

DTU



47201 Engineering thermodynamics

Lecture 2b: First law for closed systems (Ch 4.1-4.12)



- Energy has units of [Force x Distance $\frac{kg\ m^2}{s^2}$], and the SI unit is Joules – J
– kJ, GJ etc.
- Because energy changes measured in J usually become very large and for convenience, the term kWh (kWhr, kW-h) is often used as a unit of energy
 - $1\ kWh \equiv 1000\ W\ 3600\ s = 3,600,000\ J$
 - Energy can also be expressed in MWh GWh etc.
- kWh units are often convenient for calculating electrical consumption. For example, if a building consumes 2.65 kW on average over a day, then the electrical energy usage is

$$E_{el} = 2.65\ kW\ 24\ h = 63.6\ kWh$$



Power vs Energy

- Power is the rate of change in energy – equivalent to velocity and position in kinetics. For this reason, it is often represented as a rate of work, \dot{W} , or a rate of heat, \dot{Q}
- Power has units of Watts $\left[\frac{kg\ m^2}{s^3}\right]$ – W, kW and also horsepower – hp
 - $1\ W \equiv \frac{1\ J}{s}$
- The change in energy of a system can be calculated by integrating the power from both heat over time

$$\Delta E = \int (\dot{W} + \dot{Q}) dt$$



Power vs Energy examples

- Electric water boiler with a power of 2400 W
 - Power tells how fast it will boil the water
 - The change in energy depends on the properties of the water
- Electricity bills are calculated in kWh
 - Depends on how much energy was used, not how fast it was used (power)
- An electric car has a battery capacity 33 kWh and a motor power of 135 kW
 - Battery capacity indicates the driving range of the car
 - Motor power indicates how fast the car can accelerate



Power vs Energy common mistakes

- It's very common to mix up power and energy, but for those of you studying energy systems it's important to be careful with the terms
- Some common mistakes to avoid
 - “The building uses 10 kW of energy per day”
 - Stating energy in kW or power in J
 - Forgetting that most systems have independent power and energy ratings that both need to be specified.



Polytropic processes

Remember that work done by, for example, a piston during expansion is path dependent, meaning that we need to know the pressure and volume during the entire process to calculate the work correctly

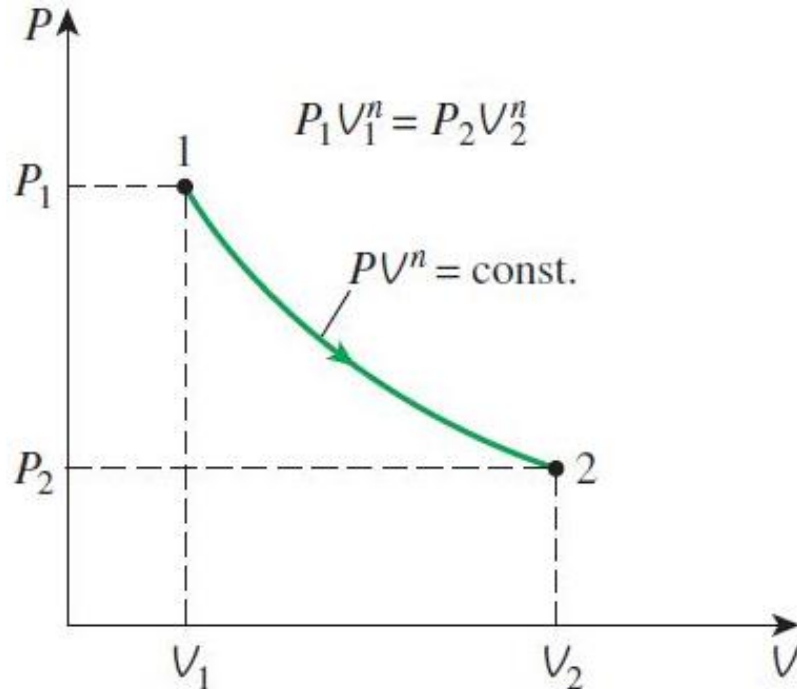
- During the expansion of a gas, the pressure and temperature often decrease simultaneously
- To solve problems with changing pressure and temperature, we introduce relations for the pressure and volume during these processes, which are called polytropic processes

$$C = P_1 V_1^n = P_2 V_2^n$$



Boundary work, polytropic processes

- The work for a polytropic process can be calculated as for any boundary work as:
- $W_{12} = - \int_{V_1}^{V_2} P dV$
- Since $P = C \frac{1}{V^n}$ we can substitute P into the work equation and get $W_{12} = -C \int_{V_1}^{V_2} \frac{dV}{V^n}$



- Evaluating the integral
- $W_{12} = \frac{1}{n-1} \left[\frac{C}{V_2^{n-1}} - \frac{C}{V_1^{n-1}} \right] \quad n \neq 1$
- Plugging in $C = P_1 V_1^n$ and $C = P_2 V_2^n$ gives

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{n-1} \quad n \neq 1 \text{ and}$$

- $W_{12} = P_2 V_2 \ln \left(\frac{V_1}{V_2} \right) \quad n = 1$

Example 4.7

Gas in a cylinder is expanded by a piston in a process for which $PV^n = C$, where C and n are constants. The initial pressure and volume are 3 bar and 0.2 m³ respectively and the final volume is 0.6 m³. Determine the work done by the gas if (a) $n = 1.4$ and (b) $n = 1.0$.

Find: Work W done by the gas.

- Part a) We will use the work equation (Eq. 4.30) for a polytropic process. For part a) n is not equal to 1, so we use:

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{n - 1}$$

- First we need P_2 , so $P_2 = P_1 \left(\frac{V_2}{V_1} \right)^{1.4} = 3 \text{ bar} \left(\frac{0.2 \text{ m}^3}{0.6 \text{ m}^3} \right)^{1.4} = 0.644 \text{ bar}$
- Then we can calculate the work

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{n - 1} = \frac{0.644 \text{ bar} \cdot 0.6 \text{ m}^3 - 3 \text{ bar} \cdot 0.2 \text{ m}^3}{1.4 - 1} \frac{100 \text{ kPa}}{1 \text{ bar}} = -53.4 \text{ kJ}$$



Example 4.7 part b)

- Part b) $n = 1$ so we use Eq. 4.32:

$$W_{12} = P_1 V_1 \ln \left(\frac{V_1}{V_2} \right)$$

- Here we don't need P_2 so we can calculate the work directly

$$W_{12} = P_1 V_1 \ln \left(\frac{V_1}{V_2} \right) = 3 \text{ bar} \frac{100 \text{ kPa}}{1 \text{ bar}} 0.2 \text{ m}^3 \ln \left(\frac{0.2 \text{ m}^3}{0.6 \text{ m}^3} \right) = -65.917 \text{ kJ}$$



Exercise 4.16

The pressure in a balloon increases linearly with its diameter. When it is filled with air at 200 kPa its volume is 1 m³. Find the work done when the balloon is inflated to a volume of 1.5 m³.

- Assume the system undergoes a polytropic process
- First draw the system



Exercise 4.16 solution

First, we need to determine the exponent, n , for the polytropic process. We know: Pressure increases linearly with diameter of balloon, initial air pressure $P_1 = 200$ kPa, initial volume of air $V_1 = 1$ m³, final volume $V_2 = 1.5$ m³ and $P V^n$ are constant for the process. And remember that the volume of a sphere is $\frac{4}{3}\pi r^3$

- From the expression for pressure in the balloon

$$P = \gamma D = C V^{\frac{1}{3}}$$

- Solving for C gives

$$C = P V^{-\frac{1}{3}}$$

- Then we need to find P_2 corresponding to the given volumes



Exercise 4.16 solution contd

We can first solve for C

- Since we know P_1 and V_1 we can calculate C

$$C = P_1 V_1^{-\frac{1}{3}} = 200 \text{ kPa} (1 \text{ m}^3)^{-\frac{1}{3}} = 200 \frac{\text{kPa}}{\text{m}}$$

- Then the final pressure is

$$P_2 = C V_2^{\frac{1}{3}} = 200 \frac{\text{kPa}}{\text{m}} (1.5 \text{ m}^3)^{-\frac{1}{3}} = 228.9 \text{ kPa}$$

- The work done is then

$$\bullet \quad W = \frac{P_2 V_2 - P_1 V_1}{n-1} = \frac{228.9 \text{ kPa} \cdot 1.5 \text{ m}^3 - 200 \text{ kPa} \cdot 1 \text{ m}^3}{-\frac{1}{3}-1} = -107.561 \text{ kJ}$$



Exercise 4.16 work on surroundings

On the outside of the balloon, the atmospheric pressure is constant at 101.3 kPa. In that case the work for an isobaric process is $W_{12} = P(V_1 - V_2)$

$$W = 101.3 \text{ kPa} (1 \text{ m}^3 - 1.5 \text{ m}^3) = -50.65 \text{ kJ}$$

- This is less than the -107.56 kJ done on the inside of the balloon.
- The difference is due to the fact that the balloon acts as a spring. Once the balloon is blown up, there is some energy stored in it (for example if you let go of one end without tying it). So the extra work that went on the inside of the balloon is stored energy or if the balloon suddenly popped, it would be dissipated as heat.



Exercises part 2

- 4.13
- 4.14
- 4.15



Exercises part 2 with answers

- 4.13 3.37 kJ
- 4.14 $T = 337 \text{ K}$, $w = 221.3 \text{ kJ/kg}$
- 4.15 -3.99 kJ

