Get the set **S** of all pending ops whose record has been previously or concurrently published



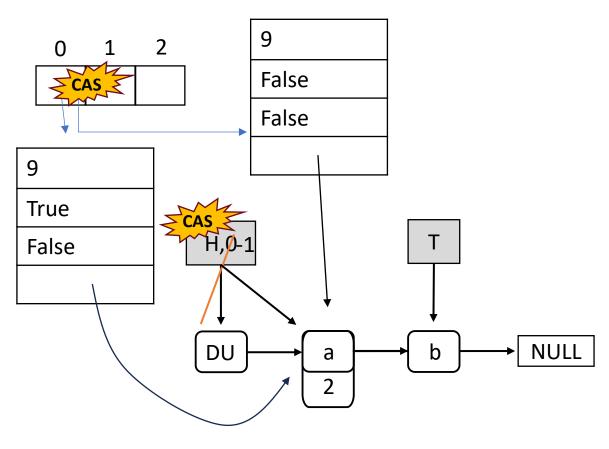
- Enqueue/Dequeue structure
  - 3. Help any operation in **S** (dequeue)
    - a. Main step



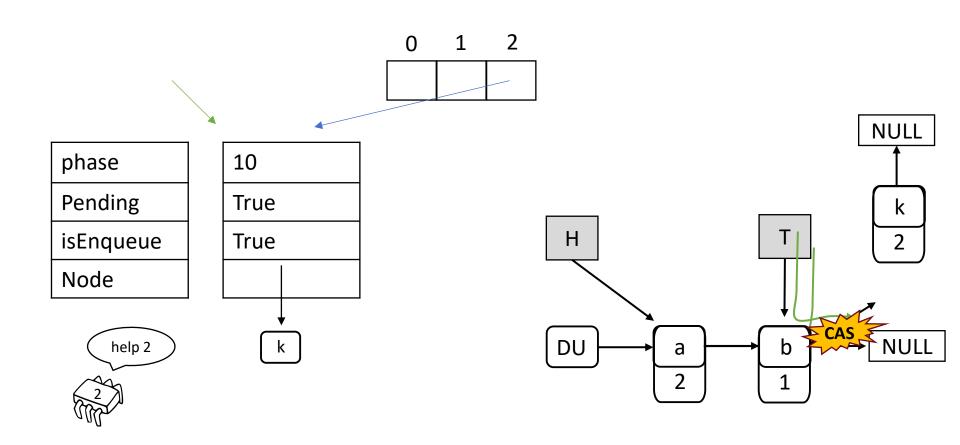
- Enqueue/Dequeue structure
  - 3. Help any operation in **S** (dequeue)
    - a. Main step
    - b. Finishing step

phase
Pending
isEnqueue
Node

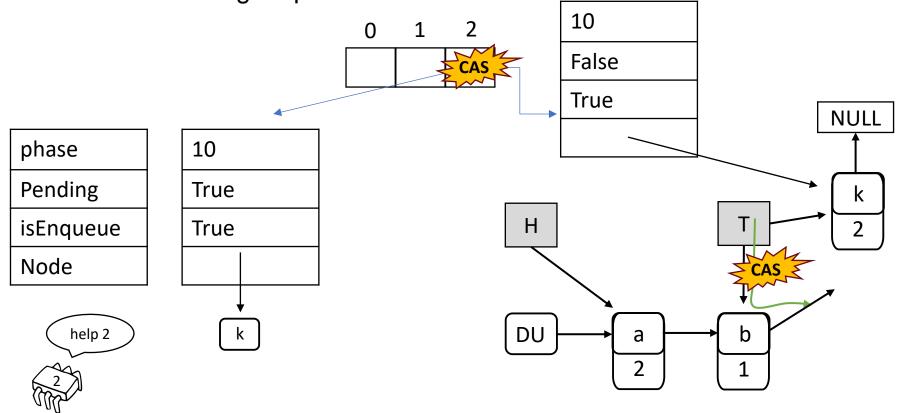




- Enqueue/Dequeue structure
  - 3. Help any operation in **S** (enqueue)
    - a. Main step

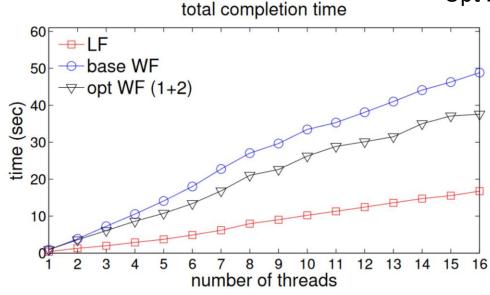


- Enqueue/Dequeue structure
  - 3. Help any operation in **S** (enqueue)
    - a. Main step
    - b. Finishing step



- Enqueue/Dequeue structure
  - 1. Publish op record
  - 2. Get the set **S** of all pending ops whose record has been previously or concurrently published
  - 3. Help any operation in S
  - 4. Do a finishing step

Opt 1: help only one pending op Opt 2: use FAD to get phase num.



#### Fast Wait-free FIFO queue

- Try with lock-free approach
- If starving, back-off to wait-free implementation

