

# MATH 141

## Hypothesis Testing III, Confidence Intervals

Chester Ismay

# What is Hypothesis Testing good for?

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are sometimes abused ( $p < 0.05$ ).
- ▶ Used to assess the degree to which data is consistent with a particular model.
- ▶ The most widely used tool in statistical inference.

# Step 1

Lay out your model(s).

$H_0$ : null model, business as usual

$H_A$ : alternative model, business not as usual

- ▶ Hypotheses are statements about the TRUE STATE of the world and should involve *parameters*, not *statistics*.

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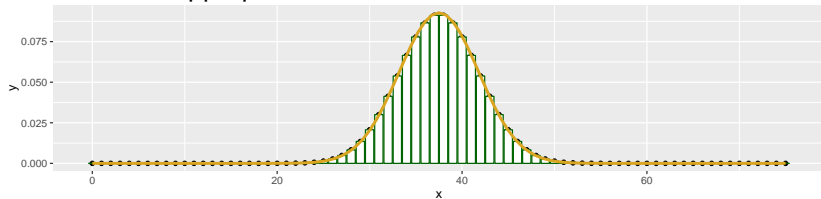
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- ▶ Hypotheses are statements about the TRUE STATE of the world and should involve *parameters*, not *statistics*.
- ▶ Hypotheses should suggest a *test statistic* that has some bearing on the claim.
- ▶ Always use two-tailed tests.

## Step 2

Construct the appropriate null distribution.

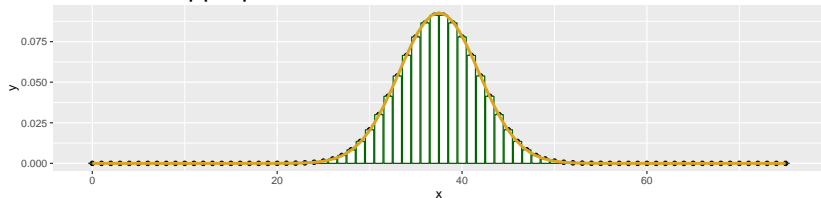


### 1. Randomization



## Step 2

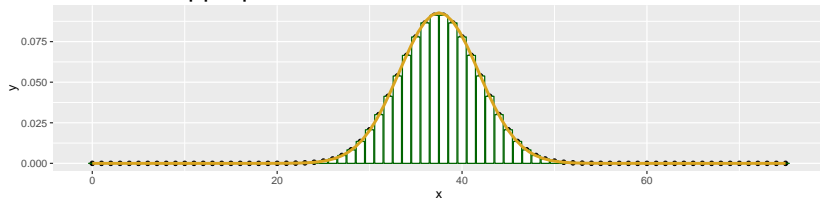
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1. Randomization
2. Simulation

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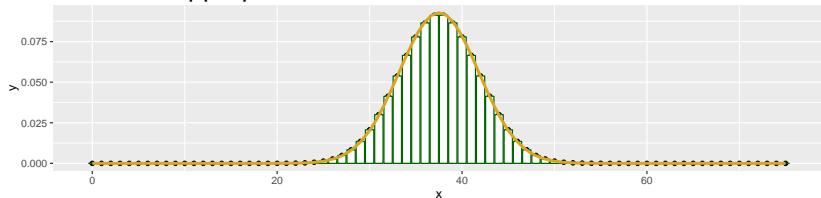
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1. Randomization
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1. Randomization
2. Simulation
3. Exact Probability Theory
4. Normal Approximation

## Step 3

Calculate a measure of consistency between the observed test statistic (the data) and the null distribution (i.e., a p-value).

- ▶ If your observed test stat is in the tails  $\rightarrow$  low p-val  $\rightarrow$  data is inconsistent with null hypothesis  $\rightarrow$  “reject null hypothesis”.

What can go wrong?

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- ▶ If your observed test stat is in the tails  $\rightarrow$  low p-val  $\rightarrow$  data is inconsistent with null hypothesis  $\rightarrow$  “reject null hypothesis”.
- ▶ If your observed test stat is in the body  $\rightarrow$  high p-val  $\rightarrow$  data is consistent with the null hypothesis  $\rightarrow$  “fail to reject the null hypothesis”.

What can go wrong?

# Confidence Intervals

**Confidence Interval:** a plausible range of values for a population parameter.







# Construction of a CI

If the distribution of the point estimate can be well approximated by the Normal:

point estimate  $\pm$  margin of error

point estimate  $\pm 1.96 \times SE$

► *point estimate:*  $\hat{p}$ ,  $\bar{x}$

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- ▶ *point estimate*:  $\hat{p}$ ,  $\bar{x}$
- ▶ *standard error (SE)*: standard deviation of the sampling distribution.

## Question

“The proportion of Americans who want to maintain planned parenthood is estimated to be between 57.1% and 62.9% with 95% confidence.” Which of the following interpretations are reasonable?

1. The interval  $[\hat{p}, \hat{p}]$  provides a plausible range for  $\hat{p}$

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2. We are certain that the population proportion lies within this interval.

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1. The interval  $[\hat{p} = .571, \hat{p} = .629]$  provides a plausible range for  $\hat{p}$
2. We are certain that the population proportion lies within this interval.
3. If we took many more samples of the same size and computed many  $\hat{p}$ s and many CIs, around 95% of those CIs would contain the population proportion.

