ISC 4220
Algorithms 1

Howework 1

Due: January 28, 2016 in the Lab

Nonlinear Equations

1. (20 points) Write a Matlab program that uses Newton's method to find the n^{th} root of any real positive number a. Hint: Consider finding the zero of $f(x) = x^n - a = 0$. Use it to find the $\sqrt{2}$, and $5^{1/5}$ to an accuracy of 10^{-4} or better.

2. The story of more and more accurate decimal or rational representations for π is quite fascinating. Archimedes, using polygons of nearly 100 sides, narrowed down the value to

$$\frac{223}{71} < \pi < \frac{22}{7}.$$

Around 1700, John Machin used trigonometry and discovered the identity:

$$\pi = 16 \tan^{-1}\left(\frac{1}{5}\right) - 4 \tan^{-1}\left(\frac{1}{239}\right).$$

- (5 points) Write down the n^{th} order polynomial $p_n(x)$ by performing a Taylor series expansion of $\tan^{-1}(x)$ around $x = 0$.
- (5 points) Use it to find the approximate value of π

  $$\pi \approx T_n = 16 p_n\left(\frac{1}{5}\right) - 4 p_n\left(\frac{1}{239}\right).$$

  for $n = 1, 3, 5, 7,$ and 9.

3. (10 points) Consider the functions $f(x) = \ln(x)$ and $g(x) = 25x^3 - 6x^2 + 7x - 88$. Use zero through third-order Taylor series expansions around the point $x = 1$ for both these functions. Use these to evaluate $f(3)$ and $g(2.5)$. Compute the true relative error for each approximation.

4. (10 points) Consider the function $f(x) = 1 + \sin(x)$. Can the bisection method be used to find it roots? Why or why not? Can Newton's method be used? What order of convergence do you expect, and why?