Solutions problems chapter 2 and 3

- Chapter 2: 2.5, 2.8, 2.12, 2.27, 2.28, 2.37

2.5: What is the weight of an object with a mass of 150 kg on a planet where g = 4.1 m/s^2?

The definition of the weight is given by equation 2.5 on p. 22 of Chapter 2:

$$F_w = mg$$

$$F_w = 150 \times 4.1 = \mathbf{615\ N}$$

2.8: A 5 kg box sliding across the floor with an initial velocity of 8 m/s is decelerated by friction to 3 m/s over 5 s. What is the force of friction acting on it?

The definition of the acceleration is in the equation 2.13, p. 28.

$$a = \frac{\Delta V}{\Delta t} = \frac{3-8}{5} = -1\ m/s^2$$

The frictional is the only force acting on the box. We can use equation 2.5:

$$F_w = mg = 5 \times (-1) = -5\ N$$

The force is in opposite direction of the sliding of the box, that is why we obtain a negative value, but we can say that the force decelerating the box is **5 N**.

2.12: Acceleration due to gravity at Earth's surface is g = 9.80665 m/s^2 and decreases by approximately 3.3 × 10^{-6} m/s^2 for each metre of height above the ground. What is the potential energy of a 100 kg mass raised to an altitude of 1000 m: (a) assuming constant g, (b) accounting for the decrease in g with height?

(a) Assuming the g is constant we have only one variable: the height. We can use equation 2.11, on p. 25.

$$\Delta PE = mg\Delta z = 100 \times 9.80665 \times 1000 = \mathbf{980\ 665\ J}$$

(b) When taking into account that g decreases with height, we have 2 variables, therefore, we have to use equation 2.10 from p. 25.

$$d(PE) = d(mgz)$$

$$\Delta PE = m\int_0^h g dz = 100 \times \int_0^h (9.80665 - 3.3 \times 10^{-6} \times \mathrm{z})\mathrm{dz}$$

$$\Delta \mathrm{PE} = 100 \times (9.80665\,\mathrm{h} - 1.65 \times 10^{-6}\,\mathrm{h}^2) = \mathbf{980550\ J}$$

2.27: Tank A has a volume of 0.5 m3 and contains air with density 1.2 kg / m3 while tank B has a volume of 0.8 m3 and contains air with density 0.9 kg / m3. The two tanks are connected to each other and their contents mixed. What is the final air density in the tanks?

First, we have to calculate the mass of air in each tank:

$$m_A = \rho_A V_A = 1.2 \times 0.5 = 0.6\ kg$$

$$m_B = \rho_B V_B = 0.9 \times 0.8 = 0.72\ kg$$

The total mass of air, combining the two tanks, is:

$$m_t = m_A + m_B = 0.6 + 0.72 = 1.32\ kg$$

The total volume is:

$$V_t = V_A + V_B = 0.5 + 0.8 = 1.3\ m^3$$

The density of air in the combined tank is then:

$$\rho = \frac{m_t}{V_t} = \frac{1.32}{1.3} = \mathbf{1.02\ kg/m^3}$$

2.28: A 0.5 m3 container is filled with a mixture of 10% by volume ethanol and 90% by volume water at 25 °C. Find the weight of the liquid.

The masses of each liquid in the container are:

$$m_{water} = \rho_{water} V \times 0.9 = 1000 \times 0.5 \times 0.9 = 450\ kg$$

$$m_{ethanol} = \rho_{ethanol} V \times 0.1 = 783 \times 0.5 \times 0.1 = 39.15\ kg$$

The total mass is :

$$m_t = m_{water} + m_{ethanol} = 450 + 39.15 = 489.15\ kg$$

$$F_w = m_t g = 489.15 \times 9.81 = \mathbf{4\,798.6\ N}$$

2.37: The inside of a house is at 20 °C while the exterior air is at –5 °C. The temperature of the walls of the house does not change with time. Are the walls at equilibrium?

On pages 35 and 36, you can find the definition of steady state and equilibrium.

As the temperatures inside the house and outside are different and the system is not isolated, there is heat transfer from the inside to the outside of the house at a constant rate. As the walls keep interacting with the inside and outside of the house, the walls are in **steady state but do not reach equilibrium** (it is illustrated on Figure 2.13 (b) on p. 35 and Figure 2.14 p. 36). Heat must be constantly added to maintain the temperature difference.

- Chapter 3: 3.3, 3.7 , 3.9, 3.15, 3.20, 3.27, 3.39

3.3: If 5 g of methane is burned in the chemical reaction $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, what mass of oxygen is consumed and what are the masses of the combustion products?

On p. 52, you can find the relation between mass, moles and molar mass:

$$m = N \times M$$

As we have the mass of methane we can calculate the mole of methane:

$$N_{CH_4} = \frac{m_{CH_4}}{M_{CH_4}} = \frac{5}{16} = 0.31\ gmol$$

From the equation we know that for one mole of methane, two moles of oxygen was used and one mole of carbon dioxyde was produced and two moles of water.

$$N_{O_2} = 2 \times N_{CH_4} = 0.62\ gmol$$

$$N_{CO_2} = N_{CH_4} = 0.31\ gmol$$

$$N_{H_2O} = 2 \times N_{CH_4} = 0.62\ gmol$$

$$m_{O_2} = N_{O_2} \times M_{O_2} = \mathbf{19.9\ g}$$

$$m_{CO_2} = N_{CO_2} \times M_{CO_2} = \mathbf{13.7\ g}$$

$$m_{H_2O} = N_{H_2O} \times M_{H_2O} = \mathbf{11.2\ g}$$

3.7: A cubical container, 10 cm long along each edge, is filled with a gas at a pressure of 350 kPa. Determine the force that the gas exerts on each wall of the container.

First, we calculate the surface area of each wall of the container:

$$A = 0.1^2 = 0.01\ m^2$$

Then, to find the force that the gas exerts on each wall of the container we use the formula 3.2 on p. 54.

$$P = \frac{F}{A}$$

$$F = P \times A = 350 \times 10^3 \times 0.01 = \mathbf{3500\ N}$$

3.9: A cylindrical drum, 1 m high, is filled with water (ρ = 1000 kg/m^3) to a depth of 0.2 m. The rest of the drum is then filled with oil (ρ = 850 kg/m^3). What is the gauge pressure on the bottom of the drum?

The gauge pressure is in this case the sum of the pressure exerted by water and oil:

$$P = \rho_{water} g h_{water} + \rho_{oil} g h_{oil} = 9.81 \times (1000 \times 0.2 + 850 \times 0.8) = \mathbf{8632.8\ Pa}$$

3.15: What is the volume in liters of 1 gmol of an ideal gas at standard temperature and pressure, defined as 0 °C and 101.325 kPa?

The ideal gas equation has to be used, it can be found on p. 58, equation 3.10:

$$V = \frac{NRT}{P} = \frac{10^{-3} \times 8.314 \times 10^{3} \times 273.15}{101325} = 0.0224\, m^3 = \mathbf{22.4}\, \boldsymbol{L}$$

3.20: An air bubble, 1 mm in diameter, is released at the bottom of a lake 20 m deep where the temperature is 10 °C. It rises to the surface of the lake where the temperature is 25 °C. What will the radius of the bubble be at the surface?

The pressure at the bottom of the lake is calculated as follow:

$$P_{20} = P_{atm} + \rho_{water} g h = 297.53\, kPa$$

Pressure at the top of the lake is the atmospheric pressure: P_0 = 101.325 kPa
The air bubble is supposed to be an ideal gas with constant mass:

$$\frac{P_{20}V_{20}}{T_{20}} = \frac{P_0 V_0}{T_0}$$

$$\frac{V_0}{V_{20}} = \frac{P_{20}T_0}{P_0 T_{20}} = 3.09$$

The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Therefore:

$$r_0 = \sqrt[3]{3.09} \times r_{20} = \mathbf{0.73}\, \boldsymbol{mm}$$

3.27: An evacuated 20 L container is filled with gas until the pressure inside reaches 800 kPa at 25 °C. By weighing the container before and after filling it is determined that its mass increased by 26 g. What gas was it filled with?

$$M = \frac{mRT}{PV} = \frac{26 \times 10^{-3} \times 8314 \times 298.15}{800 \times 10^{3} \times 20 \times 10^{-3}} = \mathbf{4.03}\, \boldsymbol{kg/kmol}$$

3.39: Five kilograms of air, initially at 0 °C, are heated until the temperature reaches 50 °C. The internal energy increases by 180 kJ during this process. Find the specific heat of air.

The relation between internal energy and specific heat is given by equation 3.30, p. 65.

$$U_2 - U_1 = mc(T_2 - T_1)$$

$$c = \frac{U_2 - U_1}{m(T_2 - T_1)} = \frac{180\ kJ}{5\ kg\ (50\ C - 0\ C)} = \mathbf{0.72}\,\frac{\boldsymbol{kJ}}{\boldsymbol{kg\ C}}$$

Solutions for selected exercises:
Chapter 4 in “Energy, Entropy and Engines”

Exercise 4.4
A 20 cm diameter pulley is driven by a belt that exerts a net tangential force of 3 kN. What is the power transmitted by the pulley when it is turning at 500 RPM?

The torque of the pullley can be found in Eq. 4.36:

$$\tau = Fr = 3000\ \text{N} \times \frac{0.2\ \text{m}}{2} = 300\ \text{Nm}.$$

By deploying the formula of the sha power corresponding to rota on speed, we get the result as:

$$\dot{W} = 2\pi\dot{n}\tau = 2\pi \times 500 \times (1/\text{min}) \times (1\ \text{min}/60\ \text{s}) \times 0.3\ \text{kNm} = 15.708\ \text{kW}.$$

No ce the unit of the rota on speed is [1/min].

Exercise 4.5
The piston in a cylinder filled with oil at a pressure of 0.8 MPa is advanced 10 cm. What is the work done if the cross-sectional area of the cylinder is 20 cm² ?

The force to the piston is F = P*A. Therefore, the work done to the piston is:

$$W = F\Delta x = PA\Delta x$$
$$W = 0.8 \times 10^{6}\ \text{Pa} \times 20 \times 10^{-4}\text{m}^2 \times 0.1\ \text{m} = 160\ \text{J}.$$

Exercise 4.7
A cylinder contains water filled on top of a piston to a depth of 1 m. As the piston is raised the water drains out of an outlet at the top of the cylinder. Find the work required to empty out all the water if the cross-sectional area of the piston is 0.2 m². Assume the density of water is 1000 kg / m³.

This is a ques on about poten al energy. When the piston goes up, the water will leave the container. Thus, the poten al energy of the water in the container is reducing. The force on the piston is also reducing in this process. The force on the piston is:

$$F = m_w g = \rho V g = \rho A h g.$$

The work can be calculated via integra on over the en re depth

$$W = \int F\,\text{d}h = \int A h g\,\text{d}h$$

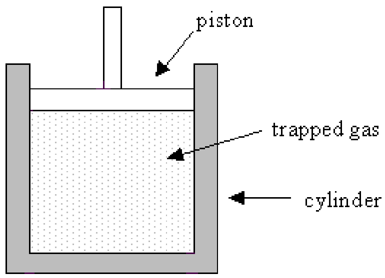
$$W = \int_0^h \rho A h g dh = \frac{1}{2}\rho A g h^2 = \frac{1}{2} \times 10^3\ \text{kg/m}^3 \times 9.81\ \text{m/s}^2 \times 0.2\ \text{m}^2 \times (1\ \text{m})^2,$$

$$W = 981\ \text{J}.$$

Exercise 4.8 - See pdf file “Lecture 2 problem 8.4”

Exercise 4.10

A 2 kg mass of air in a cylinder initially at a pressure of 100 kPa and temperature of 20° C is compressed by a piston to a pressure of 300 kPa whole being kept at constant temperature. Find the work done.



A piston compresses air in cylinder.

The work done can be calculated with

$$W = -\int_{V_1}^{V_2} P dV = -\int_{V_1}^{V_2} \frac{mRT}{V} dV = -mRT \int_{V_1}^{V_2} \frac{dV}{V} = mRT \ln \frac{V_1}{V_2}.$$

Since mass of air m = 2 kg, gas constant of air R = 0.2870 kJ/(kgK) and T = T_1 = T_2 = 20 °C are known, the only missing information is V_1/V_2. The knowledge of P_1 = 100 kPa and P_2 = 300kPa makes it to possible to use the ideal gas equation to determine V_1/V_2 with

$$P_1 V_1 = P_2 V_2,$$
$$\frac{V_1}{V_2} = \frac{P_2}{P_1}.$$

By using the known properties, the following work done results

$$W = 2\text{ kg} \times 0.2870\text{ kJ/kgK} \times 293.15\text{ K} \times \ln \frac{300\text{ kPa}}{100\text{ kPa}} = 184.861\text{ kJ}.$$

Exercise 4.13

Argon at 150 kPa, 320 K and 0.1 m³ expands in a polytropic process for which n = 1.667 to a pressure of 100 kPa. What is the work done?

To calculate the work done during a polytropic process for argon, an integration of the boundary work equation (done in chapter 4) is needed. Therefore, the final volume of the gas needs to be determined with the polytropic process equation

$$P_1 V_1^n = P_2 V_2^n$$
$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0.1\text{ m}^3 \left(\frac{150\text{ kPa}}{100\text{ kPa}}\right)^{0.6} = 0.12754\text{ m}^3.$$

Since now V_2 is known, the work done can be calculated with

$$W = \frac{P_2 V_2 - P_1 V_1}{n-1} = \frac{100\text{ kPa} \times 0.12754\text{ m}^3 - 150\text{ kPa} \times 0.1\text{ m}^3}{1.667 - 1} = -3.3688\text{ kJ}.$$

Based on standard conventions, the work done is negative since the gas does work on the surrounding.

4.14)

Air at 800 K and 1 MPa is expanded in a polytropic process for which $PV^{1.6}$ = constant until the pressure reaches 0.1 MPa. Find the final gas temperature and the work done per unit mass of air.

SOLUTION:

Based on the description of the exercise, it can be assumed that the mass of the system (Air volume) remains constant during the polytropic process and, in addition, the air behaves as an ideal gas.

Gas constant of air R = 0.2870 kJ/kgK (Found in Appendix, A1).

Using the ideal gas law for the initial and the final state of the system, we write:

Initial state: $P_1V_1 = \frac{m}{Ru}\,T_1$ **(1)**

Final state: $P_2V_2 = \frac{m}{Ru}\,T_2$ **(2)**

m = Constant and combining (1) and (2), we write:

- $\frac{P_1V_1}{T_1} = m\,R\,,\ \frac{P_2V_2}{T_2} = m\,R \ \leftrightarrow\ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \ \leftrightarrow\ \frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1}$ **(3)**

Also, $PV^{1.6}$ is constant, hence we can write:

- $P_1V_1^{1.6} = P_2V_2^{1.6} \leftrightarrow \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.6}}$ **(4)**

Replacing **(4)** to **(3)**, we write:

- $T_2 = T_1\frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{\frac{1}{1.6}} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{1.6-1}{1.6}} = 800\,K\left(\frac{0.1\,MPa}{1\,MPa}\right)^{\frac{0.6}{1.6}} = \mathbf{337.357\,K}$

The work function of a polytropic process can be obtained by integrating the boundary work equation (eq. 4.30, page 97 of the book).

- $W_{12} = \frac{P_2V_2-P_1V_1}{n-1} \xLeftrightarrow{(Using\ Ideal\ gas\ law)} W_{12} = \frac{mRT_2-mRT_1}{n-1} = \frac{mR(T_2-T_1)}{n-1}$

The work per unit mass that the system does on the surroundings is calculated as follows:

$$w_{12} = \frac{W_{12}}{m} = \frac{R(T_2 - T_1)}{n-1} = \frac{0.2870\ kJ/kgK(337.357K - 800\ K)}{1.6 - 1} = \mathbf{-221.298\ kJ/kg}$$

Final solutions:

1. **Air temperature = 337 K**
2. **Work per unit mass = 221.3 kJ/kg**

4.15)

A cylinder containing 0.05 m^3 of carbon dioxide at 200 kPa and 100°C is expanded in a polytropic process to 100 kPa and 20°C. Determine the work done.

SOLUTION:

Based on the description of the exercise, it can be assumed that the mass of the system (CO_2 mass) remains constant during the polytropic process and, in addition, CO_2 behaves as an ideal gas.

Gas constant of carbon dioxide, R = 0.1889 kJ/kgK (Found in Appendix, A1).

The work function of a polytropic process can be obtained by integrating the boundary work equation (eq. 4.30, page 97 of the book).

- $W_{12} = \frac{P_2V_2 - P_1V_1}{n-1}$ **(1)**

The system is experiencing a polytropic process, hence, we can use the main polytropic equation to describe the initial and the final state of the system.

- $P_1V_1^n = P_2V_2^n \ \leftrightarrow \ \frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^n \leftrightarrow \ ln\left(\frac{P_1}{P_2}\right) = ln\left(\frac{V_1}{V_2}\right)^n \leftrightarrow \ ln\left(\frac{P_1}{P_2}\right) = n\ ln\frac{V_1}{V_2} \ \leftrightarrow$

  $n = \frac{ln\ (P_1/P_2)}{ln\ (V_2/V_1)}$ **(2)**

Applying the ideal gas law in the initial state of the system, the mass of CO_2 can be calculated:

- $m_{CO2} = \frac{P_1 V_1}{R\,T_1} = \frac{200\,kPa\ x\ 0.05\,m^3}{0.1889\,kJ/kgK\ x\ 373.15\,K}$ = 0.14187 kg

Using the value of m_{CO2} in the ideal gas law for the final state of the system, the volume of CO_2 can also be calculated:

- $V_2 = \frac{mRT_2}{P_2} = \frac{0.14187\,kg\ x\ 0.1889\,kJ/kgK\ x\ 293.15\,K}{100\,kPa} = 0.078562\,m^3$

Using the value of V_2 to equation (2), we can write:

- $n = \frac{ln\,(P_1/P_{2)}}{ln\,(V_2/V_1)} = \frac{ln\,(200\,kPa/100\,kPa}{ln\,(0.0785m^3/0.05m^3)} = 1.5367$

All the values of the parameters of equation (1) are known, hence, we write:

- $W_{12} = \frac{P_2V_2 - P_1V_1}{n-1} = \frac{100\,kPa\ x\ 0.0785\,m^3 - 200\,kPa\ x\ 0.05\,m^3}{1.5367-1} = \boldsymbol{-3.9944\,kJ}$

Solutions for selected exercises in Lecture 3
(Chapter 4 in “Energy, Entropy and Engines”)

4.60

Notice, the “34 kW electrical heater” means Q = 34 kW. Based on first law:

$$\dot{Q}+\dot{W}=\dot{m}\left[(h_2-h_1)+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}+g(z_2-z_1)\right]=\dot{m}\left[c_p(T_2-T_1)+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}+g(z_2-z_1)\right],$$

Thus, the work done in the heater W is:

$$\dot{W}=0.8\text{ kg/s}\times[1.01344\times10^3\text{ J/kg°C}\times(200\text{°C}-60\text{°C})$$
$$+\frac{(50\text{ m/s})^2}{2}+9.81\text{ m/s}^2\times50\text{ m}]-34\times10^3\text{ W},$$
$$\dot{W}=0.8\text{ kg/s}\times[141{,}882\text{ J/kg }+1250\text{ J/kg }+490.5\text{ J/kg}]-34\times10^3\text{ W},$$
$$\dot{W}=80898\text{ W}=80.898\text{ kW}.$$

Be very careful about the unit. Many students mess up the J/kg and kJ/kg and get the wrong answer.

4.66

In this question, we should considered internal energy only:

$$\dot{m}_a h_{a,2}+\dot{m}_w h_{w,2}=\dot{m}_a h_{a,1}+\dot{m}_w h_{w,1},$$

The enthalpy of air can be interpolated from Appendix 7. The enthalpy of water can be interpolated from Appendix 8. Another equivalent approach is.

$$\dot{m}_a c_{p,a}(T_{a,2}-T_{a,1})=\dot{m}_w c_w(T_{w,2}-T_{w,1}).$$

where Cp,a is 1.0806 kJ/(kg ℃) (Appendix 4), Cw is 4.18 kJ/(kg ℃) (Appendix 3). As a result:

$$\dot{m}_w=\frac{0.5\text{ kg/s}\times1.0806\text{ kJ/kg°C}\times(600\text{°C}-300\text{°C})}{4.18\text{ kJ/kg°C}\times(80\text{°C}-20\text{°C})},$$
$$\dot{m}_w=0.64629\text{ kg/s}.$$

4.68

Air enters a nozzle at 300 kPa and 350 K with a velocity of 30 m/s and leaves at 100 kPa with a velocity of 200 m/s. The heat loss from the nozzle is 20 kJ/kg of airflow through it. What is the exit temperature of the air?

Energy balance:

$$\dot{Q}+\underbrace{\dot{W}}_{=0}=\dot{m}\left[(h_2-h_1)+\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}+\underbrace{g(z_2-z_1)}_{=0}\right]$$

There is no work done and no potential energy. Consequently, you can rearrange for h_2:

$$h_2=h_1+\frac{\dot{Q}}{\dot{m}}+\frac{\mathbf{V}_1^2-\mathbf{V}_2^2}{2}$$

Initial air specific enthalpy at T_1 = 350 K from Appendix 7: h_1 = 350.49 kJ/kg.

$$h_2=350.49\text{ kJ/kg}+\frac{(30\text{ m/s})^2-(200\text{ m/s})^2}{2\times10^3\text{ J/kJ}}-20\text{ kJ/kg}=310.94\text{ kJ/kg},$$

$$T_2=310.70\text{ K}.$$

Please note that kJ are converted in J by adding 10^3 to the denominator of the middle term.

4.82

A gas turbine operates witj 0.1 kg/s of helium that enters at 8 MPa and 600 K and leaves at 200 kPa and 350 K. If the turbine power output is 120 kW, find the rate of heat loss from the turbine. Neglect kinetic energy changes.

Energy balance:

$$\dot{Q}+\dot{W}=\dot{m}\left[(h_2-h_1)+\underbrace{\frac{\mathbf{V}_2^2-\mathbf{V}_1^2}{2}}_{=0}+\underbrace{g(z_2-z_1)}_{=0}\right]$$

$$\dot{Q}=\dot{m}c_p(T_2-T_1)-\dot{W},$$

No potential and no kinetic energy. Specific heat of helium at constant pressure c_p = 5.193 kJ/kg°C assumed (Appendix 1).

$$\dot{Q}=0.1\text{ kg/s}\times5.193\text{ kJ/kgK}\times(350\text{ K}-600\text{ K})+120\text{ kW}$$

$$\dot{Q}=-9.825\text{ kW}.$$

Please note that a consideration of task-specific temperature (600 K, 350K) and pressure (8 MPa, 200 kPa) leads to a more realistic value for the specific heat c_p but is not included here.

4.76.

Nitrogen flowing at a rate of 60 kg/min is compressed from 150 kPa and 30°C to 800 kPa and 150°C. The compressor is cooled at a rate of 15 kJ/kg of gas during operation. What is the power required to drive the compressor?

Energy balance:

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{in} = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{out}$$

Kinetic energy and potential energy are equal to zero. Specific heat of nitrogen at constant pressure c_p = 1.04179 kJ/kg°C at T_{avg} = 363.15 K interpolated from Appendix 4.

$$\dot{W}_{in} + \dot{m}(h)_{in} = \dot{Q}_{out} + \dot{m}(h)_{out}$$

$$\dot{W}_{in} = \frac{60\ kg/min}{60\ s/min}\left(1.04179\,\frac{\mathrm{kJ}}{\mathrm{kg}}\,°\mathrm{C}(150°\mathrm{C} - 30°\mathrm{C})\right) + 15\,\frac{\mathrm{kg}}{\mathrm{s}} = 140\ kW$$

4.77.

An air compressor takes air at 315 K and compresses it. For every kilogram of air passing through the compressor it does 200 kJ of work and loses 10 kJ of heat. Find the temperature of the air exiting the compressor. Do not assume that the specific heat of air is constant.

Specific enthalpy of air at 315 K, h_{in} = 315.27 kJ/kg (Appendix 7).

$$\dot{W}_{in} + \dot{m}(h)_{in} = \dot{Q}_{out} + \dot{m}(h)_{out}$$

$$(h)_{out} = \dot{W}_{in} + \dot{m}(h)_{in} - \dot{Q}_{out} = 200 + 315.27 - 10 = 505\ kJ/kg$$

Interpolating from the air tables, Appendix 7, the final temperature is T_{out} = 502.18 K.

4.80.

Argon enters an adiabatic turbine at 2 MPa and 600°C at a rate of 2.5 kg/s and leaves at 150 kPa. Determine the exit temperature of the gas when the turbine is generating 300 kW. Neglect kinetic energy changes.

Specific heat of argon at constant pressure c_p = 0.520 kJ/kg°C (Appendix 1).

$$\dot{m}(h)_{in} = \dot{W}_{out} + (h)_{out}$$

$$\dot{m}C_pT_{in} = \dot{W}_{out} + \dot{m}C_pT_{out}$$

$$T_{out} = \frac{-\dot{W}_{out}}{\dot{m}C_p} + T_{in} = \frac{-300}{2.5 \times 520} + 600 = 369°\mathrm{C}$$

4.81.

Air enters an adiabatic turbine at 800 kPa and 870 K with a velocity of 60 m/s, and leaves at 120 kPa and 520 K with a velocity of 100 m/s. The inlet area of the turbine is 90 cm^2. What is the power output?

Gas constant of air R = 0.2870 kJ/kgK (Appendix 1), specific enthalpy of air at 520 K h_2 = 523.63 kJ/kg (Appendix 7).

$$\dot{m}\left(h+\frac{V^2}{2}\right)_{in} = \dot{W}_{out} + \dot{m}\left(h+\frac{V^2}{2}\right)_{out}$$

$$v_{in} = \frac{RT_{in}}{P_{in}} = \frac{0.2870 \times 870}{800} = 0.312\ m^3/kg$$

$$\dot{m} = \frac{A_{in}V_{in}}{v_{in}} = \frac{90 \times 10^{-4} \times 60}{0.312} = 1.73\ kg/s$$

Via interpolation h_1 = 899.415 kJ/kg (Appendix 7).

$$\dot{W}_{in} = \dot{m}\left(h+\frac{V^2}{2}\right)_{in} - \dot{m}\left(h+\frac{V^2}{2}\right)_{out} = 1.73\ (899.415 - 523.63 + \frac{60^2-100^2}{2} = 644.6\ kW$$

SOLUTIONS
Chapter 5

5.1. A 0.1 A current is passed through a 1 kΩ resistor for 5 minutes. The heat generated is lost to the surrounding air at 20°C. What is the increase in entropy of the atmosphere?

Find: Increase in entropy ΔS of the atmosphere.

Known: Current in resistor I = 0.1 A, resistance R_e = 1 kΩ, duration of current Δt = 5 min, surroundings temperature T = 20°C.

Assumptions: Temperature of atmosphere remains constant.

The power of the resistor is transferred to the surroundings as heat,

$$\dot{Q} = \dot{W}_e = I^2 R_e = (0.1\ \text{A})^2 \times 1000\ \Omega = 10\ \text{W},$$
$$Q = \dot{Q}\Delta t = 10\ \text{J/s} \times (60\ \text{s/min} \times 5\ \text{min}) = 3000\ \text{J}.$$

Using the definition of entropy change and the assumption that it remains constant in the area of the resistor, the entropy increase of the atmosphere is

$$\Delta S = \frac{Q}{T} = \frac{3000\ \text{J}}{(20 + 273.15)\ \text{K}} = 10.234\ \text{J/K}.$$

The increase in entropy of the atmosphere is 10.2 J/K.

5.2. A fireplace burns 6 kg of coal which supplies 15 MJ of heat for every kilogram consumed. Find the increase in the entropy of the surrounding air at 24°C.

Find: Increase in entropy ΔS of the surroundings.

Known: Mass of coal burned m = 6 kg, heat per unit mass of coal burned q = 15 MJ/kg, temperature of surroundings T = 24°C.

Assumptions: Temperature of surroundings remains constant.

The total heat transfer to the surroundings from burning the coal is

$$Q = 6 \text{ kg} \times 15 \text{ MJ/kg} = 90 \text{ MJ},$$

Then the entropy increase is

$$\Delta S = \frac{Q}{T} = \frac{90 \text{ MJ}}{(273.15 + 24)\text{K}} = 0.30288 \text{ MJ/K}.$$

The increase in entropy of the surroundings is 0.303 MJ/K.

5.7. A rigid tank with a volume of 0.5 m^3 initially contains air at 300 kPa and 400 K. It loses heat to the surrounding air at 300 K until it reaches equilibrium. Find the increase in entropy of the atmosphere.

Find: Increase in entropy ΔS of the atmosphere.

Known: Rigid tank volume V = 0.5 m^3, initial tank air pressure P_1 = 300 kPa, initial tank air temperature T_1 = 400 K, surrounding air temperature T_s = 300 K, final tank air temperature $T_2 = T_s$.

Assumptions: Air behaves as an ideal gas.

Properties: Gas constant of air R = 0.2870 kJ/kgK (A1), specific heat of air at 350 K c_v = 0.721 kJ/kgK (A4).

The mass of air in the tank can be found using the ideal gas equation in the initial state,

$$m = \frac{PV}{RT} = \frac{300 \text{ kPa} \times 0.5 \text{ m}^3}{0.2870 \text{ kJ/kgK} \times 400 \text{ K}} = 1.3066 \text{ kg.}$$

The heat absorbed by the surroundings is lost from the air in the tank and is related to the change in potential energy,

$$Q = -Q_t = -mc_v(T_2 - T_1) = 1.3066 \text{ kg} \times 0.721 \text{ kJ/kgK} \times (300 \text{ K} - 400 \text{ K}) = 94.206 \text{ kJ,}$$

resulting in an entropy increase of the surroundings of

$$\Delta S = \frac{Q}{T_s} = \frac{94.206 \text{ kJ}}{300 \text{ K}} = 0.31402 \text{ kJ/K.}$$

The entropy of the surrounding atmosphere increases by 0.314 kJ/K.

6.3. An electric immersion heater placed inside a well-insulated tank is used to heat 500 l of water from 25°C to 50°C. The surface temperature of the heater is constant at 90°C. Find the entropy generated during this process.

Find: Entropy generated $\dot{S}_{gen}$ in this process.

Known: Insulated tank, volume of water V = 500 L, initial water temperature T_1 = 25°C, final water temperature T_2 = 50°C, heater surface temperature T_h = 90°C.

Assumptions: Water is incompressible.

Properties: Specific heat of water c = 4.18 kJ/kgK (A3).

The entropy generated during the process can be found using an entropy balance, with no entropy output from the system since the tank is insulated:

$$\Delta S = S_{in} - \cancel{S_{out}} + S_{gen},$$
$$S_{gen} = \Delta S - S_{in}.$$

The mass of water in the tank can be found, using the density of water at the average temperature T_{avg} = 37.5°C interpolated from Appendix 3 as ρ = 992.5 kg/m^3:

$$m = \rho V = 992.5 \text{ kg/m}^3 \times 500 \times 10^{-3} \text{ m}^3 = 496.25 \text{ kg}.$$

The amount of heat added to the water can be found using an energy balance,

$$Q_{in} = \Delta U = mc(T_2 - T_1) = 496.25 \text{ kg} \times 4.18 \text{ kJ/kgK} \times (50°\text{C} - 25°\text{C}) = 51858 \text{ kJ},$$

Then the entropy input to the water tank is

$$S_{in} = \frac{Q_{in}}{T_b} = \frac{51858 \text{ kJ}}{273.15 + 90 \text{ K}} = 142.80 \text{ kJ/K}.$$

The entropy change of the water in the tank during the heating process can be found using the equation for entropy change of an incompressible substance,

$$\Delta S = mc \ln \frac{T_2}{T_1} = 496.25 \text{ kg} \times 4.18 \text{ kJ/kgK} \times \ln \frac{(273.15 + 50) \text{ K}}{(273.15 + 25) \text{ K}} = 167.02 \text{ kJ/K}.$$

Then the entropy generation can be solved,

$$S_{gen} = \Delta S - S_{in} = 167.024 \text{ kJ/K} - 142.801 \text{ kJ/K} = 24.220 \text{ kJ/K}.$$

The entropy generated in the tank is 24.2 kJ/K.

6.5. A 20 mA current is passed through an insulated, 50 Ω resistor with a mass of 0.2 kg and a specific heat of 0.7 kJ/kgK. Find the entropy generated after 10 hours. The initial temperature of the resistor is 20°C.

<u>Find:</u> Entropy generated S_{gen} by the resistor after a duration.

<u>Known:</u> Resistor current I = 2 mA, insulated resistor, resistance R = 50 Ω, mass of resistor m = 0.2 kg, specific heat of resistor c = 0.7 J/kgK, duration Δt = 10 hours, initial temperature T_1 = 20°C.

<u>Assumptions:</u> The resistor is incompressible.

The entropy generation in the resistor can be found with an entropy balance, for no input or output to the resistor:

$$\Delta S = \cancel{S_{in}} - \cancel{S_{out}} + S_{gen},$$

$$S_{gen} = \Delta S = mc\ln\frac{T_2}{T_1}.$$

The final temperature of the resistor can be found using an energy balance,

$$W = mc(T_2 - T_1) = 0.2 \text{ kg} \times 0.7 \text{ J/kgK} \times (T - 293.15 \text{ K}),$$

$$T_2 = 293.15 \text{ K} + \frac{W}{0.14 \text{ J/K}}.$$

The work output of the resistor is

$$W = \dot{W}\Delta t = I^2 R\Delta t = (20\times10^{-3} \text{ A})^2 \times (50\ \Omega)\times 10 \text{ hour}\times 3600 \text{ s/hour},$$

$$W = 720 \text{ J}.$$

Then the final temperature of the resistor is

$$T_2 = 293.15 \text{ K} + \frac{720 \text{ J}}{0.14 \text{ kJ/K}},$$

$$T_2 = 298.29 \text{ K},$$

and the entropy generated during the process can be found using the equation for entropy change of an incompressible substance,

$$S_{gen} = mc\ln\frac{T_2}{T_1} = 0.2 \text{ kg } \text{´ } 0.7 \text{ kJ/kgK } \text{´ } \ln\frac{298.29 \text{ K}}{293.15 \text{ K}} = 2.433 \text{ J/K}.$$

The entropy generated during this process is 2.43 J/K.

6.7. Five kilograms of water at 20°C are poured into an insulated bucket that already contains 10 kg of water at 80°C. What is the entropy generated by the hot and cold water mixing?

<u>Find:</u> Entropy generated S_{gen} during mixing of hot and cold water.

<u>Known:</u> Mass of cool water m_c = 5 kg, temperature of cool water T_c = 20°C, insulated bucket, mass of hot water m_h = 10 kg, temperature of hot water T_h = 80°C.

<u>Assumptions:</u> Specific heat of all water is constant c = 4.18 kJ/kgK (A3), water is incompressible.

The entropy generated during this process can be found by adding the entropy change of both masses of water, since the bucket is insulated. The final temperature of the mixture can be found with an energy balance,

$$\Delta U_c - \Delta U_h = 0,$$

$$m_c c(T_f - T_c) + m_h c(T_f - T_h) = 0,$$

$$T_f = \frac{m_c T_c + m_h T_h}{m_c + m_h} = \frac{5 \text{ kg} \times 20°\text{C} + 10 \text{ kg} \times 80°\text{C} +}{5 \text{ kg} + 10 \text{ kg}} = 60°\text{C}.$$

Then the entropy change of each mass of water can be found using the equation for entropy change of an incompressible substance,

$$\Delta S_h = m_h c \ln \frac{T_f}{T_h} = 5 \text{ kg} \times 4.18 \text{ kJ/kgK } \ln \frac{333.15 \text{ K}}{293.15 \text{ K}} = 2.6733 \text{ kJ/K},$$

$$\Delta S_h = m_h c \ln \frac{T_f}{T_h} = 10 \text{ kg} \times 4.18 \text{ kJ/kgK} \times \ln \frac{333.15 \text{ K}}{353.15 \text{ K}} = -2.4369 \text{ kJ/K},$$

and the total entropy generated can be solved:

$$S_{gen} = \Delta S_A + \Delta S_B = 2.6733 \text{ kJ/K} - 2.4369 \text{ kJ/K} = 0.23640 \text{ kJ/K}.$$

The entropy generated during the mixing of the water is 0.236 kJ/K.

6.43. Hot air enters a pipe at 500 kPa and 600 K with a mass flow rate of 0.5 kg/s and exits at 450 kPa and 500 K. Find the rate of heat loss from the pipe and the rate of entropy generation in it. The ambient air temperature is 300 K.

Find: Rate of heat loss $\dot{Q}$ from pipe, rate of entropy generation $\dot{S}_{gen}$ in the pipe.

Known: Inlet air pressure P_1 = 500 kPa, inlet temperature T_1 = 600 K, mass flow rate of air $\dot{m}$ = 0.5 kg/s, outlet pressure P_2 = 450 kPa, outlet temperature T_2 = 500 K, temperature of surroundings T_s = 300 K.

Assumptions: Air behaves as an ideal gas, pipe is at steady state, temperature of surroundings is constant.

Properties: Specific enthalpy of air at 600 K h_1 = 607.02 kJ/kg (A7), specific entropy of air at 600 K s_1^o = 2.40902 kJ/kgK (A7), specific enthalpy of air at 500 K h_2 = 503.04 kJ/kg (A7), specific entropy of air at 500 K s_2^o = 2.21952 kJ/kgK (A7).

The rate of heat transfer from the air in the pipe can be found using an energy balance on the air flowing through the pipe,

$$\dot{Q} = \dot{m}(h_2 - h_1) = 0.5 \text{ kg/s} \times (503.02 \text{ kJ/kg} - 607.02 \text{ kJ/kg}) = -52.0 \text{ kW}.$$

An entropy rate balance can be used to find the rate of entropy generation, using the assumption that the pipe is at steady state:

$$\frac{\dot{Q}}{T_s} + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0,$$

$$\dot{S}_{gen} = \Delta\dot{S}_a - \frac{\dot{Q}}{T_s},$$

with the entropy change of the air in the pipe found using the equation for entropy change of an ideal gas using air tables,

$$\Delta\dot{S}_a = \dot{m}\Delta s_a = \dot{m}\left(s_2^o - s_1^o - R\ln\frac{P_2}{P_1}\right),$$

$$\Delta\dot{S}_a = 0.5 \text{ kg/s} \times \left(2.21952 \text{ kJ/kgK} - 2.40902 \text{ kJ/kgK} - 0.2870 \text{ kJ/kgK} \times \ln\frac{450 \text{ kPa}}{500 \text{ kPa}}\right),$$

$$\Delta\dot{S}_a = -0.079631 \text{ kW/K}.$$

Then the rate of entropy generation in the pipe can be found:

$$\dot{S}_{gen} = \Delta\dot{S}_a - \frac{-52.0 \text{ kJ}}{300 \text{ K}} = -0.079631 \text{ kW/K} + 0.17333 \text{ kW/K} = 0.093699 \text{ kW/K}.$$

The pipe loses heat at a rate of 52 kW, and the entropy generated in the pipe is 0.0937 kW/K.

6.48. Measurements at the inlet and outlet of an insulated turbine show air entering at 700 kPa and 900 K and leaving at 150 kPa and 600 K. Is this process possible? If it is, how much work is done per kilogram of air flowing through the turbine?

Find: Whether or not the process is possible, work done w if the process is possible.

Known: Inlet air pressure P_1 = 700 kPa, inlet temperature T_1 = 900 K, outlet air pressure P_2 = 150 kPa, outlet temperature T_2 = 600 K.

Assumptions: Air behaves as an ideal gas.

Properties: Specific enthalpy of air at 900 K h_1 = 932.93 kJ/kg (A7), specific entropy of air at 900 K $s_1^o = 2.84856$ kJ/kgK (A7), specific enthalpy of air at 600 K h_2 = 607.02 kJ/kg (A7), specific entropy of air at 600 K $s_2^o = 2.40902$ kJ/kgK (A7).

The process is possible if the entropy change during the process is non-negative. The change of entropy of the air can be found using the equation for entropy change of an ideal gas using air tables,

$$\Delta s = s_2^o - s_1^o - R \ln \frac{P_2}{P_1},$$

$$\Delta s = 2.40902 \text{ kJ/kgK} - 2.84856 \text{ kJ/kgK} - 0.2870 \text{ kJ/kgK} \times \ln \frac{150 \text{ kPa}}{700 \text{ kPa}},$$

$$\Delta s = 2.56773 \text{ J/kgK}.$$

The process is possible since the entropy change is non-negative. Then the work done per unit mass can be found with an energy balance,

$$w = h_2 - h_1 = 607.02 \text{ kJ/kg} - 932.93 \text{ kJ/kg} = -325.910 \text{ kJ/kg}.$$

The process is possible, and the work output would be 325.9 kJ/kg.

Solutions for selected exercises
(Lecture 6a and 6b)

Exercise 7.1

Similar to example 7.1 in lecture 6b, the saturation pressure $P_{sat,2}$ needs to be determined, here for T_2 = 205 °C. By using the Clausius-Clapeyron equation

$$P_{sat} = C\exp\left(\frac{-h_{fg}}{RT_{sat}}\right),$$

we are able to

- first solve for the constant C in state 1 with T_1 = 200 °C and
- then solve for the saturation pressure in state 2 with T_2 = 205 °C .

Therefore, the average enthalpy of vaporization is estimated by means of the saturated water property table:

$$h_{fg} = \frac{(h_{g,1} - h_{f,1}) + (h_{g,2} - h_{f,2})}{2} = \frac{(2793.2\ \text{kJ/kg} - 852.45\ \text{kJ/kg}) + (2796.0\ \text{kJ/kg} - 875.04\ \text{kJ/kg})}{2},$$

$$h_{fg} = 1930.855\ \text{kJ/kg},$$

This allows us to calculate C and $P_{sat,2}$:

$$C = \frac{P_{sat,1}}{\exp\left(\frac{-h_{fg}}{RT_{sat,1}}\right)} = \frac{1553.8\ \text{kPa}}{\exp\left(\frac{-1930.855\ \text{kJ/kg}}{0.4615\ \text{kJ/kgK} \times 473.15\ \text{K}}\right)} = 1.075672 \times 10^7\ \text{kPa}.$$

$$P_{sat,2} = 1.075672 \times 10^7\ \text{kPa} \times \exp\left(\frac{-1930.855\ \text{kJ/kg}}{0.4615\ \text{kJ/kgK} \times 478.15\ \text{K}}\right) = 1704.327\ \text{kPa}.$$

The value of 1723.0 kPa from Appendix 8a is close to our estimated saturation pressure of 1704.3 kPa. Please note that R in kJ/(kg*K) is used instead of R in kJ/(kmol*K).

Exercise 7.4

For this exercise, the saturation pressure of water vapour in equilibrium with ice at -20 ℃ needs to be determined.

Since temperature, pressure and latent heat of sublimation is given for water at the triple point, the Clausius-Clapeyron equation can be used to

- first find the constant *C* at the triple point and
- then solve for the saturation pressure at $T_2 = -20$ ℃ .

The calculation is similar to the one in exercise 7.1 and leads to the following results:

$$P_{sat} = C\exp\left(\frac{-h_{sg}}{RT_{sat}}\right),$$

$$C = \frac{P_{sat}}{\exp\left(\frac{-h_{sg}}{RT_{sat}}\right)} = \frac{0.6113\ \text{kPa}}{\exp\left(\frac{-2834.8\ \text{kJ/kg}}{0.4615\ \text{kJ/kgK} \times 273.16\ \text{K}}\right)} = 3.5668\times10^{9}\ \text{kPa},$$

$$P_{sat} = 3.5668\times10^{9}\ \text{kPa}\times\exp\left(\frac{-2834.8\ \text{kJ/kg}}{0.4615\ \text{kJ/kgK}\times 253.15\ \text{K}}\right) = 0.10335\ \text{kPa}.$$

Again, please note that R in kJ/(kg*K) is used instead of R in kJ/(kmol*K).

Exercise 7.8

The goal is to determine the minimum amount of time required to evaporate five kilograms of water which are boiling in an open pot placed over a 500W heater.

Keep in mind that water boils at different temperatures based on the atmospheric pressure. When assuming the atmospheric pressure to be 100 kPa, the mass flow rate of evaporated water can be estimated to:

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{\dot{Q}}{h_g - h_f} = \frac{0.5\ \text{kW}}{2258.04\ \text{kJ/kg}} = 2.2143\times10^{-4}\ \text{kg/s},$$

By considering the initial mass of 5 kilograms, the total duration is calculated as:

$$\Delta t = \frac{m}{\dot{m}} = \frac{5\ \text{kg}}{2.2143\times10^{-4}\ \text{kg/s}} = 22580\ \text{s} = 6.2722\ \text{h}.$$

Exercise 7.11

In this exercise, you need to identify the phase and specific enthalpy of water at temperature of 300 °C and pressure of 300 kPa.

There are two ways to identify the phase:

1. Determine the saturation pressure of water at 300 °C and compare to 300 kPa.
2. Determine the saturation temperature of water at 300 kPa and compare to 300 °C.

In both ways, superheated steam can be identified since the saturation pressure of water at 300 °C is 8.581 MPa (>> 300 kPa) and the saturation temperature of water at 300 kPa is 133.52 °C (<< 300°C).

The superheated steam tables in Appendix 8c give the enthalpy h = 3069.3 kJ/kg.

Exercise 7.26

2 L of saturated liquid at 100 kPa in a cyclinder with a frictionless piston is heated until its quality is 80%. How much heat is added?

The mass can be found by comparing the volume with the specific volume of liquid saturated water at 100 kPa (Appendix A8b).

$$m = \frac{V}{v} = \frac{2\times 10^{-3}\ \text{m}^3}{0.001043\ \text{m}^3/\text{kg}} = 1.917546\ \text{kg}.$$

By using the final quality and specific enthalpy of saturated water at 100 kPa for liquid and gas, the specific enthalpy in the final state is computed.

$$h_2 = h_f + x_2(h_g - h_f) = 417.46\ \text{kJ/kg} + 0.8\times(2675.5\ \text{kJ/kg} - 417.46\ \text{kJ/kg}),$$
$$h_2 = 2223.892\ \text{kJ/kg}.$$

The product of mass and specific enthakpy difference between initial and final state gives the required heat.

$$Q = m(h_2 - h_1) = 1.917546\ \text{kg}\times(2223.892\ \text{kJ/kg} - 417.46\ \text{kJ/kg}) = 3463.916\ \text{kJ}.$$

Exercise 7.46

Superheated steam at 0.5 MPa and 600 °C enters an insulated chamber with 2.5 kg/s. Liquid water at 5 MPa and 20 °C enters with unknown mass flow but condenses the steam in order to have saturated liquid at 0.5 MPa exiting the chamber.

In conclusion, the insulated chamber is characterized by two incoming streams and one exiting stream. The following energy balance for an open system shows that the specific heat capacities of the various substances at the given pressures are needed to calculate the missing mass flow of liquid water.

$$\dot{m}_w h_{1,w} + \dot{m}_s h_{1,s} = (\dot{m}_w + \dot{m}_s) h_2$$

Determining the $h_{1,w}$ (Appendix A8d) , $h_{1,s}$ (Appendix A8c) and h_2 (Appendix A8b) allows the calculation of liquid water mass flow.

$$\dot{m}_w = \frac{\dot{m}_s (h_2 - h_{1,s})}{h_{1,w} - h_2} = \frac{2.5\ \text{kg/s} \times (640.23\ \text{kJ/kg} - 3701.7\ \text{kJ/kg})}{88.7\ \text{kJ/kg} - 640.23\ \text{kJ/kg}} = 13.877\ \text{kg/s.}$$

Exercise 7.78

Use both the ideal gas equation and van der Waals equation to determine the pressure of 2 kg Oxygen occupying a volume of 0.1 m^3 at a temperature of 250 K.

First, the amount of oxygen in moles is calculated.

$$n = \frac{2\ \text{kg}}{31.999\ \text{kg/kmol}} = 0.062502\ \text{kmol},$$

The specific molar volume is calculated by diving the given volume by the amount of oxygen in moles.

$$\bar{v} = \frac{v}{n} = \frac{0.1\ \text{m}^3}{0.062502\ \text{kmol}} = 1.5999\ \text{m}^3/\text{kmol},$$

By using the ideal gas law, the pressure P_A can be calculated.

$$P_A = \frac{R_u T}{\bar{v}} = \frac{8.314\ \text{kJ/kmolK} \times 250\ \text{K}}{1.5999\ \text{m}^3/\text{kmol}} = 1.2991\ \text{MPa.}$$

Table 7.5 within chapter 7 helps to calculate the the pressure P_B with the van der Waals equation.

$$\left(P+\frac{a}{\bar{v}^2}\right)(\bar{v}-b)=R_uT,$$

$$P=\frac{R_uT}{\bar{v}-b}-\frac{a}{\bar{v}^2}=\frac{8.314\text{ kJ/kmolK}\times 250\text{ K}}{1.5999\text{ m}^3\text{/kmol}-0.0319\text{ m}^3\text{/kmol}}-\frac{136.9\text{ kPa m}^6\text{/kmol}^2}{(1.5999\text{ m}^3\text{/kmol})^2},$$

$$P_B=1.2721\text{ MPa}.$$

For this state of oxygen, the ideal gas equation approximation is close to the value calculated with the van der Waals equation. Please note that, different to exercises 7.1 and 7.4, R_u in kJ/(kmol*K) is used.

Exercise 7.79

Use both the ideal gas equation and the compressibility charts to determine the density of carbon dioxide at a pressure of 6.5 MPa and temperature of 30 °C.

First, the ideal gas equation is used to compute the density (inverse of the specific volume) with R from Appendix A1.

$$\rho=\frac{1}{v}=\frac{P}{RT}=\frac{6.5\times10^3\text{ kPa}}{0.1889\text{ kJ/kgK}\times 303.15\text{ K}}=113.507\text{ kg/m}^3.$$

The reduced temperature T_R and reduced pressure P_R are calculated with critical pressure P_c and critical temperature T_c from Appendix A6.

$$T_R=\frac{T}{T_C}=\frac{(30+273.15)\text{ K}}{304.2\text{ K}}=0.99655=1.00,$$

$$P_R=\frac{P}{P_C}=\frac{6.5\text{ MPa}}{7.39\text{ MPa}}=0.87957=0.88,$$

According to Appendix 10a, the compressibility factor is Z = 0.55.
By using the definition of the compressibility factor, we can solve for the specific volume.

$$\rho=\frac{P}{ZRT}=\frac{6.5\times10^3\text{ kPa}}{0.55\times0.1889\text{ kJ/kgK}\times 303.15\text{ K}}=206.377\text{ kg/m}^3.$$

For carbon dioxide in the given state, the density calculated by the ideal gas equation significantly differs to the more accurate value calculated from the compressibility charts.

8.1 What is the maximum thermal efficiency of an engine that operates between a heat source at 850°C and a heat sink at 35°C?

Find: Maximum thermal efficiency η_{th} of the engine.

Known: Temperature of heat source T_H = 850°C, temperature of heat sink T_C = 35°C.

Assumptions: Engine operates on a Carnot cycle.

The maximum thermal efficiency of the engine can be found if we assume that the engine operates on a Carnot cycle, and use the definition of thermal efficiency of a Carnot engine:

$$\eta_{th} = 1 - \frac{T_C}{T_H} = 1 - \frac{(35 + 273.15)\ \mathrm{K}}{(850 + 273.15)\ \mathrm{K}} = 0.72564.$$

The maximum efficiency of the engine is 72.6%.

8.2 Ocean Thermal Energy Conversion (OTEC) plants use the temperature difference between cold water near the ocean floor and warmer surface water to run a heat engine. What is the maximum efficiency of such a plant operating between deep water at 5°C and surface water at 25°C?

Find: Maximum thermal efficiency η_{th} of the OTEC plants.

Known: Temperature of surface water T_H = 25°C, temperature of deep water T_C = 5°C.

Assumptions: Engine operates on a Carnot cycle.

The maximum thermal efficiency of the engine can be found under the assumption that the plants operate on a Carnot cycle, and use the definition of thermal efficiency of a Carnot engine:

$$\eta_{\mathrm{th}} = 1 - \frac{T_C}{T_H} = 1 - \frac{278.15\ \mathrm{K}}{298.15\ \mathrm{K}} = 0.067080.$$

The maximum efficiency of the OTEC plants is 6.7%.

8.8 A car engine has a power output of 95 kW and a thermal efficiency of 25%. If gasoline gives a heat output of 47 MJ/kg when burned, find the rate of fuel consumption in kg/h.

Find: Rate of fuel consumption $\dot{m}$ in the car engine (kg/h).

Known: Power output $\dot{W}$ = 95 kW, thermal efficiency of engine η_{th} = 25%, enthalpy change for gasoline combustion Δh = 47 MJ/kg.

The rate of heat addition to the engine can be found with the definition of thermal efficiency,

$$\dot{Q}_H = \frac{\dot{W}}{\eta_{th,Carnot}} = \frac{95\text{ kW}}{0.25} = 380\text{ MW}$$

$$\dot{m}_{fuel} = \frac{380\text{ kJ/s}}{47\times 10^3\text{ kJ/kg}} = 8.1\times 10^{-3}\text{ kg/s=29.1 kg/h}$$

Then the rate of consumption of fuel is

$$Q_H = \dot{m}\Delta h,$$

$$\dot{m} = \frac{380\text{ kJ/s}}{47\times 10^3\text{ kJ/kg}} = 8.0851\times 10^{-3}\text{ kg/s} = 29.106\text{ kg/h.}$$

The car engine consumes fuel at a rate of 29.1 kg/h.

8.12 A Carnot engine with a power output of 20 kW takes 60 kW of heat from a high temperature reservoir at 800 K. What is the temperature of the heat sink?

Find: Temperature T_C of the heat sink for the Carnot engine.

Known: Carnot engine, power output $\dot{W}$ = 20 kW, rate of heat input $\dot{Q}_H$ = 60 kW, temperature of heat source T_H = 800 K.

The temperature of the heat sink can be found from the definition of thermal efficiency of a Carnot cycle,

$$\eta_{th} = \frac{\dot{W}}{\dot{Q}_H} = 1 - \frac{T_C}{T_H},$$

$$T_C = T_H\left(1 - \frac{\dot{W}}{\dot{Q}_H}\right) = 800\text{ K} \times \left(1 - \frac{20\text{ kW}}{60\text{ kW}}\right) = 533.33\text{ K.}$$

The temperature of the heat sink is 533 K.

8.13 A nuclear power plant generates 80 MW when taking heat from a reactor at 615 K. Heat is discarded to a river at 298 K. If the thermal efficiency of the plant is 73% of the maximum possible value, how much heat is discarded to the river?

Find: Rate of heat rejected $\dot{Q}_C$ from nuclear plant into river.

Known: Power output of nuclear plant $\dot{W}$ = 80 MW, temperature of heat source T_H = 615 K, temperature of river T_C = 298 K, thermal efficiency of plant η_{th} = 73% of maximum possible value.

Assumptions: Temperature of the river remains constant.

The maximum possible efficiency of the nuclear plant would be if the cycle was a Carnot cycle,

$$\eta_{th,\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{298\text{ K}}{615\text{ K}} = 0.515447.$$

then the actual efficiency of the nuclear plant can be found:

$$\eta_{th} = 0.73\eta_{th,\max} = 0.73 \times 0.515447 = 0.376276.$$

The heat input to the plant can be found using the definition of thermal efficiency,

$$\eta_{th} = \frac{\dot{W}}{\dot{Q}_H},$$

$$\dot{Q}_H = \frac{\dot{W}}{\eta_{th}} = \frac{80\text{ MW}}{0.376276} = 212.610\text{ MW},$$

and the heat rejected to the river can be found using an energy balance,

$$\dot{W} = \dot{Q}_H - \dot{Q}_C,$$

$$\dot{Q}_C = \dot{Q}_H - \dot{W} = 212.620\text{ MW} - 80\text{ MW} = 132.610\text{ MW}.$$

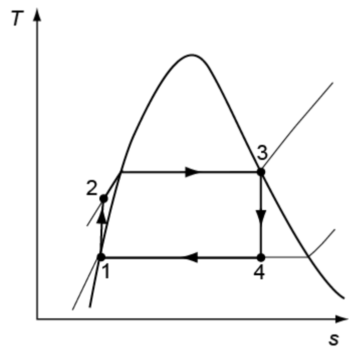
The nuclear plant rejects heat to the river at a rate of 132.6 MW.

9.1. Steam enters the turbine in an ideal Rankine cycle at 8 MPa and leaves at 10 kPa. What is the work done by the turbine per kilogram of steam?

Find: Work output w_t of the turbine per unit mass (kg) of steam.

Known: Ideal Rankine cycle using water, pressure of steam at turbine inlet P_3 = 8 MPa, pressure at turbine outlet P_4 = 10 kPa.

Properties: Specific enthalpy of saturated water at 8 MPa for gas h_3 = 2758.0 kJ/kg (A8*b*), specific entropy of saturated water at 8 MPa for gas s_3 = 5.7432 kJ/kgK (A8*b*), specific enthalpy for saturated water at 10 kPa for fluid h_f = 191.83 kJ/kg and has h_g = 2584.7 kJ/kg (A8*b*), specific entropy for saturated water at 10 kPa for fluid s_f = 0.6493 kJ/kgK and gas s_g = 8.1502 kJ/kgK (A8*b*).



The steam goes through an ideal Rankine cycle, so expansion through the turbine is isentropic and $s_4 = s_3$ = 5.7432 kJ/kgK. The quality of the steam leaving the turbine can then be found,

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{5.7432 \text{ kJ/kgK} - 0.6493 \text{ kJ/kgK}}{8.1502 \text{ kJ/kgK} - 0.6493 \text{ kJ/kgK}} = 0.679105.$$

The enthalpy at the turbine exit is then

$$h_4 = h_f + x_4(h_g - h_f),$$

$$h_4 = 191.83 \text{ kJ/kg} + 0.679105 \times (2584.7 \text{ kJ/kg} - 191.83 \text{ kJ/kg}) = 1816.84 \text{ kJ/kg}.$$

and the work output of the turbine can be found:

$$w_t = (h_3 - h_4) = 2758.0 \text{ kJ/kg} - 1816.84 \text{ kJ/kg} = 941.160 \text{ kJ/kg}.$$

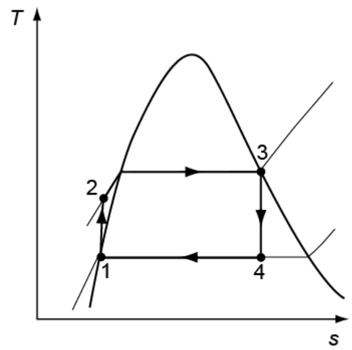
The turbine does 941.2 kJ/kg of work on the surroundings.

9.2. An ideal Rankine cycle has a condenser operating at a pressure of 50 kPa. If the steam quality at the outlet of the turbine is required to be 80%, what should the boiler pressure be?

Find: Boiler pressure $P_2 = P_3$ required to produce the necessary steam quality exiting the turbine.

Known: Ideal Rankine cycle using water, pressure of condenser $P_4 = P_1 = 50$ kPa, quality of steam exiting turbine $x_4 = 80\%$.

Properties: Specific entropy of saturated water at 50 kPa for fluid $s_f = 1.0910$ kJ/kgK and gas $s_g = 7.5939$ kJ/kgK (A8*b*).



The process occurs on an ideal Rankine cycle, so the pressure of the boiler can be found in state 2 or 3, so we will use the isentropic expansion through the turbine to find $s_3 = s_4$, using the given quality of steam exiting the turbine:

$$s_4 = s_f + x_4(s_g - s_f),$$

$$s_4 = 1.0910 \text{ kJ/kgK} + 0.80 \times (7.5939 \text{ kJ/kgK} - 1.0910 \text{ kJ/kgK}) = 6.2933 \text{ kJ/kgK}.$$

The pressure of the boiler can then be interpolated from Appendix 8*b* using entropy of saturated gas, since state 3 is saturated vapour:

$$\frac{P_3 - 2.25 \text{ MPa}}{2.50 \text{ MPa} - 2.25 \text{ MPa}} = \frac{6.2933 \text{ kJ/kgK} - 6.2575 \text{ kJ/kgK}}{6.2972 \text{ kJ/kgK} - 6.2575 \text{ kJ/kgK}},$$

$$P_3 = 2.2746 \text{ MPa}.$$

The boiler is required to be at a pressure of 2.27 MPa to achieve the desired quality.

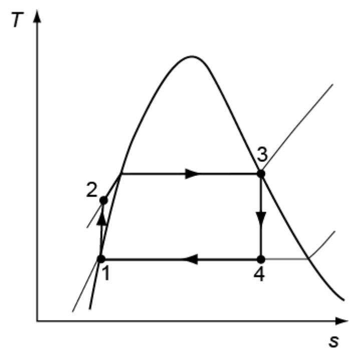
9.5. Two thermal reservoirs are available, a high temperature heat source at 350°C and a low temperature heat sink at 20°C. Calculate the maximum efficiency of *a*) a Carnot cycle, and *b*) an ideal Rankine cycle using water as a working fluid operating between these two reservoirs.

Find: Maximum efficiency *a*) η_C of a Carnot cycle, *b*) η_R of an ideal Rankine cycle.

Known: Temperature of high temperature reservoir $T_H = 350°C$, temperature of low temperature reservoir $T_C = 20°C$, Carnot cycle, ideal Rankine cycle using water.

Assumptions: Water is incompressible.

Properties: Saturation pressure of water at 350°C $P_2 = P_3 = 16.513$ MPa (A8*a*), specific enthalpy of saturated water at 350°C for gas $h_3 = 2563.9$ kJ/kg (A8*a*), specific entropy of saturated water at 20°C for gas $s_3 = 5.2112$ kJ/kgK (A8*a*), saturation pressure of water at 20°C $P_4 = P_1 = 0.002339$ MPa (A8*a*), specific enthalpy of saturated water at 20°C for fluid $h_f = 83.96$ kJ/kg and gas $h_g = 2538.1$ kJ/kg (A8*a*), specific entropy of saturated water at 20°C for fluid $s_f = 0.2966$ kJ/kgK and gas $s_g = 8.6672$ kJ/kgK (A8*a*), specific volume of saturated water at 20°C for fluid $v_1 = 0.001002$ m^3/kg (A8*a*).



a) The thermal efficiency of a Carnot cycle operating between these temperatures is

$$\eta_C = 1 - \frac{T_C}{T_H} = 1 - \frac{293.15 \text{ K}}{623.15 \text{ K}} = 0.52957.$$

b) The specific enthalpy at the turbine inlet is that of saturated water vapour at $T_3 = 350°C$, $h_3 = 2563.9$ kJ/kg. Since the cycle operates on an ideal Rankine cycle, expansion through the turbine is isentropic, so $s_4 = s_3 = 5.2112$ kJ/kgK and the quality of the steam exiting the turbine is

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{5.2112 \text{ kJ/kgK} - 0.2966 \text{ kJ/kgK}}{8.6672 \text{ kJ/kgK} - 0.2966 \text{ kJ/kgK}} = 0.58713.$$

The specific enthalpy at the turbine exit can then be found,

$$h_4 = h_f + x_4(h_g - h_4),$$

$$h_4 = 83.96 \text{ kJ/kg} + 0.58713 \times (2538.1 \text{ kJ/kg} - 83.96 \text{ kJ/kg}) = 1524.9 \text{ kJ/kg}.$$

The specific enthalpy at the condenser exit is that of saturated liquid water at $T_1 = 20°C$, $h_1 = h_f = 83.96$ kJ/kg. The specific enthalpy of the fluid at the pump exit

$$h_2 = h_1 + w_p = h_1 + v_1(P_2 - P_1),$$

$$h_2 = 83.96 \text{ kJ/kgK} + 0.001002 \text{ m}^3/\text{kg} \times (16513 \text{ kPa} - 2.339 \text{ kPa}) = 100.50 \text{ kJ/kg}.$$

The thermal efficiency of a Rankine cycle is then

$$\eta_R = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{1524.9 \text{ kJ/kg} - 83.96 \text{ kJ/kg}}{2563.9 \text{ kJ/kg} - 100.50 \text{ kJ/kg}} = 0.41506.$$

The maximum efficiency of the *a*) Carnot cycle is 53.0%, and *b*) of the ideal Rankine cycle is 41.5%.

Solutions for selected exercises
(Lecture 8)

Exercise 8.22

The percentage change in work required to remove heat to when changing the refrigerator temperature from -5 °C to -8 °C when the surrounding air is at 25 °C.

By assuming an reverse Carnot cycle and constant temperature of the surrounding air, the percentage change in work required to remove an equal amount of heat, can be expressed with just the COP of each refrigerator

$$\Delta W_p = \frac{W_2 - W_1}{W_1} \times 100\% = \frac{\dfrac{Q_C}{COP_{R,2}} - \dfrac{Q_C}{COP_{R,1}}}{\dfrac{Q_C}{COP_{R,1}}} \times 100\% = \left(\frac{COP_{R,1}}{COP_{R,2}} - 1 \right) \times 100\%.$$

Since initial refrigerator temperatures and the temperature of surroundings are given, COP's can be calculated as

$$COP_{R,1} = \frac{1}{T_H/T_{C,1} - 1} = \frac{1}{298.15\ K/268.15\ K - 1} = 8.9383$$

$$COP_{R,2} = \frac{1}{T_H/T_{C,2} - 1} = \frac{1}{298.15\ K/263.15\ K - 1} = 8.0348$$

Using both COPs in the first equation for the final percentage change in work leads to

$$\Delta W_p = \left(\frac{COP_{R,1}}{COP_{R,2}} - 1 \right) x\ 100\% = \left(\frac{8.9383}{8.0348} - 1 \right) x\ 100\% = 11.24\%$$

Exercise 8.23

Please show that a heat pump and a refrigerator between same reservoirs, both operate on a reverse Carnot cycle, transfer the same amount of heat.

The statement is proven by comparing the definition of COP for a heat pump and a refrigerator with an energy balance

$$COP_{HP} = \frac{Q_H}{W} = \frac{Q_C + W}{W} = \frac{Q_C}{W} + 1 = COP_R + 1.$$

Please note that this is valid for Carnot heat pump and Carnot refrigerator.

Exercise 8.38

The mass flow rate of a refrigerant R134a in a Carnot refrigerator which takes heat a -4 °C and rejects heat at 24 °C needs to be determined for an assumed power input of 380 W.

An energy balance around the isothermal condenser gives

$$\dot{Q}_H = \dot{m}(h_3 - h_2),$$

$$\dot{m} = \frac{\dot{Q}_H}{h_3 - h_2}.$$

In order to find the rate of heat rejected from the refrigerator, the COP is calculated as follows

$$COP_R = \frac{\dot{Q}_C}{\dot{W}} = \frac{1}{T_H / T_C - 1},$$

$$\dot{Q}_C = \frac{\dot{W}}{T_H / T_C - 1} = \frac{380 \text{ W}}{297.15 \text{ K} / 269.15 \text{ K} - 1} = 3652.8 \text{ W},$$

Since power input and rate of heat rejected are known now, the rate of heat transfer from the heat sink is

$$\dot{Q}_H = \dot{W} + \dot{Q}_C = 380 \text{ W} + 3652.8 \text{ W} = 4032.8 \text{ W}.$$

Using the specific enthalpies h_2 and h_3 of satured R134a from Appendix A9a, The resulting mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{Q}_H}{(h_3 - h_2)} = \frac{4032.8 \text{ W}}{260.45 \text{ kJ/kg} - 82.90 \text{ kJ/kg}} = 0.022714 \text{ kg/s}.$$

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}} = \frac{3.6017 \text{ kW}}{1.5 \text{ kW}} = 2.4011.$$

Exercise 9.36

Specific enthalpy of saturated R-134a at 0.20 MPa for gas h_2 = 241.30 kJ/kg (A8*b*), specific entropy of saturated R-134a at 0.20 MPa for gas s_2 = 0.9253 kJ/kg-K (A8*b*), specific enthalpy of saturated R-134a at 1.2 MPa for fluid h_4 = 115.76 kJ/kg (A8*b*).
The properties of the refrigerant entering the compressor are those of saturated R-134a vapour at P_2 = 0.20 MPa, h_2 = 241.30 kJ/kg and s_2 = 0.9253 kJ/kg-K.

Since we know that the refrigeration cycle is ideal, compression through the compressor is isentropic so that $s_3 = s_2$ = 0.9253 kJ/kg-K, so refrigerant leaving the compressor is superheated with specific enthalpy interpolated from Appendix 9*c* at P_3 = 1.2 MPa:

$$\frac{h_3 - 275.52\frac{kJ}{kg}}{287.44\frac{kJ}{kg} - 275.52\frac{kJ}{kg}} = \frac{0.9253\ \frac{kJ}{kg\ K} - 0.9164\frac{kJ}{kg\ K}}{0.9527\frac{kJ}{kg\ K} - 0.9164\ \frac{kJ}{kg\ K}}$$

And h_3 = 278.44 kJ/kg

The specific enthalpy of refrigerant leaving the condenser is that of saturated liquid R- 134a at P_4 = 1.2 MPa, h_4 = 115.76 kJ/kg.

Expansion through the throttling valve in an ideal reverse Rankine cycle is a constant enthalpy process, so specific enthalpy at the evaporator inlet is $h_1 = h_4$ = 115.76 kJ/kg. The mass flow rate of refrigerant through the cycle can be found using an energy balance around the compressor, with power supplied by the car:

$$\dot{W}_{comp} = \dot{m}_{ref}(h_3 - h_2)$$

$$\dot{m}_{ref} = \frac{\dot{W}_{comp}}{h_3 - h_2} = \frac{1.2\ kW}{278.44\frac{kJ}{kg} - 241.30\frac{kJ}{kg}} = 0.0323\frac{kg}{s}$$

then the rate of heat input to the refrigerator from the surrounding air is

$$\dot{Q}_C = \dot{m}_{ref}(h_2 - h_1) = 0.0323\frac{kg}{s}\left(241.30\frac{kJ}{kg} - 115.76\frac{kJ}{kg}\right) = 4.056\ kW$$

For air at T_{avg} = 25°C, the average specific heat at constant pressure is c_p = 1.004 kJ/kgK, so the mass flow rate of air that can be cooled by the refrigerator is

$$\dot{m}_{air} = \frac{\dot{Q}_C}{c_p\ \Delta T} = \frac{4.056\ kW}{1.004\frac{kJ}{kg\ K}\ (35 - 15)℃}0.2002\frac{kg}{s}$$

The refrigerator can cool air at a maximum mass flow rate of 0.202 kg/s.

9.37. A freezer with an internal temperature of –12°C is kept in a grocery store where the surrounding air is at 20°C. The rate of heat transfer due to conduction through the walls of the freezer is estimated to be 8 kW. What is the minimum mass flow rate of refrigerant 134a required if an ideal vapour refrigeration cycle is used in the freezer? How much power is required to drive the compressor?

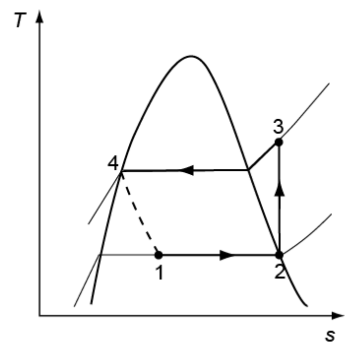
Find: Minimum mass flow rate $\dot{m}_R$ of refrigerant required, power $\dot{W}_c$ required to drive the compressor.

Known: Freezer temperature T_2 = –12°C, temperature of surrounding air T_4 = 20°C, rate of heat transfer into freezer $\dot{Q}_C$ = 8 kW, ideal Rankine refrigeration cycle.

Properties: Specific enthalpy of saturated R-134a at –12°C for gas h_2 = 240.15 kJ/kg (A9*a*), specific entropy of saturated R-134a at –12°C for gas s_2 = 0.9267 kJ/kgK (A9*a*), specific entropy of saturated R-134a at 20°C for fluid s_f = 0.2924 kJ/kgK and gas s_g = 0.9102 kJ/kgK (A9*a*), specific enthalpy of saturated R-134a at 20°C for fluid h_4 = 77.26 kJ/kg (A9*a*).

We will run a freezer which extracts heat from the internal temperature and provides it to the surrounding air at a rate equal to the heat transfer through the freezer walls.

The properties of the refrigerant entering the compressor are those of saturated R-134a vapour at T_2 = –12°C, h_2 = 240.15 kJ/kg and s_2 = 0.9267 kJ/kgK.



The Rankine refrigeration cycle is ideal, so compression through the compressor is isentropic and $s_3 = s_2$ = 0.9267 kJ/kgK, so refrigerant leaving the compressor is superheated with specific enthalpy interpolated from Appendix 9*c* at T_3 = 20°C:

$$\frac{h_3 - 262.96 \text{ kJ/kg}}{260.34 \text{ kJ/kg} - 262.96 \text{ kJ/kg}} = \frac{0.9267 \text{ kJ/kgK} - 0.9515 \text{ kJ/kgK}}{0.9264 \text{ kJ/kgK} - 0.9515 \text{ kJ/kgK}},$$

$$h_3 = 260.37 \text{ kJ/kg}.$$

The specific enthalpy of refrigerant leaving the condenser is that of saturated liquid R-134a at T_4 = 20°C, h_4 = 77.26 kJ/kg.

Expansion through the throttling valve in an ideal reverse Rankine cycle is a constant enthalpy process, so specific enthalpy at the evaporator inlet is $h_1 = h_4$ = 77.26 kJ/kg.

The minimum mass flow rate of refrigerant through the cycle can be found from an energy balance around the evaporator,

$$\dot{Q}_C = \dot{m}(h_2 - h_1),$$

$$\dot{m} = \frac{\dot{Q}_C}{(h_2 - h_1)} = \frac{8\ \text{kW}}{(240.15\ \text{kJ/kg} - 77.26\ \text{kJ/kg})} = 0.049113\ \text{kg/s}.$$

The power input to the compressor can then be found:

$$\dot{W}_c = \dot{m}(h_3 - h_2) = 0.049113\ \text{kg/s} \times (260.37\ \text{kJ/kg} - 240.15\ \text{kJ/kg}) = 0.99306\ \text{kW}.$$

To cool the refrigerator would require a minimum mass flow rate of refrigerant of 0.0291 kg/s and require 0.993 kW of power for the compressor.

SOLUTIONS
Chapter 10

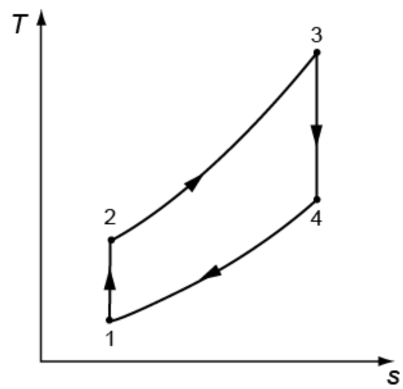
Otto Cycle

10.1. An engine operating on a cold air standard Otto cycle has a compression ratio of 8.5 and takes in air at 100 kPa and 300 K. The heat added during each cycle is 900 kJ/kg of air in the cylinder. Find the maximum temperature in the cycle.

Find: Maximum temperature T_3 reached during the cycle.

Known: Cold air standard Otto cycle, compression ratio r = 8.5, initial pressure P_1 = 100 kPa, initial temperature T_1 = 300 K, heat added q_H = 900 kJ/kg.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant volume at 300 K c_v = 0.718 kJ/kg (A4).



The air temperature at the end of isentropic compression is

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma - 1)} = 300 \text{ K} \times (8.5)^{0.400} = 706.137 \text{ K}.$$

The maximum temperature reached during the cycle occurs after constant volume heat addition and can be found using an energy balance,

$$T_3 = T_2 + \frac{q_H}{c_v} = 706.137 \text{ K} + \frac{900 \text{ kJ/kg}}{0.718 \text{ kJ/kgK}} = 1959.62 \text{ K}.$$

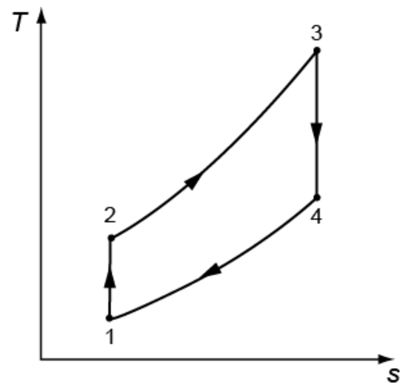
The maximum air temperature in the cycle is 1960 K.

10.4. An engine working on a cold air standard Otto cycle takes in air at 100 kPa and 295 K. The compression ratio is 9 and the maximum temperature is 2000 K. Find the heat added per kilogram of air during each cycle.

Find: Heat added q_H per unit mass (kg) of working air.

Known: Cold air standard Otto cycle, intake air pressure P_1 = 100 kPa, intake air temperature T_1 = 295 K, compression ratio r = 9, maximum temperature T_3 = 2000 K.

Properties of cold air at T_1 = 295 K are interpolated from Appendix 4: specific heat ratio γ = 1.4001 and specific heat at constant volume c_v = 0.7178 kJ/kgK.



The temperature at the end of isentropic compression is

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{(\gamma - 1)} = 295 \text{ K} \times (9)^{0.4001} = 710.582 \text{ K}.$$

The amount of heat added during heat addition is then

$$q_H = c_v (T_3 - T_2) = 0.7178 \text{ kJ/kgK} \times (2000 \text{ K} - 710.582 \text{ K}) = 925.544 \text{ kJ/kg}.$$

The heat added to the engine is 925.5 kJ/kg of working air.

Diesel Cycle

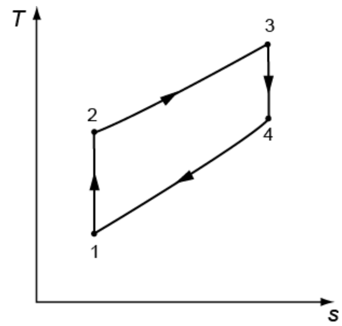
10.11. The air in an air standard Diesel cycle is at a pressure of 95 kPa and a temperature of 300 K at the start of compression. The maximum pressure in the cycle is 6500 kPa and the maximum temperature is 2200 K. Find the compression ratio and the cutoff ratio for the cycle. Use the properties of air at 300 K.

Find: Compression ratio r of the cycle, cutoff ratio r_c of the cycle.

Known: Air standard Diesel cycle, inlet air pressure P_1 = 95 kPa, inlet temperature T_1 = 300 K, maximum pressure $P_2 = P_3$ = 6500 kPa, maximum temperature T_3 = 2200 K.

Assumptions: Cold air standard Diesel cycle.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant volume at 300 K c_v = 0.717 kJ/kgK (A4).



The compression ratio can be found since compression and expansion are isentropic processes,

$$r = \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{6500 \text{ kPa}}{95 \text{ kPa}}\right)^{1/1.400} = 20.457.$$

The temperature at the end of isentropic compression is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 300 \text{ K} \times \left(\frac{6500 \text{ kPa}}{95 \text{ kPa}}\right)^{0.400/1.400} = 1003.4 \text{ K}.$$

Then the cutoff ratio can be found using the ideal gas equation during constant pressure heat addition,

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{P_3}\frac{P_2}{T_2} = \frac{T_3}{T_2} = \frac{2200 \text{ K}}{1003.4 \text{ K}} = 2.1925.$$

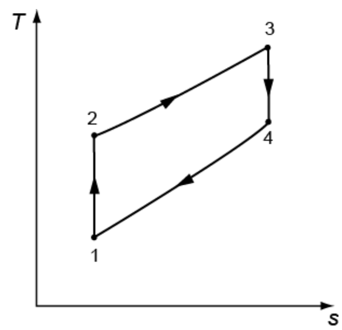
The cycle has a compression ratio of 20.5 and a cutoff ratio of 2.19.

10.16. In a cold air standard Diesel cycle the compression ratio is 18 and the cutoff ratio is 2.0. Air is at a pressure of 100 kPa, and a temperature of 300 K at the start of compression. Find the maximum temperature and pressure in the cycle, the heat added and the net work done during the cycle.

Find: Maximum temperature T_3 of air, maximum pressure P_3 of air, heat added q_H during the cycle, work done w during the cycle.

Known: Cold air standard Diesel cycle, compression ratio r = 18, cutoff ratio r_c = 2.0, intake pressure P_1 = 100 kPa, intake temperature T_1 = 300 K.

Properties: Specific heat ratio of air at 300 K γ = 1.400 (A4), specific heat of air at constant pressure at 300 K c_p = 1.005 kJ/kgK (A4), specific heat of air at constant volume at 300 K c_v = 0.718 kJ/kgK (A4).



The temperature and pressure at the end of isentropic compression are

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = 300\ \text{K} \times (18)^{0.400} = 953.3015\ \text{K},$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100\ \text{kPa} \times (18)^{1.400} = 5719.809\ \text{kPa}.$$

The maximum pressure during the process is $P_2 = P_3$ = 5719.8 kPa.

The maximum temperature occurs after constant pressure heat addition and can be found using the ideal gas equation,

$$T_3 = T_2\left(\frac{V_3}{V_2}\right) = 953.3\ \text{K} \times (2) = 1906.603\ \text{K}.$$

The temperature at the end of isentropic expansion is

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{(\gamma-1)} = T_3\left(\frac{V_3}{V_2}\frac{V_2}{V_1}\right)^{(\gamma-1)} = T_3\left(\frac{r_c}{r}\right)^{(\gamma-1)} = 1906.603\ \text{K} \times \left(\frac{2}{18}\right)^{0.400},$$

$$T_4 = 791.7048\ \text{K}.$$

The heat transfers and work done during the process can then be found:

$$q_H = c_p(T_3 - T_2) = 1.005\ \text{kJ/kgK} \times (1906.603\ \text{K} - 953.3015\ \text{K}) = 958.0680\ \text{kJ/kg},$$

$$q_C = c_v (T_4 - T_1) = 0.718 \text{ kJ/kgK} \times (791.7048 \text{ K} - 300 \text{ K}) = 353.0440 \text{ kJ/kg},$$

$$w = q_H - q_C = 958.0680 \text{ kJ/kg} - 353.0440 \text{ kJ/kg} = 605.0240 \text{ kJ/kg}.$$

The maximum air temperature is 953 K and pressure is 5719.8 kPa, and the cycle receives 958.1 kJ of heat to do 605.0 kJ of work per kilogram of working air.

Brayton Cycle

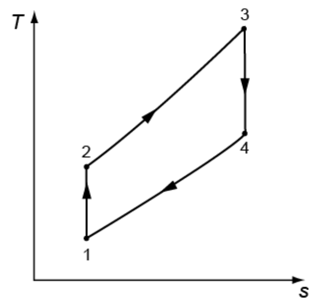
10.21. A cold air standard Brayton cycle has a minimum temperature of 320 K, a maximum temperature of 1400 K, and a compressor pressure ratio of 10. Find the back work ratio and the thermal efficiency of the cycle. Assume constant specific heats.

Find: Back work ratio *bwr* of the cycle, thermal efficiency η_{th} of the Brayton cycle.

Known: Cold air standard Brayton cycle, minimum temperature T_1 = 320 K, maximum temperature T_3 = 1400 K, compressor pressure ratio r_p = 10.

Assumptions: Cold air properties evaluated at 320 K.

Properties of cold air at T_1 = 320 K are interpolated from Appendix 4: specific heat ratio γ = 1.3992, constant heat of air at constant pressure c_p = 1.0062 kJ/kgK.



The temperature after isentropic compression is

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 320\text{ K}\times(10)^{(0.3992)/1.3992} = 617.24\text{ K}.$$

The temperature after isentropic expansion is

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(\gamma-1)/\gamma} = 1400\text{ K}\times\left(\frac{1}{10}\right)^{(0.3992)/1.3992} = 725.81\text{ K}.$$

Since we apply the cold air standard assumption, the back work ratio can be found using temperatures,

$$bwr = \frac{h_2-h_1}{h_3-h_4} = \frac{c_p(T_2-T_1)}{c_p(T_3-T_4)} = \frac{617.24\text{ K}-320\text{ K}}{1400\text{ K}-725.81\text{ K}} = 0.44088.$$

and the thermal efficiency of the cycle reduces to

$$\eta_{th} = 1-\frac{T_1}{T_2} = 1-\frac{320\text{ K}}{617.24\text{ K}} = 0.48156.$$

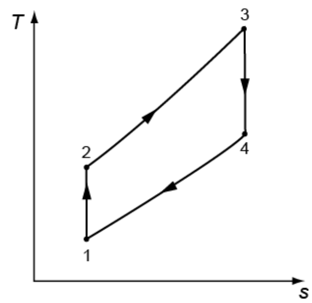
The cycle has a back work ratio of 0.441 and an efficiency of 48.2%.

10.23. A gas turbine operating on an air standard Brayton cycle has air entering the compressor with a temperature of 300 K, a pressure of 100 kPa and a mass flow rate of 4 kg/s. The compressor pressure ratio is 8 and air enters the turbine with a temperature of 1200 K. Find the thermal efficiency of the cycle and the net power supplied by it. Assume constant gas properties.

Find: Thermal efficiency η_{th} of the cycle, power $\dot{W}$ output of the engine.

Known: Air standard Brayton cycle, compressor inlet temperature T_1= 300 K, compressor inlet pressure P_1 = 100 kPa, mass flow rate $\dot{m}$ = 4 kg/s, compressor pressure ratio r_p = 8, turbine inlet temperature T_3 = 1200 K.

Assumptions: Constant specific heats.



The temperatures after isentropic compression and isentropic expansion can be found, assuming specific heat ratio γ = 1.400 at 300 K from Appendix 4:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 300\text{ K}\times(8)^{(0.4)/1.4} = 543.4342\text{ K},$$

$$T_4 = T_3\left(\frac{P_4}{P3}\right)^{(\gamma-1)/\gamma} = 1200\text{ K}\times\left(\frac{1}{8}\right)^{(0.4)/1.4} = 662.4537\text{ K}.$$

These values can be refined by using air properties at respective average process temperatures, interpolating from Appendix 4: $T_{avg,12}$ = 421.7171 K so γ_{12} = 1.393263 and $T_{avg,34}$ = 931.2269 K so γ_{34} = 1.341502, then T_2 = 539.5452 K and T_4 = 706.7803 K.

Heat transfer during the cycle can be found, using air properties at respective average process temperature interpolated from Appendix 4: $T_{avg,23}$ = 869.7726 K so c_{p23} = 1.114350 kJ/kgK and $T_{avg,41}$ = 503.3902 K so c_{p41} = 1.029746 kJ/kgK,

$$\dot{Q}_H = \dot{m}c_{p,23}(T_3 - T_2) = 4\text{ kg/s}\times 1.114350\text{ kJ/kgK}\times(1200\text{ K} - 539.5452\text{ K}),$$

$$\dot{Q}_H = 2943.911\,\text{kW},$$

$$\dot{Q}_C = \dot{m}c_{p,41}(T_4 - T_1) = 4\text{ kg/s}\times 1.029746\text{ kJ/kgK}\times(706.7803\text{ K} - 300\text{ K}),$$

$$\dot{Q}_C = 1675.522\text{ kW}.$$

The thermal efficiency and net power output of the cycle can then be found:

$$\eta_{th} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{1675.522\text{ kW}}{2943.911\text{ kW}} = 0.4308517,$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_C = 2943.911\text{ kW} - 1675.522\text{ kW} = 1268.389\text{ kW}$$

The cycle has a power output of 1268.4 kW and an efficiency of 43.1%.