Assignment 2

ISC 4232/5935

Due: September 30 at 4:00pm

Programming Languate: any sensible language
Submit write-up, code, and any auxilary files to Lukas Bystricky, lb13f@my.fsu.edu
Undergraduates: Submit all questions except 3b
Graduates: Submit all questions

---

1. Show that if we integrate the IVP

$$y'(t) = f(y(t), t),$$

from t_n to t_{n+1} and use a right Riemann sum to approximate the integral of $f(y,t)$ then we obtain the backward Euler method. What quadrature rule would result in the forward Euler method?

2. Suppose you are interested in modeling the growth of the Bread Mold Fungus, *Rhizopus stolonifer*, and comparing your numerical results to experimental data that is taken by measuring the number of square inches of mold on a slice of bread over a period of several days. Assume that the slice of bread is a square of side 5 inches.

   (a) To obtain a model describing the growth of the mold you first make the hypothesis that the growth rate of the fungus is proportional to the amount of mold present at any time with a proportionality constant of k. Assume that the initial amount of mold present is 0.25 square inches. Let $p(t)$ denote the number of square inches of mold present on day t. Write an initial value problem for the growth of the mold.

   (b) Assume that the following data is collected over a period of ten days. Assuming that k is a constant, use the data at day one to determine k. Then using the forward Euler method with Δt a fourth and an eighth of a day, obtain numerical estimates for each day of the ten day period; tabulate your results and compare with the experimental data. When do the results become physically unreasonable?

day	0	1	2	3	5	7	8	10
mold area	0.25	0.55	1.1	2.25	7.5	16.25	19.5	22.75

   (c) The difficulty with the exponential growth model is that the bread model grows in an unbounded way as you saw in 2b. To improve the model for the growth of bread mold, we want to incorporate the fact that the number of square inches of mold can not exceed the number of square inches in a slice of bread. Write a logistic differential equation which models this growth using the same initial condition and growth rate as before.

   (d) Use the forward Euler method with Δt a fourth and an eighth of a day to obtain numerical estimates for the amount of mold present on each of the ten days using your logistic model. Tabulate your results as in 2b and compare your results to those from the exponential growth model.

3. Suppose we integrate the IVP

$$y'(t) = f(y(t), t)$$

from t_n to t_{n+1} and use the trapezoid rule to approximate the integral of $f(y, t)$. Write down the resulting scheme, which is known as Crank-Nicolson.

(a) Is this method implicit or explicit?

(b) (**Grad Students Only**) Show that the local truncation error is order Δt^3, but not order Δt^4.

(c) Knowing that the local truncation error is third-order accurate, what error do you expect for the global discretization error?

(d) Apply this scheme to the linear IVP

$$\begin{aligned} y'(t) &= 1 - y(t), \quad 0 < t \leq 1, \\ y(0) &= 0, \end{aligned}$$

and check that the global discretization error is as you expect. Did you require a nonlinear solver to compute Y_{n+1}? Why or why not?

4. Consider the logistic growth model

$$\begin{aligned} p'(t) &= (1 - p(t))p(t), \quad 0 < t \leq 6, \\ p(0) &= \frac{1}{2}. \end{aligned}$$

(a) What is the exact solution?

(b) Write down the scheme when Crank-Nicolson from Question 3 is applied to the logistic growth model.

(c) What nonlinear equation must be solved at each time step? How many solutions does this nonlinear equation have?

(d) Implement Crank-Nicolson for the logistic growth model. To solve the nonlinear equation, use a Newton solver with a tolerance of 10^{-8} and an initial guess of Y_n to compute Y_{n+1}.

(e) Do a convergence study on the global discretization error at $t = 6$ and report the observed order of convergence. Does this agree with your results from Question 3?