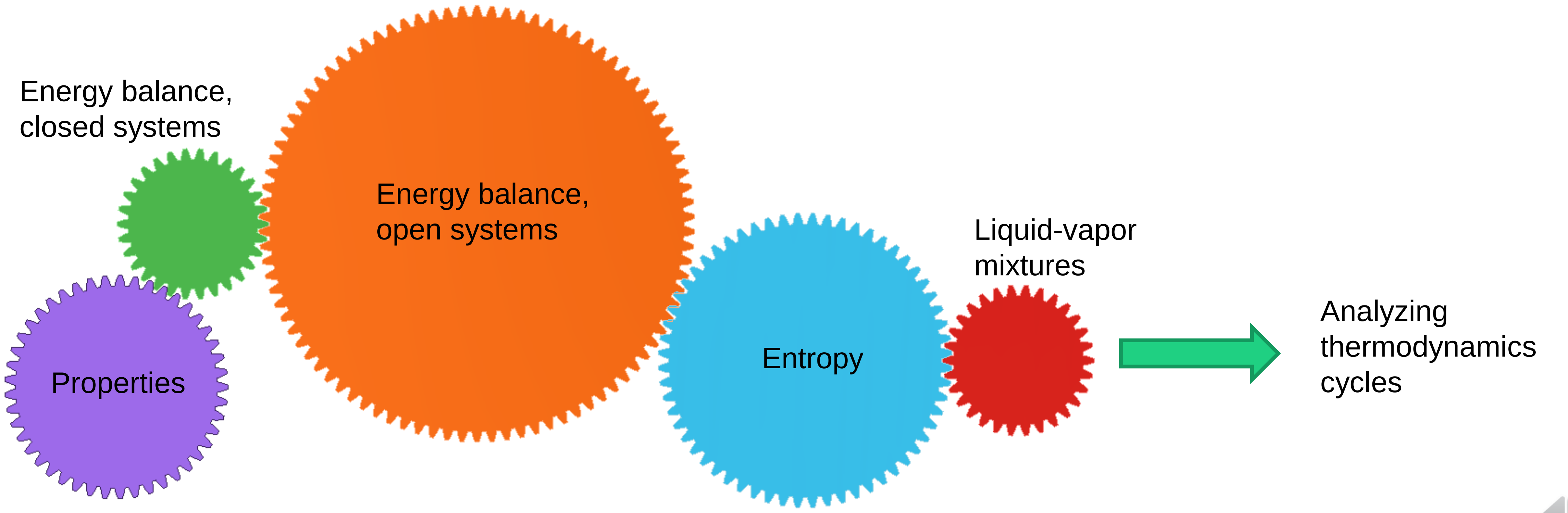
47201 Engineering thermodynamics

Lecture 7a: Ideal heat engines (Ch. 8.1-3)

Thermodynamic cycles

- What we have done in this course up to now has prepared us to look at how a thermodynamic cycle can be applied, in the first case, to produce work/electricity from heat



Cycles

<u>Definition</u>

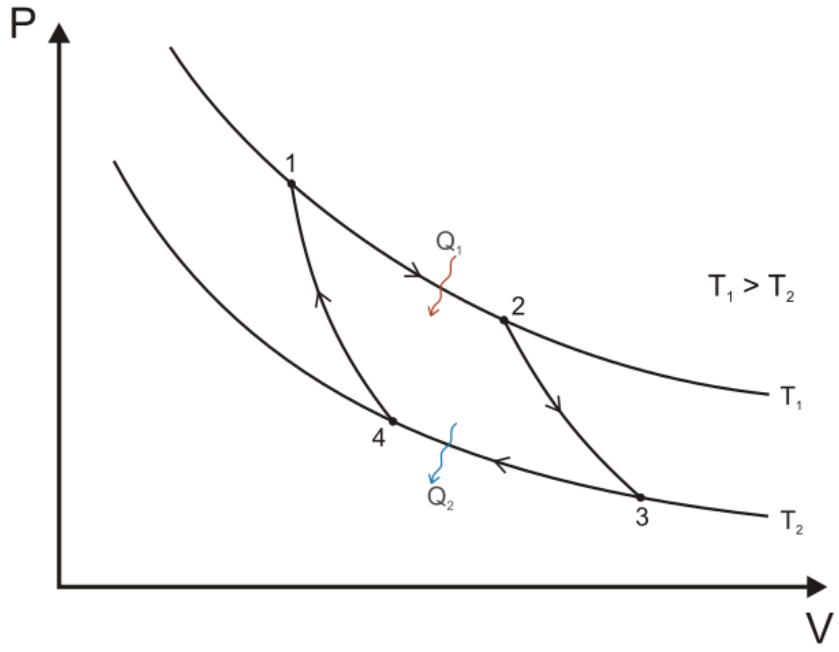
A **cycle** is a process or **series of processes** that **restore** a system to its **initial state** and does something useful

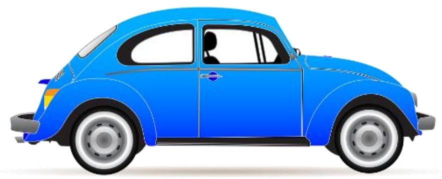
<u>Implications:</u>

Integrating a **property over a cycle will always give 0**

However, the **transfer of energy in the form if heat or work will not be 0**!

All **processes** that do **work repeatedly** must be cycles. This makes the analysis of cycles immensely important!





Heat engine

A heat engine is any device that operates in a cycle and does work on the surroundings when heat is input.

- By operating in a cycle, the device do work repeatedly

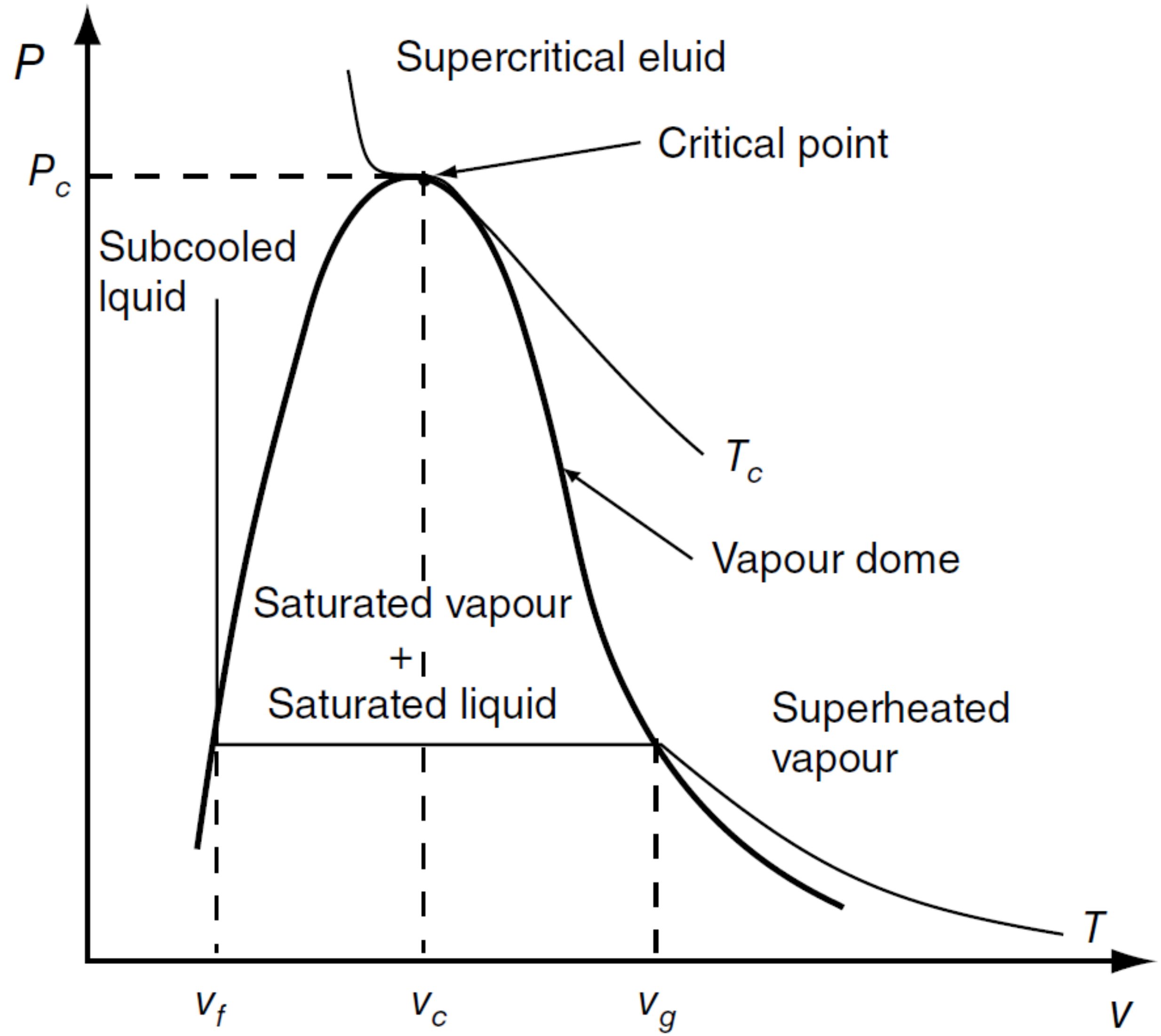
- Examples of heat engines include internal combustion engines, locomotives, gas turbines, and power plants

- Batteries and fuel cells produce electricity from chemical reactions and are not heat engines, although there are still heat transfer interactions

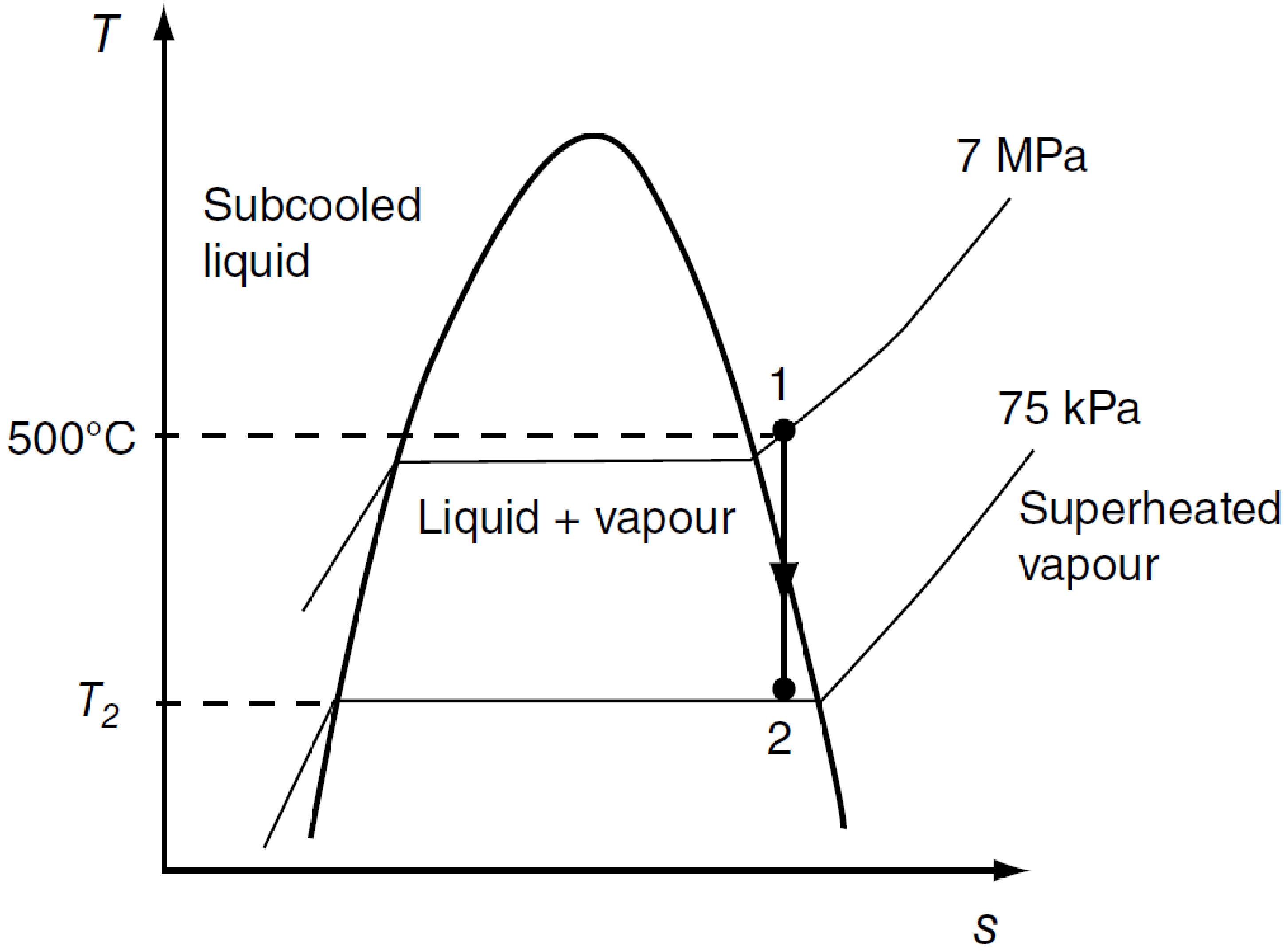
Phase diagrams and cycles

- We use phase diagrams when analyzing cycles because they help visualize the operating points
- Plotting state points on a phase diagram can also be a way to double check calculations and property values
- For power production and refrigeration/heat pump cycles we will visualize the cycles using both T-s and p-v diagrams
  - The book focuses more on T-s diagrams and therefore the lectures will as well

Property charts from the book



Property charts from the book



Heat engine - efficiency

One of the most important figures of merit for any thermodynamic device is the efficiency, η.

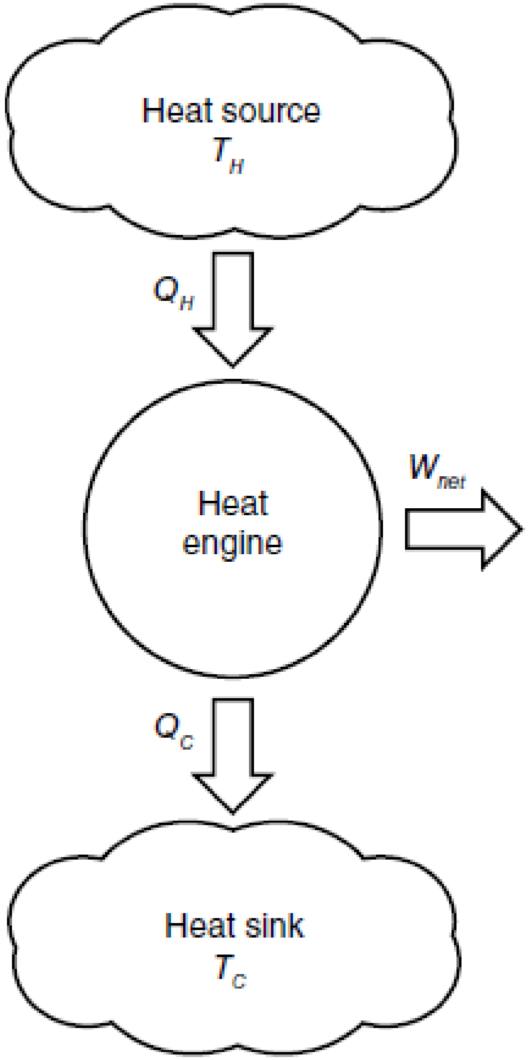
$$\eta = \frac{\text{net work output}}{\text{heat input}} = \frac{W_{net}}{Q_{in}}$$

- We can also define the efficiency on a rate basis

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

- This is also called a 1st law efficiency because it is based purely on energy

Power cycles – overall analysis



The overall energy balance for any similar heat engine is:

- $\dot{Q}_{in} + \dot{W}_{in} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{in} = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{out}$

$$\dot{Q}_H = \dot{W}_{out} + \dot{Q}_C$$

- And on an energy basis

$$Q_H = W_{out} + Q_C$$

What is the maximum efficiency of a heat engine?

According to the 1st law of thermodynamics, it is possible to put in some energy in the form of heat and get that same energy out in the form of work. This would correspond to an efficiency of 100%.

- However, we are constrained by the physics of the working fluids. The heat added has a lower thermodynamic value (higher entropy) and it cannot be converted completely to work in a heat engine.

- From the Kelvin-Planck statement: it is impossible for for any device operating in a cycle to receive heat from a high temperature source and produce a net amount of work without rejecting heat to a low temperature sink.
  - Since some heat also leaves the system, the efficiency must be below 100%.

Carnot engine – historical perspective

- Sadi Carnot – French physicist – 1796-1832

- Carnot's book (the only one he published) "*Reflections on the motive power of fire*" – 1824
- Based on his desire to understand and optimize steam engines

- Work was the basis of the 2nd law of thermodynamics (before 1st law)

- Some false theory (caloric etc.) but correct conclusions:

1) A heat engine must lose heat to surroundings to operate ($\neq$ 100% efficiency) – 2nd law.
2) Efficiency only depends on the temp. of heat source and sink (not on the type of working fluid)

- Carnot's book not read by many scientists at the time of publication.
- Later (and translated) became part of the foundation for modern thermodynamics

The Carnot cycle [Carnot engine]

The Carnot cycle uses four processes where each step is as efficient as possible. Carnot found that the most efficient process is independent of working fluid so the Carnot cycle is specified only in terms of the operating temperatures.

- The cycle is built between a hot reservoir and a cold reservoir.
- The compressor (work input) and expander (work generation) operate isentropically (reversibly) which is the most efficient way for them to operate
- Heat is added isothermally at the hot reservoir and removed isothermally at the cold reservoir

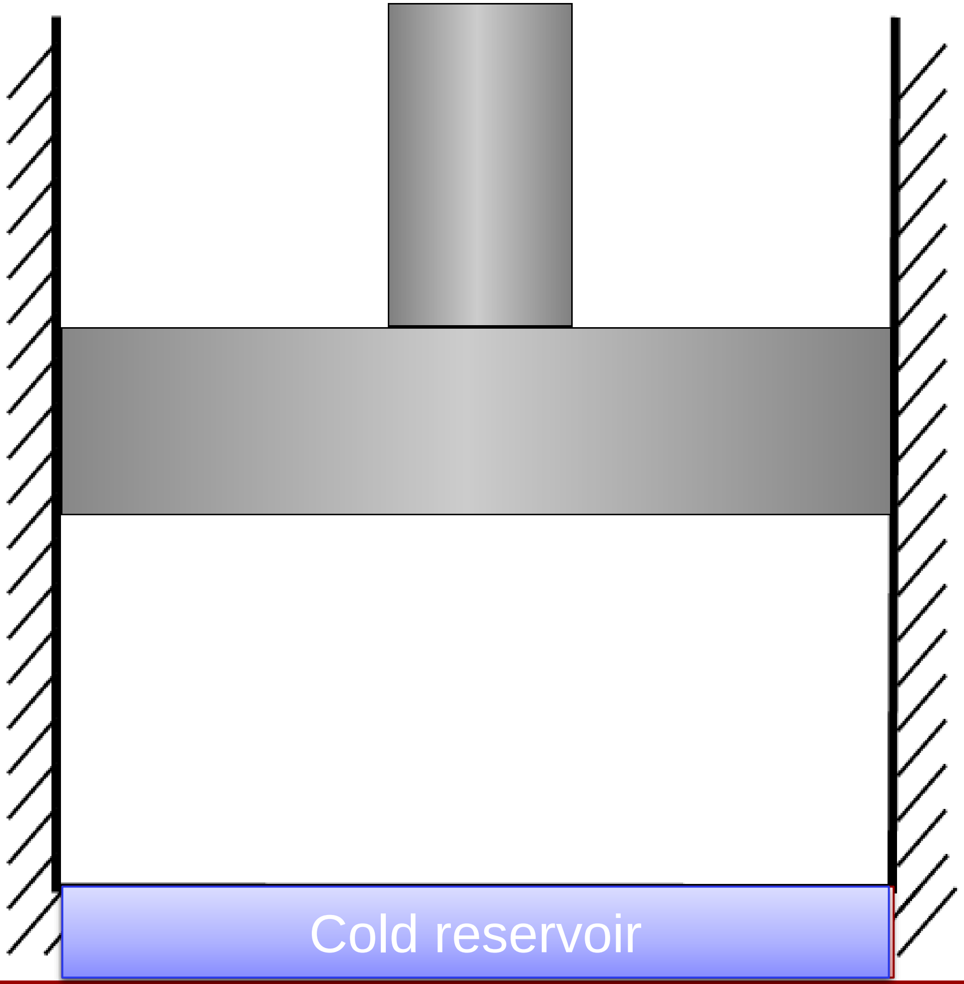
The single-phase (closed system) Carnot cycle

State 1

State 2

State 3

State 4



Adiabatic compression
Isothermal expansion
Adiabatic expansion
Isothermal compression

Entropy addition for the Carnot cycle

- If the heat is added reversibly and the expansion and compression are isentropic, then the change in entropy of the system is 0

$$\Delta S_{cycle} = \oint \frac{\delta Q_{int,rev}}{T} = 0$$

- At the hot reservoir, the entropy addition is

$$\Delta S_{addition} = \frac{Q_H}{T_H}$$

- Similarly at the cold reservoir

$$\Delta S_{rejection} = \frac{Q_C}{T_C}$$

Carnot cycle analysis

- Since the overall cycle is reversible and there is no entropy associated with work, the entropy associated with the heat addition is equal to the entropy associated with the heat rejection

$$\Delta S_{addition} = \Delta S_{rejection} \Rightarrow \frac{Q_H}{T_H} = \frac{Q_C}{T_C}$$

- The ratio of the heat additions is then

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

- Remembering that $Q_H = W_{out} + Q_C$ for a heat engine, we can express the work as

$$W_{out,Carnot} = Q_H - Q_H \frac{T_C}{T_H} = Q_H \left(1 - \frac{T_C}{T_H}\right)$$

- Then the efficiency of the Carnot cycle is

$$\eta_{th,Carnot} = \frac{W_{out}}{Q_H} = \frac{Q_H \left(1 - \frac{T_C}{T_H}\right)}{Q_H} = \left(1 - \frac{T_C}{T_H}\right)$$

Two-phase Carnot cycle

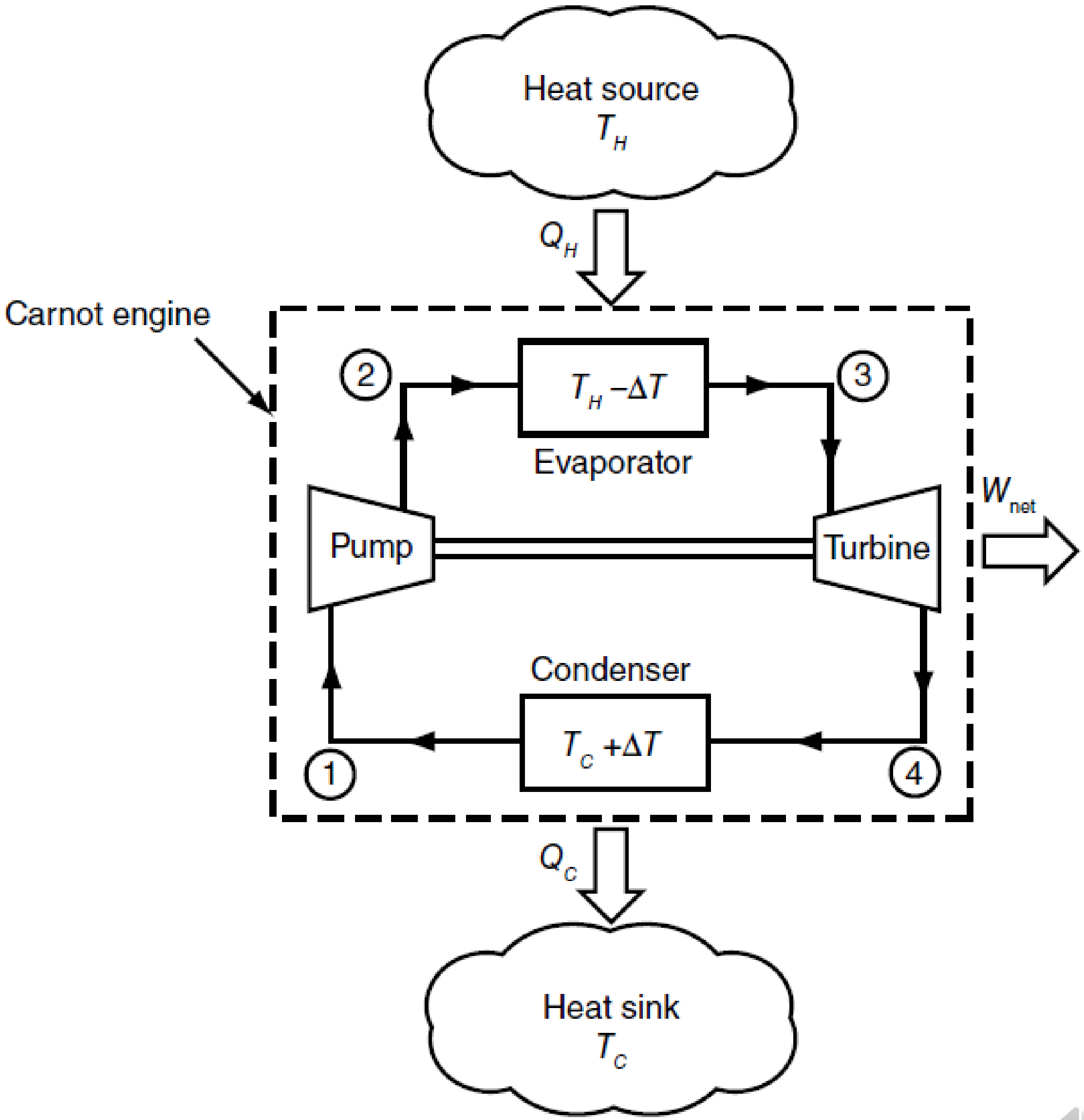
The two-phase Carnot cycle is built on the same principles as the single phase cycle, but now four separate devices are used, which allows the cycle to produce work constantly

1-2 a two-phase fluid is compressed adiabatically to a higher temperature and pressure

2-3 heat is added at a high pressure and constant temperature while the fluid is always within the vapor dome

3-4 the working fluid is expanded isentropically to produce work

4-1 the working fluid is condensed isothermally and at low pressure to return it to its original state



Two-phase vs single phase Carnot cycles

- The single phase cycle uses a gas in a control mass that completes the four steps of the cycle sequentially
  - The cycle produces work approximately half of the time and consumes work approximately half the time

- The two phase cycle uses four separate control volumes, with each one operating continuously.
  - The two phase cycle makes use of the fact that a fluid can increase in energy while remaining at a constant temperature as it is converted from a saturated liquid to a saturated vapor
  - Since all four devices operate continuously, the cycle outputs and consumes work continuously

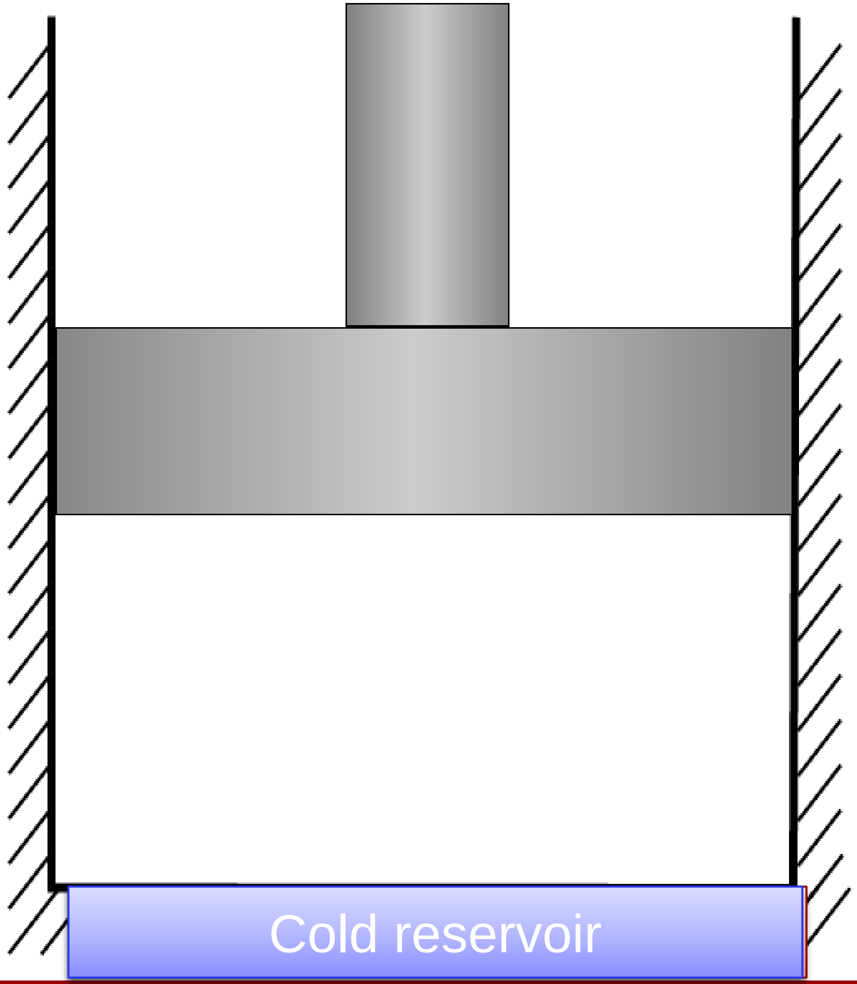
The single-phase Carnot cycle P-V diagram

State 1

State 2

State 3

State 4



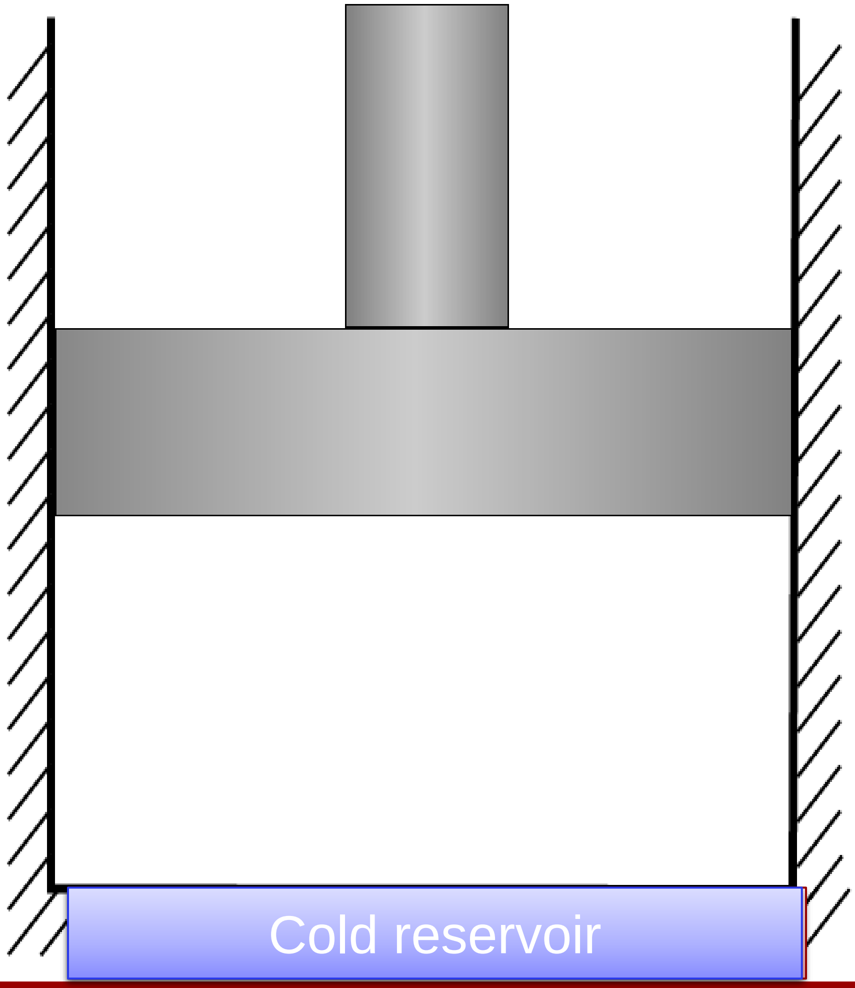
The single-phase Carnot cycle T-s diagram

tate 1

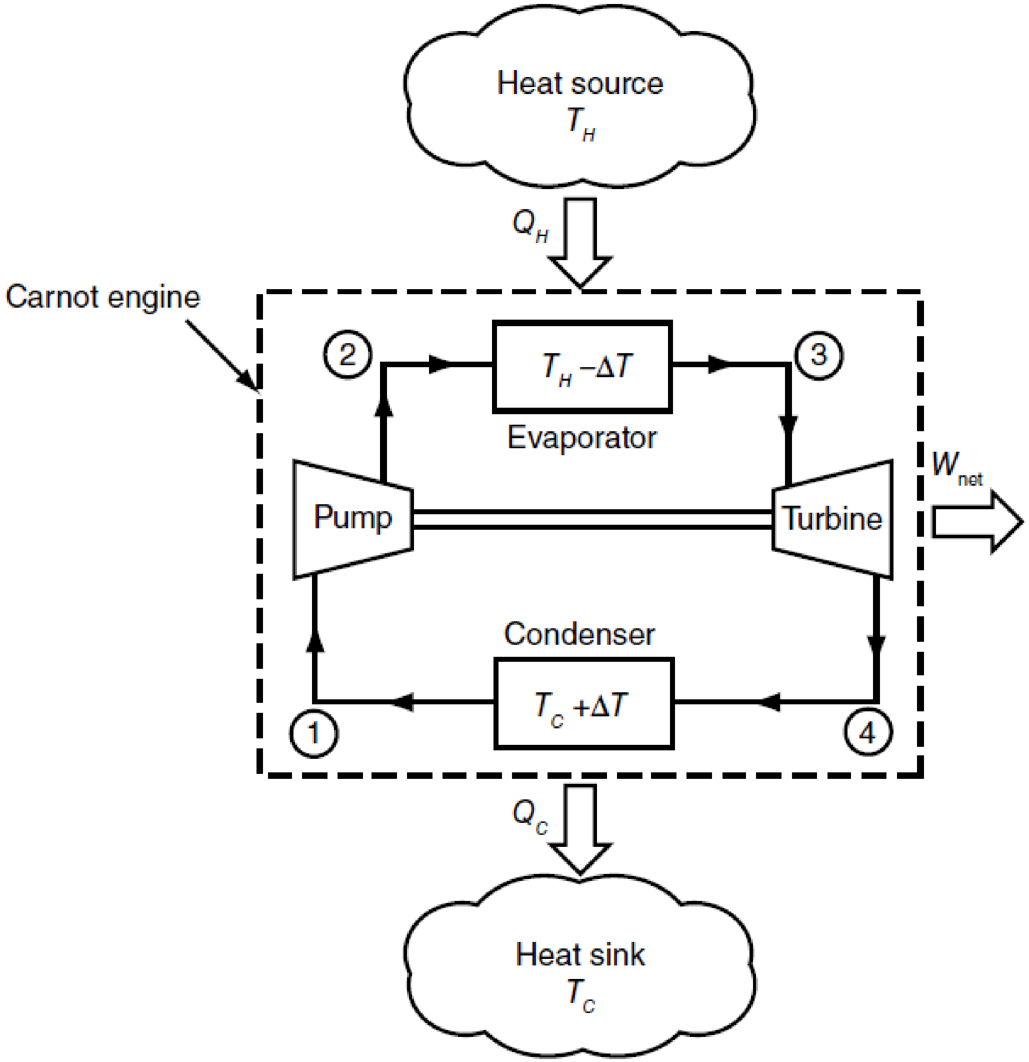
tate 2

tate 3

tate 4



Two-phase Carnot cycle T-s diagram



Review: quality

Many heat engines use the fact that fluids can absorb a large amount of heat at a constant temperature during the phase change from liquid to gas. Inside the vapor dome, an additional term can be used to specify the state: the quality, which is denoted as *x*. The book uses a subscript *g* for vapor and *f* for liquid.

- A quality of 0 corresponds to saturated liquid and a quality of 1 corresponds to saturated vapor.

$$x = \frac{m_g}{m}$$

- Within the vapor dome, the properties can be calculated based on the the quality

$$x = \frac{u - u_f}{u_g - u_f}$$

Example 8.2

A Carnot engine using 10 kg / s of water as the working fluid operates between an evaporator temperature of 300 C and a condenser temperature of 80 C. Find the rate of heat addition in the boiler and the steam quality at the turbine exhaust.

Find: Rate of heat addition Q_H and steam quality x_4 at the turbine exhaust.

Assume:

1. Carnot cycle with isentropic expansion and compression
2. Quality entering the turbine is 1
3. Quality leaving the compressor is 0

Example 8.2 solution: heating

1st law analysis of the evaporator (boiler)

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{in} = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{out}$$

The change in enthalpy is from saturated liquid to saturated gas (state 2-3)

$$\dot{Q}_H = \dot{m}(h_3 - h_2) \quad = \dot{m}\left(h_g - h_f\right)$$

Looking up the values in Appendix 8a gives

$$\dot{Q}_H = 10\frac{kg}{s}\left(2749.0\frac{kJ}{kg} - 1344.0\frac{kJ}{kg}\right)\frac{1\ MW}{1000\ kW} = 14.05\ MW$$

Example 8.2 solution: quality exiting the turbine

Since the expansion is isentropic, the entropy at the inlet and exit are equal.

- The entropy entering the turbine (state 3) is the entropy of saturated vapor from Table 8a

$$s_3 = s_g(T = 300\ C) = 5.7045 \frac{kJ}{kg\ K}$$

The entropy at the turbine exit can be calculated based on the quality

$$x = \frac{s_4 - s_f}{s_g - s_f} = \frac{5.7045 \frac{kJ}{kg\ K} - 1.0753 \frac{kJ}{kg\ K}}{7.6122 \frac{kJ}{kg\ K} - 1.0753 \frac{kJ}{kg\ K}} = 0.7078$$

Example 8.3

Problem: A Carnot engine using air as the working fluid works between temperatures of 573 K and 293 K. The pressure at the start of isothermal expansion is 100 kPa and at the end is 50 kPa. Find the work output and heat added per kilogram of air.

Find: Work output *W* and heat added *Q* both per kilogram of air in the Carnot engine.

Assume:

1. Air is an ideal gas

Example 8.3 solution: Input heat

The isothermal expansion is where heat is added. We can use Eq. 6.40 to find the change in entropy during the process

$$\Delta s_{23} = c_p \ \ln\left(\frac{T_3}{T_2}\right) - R \ \ln\left(\frac{P_3}{P_2}\right) = -0.287 \frac{kJ}{kg\ K} \ln \frac{50\ kPa}{100\ kPa} = 0.1989 \frac{kJ}{kg\ K}$$

Then the heat addition can be calculated using Eq. 6.63

$$Q_{23} = T_H \Delta s_{23} = 573\ K\ 0.1989 \frac{kJ}{kg\ K} = 113.99 \frac{kJ}{kg}$$

Example 8.3 solution: Work output

Work is done during the adiabatic expansion. We can calculate the work from the two heat flows Eq. 8.10

$$W_{net} = Q_H - Q_C$$

Knowing that the entropy entering via Q_H is the same as exiting via Q_C because it is a Carnot cycle, we can use the mass specific version of Eq. 8.16 to calculate the work

$$W_{net} = (T_H - T_C)\,(s_3 - s_2) = (573\ K - 293\ K)\ 0.1989\,\frac{J}{kg\ K} = 55.70\,\frac{kJ}{kg}$$