Engineering Thermodynamics - Final project

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Question 1

Instead of developing a straightforward script as initially required by Question 1, our team created an comprehensive computational solver for the entire inverse Rankine cycle. This solver accepts any combination of known thermodynamic state variables (such as temperatures, pressures, specific entropies, and enthalpies, for different points in the cycle, i.e., T_2, T_4, $\dot{m}$) as inputs. The solver iteratively applies all known thermodynamic relations defined within the code, repeatedly cycling through these relationships to progressively compute all accessible unknown variables until the system is completely resolved. Designing this complete solver proved particularly useful in addressing the subsequent questions in the assignment.

The solver was validated against the textbook problem 9.5, yielding the following results:

- Compressor input power: $\dot{W}_c = 2.695$ kW (validated with 9.5's solution)
- Heating power: $\dot{Q}H = 17.61$ kW
- Heating coefficient of performance: $\mathrm{COP}_{\mathrm{hp}} = 6.533$

Additionally, the solver enabled plotting of the cycle on the T-s diagram, illustrating the states (1, 2, 3, 3b, and 4), interconnected through line segments. The vapor dome of R-134a refrigerant was also included in the plot using the data from appendix 9a. This T-s diagram for problem 9.5 is presented in Figure 1.

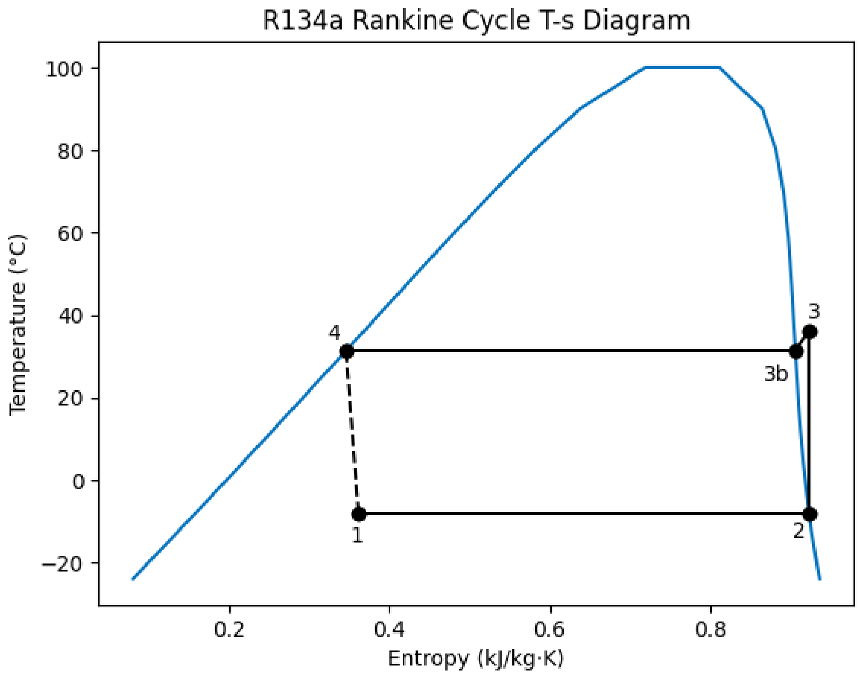


Figure 1: T-s Diagram of the R134a Reverse Rankine Cycle Heat Pump with State Points

Question 2

In this question, we investigate how the heating coefficient of performance (COP) of the heat pump varies with the evaporator temperature T_2, for three fixed values of the condenser temperature T_4. Specifically, the heating COP is plotted as a function of T_2 over the range $[-10°\mathrm{C}, 15°\mathrm{C}]$ for $T_4 \in 25°\mathrm{C}, 30°\mathrm{C}, 35°\mathrm{C}$.

The resulting plot is shown in Figure 2. For all three T_4 values, the heating COP increases monotonically with increasing T_2. In each case, the highest COP is achieved at the upper bound of the considered T_2 range, that is, at $T_2 = 15°\mathrm{C}$.

The maximum COP values observed for each T_4 are:

- For $T_4 = 25°\mathrm{C}$: $\mathrm{COP} = 28.3$
- For $T_4 = 30°\mathrm{C}$: $\mathrm{COP} = 18.7$
- For $T_4 = 35°\mathrm{C}$: $\mathrm{COP} = 13.8$

As expected, the heating COP is higher when the temperature difference between T_2 and T_4 is smaller. This behavior reflects the lower amount of work required by the compressor when the temperature delta is reduced.

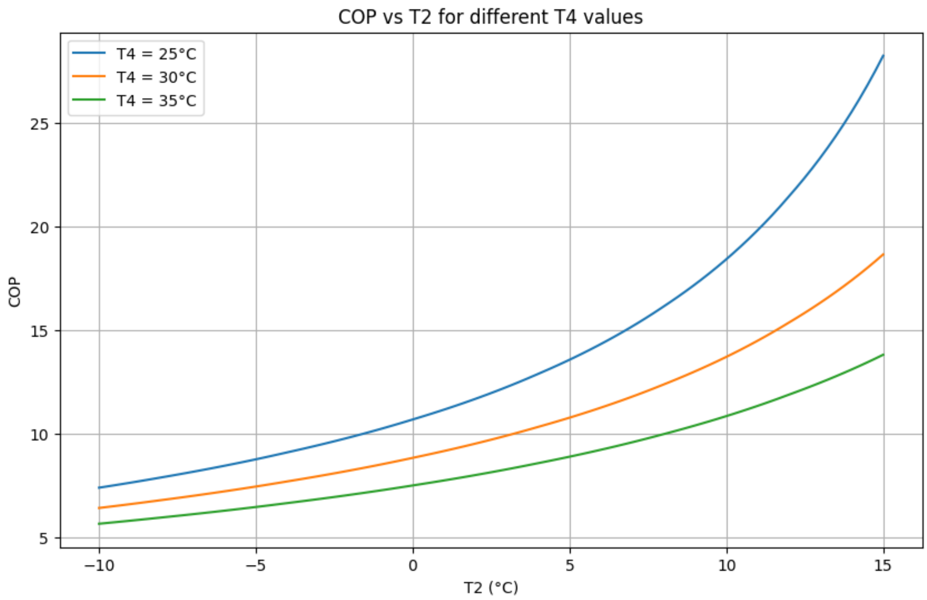


Figure 2: Heating COP as a function of T_2 for various values of T_4

Question 3

In reality the flow rate is limited by the time it takes the working fluid to exchange enough heat with the heat reservoir to complete a phase transition. We would like to estimate this maximum flow rate $\dot{m}^{\text{Max}}$. Before doing so we need to calculate the thermal conductance between the heat exchangers and the surrounding air. We consider a simple serpentine heat exchanger with length $L = 1$m, inner radius $r_1 = 3$mm and $r_2 = 5$mm. The working fluid is R-134a and the tube is made of copper, so $k = 398$ W/mK. The thermal conductance of a hollow cylinder through its lateral surface is :

$$G = \frac{2\pi k L}{\ln(r_2/r_1)} \tag{1}$$

We plug in the known values,

$$G = \frac{2\pi \cdot 398\,\text{W/mK} \cdot 1\,\text{m}}{\ln(5\,\text{mm}/3\,\text{mm})} = 4895.42\ \text{W/K} \tag{2}$$

Question 4

Now that we know the conductance of the evaporator and condenser we can calculate the maximum flow rate. We introduce in this question the internal temperature T_H and external temperature T_C of the house. We start by focusing on the evaporator, i.e process $1 \to 2$. The working fluid inside the evaporator has a fixed temperature T_2 and is surrounded by air at the temperature T_C. The total heat flow rate between the fluid and the air around it is given by

$$\dot{Q}_C = G \cdot \Delta T = \dot{m} \cdot q_C \tag{3}$$

where q_C is the amount of heat (in kJ/kg) that the fluid must receive in order to complete the phase transition. We can therefore isolate $\dot{m}$ to get an expression for the maximum flow rate on the cold side $\dot{m}_C^{\text{Max}}$.

$$\dot{m}_C^{\text{Max}} = \frac{G\Delta T}{q_C} = \frac{G \cdot |T_C - T_2|}{q_C} \tag{4}$$

where $q_C = h_2 - h_1$

We use the same reasoning to calculate the maximum flow rate of the condenser $\dot{m}_H^{\text{Max}}$, i.e process $3 \rightarrow 4$. We get the following expression :

$$\dot{m}_H^{\text{Max}} = \frac{G\Delta T}{q_H} = \frac{G \cdot |T_H - T_4|}{q_H} \tag{5}$$

where $q_H = h_3 - h_4$

The overall maximum value of mass flow rate, $\dot{m}^{\text{Max}}$, will be the minimum between these two values. We now calculate these two values using the values $T_H = 22°C, T_C = 10°C, T_2 = 5°C$ and $T_4 = 35°C$. The code we created extracts the enthalpy values necessary for each state and calculate q_C and q_H. We get $q_C \approx 151$ kJ/kg and $q_H \approx 171$ kJ/kg. We then plug in all the values into equations 4 and 5 and end up with :

$$\dot{m}_C^{\text{Max}} = \frac{4.895\,\text{kW/K} \cdot |10°C - 5°C|}{151\,\text{kJ/kg}} \approx 0.16 \text{ kg/s} \tag{6}$$

and

$$\dot{m}_H^{\text{Max}} = \frac{4.895\,\text{kW/K} \cdot |22°C - 35°C|}{171\,\text{kJ/kg}} \approx 0.37 \text{ kg/s} \tag{7}$$

Therefore, for these temperatures, the maximum mass flow rate is 0.16 kg/s.

We now want to plot the maximum heating power $\dot{Q}_H$ as a function of $T_2 \in [-10°C, 15°C]$ for $T_4 = \{25°C, 30°C, 35°C\}$. The code will compute the maximum mass flow rate for each temperature and extract the values q_H and q_C in order to calculate $\dot{Q}_H$. The result is shown below.

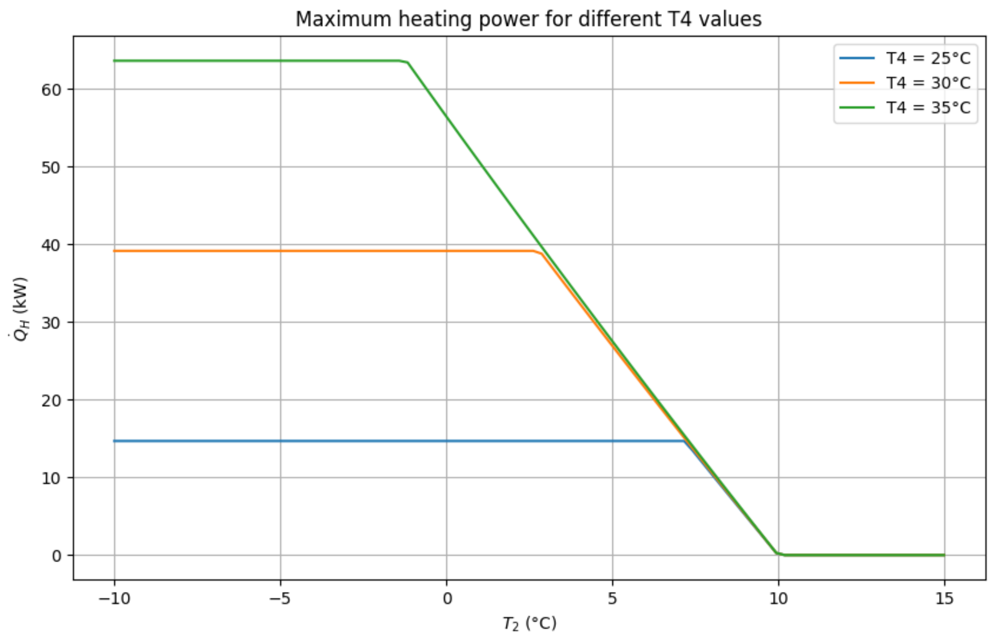


Figure 3: Maximum heating power as a function of T_2 for different values of T_4

As the temperature T_2 increases the maximum heating power starts to decrease. As $T_2 \to T_C$ the heating power decreases to 0 and flattens. This is because when $T_2 = T_C$ the working fluid is hotter than the cold reservoir and heat flows the wrong way (into the cold reservoir), which violates the operating principle of a heat pump.

Question 5

To calculate the heat per unit mass (q_H) of the refrigerant added to the hot heat reservoir, we can use the relation between the enthalpies of h_3 and h_4, which correlates to $q_H = h_3 - h_4 = 269.29\ \mathrm{kJ/kg} - 98.78\ \mathrm{kJ/kg}$, getting a value of $q_H = 170.75\ \mathrm{kJ/kg}$. Now, we can move on to compute the mass flow rate for the given heating power $\dot{Q}_H = 8.18\ \mathrm{kW}$. We know that

$$\dot{m}_H = \frac{\dot{Q}_H}{q_H} = \frac{8.18\ \mathrm{kW}}{170.51\ \mathrm{kJ/kg}} = 0.0478\ \mathrm{kg/s} \tag{8}$$

The value obtained in this question is smaller than the one above $\dot{m}_H^{max} = 0.37\ \mathrm{kg/s}$.

Question 6

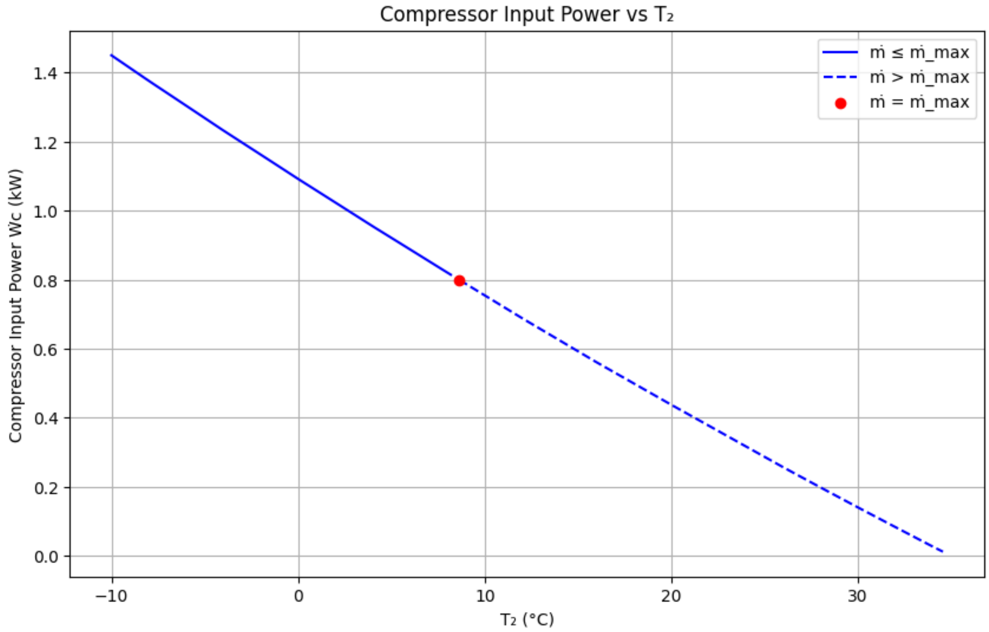


Figure 4: Compressor Input Power $\dot{W}_c$ vs. Evaporator Outlet Temperature T_2 for $\dot{Q}_H = 8.18\,\text{kW}$ and $T_4 = 35°\text{C}$

Figure 4 shows the compressor input power $\dot{W}_c$ as a function of the evaporator outlet temperature T_2, over the range $[-10°\text{C}, 35°\text{C}]$, with the condenser inlet temperature held constant at $T_4 = 35°\text{C}$. The compressor power is computed to maintain a constant heat delivery rate of $\dot{Q}_H = 8.18\,\text{kW}$, which in turn determines the required mass flow rate $\dot{m}$ through the relation $\dot{m} = \dot{Q}_H/q_H$, where q_H is the specific enthalpy gain at each T_2. Because q_H varies with T_2, the mass flow rate $\dot{m}$ also varies as a function of T_2. The maximum allowable mass flow rate, denoted $\dot{m}_{\text{max}}$, is thus a threshold that is evaluated with respect to each T_2.

In the figure, the solid blue segment of the curve represents values of T_2 for which the required $\dot{m}$ remains below or equal to $\dot{m}_{\text{max}}$, while the dashed blue line corresponds to points where the required $\dot{m}$ exceeds the maximum allowable for that temperature. The red point marks the transition, occurring at $T_2 \approx 8.64°\text{C}$, where $\dot{m} = \dot{m}_{\text{max}}$ and the corresponding compressor input power is approximately $0.80\,\text{kW}$.

As expected from thermodynamic principles, $\dot{W}_c$ decreases as T_2 increases. This reflects the reduced temperature lift (and thus lower pressure difference) the compressor must overcome. In the idealized limit where $T_2 \to T_4$, the compression ratio approaches unity, and the compressor would theoretically require no power: $\dot{W}_c \to 0$. However, in this limit, the cycle becomes non-functional. Without a temperature or pressure difference between evaporator and condenser, there is no driving force for heat absorption, no phase change in the evaporator, and no useful work from the compressor.

Thus, while the lowest possible power consumption occurs theoretically at $T_2 = T_4$, this point is physically meaningless in a real system. The practical minimum power in the setup occurs at the point where the required mass flow rate just reaches the maximum allowable, at $T_2 \approx 8.64°\mathrm{C}$ with $\dot{W}_c \approx 0.80\,\mathrm{kW}$. Beyond this, the required flow rate would exceed system capabilities, rendering operation infeasible despite the potential for reduced power.

Question 7

To understand how the required compressor power $\dot{W}_c$ varies with outdoor temperature T_C, we used the heat loss model from the mid-term project and computed the total heating power using the conductance and the difference between the outdoor and indoor temperatures of the house, given by:

$$\dot{Q}_H = G_{\text{house}} \cdot (T_H - T_C) \tag{9}$$

where $G_{house} = 682.27W/K$ and $T_h = 22C$. This expression basically gives the amount of heat that the heat pump must deliver in order to keep the house at $22C$.

We computed, for each T_c, the required $\dot{Q}_H$ and then computed the work fluid temperatures T_2 and T_4 such that we minimized $\dot{W}_c$ as much as possible, taking into account the constraint on maximum allowable mass flow rate. This mass flow rate was computed in the same way as on exercise 4, that is:

$$\dot{m}_{\max} = \min\left(\frac{G_{\text{HP}}(T_H - T_4)}{q_h}, \frac{G_{\text{HP}}(T_C - T_2)}{q_c}\right) \tag{10}$$

where $G_{\text{HP}} = 4895.42$ W/K is the heat pump conductance and q_h and q_c are the specific heat transfers at the evaporator and condenser, respectively.

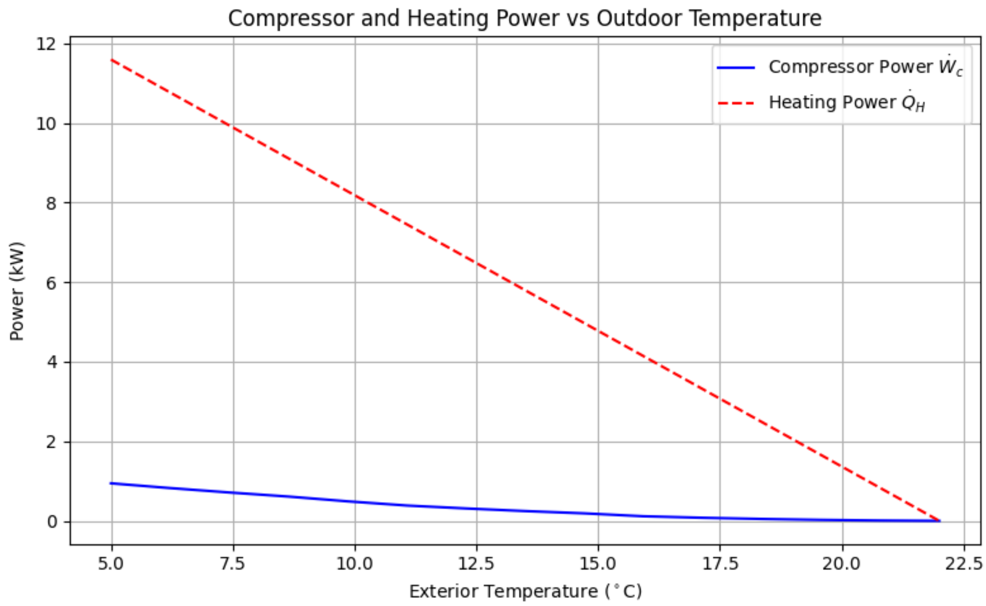


Figure 5: Optimized compressor work $\dot{W}_c$ vs outdoor temperature T_C

As we can see, the compressor power needed for the heat pump decreases as the exterior temperature increases. This makes sense as the warmer it is outside, the lower are the heat losses from the house and therefore less working power is needed to maintain the house at $22C$.

Question 8

To use the function programmed in Question 7, we defined an additional function that returned $T_C(t) = (10°C) + (5°C)sin(\omega t)$ with $\omega = 2\pi/24h$ for every hour in a day. We thus replaced T_C by this time-varying version to compute again the power consumption over a day, which, on average, was $\dot{W}_c = 0.518$ kW.

We also computed the effect that a time-varying T_C would have on the power consumed by an electric heater, which can be obtained, within the heat pump scenario, from $\dot{Q}_H$. This value was $\dot{Q}_H = 8.187$ kW which is very similar to the value given in the project description $\dot{Q}_H = 8.2$ kW.

When comparing the two values, the power consumption of the electric heater is 14.79 times higher than that of the heat pump. Such an abysmal difference that would entail higher electricity bills and higher emissions depending on the mix of the local energy grid for the same increase in temperature in the household continues to support the preference of a heat pump over an electric heater.

Question 9

We now test the daily carbon footprint of our heat pump against the carbon footprint of the electric heater and direct combustion of methane. The two heaters run on electricity from an energy mix of 60% renewables and 40% from the combustion of methane - with a conversion efficiency of 55%. We know that we need a heating power of 8.2 kW to supply the house with sufficient heating. That means that the work of the electric heater is equal to the heating power if we consider that the power gets converted with 100% efficiency from electricity to heat. The work of the heat pump was calculated in Question 8: $\dot{W}_c = 0.518$ To find the carbon footprint of methane combustion, we want to convert the value for the heat of combustion from kJ/mol methane to kJ/kg CO2.

Let's consider the reaction for the combustion of methane:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \tag{11}$$

We therefore know that one mol of methane combusts to one mol of CO_2.

$$\Delta H° = -890.3 \text{ kJ/mol CH4} = -890.3 \text{ kJ/mol CO2} \tag{12}$$

Therefore we can divide by the molar mass of carbon dioxide to get the heat of combustion per kg CO_2.

$$\Delta H° = \frac{-890.3 \text{ kJ/mol CO2}}{16.04 \cdot 10^{-3} \text{kg CO2/mol CO2}} = 20.23 \text{ MJ/kg CO2} \tag{13}$$

This is the energy gained by methane combustion per kg of CO_2. For the electric heater and the heat pump, we have to consider a conversion efficiency of 55% from the heat of the methane combustion to electricity.

$$I_{CO_2} = 20.23 \text{ MJ/kg CO2} \cdot 0.55 = 11.13 \text{ MJ/kg CO2} \tag{14}$$

We now use the power consumption and multiply it with the seconds in a day to find the total energy consumption of both our heat pump and electric heater. Since this electricity makes up 40% of our energy grid we will multiple the power $\dot{W}$ by 0.4 as this is the part of the power that has to get covered by the combustion of methane. We therefore calculate the amount of energy a day that has to be supplied by the combustion of methane.

- $E_{\text{Heat pump}} = 0.518 \text{ W} \cdot 0.4 \cdot 86400 \frac{\text{s}}{\text{day}} = 17.9 \text{ kJ}$

- $E_{\text{Electric Heater}} = 8.2 \text{ W} \cdot 0.4 \cdot 86400 \frac{\text{s}}{\text{day}} = 283.4 \text{ kJ}$

The energy consumed if we simply use the combustion of methane directly to supply the heat demand is exactly equal to that of the heat demand. We do not consider any losses from transport of heat, meaning that we consider a situation where the methane is combusted in the house. We advise caution at replicating this in your home.

- $E_{\text{Methane}} = 8.2 \text{ W} \cdot 86400 \frac{\text{s}}{\text{day}} = 708.5 \text{ kJ}$

The CO2 intensity is therefore the exactly that of the heat of combustion. We can now calculate the carbon intensity for all three technologies:

- Heat Pump: $m_{CO_2} = E_{\text{Heat pump}} \cdot I_{CO_2} = 1.61 \text{ kg}$

- Electric Heater: $m_{CO_2} = E_{\text{Electric Heater}} \cdot I_{CO_2} = 25.4 \text{ kg}$

- Methane Combustion: $m_{CO_2} = E_{\text{Methane}} \cdot \Delta H^\circ = 34.9 \text{ kg}$

As expected, the CO_2 emissions from the heat pump are the lowest, followed by the electric heater and those from the direct methane combustion.

Question 10

To determine the carbon footprints of the heat pump and the electric heater throughout their 15-year lifespans, we took as reference the 24-hour CO_2 emissions computed in Question 9 and increased them by a factor of $365 * 15$ for the days in a year and the established lifetime, respectively.

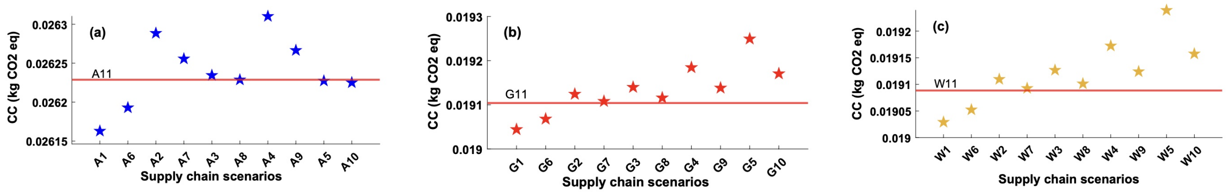
This yielded the following CO_2 footprints for the device lifetime of 15 years:

- Heat pump: 8.82 tons of CO_2

- Electric heater: 139.09 tons of CO_2

The emission of the heat pump is lower than the heater's by 130.28 tons of CO_2. This negative difference was expected because heat pumps provide the same amount of heat with a lower amount of input work since the temperature difference between the outside and the inside of the house also provides energy.

Still, for the heat pump to remain preferable in terms of emissions, its manufacturing process must produce the same or less CO_2 than the amount stated as the difference between the two technologies. Satisfactorily, this seems to be the case for most heat pump technologies. Traditional technologies, which have a bigger market share, like the Air-Source Heat Pump (ASHP), the Gas-Source Heat Pump (GSHP), and the Water-Source Heat Pump (WSHP), have supply chains that emit between 0.019 and 0.0263 units equivalent to kg of CO_2 [1]. The paper we consulted considered "supply chain emissions" as those emitted during the raw material extraction, the transportation, the manufacturing, the Operation & Maintenance (O&M), and the Assembly and Installation (A &I), and under different scenarios, which varied in the amount of heat pump components manufactured locally, i.e., in the UK where the research was conducted. We focused on scenarios 1-5 which looked at a manufacturing process conducted 100 % abroad since Denmark is not a major player in the heat-pump-parts industry, making it the likelier case for installations anywhere in the nation. Figure 6 summarizes the equivalent tonnes of CO_2 emissions emitted by the ASHP, GSHP, and WSHP technologies.



(a) CO_2 Emissions of Air-Source Heat Pumps

(b) CO_2 Emissions of Gas-Source Heat Pumps

(c) CO_2 Emissions of Water-Source Heat Pumps

Figure 6: Scenarios A, G, or W numbered from 1 through 5 refer to those where 100 % of the heat pump equipment is manufacture abroad [1].

Thus, since the above refer to the emissions of producing one heat pump unit, and they are smaller by about 7 orders of magnitude, we can predict that heat pumps will remain preferable over traditional electric heaters in terms of their CO_2 footprints.

References

[1] M. Shamoushaki, "Heat pump supply chain environmental impact reduction to improve the uk energy sustainability, resiliency, and security," *Nature*, 2023.