Lecture 11. Propositional Logic (Section 1.2.2 and 1.2.6)

Terminology

Propositional logic - deals with statements

Propositional constants:

T - True
F - False

Propositional variables - can have T or F value.

Simple statements: They cannot be further subdivided.

"The sun is shining."

Compound statements: Not simple, contain at least one logical connective.

"The sun is shining and the sky is blue."

English to Logic

- English is often ambiguous. Translating sentences into *symbols with logical connectives* [1] removes the ambiguity:
  1. **Identify** *simple statements* and **represent** using *propositional variables* such as p, q, r....
  2. **Determine** appropriate *logical connectives*.

[1] Propositional logic

[2] The propositional form is a compound statement with propositional variables (such as p, q, r) and logical connectives (such as $\wedge, \vee, \rightarrow$).

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- **Example 11.1.** Translate the following sentence into a propositional form: [2]

  "You can access the AI research lab if you are a robotics specialist or you are not a first-year student."

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  "You can access the AI research lab if you are a robotics specialist or you are not a first-year student."
  - **a**: You can access the AI research lab.
  - **r**: You are a robotics specialist.
  - **f**: You are not a first-year student
  - Propositional logic: $(r \vee \neg f) \rightarrow a$
  - Truth value of the propositional logic

(Truth Table to the NEXT PAGE $\longrightarrow$)

[1] Propositional logic

[2] The propositional form is a compound statement with propositional variables (such as p, q, r) and logical connectives (such as $\wedge, \vee, \rightarrow$).

- **a**: You can access the AI research lab.
- **r**: You are a robotics specialist.
- **f**: You are not a first-year student
- Propositional logic: $(r \vee \neg f) \rightarrow a$

r	f	a	$\neg f$	$r \vee \neg f$	$(r \vee \neg f) \rightarrow a$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	F

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- **r**: You are a robotics specialist.
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- Propositional logic: $(r \vee \neg f) \rightarrow a$

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T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	F

Note: This does not say anything about the fact when $(r \vee \neg f)$ is false, you might or might not have access.

Practice 11.2. Translate the following statement into a propositional form and construct its truth table:

"You can't vote if you are under 18 years old or you are from Mars."

Priority Order of Logic Operations

- **Parentheses**. Before operating on anything else, you must evaluate all parentheticals starting at the innermost level.

Priority	Operator	Comments
1	$\sim$	Evaluate $\sim$ first
2	$\wedge$ $\vee$	Evaluate $\wedge$ and $\vee$ next; Use parenthesis to avoid ambiguity
3	$\rightarrow$ $\leftrightarrow$	Evaluate $\rightarrow$ and $\leftrightarrow$ next; Use parenthesis to avoid ambiguity

- **Example 11.3** For each propositional logic in Python, indicate the value returned.
  - $3 > 4$ or $(2 < 3$ and $9 > 10)$
  - $4 > 5$ or $3 < 4$ and $9 > 8$
  - $\neg(4 > 3$ and $100 > 6)$

Activity 11.4 Consider the statement "If Ahmed is late, then Mustafa is late, and, if both Ahmed and Mustafa are late, then the class is boring."

Given the statement is True, if the class is not boring, what can you conclude about Ahmed? What about Mustafa?