Physics for Materials and Energy Formula Booklet

1. Thermodynamics Concept

$W = F \cdot x$

$P \cdot V = n \cdot R \cdot T$

$m = \rho \cdot V$

$Q = m \cdot c \cdot \Delta T$

2. The First Law (Closed Systems)

Q is heat added to system, W is work done on system

$E = Q + W = pe + ke + u$

$W = \dot{Q} = \frac{\partial Q}{\partial t}$ Heat Transfer Rate

$W_{boundary} = -\int_{V_1}^{V_2} P_{ex}\, \mathrm{d}V$ Isobaric (Constant Pressure)

Isochoric (Constant Volume) = 0

$W = m \cdot R \cdot T \cdot \ln\left(\frac{V_1}{V_2}\right) = P_1 V_1 \cdot \ln\left(\frac{V_1}{V_2}\right)$ Isothermal (Constant Temp)

$Q + W = E$

3. The First Law (Open Systems)

$h = u + P \cdot v$

$\dot{m} = \rho \cdot V \cdot A$ mass flow

$\dot{W} = \dot{m} \cdot \left(h_{out} - h_{\mathbf{in}}\right)$ Steady flow devices (Turbines and Compressors)

$\dot{W} = \dot{m} \cdot \left[v \cdot \left(P_2 - P_1\right) + g\left(z_2 - z_1\right)\right]$ Steady flow devices (Pumps)

V = $\sqrt{2 \cdot \left(h_1 - h_2\right)}$ Steady flow devices (Nozzles and diffusers)

When you want to calculate the change in internal energy, use c_v and when you want to calculate the change in enthalpy, use c_p

$h(T) = h_1 + \frac{(T - T_1)}{T_2 - T_1} \cdot (h_2 - h_1)$ Interpolation for properites table

$\Delta u = c(T_2 - T_1), \Delta h = c(T_2 - T_1) + v(P_2 - P_1)$ Incompressible substance approximations

hf (enthalpy of saturated liquid) - represents the amount of energy contained in the liquid water just before it begins to vaporize.

hg (enthalpy of saturated vapor) - indicates the amount of energy contained in the steam (vapor) when it has completely vaporized from the liquid state.

hfg (enthalpy of vaporisation) - represents the amount of energy required to convert a unit mass of saturated liquid into saturated vapor at the same temperature and pressure.

4. Heat Transfer Mechanisms

$\boldsymbol{q} \cdot \boldsymbol{n} = -h\Delta T$ Newton's Law of Cooling, h is convection heat transfer coefficient

$\boldsymbol{q} = -k\Delta T$ Fourier's law of heat conduction, k is thermal conductivity, q is heat flux density

$\mathrm{R} = \frac{L}{kA}$ Thermal Resistance

$G = \frac{kA}{L}$ Thermal conductance

$||\boldsymbol{q}|| = \frac{\dot{Q}}{A}$ Heat flux density

$||\nabla T|| = \frac{\Delta T}{L}$ Temperature Gradient

$\boldsymbol{Q} = A \cdot \varepsilon \cdot \sigma \cdot T^4$ Black-body radiation

$\dot{Q} = \frac{kA(T1 - T2)}{L}$ Transfer of heat through conduction

5. The Second Law: Spontaneity and Reversibility. Entropy

PV Diagrams

$\delta W = -P\,dV$

$dS = \dfrac{\delta Q}{T}$ Infinitesmly small entropy variation

$\Delta S = \displaystyle\int_1^2 \frac{\delta Q}{T} + S_{gen}$ Variation during finite process

$\Delta S = \dfrac{Q}{T}$ At constant temperature

$\Delta S = m \cdot c \cdot \ln\left(\dfrac{T_2}{T_1}\right)$ Entropy change for a reversible process (isobaric heating or cooling)

$\Delta s = c \ln\left(\dfrac{T_2}{T_1}\right) = c_v \ln\left(\dfrac{T_2}{T_1}\right) + R \ln\left(\dfrac{v_2}{v_1}\right) = c_p \ln\left(\dfrac{T_2}{T_1}\right) - R \ln\left(\dfrac{p_2}{p_1}\right)$ Entropy change for ideal gases volume form and pressure form

$\dot{S} = S_{gen} + \displaystyle\sum_j \left(\frac{\dot{Q}_j}{T_j}\right) + \sum_i \left(m_i s_i\right) = S\_generated\ within\ the\ system + Heat\ Transfer$ to $system$
$+\ mass\ transfer$ to $system$

Total entropy variation of the system

$TV^{\gamma-1} = constant$

$PV^{\gamma} = constant$

$T_2 = T_1 \left(\dfrac{V_1}{V_2}\right)^{\gamma-1},\ T_2 = T_1 \left(\dfrac{P2}{P1}\right)^{\frac{(\gamma-1)}{\gamma}},\ \gamma = \dfrac{c_p}{c_v}$

$W = \dfrac{P_2 V_2 - P_1 V_1}{1-\gamma} = \dfrac{mR\left(T_2 - T_1\right)}{1-\gamma}$ Work in isentropic Ideal gas processes

6. Phase diagrams and phase transitions

$x = \dfrac{m_{vapor}}{m_{total}} = \dfrac{m_{vapor}}{m_{vapor} + m_{liquid}} = \left(\dfrac{s - s1}{s2 - s1}\right)$ Quality (use for part vapor/liquid questions)

$y = y_f + x \cdot \left(y_{fg}\right)$ Interpolation for any thermodyanmic property

$h = h_1 + \left(\frac{s - s1}{s2 - s1}\right) \cdot (h2 - h1)$ Example: h1, h2, s1, s2 are just chosen values from the table, while h is the matching enthalpy for entropy s

$h_{fg} = u_g - u_f + P(v_g - v_f)$ Latent heat of vaporization

$\frac{dP_{sat}}{dT} = \frac{h_{fg}}{T(v_g - v_f)}$ Calpeyron Equation, Describes how saturation pressure varies with temperature

$P_{sat} = C\, e^{\left(\frac{-\Delta h_{vap}}{RT_{sat}}\right)}$ Most common use of Clausius-Clapeyron Equation (Integrated form for P)

$\left(P + \frac{a}{v^2}\right)(v - b) = RT$ Van der Waals Equation

$Z = \frac{Pv}{RT}$ Generalized compressability factor

$G = H - TS$ Gibs Free energy

7. Heat Engines: Rankine Cycle

$\eta = \frac{W_{net}}{Q_{\mathbf{in}}} = 1 - \frac{Q_C}{Q_H}$ Thermal Efficiency of Thermal Engines

1. Carnot Cycle

$$\eta_{carnot} = 1 - \frac{T_C}{T_H}$$

$$\frac{Q_H}{T_H} = \frac{Q_C}{T_C}$$

2. Rankine Cycle

$$\eta_{rankine} = \frac{\dot{W}_{turb} - \dot{W}_{pump}}{\dot{Q}_{\mathbf{in}}}$$

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4)$$

$$\dot{W}_{pump} = \dot{m}v(P_2 - P_1)$$

$$\dot{Q}_{\mathbf{in}} = \dot{m}\left(h_3 - h_2\right)$$

8. Refrigeration cycles

$$COP_{carnot,\, ref} = \frac{T_C}{T_H - T_C}$$

$$COP_{carnot,\, hp} = \frac{T_H}{T_C - T_H}$$

$$Q_H = Q_C + W \;\Rightarrow\; Q_C = Q_H - W_{net}$$

$$COP_{ref} = \frac{Q_C}{W} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$COP_{hp} = \frac{Q_H}{W} = \frac{Q_C + W}{W}$$

9. Gas Power Cycles

Otto Cycle
1>2 Isentropic Compression, 2>3 Constant Volume heat addition, 3>4 Isentropic Expansion, 4>1 Constant Volume heat rejection

$$r = \frac{V_1}{V_2} = \frac{V_{\max}}{V_{\min}}$$ Compression Ratio

$$\eta_{th,\, otto} = 1 - \frac{1}{r^{\gamma - 1}}$$ Thermal Efficiency

Diesel Cycle
1>2 Isentropic Compression, 2>3 Constant pressure heat addition, 3>4 Isentropic Expansion, 4>1 Constant Volume heat rejection

$$r_c = \frac{V_3}{V_4}$$ cut-off ratio

$$\eta_{th,\, diesel} = 1 - \frac{1}{r^{\gamma - 1}} \cdot \frac{r_c^{\gamma} - 1}{\gamma \cdot \left(r_c - 1\right)}$$ Thermal Efficiency

$$Q_H = h_3 - h_2$$ Heat added

Brayton Cycle
1>2 Isentropic Compression, 2>3 Constant pressure heat addition, 3>4 Isentropic Expansion, 4>1 Constant Pressure heat rejection

$$bwr = \frac{w_{comp}}{w_{turb}} = \frac{h_2 - h_1}{h_3 - h_4}$$ Back Work Ratio

$$\eta_{th,\ brayton} = 1 - \left(\frac{T_1}{T_2}\right)^{\frac{\gamma - 1}{\gamma}}$$ or $\eta = \frac{w_{net}}{q_{\mathbf{in}}}$ Thermal Efficiency

10. Gas Power Cycles