Spectrum of Mass of Neutrons' Hollow Super-Massive Clusters with High-Temperature

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ABSTRACT

The attempt to construct the gaseous cluster's model of high-temperature gravitating particles, being confined at the restricted area by self-consistent field, has been made in the paper. The complete system of equations, being solved with respect to potential, can be reduced to a three-dimensional equation. The fundamental conservation law of total pressure, existing in the system with plane symmetry, forms the field boundary conditions. They define the existence of equipotential surface, where the pressure of self-consistent field turns into zero and potential has the minimal value. The conservation law points out on compensation of gravitational force by Bernoulli force at any unit volume of the system. In the spherical symmetry the same three-dimensional equation comes to the known Emden's *E*-equation. The mathematical problem of bounder conditions' choice under integration of this equation is discussed. Two classes of analytical solutions, describing the distributions of fields and particles in spherical systems with potential well and potential slot, have been found. The possibility of existence of neutrons' hollow clusters, which mass spectrum for temperature span of 10^{11} - 10^{12} K can be identified with mass spectrum and dimensions of objects, being observed in the centers of galaxies and named as "super-massive black holes", follows from solutions obtained.

Keywords: black hole physics, dense matter, equation ofstate, gravitation, stars neutron, Galaxy nucleus

1. INTRODUCTION

Emden *E*-equation, derived in [1], belongs to the class of non-linear ordinary second order differential equations. Together with Lane-Emden equation it has played an important role at the first stage of stellars' structure determination [2]. Stars have been considered as gaseous formations that stay either in polytropic equilibrium or in the balance with uniform temperature distribution. While solving the spherical problem of density distribution in a system with uniform temperature, Emden hasn't found the analytical solution, which would reach a maximum density in the centre of the sphere.

Independently of Emden's research and, seemingly, was being unaware of it, Frenkel in 1948 introduced the similar method of calculating the fields of gravitating particle for systems with permanent temperature, and named the macroscopic fields created by them as self-consistent ones [3]. While Emden derived the equations for the density of stellar matter, Frenkel expressed them in terms of potential of gravitational field formed by the cluster. Solving the problem of density distribution in a spherical cluster, he came to unexpected conclusion that this exact solution obtained had led to meaningless physical results.

As it is shown in the notice, the correct physical interpretation of analytic solutions, existing in spherical symmetry, can be obtained after detail analysis of conditions and reasons of substance's equilibrium with self-consistent field at the plane symmetry only.

2. THE MAIN EQUATIONS OF THE PROBLEM

To generalize Emden's paper [1] and to exploit Frenkel's approach [3], let us write three-dimensional equations of static gravitation in the modern notation of vector analysis

$$\rho \boldsymbol{g} + \boldsymbol{f} = 0 ; \tag{2.1}$$

$$\nabla \cdot \boldsymbol{g} = -4\pi\gamma\rho ; \tag{2.2}$$

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$$\boldsymbol{g} = -\nabla \varphi ; \tag{2.3}$$

$$p = \rho kT / m ; \tag{2.4}$$

$$\boldsymbol{f} = -\nabla p . \tag{2.5}$$

There ρ is the mass density of elementary unit, $\boldsymbol{g}$ – the strength of macroscopic collective gravitational field, p – the gas-kinetic pressure inside the system, T – the absolute temperature of the system, φ – the potential of self-consistent field, γ – the gravitation constant, m – the mass of gravitating particle, k – Boltzmann constant, $\boldsymbol{f}$ – Bernoulli's force.

The first equation of the system represents the balance condition for elementary volume of gravitating particles' system. The second one is the differential form of Newton's law that permits to calculate divergent static fields of smeared mass. The equation (2.3) relates the potential with the strength of gravitation field, and (2.4) – the equation of state with uniform temperature. The equation (2.5) defines Bernoulli's gas-static force. Let's notice, that the relation between the strength of the field and the potential (2.3) was not been applied by Emden in his study.

Let us show that the complete system of equations (2.1-2.5) describes the collective interaction among gravitating particles, where the back action of the field on the particles, that generate this field, is manifested.

3. SELF-CONSISTENT GAS-STATIC'S EQUATION OF NON-RADIATING GRAVITATING PARTICLES

Substituting (2.5) and (2.3) into (2.1), we get

$$\rho \nabla \varphi + \nabla p = 0 . \tag{3.1}$$

Take into consideration the equation of state (2.4) and the fact that the temperature is uniform, let us reduce (3.1) to the form

$$\nabla \left(\frac{m\varphi}{kT} + \boldsymbol{log}\, \rho \right) = 0 . \tag{3.2}$$

It is obvious from (3.2) that any equilibrium of gravitating particles with the uniform temperature is characterized by scalar integral

$$\frac{m\varphi}{kT} + \boldsymbol{log}\, \rho = \frac{m\varphi_0}{kT} + \boldsymbol{log}\, \rho_0 = const , \tag{3.3}$$

where ρ_0 and φ_0 are constants. Bolzmann distribution function follows from (3.3)

$$\rho = \rho_0\, \boldsymbol{exp}\left[-m(\varphi - \varphi_0) / kT \right] . \tag{3.4}$$

This function shows the back action of the field on the distribution of the particles.

By substituting (3.4) into (2.2) we express everything in terms of φ and convolve the system of equations (2.1) – (2.5) into one equation (i.e., execute the matching)

$$\Delta\varphi = 4\pi\gamma\rho_0\, \boldsymbol{exp}\left[-m(\varphi - \varphi_0) / kT \right] . \tag{3.5}$$

The equation (3.5) is three-dimensional field analog of Emden's *E*– equation. This equation describes the distribution of macroscopic potential in dynamic systems of particles with uniform temperature. The particles are in static equilibrium with the self-consistent field. The positive sign of the right-hand side of equation (3.5) points that the system consists from the particles, which interact according to the Newton's law. The equation (3.5) was given by Frenkel in [3].

4. FIRST INTEGRAL OF EMDEN'S *E*– EQUATION FOR PLANE-SYMMETRIC CASE

The equation (3.5) for plane-symmetric case has the form

$$\varphi'' = 4\pi\gamma\rho_0\, \boldsymbol{exp}\left[-m(\varphi - \varphi_0) / kT \right] , \tag{4.1}$$

where the primes denote derivatives with respect to the *x*-coordinate. The order of the equation (4.1) can be reduced. It has a first integral, corresponding to the total pressure of the system [4]:

$$\frac{(\varphi')^2}{8\pi\gamma} + p(\varphi) = P = H(\varphi', \varphi) = const , \tag{4.2}$$

where $p(\varphi)=p_0\,\boldsymbol{exp}\left[-m(\varphi-\varphi_0)/kT\right]$ – the pressure of gravitating particles of the system at the plane with the potential φ, $p_0=\rho_0 kT/m=n_0 kT$ –the pressure of gravitating particles of the system at the plane $\varphi=\varphi_0$ and n_0 – the number density of particles at the plane $\varphi=\varphi_0$.

The total pressure *P* of the system in (4.2) consists of two summands: the first one represents the pressure of the self-consistent field of the system, and the second one – the pressure of the particles. The first integral (4.2) coincides with $H(\varphi',\varphi)$ Hamiltonian function, where generalized momentum $\varphi'/4\pi\gamma$ and generalized coordinate φ are canonically conjugated quantities. Coordinate *x* acts the role of generalized time.

Equality (4.2) is carried out under the condition of absence of any external static gravitational fields, considered with respect to the self-consistent field of the system. The class of even functions and their derivatives is always set in such way that the sum of pressures of the field and particles of the system would be invariant at any plane inside the system.

The conservation law (4.2) also means that at any plane of the system being considered the gradients of self-consistent field and particles of the system are equal and have opposite directions. The volume density of Bernoulli force (further denoted just as Bernoulli force) (2.5) is opposite to the pressure gradient of the particles. It receives a new mathematical definition in the problem, being considered. The Bernoulli force: 1) has the same value and direction as the pressure gradient of the self-consistent field; 2) compensates forces of gravitation; and 3) provides a class of equilibrium states of particles with the field, being generated by these particles.

Substituting the mass density from the equation (2.2) into (2.1), we derive the connection between the Bernoulli force and the pressure gradient of self-consistent field $\boldsymbol{G}$

$$f=-\rho\,\boldsymbol{g}=\frac{\boldsymbol{g}}{4\pi\gamma}(\nabla\cdot\boldsymbol{g})=\boldsymbol{G}\,, \tag{4.3}$$

which together with (2.5) gives the physical condition of confinement of the substance by self-consistent field

$$\boldsymbol{G}+\nabla p=0\,. \tag{4.4}$$

If the field of the system, being researched, is unary and plane, i.e. $\boldsymbol{g}=[g_x(x),0,0]$, the equality (4.4) has a form

$$G_x+\frac{\mathrm{d}p}{\mathrm{d}x}=\frac{g_x}{4\pi\gamma}\frac{\mathrm{d}g_x}{\mathrm{d}x}+\frac{\mathrm{d}p}{\mathrm{d}x}=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{g_x^2}{8\pi\gamma}+p\right)=0$$

and it reduces to total pressure integral (4.2).

Equality (4.4) points to the earlier unknown property of gravitation's self-consistent field: to hold an inhomogeneous system of particles in a restricted space by static forces of field origin.

It follows from this equality that the system of collectively interacting particles is in a static equilibrium with the self-consistent field of gravitation only when the sum of gradients of field pressure and particle pressure is equal to zero. This equality has to be satisfied in any arbitrarily elementary volume of the system.

5. DISTRIBUTION OF PHYSICAL PARAMETERS IN A PLANE-SYMMETRIC SYSTEM

After integrating (4.2) under such condition that the potential reaches the minimum $\varphi'=0$ with the value φ_0 (it's realized, when $P=p_0$), we put this minimum in the origin of coordinates *x=0* and derive the law of potential distribution along the coordinate

$$\varphi=\varphi_0+\varphi_*\,\boldsymbol{log}\left[\boldsymbol{cosh}(x/l)\right], \tag{5.1}$$

where

$$l=\sqrt{kT/(2\pi\gamma m\rho_0)}=\sqrt{kT/(2\pi\gamma n_0)}\,/\,m \tag{5.2}$$

is the spatial scale of the system and $\varphi_*=2kT/m$ is the scale of potential.

The potential distributions along the coordinate of the system have a form of potential wells with infinite walls, as one can see from (5.1). The wells have the minimum with a value φ_0 at the plane *x*=0.

The projection of the strength of the gravitational self-consistent field along the coordinate of the system is distributed according the following law:

$$g_x = -\varphi' = -g_0 \tanh(x/l), \tag{5.3}$$

where $g_0 = \varphi_* / l = 2kT/(ml)$ – scale of the strength. One can see from (5.3) that the field strength of the system vanishes at the plane x=0, and when $x/l \to \pm\infty$ $g_x \to \mp g_0$.

The system has blur borders, since the mass density, the number density and the pressure of particles have soliton-like distribution with maximum value at the bottom of the well

$$\rho/\rho_0 = n/n_0 = p/p_0 = \cosh^{-2}(x/l). \tag{5.4}$$

As it is seen from (5.3) and (5.4), the field pushes out the particles into the regions in which the potential energy of the system is minimal. In the regions with absence of the matter the value of field strength stays the same.

The distribution of the field pressure along the coordinate of the system follows from (5.3):

$$D = (\varphi')^2/(8\pi\gamma) = D_0 \tanh^2(x/l), \tag{5.5}$$

where $D_0 = p_0 = g_0^2/(8\pi\gamma)$ is the scale of pressure.

One can see from (5.5) and (5.4) that the sum of pressures of particles and field of the system at any plane of interaction space stays constant and equal to the total pressure of the system P=p_0, which is the integral of the system. The result of derivation (5.5) shows that the pressure gradient of the field at any plane of the system is opposite to the particles' pressure gradient, following from (5.4), and is equal to field's pressure gradient by modulo:

$$\mathrm{d}D/\mathrm{d}x = -\mathrm{d}p/\mathrm{d}x = f_0 \tanh(x/l)/\cosh^2(x/l), \tag{5.6}$$

where $f_0 = 2p_0/l$ – the scale of pressure gradient.

The directions of gradients permit to ascertain the directions of the volume forces that hold the system in balance. The forces of gravitation, compressing the system of the particles, are directed towards the plane x=0 and coincide with the direction of $\boldsymbol{g}$ vector. Bernoulli forces, expanding the system, are being created by pressure gradient of self-consistent field (4.3).This gradient compensates the action of pressure gradient of the particles.

The mathematical equalities (2.5) and (4.3) point the dual role of self-consistent field, generating the configuration of the trap. On one hand, the field creates the pressure gradient in the substance which is co-directional with the vector of field strength (2.3). On the other hand, this field creates the static force (4.3), which compensates the arising gradient.

One can formulate the field boundary conditions that are adequate to the problem considered as following: there must be a surface in the system, where the potential is minimal and the pressure of self-consistent field vanishes.

In the plane-symmetric case this surface is at the plane x=0. As it is shown in the next section, the formulated initial conditions in the spherically-symmetric case can be realized at the finite distance from the centre of the system only.

6. SPHERICAL FIELD TRAP OF FIRST KIND

Let us write down the equation (3.5) for spherical symmetry, taking into account only the radial dependence of the potential:

$$\varphi'' + 2\varphi'/r = 4\pi\gamma m n_0 \,\boldsymbol{exp}\left[-m(\varphi-\varphi_0)/kT\right], \tag{6.1}$$

where $n_0 = \rho_0/m$ is the value of number density of particles of the system at the sphere $\varphi = \varphi_0$, and primes denote derivatives with respect to r.

In (6.1) passing on to the function $y(x) = (\varphi_0 - \varphi)/\varphi_*$ with respect to variable x=r/R, where R is the radius of the sphere on which field boundary conditions are being specified, we reduce equation (6.1) to the form

$$xy'' + 2y' + \alpha^2 x\,\boldsymbol{exp}(2y) = 0, \tag{6.2}$$

where

$$\alpha = mR\sqrt{2\pi\gamma n_0/(kT)} = \sqrt{T_*/T} = R/l \tag{6.3}$$

is a new parameter of the state of the system and

$$T_* = 2\pi\gamma m^2 n_0 R^2 / k \qquad (6.4)$$

is a scale of the temperature (primes denote derivatives with respect to *x*). The parameter of state of the system *α* can be interpreted in two different ways. On one hand it allows one to compare the temperature of the system with its scale. On the other hand it allows one to compare the radius of the sphere on which the boundary conditions are specified with the spatial scale of the system *l* (5.2).

Equation (6.2) belongs to the class of *E*-equation of Emden. It describes heterogeneous distribution of the substance in the gaseous sphere with uniform temperature and is the field analog of *E*-equation. As it is known, it has no exact solutions in the elementary functions for the boundary conditions, representing physical interest.

6.1 Approximate solutions of the problem

Let us look for approximate solutions of (6.2) with the field boundary conditions $x=1,\ y(1)=0,\ y'(1)=0,$ which assume the existence of the sphere with zero field pressure in the cluster. These solutions describe the distribution of physical parameters of the system in the field traps of fist kind.

Pass on to the new function

$$y(x)=\eta(\xi)-\xi, \quad y' = \mathrm{d}\xi/\mathrm{d}x(\mathrm{d}\eta/\mathrm{d}\xi - 1),$$

where $\xi = \log x,$ we obtain the equation

$$\mathrm{d}^2\eta/\mathrm{d}\xi^2 + \mathrm{d}\eta/\mathrm{d}\xi = 1-\alpha^2 \exp(2\eta) \qquad (6.5)$$

withinitial conditions ξ=0, $\eta(0)$=0, $\mathrm{d}\eta/\mathrm{d}\xi = 1,$ which assumes the reduction of order by introduction of new function

$$p(\eta) = \mathrm{d}\eta/\mathrm{d}\xi; \quad \mathrm{d}^2\eta/\mathrm{d}\xi^2 = p\mathrm{d}p/\mathrm{d}\eta\ .$$

The first-order differential equation

$$p\mathrm{d}p/\mathrm{d}\eta + p = 1-\alpha^2 \exp(2\eta) \qquad (6.6)$$

has η=0, $p(0)$=1 as new initial conditions. The integration of this equation in elementary functions by the author has no success.

The numerical solution of equation (6.6) in [5] has been executed for a set of integral curves which pass through the initial conditions' point. The solution shows that a singular point of Emden exists under $\eta=\eta_s>0$. In this point $p \to 0_+$ and $\mathrm{d}p/\mathrm{d}\eta \to -\infty$. Figure 1 shows four integral curves in $p = p(\eta)$ coordinates. The curve 1 has been calculated for *α*=0.5; the curve 2 – for *α*=1.0; the curve 3 – for *α*=1.5; the curve 4 – for *α*=2.0.

As one can see from figure 1, the position of Emden's singular point $\eta=\eta_s$ depends on the value of α^2. For small α^2 this point is located far from the origin of coordinates $z=\eta=0$. For values α^2>>1 the singular point approaches the origin of coordinates from the right. This allows one to find an approximate solution (6.6) under condition α^2>>1.

Let us solve (6.6) with respect to derivative:

$$\mathrm{d}p/\mathrm{d}\eta = [1-\alpha^2 \exp(2\eta)]/p - 1. \qquad (6.7)$$

Under condition

$$\alpha^2 \exp(2\eta)/p >> 1/p - 1 \qquad (6.8)$$

the equation (6.7) can be shortened

$$\mathrm{d}p/\mathrm{d}\eta \approx -\alpha^2 \exp(2\eta)/p \qquad (6.9)$$

and integrated in the elementary functions

$$p \approx \sqrt{1-\alpha^2[\exp(2\eta)-1)]}\ . \qquad (6.10)$$

To define values *α*, for which approximation (6.8) is valid, let us put (6.8) in the following form:

$$\alpha^2 >> (1-p)\exp(-2\eta) = f_1(\eta)\ .$$

The most value of function $f_1(\eta)$ from the right side of inequality is achieved at the value $\eta=\eta_s$, which is the singular point of Emden. Then approximation (6.8) is realized when

$$1/(\alpha^2+1) << 1\ , \qquad (6.11)$$

that is satisfactory even for *α*=3 and is improved with α growing.

Coming back to the original function $\varphi(r)$ in (6.10), we obtain two-parameter law of potential distribution for spherical case, when the temperature of the system is less than the scale (cold cluster):

$$\frac{\varphi}{\varphi_*} \approx \frac{\varphi_0}{\varphi_*} + \log\left[\frac{\alpha r}{R\sqrt{1+\alpha^2}}\cosh(A)\right], \tag{6.12}$$
$$A = \text{Apeacosh}\left(\frac{\sqrt{1+\alpha^2}}{\alpha}\right) - \sqrt{1+\alpha^2}\log\left(\frac{r}{R}\right),$$

whereα and φ_0 are the parameters of distribution.

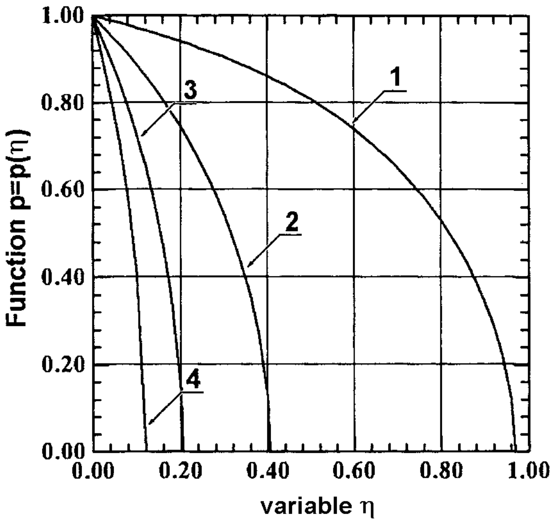

Fig. 1.The integral curves p=p (η).

As it is seen from (6.12), the potential of self-consistent field, being created by cluster's particles, has a form of potential well with infinite walls and with minimum $\varphi = \varphi_0$ at the sphere of zero field pressure*.*

The expression (6.12) allows one to derive an analytic form of the main gravity-static and kinetic characteristics of the cluster in the case when α^2>1. The projection of *r*-component of the strength of self-consistent field is derived from (6.12) and has the following form:

$$g_r / g_* \approx RB / r, \tag{6.13}$$

where $g_* = \varphi_* / R$ is a new scale of the strength and $B = \sqrt{1+\alpha^2}\tanh(A) - 1$.

The distribution of the mass density, the number density and the pressure of particles in the cluster are the following:

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} = \frac{n}{n_0} \approx \left[\frac{\alpha r}{R\sqrt{1+\alpha^2}}\cosh(A)\right]^{-2}. \tag{6.14}$$

The pressure of self-consistent field inside the cluster follows the law:

$$D = \frac{g_r^2}{8\pi\gamma} \approx D_* \frac{R^2 B^2}{r^2}, \tag{6.15}$$

where $D_* = p_0 / \alpha^2 = g_*^2 / (8\pi\gamma)$ is a new scale of the field pressure.

The projection of radial component of Bernoulli's compensating force has the form

$$f_r = -\frac{\mathrm{d}p}{\mathrm{d}r} \approx -\frac{f_* R^3 (1+\alpha^2) B}{\alpha^2 r^3 \cosh^2(A)}, \tag{6.16}$$

where $f_* = 2p_0 / R$ is a new scale of pressure gradient.

The radial projection of field's pressure gradient has been distributed by the law

$$G_r = \frac{g_r}{4\pi\gamma r^2}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 g_r\right)\right] = \frac{4D}{r} + \frac{\mathrm{d}D}{\mathrm{d}r} \approx \frac{f_* R^3 B}{\alpha^2 r^3}\left(B - \frac{1+\alpha^2}{\cosh^2 A}\right). \tag{6.17}$$

The condition of substance's confinement by self-consistent field (4.4) assumes the form

$$G_r + \frac{\mathrm{d}p}{\mathrm{d}r} \approx \frac{f_* R^3 B^2}{\alpha^2 r^3} = 0 \,. \tag{6.18}$$

As it is seen from (6.12) – (6.16), the sphere of zero field pressure divides the entire interaction space of the cluster in two regions: the interior $0 < r / R < 1$ and the exterior $r / R \geq 1$ one. In the interior region the strength of self-consistent field and radius-vector have the same direction. When *r* grows, the pressure and the number density of particles in this region increase while the potential decreases.

In the exterior region the direction of field's strength vector is opposite to the direction of radius-vector. When *r* grows the potential increases, while the pressure and the number density of particles in this region decrease. The system has blur borders. Since the field pushes out the substance to the region with minimal potential energy, so the hollow, where the substance is virtually absent, is formed inside the cold cluster.

To analyse the behavior of the system close to the bottom of potential well one will introduce ξaxis, directedalong the radius-vector $r=R+\xi$. Expanding into Taylor series (6.12) accurate within quadratic term by the small $\xi / R \ll 1$ parameter, we obtain the dependences of physical parameters:

$$\varphi(\xi) \approx \varphi_0 + \varphi_* \alpha^2 \xi^2 /(2R^2) \, ; \; g_\xi \approx -g_* \alpha^2 \xi / R \, ;$$

$$D(\xi) \approx D_* \alpha^4 \xi^2 / R^2 \, ; \; p(\xi) \approx p_0 \left(1 - \alpha^2 \xi^2 / R^2\right);$$

$$P = D(\xi) + p(\xi) = p_0 = const \, ; \; \frac{\mathrm{d}}{\mathrm{d}\xi}\left[D(\xi) + p(\xi)\right] = 0 \,.$$

The condition of confinement (6.18) is executed very well in the thin quasi-plane layers close to the bottom of potential well ($B^2 \approx 0$).The condition of confinement is also executed under $r \to \infty$ in the system's singular point, because (6.18) tends to zero by $\sim r^{-3}$ law:

$$G_r + \frac{\mathrm{d}p}{\mathrm{d}r} \sim \frac{f_* R^3}{\alpha^2 r^3}\left(1 + \sqrt{1+\alpha^2}\right)^2 \to 0_+ \,.$$

In the other singular point of system the condition of confinement doesn't execute, because under $r \to 0_+$

$$G_r + \frac{\mathrm{d}p}{\mathrm{d}r} \sim \frac{f_* R^3}{\alpha^2 r^3}\left(\sqrt{1+\alpha^2} - 1\right)^2 \to +\infty .$$

It points to the limited applicability of approximate solution in the region, which abuts on the origin of coordinates. It has been shown in [6] that approximate solution obtained (6.12) is exact for any α in the cylindrical symmetry.

The estimation of the number of cluster's particles, being coupled by self-consistent field, is obtained after integration of expression

$$N_1 = \int_0^\infty n(r) 4\pi r^2 \mathrm{d}r = \frac{3n_0 V(1+\alpha^2)}{\alpha^2 R} \int_0^\infty \mathrm{d}r / \cosh^2(A) \tag{6.19}$$

in which (6.14) has been substituted. In (6.19) $V = 4\pi R^3 / 3$ is the volume of the sphere with zero field pressure and *A* value has been defined in (6.12). By substitution of variables in (6.19) the integration can be reduced to the calculation of ψ - function values, where $\Psi(z) = \Gamma'(z) / \Gamma(z)$ – the derivative of gamma-function logarithm. There we taking into account the existence of following integrals:

$$\beta(z) = \int_0^1 \frac{t^{z-1}\mathrm{d}t}{1+t} \,,$$

$$I(\mu) = \int_0^\infty \frac{\exp(\pm \mu t)\mathrm{d}t}{\cosh^2 t} = \mp \mu \, \beta(\mp \mu / 2) - 1 \,,$$

$$\beta(z) = \frac{1}{2}\left[\Psi\left(\frac{z+1}{2}\right) - \Psi\left(\frac{z}{2}\right)\right].$$

Not to violate the convergence of $I(\mu)$ integral, $-2 < \mu < 2$ condition with $\mu = (1+\alpha^2)^{-1/2} > 0$ must be executed. This condition is executed for the whole region of change of $3 \leq \alpha < \infty$ parameter. Then the number of particles (6.19) can be estimated from the following relation

$$N_1 = \frac{N_*}{2\alpha^2} \exp(\mu d)[\Psi(1/2 + \mu/4) - \Psi(\mu/4) - \Psi(1/2 - \mu/4) + \Psi(1 - \mu/4)] \,, \tag{6.20}$$

where $N_* = 3n_0 V$ – the scale of particles' number, $d = \mathrm{Apeacosh}\left(\sqrt{1+\alpha^2} / \alpha\right)$.The recurrent formula $\Psi(z+1) = \Psi(z) + 1/z$ has beenexploited for reception of (6.20). We shall transform (6.20) to the form

$$N_1 = \frac{N_*}{2\alpha^2} exp(\mu d)[\Psi(3/2+\mu/4)+\Psi(2-\mu/4)$$
$$-\Psi(1+\mu/4)-\Psi(3/2-\mu/4)+\frac{8\mu}{4-\mu^2}+\frac{8(2-\mu)}{\mu(4-\mu)}]. \quad (6.21)$$

In (6.21) the table values of Ψ - function, assigned in the interval from 1 to 2, can be used. It is seen from (6.21) that reduced number of particles, being confined by the field, at $\alpha \to \infty$ decreases according to $\frac{N_1}{N_*} \sim \frac{2}{\alpha}$ law.

The additive mass of the cluster can be easy estimated from
$$M_1 = mN_1 = M_0 f(\alpha), \quad (6.22)$$
where $M_0 = mN_*$ – the scale of mass of the first kind traps.

6.2 The results of numerical simulation

The numerical simulation of equation (6.1) (details are in [7]) has been executed for $\varphi(R)=\varphi_0=0,\ \varphi'(R)=0$ field initial conditions with the step 10^{-4}. The special formulas of Runge-Kutta method with 4-order accuracy have been chosen to do it. During simulation equal-zero condition of the sum (4.4) of field's pressure and particles' pressure gradients has been executed automatically. Here, under radial dependence, this condition is reduced to the equation (6.1).

Figure 2 shows the radial distributions of normalized particle number density n/n_0 of a spherical cluster for the various parameters of state. The curve 1 has been calculated for the value α=0.5; the curve 2 – for α=0.7; the curve 3 – for α=1.0; the curve 4 – for α=3.0. From figure 2 one can see the way of filling the cluster by particles under the change of cluster's temperature in the vicinity of the temperature scale value $T \sim T_*$. The curve 4 shows the existence of hollow inside the cluster, and curves 1, 2, 3 point out the fact that when the temperature grows, the whole volume of cluster has been filled by the particles completely. The distribution function of particles' number density in the centre of the cluster and derivative of this distribution tend to zero for all curves.
The details of numerical simulation of radial distributions of spherical cluster's major physical parameters one can find in [7].

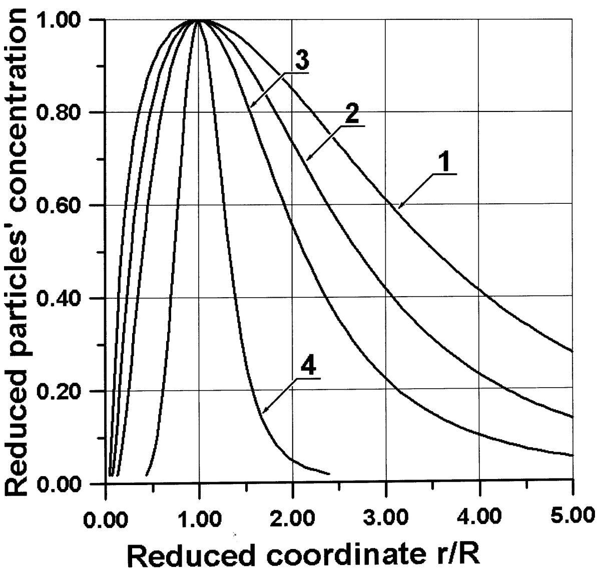

Fig. 2.The number density distribution of particles
In the field trap of first kind

The normalized values of the hollow r_1/R and external border r_2/R of the cluster have been submitted in comparison table 1. These values have been defined via approximate solutions and numerical simulation out of $n_{1,2} = 0{,}01 n_0$ condition.

The values of function $N_1 / N_* = f(\alpha)$ for the same values of state parameter *α* have been submitted in the table 1. Three lower rows contain the results of numerical simulation accurate within $1{\cdot}10^{-3}$, and three rows above them – the results, being given by approximate solution with the same accuracy.

Table 1. The results of numerical simulation

α	3	4	5	6	7	8	9	10
r_1/R	0,284	0,411	0,503	0,572	0,624	0,665	0,699	0,726
r_2/R	2,244	1,891	1,692	1,564	1,476	1,412	1,362	1,323
N_1/N_*	0,812	0,561	0,431	0,351	0,297	0,257	0,227	0,204
r_1/R	0,399	0,494	0,564	0,618	0,660	0,694	0,722	0,746
r_2/R	2,689	2,123	1,831	1,656	1,541	1,460	1,400	1,353
N_1/N_*	1,220	0,640	0,463	0,368	0,307	0,264	0,232	0,207

7. SPHERICAL FIELD TRAP OF SECOND KIND

The exact solution (6.2) for notation accepted has a form

$$y(x) = -\log(\alpha x)\,. \tag{7.1}$$

Coming back to the potential, we receive one-parametric law for $\varphi(r)$ distribution

$$\varphi = \varphi_0 + \varphi_* \log(r / l), \tag{7.2}$$

where*l* – the spatial scale (5.2), and φ_0 – the parameter of distribution.

As one can see from (7.2), potentials' difference and its equivalent potential energy of the system $U = m(\varphi - \varphi_0)$ have two signs. They are negative for the substance in the region $r / l < 1$ and are positive – in the $r / l \geq 1$ region.

The potential distribution has two singular points: $r = 0$, $r \to \infty$. The value of potential $\varphi = \varphi_0$ belongs to the sphere with $r = l$ radius. The potential energy has no familiar minimum, but it looks like infinitely deep logarithmic slot.

The basic gravity-static and kinetic characteristics of the second kind traps follow from (7.2). The projected radial component of the field strength has a form

$$g_r / g_0 = -l / r, \tag{7.3}$$

where $g_0 = \varphi_* / l$ coincides with the scale of field strength at the plane case. The strength of trap's self-consistent field has been always directed to the center of the system and has the singularity of ~ $-r^{-1}$ type at zero.

The system has been unlimited in the space. The density, the number density and the pressure of particles have the distribution with singularity of ~ r^{-2} type at zero

$$\rho / \rho_0 = n / n_0 = p / p_0 = l^2 / r^2\,. \tag{7.4}$$

As it is seen from (7.4), in traps of the second kind the field pushes out the particles into infinitely deep slot, formed by the potential energy in the origin of coordinates.

The substance of the cluster has disappeared at infinitely large distance from the centre of the system only. It points out at the principle difference of the second kind states and the states, observed earlier. In the field traps of second kind the number density is decreasing function of increasing radius.

The pressure of the field has singularity of ~ r^{-2} kind at zero

$$D / D_0 = l^2 / r^2\,, \tag{7.5}$$

where $D_0 = p_0 = g_0^2 / (8\pi\gamma)$ coincides with the scale of field pressure at the plane case.

The total pressure of the system is not the integral of equation now, as it varies by the law

$$P = D + p = 2p_0 l^2 / r^2 \neq const \, . \tag{7.6}$$

The pressure gradient of the particles has been always directed along the strength of system's self-consistent field and has singularity of ~ $-r^{-3}$ type at zero

$$\mathrm{d}p / \mathrm{d}r = -f_0 l^3 / r^3 \, , \tag{7.7}$$

where $f_0 = 2p_0 / l$ coincides with the scale of pressure gradient at the plane case.

The radial component of self-consistent field's pressure gradient (Bernoulli compensating force) has been directed opposite to the strength and has singularity of ~ r^{-3} type at zero

$$G_r = 4D / r + \mathrm{d}D / dr = f_0 l^3 / r^3 \, . \tag{7.8}$$

It is obvious from (7.7) and (7.8) that the condition of confinement of arbitrary elementary volume in the field trap of second kind has been realized at all points, except the singular one, being tended to $r \to 0_+$:

$$G_r + \mathrm{d}p / dr = 0 \, . \tag{7.9}$$

Due to this fact the system, occupying the infinity volume, can be limited artificially by choose of internal system's radius r_1 (radius of the hollow), inside which the substance is absent, and of external system's radius r_2, outside of which the substance is absent too.

Let us calculate from (7.4) the overall number of particles, being confined by the system

$$N_2 = \int_{r_1}^{r_2} n(r) 4\pi r^2 \mathrm{d}r = 2kTd / (m^2 \gamma) \, , \tag{7.10}$$

where $d = r_2 - r_1$ is the thickness of layer, occupied by the particles. It is seen from (7.10), as gravitating particle's mass is less, so the number of particles, being confined by cluster's field, is more under the other equal conditions.

The additive mass of the cluster is obtained from (7.10)

$$M_2 = mN_2 = 2kTd / (m\gamma) \, . \tag{7.11}$$

This mass increases with growing of the temperature, layer's thickness and turns out to be as more as the mass of gravitating particle is less.

8. THE SIMILARITY RELATION IN THE FIELD TRAPS

Let us study the similarity properties of the field traps, consisting from gravitating particles with identical masses, and express the spatial scale of length (5.2) in the form

$$l = C(T / n_0)^{1/2} \, , \tag{8.1}$$

where

$$C = m^{-1}(k / 2\pi\gamma)^{1/2} = const_1 . \tag{8.2}$$

Introduce the scale of mass of the field trap

$$M_* = 2kTl / m\gamma = FT^{3/2} n_0^{-1/2} \, , \tag{8.3}$$

where

$$F = m^{-2}(2/\pi)^{1/2}(k/\gamma)^{3/2} = const_2 . \tag{8.4}$$

By exception of number density from (8.1) and (8.3), we come to the linear connection between the spatial scale and the scale of mass under given value of the temperature:

$$l = EM_* / T \, , \tag{8.5}$$

Where $E = C/F = m\gamma / (2k) = const_3$.

The next similarity relations are followed from (8.5):

a) the relation of the scale of mass to the scale of length is the same for the systems with identical temperature. So, the scale of mass is greater, the scale of length is greater too. The inverse is also true.

b) the more hot object has the less spatial scale under identical scales of masses of objects. The inverse is also true.

c) the objects with identical relation of the scale of mass to the temperature have the same spatial scale.

By exception of object's temperature from (8.1) and (8.3) we obtain, that the scale of object's mass is always proportional to the scale of its $n_0 l^3$ particles' number:

$$M_* = 4\pi m n_0 l^3 . \qquad (8.6)$$

Let's attract from (6.22) the value of the scale of mass M_0 in the field traps of the first kind

$$M_0 = mN_* = 3n_0 mV = 4\pi R^3 m n_0 , \qquad (8.7)$$

determine the ratio of this scale to the scale of mass

$$M_0 / M_* = 2\pi R^3 m^2 n_0 \gamma /(kTl) = \alpha^3 \qquad (8.8)$$

and write the trap's mass in account with (8.8)

$$M_1 / M_* = \alpha^3 N_1 / N_* . \qquad (8.9)$$

It is seen from (8.9) that field traps with potential well have a large capacity. Their masses can exceed the scale of mass on one or two orders under change of the state parameter in $3 \le \alpha \le 10$ range and at the other identical conditions.

Let's discriminate three types of states, in which the field trap of second kind can stay. The first state is the state with negative potential energy, i.e. $U = m(\varphi - \varphi_0) < 0$. In this state the field trap has $r_2 < l$ external radius. The radius of internal hollow r_1 one can find from the largest values of number density of the particles and attainable in nucleus $n_1 \approx n_n \approx 10^{38}$ cm^{-3}.

Then the mass of such closely packed object can be submitted as

$$M_2 = M_*(\chi - \zeta) , \qquad (8.10)$$

where $\zeta = r_1 / l = \sqrt{n_0 / n_n}$ – normalized radius of the hollow and $\chi = r_2 / l = \sqrt{n_0 / n_2} < 1$ – normalized external radius. It is seen from (8.10) that the mass is $M_2 < M_*$ in such states.

The states with positive potential energy, having large hollow, are realized under $\zeta \ge 1$ condition. In such states under $\chi > 1 + \zeta$ the mass is $M_2 \ge M_*$ and under $0 < \chi < 1 + \zeta$ the mass is $M_2 \le M_*$.

At the intermediate states the ratio of mass can be an arbitrarily value under execution of the following inequalities: $0 < \zeta < 1$, $\chi > 1$. In the presence of close packing ($\zeta \to 0$) at the intermediate states the mass of object will be large under $\chi >> 1$ condition:

$$M_2 = M_* \chi . \qquad (8.11)$$

If the value of thickness of the layer, occupied by the particles, is equal to the (value of spatial scale $d=l$, so the variable is $\chi = 1 + \zeta$ and the mass is $M_2 = M_*$.

9. PARAMETERS OF THE FIELD TRAPS, CONSISTING OF NEUTRONS WITH HIGH TEMPERATURE

For the field trap, holding neutrons, the values of permanent coefficients are following: C=1,0837·10^{19} (cm K)$^{-1/2}$; F=2,6785·10^{34} g (cm K)$^{-3/2}$; E=4,0459·10^{-16}cm K g^{-1}. It is followed from (8.5) that for neutrons, being under the temperature T=10^{12} K, while the scale of mass M_* is changed from 2·10^{39} till 2·10^{42} g,the spatial scale l is changed in the limits of 8·10^{11} to 8·10^{14} cm. The inequalities limit the interval of possible changes of value n_0 in the range from 1,8·10^{20} till 1,8·10^{26} cm^{-3}. They are far from n_n - the average nuclear number density of the substance. The estimates, obtained from relations of similarity, permit to execute the correct calculations.

At the table 2 the dependences of the scales of the field traps from n_0 have been adduced in assumption that they consist of neutrons, heated to the temperature T=10^{11} K (i.e., on one or two orders more than in neutrons stars), and the value n_0 can changes in the range of 2,0·10^{20} to 2,0·10^{26} cm^{-3}.

At the table 3 the results of numerical simulation of super-massive clusters' parameters for the traps of first and second kinds, being under the same conditions as in the table 2, are placed. The next algorithm of simulation has been accepted. One must:

1. Calculate the spatial scale (5.2) by given n_0 and T.
2. Determine radius R of the sphere of zero field pressure for given α from the formula (6.3).
3. Break down the distribution of number density in the points with $n_{1,2} = 0{,}01 n_0$ and find normalized values of the hollow's radius r_1/R and external border's radius r_2/R, depending on the state parameter α (see the second and the third row from the bottom of the table 1 for the field traps of first kind).

Table 2. The dependence of scale of mass and spatial scale from number density

n_0(cm^{-3})	M_*(g)	l (cm)
$0{,}2\cdot10^{21}$	$0{,}599\cdot10^{41}$	$0{,}242\cdot10^{15}$
$0{,}2\cdot10^{22}$	$0{,}189\cdot10^{41}$	$0{,}766\cdot10^{14}$
$0{,}2\cdot10^{23}$	$0{,}599\cdot10^{40}$	$0{,}242\cdot10^{14}$
$0{,}2\cdot10^{24}$	$0{,}189\cdot10^{40}$	$0{,}766\cdot10^{13}$
$0{,}2\cdot10^{25}$	$0{,}599\cdot10^{39}$	$0{,}242\cdot10^{13}$
$0{,}2\cdot10^{26}$	$0{,}189\cdot10^{39}$	$0{,}766\cdot10^{12}$
$0{,}2\cdot10^{27}$	$0{,}599\cdot10^{38}$	$0{,}242\cdot10^{12}$

4. Calculate the scale of $N_* = 4\pi R^3 n_0$ particles' number.
5. Find the values of normalized number of particles $N_1(3)/N_* = 1{,}220$ under α=3 and $N_1(10)/N_* = 0{,}207$ under α=10 from the table 1 (the lower row).
6. Calculate the additive mass of the field trap of first kind from (6.22).
7. Assume that for the field trap of second kind the radius of the field trap's hollow coincides with r_1 and its external radius – with r_2.
8. Find the number of particles and the mass for the field trap of second kind from relations (7.10), (7.11).

We record into the first column of the table 3 the value of n_0, to the second one – the radius of hollow, to the third column – the external radius of cluster, to the fourth one – the number of particles in the trap of first kind, to the fifth – the number of particles in the trap of second kind, in the six column – the mass of cluster in the trap of first kind, related to the mass M_s of the Sun, to the seven - the mass of cluster in the trap of second kind, related to the mass M_s of the Sun.

It is seen from the table 3, that the mass of the field trap of first kind is more than the mass of the field trap of second kind under the same parameters. This distinction increases with α growth. The relation (8.9) explains the reason of it. The mass of the field trap of first kind increases with the growth of state parameter α. The increase of the number density n_0 leads to the decrease of the trap's mass and of external radius of the cluster (see 8.5) under the permanent α. The inverse is also true.

The simulation shows that the growth of cluster's temperature on the order (for identical state parameters) leads to the growth of mass of the field traps of second kind more than an order. At the same time the range of their masses exceeds the range of masses of the field traps of first kind under original temperature with negligible changing of external radius of the cluster. The external dimensions of clusters under the lower values n_0 exceed the mean radius of Solar system from 10-th to 18-th times only. The value of greater semi-axis of Pluto's orbit ~ $0{,}6\cdot10^{15}$ cm has been taken as the mean radius of Solar system.

Since it has been appeared in accepted algorithm, that ζ value is $\zeta = r_1 / l > 1$, so only masses of the field trap of second kind with positive potential energy have entered into the table 3. Under the temperature T=10^{11} K and at the close packing in intermediate states, the radius of the hollow is r_1=$3{,}42\cdot10^5$ cm and the parameter is ζ=$1{,}41\cdot10^{-6}$<<1 at the upper value of n_0. Then masses of the field traps of second kind get into $10^6 M_s < M_2 < 10^9 M_s$ range. They have the external radius r_2, which value gets into the range from $8\cdot10^{12}$ till $8\cdot10^{15}$ cm. Under identical conditions the parameter is ζ=$1{,}41\cdot10^{-9}$<<1 at the lower value of n_0, but the value of radius r_2 gets into the same range as above.

Table 3. The results of numerical simulation of super-massive clusters' parameters for the traps of first and second kinds

The state parameter α=3						
n_0, cm^{-3}	r_1, cm	r_2, cm	N_1	N_2	M_1/M_s	M_2/M_s
$0{,}2{\cdot}10^{21}$	$0{,}290{\cdot}10^{15}$	$0{,}195{\cdot}10^{16}$	$0{,}118{\cdot}10^{67}$	$0{,}246{\cdot}10^{66}$	$0{,}991{\cdot}10^{9}$	$0{,}207{\cdot}10^{9}$
$0{,}2{\cdot}10^{22}$	$0{,}918{\cdot}10^{14}$	$0{,}618{\cdot}10^{15}$	$0{,}373{\cdot}10^{66}$	$0{,}777{\cdot}10^{65}$	$0{,}314{\cdot}10^{9}$	$0{,}654{\cdot}10^{8}$
$0{,}2{\cdot}10^{23}$	$0{,}290{\cdot}10^{14}$	$0{,}195{\cdot}10^{15}$	$0{,}118{\cdot}10^{66}$	$0{,}246{\cdot}10^{65}$	$0{,}991{\cdot}10^{8}$	$0{,}207{\cdot}10^{8}$
$0{,}2{\cdot}10^{24}$	$0{,}918{\cdot}10^{13}$	$0{,}618{\cdot}10^{14}$	$0{,}373{\cdot}10^{65}$	$0{,}777{\cdot}10^{64}$	$0{,}314{\cdot}10^{8}$	$0{,}654{\cdot}10^{7}$
$0{,}2{\cdot}10^{25}$	$0{,}290{\cdot}10^{13}$	$0{,}195{\cdot}10^{14}$	$0{,}118{\cdot}10^{65}$	$0{,}246{\cdot}10^{64}$	$0{,}991{\cdot}10^{7}$	$0{,}207{\cdot}10^{7}$
$0{,}2{\cdot}10^{26}$	$0{,}918{\cdot}10^{12}$	$0{,}618{\cdot}10^{13}$	$0{,}373{\cdot}10^{64}$	$0{,}777{\cdot}10^{63}$	$0{,}314{\cdot}10^{7}$	$0{,}654{\cdot}10^{6}$
$0{,}2{\cdot}10^{27}$	$0{,}290{\cdot}10^{12}$	$0{,}195{\cdot}10^{13}$	$0{,}118{\cdot}10^{64}$	$0{,}246{\cdot}10^{63}$	$0{,}991{\cdot}10^{6}$	$0{,}207{\cdot}10^{6}$
The state parameterα=10						
n_0, cm^{-3}	r_1, cm	r_2, cm	N_1	N_2	M_1/M_s	M_2/M_s
$0{,}2{\cdot}10^{21}$	$0{,}181{\cdot}10^{16}$	$0{,}328{\cdot}10^{16}$	$0{,}740{\cdot}10^{67}$	$0{,}217{\cdot}10^{66}$	$0{,}623{\cdot}10^{10}$	$0{,}183{\cdot}10^{9}$
$0{,}2{\cdot}10^{22}$	$0{,}571{\cdot}10^{15}$	$0{,}104{\cdot}10^{16}$	$0{,}234{\cdot}10^{67}$	$0{,}687{\cdot}10^{65}$	$0{,}197{\cdot}10^{10}$	$0{,}578{\cdot}10^{8}$
$0{,}2{\cdot}10^{23}$	$0{,}181{\cdot}10^{15}$	$0{,}328{\cdot}10^{15}$	$0{,}740{\cdot}10^{66}$	$0{,}217{\cdot}10^{65}$	$0{,}623{\cdot}10^{9}$	$0{,}183{\cdot}10^{8}$
$0{,}2{\cdot}10^{24}$	$0{,}571{\cdot}10^{14}$	$0{,}104{\cdot}10^{15}$	$0{,}234{\cdot}10^{66}$	$0{,}687{\cdot}10^{64}$	$0{,}197{\cdot}10^{9}$	$0{,}578{\cdot}10^{7}$
$0{,}2{\cdot}10^{25}$	$0{,}181{\cdot}10^{14}$	$0{,}328{\cdot}10^{14}$	$0{,}740{\cdot}10^{65}$	$0{,}217{\cdot}10^{64}$	$0{,}623{\cdot}10^{8}$	$0{,}183{\cdot}10^{7}$
$0{,}2{\cdot}10^{26}$	$0{,}571{\cdot}10^{13}$	$0{,}104{\cdot}10^{14}$	$0{,}234{\cdot}10^{65}$	$0{,}687{\cdot}10^{63}$	$0{,}197{\cdot}10^{8}$	$0{,}578{\cdot}10^{6}$
$0{,}2{\cdot}10^{27}$	$0{,}181{\cdot}10^{13}$	$0{,}328{\cdot}10^{13}$	$0{,}740{\cdot}10^{64}$	$0{,}217{\cdot}10^{63}$	$0{,}623{\cdot}10^{7}$	$0{,}183{\cdot}10^{6}$

The non-radiating neutrons with high temperature can be in equilibrium states of particles with the field,and these states should be regarded asthe passive states of nuclei of galaxies.The neutrons, connected by self-consistent field, can radiate or absorb the energy in the next cases: 1) under absorption (ejection) of the substance by exter-nalsurface of cluster; 2) under transition of field traps of second kind from the states with positive potential energy to the states with negative potential energy and vice versa; 3) under transition from the field traps of the first kind to the field traps of the second kind and vice versa.

10. CONCLUSION

- The total system of equations, which describes the class of static equilibriums of non-radiating gravitating particles, has been proposed. These particles have been weighted in self-consistent field and interact between themselves.
- Bernoulli force, compensating the gravity, obtains a new mathematic definition. Bernoulli force is created by the force of field origin, coinciding in the direction and value with pressure gradient of self-consistent field of gravitation.
- Equilibrium distribution function of the system coincides with Bolzmann's one and points out at the back action of the field on particles, which create this field.
- The self-consistent field, forming configuration of the trap, plays dual role. From one hand the field creates pressure gradient of the particles, which is co-directional with the vector of field's strength. From another hand this field creates the static force, compensating gradient, being created.
- The fundamental law of total pressureconservation, existing in the system with plane symmetry, forms the field's boundary conditions. These conditions correspond to existence of equipotential surface in the system. The pressure of self-consistent field is equal to zero and the potential has minimal value at this surface.
- The analytic solutions have been discovered in spherical symmetry. They are following: one class of approximate solutions (the field traps of first kind) and one class of exact solutions (the field traps of second kind). They describe the distribution of physical parameters in the systems with potential well and potential slot accordingly.
- In the traps of first kind the field pushes out the particles in the region with minimal potential energy for large values of the state parameter. In result the hollow, where the substance is virtually absent, is formed inside the cold spherical cluster.
- In the traps of second kind the field pushes out the particles into infinitely deep logarithmic slot, formed by potential energy of the system in its centre.
- For the field traps the relations of similarity have been obtained. In these relations univocal connection between the spatial scale, the scale of mass and absolute temperature of the system has been discovered.
- The field traps with potential well hold more particles than the traps with potential slot under other identical conditions.
- The three types of states have been distinguished in the field traps. They are following: the states with negative potential energy, the states with positive potential energy and intermediate states.
- The possibility of existence of neutrons' hollow clusters, which mass spectrum for temperature span of 10^{11} - 10^{12} K can be identified with mass spectrum and dimensions of objects, being observed in the centers of galaxies and named as "super-massive black holes", follows from solutions obtained.
- The non-radiating neutrons with high temperature can be in equilibrium states of particles with the field, and these states should be regarded as the passive states of nuclei of galaxies.
- The neutrons, connected by self-consistent field, can radiate or absorb the energy in the next cases: 1) under absorption (ejection) of the substance by external surface of cluster; 2) under transition of field traps of second kind from the states with positive potential energy to the states with negative potential energy and vice versa; 3) under transition from the field traps of the first kind to the field traps of the second kind and vice versa.
- The known indirect methods of measurement of masses of nuclei in the centers of galaxies over the speed of nearest stars [8-9] don't permit to answer the question: in which states of the field trap these objects are?

ACKNOWLEDGMENTS

The authors express the gratitude to the Chief Executive Officer of The Institute of Magnetism of National Academy of Sciences and Ministry of Education of Ukraine (IM NAS and ME, city Kiev) academician of NAS of Ukraine V.G. Barjyahtar for sincere interest and moral support of the new scientific area: the cluster's states of the matter, being confined by the self-consistent field.

The authors are very thankful to the Vice-Chancellor of South-Russian State University of Economy and Service (SRSUES, city Shakhty)) professor Prokopenko N.N and to professor of Taganrog Institute of Technology of Southern Federal University Krutchinkiy S.G. for beneficial discussion of results, obtained in the paper.

The authors want to express the special thanks to Sapogina L.K for the help in translation of this paper.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

Authors' contributions

The work was executed in collaboration between the authors.

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