1 Understanding Outer Products

1. Consider two vectors: $\mathbf{a} = [1, 2, 3]$ and $\mathbf{b} = [1, 3, 1]$. Calculate their outer product $\mathbf{a} \otimes \mathbf{b}$.

2. Without exact calculations, sketch the resulting matrix from the outer product $\mathbf{u} \otimes \mathbf{v}$ of the following pairs of vectors.

   (a) $\mathbf{u} = [5, 4, 3, 2, 1]$ and $\mathbf{v} = [5, 4, 3, 2, 1]$

   (b) $\mathbf{u} = [1, 2, 4, 0, 0]$ and $\mathbf{v} = [3, 1, 2, 0, 0]$

   (c) $\mathbf{u} = [1, 1, 1, -1, -1]$ and $\mathbf{v} = [1, 1, 1, -1, -1]$

   Use shading to represent the relative values in the matrix (darker shades for larger values).

(a)

	1	2	3	4	5
1					
2					
3					
4					
5					

(b)

	1	2	3	4	5
1					
2					
3					
4					
5					

(c)

	1	2	3	4	5
1					
2					
3					
4					
5					

3. Look at the matrices you've sketched. If these matrices represented networks, what kind of network structures might each of them represent?

2 Decomposing Matrices

4. Consider the following matrix representing a small network:

$$\begin{bmatrix} 3 & 2 & 1 & 0 & 1 \\ 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 3 & 2 & 1 \\ 0 & 1 & 2 & 4 & 2 \\ 1 & 0 & 1 & 2 & 3 \end{bmatrix}$$

Try to "decompose" this matrix into the outer product of two vectors with minimal error. Sketch the vectors and use shading to represent their values.

5. Now consider this matrix:

$$\begin{bmatrix} 8 & 4 & 2 & 1 & 0 \\ 4 & 4 & 2 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Can you decompose this into a sum of two outer products? Sketch the vectors for each outer product and use shading.

6. For the matrix in question 7, if you had to keep only one of the two outer products, which would you choose and why? What information about the network would be preserved?