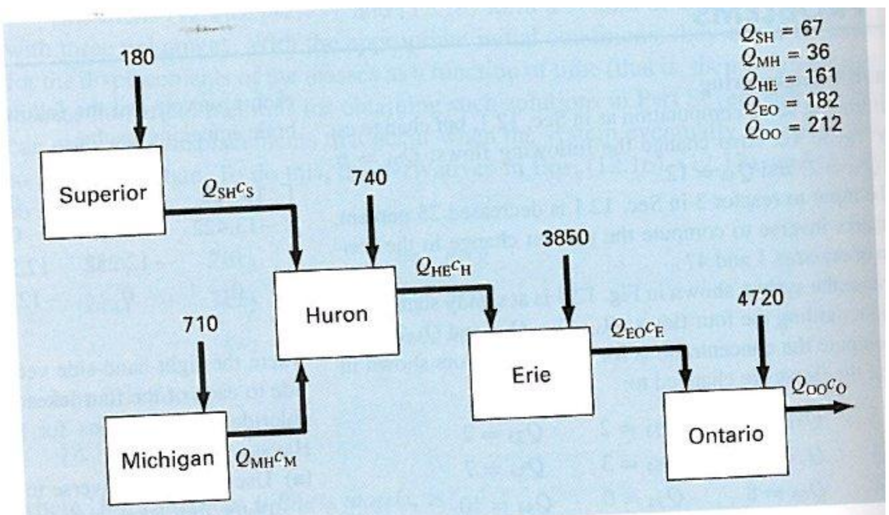
ISC 4220
Algorithms 1
Lab 2

Due: Feb 4, 2016 in the Lab

Linear Systems and Gauss Elimination

1. Consider the following data on the concentration and transport of chlorides in each of the Great Lakes. Here the "Q" refers to the flow rate in some units, and "c" the concentration of chloride in a particular lake in compatible units. Numbered arrows are direct chloride inputs from the land area surrounding a particular lake. (from Chapra and Canale)



$$
\begin{aligned}
180 - Q_{SH}c_s &= 0 \\
Q_{SH}c_s + 740 + Q_{MH}c_M - Q_{HE}c_H &= 0 \\
\cdots &= 0
\end{aligned}
$$

- Write the complete set of balance equations (5 points)
- Rewrite the balance equations in matrix form $\mathbf{Ac} = \mathbf{b}$, where, $\mathbf{c} = [c_S\ c_H\ c_M\ c_E\ c_O]^T$ (5 points)
- What is the condition number of the resulting matrix? You may use Matlab's intrinsic function for this part. (5 points)
- Write a Gaussian elimination routine with partial pivoting to solve for $\mathbf{c}$. (15 points)
- If the direct input into Lake Michigan increases from 710 to 1000, due to the increasing population of Chicago, how would it affect the other lakes? (5 points)

2. Consider the linear system:

$$\begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+(1+\epsilon)\epsilon \\ 1 \end{bmatrix}$$

where ϵ is a small parameter. The exact solution is $\mathbf{x} = [1\ \epsilon]^T$.

- Using intrinsic Matlab functions to solve the linear system, experiment with various values of ϵ especially those near $\sqrt{\epsilon_{\text{mach}}}$. (5 points)
- For each value you try estimate the condition number of the matrix (again use Matlab to estimate it), and the relative error in each component of the solution. (5 points)
- How does the accuracy attained for each component compare with expectations based on the condition number of the matrix? (5 points)