# Lab Assignments Computational Finance

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**Submission guidelines**

These assignments can be done in groups of three students. Reports with a *clear description of the assignment, the methods, the results and discussion* should be submitted before the deadlines. You are free to choose the programming language/environment in which you would like to write your computer programs. If you have questions about the assignments do not hesitate to contact the teaching assistant or the lecturer.

**Grading scheme**

- Each of the three assignments carries equal weight of 20% and the exam is worth 40%;
- The score of the exam should be 5 points (on the scale of 1 to 10) and higher for passing the course;
- The fourth assignment is a bonus assignment and has to be submitted before the exam. With the bonus assignment the final grade can be increased by at most 1 point (on the scale of 1 to 10), but only if this assignment has been graded sufficient ($\geq 50\%$).

| Assignment 1 | Assignment 2 | Assignment 3 | Exam | Assignment 4 (Bonus) |
|---|---|---|---|---|
| 20% | 20% | 20% | 40% | 10% |

# Assignment 2: Monte Carlo (MC) Methods in Finance

## Part I
# Basic Option Valuation

As we derived in the class, for the purpose of pricing options, we can assume that the stock price $S$ evolves in the risk neutral world:

$$dS = rSdt + \sigma SdZ \tag{1}$$

where $r$ is the risk free return, $\sigma$ is the volatility, and $dZ$ is the increment of a Wiener process. Let the expiry time of an option be $T$, and let

$$N = \frac{T}{\Delta t}$$

$$S^n = S(n\Delta t)$$

Then, given an initial price $S^0$, $M$ realizations of the path of a risky asset are generated using the algorithm (Euler method)

$$S^{n+1} = S^n + S^n(r\Delta t + \sigma\varphi\sqrt{\Delta t})$$

where $\varphi$ is a normally distributed random variable with mean zero and unit variance.
For special cases of constant coefficients, we can avoid time stepping errors for geometric Brownian motion, since we can integrate Equation 1 exactly to get

$$S^T = S^0 e^{(r-0.5\sigma^2)T+\sigma\sqrt{T}Z} \tag{2}$$

The price of an option can be calculated by computing the discounted value of the average pay-off, i.e.

$$V(S^0, t=0) = e^{-rT}\frac{\sum_{m=1}^{M} payoff^m(S^N)}{M}$$

Write a computer program for the Monte Carlo method. Price European put option with (T = 1 year, K =€99, r = 6%, S = €100 and $\sigma$ = 20%). Carry out convergence studies by increasing the number of trials. How do your results compare with the results obtained in assignment 1? Perform numerical tests for varying values for the strike and the volatility parameter. What is the standard error of your estimate and what does this tell you about the accuracy?

## Part II
# Estimation of Sensitivities in MC

1. The hedge parameter $\delta$ in Monte Carlo can be estimated by the bump-and-revalue method. Calculate the $\delta$ by applying the following methods:

   - Use different seeds for the bumped and unbumped estimate of the value;

- Use the same seed for the bumped and unbumped estimate of the value;

Compare your results which the values obtained in Assignment I.

2. Consider a digital option which pays 1 euro if the stock price at expiry is higher than the strike and otherwise nothing. Calculate the hedge parameter $\delta$ using the method used in 1. Explain your results and use the **sophisticated methods** discussed in the lectures to improve your results.

## Part III

# Variance Reduction

A major drawback of Monte Carlo simulation is that a large number of realizations are typically required to obtain accurate results. Therefore techniques to speed-up the simulations are quite useful. In this assignment students will work on different variance reduction techniques and quasi-Monte Carlo methods to accelerate numerical valuation of financial derivatives.

**Variance Reduction by Control Variates.** For the control variates technique an accurate estimate of the value of an option that is similar to the one that you would like to price is required. For valuation of an Asian option based on *arithmetic averages* one can use the value of an Asian option based on *geometric averages*. This case can be solved analytically.

1. Derive an analytical expression for the price of an Asian option that is based on geometric averages.

   Hint: First, recall that the geometric average is defined as:

$$\widetilde{A}_N = \left( \prod_{i=1}^{N} S_i \right)^{\frac{1}{N}}$$

   Use the following property:

$$\prod_{i=1}^{N} S_i = \frac{S_N}{S_{N-1}} \left( \frac{S_{N-1}}{S_{N-2}} \right)^2 \left( \frac{S_{N-2}}{S_{N-3}} \right)^3 \cdots \left( \frac{S_2}{S_1} \right)^{N-1} \left( \frac{S_1}{S_0} \right)^N S_0^N$$

   Show that the following identity is true:

$$\ln \left( \frac{\left( \prod_{i=1}^{N} S_i \right)^{\frac{1}{N}}}{S_0} \right) = Normal\_dist \left( (r - 0.5\sigma^2) \frac{(N+1)}{2N} T, \sigma^2 \frac{(N+1)(2N+1)}{6N^2} T \right)$$

   Note that this problem is very similar to derivation of the Black-Scholes formula for the price of an option on a stock. This has been derived in detail in the lectures. Using this similarity, it is straightforward to derive the analytical Black-Scholes formula for the Asian option? Check your analytical expression by comparing with values obtained by using Monte-Carlo simulations.

2. Explain how this strategy of using this as a control variate works.