MAD Assignment 2

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Exercise 1 (Weighted Average Loss)

a)

$\mathcal{L} = \frac{1}{N}(Xw - t)^\intercal A(Xw - t)$
$\mathcal{L} = \frac{1}{N}(w^\intercal X^\intercal AXw - 2w^\intercal X^\intercal At + t^\intercal At)$
$\frac{\partial \mathcal{L}}{\partial w} = \frac{2}{N}X^\intercal AXw - \frac{2}{N}X^\intercal At = 0$
$w = (X^\intercal AX)^{-1}X^\intercal At$

b)

I expect it will show more differences compared with Assignment 1. The following figures also show that figure 2 has more points differences compared with figure 1. So the additional weights hace an influence on the outcome.

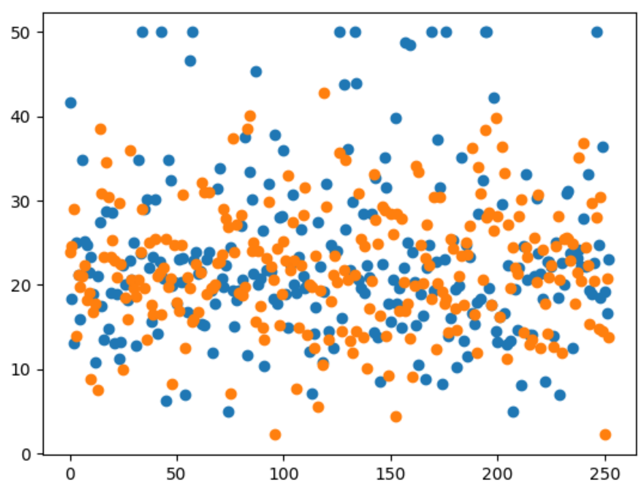

Figure 1: Scattor plot from Assignment 1

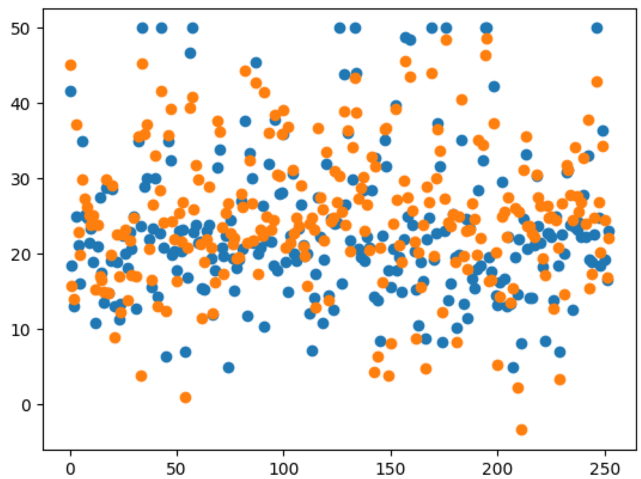

Figure 2: Scattor plot from Assignment 2

Exercise 2 (Polynomial Fitting with Regularized Linear Regression and Cross-Validation)

a)

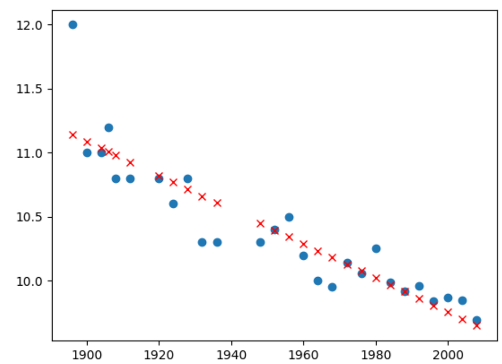

Figure 3:

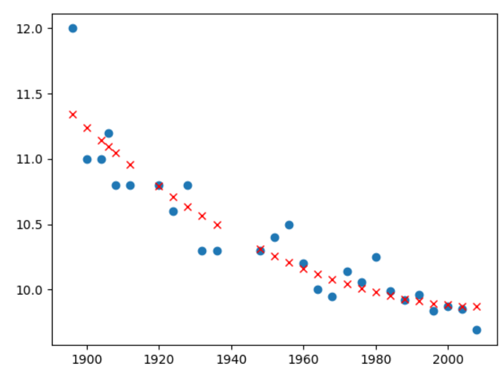

Figure 4:

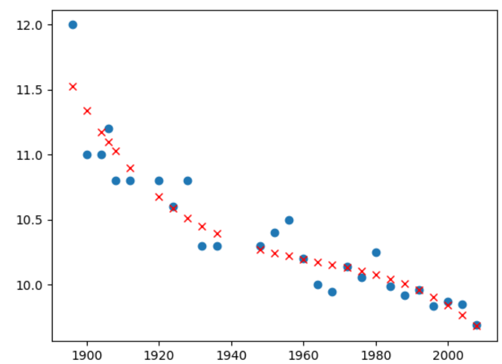

Figure 5:

b)

Exercise 3 (Pdf and Cdf)

a)

Pdf is the derivative of the cdf.

$F'(x) = 0,\ x \leq 0$

$F'(x) = \alpha\beta x^{a-1}\mathrm{e}^{-\beta x^{\alpha}},\ x > 0$

b)

Possibility is the integration of pdf, which is cdf.

$\alpha = 2,\ \beta = \frac{1}{4}$

$F(x) = 1 - \mathrm{e}^{-\frac{1}{4}x^2}$

$x = 4,\ F(4) = 1 - \mathrm{e}^{-\frac{1}{4}4^2} = 1 - \mathrm{e}^{-\frac{1}{4}16} = 1 - \mathrm{e}^{-4}$

$x \geq 4,\ P = 1 - (1 - e^{-4}) = e^{-4} \approx 0.0183$

$x = 5,\ F(5) = 1 - \mathrm{e}^{-\frac{1}{4}5^2} = 1 - \mathrm{e}^{-\frac{25}{4}}$

$x = 10,\ F(10) = 1 - \mathrm{e}^{-\frac{1}{4}10^2} = 1 - \mathrm{e}^{-25}$

$5 \leq x \leq 10,\ P = F(10) - F(5) = (1 - \mathrm{e}^{-25}) - (1 - \mathrm{e}^{-\frac{25}{4}}) = \mathrm{e}^{-\frac{25}{4}} - \mathrm{e}^{-25} \approx 0.00193$

c)

$F(x) = 0.5$

$1 - \mathrm{e}^{-\beta x^{\alpha}} = 0.5$

$\mathrm{e}^{-\beta x^{\alpha}} = 0.5$

$\ln \mathrm{e}^{-\beta x^{\alpha}} = \ln 0.5$

$-\beta x^{\alpha} = \ln 1 - \ln 2$

$\beta x^{\alpha} = \ln 2$

$x^{\alpha} = \frac{\ln 2}{\beta}$

$x = (\frac{\ln 2}{\beta})^{\frac{1}{\alpha}},\ \alpha \neq 0$

Exercise 4 (Conditional Probability and Expectations)

$P(to\ court \mid silent,\ NC) = P(to\ court \mid silent,\ C) = 0.001$
$P(to\ court \mid speak,\ NC) = 0.002$
$P(to\ court \mid speak,\ C) = 0.005$
$P(not\ convicted \mid to\ court,\ NC) = 0.5$
$P(not\ convicted \mid to\ court,\ C) = 0.1$
$P(not\ convicted \mid silent) = \frac{1}{4}P(not\ convicted \mid speak)$
$P(convicted) = 1, P(speak) = 1,\ then\ X_{speak} = 0.25 \times sentence$

a)

$P(convicted \mid to\ court,\ NC) = 1 - P(not\ convicted \mid to\ court,\ NC) = 1 - 0.5 = 0.5$
$P(convicted,\ to\ court,\ speak,\ NC) = P(convicted \mid to\ court,\ speak,\ NC) \times P(to\ court \mid speak,\ NC) \times P(speak = 0.5 \times 0.002 \times 1 = 0.001$
$X_{speak}(convicted) = (1 - 0.75) \times 0.001 \times 5 = 0.00125$
$X_{speak}(not\ convicted) = 0$
$E(X_{speak}) = X_{speak}(convicted) \times P(X_{speak}(convicted)) + X_{speak}(not\ convicted) \times P(X_{speak}(not\ convicted)) = 0.00125 \times 1 + 0 = 0.00125$

b)

$P(convicted \mid to\ court,\ NC) = 1 - P(not\ convicted \mid to\ court,\ NC) = 1 - 0.5 = 0.5$
$P(convicted,\ to\ court,\ silent,\ NC) = P(convicted \mid to\ court,\ silent,\ NC) \times P(to\ court \mid silent,\ NC) \times P(silent) = \frac{1}{4} \times 0.5 \times 0.001 \times 1 = 0.000125$
$X_{silent}(convicted) = 0.000125 \times 5 = 0.000625$
$X_{silent}(not\ convicted) = 0$
$E(X_{silent}) = X_{silent}(convicted) \times P(X_{silent}(convicted)) + X_{silent}(not\ convicted) \times P(X_{silent}(not\ convicted)) = 0.000625 \times 1 + 0 = 0.000625$

Peter should remain silent to get the least sentence duration.

c)

$P(convicted \mid to\ court,\ C) = 1 - P(not\ convicted \mid to\ court,\ C) = 1 - 0.1 = 0.9$

$P(convicted,\ to\ court,\ speak,\ C) = P(convicted \mid to\ court,\ speak,\ C) \times P(to\ court \mid speak,\ C) \times P(speak) = 0.9 \times 0.005 \times 1 = 0.0045$

$Y_{speak}(convicted) = (1 - 0.75) \times 0.0045 \times 5 = 0.005625$

$Y_{speak}(not\ convicted) = 0$

$E(Y_{speak}) = Y_{speak}(convicted) \times P(Y_{speak}(convicted)) + Y_{speak}(not\ convicted) \times P(Y_{speak}(not\ convicted)) = 0.005625 \times 1 + 0 = 0.005625$

$P(convicted,\ to\ court,\ silent,\ C) = P(convicted \mid to\ court,\ silent,\ C) \times P(to\ court \mid silent,\ C) \times P(silent) = \frac{1}{4} \times 0.9 \times 0.001 \times 1 = 0.000225$

$Y_{silent}(convicted) = 0.000225 \times 5 = 0.001125$

$Y_{silent}(not\ convicted) = 0$

$E(Y_{silent}) = Y_{silent}(convicted) \times P(Y_{silent}(convicted)) + Y_{silent}(not\ convicted) \times P(Y_{silent}(not\ convicted)) = 0.001125 \times 1 + 0 = 0.001125$

Brian should remain silent to get the least sentence duration.