DTU

47201 Engineering thermodynamics

Module 4

Heat transfer

What is heat?

Heat is a mechanism of energy *transfer* between bodies or different regions within a body.

It has the unit of energy, but it’s not to be confused with the internal energy *U* of a body.

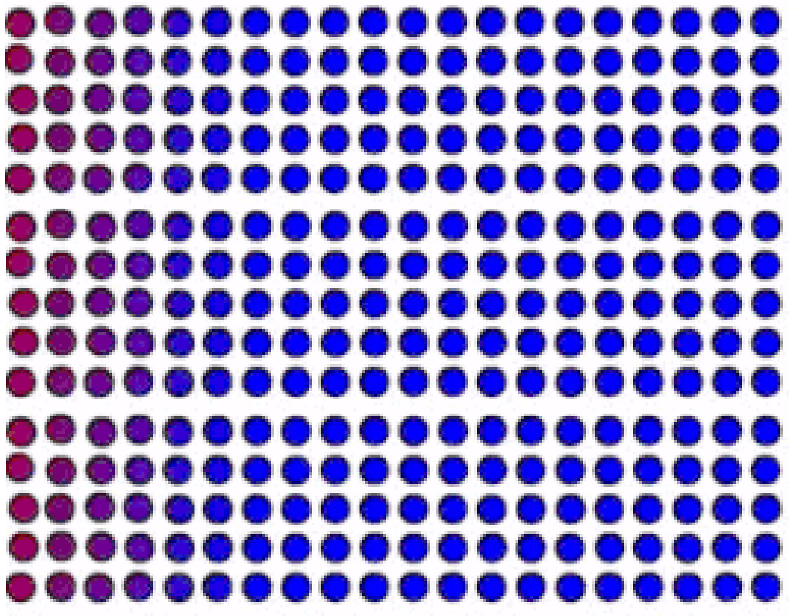
It does *not* make sense to ask how much heat is within a body, but only how much heat has been transferred between two bodies.

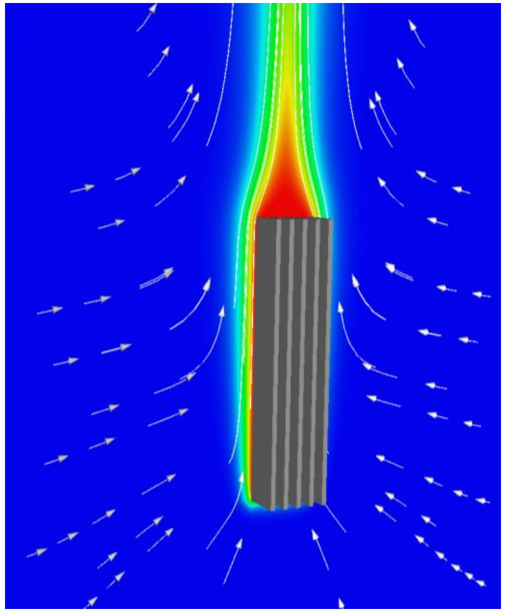
This process happens spontaneously: it cannot be controlled

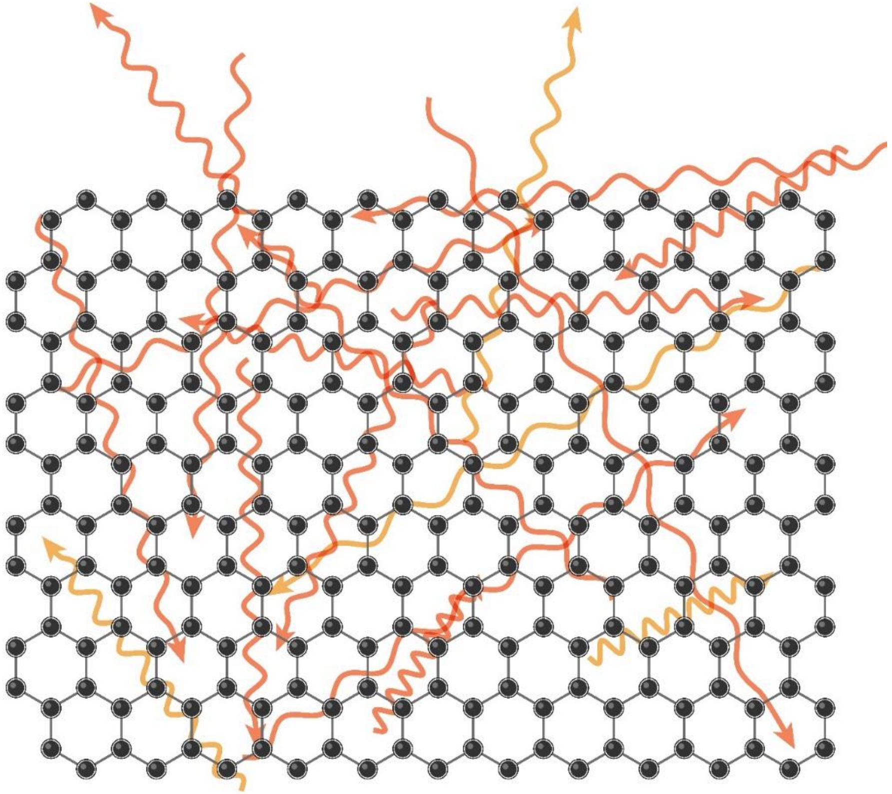
It happens at the microscopic level: it cannot be directly observed

It happens because of the randomness of microscopic interactions. When there is a lot of these interaction, the overall effect becomes predictable.

The three mechanisms of heat transfer







Conduction

Vibration and collisions of molecules and e^- in solids or fluids

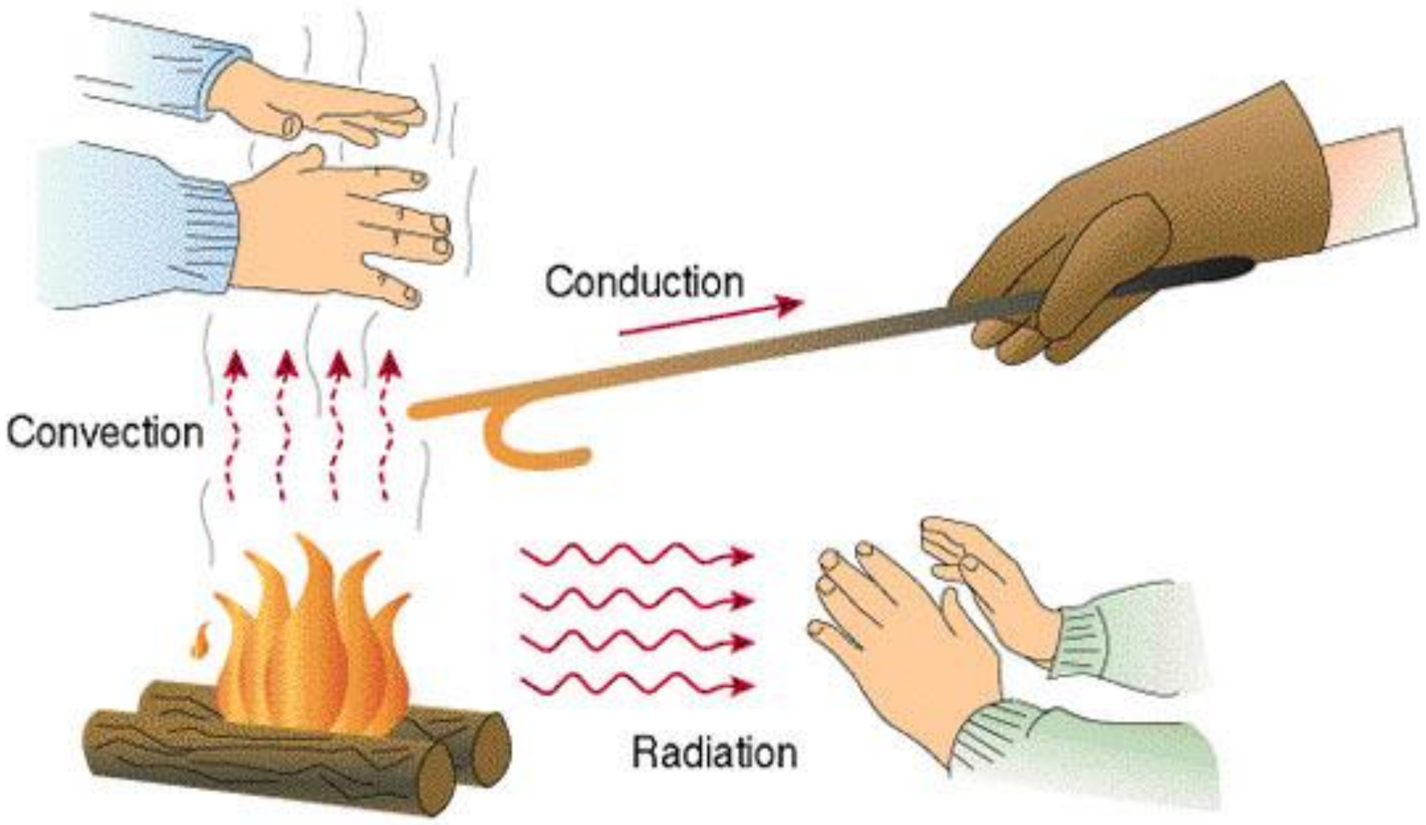
Convection

Associated to the flow in fluids (forced or natural flow)

Radiation

Mediated by (thermal) photons emitted and absorbed by solids

The three mechanisms of heat transfer



How does it work? It follows the second law

- Heat flows spontaneously from warmer bodies (higher temperature) to colder ones (lower T).
- Within a body, if temperature gradients are present, it flows from warmer regions to colder ones.
- The higher the temperature difference, the higher is the rate of heat transfer.
- When two bodies are in thermal equilibrium, i.e. same T, or when within a body there are no temperature gradients, there is no heat flow.

Physical quantities

Q	Heat [J]
$\dot{Q}$	Heat flux (or heat transfer rate) [W = J/s]
$\boldsymbol{q}$	Heat flux (surface) density [W/m^2 = J/(s m^2)]
T	Temperature [K]
$\boldsymbol{\nabla} T$	Temperature gradient [K/m]
$\dot{Q}_{\mathrm{vol}}$	Volumetric heat source [W/m^3]

Physical quantities

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How do we compute each of these two quantities from the other one?

T Temperature [K]

$\boldsymbol{\nabla} T$ Temperature gradient [K/m]

$\dot{Q}_{\mathrm{vol}}$ Volumetric heat source [W/m^3]

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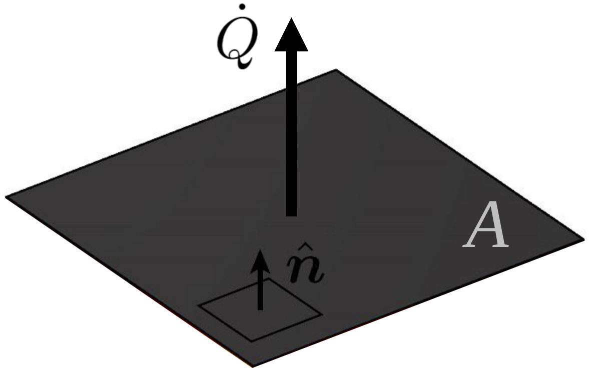

$$\boldsymbol{q} = \frac{\dot{Q}}{A}\hat{\boldsymbol{n}}$$

$$\dot{Q} = \iint \hat{\boldsymbol{n}} \cdot \boldsymbol{q}\, dS$$

Physical quantities

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Properties of materials / bodies

*extensive – ‡intensive

C	Heat capacity* [J/K]
c	Specific heat capacity‡ [J/(kg K)]
R_{heat}	Thermal (or heat) resistance* [K/W]
$1/R_{\text{heat}}$	Thermal conductance* [W/K]
k	Thermal (or heat) conductivity‡ [W/(m K)]
$1/k$	Thermal (or heat) resistivity‡ [m K/ W]
$\alpha = \dfrac{k}{\rho c}$	Thermal diffusivity‡ [m^2/s]

Heat conduction

Microscopic view of heat conduction

It is a diffusion phenomenon: caused by probabilistic effects.

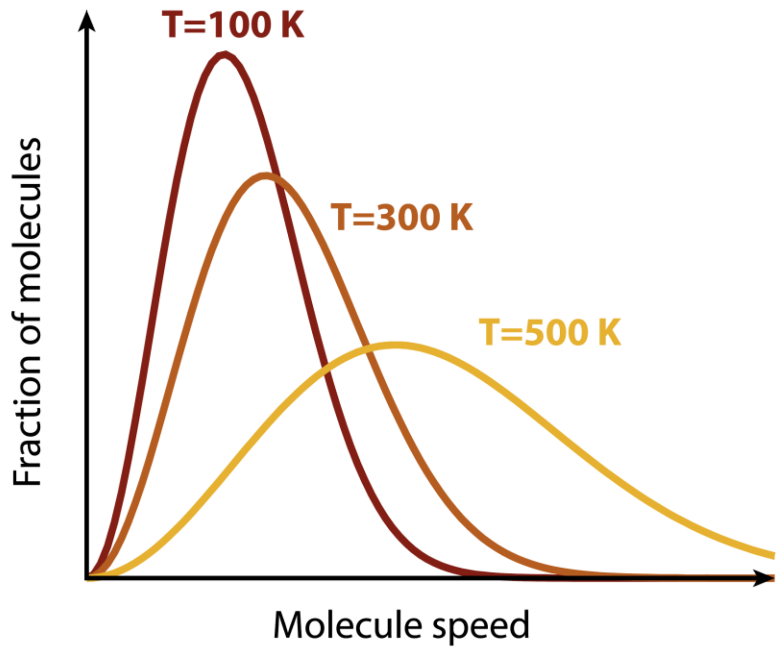
- In a *fluid*: random thermal motion of free molecules
- In a *solid*: random thermal vibration of ions in the lattice

The warmer the material (i.e. higher the temperature),
the faster are the molecules moving.

If initially they are faster in a certain region than elsewhere,
(i.e. if temperature gradients are present), then random collisions
and other interactions will eventually equilibrate things
(i.e. the temperature will become uniform).

In solids, the quantum mechanical description of vibration is given
in terms of phonons: collective modes of vibrations of the ions in the lattice.

Example: ideal gas
Maxwell-Boltzmann distribution



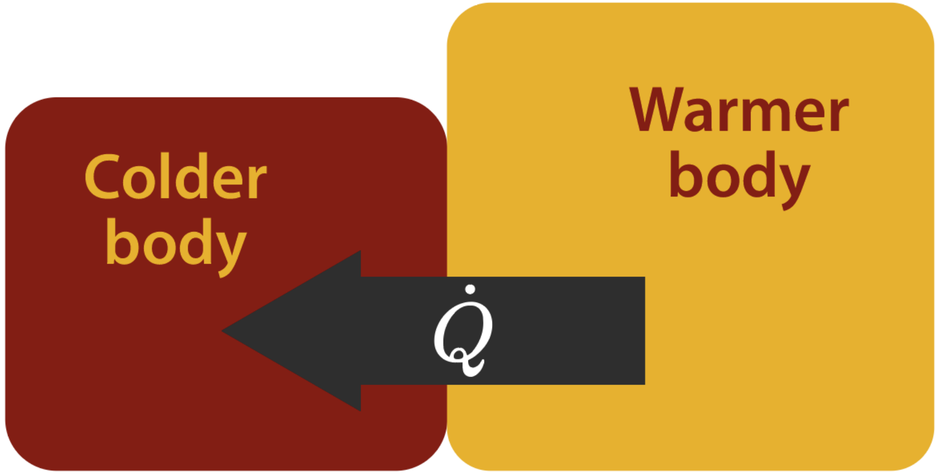
Newton's law of cooling

Heat flows from warmer bodies (higher temperature) to colder ones (lower temperature).

The normal component of the heat flux density is proportional to the temperature difference.

$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = -h \Delta T$$

h is the heat transfer coefficient. *What are its units?*



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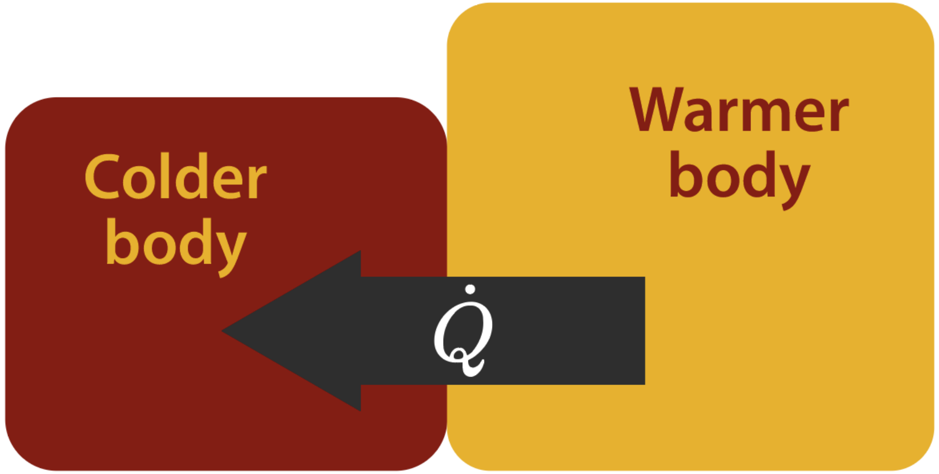
Heat transfer coefficient *between two bodies*.
Clearly, it depends on a lot of things:

- roughness
- materials involved
- …



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As the colder body absorbs heat, and the warmer body releases heat, the temperature difference decreases until thermal equilibrium is reached.

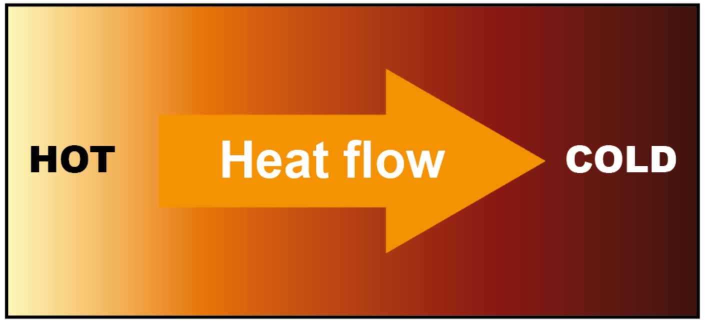
Thermal equilibrium:
same temperature
no heat transfer

Fourier's law of heat conduction

The heat flux density, $\boldsymbol{q}$, is directed opposite to the temperature gradient $\boldsymbol{\nabla} T$.

The proportionality factor is the thermal conductivity, *k,* of the material. *What are its units?*

$$\boldsymbol{q} = -k\boldsymbol{\nabla} T \xrightarrow{\text{in 1D}} q_x = -k\frac{\partial T}{\partial x}$$

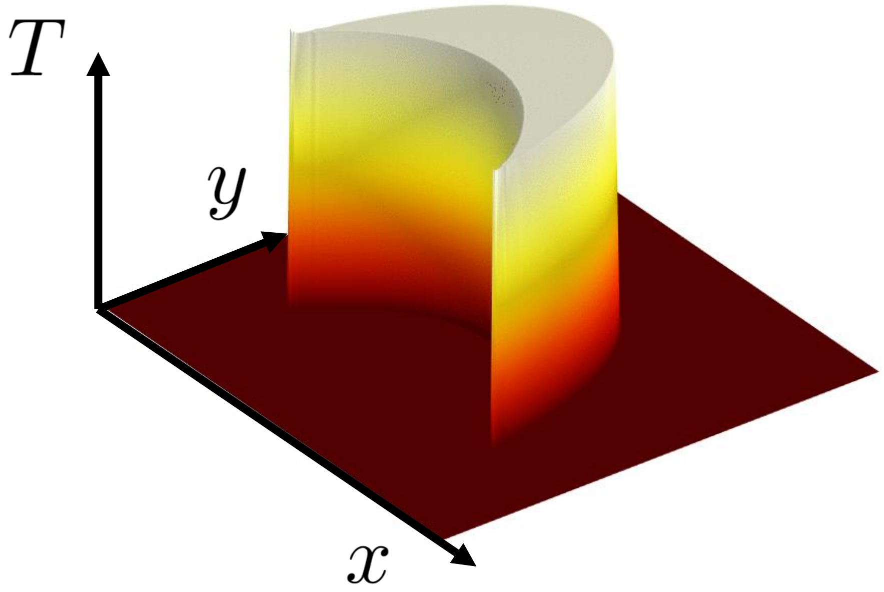


Heat equation and thermal circuits

The heat equation

How does the temperature, T, evolve inside a solid material?

$T = T(x, y, z, t)$ is a **continuous** function.



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The heat equation

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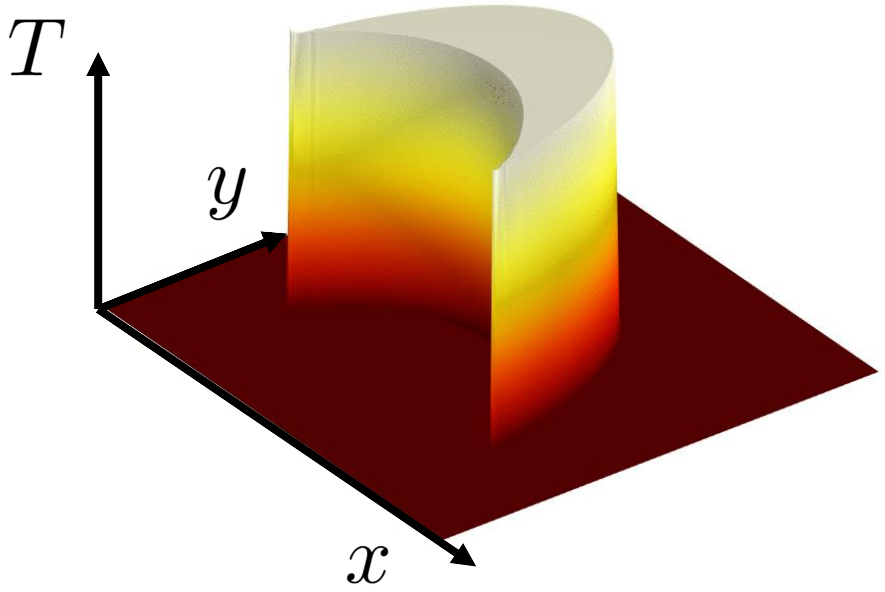
The time-evolution is governed by:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \,; \qquad \alpha \text{ : diffusivity}$$



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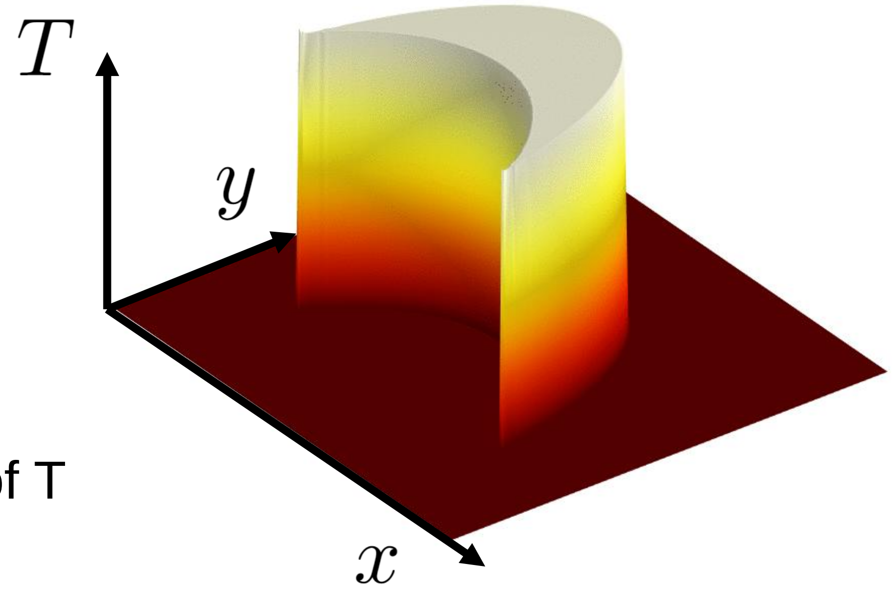
This equation relates the first-order time-derivative of T with its second-order space-derivative.

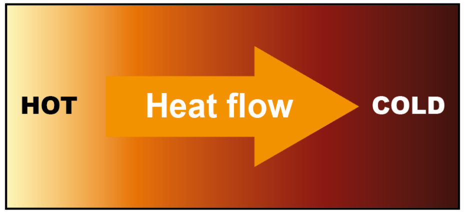
Laplace operator $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$



$T = T(x, y, t)$

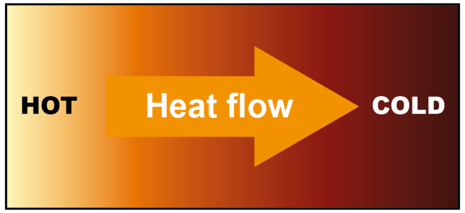
Deconstructing the heat equation

- **Fourier's law of heat conduction** $$\boldsymbol{q} = -k\boldsymbol{\nabla}T$$



The heat flux density, $\boldsymbol{q}$, is directed opposite to the temperature gradients.
The proportionality factor is the thermal conductivity, k, of the material.

Deconstructing the heat equation



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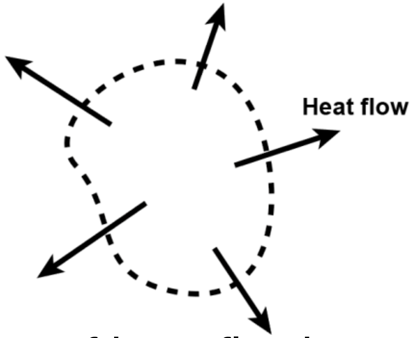
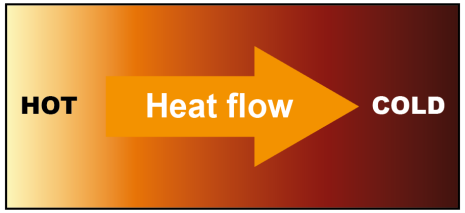


- **Energy conservation**

$$\frac{\partial U_{\mathrm{vol}}}{\partial t} = -\boldsymbol{\nabla} \cdot \boldsymbol{q}$$

The time derivative of the internal energy density, U_{vol} in a point is the amount of heat flowing “into that point”, i.e. the opposite of the divergence of the heat flux density.

Deconstructing the heat equation



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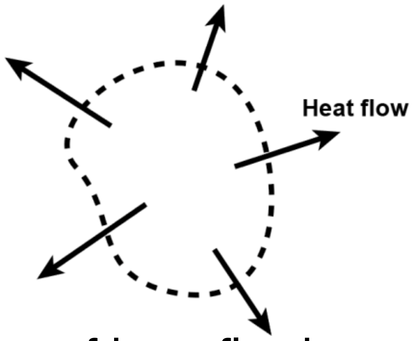

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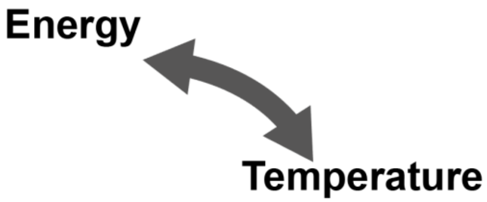

- **Heat capacity** $$\frac{\partial T}{\partial t} = \frac{1}{c\rho}\frac{\partial U_{\mathrm{vol}}}{\partial t}$$

The temperature variation is related to the variation of the internal energy via the specific heat capacity, *c*, and the mass density, *ρ*.

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \quad \alpha = \frac{k}{c\rho}$$

Deconstructing the heat equation

- **Fourier's law of heat conduction** $\boldsymbol{q} = -k\boldsymbol{\nabla}T$

- **Energy conservation** $$\frac{\partial U_{\mathrm{vol}}}{\partial t} = -\boldsymbol{\nabla} \cdot \boldsymbol{q}$$

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Deconstructing the heat equation

- **Fourier's law of heat conduction** $\boldsymbol{q} = -k\boldsymbol{\nabla} T$

$$\frac{\partial T}{\partial t} = \frac{1}{c\rho}\boldsymbol{\nabla}\cdot(k\boldsymbol{\nabla} T)$$

- **Energy conservation** $$\frac{\partial U_{\mathrm{vol}}}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{q}$$

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Deconstructing the heat equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \quad \alpha = \frac{k}{c\rho}$$

Uniform k

$$\frac{\partial T}{\partial t} = \frac{1}{c\rho} \boldsymbol{\nabla} \cdot (k \boldsymbol{\nabla} T)$$

- **Fourier's law of heat conduction** $\boldsymbol{q} = -k \boldsymbol{\nabla} T$

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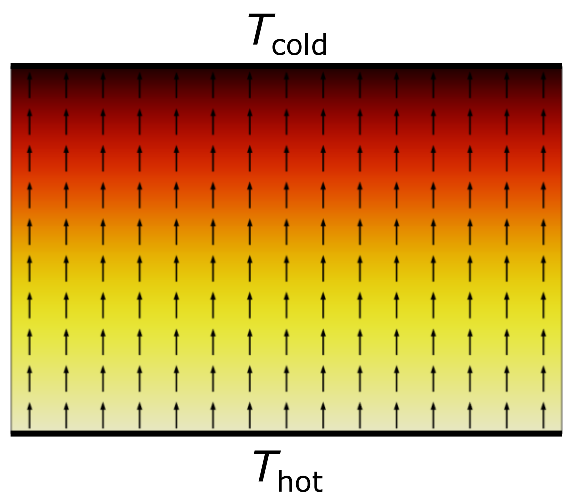
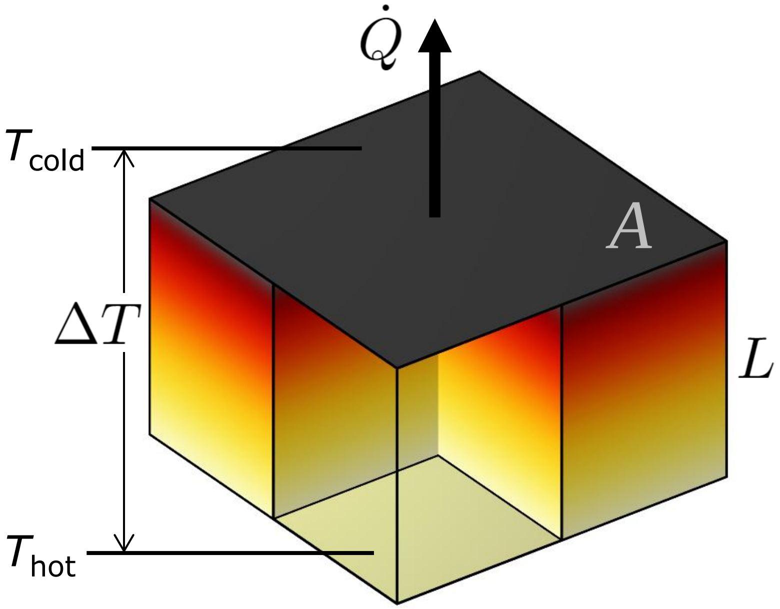
From continuous to discrete

Let's consider a block of conductive material surrounded by an insulating medium.
The bottom and top faces are *kept at fixed temperatures* T_{hot} and T_{cold}, respectively.

A temperature gradient will thus be established along the vertical direction, accompanied by a corresponding heat flow.



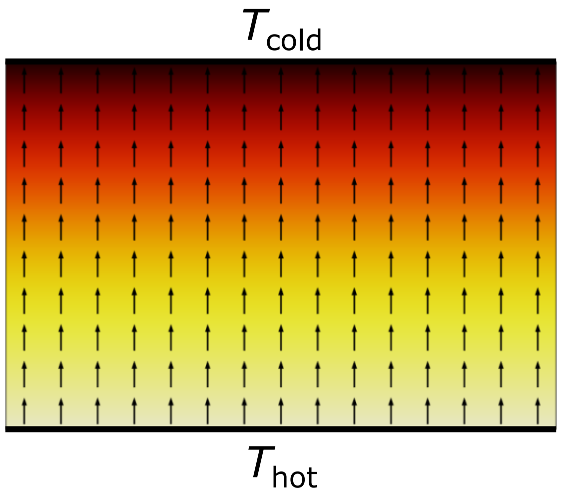


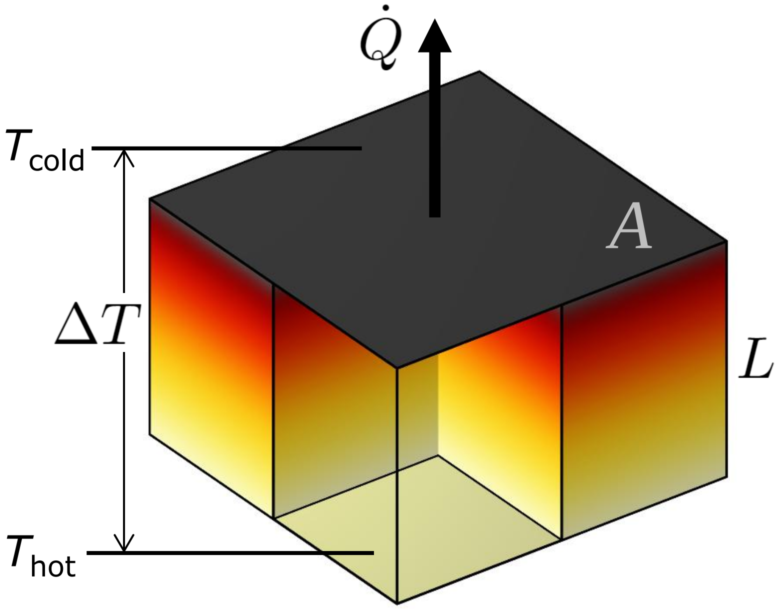
The lumped-element model: Circuits

This situation is the most essential. We can express it in terms of the net amounts:

$\|\boldsymbol{q}\| = \dot{Q}/A$, total heat transfer rate $\dot{Q}$ ↔ heat flux density $\boldsymbol{q}$, with cross section area A

$\|\boldsymbol{\nabla}T\| = \Delta T/L$, temperature difference ΔT ↔ temperature gradient $\boldsymbol{\nabla}T$, with length L





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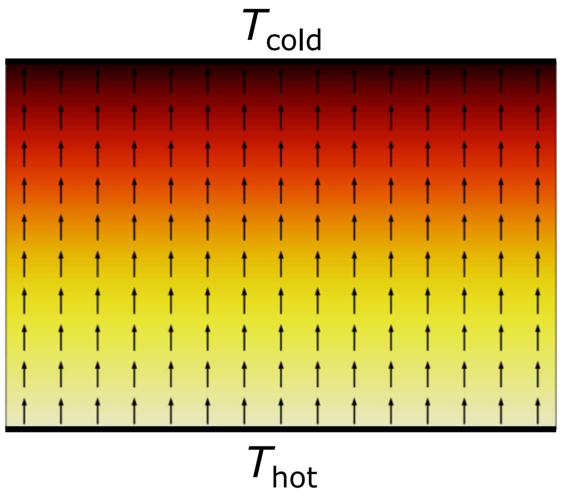

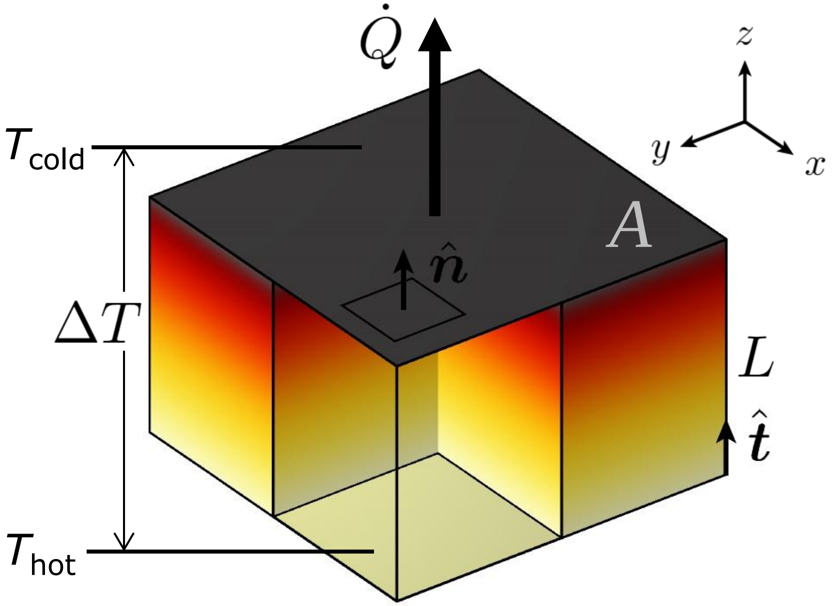

$$\dot{Q} = \iint \hat{\boldsymbol{n}} \cdot \boldsymbol{q} \, dS$$

$$\dot{Q} = \iint \sum_k n_k q_k \, dS$$

z dir.

$$\dot{Q} = \iint n_z q_z \, dx dy$$

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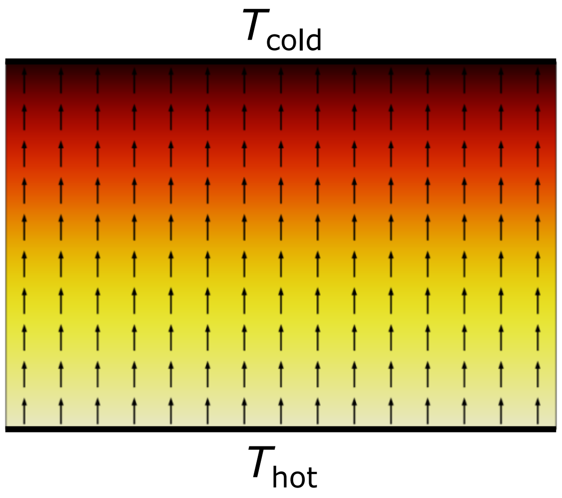
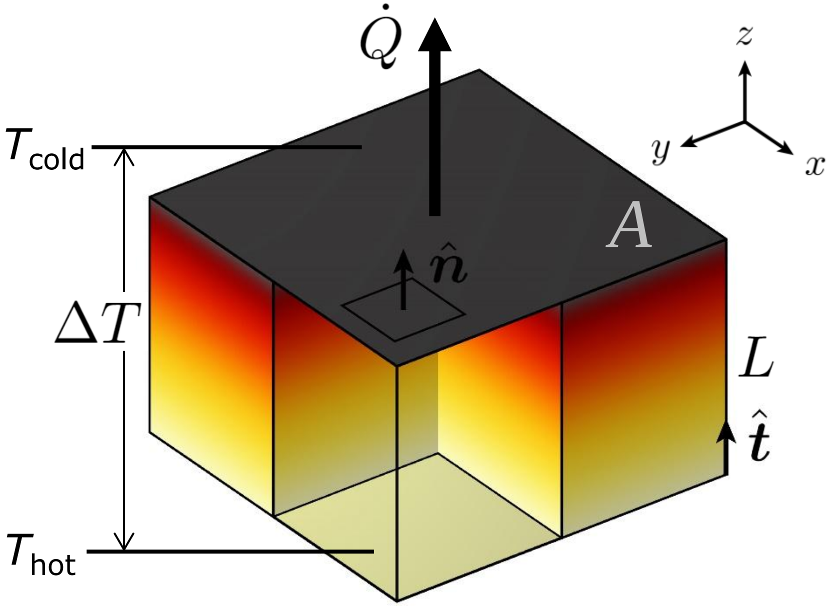




$$\Delta T = \int \hat{\boldsymbol{t}} \cdot (\boldsymbol{\nabla} T)\, d\ell$$

$$\Delta T = \int \sum_k t_k \frac{\partial T}{\partial x_k}\, d\ell$$

z dir.

$$\Delta T = \int t_z \frac{\partial T}{\partial z}\, dz$$

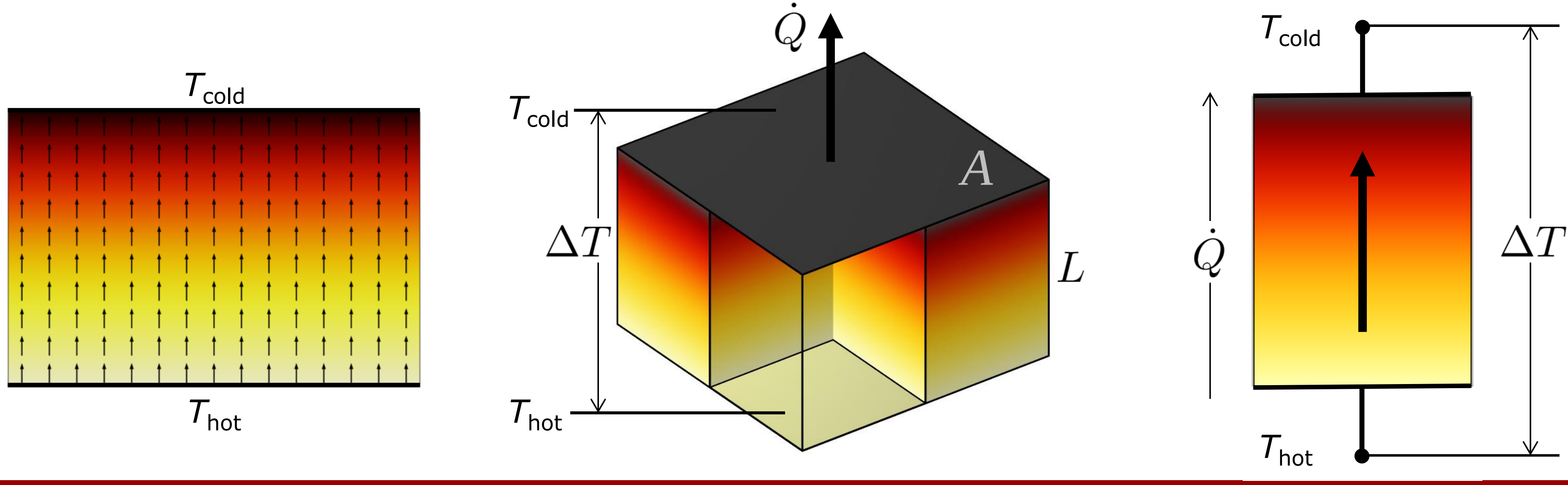
The lumped-element model: Circuits

Thermal resistance

$$R = \frac{L}{kA}$$

$$\boldsymbol{q} = -k\boldsymbol{\nabla}T$$

$$R\dot{Q} = \Delta T$$



Thermal circuits

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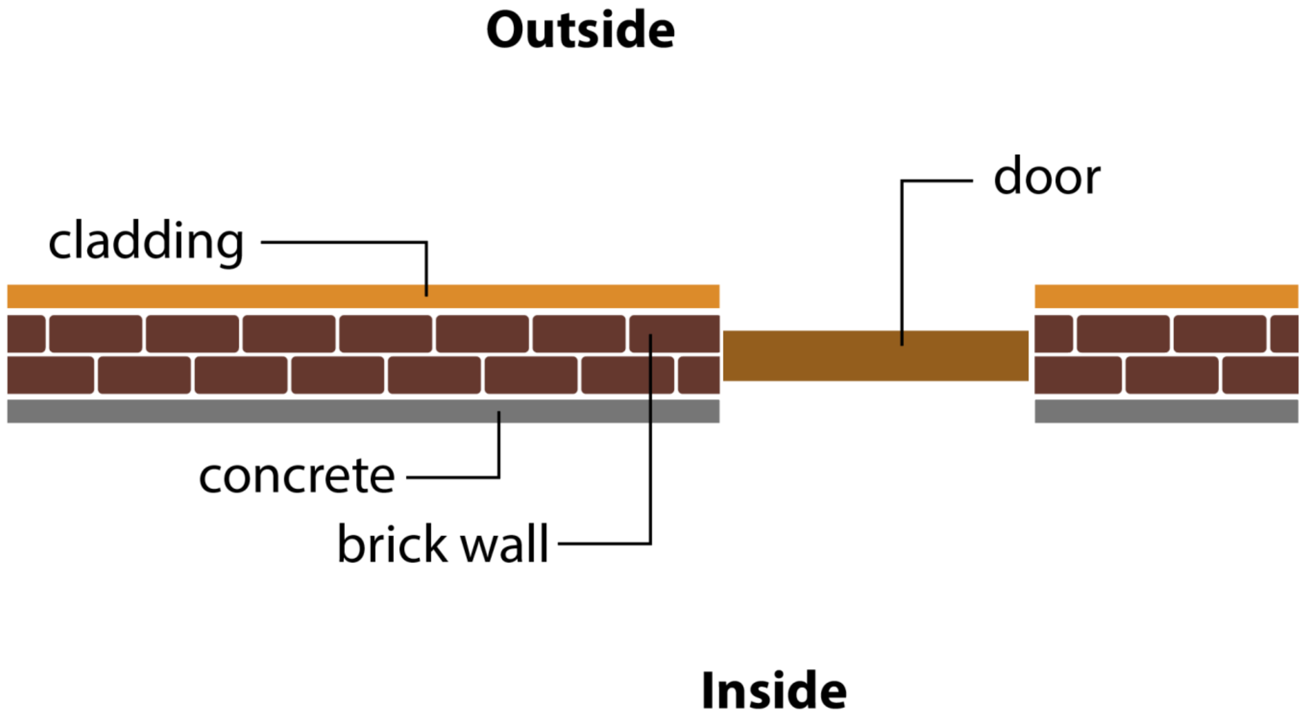
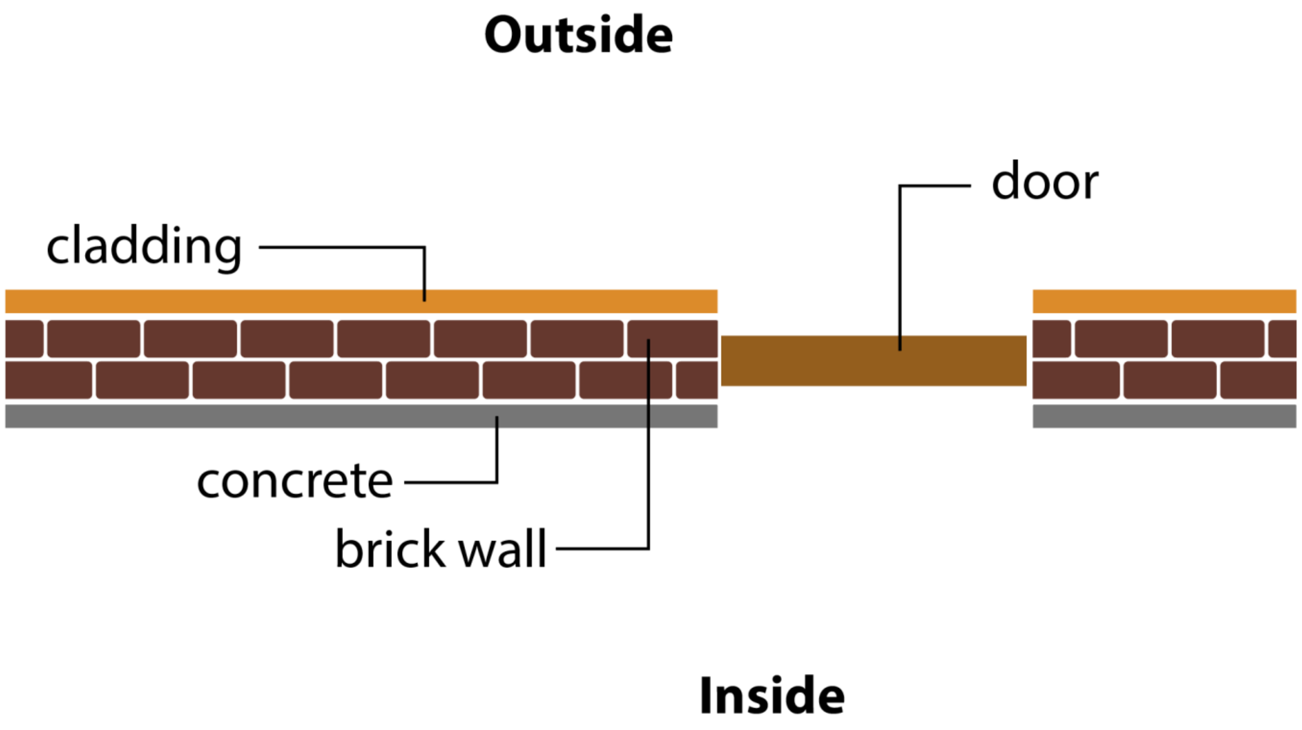


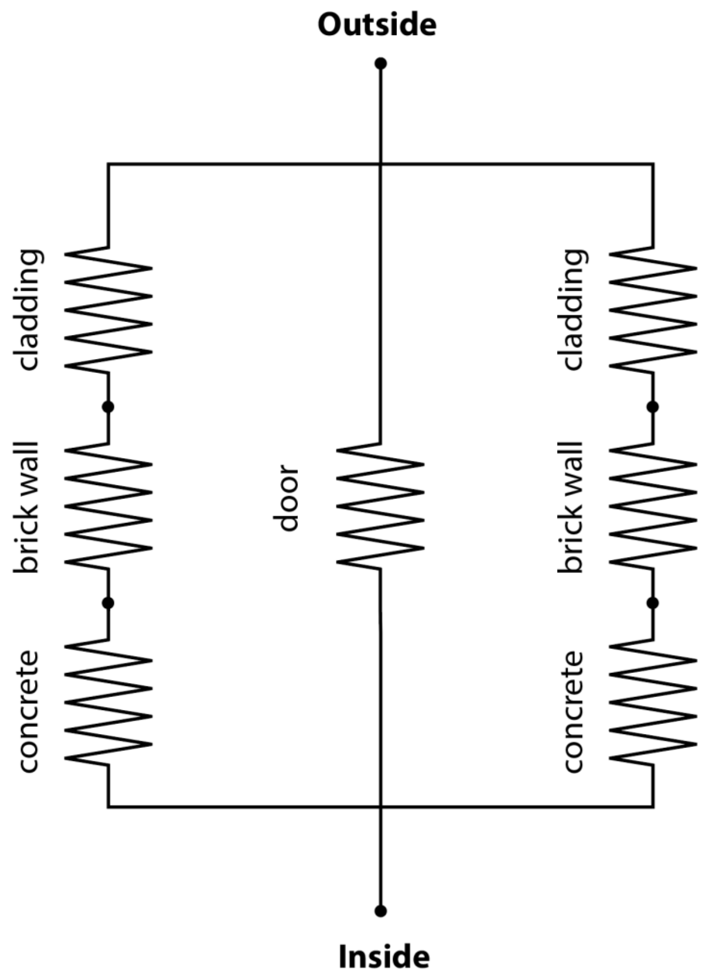
Example: how do we calculate the thermal resistance between the inside and the outside of a house?

Thermal circuits

Thermal resistance

$$R = \frac{L}{kA}$$





Example

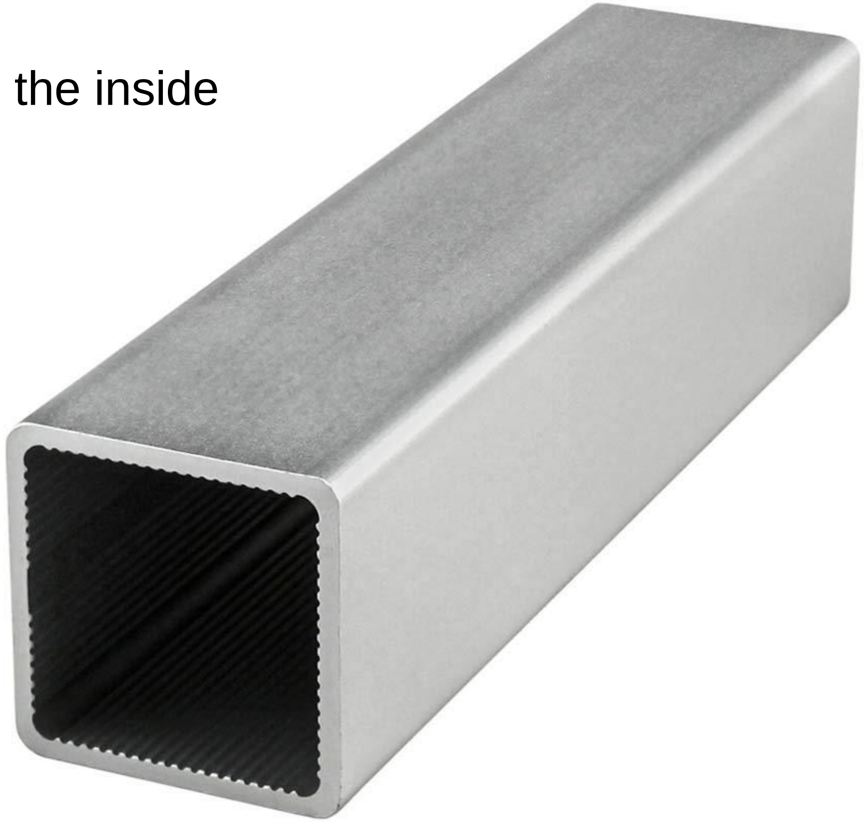
Look up the thermal conductivity of aluminum.

1) What is the thermal resistance per unit length between the inside and the outside of this profile?

Assume that the side is 5 cm and the thickness is 5 mm.

2) What is the thermal resistance for 20 cm length?

3) If the temperature difference between inside and outside is kept at 30 K, how much heat will be transferred during 1 min?



Convection

Convection vs conduction

- **Conduction** – it is a diffusion phenomenon: caused by probabilistic effects.
  - In a solid: random thermal vibration of ions in the lattice (phonons)
  - In a fluid: random thermal motion of free molecules

- **Convection** – it is a transport phenomenon: caused by deterministic effects
  - In a fluid: organized motion (flow) of the molecules
  - There can’t be convection in a solid.

Heat diffusion vs heat transport

Mathematically, we just need to include an extra contribution term to the flux density.

Diffusion only: $\boldsymbol{J} = -k\boldsymbol{\nabla} c$

heat

$$\boldsymbol{q} = -k\boldsymbol{\nabla} T$$

- Driven by concentration gradients
- It counteracts the gradients over time
- Eventually leads to equilibrium, i.e. uniform concentration, i.e. no gradients

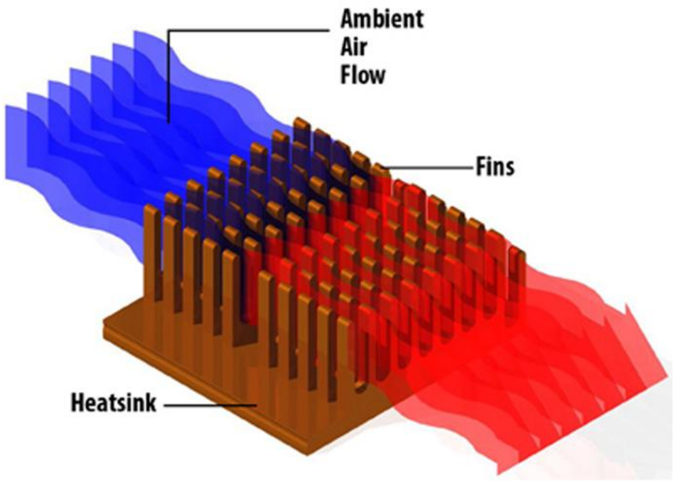
Diffusion + transport $\boldsymbol{J} = \boldsymbol{u} - k\boldsymbol{\nabla} c$

heat

$$\boldsymbol{q} = \boldsymbol{u} - k\boldsymbol{\nabla} T$$

- In a reference frame that moves together with the fluid, the perceived flow velocity $\boldsymbol{u}$ is zero, and there is no transport.
- It is just expressing the fact that when the fluid moves, its properties move with it.
- These properties can be: local concentration of a species mixed with the fluid, or the local temperature of the fluid.

Coupling with fluid dynamics



- Clearly, unless we know the flow velocity field $\boldsymbol{u}(\boldsymbol{x})$ we need to calculate it in order to then solve the heat equation.

- The flow is governed by the equation of fluid dynamics, i.e. the Navier stokes equations, which are again similar to the other equations mentioned so far, except that:
  - The rank is greater (rank-1 tensors → rank-2, and rank-2 → rank-4)
  - The phenomenon is inertial (the time dependent equation involves second order time derivative), instead of relaxation (first order time derivative)
  - The equation also involves an advective derivative term that is non-linear, specifically quadratic, with respect to $\boldsymbol{u}(\boldsymbol{x})$

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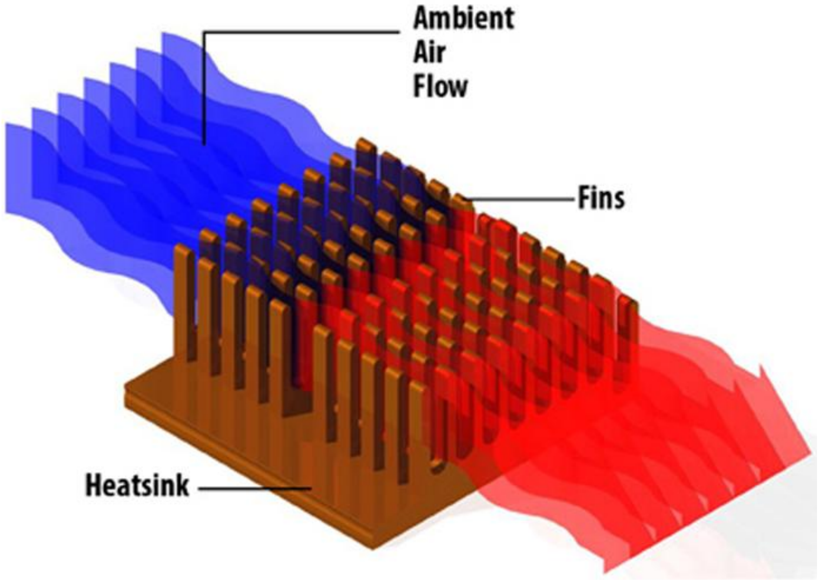
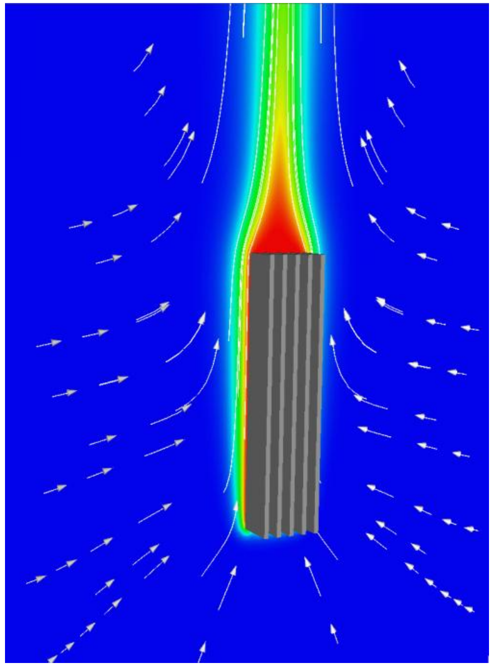
- The inverse coupling can also be of interest, i.e.: temperature dependence of flow property, e.g. viscosity, density (thermal expansion), etc.

Forced vs Natural convection

- **Forced convection**: the fluid flow is deliberately created by human intervention
  with the purpose of enhancing heat transfer, generally between the fluid and a solid that the fluid is flowing past. Examples: water inside radiators, air in heat exchangers, heat sinks, dissipators, fans.

- **Natural convection**, the fluid flow is occurring spontaneously due to the combined effect of thermal expansion (temperature dependent pressure) and buoyancy (pressure mediated gravitational forces in fluids). Example: air *outside* radiators.





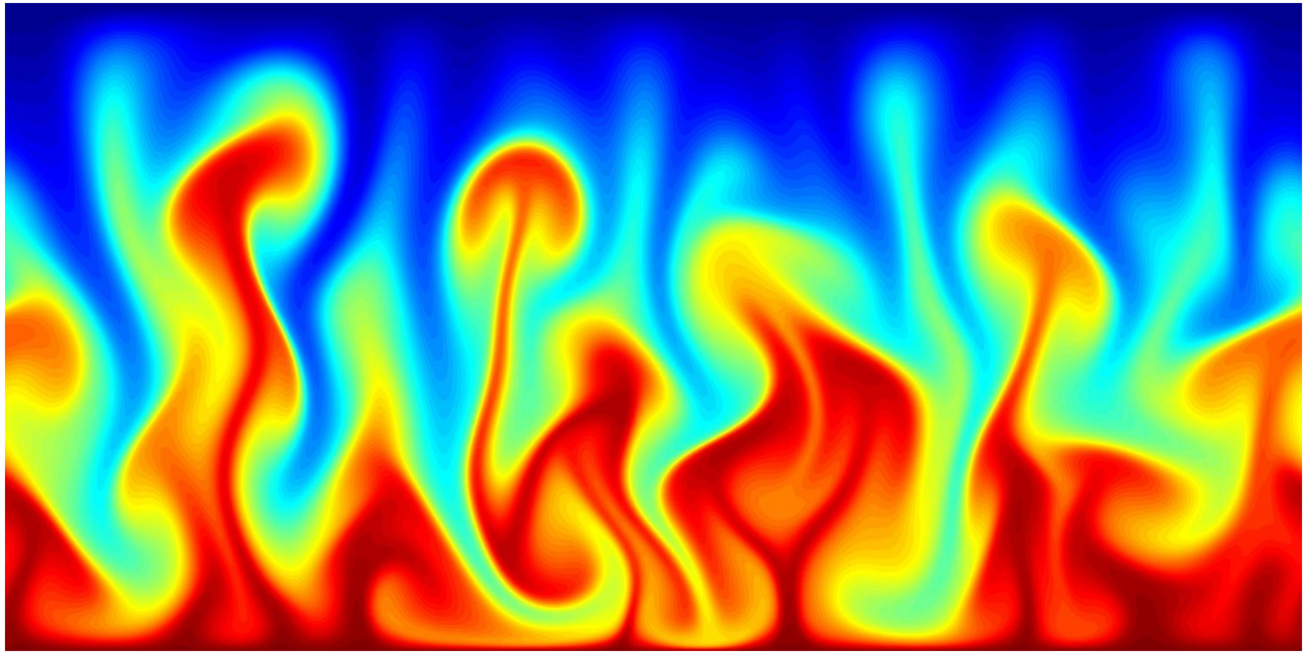
Convection coefficient

The exact calculation of the convection between an object and the surrounding fluid requires to solve the coupled equations of heat transfer and fluid dynamics (by numerical simulation).

However, we can approximate with a simple formula analogous to Newton's law of cooling:

$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = -h\Delta T$$

h is the convection coefficient.



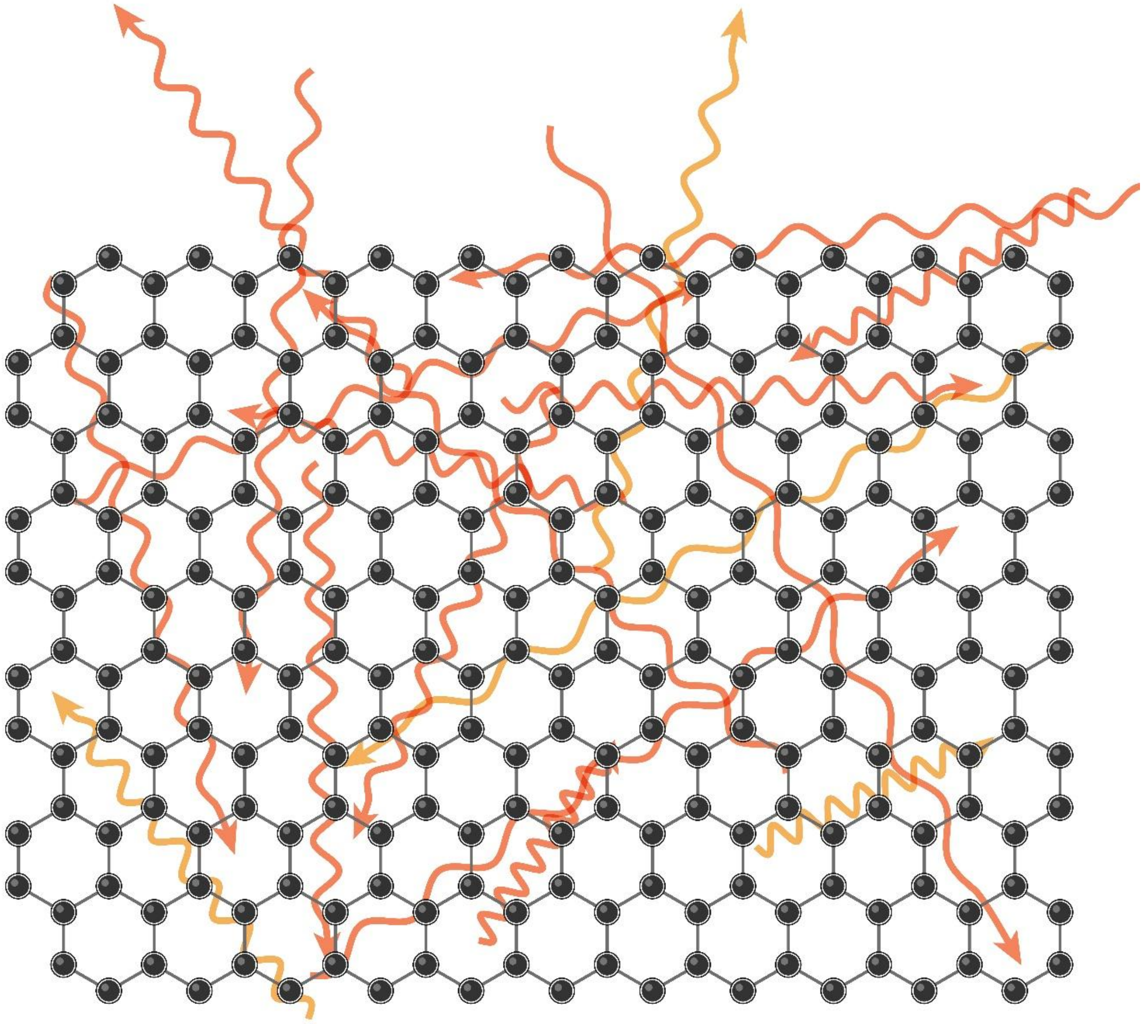
Radiation

Black-body radiation

Within a body, photons are continuously being emitted and absorbed.

We can visualize photons just as the molecules in a gas:

- they travel in a straight line until they interact with the body.
- on average, there are an equal number of photons traveling in any direction
- two photons may have different energies *E*
- the probability *P* depends on the energy: *P = P(E)*
- the probability distribution *P(E)* can be calculated from thermodynamics



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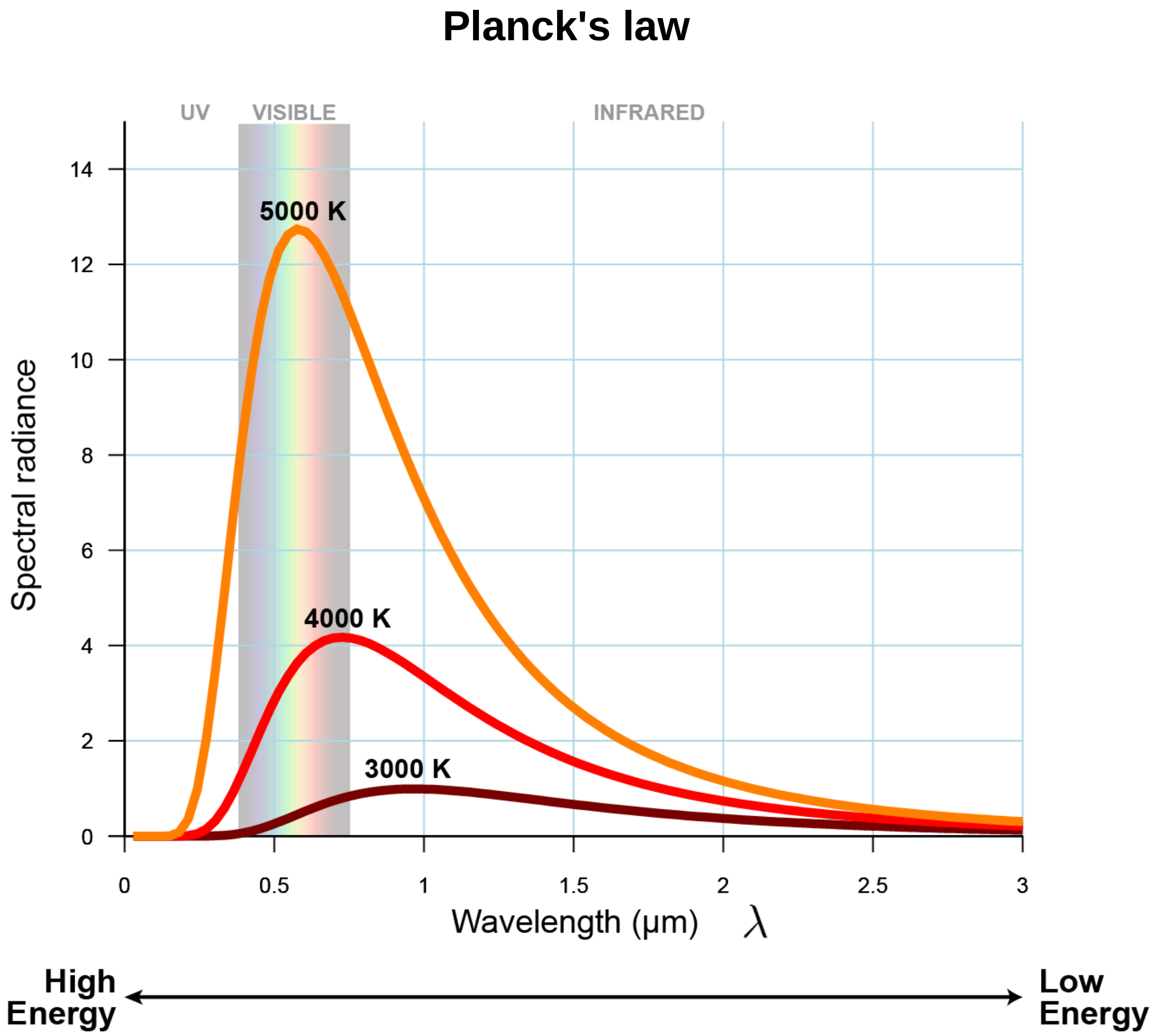


Black-body radiation

Within a body, photons are continuously being emitted and absorbed.

Plank's law – energy spectrum P(E) – is obtained from these considerations:

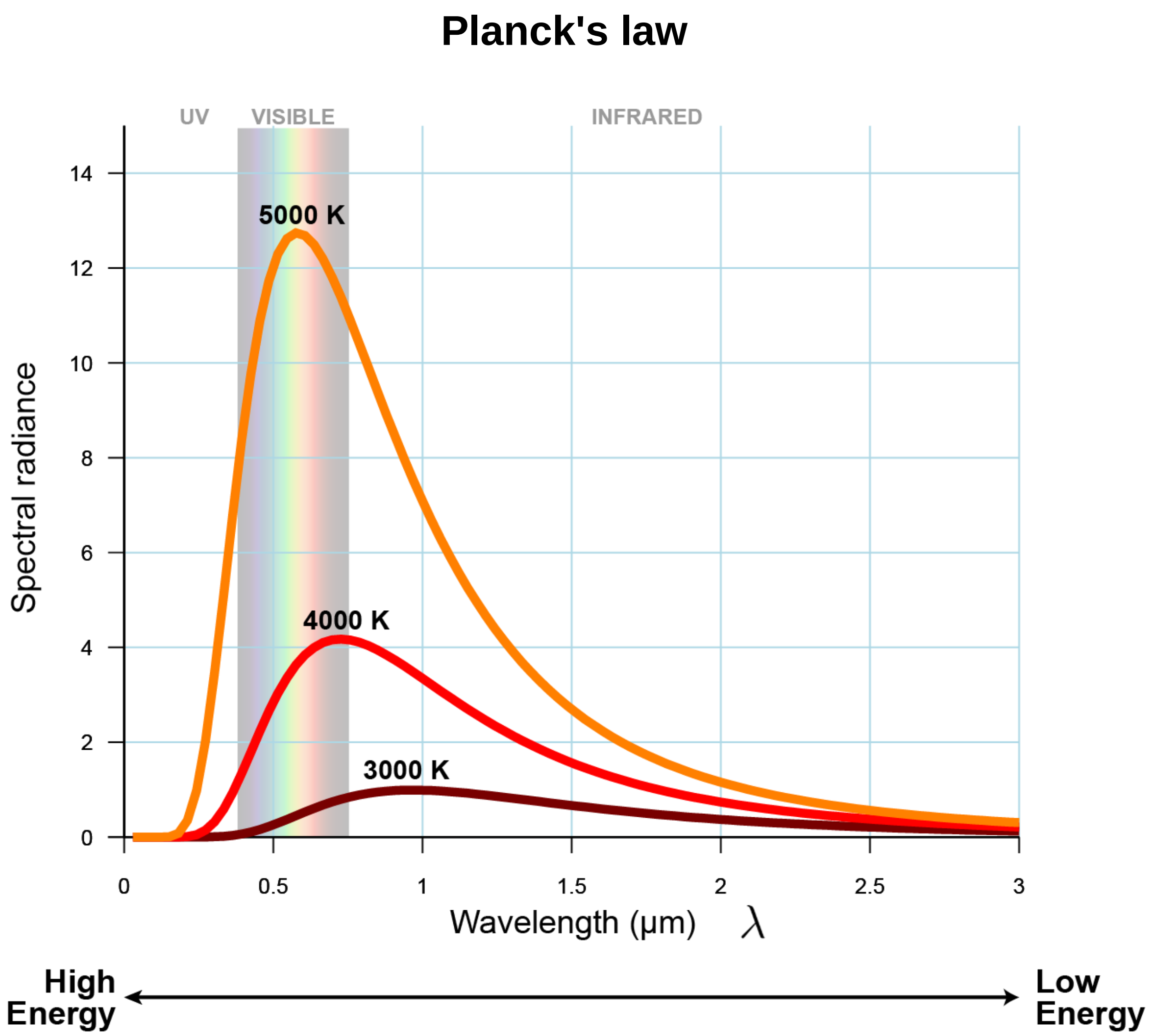
- Boltzmann factor (probability)

$$P \propto e^{-\frac{E}{k_B T}}$$

- Energy of a photon

$$E = h\nu = \frac{hc}{\lambda}$$

- Density of states, $\frac{dE}{d\boldsymbol{k}}$, where $\boldsymbol{k}$ is the wavenumber



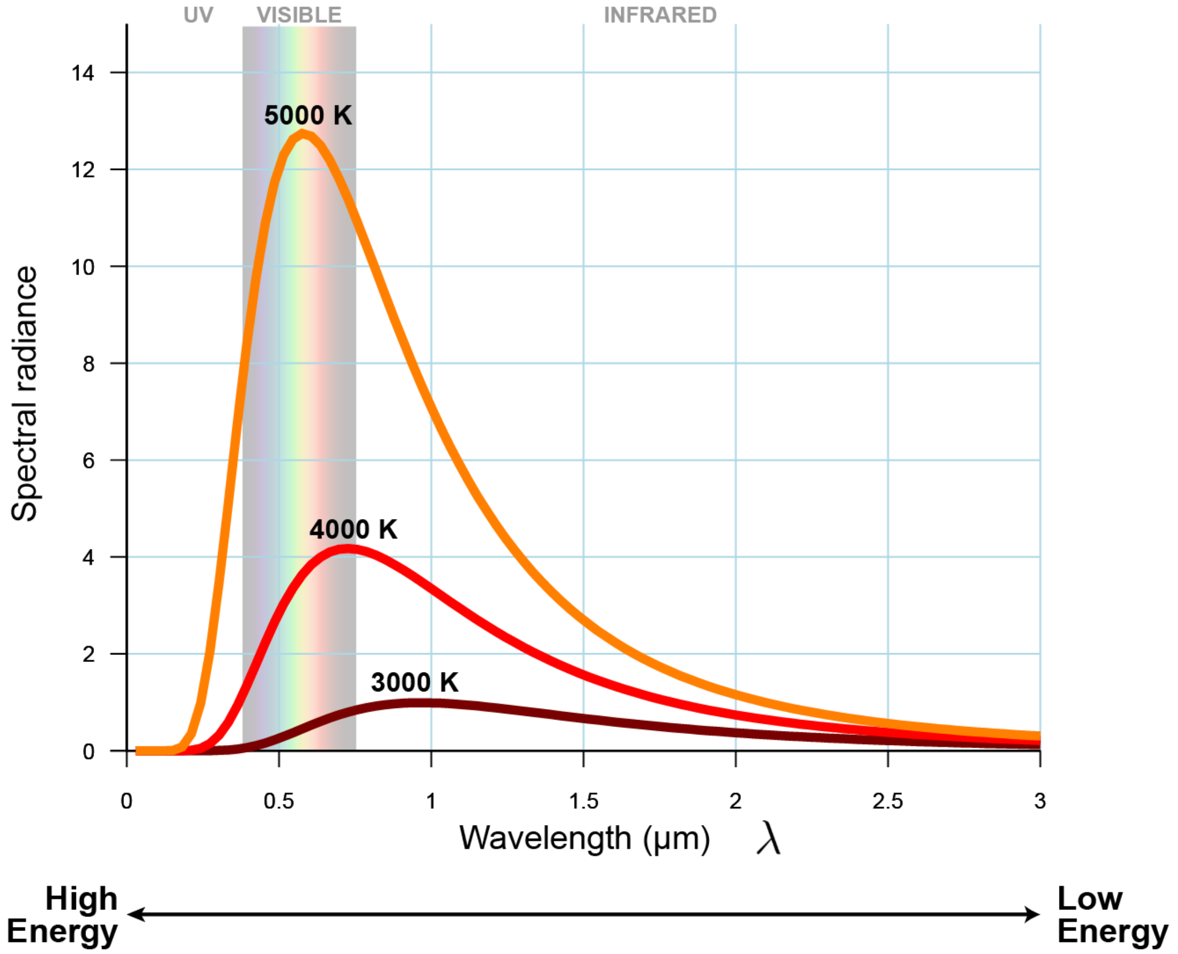
Black-body radiation

Within a body, photons are continuously being emitted and absorbed.

Stefan–Boltzmann law: energy radiated per unit-area [W/m^2], over the whole spectrum

$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = \varepsilon\sigma\, T_{\text{body}}^4$$

Planck's law



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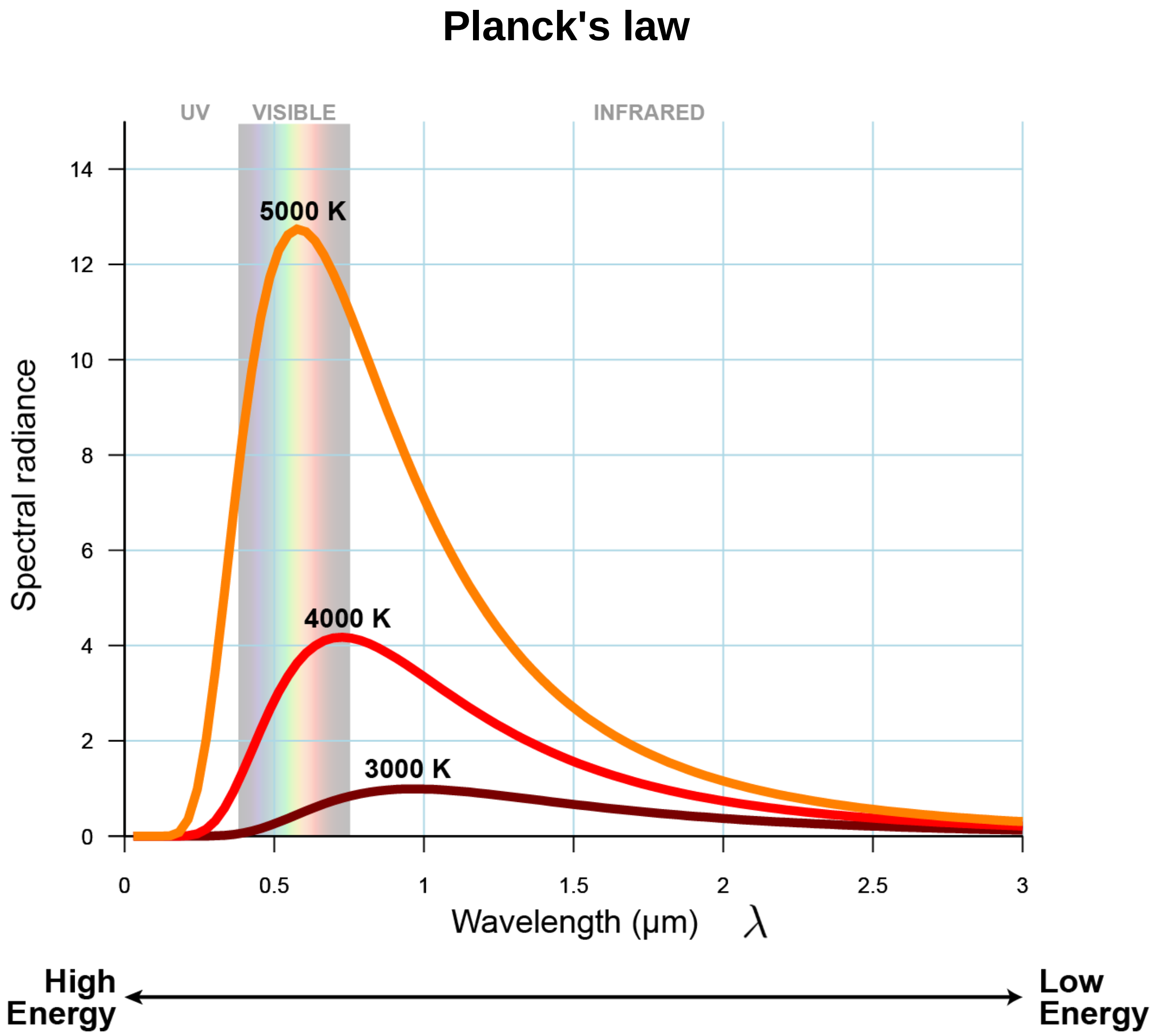
Stefan–Boltzmann law: energy radiated per unit-area [W/m^2], over the whole spectrum

$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = \varepsilon\sigma \, T_{\text{body}}^4$$

Total energy radiated, [W], by a body of surface area *A*

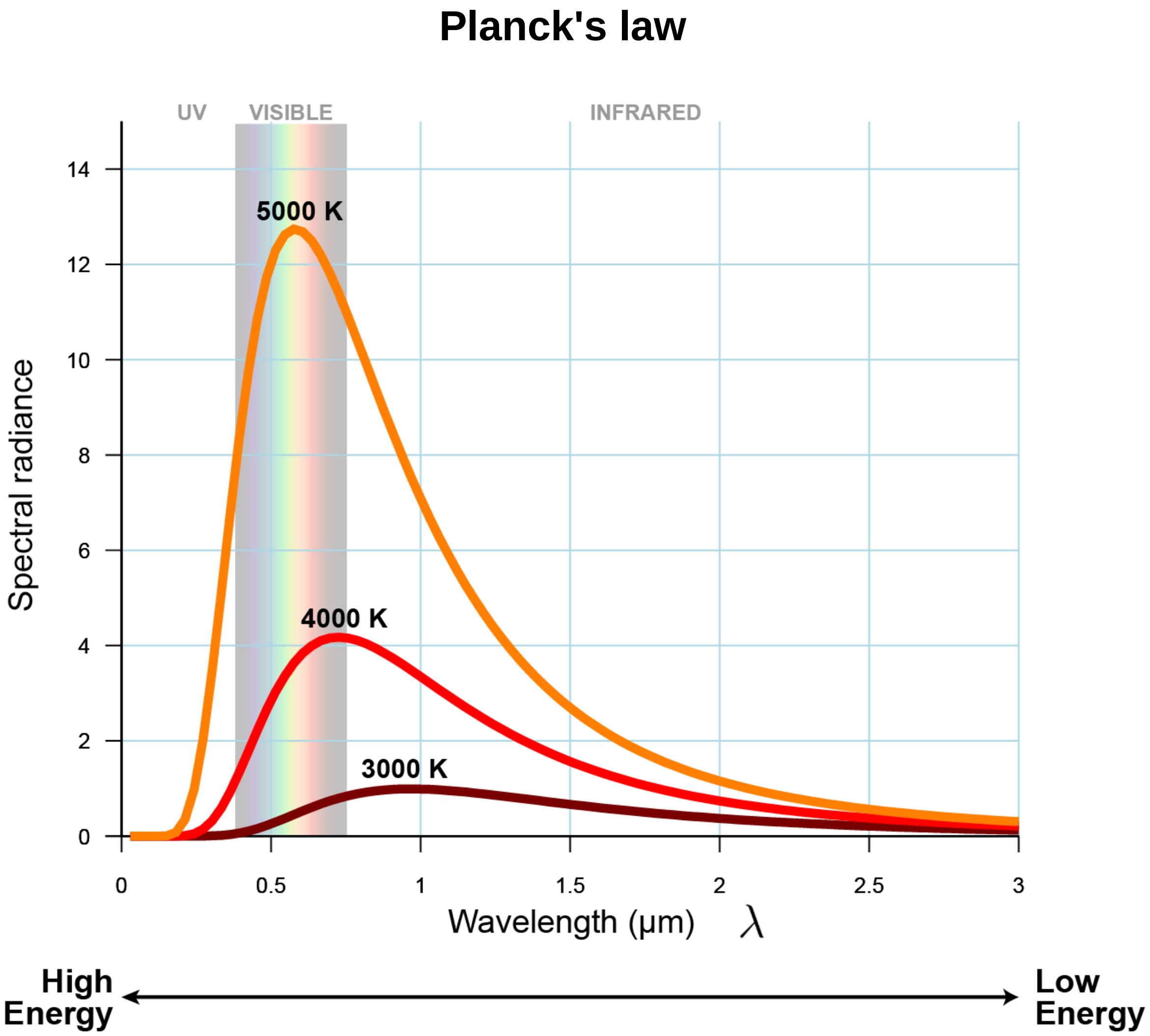
$$\dot{Q}_{\text{emitted}} = A\varepsilon\sigma \, T_{\text{body}}^4$$

Where $\varepsilon \in [0, 1]$ is the emissivity of the material.
It is a dimensionless quantity that goes to 1
for an ideal blackbody. Less than 1 for “gray bodies”.




NORMAL CAMERA

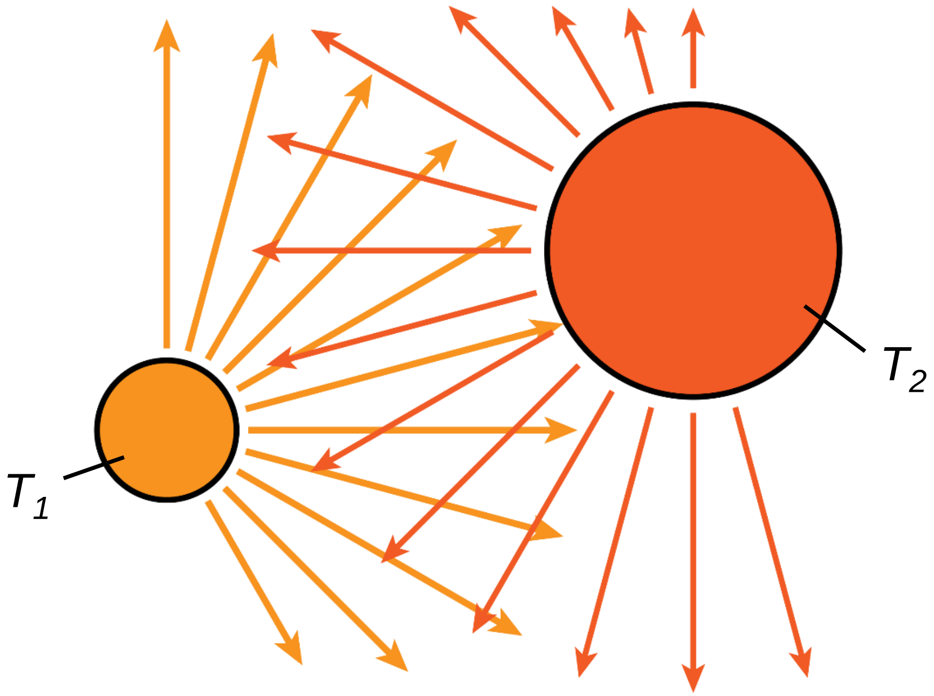

INFRARED VIEW

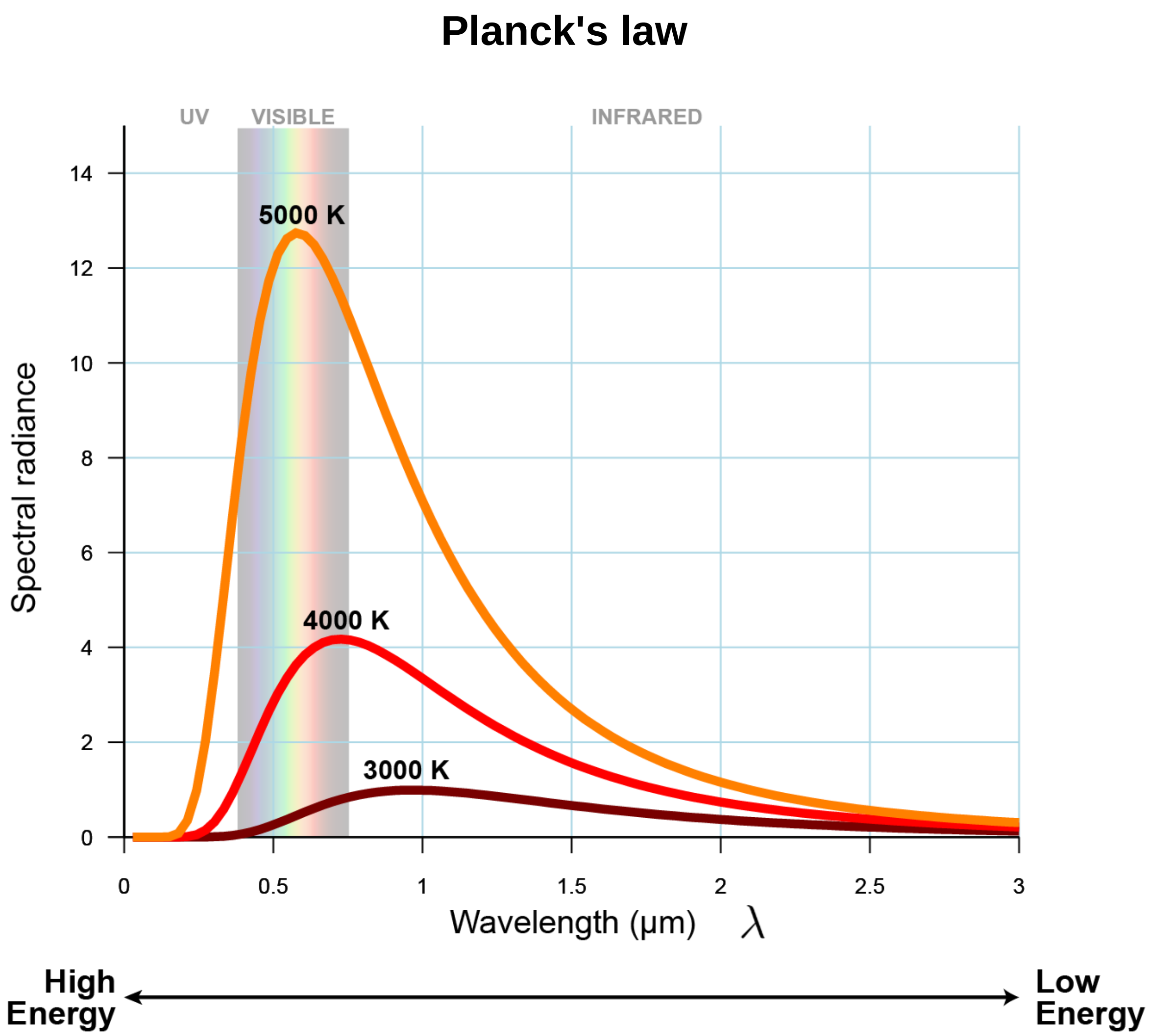

Planck's law
UV
VISIBLE
INFRARED
5000 K
4000 K
3000 K
Spectral radiance
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
Wavelength (μm) λ
High Energy
Low Energy

Black-body radiation

Total energy radiated by a body:

$$\dot{Q}_{\text{emitted}} = A\varepsilon\sigma\, T_{\text{body}}^{4}$$





Black-body radiation

Total energy radiated by a body:

$$\dot{Q}_{\text{emitted}} = A\varepsilon\sigma\, T_{\text{body}}^4$$

In many cases, it is realistic to assume that a body is also receiving radiation from all directions:

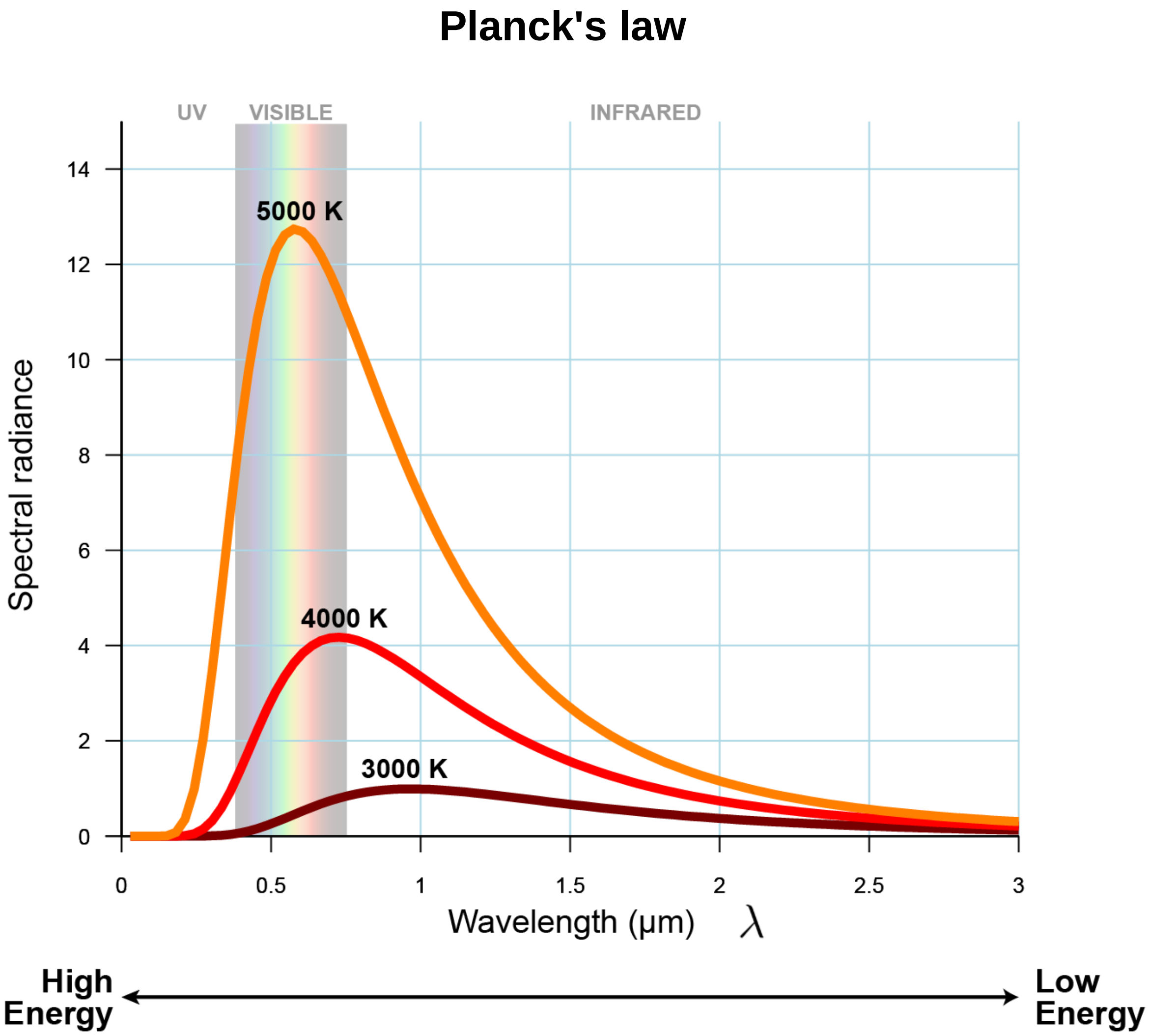
$$\dot{Q}_{\text{absorbed}} = A\varepsilon\sigma\, T_{\text{ambient}}^4$$

The net absorbed energy is thus given by:

$$\dot{Q}_{\text{absorbed, net}} = \dot{Q}_{\text{absorbed}} - \dot{Q}_{\text{emitted}}$$

At equilibrium:

$$\dot{Q}_{\text{absorbed}} = \dot{Q}_{\text{emitted}}$$
$$\Updownarrow$$
$$T_{\text{body}} = T_{\text{ambient}}$$



Summary of heat transfer mechanisms

Fourier's law of heat conduction

$$\boldsymbol{q} = -k\boldsymbol{\nabla}T$$

Convection at the boundary of a solid

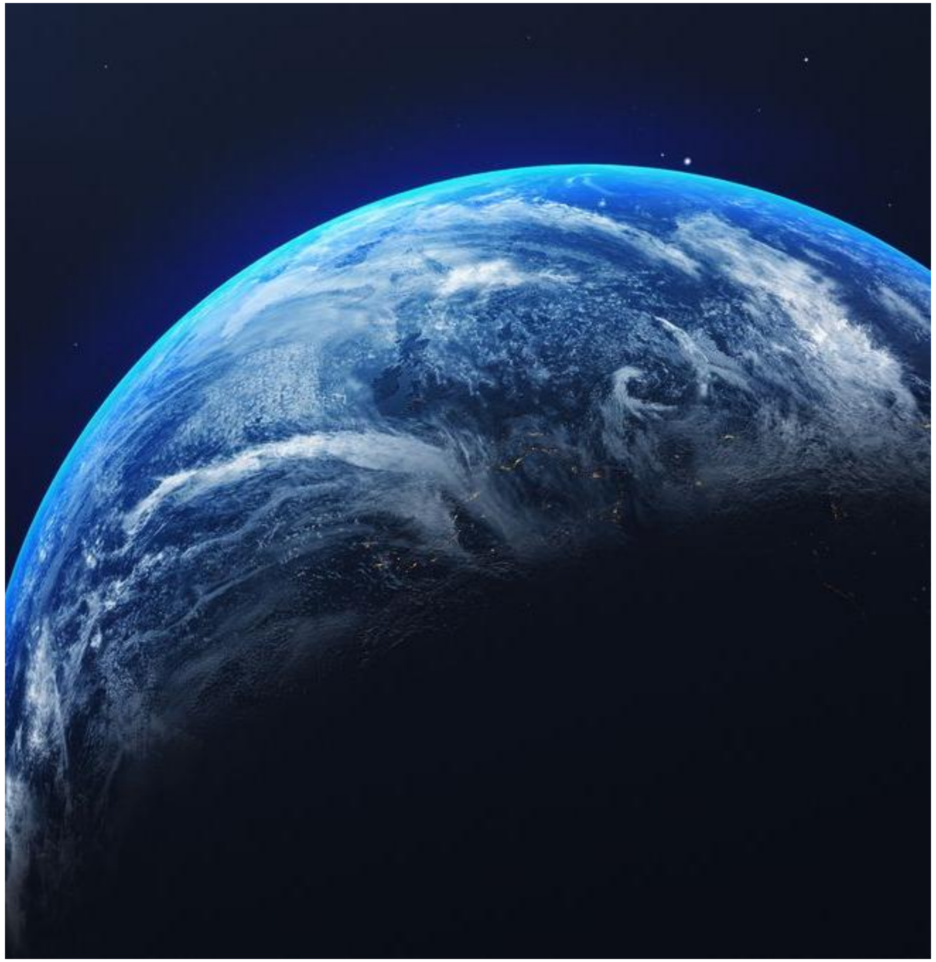
$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = h(T - T_{\text{fluid}})$$

Radiation

$$\boldsymbol{q} \cdot \hat{\boldsymbol{n}} = \varepsilon\sigma(T^4 - T^4_{\text{ambient}})$$

$\boldsymbol{q}$	Heat flux density [W/m^2 = J/(s m^2)]
$\boldsymbol{q} \cdot \hat{\boldsymbol{n}}$	Rate of heat *loss* of a body per unit area [W/m^2 = J/(s m^2)]
T	Temperature of the body
k	Thermal conductivity [W/(mK)]
h	Convection coefficient [W/(m^{2}K)]

Earth’s equilibrium temperature



- Sunlight arrives to earth with a solar irradiance (power per unit area) of 1366 W/m^2
- The average albedo of earth (fraction of incident light that is absorbed) is 0.3.
- The average emissivity of the earth’s surface is $\varepsilon = 0.95$.
- What is the equilibrium temperature of earth?