Solutions for selected exercises
(Lecture 8)

Exercise 8.22

The percentage change in work required to remove heat to when changing the refrigerator temperature from -5 °C to -8 °C when the surrounding air is at 25 °C.

By assuming an reverse Carnot cycle and constant temperature of the surrounding air, the percentage change in work required to remove an equal amount of heat, can be expressed with just the COP of each refrigerator

$$\Delta W_p = \frac{W_2 - W_1}{W_1} \times 100\% = \frac{\frac{Q_C}{COP_{R,2}} - \frac{Q_C}{COP_{R,1}}}{\frac{Q_C}{COP_{R,1}}} \times 100\% = \left(\frac{COP_{R,1}}{COP_{R,2}} - 1 \right) \times 100\%.$$

Since initial refrigerator temperatures and the temperature of surroundings are given, COP's can be calculated as

$$COP_{R,1} = \frac{1}{T_H/T_{C,1} - 1} = \frac{1}{298.15\ K/268.15\ K - 1} = 8.9383$$

$$COP_{R,2} = \frac{1}{T_H/T_{C,2} - 1} = \frac{1}{298.15\ K/263.15\ K - 1} = 8.0348$$

Using both COPs in the first equation for the final percentage change in work leads to

$$\Delta W_p = \left(\frac{COP_{R,1}}{COP_{R,2}} - 1 \right) x\ 100\% = \left(\frac{8.9383}{8.0348} - 1 \right) x\ 100\% = 11.24\%$$

Exercise 8.23

Please show that a heat pump and a refrigerator between same reservoirs, both operate on a reverse Carnot cycle, transfer the same amount of heat.

The statement is proven by comparing the definition of COP for a heat pump and a refrigerator with an energy balance

$$COP_{HP} = \frac{Q_H}{W} = \frac{Q_C + W}{W} = \frac{Q_C}{W} + 1 = COP_R + 1.$$

Please note that this is valid for Carnot heat pump and Carnot refrigerator.

Exercise 8.38

The mass flow rate of a refrigerant R134a in a Carnot refrigerator which takes heat a -4 °C and rejects heat at 24 °C needs to be determined for an assumed power input of 380 W.

An energy balance around the isothermal condenser gives

$$\dot{Q}_H = \dot{m}(h_3 - h_2),$$
$$\dot{m} = \frac{\dot{Q}_H}{h_3 - h_2}.$$

In order to find the rate of heat rejected from the refrigerator, the COP is calculated as follows

$$COP_R = \frac{\dot{Q}_C}{\dot{W}} = \frac{1}{T_H / T_C - 1},$$
$$\dot{Q}_C = \frac{\dot{W}}{T_H / T_C - 1} = \frac{380\ \mathrm{W}}{297.15\ \mathrm{K} / 269.15\ \mathrm{K} - 1} = 3652.8\ \mathrm{W},$$

Since power input and rate of heat rejected are known now, the rate of heat transfer from the heat sink is

$$\dot{Q}_H = \dot{W} + \dot{Q}_C = 380\ \mathrm{W} + 3652.8\ \mathrm{W} = 4032.8\ \mathrm{W}.$$

Using the specific enthalpies h_2 and h_3 of satured R134a from Appendix A9a, The resulting mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{Q}_H}{(h_3 - h_2)} = \frac{4032.8\ \mathrm{W}}{260.45\ \mathrm{kJ/kg} - 82.90\ \mathrm{kJ/kg}} = 0.022714\ \mathrm{kg/s}.$$

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}} = \frac{3.6017 \text{ kW}}{1.5 \text{ kW}} = 2.4011.$$

Exercise 9.36

Specific enthalpy of saturated R-134a at 0.20 MPa for gas h_2 = 241.30 kJ/kg (A8*b*), specific entropy of saturated R-134a at 0.20 MPa for gas s_2 = 0.9253 kJ/kg-K (A8*b*), specific enthalpy of saturated R-134a at 1.2 MPa for fluid h_4 = 115.76 kJ/kg (A8*b*).
The properties of the refrigerant entering the compressor are those of saturated R-134a vapour at P_2 = 0.20 MPa, h_2 = 241.30 kJ/kg and s_2 = 0.9253 kJ/kg-K.

Since we know that the refrigeration cycle is ideal, compression through the compressor is isentropic so that $s_3 = s_2$ = 0.9253 kJ/kg-K, so refrigerant leaving the compressor is superheated with specific enthalpy interpolated from Appendix 9*c* at P_3 = 1.2 MPa:

$$\frac{h_3 - 275.52\,\frac{kJ}{kg}}{287.44\,\frac{kJ}{kg} - 275.52\,\frac{kJ}{kg}} = \frac{0.9253\,\frac{kJ}{kg\,K} - 0.9164\,\frac{kJ}{kg\,K}}{0.9527\,\frac{kJ}{kg\,K} - 0.9164\,\frac{kJ}{kg\,K}}$$

And h_3 = 278.44 kJ/kg

The specific enthalpy of refrigerant leaving the condenser is that of saturated liquid R- 134a at P_4 = 1.2 MPa, h_4 = 115.76 kJ/kg.

Expansion through the throttling valve in an ideal reverse Rankine cycle is a constant enthalpy process, so specific enthalpy at the evaporator inlet is $h_1 = h_4$ = 115.76 kJ/kg. The mass flow rate of refrigerant through the cycle can be found using an energy balance around the compressor, with power supplied by the car:

$$\dot{W}_{comp} = \dot{m}_{ref}(h_3 - h_2)$$

$$\dot{m}_{ref} = \frac{\dot{W}_{comp}}{h_3 - h_2} = \frac{1.2\,kW}{278.44\,\frac{kJ}{kg} - 241.30\,\frac{kJ}{kg}} = 0.0323\,\frac{kg}{s}$$

then the rate of heat input to the refrigerator from the surrounding air is

$$\dot{Q}_C = \dot{m}_{ref}(h_2 - h_1) = 0.0323\,\frac{kg}{s}\left(241.30\,\frac{kJ}{kg} - 115.76\,\frac{kJ}{kg}\right) = 4.056\,kW$$

For air at T_{avg} = 25°C, the average specific heat at constant pressure is c_p = 1.004 kJ/kgK, so the mass flow rate of air that can be cooled by the refrigerator is

$$\dot{m}_{air} = \frac{\dot{Q}_C}{c_p\,\Delta T} = \frac{4.056\,kW}{1.004\,\frac{kJ}{kg\,K}\,(35-15)°\mathrm{C}} 0.2002\,\frac{kg}{s}$$

The refrigerator can cool air at a maximum mass flow rate of 0.202 kg/s.

9.37. A freezer with an internal temperature of –12°C is kept in a grocery store where the surrounding air is at 20°C. The rate of heat transfer due to conduction through the walls of the freezer is estimated to be 8 kW. What is the minimum mass flow rate of refrigerant 134a required if an ideal vapour refrigeration cycle is used in the freezer? How much power is required to drive the compressor?

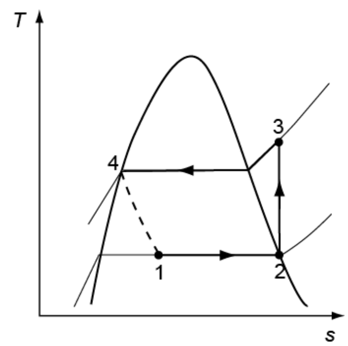
Find: Minimum mass flow rate $\dot{m}_R$ of refrigerant required, power $\dot{W}_c$ required to drive the compressor.

Known: Freezer temperature T_2 = –12°C, temperature of surrounding air T_4 = 20°C, rate of heat transfer into freezer $\dot{Q}_C$ = 8 kW, ideal Rankine refrigeration cycle.

Properties: Specific enthalpy of saturated R-134a at –12°C for gas h_2 = 240.15 kJ/kg (A9*a*), specific entropy of saturated R-134a at –12°C for gas s_2 = 0.9267 kJ/kgK (A9*a*), specific entropy of saturated R-134a at 20°C for fluid s_f = 0.2924 kJ/kgK and gas s_g = 0.9102 kJ/kgK (A9*a*), specific enthalpy of saturated R-134a at 20°C for fluid h_4 = 77.26 kJ/kg (A9*a*).

We will run a freezer which extracts heat from the internal temperature and provides it to the surrounding air at a rate equal to the heat transfer through the freezer walls.

The properties of the refrigerant entering the compressor are those of saturated R-134a vapour at T_2 = –12°C, h_2 = 240.15 kJ/kg and s_2 = 0.9267 kJ/kgK.



The Rankine refrigeration cycle is ideal, so compression through the compressor is isentropic and $s_3 = s_2$ = 0.9267 kJ/kgK, so refrigerant leaving the compressor is superheated with specific enthalpy interpolated from Appendix 9*c* at T_3 = 20°C:

$$\frac{h_3 - 262.96 \text{ kJ/kg}}{260.34 \text{ kJ/kg} - 262.96 \text{ kJ/kg}} = \frac{0.9267 \text{ kJ/kgK} - 0.9515 \text{ kJ/kgK}}{0.9264 \text{ kJ/kgK} - 0.9515 \text{ kJ/kgK}},$$

$$h_3 = 260.37 \text{ kJ/kg}.$$

The specific enthalpy of refrigerant leaving the condenser is that of saturated liquid R-134a at T_4 = 20°C, h_4 = 77.26 kJ/kg.

Expansion through the throttling valve in an ideal reverse Rankine cycle is a constant enthalpy process, so specific enthalpy at the evaporator inlet is $h_1 = h_4$ = 77.26 kJ/kg.

The minimum mass flow rate of refrigerant through the cycle can be found from an energy balance around the evaporator,

$$\dot{Q}_C = \dot{m}(h_2 - h_1),$$

$$\dot{m} = \frac{\dot{Q}_C}{(h_2 - h_1)} = \frac{8\ \text{kW}}{(240.15\ \text{kJ/kg} - 77.26\ \text{kJ/kg})} = 0.049113\ \text{kg/s}.$$

The power input to the compressor can then be found:

$$\dot{W}_c = \dot{m}(h_3 - h_2) = 0.049113\ \text{kg/s} \times (260.37\ \text{kJ/kg} - 240.15\ \text{kJ/kg}) = 0.99306\ \text{kW}.$$

To cool the refrigerator would require a minimum mass flow rate of refrigerant of 0.0291 kg/s and require 0.993 kW of power for the compressor.