SOLUTIONS
Chapter 10

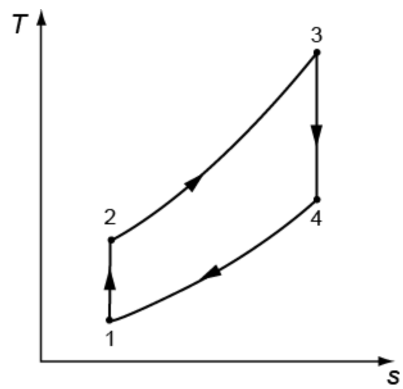
Otto Cycle

10.1. An engine operating on a cold air standard Otto cycle has a compression ratio of 8.5 and takes in air at 100 kPa and 300 K. The heat added during each cycle is 900 kJ/kg of air in the cylinder. Find the maximum temperature in the cycle.

Find: Maximum temperature T_3 reached during the cycle.

Known: Cold air standard Otto cycle, compression ratio r = 8.5, initial pressure P_1 = 100 kPa, initial temperature T_1 = 300 K, heat added q_H = 900 kJ/kg.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant volume at 300 K c_v = 0.718 kJ/kg (A4).



The air temperature at the end of isentropic compression is

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma - 1)} = 300 \text{ K} \times (8.5)^{0.400} = 706.137 \text{ K.}$$

The maximum temperature reached during the cycle occurs after constant volume heat addition and can be found using an energy balance,

$$T_3 = T_2 + \frac{q_H}{c_v} = 706.137 \text{ K} + \frac{900 \text{ kJ/kg}}{0.718 \text{ kJ/kgK}} = 1959.62 \text{ K.}$$

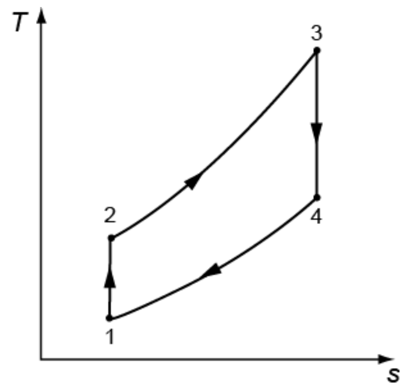
The maximum air temperature in the cycle is 1960 K.

10.4. An engine working on a cold air standard Otto cycle takes in air at 100 kPa and 295 K. The compression ratio is 9 and the maximum temperature is 2000 K. Find the heat added per kilogram of air during each cycle.

Find: Heat added q_H per unit mass (kg) of working air.

Known: Cold air standard Otto cycle, intake air pressure P_1 = 100 kPa, intake air temperature T_1 = 295 K, compression ratio r = 9, maximum temperature T_3 = 2000 K.

Properties of cold air at T_1 = 295 K are interpolated from Appendix 4: specific heat ratio γ = 1.4001 and specific heat at constant volume c_v = 0.7178 kJ/kgK.



The temperature at the end of isentropic compression is

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = 295\text{ K}\times(9)^{0.4001} = 710.582\text{ K.}$$

The amount of heat added during heat addition is then

$$q_H = c_v(T_3 - T_2) = 0.7178\text{ kJ/kgK}\times(2000\text{ K} - 710.582\text{ K}) = 925.544\text{ kJ/kg.}$$

The heat added to the engine is 925.5 kJ/kg of working air.

Diesel Cycle

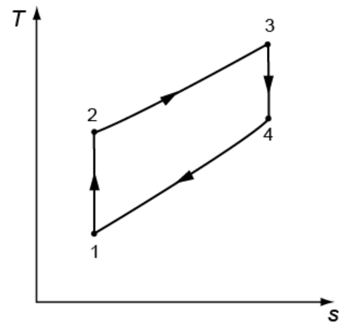
10.11. The air in an air standard Diesel cycle is at a pressure of 95 kPa and a temperature of 300 K at the start of compression. The maximum pressure in the cycle is 6500 kPa and the maximum temperature is 2200 K. Find the compression ratio and the cutoff ratio for the cycle. Use the properties of air at 300 K.

Find: Compression ratio r of the cycle, cutoff ratio r_c of the cycle.

Known: Air standard Diesel cycle, inlet air pressure P_1 = 95 kPa, inlet temperature T_1 = 300 K, maximum pressure $P_2 = P_3$ = 6500 kPa, maximum temperature T_3 = 2200 K.

Assumptions: Cold air standard Diesel cycle.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant volume at 300 K c_v = 0.717 kJ/kgK (A4).



The compression ratio can be found since compression and expansion are isentropic processes,

$$r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{6500 \text{ kPa}}{95 \text{ kPa}}\right)^{1/1.400} = 20.457.$$

The temperature at the end of isentropic compression is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 300 \text{ K} \times \left(\frac{6500 \text{ kPa}}{95 \text{ kPa}}\right)^{0.400/1.400} = 1003.4 \text{ K}.$$

Then the cutoff ratio can be found using the ideal gas equation during constant pressure heat addition,

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{P_3}\frac{P_2}{T_2} = \frac{T_3}{T_2} = \frac{2200 \text{ K}}{1003.4 \text{ K}} = 2.1925.$$

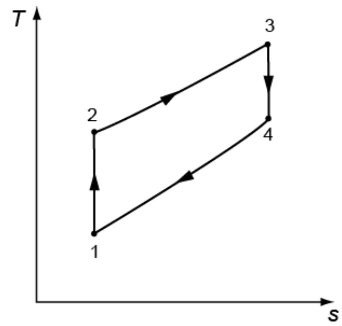
The cycle has a compression ratio of 20.5 and a cutoff ratio of 2.19.

10.16. In a cold air standard Diesel cycle the compression ratio is 18 and the cutoff ratio is 2.0. Air is at a pressure of 100 kPa, and a temperature of 300 K at the start of compression. Find the maximum temperature and pressure in the cycle, the heat added and the net work done during the cycle.

Find: Maximum temperature T_3 of air, maximum pressure P_3 of air, heat added q_H during the cycle, work done w during the cycle.

Known: Cold air standard Diesel cycle, compression ratio r = 18, cutoff ratio r_c = 2.0, intake pressure P_1 = 100 kPa, intake temperature T_1 = 300 K.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant pressure at 300 K c_p = 1.005 kJ/kgK (A4), specific heat of air at constant volume at 300 K c_v = 0.718 kJ/kgK (A4).



The temperature and pressure at the end of isentropic compression are

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = 300\ \text{K} \times (18)^{0.400} = 953.3015\ \text{K},$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100\ \text{kPa} \times (18)^{1.400} = 5719.809\ \text{kPa}.$$

The maximum pressure during the process is $P_2 = P_3$ = 5719.8 kPa.

The maximum temperature occurs after constant pressure heat addition and can be found using the ideal gas equation,

$$T_3 = T_2\left(\frac{V_3}{V_2}\right) = 953.3\ \text{K} \times (2) = 1906.603\ \text{K}.$$

The temperature at the end of isentropic expansion is

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{(\gamma-1)} = T_3\left(\frac{V_3}{V_2}\frac{V_2}{V_1}\right)^{(\gamma-1)} = T_3\left(\frac{r_c}{r}\right)^{(\gamma-1)} = 1906.603\ \text{K} \times \left(\frac{2}{18}\right)^{0.400},$$

$$T_4 = 791.7048\ \text{K}.$$

The heat transfers and work done during the process can then be found:

$$q_H = c_p(T_3 - T_2) = 1.005\ \text{kJ/kgK} \times (1906.603\ \text{K} - 953.3015\ \text{K}) = 958.0680\ \text{kJ/kg},$$

$$q_C = c_v(T_4 - T_1) = 0.718 \text{ kJ/kgK} \times (791.7048 \text{ K} - 300 \text{ K}) = 353.0440 \text{ kJ/kg},$$

$$w = q_H - q_C = 958.0680 \text{ kJ/kg} - 353.0440 \text{ kJ/kg} = 605.0240 \text{ kJ/kg}.$$

The maximum air temperature is 953 K and pressure is 5719.8 kPa, and the cycle receives 958.1 kJ of heat to do 605.0 kJ of work per kilogram of working air.

Brayton Cycle

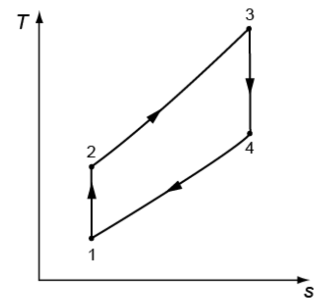
10.21. A cold air standard Brayton cycle has a minimum temperature of 320 K, a maximum temperature of 1400 K, and a compressor pressure ratio of 10. Find the back work ratio and the thermal efficiency of the cycle. Assume constant specific heats.

Find: Back work ratio *bwr* of the cycle, thermal efficiency η_{th} of the Brayton cycle.

Known: Cold air standard Brayton cycle, minimum temperature T_1 = 320 K, maximum temperature T_3 = 1400 K, compressor pressure ratio r_p = 10.

Assumptions: Cold air properties evaluated at 320 K.

Properties of cold air at T_1 = 320 K are interpolated from Appendix 4: specific heat ratio γ = 1.3992, constant heat of air at constant pressure c_p = 1.0062 kJ/kgK.



The temperature after isentropic compression is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 320\text{ K}\times(10)^{(0.3992)/1.3992} = 617.24\text{ K}.$$

The temperature after isentropic expansion is

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} = 1400\text{ K}\times\left(\frac{1}{10}\right)^{(0.3992)/1.3992} = 725.81\text{ K}.$$

Since we apply the cold air standard assumption, the back work ratio can be found using temperatures,

$$bwr = \frac{h_2-h_1}{h_3-h_4} = \frac{c_p(T_2-T_1)}{c_p(T_3-T_4)} = \frac{617.24\text{ K}-320\text{ K}}{1400\text{ K}-725.81\text{ K}} = 0.44088.$$

and the thermal efficiency of the cycle reduces to

$$\eta_{th} = 1-\frac{T_1}{T_2} = 1-\frac{320\text{ K}}{617.24\text{ K}} = 0.48156.$$

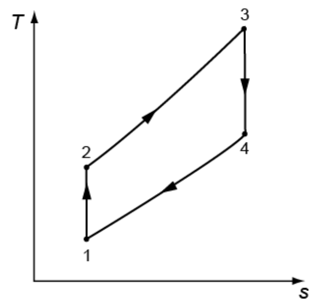
The cycle has a back work ratio of 0.441 and an efficiency of 48.2%.

10.23. A gas turbine operating on an air standard Brayton cycle has air entering the compressor with a temperature of 300 K, a pressure of 100 kPa and a mass flow rate of 4 kg/s. The compressor pressure ratio is 8 and air enters the turbine with a temperature of 1200 K. Find the thermal efficiency of the cycle and the net power supplied by it. Assume constant gas properties.

Find: Thermal efficiency η_{th} of the cycle, power $\dot{W}$ output of the engine.

Known: Air standard Brayton cycle, compressor inlet temperature T_1= 300 K, compressor inlet pressure P_1 = 100 kPa, mass flow rate $\dot{m}$ = 4 kg/s, compressor pressure ratio r_p = 8, turbine inlet temperature T_3 = 1200 K.

Assumptions: Constant specific heats.



The temperatures after isentropic compression and isentropic expansion can be found, assuming specific heat ratio γ = 1.400 at 300 K from Appendix 4:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 300 \text{ K} \times (8)^{(0.4)/1.4} = 543.4342 \text{ K},$$

$$T_4 = T_3\left(\frac{P_4}{P3}\right)^{(\gamma-1)/\gamma} = 1200 \text{ K} \times \left(\frac{1}{8}\right)^{(0.4)/1.4} = 662.4537 \text{ K}.$$

These values can be refined by using air properties at respective average process temperatures, interpolating from Appendix 4: $T_{avg,12}$ = 421.7171 K so γ_{12} = 1.393263 and $T_{avg,34}$ = 931.2269 K so γ_{34} = 1.341502, then T_2 = 539.5452 K and T_4 = 706.7803 K.

Heat transfer during the cycle can be found, using air properties at respective average process temperature interpolated from Appendix 4: $T_{avg,23}$ = 869.7726 K so c_{p23} = 1.114350 kJ/kgK and $T_{avg,41}$ = 503.3902 K so c_{p41} = 1.029746 kJ/kgK,

$$\dot{Q}_H = \dot{m}c_{p,23}(T_3 - T_2) = 4 \text{ kg/s} \times 1.114350 \text{ kJ/kgK} \times (1200 \text{ K} - 539.5452 \text{ K}),$$

$$\dot{Q}_H = 2943.911 \text{ kW},$$

$$\dot{Q}_C = \dot{m}c_{p,41}(T_4 - T_1) = 4 \text{ kg/s} \times 1.029746 \text{ kJ/kgK} \times (706.7803 \text{ K} - 300 \text{ K}),$$

$$\dot{Q}_C = 1675.522 \text{ kW}.$$

The thermal efficiency and net power output of the cycle can then be found:

$$\eta_{th} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{1675.522 \text{ kW}}{2943.911 \text{ kW}} = 0.4308517,$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_C = 2943.911 \text{ kW} - 1675.522 \text{ kW} = 1268.389 \text{ kW}$$

The cycle has a power output of 1268.4 kW and an efficiency of 43.1%.