SOLUTIONS
Chapter 5

5.1. A 0.1 A current is passed through a 1 kΩ resistor for 5 minutes. The heat generated is lost to the surrounding air at 20°C. What is the increase in entropy of the atmosphere?

Find: Increase in entropy ΔS of the atmosphere.

Known: Current in resistor I = 0.1 A, resistance R_e = 1 kΩ, duration of current Δt = 5 min, surroundings temperature T = 20°C.

Assumptions: Temperature of atmosphere remains constant.

The power of the resistor is transferred to the surroundings as heat,

$$\dot{Q} = \dot{W}_e = I^2 R_e = (0.1\ \text{A})^2 \times 1000\ \Omega = 10\ \text{W},$$
$$Q = \dot{Q}\Delta t = 10\ \text{J/s} \times (60\ \text{s/min} \times 5\ \text{min}) = 3000\ \text{J}.$$

Using the definition of entropy change and the assumption that it remains constant in the area of the resistor, the entropy increase of the atmosphere is

$$\Delta S = \frac{Q}{T} = \frac{3000\ \text{J}}{(20 + 273.15)\ \text{K}} = 10.234\ \text{J/K}.$$

The increase in entropy of the atmosphere is 10.2 J/K.

5.2. A fireplace burns 6 kg of coal which supplies 15 MJ of heat for every kilogram consumed. Find the increase in the entropy of the surrounding air at 24°C.

<u>Find:</u> Increase in entropy ΔS of the surroundings.

<u>Known:</u> Mass of coal burned m = 6 kg, heat per unit mass of coal burned q = 15 MJ/kg, temperature of surroundings T = 24°C.

<u>Assumptions:</u> Temperature of surroundings remains constant.

The total heat transfer to the surroundings from burning the coal is

$$Q = 6 \text{ kg} \times 15 \text{ MJ/kg} = 90 \text{ MJ},$$

Then the entropy increase is

$$\Delta S = \frac{Q}{T} = \frac{90 \text{ MJ}}{(273.15 + 24)\text{K}} = 0.30288 \text{ MJ/K}.$$

The increase in entropy of the surroundings is 0.303 MJ/K.

5.7. A rigid tank with a volume of 0.5 m^3 initially contains air at 300 kPa and 400 K. It loses heat to the surrounding air at 300 K until it reaches equilibrium. Find the increase in entropy of the atmosphere.

Find: Increase in entropy ΔS of the atmosphere.

Known: Rigid tank volume V = 0.5 m^3, initial tank air pressure P_1 = 300 kPa, initial tank air temperature T_1 = 400 K, surrounding air temperature T_s = 300 K, final tank air temperature $T_2 = T_s$.

Assumptions: Air behaves as an ideal gas.

Properties: Gas constant of air R = 0.2870 kJ/kgK (A1), specific heat of air at 350 K c_v = 0.721 kJ/kgK (A4).

The mass of air in the tank can be found using the ideal gas equation in the initial state,

$$m = \frac{PV}{RT} = \frac{300 \text{ kPa} \times 0.5 \text{ m}^3}{0.2870 \text{ kJ/kgK} \times 400 \text{ K}} = 1.3066 \text{ kg}.$$

The heat absorbed by the surroundings is lost from the air in the tank and is related to the change in potential energy,

$$Q = -Q_t = -mc_v(T_2 - T_1) = 1.3066 \text{ kg} \times 0.721 \text{ kJ/kgK} \times (300 \text{ K} - 400 \text{ K}) = 94.206 \text{ kJ},$$

resulting in an entropy increase of the surroundings of

$$\Delta S = \frac{Q}{T_s} = \frac{94.206 \text{ kJ}}{300 \text{ K}} = 0.31402 \text{ kJ/K}.$$

The entropy of the surrounding atmosphere increases by 0.314 kJ/K.

6.3. An electric immersion heater placed inside a well-insulated tank is used to heat 500 l of water from 25°C to 50°C. The surface temperature of the heater is constant at 90°C. Find the entropy generated during this process.

Find: Entropy generated $\dot{S}_{gen}$ in this process.

Known: Insulated tank, volume of water V = 500 L, initial water temperature T_1 = 25°C, final water temperature T_2 = 50°C, heater surface temperature T_h = 90°C.

Assumptions: Water is incompressible.

Properties: Specific heat of water c = 4.18 kJ/kgK (A3).

The entropy generated during the process can be found using an entropy balance, with no entropy output from the system since the tank is insulated:

$$\Delta S = S_{in} - \cancel{S_{out}} + S_{gen},$$
$$S_{gen} = \Delta S - S_{in}.$$

The mass of water in the tank can be found, using the density of water at the average temperature T_{avg} = 37.5°C interpolated from Appendix 3 as ρ = 992.5 kg/m^3:

$$m = \rho V = 992.5 \text{ kg/m}^3 \times 500 \times 10^{-3} \text{ m}^3 = 496.25 \text{ kg}.$$

The amount of heat added to the water can be found using an energy balance,

$$Q_{in} = \Delta U = mc(T_2 - T_1) = 496.25 \text{ kg} \times 4.18 \text{ kJ/kgK} \times (50°\text{C} - 25°\text{C}) = 51858 \text{ kJ},$$

Then the entropy input to the water tank is

$$S_{in} = \frac{Q_{in}}{T_b} = \frac{51858 \text{ kJ}}{273.15 + 90 \text{ K}} = 142.80 \text{ kJ/K}.$$

The entropy change of the water in the tank during the heating process can be found using the equation for entropy change of an incompressible substance,

$$\Delta S = mc \ln \frac{T_2}{T_1} = 496.25 \text{ kg} \times 4.18 \text{ kJ/kgK} \times \ln \frac{(273.15 + 50) \text{ K}}{(273.15 + 25) \text{ K}} = 167.02 \text{ kJ/K}.$$

Then the entropy generation can be solved,

$$S_{gen} = \Delta S - S_{in} = 167.024 \text{ kJ/K} - 142.801 \text{ kJ/K} = 24.220 \text{ kJ/K}.$$

The entropy generated in the tank is 24.2 kJ/K.

6.5. A 20 mA current is passed through an insulated, 50 Ω resistor with a mass of 0.2 kg and a specific heat of 0.7 kJ/kgK. Find the entropy generated after 10 hours. The initial temperature of the resistor is 20°C.

Find: Entropy generated S_{gen} by the resistor after a duration.

Known: Resistor current I = 2 mA, insulated resistor, resistance R = 50 Ω, mass of resistor m = 0.2 kg, specific heat of resistor c = 0.7 J/kgK, duration Δt = 10 hours, initial temperature T_1 = 20°C.

Assumptions: The resistor is incompressible.

The entropy generation in the resistor can be found with an entropy balance, for no input or output to the resistor:

$$\Delta S = \cancel{S_{in}} - \cancel{S_{out}} + S_{gen},$$

$$S_{gen} = \Delta S = mc \ln \frac{T_2}{T_1}.$$

The final temperature of the resistor can be found using an energy balance,

$$W = mc(T_2 - T_1) = 0.2 \text{ kg} \times 0.7 \text{ J/kgK} \times (T - 293.15 \text{ K}),$$

$$T_2 = 293.15 \text{ K} + \frac{W}{0.14 \text{ J/K}}.$$

The work output of the resistor is

$$W = \dot{W}\Delta t = I^2 R \Delta t = (20 \times 10^{-3} \text{ A})^2 \times (50\ \Omega) \times 10 \text{ hour} \times 3600 \text{ s/hour},$$

$$W = 720 \text{ J}.$$

Then the final temperature of the resistor is

$$T_2 = 293.15 \text{ K} + \frac{720 \text{ J}}{0.14 \text{ kJ/K}},$$

$$T_2 = 298.29 \text{ K},$$

and the entropy generated during the process can be found using the equation for entropy change of an incompressible substance,

$$S_{gen} = mc \ln \frac{T_2}{T_1} = 0.2 \text{ kg} ´ \ 0.7 \text{ kJ/kgK} ´ \ \ln \frac{298.29 \text{ K}}{293.15 \text{ K}} = 2.433 \text{ J/K}.$$

The entropy generated during this process is 2.43 J/K.

6.7. Five kilograms of water at 20°C are poured into an insulated bucket that already contains 10 kg of water at 80°C. What is the entropy generated by the hot and cold water mixing?

Find: Entropy generated S_{gen} during mixing of hot and cold water.

Known: Mass of cool water m_c = 5 kg, temperature of cool water T_c = 20°C, insulated bucket, mass of hot water m_h = 10 kg, temperature of hot water T_h = 80°C.

Assumptions: Specific heat of all water is constant c = 4.18 kJ/kgK (A3), water is incompressible.

The entropy generated during this process can be found by adding the entropy change of both masses of water, since the bucket is insulated. The final temperature of the mixture can be found with an energy balance,

$$\Delta U_c - \Delta U_h = 0,$$

$$m_c c(T_f - T_c) + m_h c(T_f - T_h) = 0,$$

$$T_f = \frac{m_c T_c + m_h T_h}{m_c + m_h} = \frac{5 \text{ kg} \times 20°\text{C} + 10 \text{ kg} \times 80°\text{C} +}{5 \text{ kg} + 10 \text{ kg}} = 60°\text{C}.$$

Then the entropy change of each mass of water can be found using the equation for entropy change of an incompressible substance,

$$\Delta S_h = m_h c \ln\frac{T_f}{T_h} = 5 \text{ kg} \times 4.18 \text{ kJ/kgK } \ln\frac{333.15 \text{ K}}{293.15 \text{ K}} = 2.6733 \text{ kJ/K},$$

$$\Delta S_h = m_h c \ln\frac{T_f}{T_h} = 10 \text{ kg} \times 4.18 \text{ kJ/kgK} \times \ln\frac{333.15 \text{ K}}{353.15 \text{ K}} = -2.4369 \text{ kJ/K},$$

and the total entropy generated can be solved:

$$S_{gen} = \Delta S_A + \Delta S_B = 2.6733 \text{ kJ/K} - 2.4369 \text{ kJ/K} = 0.23640 \text{ kJ/K}.$$

The entropy generated during the mixing of the water is 0.236 kJ/K.

6.43. Hot air enters a pipe at 500 kPa and 600 K with a mass flow rate of 0.5 kg/s and exits at 450 kPa and 500 K. Find the rate of heat loss from the pipe and the rate of entropy generation in it. The ambient air temperature is 300 K.

Find: Rate of heat loss $\dot{Q}$ from pipe, rate of entropy generation $\dot{S}_{gen}$ in the pipe.

Known: Inlet air pressure P_1 = 500 kPa, inlet temperature T_1 = 600 K, mass flow rate of air $\dot{m}$ = 0.5 kg/s, outlet pressure P_2 = 450 kPa, outlet temperature T_2 = 500 K, temperature of surroundings T_s = 300 K.

Assumptions: Air behaves as an ideal gas, pipe is at steady state, temperature of surroundings is constant.

Properties: Specific enthalpy of air at 600 K h_1 = 607.02 kJ/kg (A7), specific entropy of air at 600 K s_1^o = 2.40902 kJ/kgK (A7), specific enthalpy of air at 500 K h_2 = 503.04 kJ/kg (A7), specific entropy of air at 500 K s_2^o = 2.21952 kJ/kgK (A7).

The rate of heat transfer from the air in the pipe can be found using an energy balance on the air flowing through the pipe,

$$\dot{Q} = \dot{m}(h_2 - h_1) = 0.5 \text{ kg/s} \times (503.02 \text{ kJ/kg} - 607.02 \text{ kJ/kg}) = -52.0 \text{ kW}.$$

An entropy rate balance can be used to find the rate of entropy generation, using the assumption that the pipe is at steady state:

$$\frac{\dot{Q}}{T_s} + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0,$$

$$\dot{S}_{gen} = \Delta\dot{S}_a - \frac{\dot{Q}}{T_s},$$

with the entropy change of the air in the pipe found using the equation for entropy change of an ideal gas using air tables,

$$\Delta\dot{S}_a = \dot{m}\Delta s_a = \dot{m}\left(s_2^o - s_1^o - R\ln\frac{P_2}{P_1}\right),$$

$$\Delta\dot{S}_a = 0.5 \text{ kg/s} \times \left(2.21952 \text{ kJ/kgK} - 2.40902 \text{ kJ/kgK} - 0.2870 \text{ kJ/kgK} \times \ln\frac{450 \text{ kPa}}{500 \text{ kPa}}\right),$$

$$\Delta\dot{S}_a = -0.079631 \text{ kW/K}.$$

Then the rate of entropy generation in the pipe can be found:

$$\dot{S}_{gen} = \Delta\dot{S}_a - \frac{-52.0 \text{ kJ}}{300 \text{ K}} = -0.079631 \text{ kW/K} + 0.17333 \text{ kW/K} = 0.093699 \text{ kW/K}.$$

The pipe loses heat at a rate of 52 kW, and the entropy generated in the pipe is 0.0937 kW/K.

6.48. Measurements at the inlet and outlet of an insulated turbine show air entering at 700 kPa and 900 K and leaving at 150 kPa and 600 K. Is this process possible? If it is, how much work is done per kilogram of air flowing through the turbine?

Find: Whether or not the process is possible, work done *w* if the process is possible.

Known: Inlet air pressure P_1 = 700 kPa, inlet temperature T_1 = 900 K, outlet air pressure P_2 = 150 kPa, outlet temperature T_2 = 600 K.

Assumptions: Air behaves as an ideal gas.

Properties: Specific enthalpy of air at 900 K h_1 = 932.93 kJ/kg (A7), specific entropy of air at 900 K $s_1^o = 2.84856$ kJ/kgK (A7), specific enthalpy of air at 600 K h_2 = 607.02 kJ/kg (A7), specific entropy of air at 600 K $s_2^o = 2.40902$ kJ/kgK (A7).

The process is possible if the entropy change during the process is non-negative. The change of entropy of the air can be found using the equation for entropy change of an ideal gas using air tables,

$$\Delta s = s_2^o - s_1^o - R\ln\frac{P_2}{P_1},$$

$$\Delta s = 2.40902 \text{ kJ/kgK} - 2.84856 \text{ kJ/kgK} - 0.2870 \text{ kJ/kgK} \times \ln\frac{150 \text{ kPa}}{700 \text{ kPa}},$$

$$\Delta s = 2.56773 \text{ J/kgK}.$$

The process is possible since the entropy change is non-negative. Then the work done per unit mass can be found with an energy balance,

$$w = h_2 - h_1 = 607.02 \text{ kJ/kg} - 932.93 \text{ kJ/kg} = -325.910 \text{ kJ/kg}.$$

The process is possible, and the work output would be 325.9 kJ/kg.