ISC 4220
Algorithms 1
Due: April 14 (Thursday), 2016

Numerical Differentiation

1. Let us numerically evaluate the derivative of $f(x) = \sin(x)$ at $x = \pi/3$, using two different numerical formulae:

Centered-difference formula:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h},$$

Forward-difference formula:

$$f'(x) = \frac{f(x+h) - f(x)}{h}.$$

Vary the step-size h between 10^{-15} to 10^{-1} (consider using the `logspace` command), and construct a `log-log` plot of absolute error and h. Note that we can compute the absolute error, since we know that the true value of $f'(\pi/3) = 0.5$.

Using the graphs above, find the (approximately) optimum value of h for the two difference formula, and compare the absolute errors of the two numerical formulae.

2. Consider the integral

$$I = \int_0^1 \sqrt{x} \log(x) dx$$

The exact value is $I = -4/9$.
(a) Use trapezoidal rule with $n = 1$, 2, 4, and 8 equal intervals to numerically evaluate the integral. Report the relative and absolute error.
(b) Use Romberg integration with successive refinement to report the most "refined" estimate you can, based on the values computed from trapezoidal rule above.

3. Evaluate the integral

$$I = \int_0^2 \exp(-x^2) dx,$$

which is related (but differs by a known constant) to the "error function", defined intrinsically in Matlab as `erf`.
(a) Use $n = 4$ equispaced intervals, and apply Simpson's 1/3 rule to evaluate the integral.
(b) Using Gauss quadrature with 4 nodes, recompute the integral above.

In both cases report the absolute error.