

## Solutions for selected exercises (Lecture 6a and 6b)

### Exercise 7.1

Similar to example 7.1 in lecture 6b, the saturation pressure  $P_{\text{sat},2}$  needs to be determined, here for  $T_2 = 205^\circ\text{C}$ . By using the Clausius-Clapeyron equation

$$P_{\text{sat}} = C \exp\left(\frac{-h_{fg}}{RT_{\text{sat}}}\right),$$

we are able to

- first solve for the constant  $C$  in state 1 with  $T_1 = 200^\circ\text{C}$  and
- then solve for the saturation pressure in state 2 with  $T_2 = 205^\circ\text{C}$ .

Therefore, the average enthalpy of vaporization is estimated by means of the saturated water property table:

$$h_{fg} = \frac{(h_{g,1} - h_{f,1}) + (h_{g,2} - h_{f,2})}{2} = \frac{(2793.2 \text{ kJ/kg} - 852.45 \text{ kJ/kg}) + (2796.0 \text{ kJ/kg} - 875.04 \text{ kJ/kg})}{2},$$

$$h_{fg} = 1930.855 \text{ kJ/kg},$$

This allows us to calculate  $C$  and  $P_{\text{sat},2}$ :

$$C = \frac{P_{\text{sat},1}}{\exp\left(\frac{-h_{fg}}{RT_{\text{sat},1}}\right)} = \frac{1553.8 \text{ kPa}}{\exp\left(\frac{-1930.855 \text{ kJ/kg}}{0.4615 \text{ kJ/kgK} \times 473.15 \text{ K}}\right)} = 1.075672 \times 10^7 \text{ kPa}.$$

$$P_{\text{sat},2} = 1.075672 \times 10^7 \text{ kPa} \times \exp\left(\frac{-1930.855 \text{ kJ/kg}}{0.4615 \text{ kJ/kgK} \times 478.15 \text{ K}}\right) = 1704.327 \text{ kPa}.$$

The value of 1723.0 kPa from Appendix 8a is close to our estimated saturation pressure of 1704.3 kPa. Please note that  $R$  in  $\text{kJ}/(\text{kg}\cdot\text{K})$  is used instead of  $R$  in  $\text{kJ}/(\text{kmol}\cdot\text{K})$ .

### Exercise 7.4

For this exercise, the saturation pressure of water vapour in equilibrium with ice at -20 °C needs to be determined.

Since temperature, pressure and latent heat of sublimation is given for water at the triple point, the Clausius-Clapeyron equation can be used to

- first find the constant  $C$  at the triple point and
- then solve for the saturation pressure at  $T_2 = -20\text{ °C}$ .

The calculation is similar to the one in exercise 7.1 and leads to the following results:

$$P_{sat} = C \exp\left(\frac{-h_{sg}}{RT_{sat}}\right),$$
$$C = \frac{P_{sat}}{\exp\left(\frac{-h_{sg}}{RT_{sat}}\right)} = \frac{0.6113 \text{ kPa}}{\exp\left(\frac{-2834.8 \text{ kJ/kg}}{0.4615 \text{ kJ/kgK} \times 273.16 \text{ K}}\right)} = 3.5668 \times 10^9 \text{ kPa},$$
$$P_{sat} = 3.5668 \times 10^9 \text{ kPa} \times \exp\left(\frac{-2834.8 \text{ kJ/kg}}{0.4615 \text{ kJ/kgK} \times 253.15 \text{ K}}\right) = 0.10335 \text{ kPa}.$$

Again, please note that  $R$  in  $\text{kJ}/(\text{kg}\cdot\text{K})$  is used instead of  $R$  in  $\text{kJ}/(\text{kmol}\cdot\text{K})$ .

### Exercise 7.8

The goal is to determine the minimum amount of time required to evaporate five kilograms of water which are boiling in an open pot placed over a 500W heater.

Keep in mind that water boils at different temperatures based on the atmospheric pressure. When assuming the atmospheric pressure to be 100 kPa, the mass flow rate of evaporated water can be estimated to:

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{\dot{Q}}{h_g - h_f} = \frac{0.5 \text{ kW}}{2258.04 \text{ kJ/kg}} = 2.2143 \times 10^{-4} \text{ kg/s},$$

By considering the initial mass of 5 kilograms, the total duration is calculated as:

$$\Delta t = \frac{m}{\dot{m}} = \frac{5 \text{ kg}}{2.2143 \times 10^{-4} \text{ kg/s}} = 22580 \text{ s} = 6.2722 \text{ h}.$$

### Exercise 7.11

In this exercise, you need to identify the phase and specific enthalpy of water at temperature of 300 °C and pressure of 300 kPa.

There are two ways to identify the phase:

1. Determine the saturation pressure of water at 300 °C and compare to 300 kPa.
2. Determine the saturation temperature of water at 300 kPa and compare to 300 °C.

In both ways, superheated steam can be identified since the saturation pressure of water at 300 °C is 8.581 MPa ( $\gg$  300 kPa) and the saturation temperature of water at 300 kPa is 133.52 °C ( $\ll$  300 °C).

The superheated steam tables in Appendix 8c give the enthalpy  $h = 3069.3$  kJ/kg.

### Exercise 7.26

2 L of saturated liquid at 100 kPa in a cylinder with a frictionless piston is heated until its quality is 80%. How much heat is added?

The mass can be found by comparing the volume with the specific volume of liquid saturated water at 100 kPa (Appendix A8b).

$$m = \frac{V}{v} = \frac{2 \times 10^{-3} \text{ m}^3}{0.001043 \text{ m}^3/\text{kg}} = 1.917546 \text{ kg}.$$

By using the final quality and specific enthalpy of saturated water at 100 kPa for liquid and gas, the specific enthalpy in the final state is computed.

$$h_2 = h_f + x_2(h_g - h_f) = 417.46 \text{ kJ/kg} + 0.8 \times (2675.5 \text{ kJ/kg} - 417.46 \text{ kJ/kg}),$$
$$h_2 = 2223.892 \text{ kJ/kg}.$$

The product of mass and specific enthalpy difference between initial and final state gives the required heat.

$$Q = m(h_2 - h_1) = 1.917546 \text{ kg} \times (2223.892 \text{ kJ/kg} - 417.46 \text{ kJ/kg}) = 3463.916 \text{ kJ}.$$

### Exercise 7.46

Superheated steam at 0.5 MPa and 600 °C enters an insulated chamber with 2.5 kg/s. Liquid water at 5 MPa and 20 °C enters with unknown mass flow but condenses the steam in order to have saturated liquid at 0.5 MPa exiting the chamber.

In conclusion, the insulated chamber is characterized by two incoming streams and one exiting stream. The following energy balance for an open system shows that the specific heat capacities of the various substances at the given pressures are needed to calculate the missing mass flow of liquid water.

$$\dot{m}_w h_{1,w} + \dot{m}_s h_{1,s} = (\dot{m}_w + \dot{m}_s) h_2$$

Determining the  $h_{1,w}$  (Appendix A8d),  $h_{1,s}$  (Appendix A8c) and  $h_2$  (Appendix A8b) allows the calculation of liquid water mass flow.

$$\dot{m}_w = \frac{\dot{m}_s (h_2 - h_{1,s})}{h_{1,w} - h_2} = \frac{2.5 \text{ kg/s} \times (640.23 \text{ kJ/kg} - 3701.7 \text{ kJ/kg})}{88.7 \text{ kJ/kg} - 640.23 \text{ kJ/kg}} = 13.877 \text{ kg/s.}$$

### Exercise 7.78

Use both the ideal gas equation and van der Waals equation to determine the pressure of 2 kg Oxygen occupying a volume of 0.1 m<sup>3</sup> at a temperature of 250 K.

First, the amount of oxygen in moles is calculated.

$$n = \frac{2 \text{ kg}}{31.999 \text{ kg/kmol}} = 0.062502 \text{ kmol,}$$

The specific molar volume is calculated by dividing the given volume by the amount of oxygen in moles.

$$\bar{v} = \frac{v}{n} = \frac{0.1 \text{ m}^3}{0.062502 \text{ kmol}} = 1.5999 \text{ m}^3/\text{kmol,}$$

By using the ideal gas law, the pressure  $P_A$  can be calculated.

$$P_A = \frac{R_u T}{\bar{v}} = \frac{8.314 \text{ kJ/kmolK} \times 250 \text{ K}}{1.5999 \text{ m}^3/\text{kmol}} = 1.2991 \text{ MPa.}$$

Table 7.5 within chapter 7 helps to calculate the the pressure  $P_B$  with the van der Waals equation.

$$\left(P + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = R_u T,$$

$$P = \frac{R_u T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \frac{8.314 \text{ kJ/kmolK} \times 250 \text{ K}}{1.5999 \text{ m}^3/\text{kmol} - 0.0319 \text{ m}^3/\text{kmol}} - \frac{136.9 \text{ kPa m}^6/\text{kmol}^2}{(1.5999 \text{ m}^3/\text{kmol})^2},$$

$$P_B = 1.2721 \text{ MPa}.$$

For this state of oxygen, the ideal gas equation approximation is close to the value calculated with the van der Waals equation. Please note that, different to exercises 7.1 and 7.4,  $R_u$  in kJ/(kmol\*K) is used.

### Exercise 7.79

Use both the ideal gas equation and the compressibility charts to determine the density of carbon dioxide at a pressure of 6.5 MPa and temperature of 30 °C.

First, the ideal gas equation is used to compute the density (inverse of the specific volume) with  $R$  from Appendix A1.

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{6.5 \times 10^3 \text{ kPa}}{0.1889 \text{ kJ/kgK} \times 303.15 \text{ K}} = 113.507 \text{ kg/m}^3.$$

The reduced temperature  $T_R$  and reduced pressure  $P_R$  are calculated with critical pressure  $P_c$  and critical temperature  $T_c$  from Appendix A6.

$$T_R = \frac{T}{T_c} = \frac{(30 + 273.15) \text{ K}}{304.2 \text{ K}} = 0.99655 = 1.00,$$

$$P_R = \frac{P}{P_c} = \frac{6.5 \text{ MPa}}{7.39 \text{ MPa}} = 0.87957 = 0.88,$$

According to Appendix 10a, the compressibility factor is  $Z = 0.55$ .

By using the definition of the compressibility factor, we can solve for the specific volume.

$$\rho = \frac{P}{ZRT} = \frac{6.5 \times 10^3 \text{ kPa}}{0.55 \times 0.1889 \text{ kJ/kgK} \times 303.15 \text{ K}} = 206.377 \text{ kg/m}^3.$$

For carbon dioxide in the given state, the density calculated by the ideal gas equation significantly differs to the more accurate value calculated from the compressibility charts.