47201 Engineering thermodynamics

Lecture 7b: Vapour power cycles (Ch. 9.1-3)

Practical issues with the Carnot cycle

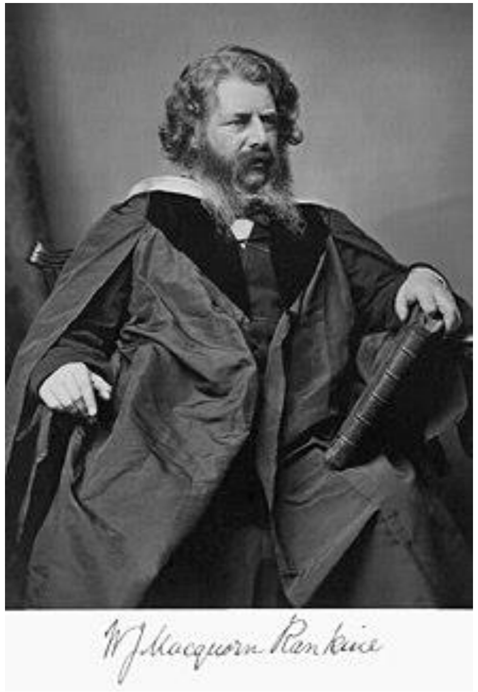
- Compressors, especially, do not like two phase fluids. Water droplets can damage blades on turbomachinery and can cause difficulties in designing an efficienct compressor

- Isothermal heat exchange is efficient but not so fast

- A more realistic but still idealized power cycle is the ideal Rankine cycle

William John Macquorn Rankine

- July 1820 – December 1876
- Professor at Glasgow University
- One of the founding fathers of modern thermodynamics
- Proposed the absolute temperature scale now called the Rankine scale.
- Developed a model of a thermal power cycle that has been named the Rankine cycle



Basic principles of the Rankine cycle (Chapter 9)

The main purpose of the Rankine cycle is to avoid two phase fluids in the compressor. The main modification to the Carnot cycle is that the working fluid is cooled to a saturated liquid before entering a pump. It is then pumped to the higher pressure, where it is a subcooled liquid.

- Whereas the Carnot cycle refers to specific operating conditions, the Rankine cycle exists in several configurations.

Ideal Rankine cycle overview

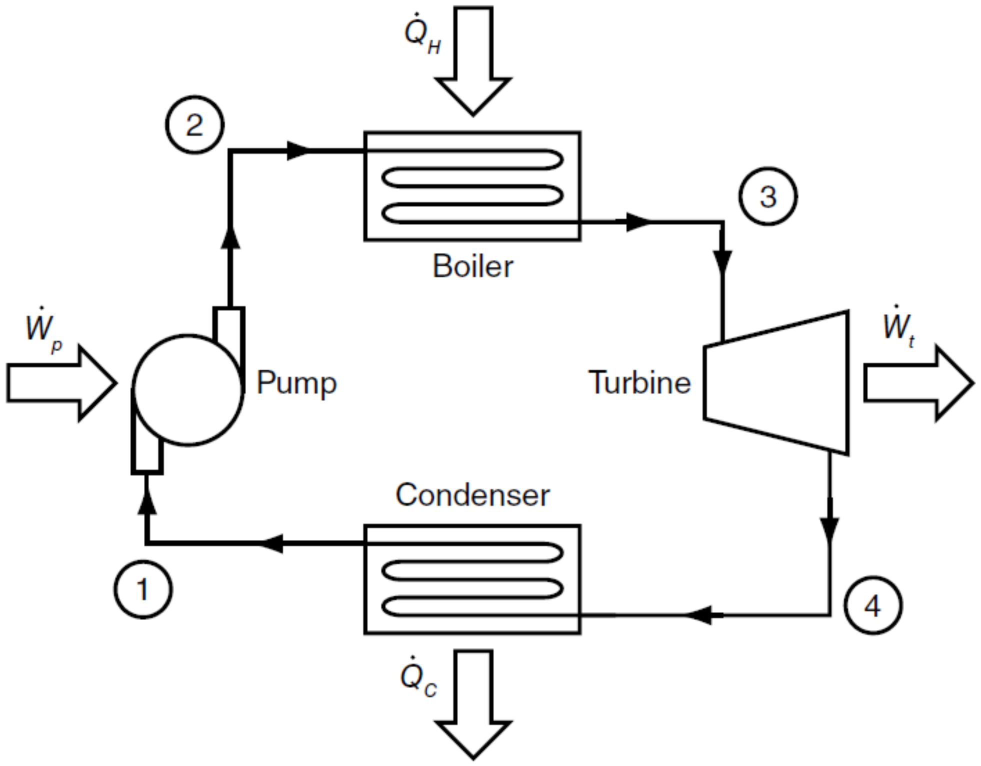
The most basic form is the ideal Rankine cycle, which operates using a two-phase fluid. The cycle consists of the following processes:

1-2 Saturated liquid enters a pump at the lower pressure and is pumped to the high pressure.

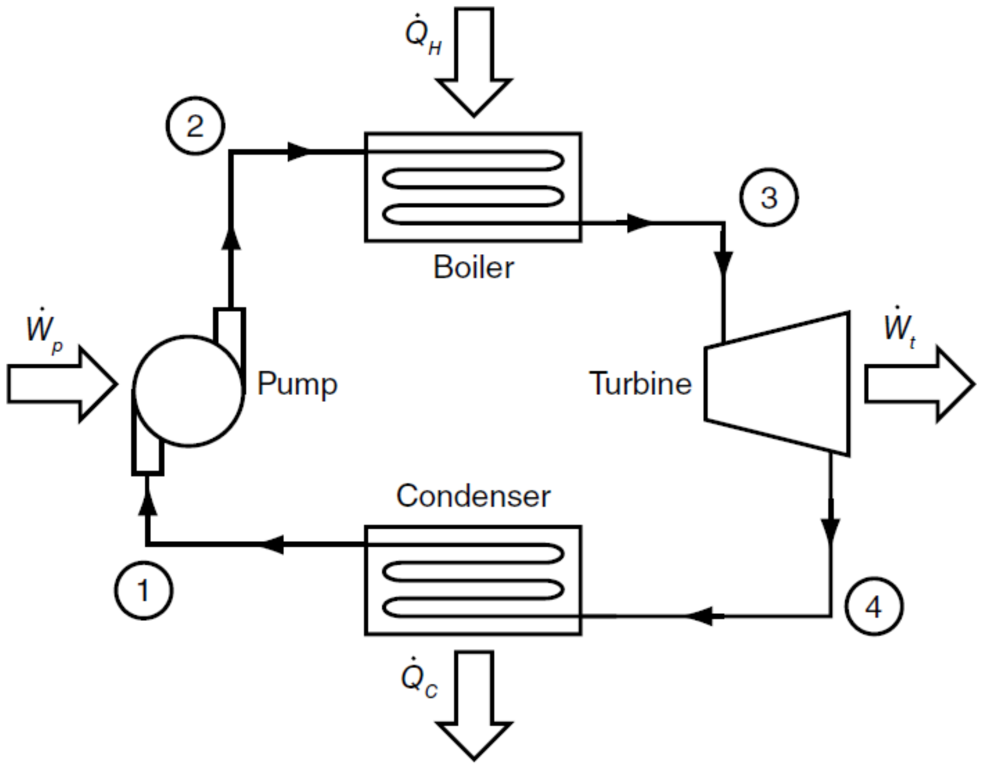
2-3 Heat is added from the hot reservoir until the fluid becomes a saturated vapor.

3-4 The fluid is expanded isentropically to the lower pressure (condensor pressure)

4-1 Heat is rejected to the cold reservoir until it reaches the state of a saturated liquid.



Ideal Rankine cycle T-s diagram



Analyzing the Rankine cycle

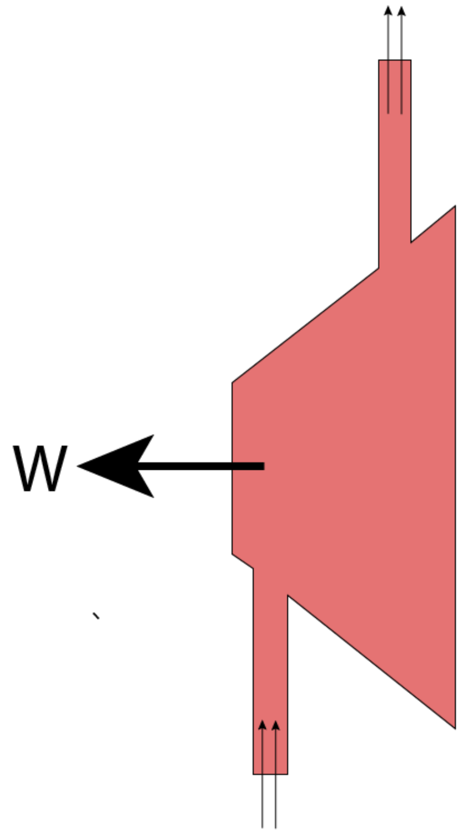
The basic concept is to start from a few known states and move around the cycle component-by-component using techniques that have already been covered.

1. Turbine (Ch 6.13.1)
2. Boiler (Ch 8.3)
3. Pump or compressor (Ch 6.13.3)

Modelling the turbine

Assuming steady state, no changes in kinetic or potential energy and no heat losses, the work output becomes:

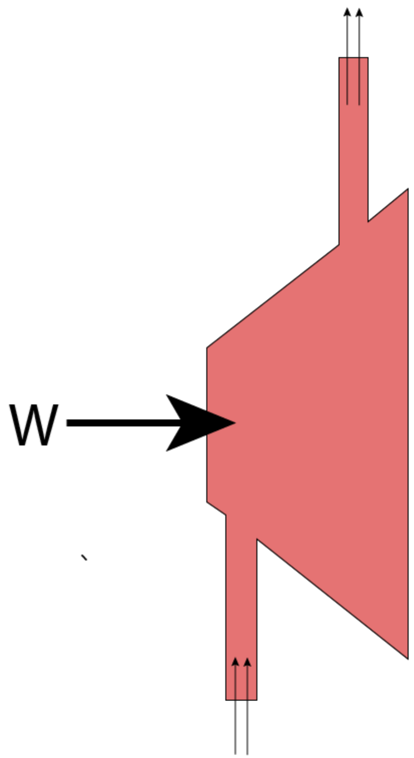
$$\dot{W}_{turb} = \dot{m}(h_i - h_o) \quad \text{Eq. 6.81}$$



Modelling compressors and pumps

Again, we often Assume steady state, no changes in kinetic or potential energy and no heat losses. For a compressor, the work input is:

$$\dot{W}_{comp} = \dot{m}(h_o - h_i) \quad \text{Eq. 6.85}$$



And for a pump with an incompressible fluid the work is:

$$\dot{W}_{pump} = \dot{m}v(P_o - P_i) \quad \text{Eq. 9.2}$$

$$\dot{W}_{pump} = \dot{m}(h_o - h_i)$$

Modelling the heat exchangers (boiler and condenser)

Assuming steady state, no changes in kinetic or potential energy and no heat losses, the heat transfer in the heat exchangers is:

$$\dot{Q}_{HX} = \dot{m}\,(h_o - h_i) \qquad \text{Eq. 9.5}$$

Efficiency of the Rankine cycle

The efficiency in thermodynamics is generally defined as the desired output over the required input.

- In the case of the Rankine cycle, the desired output is power and the required input is heat to the hot reservoir.

$$\eta_{Rankine} = \frac{\dot{W}_{Turb} - \dot{W}_{pump}}{\dot{Q}_H} \qquad \text{Eq. 9.7}$$

A measure of the work required to power the pump compared to that produced by the turbine is the back work ratio (*bwr*)

$$bwr = \frac{\dot{W}_{pump}}{\dot{W}_{Turb}} \qquad \text{Eq. 9.6}$$

Basic steps to analyzing cycles

1. Read the problem carefully to determine as many known states as possible

2. Draw a schematic of the system and label each state with a number (starting point does not matter)

3. Start from known states and perform analysis on the different components in order to determine the state of the working fluid at the other side of that component.

4. Step all the way around the cycle until all the states are known.

5. Calculate the inputs, outputs and overall efficiency

Example 9.1

Problem: Saturated steam at a pressure of 6 MPa enters the turbine of a Rankine cycle and leaves at a condenser pressure of 30 kPa. What is the back work ratio? Find the thermal efficiency of the cycle and compare it to that of a Carnot cycle operating between the same temperatures.

Find: Back work ratio *bwr* of the Rankine cycle, thermal efficiency of the Rankine cycle $\eta_{th,\text{Rankine}}$, thermal efficiency of a Carnot cycle $\eta_{th,\text{Carnot}}$ operating between the same temperatures.

Assume:

1. All components are adiabatic with no significant changes in KE or PE
2. Expansion in the turbine is isentropic

Example 9.1 Solution

Draw the T-s diagram

Let's start at state 3 because it is fully defined and we can use it to find state 4.

- State 3 is saturated vapor, so we can look up the value of enthalpy and entropy directly from Appendix 8b. The enthalpy entering the turbine is $h_3 = h_g = 2784.3\,\frac{kJ}{kg}$ and the entropy is $s_3 = s_g = 5.8892\,\frac{kJ}{kg\ K}$

Example 9.1 Solution continued

The expansion is isentropic so $s_4 = s_3$. We also know that state 4 is within the vapor dome, so

$$x_4 = \frac{s_4 - s_{4,f}}{s_{4,g} - s_{4,f}} = \frac{5.8892\,\frac{kJ}{kg\ K} - 0.9439\,\frac{kJ}{kg\ K}}{7.7686\,\frac{kJ}{kg\ K} - 0.9439\,\frac{kJ}{kg\ K}} = 0.7246$$

Now that we know the quality and pressure, we can find the enthalpy

$$h_4 = h_{4,f} + x_4\,(h_{4,g} - h_{4,f}) = 289.23\,\frac{kJ}{kg} + 0.7246\left(2625.3\,\frac{kJ}{kg} - 289.23\,\frac{kJ}{kg}\right) = 1982.0\,\frac{kJ}{kg}$$

State 1 if fully defined because it is saturated liquid at the condenser pressure

$$h_1 = h_{1,f} = 289.23\,\frac{kJ}{kg}$$

Example 9.1 Solution continued II

Then we can find the state after the pump by applying Eq 9.2

$$h_2 = h_1 + v_1(P_2 - P_1) = 289.23\,\frac{kJ}{kg} + 0.001022\,\frac{m^3}{kg}(6000\;kPa\; - 30\;kPa) = 295.3\,\frac{kJ}{kg}$$

Where v_1 is the specific volume entering the pump.

Now all states are defined and we can evaluate the entire cycle. No mass flow rate is given, but it is equal in each component. We will calculate each energy interaction on a mass specific basis. Let's look at the heat input, work output and pump work input separately.

$$\dot{q}_H = h_3 - h_2 = 2784.3\,\frac{kJ}{kg} - 295.33\,\frac{kJ}{kg} = 2489.0\,\frac{kJ}{kg}$$

Example 9.1 Solution continued III

The work out of the turbine can be calculated as:

$$\dot{w}_{out} = h_3 - h_4 = 2784.3\frac{kJ}{kg} - 1982.0\frac{kJ}{kg} = 802.3\frac{kJ}{kg}$$

And the pump work is

$$\dot{w}_{pump} = h_2 - h_1 = 295.33\frac{kJ}{kg} - 289.23\frac{kJ}{kg} = 6.1\frac{kJ}{kg}$$

The back work ratio is

$$bwr = \frac{\dot{w}_{pump}}{\dot{w}_{out}} = \frac{6.1\frac{kJ}{kg}}{802.3\frac{kJ}{kg}} = 0.00761$$

And the system efficiency is

$$\eta = \frac{\dot{w}_{out}}{\dot{q}_{in}} = \frac{802.3\frac{kJ}{kg} - 6.1\frac{kJ}{kg}}{2489.0\frac{kJ}{kg}} = 0.320$$

Example 9.1 Solution continued IV

For a Carnot cycle operating at the same conditions, we need the temperatures at the hot and cold sides of the cycle. To find those, we look up the saturation temperature as a function of pressure in Appendix 8b.

Using those values, the efficiency would be:

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{342.25\ K}{548.79\ K} = 0.374$$

So the efficiecy of the ideal Rankine cycle is fairly close to the Carnot cycle

Rankine cycle with superheat

The ideal Rankine cycle can be modified by superheating the fluid in the boiler, which can increase the efficiency and power output. Isentropic efficiencies of the components may be included to model more realistic conditions. The cycle consists of the following processes:

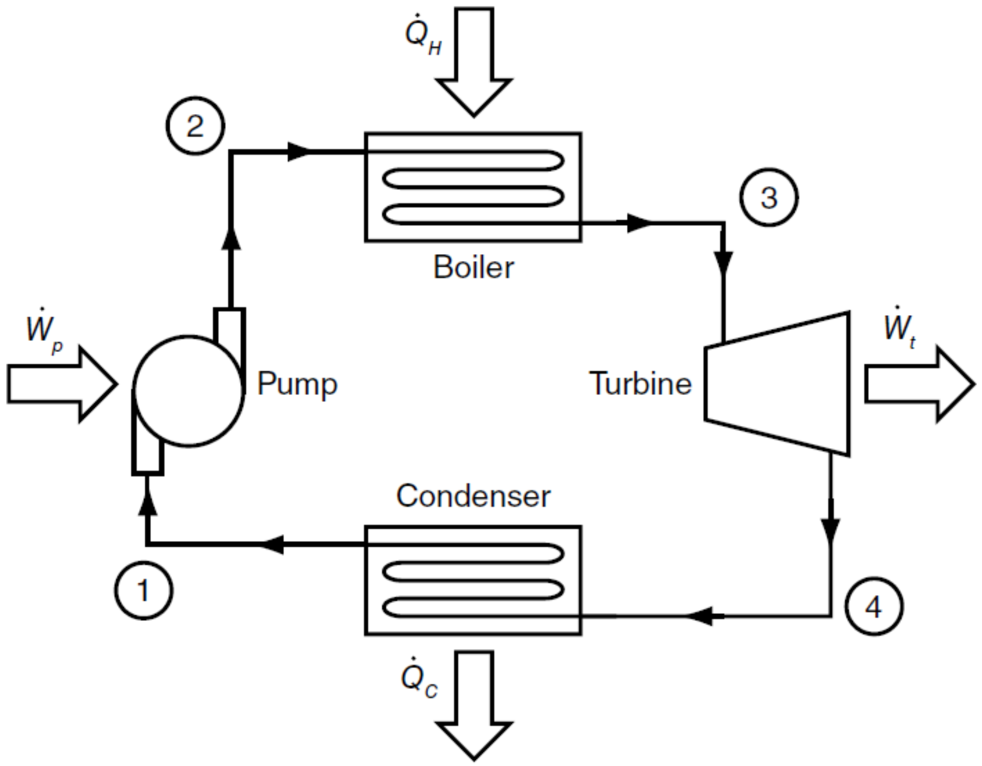
1-2 Saturated liquid enters a pump at the lower pressure and is pumped to the high pressure.

2-3 Heat is added from the hot reservoir until it reaches a desired temperature.

3-4 The fluid is expanded to the lower pressure (condensor pressure)

4-1 Heat is rejected to the cold reservoir until it reaches the state of a saturated liquid.

Rankine cycle with superheat T-s diagram



Isentropic efficiency of components

- Entropy can be used to find the most efficient way that many processes can occur. One method that will be used quite often is the concept of isentropic efficiency. For a turbine the form is:

$$\eta_t = \frac{h_i - h_o}{h_i - h_{o,s}} \qquad \text{Eq (9.10)}$$

- One interperatation of the 2nd law of thermodynamics is that entropy always increases with any process so entropy is always generated in turbines, compressors, heat exchangers, etc.

Example 9.2

Problem: Superheated steam at a pressure of 6 MPa and temperature of 400 ° C enters the turbine of a Rankine cycle and leaves at a condenser pressure of 30 kPa. Find the thermal efficiency of the cycle if (a) the turbine is isentropic and (b) the turbine isentropic efficiency is 92%.

Find: Thermal efficiency of the Rankine cycle for (a) an isentropic turbine $\eta R,s$ (b) a non-isentropic turbine $\eta_{R,a}$.

Assume:

1. All components are adiabatic with no significant changes in KE or PE
2. Expansion in the turbine is isentropic for part (a)