CAB 203 Problem Solving

Rona Rosal: N10360387

1. Linear Algebra

Suppose we have 2 linear equations with 2 unknowns, in this scenario we first want to solve for the nitrogen and phosphate needed by the crop. We will follow the techniques used in CAB203 Lecture 11 and use the equation:

$$ax + by = e$$

$$cx + dy = f$$

applying this equation to our problem, we will represent it as,

$$an * A + bn * B = n \text{ (Nitrogen)}$$

$$ap * A + bp * B = p \text{ (Phosphate)}$$

for x and y. We can represent this as an equation in vectors:

$$\begin{pmatrix} an & bn \\ ap & bp \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} n \\ p \end{pmatrix}$$

We will then solve the given problem by finding the inverse A of the matrix. However, this is only possible if the determinant is not equal to 0. We will check the determinant A first using the equation:

$$\det \begin{pmatrix} an & bn \\ ap & bp \end{pmatrix} := (an)(bp) - (bn)(ap)$$

If the determinant A is not less than or equal to 0. Then we can proceed in solving for the inverse A of the matrices using the following equation:

$$\begin{pmatrix} an & bn \\ ap & bp \end{pmatrix}^{-1} = \frac{1}{(an)(bp)-(bn)(ap)} \begin{pmatrix} bp & -bn \\ -ap & an \end{pmatrix}$$

Once we have determined the inverse A we will use matrix-vector multiplication to find the result by multiplying the inverse A and B (which is (n p)). We will use the '@' operation in this case.

$$\text{result} = \begin{pmatrix} bp & -bn \\ -ap & an \end{pmatrix} @ \begin{pmatrix} n \\ p \end{pmatrix}$$

2. Probability

Using the Bayesian approach, we will calculate the likelihood of the event when the two biased coins are tossed. We will make use of the Bayes rule equation from CAB203 Lecture 12.

$$P(B \mid A) = \frac{P(A|B)\,P(B)}{P(A)}$$

Given the problem, we have two biased coins. We can write the information as follows:

P(H) = 0.7, P(T) = 0.3 where heads is the biased

PH(T) = 0.6, P(H) = 0.4 where tails is the biased

Since we do not have enough knowledge about the coins, and not sure which is which when tossed. We can represent this information as our prior state:

P(H) = 0.5, P(T) = 0.5

Applying the bayes rule to our problem it gives us:

$$P(B|H) = \frac{P(H|B)P(B)}{P(H)}$$

$$= \frac{0.7 \times 0.5}{0.6} = 0.58$$

where $P(H) = P(H|B)P(B) + P(H|U)P(U)$

$$= 0.7 \times 0.5 + 0.5 \times 0.5$$

$$= 0.6$$

This gives us that the likelihood that the coin is biased is 0.58.

We can now calculate the probability of the heads using the formula:

$$P(H) = P(H|B)P(B) + P(H|U)P(U)$$

$$= 0.7 \times 0.5 + 0.5 \times (1 - 0.58)$$

$$= 0.56$$

According to McKague (2022), When finding out the posterior probability of an event this can be achieved using the equation:

$$\text{posterior probability} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}$$

$$P(H_j|E) = \frac{P(E|H_j)P(H_j)}{P(E)}$$

The marginal likelihood $P(E)$ can be found as

$$P(E) = \sum_{j=1}^{n} P(E|H_j)P(H_j)$$

Using the normative decision theory, we can maximize our expected utility using the function:

$$\mathcal{E}_P(u) = \sum_{s \in S} u(s)P(s)$$

We will then choose the highest given utility in our sample space.

Lastly, we will make a bet on the event. Given the problem, we will consider betting on heads on the coin toss game.

H means Heads wins, and T means Tails wins. We have the following choices to make a bet, heads, tails, and no bets.

Given P(H) = 0.7

CHOICE	HEADS	TAILS	EXPECTED UTILITY
Heads	1.1	-1	1.1 x 0.7 + (-1) x 0.3 = 0.47
Tails	-1	0.5	(-1) x 0.7 + 0.5 x 0.3 = -0.55
No Bet	0	0	0

The best choice is to bet on heads.

References:

Mckague, M. (2022). *CAB230 Discrete Structures - Lecture 11: Linear Algebra* [PowerPoint Slides]. Faculty of Science. Queensland University of Technology.

Mckague, M. (2022). *CAB230 Discrete Structures - Lecture 12: Probability* [PowerPoint Slides]. Faculty of Science. Queensland University of Technology.