Numerical Optimization

Assignment 1

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Introduction

This assignment shows testing results of different algorithms' performances by using python.

Theoretical Exercises

Exercise 1

When $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is a strictly convex function, for all $\alpha \in (0,1)$ holds $f(\alpha x + (1-\alpha)y) < \alpha f(x) + (1-\alpha)f(y)$. Then, $f(Ax) = f(A\alpha x + A(1-\alpha)y) < \alpha f(Ax) + (1-\alpha)f(Ay)$, so $f(Ax)$ is also a strictly convex function.

Now we know Q is a symmetric positive definite matrix, $Q = LL^\intercal$, where L is a lower triangular matrix with unit diagonal elements. $x^\intercal Qx = x^\intercal LL^\intercal x = ||L^\intercal x||^2$. Since $||x||^2$ is a strictly convex function and $L^\intercal$ is an invertible matrix. by using previous proved statement, $x^\intercal Qx$ is strictly convex.

Exercise 2

$f_3(x) = \log(\epsilon + f_1(x)), \epsilon = 10^{-4}$

$f_1(x) = \sum_{i=1}^{d} \alpha^{\frac{i-1}{d-1}} x_i^2, \alpha = 1000$

When $d = 1$, $\alpha^{\frac{0}{0}} = 1$, $f_3(x) = \log(\epsilon + x_i^2), \epsilon = 10^{-4}$

$\nabla f_3(x) = \frac{2x_i}{\epsilon + x_i^2}$

$\nabla^2 f_3(x) = \frac{2(\epsilon + x_i^2) - (2x_i)^2}{(\epsilon + x_i^2)^2}$

When $|x| < \sqrt{\epsilon}$, $\nabla^2 f_3(x) > 0$, so $f_3(x)$ is a strictly convex function.

Exercise 3

$f_4(x) = \sum_{i=1}^{d} h(x_i)^2 + 100 \cdot h(-x_i)^2,$

$h(x) = \frac{\log(1+\exp(q \cdot x))}{q}, q = 10^4$

Conclusion

From this assignment, I had a deeper view on the benchmark functions from previous assignment. Also, used python to test different algorithms' performances in common optimization toolboxes in order to get a visualized and further understanding behind the algorithms.