Lab 4: Finding Roots
Assigned on 11/10/2015 (Tuesday) and due on 11/12/2015 (Thursday)

The specific energy equation is $h^3 - Eh^2 + \frac{q^2}{2g} = 0$,

where h is water depth, E is specific energy, q is specific discharge (Q/w) and g is gravitational acceleration [Box 2.1]. For E=1 m and $q^2 / (2g)$ = 0.05 m^3, find the roots of the equation for h using the following methods:

(a) Plot the function and roughly estimate the roots. (5 points)

(b) Use the MATLAB commands **roots** and **fzero** to obtain the roots. Plot the roots on the figure of problem (a) above. (5 points)

(c) Write an m-file to find the roots using the secant method. Use x_0=0.5 and x_1=1.3. List the values of x at the beginning and end of all the iterations (e.g., x_1 and x_2 values for iteration 1). <u>Discuss the effect of tolerance on the number of iterations and select the optimum tolerance value for the next question. You need to explain the basis for selecting the optimum value</u>. (5 points)

Bonus Question:

(d) Theoretical analysis shows that the secant method is normally superlinearly onvegent with r = 1.618 (the golden ratio), if the starting values are close enough to the solution. Examine this convergence rate using the initial values and selected tolerance in problem (c). (5 points)

Box 2.1. Specific energy

Specific energy, E, is the energy per unit weight of water flowing through a channel relative to the channel bottom, $U^2/(2g)+h$, where U is channel mean velocity, g is gravitational acceleration, and h is flow depth. If the flow is steady and uniform and the channel cross section is rectangular, then $U = Q/(wh) = q_w/h$, where Q is channel discharge, w is channel width, and q_w is discharge per unit width or specific discharge. Combining these equations gives the specific energy equation

$$E = \frac{q_w^2}{2gh^2} + h \quad \text{or} \quad h^3 - Eh^2 + \frac{q^2}{2g} = 0$$

Of the three roots to the specific energy equation, two are positive and one is negative. The negative root has no physical meaning. The two positive roots, called *alternate depths*, are both possible, depending on whether the Froude number, $\mathbf{F} = U/\sqrt{gh}$, is subcritical ($\mathbf{F}$<1) or supercritical ($\mathbf{F}$>1). (See Hornberger et al. (1998) for the more information about specific energy and alternate depths.)