Solutions for selected exercises in Lecture 3
(Chapter 4 in "Energy, Entropy and Engines")

4.60

Notice, the "34 kW electrical heater" means Q = 34 kW. Based on first law:

$$\dot{Q}+\dot{W}=\dot{m}\left[(h_2-h_1)+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}+g(z_2-z_1)\right]=\dot{m}\left[c_p(T_2-T_1)+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}+g(z_2-z_1)\right],$$

Thus, the work done in the heater W is:

$$\dot{W}=0.8\text{ kg/s}\times[1.01344\times10^3\text{ J/kg°C}\times(200\text{°C}-60\text{°C})$$
$$+\frac{(50\text{ m/s})^2}{2}+9.81\text{ m/s}^2\times50\text{ m}]-34\times10^3\text{ W},$$
$$\dot{W}=0.8\text{ kg/s}\times[141{,}882\text{ J/kg }+1250\text{ J/kg }+490.5\text{ J/kg}]-34\times10^3\text{ W},$$
$$\dot{W}=80898\text{ W}=80.898\text{ kW}.$$

Be very careful about the unit. Many students mess up the J/kg and kJ/kg and get the wrong answer.

4.66

In this question, we should considered internal energy only:

$$\dot{m}_a h_{a,2}+\dot{m}_w h_{w,2}=\dot{m}_a h_{a,1}+\dot{m}_w h_{w,1},$$

The enthalpy of air can be interpolated from Appendix 7. The enthalpy of water can be interpolated from Appendix 8. Another equivalent approach is.

$$\dot{m}_a c_{p,a}(T_{a,2}-T_{a,1})=\dot{m}_w c_w(T_{w,2}-T_{w,1}).$$

where Cp,a is 1.0806 kJ/(kg ℃) (Appendix 4), Cw is 4.18 kJ/(kg ℃) (Appendix 3). As a result:

$$\dot{m}_w=\frac{0.5\text{ kg/s}\times1.0806\text{ kJ/kg°C}\times(600\text{°C}-300\text{°C})}{4.18\text{ kJ/kg°C}\times(80\text{°C}-20\text{°C})},$$
$$\dot{m}_w=0.64629\text{ kg/s}.$$

4.68

Air enters a nozzle at 300 kPa and 350 K with a velocity of 30 m/s and leaves at 100 kPa with a velocity of 200 m/s. The heat loss from the nozzle is 20 kJ/kg of airflow through it. What is the exit temperature of the air?

Energy balance:

$$\dot{Q} + \underbrace{\dot{W}}_{=0} = \dot{m}\left[(h_2 - h_1) + \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} + \underbrace{g(z_2 - z_1)}_{=0}\right]$$

There is no work done and no potential energy. Consequently, you can rearrange for h_2:

$$h_2 = h_1 + \frac{\dot{Q}}{\dot{m}} + \frac{\mathbf{V}_1^2 - \mathbf{V}_2^2}{2}$$

Initial air specific enthalpy at T_1 = 350 K from Appendix 7: h_1 = 350.49 kJ/kg.

$$h_2 = 350.49\ \text{kJ/kg} + \frac{(30\ \text{m/s})^2 - (200\ \text{m/s})^2}{2 \times 10^3\ \text{J/kJ}} - 20\ \text{kJ/kg} = 310.94\ \text{kJ/kg},$$

$$T_2 = 310.70\ \text{K}.$$

Please note that kJ are converted in J by adding 10^3 to the denominator of the middle term.

4.82

A gas turbine operates witj 0.1 kg/s of helium that enters at 8 MPa and 600 K and leaves at 200 kPa and 350 K. If the turbine power output is 120 kW, find the rate of heat loss from the turbine. Neglect kinetic energy changes.

Energy balance:

$$\dot{Q} + \dot{W} = \dot{m}\left[(h_2 - h_1) + \underbrace{\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2}}_{=0} + \underbrace{g(z_2 - z_1)}_{=0}\right]$$

$$\dot{Q} = \dot{m}c_p(T_2 - T_1) - \dot{W},$$

No potential and no kinetic energy. Specific heat of helium at constant pressure c_p = 5.193 kJ/kg°C assumed (Appendix 1).

$$\dot{Q} = 0.1\ \text{kg/s} \times 5.193\ \text{kJ/kgK} \times (350\ \text{K} - 600\ \text{K}) + 120\ \text{kW}$$

$$\dot{Q} = -9.825\ \text{kW}.$$

Please note that a consideration of task-specific temperature (600 K, 350K) and pressure (8 MPa, 200 kPa) leads to a more realistic value for the specific heat c_p but is not included here.

4.76.

Nitrogen flowing at a rate of 60 kg/min is compressed from 150 kPa and 30°C to 800 kPa and 150°C. The compressor is cooled at a rate of 15 kJ/kg of gas during operation. What is the power required to drive the compressor?

Energy balance:

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{in} = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{out}$$

Kinetic energy and potential energy are equal to zero. Specific heat of nitrogen at constant pressure c_p = 1.04179 kJ/kg°C at T_{avg} = 363.15 K interpolated from Appendix 4.

$$\dot{W}_{in} + \dot{m}(h)_{in} = \dot{Q}_{out} + \dot{m}(h)_{out}$$

$$\dot{W}_{in} = \frac{60\ kg/min}{60\ s/min}\left(1.04179\frac{\mathrm{kJ}}{\mathrm{kg}}°\mathrm{C}(150°\mathrm{C} - 30°\mathrm{C})\right) + 15\frac{\mathrm{kg}}{\mathrm{s}} = 140\ kW$$

4.77.

An air compressor takes air at 315 K and compresses it. For every kilogram of air passing through the compressor it does 200 kJ of work and loses 10 kJ of heat. Find the temperature of the air exiting the compressor. Do not assume that the specific heat of air is constant.

Specific enthalpy of air at 315 K, h_{in} = 315.27 kJ/kg (Appendix 7).

$$\dot{W}_{in} + \dot{m}(h)_{in} = \dot{Q}_{out} + \dot{m}(h)_{out}$$

$$(h)_{out} = \dot{W}_{in} + \dot{m}(h)_{in} - \dot{Q}_{out} = 200 + 315.27 - 10 = 505\ kJ/kg$$

Interpolating from the air tables, Appendix 7, the final temperature is T_{out} = 502.18 K.

4.80.

Argon enters an adiabatic turbine at 2 MPa and 600°C at a rate of 2.5 kg/s and leaves at 150 kPa. Determine the exit temperature of the gas when the turbine is generating 300 kW. Neglect kinetic energy changes.

Specific heat of argon at constant pressure c_p = 0.520 kJ/kg°C (Appendix 1).

$$\dot{m}(h)_{in} = \dot{W}_{out} + (h)_{out}$$

$$\dot{m}C_pT_{in} = \dot{W}_{out} + \dot{m}C_pT_{out}$$

$$T_{out} = \frac{-\dot{W}_{out}}{\dot{m}C_p} + T_{in} = \frac{-300}{2.5 \times 520} + 600 = 369°\mathrm{C}$$

4.81.

Air enters an adiabatic turbine at 800 kPa and 870 K with a velocity of 60 m/s, and leaves at 120 kPa and 520 K with a velocity of 100 m/s. The inlet area of the turbine is 90 cm^2. What is the power output?

Gas constant of air R = 0.2870 kJ/kgK (Appendix 1), specific enthalpy of air at 520 K h_2 = 523.63 kJ/kg (Appendix 7).

$$\dot{m}\left(h+\frac{V^2}{2}\right)_{in} = \dot{W}_{out} + \dot{m}\left(h+\frac{V^2}{2}\right)_{out}$$

$$v_{in} = \frac{RT_{in}}{P_{in}} = \frac{0.2870 \times 870}{800} = 0.312\ m^3/kg$$

$$\dot{m} = \frac{A_{in}V_{in}}{v_{in}} = \frac{90 \times 10^{-4} \times 60}{0.312} = 1.73\ kg/s$$

Via interpolation h_1 = 899.415 kJ/kg (Appendix 7).

$$\dot{W}_{in} = \dot{m}\left(h+\frac{V^2}{2}\right)_{in} - \dot{m}\left(h+\frac{V^2}{2}\right)_{out} = 1.73\ (899.415 - 523.63 + \frac{60^2-100^2}{2} = 644.6\ kW$$