Chapter 2
Mathematics Review

Key Terms

Average
Decimal
Denominator
Fraction
Numerator
Percentage
Proportion
Quotient
Rate
Ratio
Rounding
Whole number

Objectives

At the conclusion of this chapter, you should be able to:

- Explain fraction, quotient, decimal, ratio, proportion, rate, and percentage
- Understand the difference between a numerator and denominator
- Understand how to round whole numbers and decimals
- Convert fractions and decimals to percentages
- Understand how to average a group of numbers

Review of Basic Mathematical Expressions

Numbers may be expressed in a variety of ways for use in calculating statistics. The following sections discuss fractions, quotients, decimals, proportions, how to round numbers, percentages, ratios, rates, and averages.

Fractions

A **fraction** is one or more parts of the whole. Figure 2.1 shows two circles; the first circle is split into two equal parts, and the second shows one part of the circle larger than the other part. The fraction of the first circle is $\frac{1}{2}$; the fraction of the second circle (in darker color) is $\frac{3}{4}$. The top number is called the **numerator** and the bottom number is called the **denominator.**

> **Example:** Of the 40 patients with diabetes seen last month in a physician's clinic, 20 were Caucasian, 10 were African-American, and 10 were Asian-American. The following fractions show the number of patients of each race compared to the total number of patients who visited the clinic: Caucasian, $\frac{20}{40}$; African-American, $\frac{10}{40}$; and Asian-American, $\frac{10}{40}$. Fractions should be converted to their simplest form. Each fraction can be converted by dividing both top (numerator) and bottom (denominator) by a common factor. In this example, both the numerator and the denominator can be divided by 10: Caucasians, $\frac{2}{4}$; African-American, $\frac{1}{4}$; and Asian-American $\frac{1}{4}$. The first fraction can be further simplified with the common factor of 2; thus, $\frac{2}{4}$ can be expressed as $\frac{1}{2}$.

Exercise 2.1

Find the simplest form of each of the following fractions:

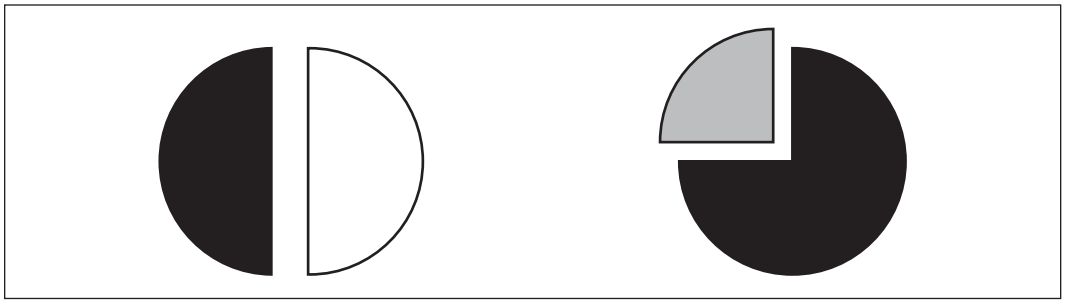
1. $\frac{40}{80}$
2. $\frac{3}{9}$
3. $\frac{75}{150}$
4. $\frac{6}{36}$
5. $\frac{20}{100}$

Quotient

A **quotient** is the number obtained by dividing the numerator of a fraction by the denominator. This number may be expressed in **decimals.**

> **Example:** The 14 members of your health information class decide to participate in your college's information day. The booth is going to be open for 21 hours over a three-day period. To find out how many hours each student would need to attend to the booth, you would divide the numerator (21 hours) by the denominator (14 students). This calculation gives a quotient of 1.5. Thus, each student would have to attend the booth for 1.5 hours.

Figure 2.1. Fractions of a circle



Decimal Fraction

Decimal fractions are simply referred to as decimals. The notation indicates a value that is less than one. In 14.37, for example, the digits to the right of the decimal point (3 and 7) are called decimal digits. The decimal point is used to separate the fraction of a **whole number** (.37) from the whole number itself (14). The decimal point is not ordinarily used in whole numbers (for example, 14.0) unless the healthcare facility has a particular reason for doing so.

Handy Tip: The decimal .5 is usually written as 0.5 in order to call attention to the decimal point.

Rounding Numbers

Rounding is a process of approximating a number. Numbers may be rounded to the nearest 10, 100, and so on.

Rounding to the Nearest Ten

Rounding to the nearest ten means that any number between multiples of ten (10, 20, 30, 40, and so on) is rounded to the multiple it is closest to. For example, 31 falls between 30 and 40 but is closer to 30, so it is rounded to 30. However, 37 is closer to 40, so it is rounded to 40. When a number is exactly between the two multiples of ten, the rule of thumb is to round up. Thus, the 5 in the ones place indicates 35 would be rounded up to 40 as in the example below.

3	5
↓	↓
tens	ones

Rounding to the Nearest Hundred

Rounding to the nearest hundred refers to rounding numbers that fall between multiples of 100. For example, 327 falls between 300 and 400 but is closer to 300. Thus, the 2 in the tens place indicates that 327 should be rounded down to 300.

3	2	7
↓	↓	↓
hundreds	tens	ones

The number 7,868 falls between 7,800 and 7,900. In this case, the 6 (in the tens place) indicates that 7,868 should be rounded up to 7,900.

7	8	6	8
↓	↓	↓	↓
thousands	hundreds	tens	ones

Rounding Decimals

Most healthcare statistics are reported as decimals, and each healthcare facility has its own policy on the number of decimal places to be used in computing and reporting percentages. The principles that apply to rounding whole numbers also apply to rounding decimals. To round to the nearest whole number, look at the first digit to the right of the decimal point (tenths); if the number is 5 or more, the whole number should be rounded up; if the number is less than 5, the whole number should be left as it is. Thus, the whole number in 14.4 should remain at 14; however, 14.5 should be rounded up to 15.

To round to the nearest tenth, you do the same thing except use the hundredths digit rather than the tenths. The hundredths is the second digit to the right of the decimal point. For example, 14.46 would be rounded up one to 14.5 because the 6 in .46 is greater than 5. In the case of 14.13, the 3 in .13 is less than 5, so the .1 is kept rather than rounding up. To round to the nearest hundredths, the calculation must be carried out to three decimal places (the thousandths digit) and then rounded. For example, 14.657 would be rounded up to 14.66 because the 7 in .657 is greater than 5. In the case of 14.654, the 4 in .654 is less than 5, therefore the decimal is rounded down to 14.65.

Handy Tip: When the decimal points must be carried out to two places, the calculation should be carried out to one more place in the quotient and rounded back. For example:

$$\frac{1}{7} = 0.142 = 0.14$$

Exercise 2.2

Find the quotient in the following fractions. Round to two decimal places.

1. $\frac{3}{5}$
2. $\frac{8}{12}$
3. $\frac{34}{56}$
4. $\frac{107}{98}$
5. $\frac{545}{654}$

Exercise 2.3

Round the following numbers to the nearest 10.

1. 48
2. 356
3. 311
4. 5,896
5. 3,258
6. 9,631
7. 232,563
8. 2,634
9. 48,605
10. 8,563

Round the following numbers to the nearest hundred.

11. 651
12. 123
13. 8,307
14. 7,534
15. 5,781

Round to the nearest whole number.

16. 18.3
17. 32.5
18. 23.1
19. 152.6
20. 99.4

Round to one decimal place.

21. 15.89
22. 18.58
23. 32.62
24. 99.98
25. 124.07

(continued on next page)

Exercise 2.3 (continued)

Round to two decimal places.

26. 7.897438
27. 12.14526
28. 0.569888
29. 27.99999

Percentage

The ratio of a part to the whole is often expressed as a **percentage.** A percentage is a fraction expressed in hundredths. Percent means "per 100." There is a specific way to write this. For example, .34 would be written as $\frac{34}{100}$ and is equal to 34 percent.

Percentages are a useful way to make fair comparisons. For example, if 20 patients died in hospital A last month and 50 patients died in hospital B during the same period, one might conclude that it would be better to use the services at hospital A because hospital A had fewer deaths. However, that conclusion would be wrong if hospital A had 100 discharges during the month and hospital B had 500 discharges for the same period.

Hospital A: 20/100 = 20%
Hospital B: 50/500 = 10%

Not all percentages are converted to whole numbers. For example:

$$\frac{1}{8} = .125 = 12.5\%$$

Changing a Fraction to a Percentage

To change a fraction to a percentage, divide the numerator by the denominator and multiply by 100. For example, to change $\frac{1}{2}$ to a percentage, divide 1 by 2 and multiply by 100. The calculation is as follows:

$$\frac{1}{2} = 0.5 \times 100 = 50\%$$

Changing a Decimal to a Percentage

To change a decimal to a percentage, simply multiply the decimal by 100. The calculation changes the position of the decimal point. For example:

$$0.29 \times 100 = 29\%$$

Changing a Percentage to a Fraction

To convert a percentage to a fraction, eliminate the percent sign and multiply the number by $\frac{1}{100}$. A simpler version of this is to place the number in the numerator and 100 in the denominator. For example:

$$5\% \text{ would be } 5 \times \frac{1}{100} = \frac{5}{100}$$

$$15\% \text{ would be } 15 \times \frac{1}{100} = \frac{15}{100}$$

Fractions are usually converted to their simplest form. In the example of 5 percent, both 5 and 100 can be divided by 5, which would result in a fraction of $\frac{1}{20}$. In the example 15 percent, again, both 15 and 100 are divisible by 5, resulting in a fraction of $\frac{3}{20}$.

Changing a Percentage to a Decimal

To convert a percentage to a decimal, eliminate the percent sign and place a decimal point two places to the left. If the percentage is only one digit, place a 0 in front of it and place the decimal point in front of the 0. For example:

76 percent would be 0.76

4 percent would be 0.04

104 percent would be 1.04

Exercise 2.4

Complete the following conversions.

Fractions to percentages (round to one decimal place):

1. $\frac{3}{4}$
2. $\frac{4}{5}$
3. $\frac{7}{8}$
4. $\frac{9}{10}$
5. $\frac{1}{15}$

Decimals to percentages:

6. 0.75
7. 0.09
8. 0.16
9. 0.325
10. 0.965

Percentages to fractions:

11. 35%
12. 82%
13. 25%
14. 4%
15. 33%

(continued on next page)

Exercise 2.4 (continued)

Percentages to decimals:

16. 17%
17. 3%
18. 0.5%
19. 118%
20. 99%
21. A pediatrician in your local physician's clinic saw 50 children in one week for their preventive well-baby checkup. 32 received the MMR vaccine.

    Express the rate of the MMR administration in percent. Round to one decimal place.
22. During the year 2005, there were 884,974 physicians of various specialties in the United States. The following specialties made up a part of that figure. Using the information below, what percentage of the whole did these specialties make up? Round to two decimal places.

    - Forensic Pathologists 620
    - Medical Genetics 476
    - Ophthalmologists 18,706
    - Internal Medicine 150,933
23. The physician practice you work for needs a new shredder. They want a cross-cut shredder that can cut 20 sheets at a time. It must be strong enough to destroy staples and small paper clips with a waste bin of at least eight gallons. You did some investigation and secured five offers from different companies. The shredders are all of equal quality. Using the information below, which company is giving you the best deal?

	List Price	Discount	Cost of Two-Year Replacement Warranty	Shipping and Handling	TOTAL COST
Company A	$579.00	20%	$20.00	Local $00.00	
Company B	$625.00	30%	$25.00	$25.00	
Company C	$600.00	20%	$20.00	Free $00.00	
Company D	$551.00	15%	$18.00	$33.00	
Company E	$584.00	25%	$25.00	$30.00	

Exercise 2.5

Using the information found in the scenario below, answer the following questions.

Forty patients were seen in the Hematology/Oncology Clinic last Tuesday. Twenty patients had sickle-cell anemia, twelve patients had hemophilia, six patients had Ewing's sarcoma and two had Wilms' tumor.

1. Express in fractions the number of patients with each condition compared to the number of patients who visited the clinic last Tuesday. Remember to convert each fraction to its simplest form.

Sickle-Cell	Hemophilia	Ewing's Sarcoma	Wilms' Tumor

2. Express in decimals the number of patients with each condition compared to the number of patients who visited the clinic last Tuesday.

Sickle-Cell	Hemophilia	Ewing's Sarcoma	Wilms' Tumor

3. Express in percents the number of patients with each condition compared to the number of patients who visited the clinic last Tuesday.

Sickle-Cell	Hemophilia	Ewing's Sarcoma	Wilms' Tumor

Ratio

Three general classes of mathematical parameters are used to relate the number of cases, diseases, patients, or outcomes in the healthcare environment to the size of the source population in which they occur. The most basic measure is the **ratio.** A ratio expresses the relationship of one quantity to another.

Calculating Ratio

To calculate a ratio, one quantity is divided by another. The number can be greater than 1 or less than 1. For example, if seven men and five women were in a group, the ratio of men to women would be $\frac{7}{5}$. This ratio also may be written as 7:5 and verbalized as 7 to 5. The numbers 7 and 5 have no common factors, so this ratio cannot be simplified. However, if the group consisted of 6 men and 10 women, for example, the ratio would be 6:10. Because the numbers in this ratio have a common factor of 2, the ratio can be simplified by dividing

each number by 2, which simplifies the ratio to 3:5. This is done the same as before in the section on fractions. This is converted by dividing both the top and bottom numbers by a common factor. In this example, both numbers can be divided by 2.

$$\text{Ratio} = \frac{x}{y} = \frac{6}{10} = \frac{\frac{6}{2} = 3}{\frac{10}{2} = 5}$$

or

$$\text{Ratio} = \frac{y}{x} = \frac{10}{6} = \frac{\frac{10}{2} = 5}{\frac{6}{2} = 3}$$

Exercise 2.6

Express the following ratios in their simplest form.

1. 14:28
2. 3:12
3. 5:10
4. 7:49
5. 1:16
6. A group of 12 men and 18 women have diabetes. Express the ratio of men with diabetes to women with diabetes. Calculate it to its simplest form.

Proportion

A **proportion** is a type of ratio in which x is a portion of the whole $(x + y)$. In a proportion, the numerator is always included in the denominator. For example, if two women out of a group of 10 over the age of 50 have had breast cancer, where $x = 2$ (women who have had breast cancer) and $y = 8$ (women who have not had breast cancer), the calculation would be 2 divided by 10. The proportion of women who have had breast cancer is 0.2.

$$\frac{x}{(x+y)} = \frac{2}{(2+8)} = 0.2$$

Exercise 2.7

1. A school district wants to know the proportion of students who have deferrals for mandated vaccines. School #2 has 450 students. Of the 450 students, 435 students are up to date on their vaccines. There are 15 students with deferrals. What is the proportion of students with deferrals who have not been vaccinated? Round to two decimal places.
2. In a group of 50 persons, 12 have Type II diabetes mellitus.

   What is the proportion of people in the group that have diabetes? Express as a decimal and round to two decimal places.

Rate

A **rate** is a ratio in which there is a distinct relationship between the numerator and denominator and the denominator often implies a large base population. A measure of time is often an intrinsic part of the denominator. Healthcare facilities calculate many types of rates in order to determine how the facilities are performing.

> **Handy Tip:** Misplaced decimal points can result in mathematical errors. All calculations should be checked for sensible answers. For example, a hospital death rate of 25 percent should seem unreasonable because it indicates that one of every four patients treated at this hospital died. Why would anyone want to be treated at a hospital that had a 25 percent death rate? Thus, the decimal placement in this calculation should be checked. The correct death rate for this hospital may be 2.5 percent or 0.25 percent, which would be more realistic.

The term *rate* is often used loosely to refer to rate, proportion, percentage, and ratio. Indeed, many books and organizations use these terms interchangeably. For this reason, it is important to be aware of how any measure being reported has actually been defined and calculated.

Calculating Rate

The basic rule of thumb for calculating rate is to indicate the number of times something *actually* happened in relation to the number of times it *could have* happened (actual/potential). For example, let's say you have been eating out often in the past few weeks. To calculate the rate of meals you have eaten out in one week, divide the number of meals that you ate out (for example, 13) by the number of meals you could have eaten out (21). The rate is $\frac{13}{21}$, or 61.9 percent. The formula for determining rate is as follows:

$$Rate = \frac{Part}{Base}, \text{ or } R = \frac{P}{B}$$

Table 2.1 shows the equations for calculating ratio, proportion, percentage, and rate.

Table 2.2 shows a sample computerized statistical report provided by the information systems (IS) department of an acute care facility and illustrates how the department uses percentages.

> **Handy Tip:** Everyone has heard this saying about computers: "garbage in, garbage out," or "GIGO." Computers are great for many things, including performing statistical calculations, but they must be programmed accurately to calculate correctly. Even the function of rounding needs to be validated.

Table 2.1. Review of equations for calculating ratio, proportion, percentage, and rate

The following list of equations differentiates among ratio, proportion, percentage, and rate, where $x = 5$ men and $y = 3$ women.

Ratio: $\frac{x}{y} = \frac{5}{3}$

Proportion: $\frac{x}{(x + y)} = \frac{5}{(5 + 3)}$

Percentage: $\left[\frac{x}{(x + y)}\right] \times 100 = \left[\frac{5}{(5 + 3)}\right] \times 100$

Rate: $R = \frac{Part}{Base}$ or $R = \frac{P}{B}$

Table 2.2. Administrator's semiannual reference report

Administrator's Semiannual Reference Report Admissions by Day of Week 1/1/20XX–6/30/20XX		
Day	**Number of Patients**	**Percent of Patients**
Sunday	1,187	19.1
Monday	755	11.3
Tuesday	1,085	16.3
Wednesday	1,031	15.5
Thursday	1,024	17.0
Friday	808	12.1
Saturday	773	11.6
Total	**6,663**	**100.0**

For example, it is not unheard of for an IS manager to ask a health information management (HIM) professional for the formula for death rate because the computer system crashed and all new formulas and specifications had to be reprogrammed.

Averages

An **average** is the value obtained by dividing the sum of a set of numbers by the number of values. Average is generally referred to as the arithmetic mean to distinguish it from the mode or median. (This is covered in more detail in chapter 10.)

The symbol $\overline{X}$ (pronounced "ex bar") is used to represent the mean in this formula.

$$\frac{\textit{Sum of all the values}}{\textit{Number of all the values involved}} = \overline{X}$$

Example: Let's say that you have taken six medical terminology tests. Your scores are 82, 78, 94, 56, 91, and 85. Adding together the scores gives you a total score of 486. Now, divide this by 6 (the number of tests you have taken). This equals 81. This means that your average score on the medical terminology tests is 81.

Exercise 2.8

The following were the recorded birth weights for babies born Jan. 22, 20XX: 5.5 lbs, 5.0 lbs, 7.7 lbs, 8.9 lbs, 4.6 lbs, 7.3 lbs, 6.5 lbs, 6.8 lbs, 8.0 lbs, and 8.13 lbs.

What was the average birth weight for the day? Round to two decimal places.

Chapter 2 Test

Complete the following exercises.

1. Convert the fraction $\frac{1}{5}$ to a quotient and then a percentage.

Chapter 2 Test (continued)

2. Round the following percentages to two decimal places.
   a. 15.894%
   b. 13.256%
   c. 0.765%
   d. 0.068%
   e. 56.325%
3. Convert $\frac{1}{6}$ to a percentage to two decimal points.
4. Review table 2.2 to verify that the calculations are correct. Note that the percent of patients listed in this report is the actual/potential.

$$\left(\frac{\textit{Actual \# of patients}}{\textit{Potential \# of patients}}\right) \times 100$$

5. It was reported in your department meeting that over the past year your hospital increased the number of employees by 12 percent. Last year there were 347 people employed; how many are employed this year? (Round to a whole number.)
6. Your manager needs to purchase a personal computer for the new receptionist in your department. The usual price is $1,100. The local supply company gives the facility an 11.5 percent reduction on all items they purchase. What price will your manager pay?
7. Your beginning salary as an analyst in the HIM department is $12.50 per hour. You are due to receive a 3.5 percent cost-of-living raise in your next paycheck. Your performance evaluation is coming up in one month, and you believe you should get an additional 4 percent increase based on your excellent performance. What should your hourly wage be after your next paycheck, and what do you anticipate it will be after your performance evaluation?
8. Last year, you purchased equipment in the HIM department for $10,250. You have been told that the equipment you bought has depreciated in value by 15 percent. What is the value of the equipment now?
9. You just scored 15 correct out of a possible 40 on your health information test. What percentage did you earn?
10. Last year, the number of hospitals in your state decreased from 320 to 240. What is the percentage of decrease?
11. During the last week Community Hospital reported that the following numbers of patients were discharged:

    Sunday—18; Monday—22; Tuesday—37; Wednesday—25; Thursday—22; Friday—28; Saturday—12

    What is the average number of patients discharged during the week? (Round to a whole number.)

For the following questions, refer to the Quarterly Coder Accuracy Report on the next page.

12. Are the calculations of the percentage of records accurately coded for each month and the total for the quarter correct?
13. Coder D determined her accuracy rate for the quarter to be 95.9 percent. She would like you to recalculate her accuracy rate because she thinks it is incorrect in the report.

(continued on next page)

Chapter 2 Test (continued)

Community Hospital
Quarterly Coder Accuracy Report
January–March 20XX

	January			February			March			Total		
	# Records	# Correct	% Correct	# Records	# Correct	% Correct	# Records	# Correct	% Correct	# Records	# Correct	% Correct
Coder A	560	504	90.0%	544	495	91.0%	270	243	90.0%	1,374	1,242	90.4%
Coder B	540	503	93.1%	523	491	93.9%	531	494	93.0%	1,594	1,488	93.4%
Coder C	500	440	88.0%	445	401	90.1%	493	435	88.2%	1,438	1,276	88.7%
Coder D	620	583	94.0%	588	570	96.9%	584	551	94.3%	1,792	1,704	95.1%
Coder E	480	408	85.0%	432	392	90.7%	465	397	85.4%	1,377	1,197	86.9%
Total	**2,700**	**2,438**	**90.3%**	**2,532**	**2,349**	**92.8%**	**2,343**	**2,120**	**90.5%**	**7,575**	**6,907**	**91.2%**