Solutions for selected exercises:
Chapter 4 in "Energy, Entropy and Engines"

Exercise 4.4
A 20 cm diameter pulley is driven by a belt that exerts a net tangential force of 3 kN. What is the power transmitted by the pulley when it is turning at 500 RPM?

The torque of the pullley can be found in Eq. 4.36:

$$\tau = Fr = 3000 \text{ N} \times \frac{0.2 \text{ m}}{2} = 300 \text{ Nm}.$$

By deploying the formula of the sha power corresponding to rota on speed, we get the result as:

$$\dot{W} = 2\pi\dot{n}\tau = 2\pi \times 500 \times (1/\text{min}) \times (1 \text{ min}/60 \text{ s}) \times 0.3 \text{ kNm} = 15.708 \text{ kW}.$$

No ce the unit of the rota on speed is [1/min].

Exercise 4.5
The piston in a cylinder filled with oil at a pressure of 0.8 MPa is advanced 10 cm. What is the work done if the cross-sectional area of the cylinder is 20 cm² ?

The force to the piston is F = P*A. Therefore, the work done to the piston is:

$$W = F\Delta x = PA\Delta x$$
$$W = 0.8 \times 10^6 \text{ Pa} \times 20 \times 10^{-4} \text{m}^2 \times 0.1 \text{ m} = 160 \text{ J}.$$

Exercise 4.7
A cylinder contains water filled on top of a piston to a depth of 1 m. As the piston is raised the water drains out of an outlet at the top of the cylinder. Find the work required to empty out all the water if the cross-sectional area of the piston is 0.2 m². Assume the density of water is 1000 kg / m³.

This is a ques on about poten al energy. When the piston goes up, the water will leave the container. Thus, the poten al energy of the water in the container is reducing. The force on the piston is also reducing in this process. The force on the piston is:

$$F = m_w g = \rho V g = \rho A h g.$$

The work can be calculated via integra on over the en re depth

$$W = \int F \,\mathrm{d}h = \int Ahg \,\mathrm{d}h$$

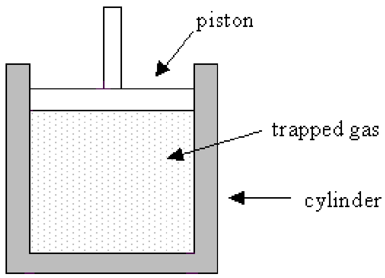
$$W = \int_0^h \rho Ahg dh = \frac{1}{2}\rho A g h^2 = \frac{1}{2} \times 10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 0.2 \text{ m}^2 \times (1 \text{ m})^2,$$

$$W = 981 \text{ J}.$$

Exercise 4.8 - See pdf file "Lecture 2 problem 8.4"

Exercise 4.10

A 2 kg mass of air in a cylinder initially at a pressure of 100 kPa and temperature of 20° C is compressed by a piston to a pressure of 300 kPa whole being kept at constant temperature. Find the work done.



A piston compresses air in cylinder.

The work done can be calculated with

$$W = -\int_{V_1}^{V_2} PdV = -\int_{V_1}^{V_2} \frac{mRT}{V} dV = -mRT \int_{V_1}^{V_2} \frac{dV}{V} = mRT \ln \frac{V_1}{V_2}.$$

Since mass of air m = 2 kg, gas constant of air R = 0.2870 kJ/(kgK) and $T = T_1 = T_2 = 20$ °C are known, the only missing information is V_1/V_2. The knowledge of $P_1 = 100$ kPa and $P_2 = 300$kPa makes it to possible to use the ideal gas equation to determine V_1/V_2 with

$$P_1V_1 = P_2V_2,$$
$$\frac{V_1}{V_2} = \frac{P_2}{P_1}.$$

By using the known properties, the following work done results

$$W = 2\text{ kg} \times 0.2870\text{ kJ/kgK} \times 293.15\text{ K} \times \ln \frac{300\text{ kPa}}{100\text{ kPa}} = 184.861\text{ kJ}.$$

Exercise 4.13

Argon at 150 kPa, 320 K and 0.1 m³ expands in a polytropic process for which n = 1.667 to a pressure of 100 kPa. What is the work done?

To calculate the work done during a polytropic process for argon, an integration of the boundary work equation (done in chapter 4) is needed. Therefore, the final volume of the gas needs to be determined with the polytropic process equation

$$P_1V_1^n = P_2V_2^n$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0.1\text{ m}^3 \left(\frac{150\text{ kPa}}{100\text{ kPa}}\right)^{0.6} = 0.12754\text{ m}^3.$$

Since now V_2 is known, the work done can be calculated with

$$W = \frac{P_2V_2 - P_1V_1}{n-1} = \frac{100\text{ kPa} \times 0.12754\text{ m}^3 - 150\text{ kPa} \times 0.1\text{ m}^3}{1.667 - 1} = -3.3688\text{ kJ}.$$

Based on standard conventions, the work done is negative since the gas does work on the surrounding.

4.14)

Air at 800 K and 1 MPa is expanded in a polytropic process for which $PV^{1.6}$ = constant until the pressure reaches 0.1 MPa. Find the final gas temperature and the work done per unit mass of air.

SOLUTION:

Based on the description of the exercise, it can be assumed that the mass of the system (Air volume) remains constant during the polytropic process and, in addition, the air behaves as an ideal gas.

Gas constant of air R = 0.2870 kJ/kgK (Found in Appendix, A1).

Using the ideal gas law for the initial and the final state of the system, we write:

Initial state: $P_1V_1 = \frac{m}{Ru}\, T_1$ **(1)**

Final state: $P_2V_2 = \frac{m}{Ru}\, T_2$ **(2)**

m = Constant and combining (1) and (2), we write:

- $\frac{P_1V_1}{T_1} = m\,R\,,\ \frac{P_2V_2}{T_2} = m\,R \quad \leftrightarrow \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \leftrightarrow \quad \frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1}$ **(3)**

Also, $PV^{1.6}$ is constant, hence we can write:

- $P_1V_1^{1.6} = P_2V_2^{1.6} \leftrightarrow \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.6}}$ **(4)**

Replacing **(4)** to **(3)**, we write:

- $T_2 = T_1\frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{\frac{1}{1.6}} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{1.6-1}{1.6}} = 800\,K\left(\frac{0.1\,MPa}{1\,MPa}\right)^{\frac{0.6}{1.6}} = \mathbf{337.357\,K}$

The work function of a polytropic process can be obtained by integrating the boundary work equation (eq. 4.30, page 97 of the book).

- $W_{12} = \frac{P_2V_2 - P_1V_1}{n-1} \xLeftrightarrow{(Using\ Ideal\ gas\ law)} W_{12} = \frac{mRT_2 - mRT_1}{n-1} = \frac{mR(T_2 - T_1)}{n-1}$

The work per unit mass that the system does on the surroundings is calculated as follows:

$$w_{12} = \frac{W_{12}}{m} = \frac{R(T_2 - T_1)}{n-1} = \frac{0.2870\ kJ/kgK(337.357K - 800\ K)}{1.6 - 1} = \mathbf{-221.298\ kJ/kg}$$

Final solutions:

1. **Air temperature = 337 K**
2. **Work per unit mass = 221.3 kJ/kg**

4.15)

A cylinder containing 0.05 m^3 of carbon dioxide at 200 kPa and 100°C is expanded in a polytropic process to 100 kPa and 20°C. Determine the work done.

SOLUTION:

Based on the description of the exercise, it can be assumed that the mass of the system (CO_2 mass) remains constant during the polytropic process and, in addition, CO_2 behaves as an ideal gas.

Gas constant of carbon dioxide, R = 0.1889 kJ/kgK (Found in Appendix, A1).

The work function of a polytropic process can be obtained by integrating the boundary work equation (eq. 4.30, page 97 of the book).

- $W_{12} = \frac{P_2 V_2 - P_1 V_1}{n-1}$ **(1)**

The system is experiencing a polytropic process, hence, we can use the main polytropic equation to describe the initial and the final state of the system.

- $P_1 V_1^n = P_2 V_2^n \leftrightarrow \frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^n \leftrightarrow ln\left(\frac{P_1}{P_2}\right) = ln\left(\frac{V_1}{V_2}\right)^n \leftrightarrow ln\left(\frac{P_1}{P_2}\right) = n\ ln\frac{V_1}{V_2} \leftrightarrow$

  $n = \frac{ln\,(P_1/P_2)}{ln\,(V_2/V_1)}$ **(2)**

Applying the ideal gas law in the initial state of the system, the mass of CO_2 can be calculated:

- $m_{CO2} = \frac{P_1 V_1}{R\, T_1} = \frac{200\ kPa\ x\ 0.05\ m^3}{0.1889\ kJ/kgK\ x\ 373.15\ K}$ = 0.14187 kg

Using the value of m_{CO2} in the ideal gas law for the final state of the system, the volume of CO_2 can also be calculated:

- $V_2 = \frac{mRT_2}{P_2} = \frac{0.14187\ kg\ x\ 0.1889\ kJ/kgK\ x\ 293.15\ K}{100\ kPa} = 0.078562\ m^3$

Using the value of V_2 to equation (2), we can write:

- $n = \frac{ln\ (P_1/P_{2)}}{ln\ (V_2/V_1)} = \frac{ln\ (200\ kPa/100\ kPa}{ln\ (0.0785 m^3/0.05 m^3)} = 1.5367$

All the values of the parameters of equation (1) are known, hence, we write:

- $W_{12} = \frac{P_2 V_2 - P_1 V_1}{n-1} = \frac{100\ kPa\ x\ 0.0785\ m^3 - 200\ kPa\ x\ 0.05\ m^3}{1.5367-1} = \boldsymbol{-3.9944\ kJ}$