ISC 4220
Algorithms 1
Lab 4
Due: March 3, 2015

Optimization

1. The growth rate, g , of yeast used to produce an antibiotic depends on the concentration of the food c as:

$$g(c) = \frac{2c}{4 + 0.8c + c^2 + 0.2c^3}.$$

The units of g and c, in the above equation, are per day and mg/L, respectively. The growth rate is small when c is small (as the yeast starve), and when c is very large (toxicity effects). We want to find the c at which g is maximum.

- Plot the function $g(c)$, and identify a suitable interval which contains the maximum. [10 points]
- Using the interval determined above as the starting guess, find the maximum using the Golden Section Search method. [20 points]
- Take the derivative of $g(c)$, $f(c) = g'(c)$, and determine the maximum using bisection to solve the nonlinear equation $f(c) = 0$. [20 points]

2. The Rosenbrock function is often used to test the performance of optimization algorithms, because its global minimum, which occurs at (1,1), lies within a narrow parabolic valley.

$$f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2.$$

- Plot the function using Matlab's meshc function (or equivalent) for $-2 < x_1 < 2$ and $-1 < x_2 < 3$. An example can be found here:

  http://www.malinc.se/math/octave/threeden.php [5 points]

- Plot the contours of $f(x_1, x_2)$ for $-2 < x_1 < 2$ and $-1 < x_2 < 3$ using the contour function of Matlab. Examples are at

  http://www.mathworks.com/help/matlab/ref/contour.html [5 points]

- Write a steepest decent program to find the minimum of the function, from the starting point $[-1,\ 1]^T$. For the line search, use the fminsearch trick that we showed in the class. You can base your program on the sample code from the class. If you do not converge, stop after 100 iterations. Report function values, x_1 and x_2, the norm of the gradient at each iteration. [20 points]
- Modify your code to perform the Newton's method to again optimize this problem *with a line search*, from the starting point $[-1,\ 1]^T$. Report function values, x_1 and x_2, and the norm of the gradient at each iteration. [20 points]