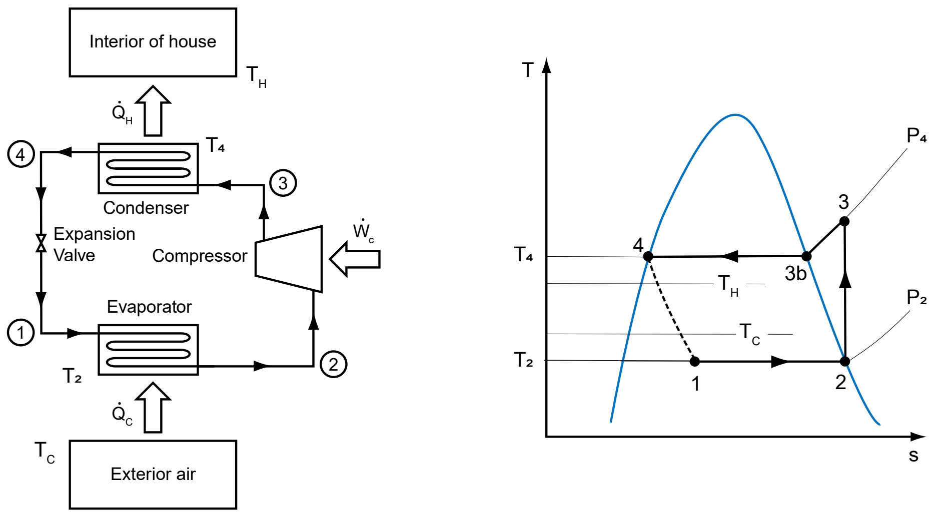
Final project

This project is a continuation of the midterm project. We will now investigate how to more efficiently keep the house warm, by replacing the electric heater with a heat pump.



(a) Schematic

(b) T-s diagram

Figure 1: Heat pump in heating mode, operating with reverse Rankine cycle.

In particular, we consider a device operating by a vapour refrigeration Reverse Rankine cycle. The schematic of the device is shown in Fig. 1a and the cycle is represented on the T-s diagram in Fig. 1b. The working medium is refrigerant 134a.

Performing this assignment requires you to develop a computer program that does the calculations (**N.B.:** you don't need to include the code in your report!). The main reason for automating the computation is that the data describing fluid properties (e.g. the shape vapour dome) is only available in numerical form, as opposed to having simple analytical formulas available. While looking up the data tables is a suitable approach for solving a single exercise, when performing a more extensive investigation this process needs to be automated.

You will thus be provided with the data in txt format (here on Learn), and you will need to implement in your program a routine that loads these datasets, You will also need to use 1D and 2D interpolations methods. Such functions are commonly available in most computational frameworks. For example, for 2D interpolation of scattered data, you can use the Matlab/Python-SciPy functions `griddata`, or the Maple function `Interpolation`, or the `ScatteredInterpolation` Mathematica function (uploaded by a user). For 1D interpolation, you may use, e.g. the Matlab function `interp1` or the Python function `interp1d` or equivalent functions. After the questions, you will find information on the formats of the datasets and how to import them into your program with the correct units.

The logic of the assignment is to investigate the trade-off between efficiency and power. Denoting by T_2 and T_4 the cold and hot operating temperature of the working fluid, and by T_C and T_H the interior and exterior temperature of the house, for a heat pump in heating mode, these must satisfy the relation: $T_2 < T_C < T_H < T_4$. While the maximum efficiency is obtained when $T_2 \to T_4$, in those conditions the heat pump would not provide any heating power.

Tasks

Let us now start by considering example 9.5 from the textbook.

Question 1 Develop a program that, given the input values of the problem (i.e. T_2, P_4 and $\dot{m}$), computes the heating power $\dot{Q}_H$, the input power $\dot{W}_c$ and the heating COP (defined as $|\dot{Q}_H|/\dot{W}_c$). Note that in the textbook, the refrigerating COP is considered instead of the heating COP. Moreover, the result for the COP in the book is not correct.

Once you confirmed that the program is working correctly (by comparing the computed $\dot{W}_c$ with the value from the textbook), use the program to plot the cycle diagram on the T-s diagram. Be sure to include all the points of the cycles (1, 2, 3, 3b, and 4) connected by line segments, and to show the vapour dome. Instead, the lines representing the isobaric processes as they extend outside the vapour dome are optional.

Hints: in the procedure, you will need to (for example) compute the value of specific enthalpy of saturated liquid R-134a at a specific pressure (P_4). In Matlab, this is done with the following line of code:

```
h4 = interp1(P,hf,P4)
```

where `P` and `hf` correspond to the data of Appendix 9a or 9b.

You will also need to find the specific enthalpy of superheated R-134a vapour having a given value of specific entropy (s_2) at a given pressure (P_3). This is done with the following line of code:

```
h3 = griddata(Pgs,sgs,hgs,P3,s2)
```

where `Pgs`, `sgs`, and `hgs` correspond to the data of Appendix 9c.

Question 2 Now we change the value of the input parameter T_2. Plot the heating COP as function of T_2 in the range $T_2 \in [-10°\mathrm{C}, 15°\mathrm{C}]$. Modify the program such that the inputs are T_2, T_4 and $\dot{m}$. On the same graph as above, plot curves for the following values of T_4: {25°C, 30°C, 35°C}. Make sure to clearly indicate which value of T_4 each curve corresponds to (e.g. using legends or annotations). Discuss the results: for each value of T_4, what is the value of T_2 that gives the best COP in this range?

Question 3 So far, we assumed that the mass flow rate can be increased indefinitely, thus providing additional heating power without changing the COP. In reality, the mass flow rate is also limited by the time that it takes to the working fluid to exchange enough eat with the heat reservoirs to fully undergo the phase transitions during the two isobaric processes. We will thus try to estimate that. In order to do so, we first need to calculate the thermal conductance between the heat exchangers (of condenser and evaporator) and the surrounding air. Here we consider a simple serpentine heat exchanger (i.e. a long tube) that contains the working fluid R-134a. Let $L = 1$ m be the length of the tube, and $r_2 = 5$ mm and $r_1 = 3$ mm be the external and internal radius, respectively. Look up the formula for the formula for the thermal conductance of a hollow cylinder though its lateral surface (it is not the usual formula) and compute the conductance G if the tube is made of copper, assuming thermal conductivity $k = 398$ W/(m K).

Question 4 Now that we know the conductance G of evaporator and condenser (assumed equal to each other) we can try to calculate the maximum mass flow rate.

We need to remember that the temperature of the heat reservoirs is different from the temperature of the working fluid (such that heat can actually be transferred). We thus need to introduce two additional variables T_H and T_C. These correspond to the temperature inside and outside the house, respectively. Let's focus for example on the cold side (i.e. the evaporator).

We consider that the working fluid in the evaporator is at fixed temperature T_2 throughout the process and is surrounded by air which is at the exterior temperature $T_C > T_2$.

The relation $\dot{Q}_C = G\Delta T$ gives us the total heat flow rate between the fluid in the evaporator and the air around it. We already know that $\dot{Q}_C = \dot{m}\, q_C$ where q_C is the amount of heat per unit mass that the fluid must receive in order to complete the phase transition. By combining the two previous equations and solving for $\dot{m}$, we can thus compute the maximum flow rate that is compatible with the required cycle. This depends on the temperature difference ΔT, i.e. it is a function of T_2 and T_C. We can apply the same procedure to the process $3 \to 4$, approximating the temperature as constant throughout the process (i.e. equal to T_4). Calculate the maximum value of $\dot{m}$ for the two processes, assuming these conditions: $T_H = 22^\circ$ C, $T_C = 10^\circ$ C, $T_2 = 5^\circ$ C, $T_4 = 35^\circ$ C. The overall maximum value of mass flow rate, $\dot{m}^{\text{Max}}$, is the minimum between these two values. You should obtain: $\dot{m}_H^{\text{Max}} \approx 0.37$ kg/s, and $\dot{m}_C^{\text{Max}} \approx 0.16$ kg/s.

Now plot the maximum heating power for the same temperatures considered in question 2, i.e. $T_2 \in [-10^\circ\text{C}, 15^\circ\text{C}]$ and $T_4 = \{25^\circ\text{C}, 30^\circ\text{C}, 35^\circ\text{C}\}$. Discuss the result: is the maximum power an increasing or decreasing function of T_2? What happens when $T_2 \to T_C$?

Hint: for $T_4 = 35^\circ$C and $T_2 = 0^\circ$C, you should obtain: $\dot{Q}_H \approx 56.5$ kW.

Question 5 For the conditions $T_H = 22^\circ$ C, $T_C = 10^\circ$ C, $T_2 = 5^\circ$ C, $T_4 = 35^\circ$ C, compute the heat per unit mass of refrigerant added to the hot heat reservoir, i.e. q_H. What mass flow rate would we need, to obtain a heating power of $\dot{Q}_H = 8.18$ kW? Is that smaller or larger than the maximum possible mass flow rate that you computed in the previous question?

Question 6 For $T_2 \in [-10^\circ\text{C}, 35^\circ\text{C}]$ and $T_4 = 35^\circ$C, plot as function of T_2 the input power $\dot{W}_c$ corresponding to the value of $\dot{m}$ that results in $\dot{Q}_H = 8.18$ kW. Mark the point where $\dot{m} = \dot{m}^{\text{Max}}$, and plot with a solid line the part of the curve for which $\dot{m} \le \dot{m}^{\text{Max}}$ and with a dashed line the part of the curve for which $\dot{m} > \dot{m}^{\text{Max}}$. Discuss the results: what is the value of T_2 corresponding to the minimal power consumption? How much power would we need, if we could run with $T_2 \to T_4$? Is that actually possible?

Hint: at $T_2 = 0^\circ$C you should obtain $\dot{W}_c \approx 1.09$ kW.

Question 7 Now program a function that given T_H and T_C, and a target $\dot{Q}_H$, finds the values of T_2 and T_4 that would result in the minimum value of $\dot{W}_c$ among those compatible with the maximum possible mass flow rate. In other words, for fixed temperature of heat reservoirs and target heating power, we optimize the operating temperatures T_2 and T_4 to minimize the power consumption $\dot{W}_c$, while avoiding the situation $\dot{m} > \dot{m}^{\text{Max}}$. This optimal value of $\dot{W}_c$ should be the output of the function.

We now assume $T_H = 22^\circ$ C and we vary T_C in the range $\{5^\circ\text{C}, 22^\circ\text{C}\}$. The required value of heating power $\dot{Q}_H$ for each temperature T_C will be given by the conductive heat losses that you calculated in the midterm project (i.e. $\dot{Q}_H$ is itself a function of $\Delta T = T_H - T_C$). Plot $\dot{W}_c$, as outputted from the function you just programmed, as function of T_C. Also plot $\dot{Q}_H$ on the same graph.

Hint: at $T_C = 5^\circ$C you should obtain $\dot{W}_c \approx 0.93$ kW.

Question 8 Assume that the external temperature varies over 24 hours according to:

$$T_C(t) = (10^\circ\text{C}) + (5^\circ\text{C})\sin(\omega t)$$

with $\omega = (2\pi)/(24 \text{ h})$. Use the function you programmed in the previous question to calculate the power consumption averaged over a day. Compare with the power that is consumed with the same

$T_C(t)$ dependence if an electric heater is used instead of the heat pump.

Question 9 Assume that the heat pump is powered by electrical energy of which 60% is renewable and 40% is converted from combustion of methane (with a conversion efficiency of 55% percent). What is the mass of CO_2 emissions (in kg) over one day? What would the amount have been if the electric heater was used, instead of the heat pump? What if methane was combusted to produce heat directly? For all three cases, assume that the molar combustion enthalpy of methane given by $\Delta H^\circ = -890.3$ kJ/mol.

Question 10 The heat pump is expected to operate for about 15 years. Producing the device has a certain carbon footprint (equivalent amount of CO_2 emitted). What would be the maximum carbon footprint that would break even with the savings in emissions of the heat pump compared to the electric heater? (neglect the carbon footprint of the heater).

Datasets

The two files `R134a_PressureTable.txt` and `R134a_SuperHeated01_test02.txt` (available here on Learn) correspond to Appendix 8b and 9c, respectively. The units are the same as in the textbook. Therefore, in order to import them in S.I. units (e.g. Pa instead of MPa, etc.), you can use the following code (example is given in Matlab syntax). The units are indicated between square brackets.

```
A = readmatrix('R134a_PressureTable.txt') ;
P = A(:,1).*1e6 ; % [Pa]
T = A(:,2) + 273.15 ; % [K]
vf = A(:,3) ; % [m3/kg]
vg = A(:,4) ; % [m3/kg]
uf = A(:,5).*1e3 ; % [J/kg]
ug = A(:,6).*1e3 ; % [J/kg]
hf = A(:,7).*1e3 ; % [J/kg]
hg = A(:,8).*1e3 ; % [J/kg]
sf = A(:,9).*1e3 ; % [J/(kg K)]
sg = A(:,10).*1e3 ; % [J/(kg K)]


Asup = readmatrix('R134a_SuperHeated01_test02.txt') ;
Pgs = Asup(:,1).*1e6 ; % [Pa]
Tgs = Asup(:,2) + 273.15 ; % [K]
vgs = Asup(:,3) ; % [m3/kg]
ugs = Asup(:,4).*1e3 ; % [J/kg]
hgs = Asup(:,5).*1e3 ; % [J/kg]
sgs = Asup(:,6).*1e3 ; % [J/(kg K)]
```