3.1.4 Hint: Reverse KL minimization

You are instructed to use $q(x,y) = q(x)q(y)$ approximation. Therefore,

$$\mathrm{KL}(q(x,y)||p(x,y)) = \sum_{x,y} q(x,y) \log \frac{q(x,y)}{p(x,y)} = \sum_{x,y} q(x)q(y) \log \frac{q(x)q(y)}{p(x,y)}$$
$$= \sum_{x} q(x) \log q(x) + \sum_{y} q(y) \log q(y) - \sum_{x,y} q(x)q(y) \log p(x,y)$$

where we have used $\sum_y q(y) = \sum_x q(x) = 1$.

Setting derivative wrt $q(x)$ to zero:

$$\frac{\partial}{\partial q(x)} \mathrm{KL}(q(x,y)||p(x,y)) = 0$$
$$\frac{\partial}{\partial q(x)} \sum_{x} q(x) \log q(x) + \frac{\partial}{\partial q(x)} \sum_{y} q(y) \log q(y) = \frac{\partial}{\partial q(x)} \sum_{x} q(x) \sum_{y} q(y) \log p(x,y)$$

Applying the partial derivatives yields (look up functional derivatives for more details),

$$\big(1 + \log q(x)\big) + 0 = \sum_{y} q(y) \log p(x,y)$$
$$\big(1 + \log q(x)\big) = \mathbb{E}_{q_y}(\log p(x,y))$$

Notice that the terms on the right hand side are only related to x due to the expectation wrt $q(y)$.

Similarly setting derivative wrt $q(y)$ to zero:

$$\big(1 + \log q(y)\big) = \mathbb{E}_{q_x}(\log p(x,y))$$

These systems of equations can be solved to obtain three minima.