1 a) Lecture Notes / Scripts / online resources

b) Check $s = 101$

$000 \oplus 101 = 101 \rightarrow f(000) = f(101)$ ✓

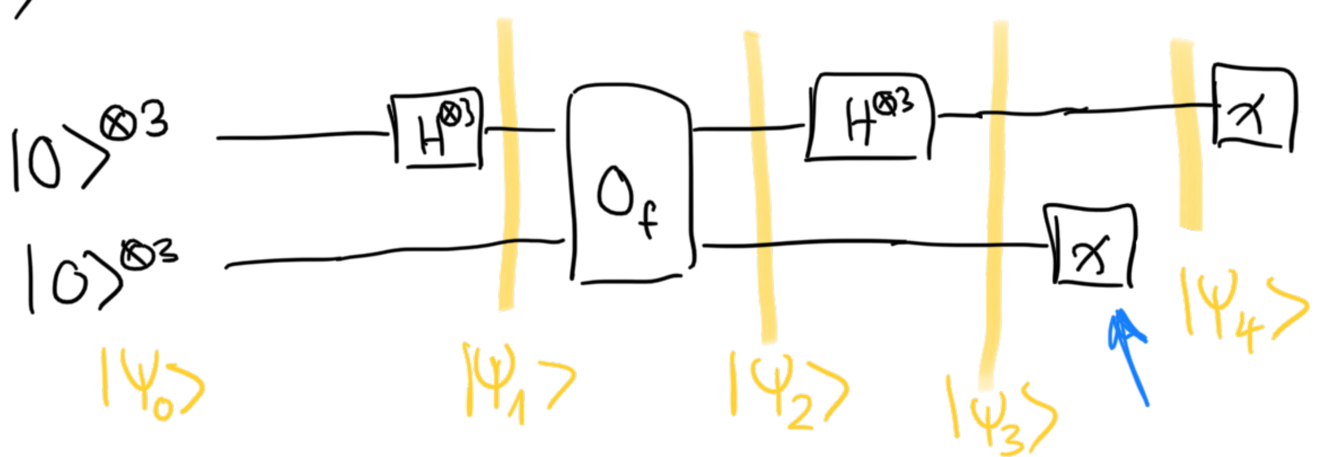
$001 \oplus 101 = 100 \rightarrow f(001) = f(100)$ ✓

$010 \oplus 101 = 111 \rightarrow f(010) = f(111)$ ✓

$011 \oplus 101 = 110 \rightarrow f(011) = f(110)$ ✓

fits w/ given function

c)



d) $|\psi_0\rangle = |000\rangle \otimes |000\rangle$

After first Hadamards:

$$|\psi_1\rangle = |{+}{+}{+}\rangle = \frac{1}{\sqrt{2^3}}\left(|000\rangle + |100\rangle + |010\rangle + |001\rangle + |011\rangle + |110\rangle + |101\rangle + |111\rangle\right)$$

After oracle: $|x\rangle|0\rangle \mapsto |x\rangle|f(x)\rangle$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}^3}\Big[(|000\rangle + |101\rangle)\otimes|01\rangle + (|001\rangle + |100\rangle)\otimes|10\rangle + (|010\rangle+|111\rangle)\otimes|11\rangle + (|011\rangle+|110\rangle)\otimes|00\rangle\Big]$$

Second Hadamards: just the terms w/ $|01\rangle$

$$\frac{1}{\sqrt{2}^3} H\otimes H\otimes H\,(|000\rangle + |101\rangle)$$

$$= \frac{1}{\sqrt{2}^3}\frac{1}{\sqrt{2}^3}\Big[|000\rangle+|010\rangle+|100\rangle+|001\rangle+|110\rangle + |101\rangle+|011\rangle+|111\rangle + \underbrace{(|0\rangle-|1\rangle)\otimes|0\rangle+|1\rangle\otimes(|0\rangle-|1\rangle)}\Big]$$

$$|000\rangle+|010\rangle-|001\rangle-|011\rangle$$
$$-|100\rangle-|110\rangle+|101\rangle+|111\rangle$$

$$= \frac{1}{\sqrt{2}^6}\Big[2|000\rangle + 2|010\rangle + 2|101\rangle + 2|111\rangle\Big]$$

$$= \frac{1}{2^3}\Big[|000\rangle + |010\rangle + |101\rangle + |111\rangle\Big]$$

After 2nd Hadamards:

$$|\psi\rangle = \frac{1}{\ }\ \frac{1}{\ }\Big[(|000\rangle \ldots$$

$$|\psi_3\rangle = \underbrace{\frac{1}{\sqrt{2^3}}\frac{1}{\sqrt{2^3}}}_{\frac{1}{2^3} = \frac{1}{8}}\Big[(|000\rangle + |101\rangle + |010\rangle + |111\rangle)\otimes|01\rangle$$
$$+ \underbrace{\ldots}\,|00\rangle + \underbrace{\ldots}_{\text{other terms}}\otimes|10\rangle + \underbrace{\ldots}\otimes 11\Big]$$

Oracle measurement: get 01

$$\rightarrow |\psi_4\rangle = \frac{1}{2}(|000\rangle + |101\rangle + |010\rangle + |111\rangle)$$

e+f)

If f was one-to-one, there would not be any terms like $(|000\rangle + |101\rangle)\otimes f_{000}$

only $\sum_x |x\rangle f_x$ with one x (bitstring of length 3) for each f.

Then, measuring f_x leads to a single state $|x\rangle$. Applying Hadamards $\rightarrow$ for each f_x, get a superposition of $|000\rangle, |001\rangle, |100\rangle, \ldots$ with prefactors $\pm\frac{1}{2^3} = \pm\frac{1}{8}$

$\rightarrow$ probability would be equally distributed, getting each bitstring x w/ prob $1/16$.

If f is not one way, some results appear w/ prob 0.

Case: Oracle measurement gives 01.

How to infer s from the results?

$$P_{000} = \frac{1}{4} \xrightarrow[\text{know}]{\text{we}} s \cdot 000 \text{ even}$$

[others: $p = 0$; $s \cdot 110$ odd]

$$P_{101} = \frac{1}{4} \longrightarrow s \cdot 101 \text{ even}$$

$$P_{010} = \frac{1}{4} \longrightarrow s \cdot 010 \text{ even}$$

$$P_{111} = \frac{1}{4} \longrightarrow s \cdot 111 \text{ even}$$

$$s \cdot 000 = 0$$

$$s \cdot 101 = 0 = s_1 + s_3$$

$$s \cdot 111 = s_1 + s_2 + s_3 = 0$$

$$s \cdot 010 = s_2 = 0$$

$$\Rightarrow s_2 = 0$$

$$\Rightarrow s_1 + s_3 = 0 \Rightarrow s_1 = s_3 = 1$$

$$\Longrightarrow s = 101.$$