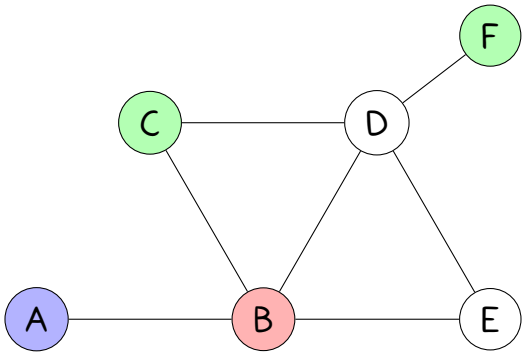
Color Spreading in Social Networks

In a network of students forming study groups, each student starts with a preferred subject, shown by colors - Math (Blue), Science (Green), or Arts (Red). Through their study group interactions, students influence each other's interests. The network diagram below shows students as circles connected by lines representing their study partnerships.



Each color can be represented as a one-hot vector as follows:

- Blue (Math) = [1,0,0]
- Green (Science) = [0,1,0]
- Red (Arts) = [0,0,1]
- White (Undecided) = [0,0,0]

Question 1

Step 1. Let's start by looking at student D, who is currently undecided. Look at D's study partners (connected by lines). What colors do they have?

Think: How could we represent D's neighboring colors as a collection of vectors?

Step 2. For each undecided student, we'll follow this rule: "Adopt the most common subject among your study partners."

Think: What subject would student D choose? Notice that D has four neighbors:

- B (Red): [0,0,1]
- C (Green): [0,1,0]
- E (Undecided): [0,0,0]
- F (Green): [0,1,0]

Step 3. Now look at student E. Using the same rule, what subject would they choose?

Think: Notice how information from distant nodes (like C and F) can influence E through intermediate nodes.

Step 4. Let's formalize this process. For each node:

1. Collect vectors from neighbors: $\{v_1, v_2, ..., v_k\}$
2. Stack them into a matrix: $M = [v_1^T; v_2^T; ...; v_k^T]$
3. Take column-wise maximum: $\max(M)$

Try writing out this process for node D:

$$M_D = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Step 5. If we repeat this process multiple times, what do you notice about the final pattern of colors?

Think: How does this relate to how information or influence spreads in real social networks?

Key Insights

What we've just explored is the foundation of graph convolutional networks! The key insights are:

1. Each node's features (colors) can be represented as vectors 2. The "spreading" of information follows a specific pattern: - Gather information from neighbors (convolution) - Aggregate using a function (max pooling) 3. This process can be repeated multiple times, allowing information to flow through the entire network 4. The final state often depends on both local connections and the overall network structure

This simple model shows how local interactions can lead to global patterns - a key principle in both social networks and neural networks!