Homework 4: Finding Zeros
Assigned on 11/5/2015 (Thursday) and Due on 11/19/2015 (Thursday)

Problem 1: Bracketing algorithm (5 points)

$\boxed{\textbf{6.2}}$ The function $f(x) = \sin(x^2) + x^2 - 2x - 0.09$ has four roots in the interval $-1 \le x \le 3$. Given the m-file **fx.m**, which contains

```
function f = fx(x)
f = sin(x.^2) + x.^2 - 2*x - 0.09;
```

the statement

```
>> brackPlot('fx',-1,3)
```

produces only two brackets. Is this result due to a bug in `brackPlot` or `fx`? What needs to be changed so that all four roots are found? Demonstrate that your solution works.

The program brackPlot.m is given in Listing 6.1 of the textbook.

Problem 2: Fixed-point iteration (5 point)

$\boxed{\textbf{6.7}}$ Verify that the behavior of the iteration functions in Example 6.4 is consistent with the convergence criterion $|g'(x)| < 1$ for fixed-point iteration.

You can pick one of the three *g* functions (given in page 251 of the textbook) to verify.

Correction: In the displayed equation in §6.2.1, the convergence criteria for fixed point iteration should read

$$|g'(x)| < 1, \quad \text{and} \quad a \le g(x) \le b, \quad \text{for all } x : a \le x \le b.$$

Problem 3: Newton's method (5 points)

$\boxed{\textbf{6.14}}$ Derive an iterative formula for finding the roots of $\cos(x) = x$ with Newton's method. Starting with an initial guess of $x = 5$ radians, determine the estimate of the root after five iterations. How many iterations are needed to get $f(x) < 5 \times 10^{-10}$ for initial guesses $x_0 = \pi$, $x_0 = 3\pi/2$, and $x_0 = 2\pi$?

Problem 4: Secant method (5 points)

$\boxed{\textbf{6.28}}$ Implement the secant method using Algorithm 6.5 and Equation (6.14). Test your program by re-creating the results in Example 6.10. What happens if 10 iterations are performed? Replace the formula in Equation (6.13) with

$$x_{k+1} = \frac{f(x_k)x_{k-1} - f(x_{k-1})x_k}{f(x_k) - f(x_{k-1}) + \varepsilon},$$

where ε is a small number on the order of ε_m. How does this compare to the results of Exercise 27? Which formulation has better numerical properties?