Introduction to Quantum Computing

Assignment 2: quantum algorithms and error correction
University of Copenhagen

This assignment is 2 pages long and consists of 3 problems. Students are requested to formulate their solutions individually. Cases of overlaps that indicate copying of other students' work will not be accepted and are handled by the general rules of the faculty. Please hand in your solutions as a single PDF file via Absalon — both LaTeX and readable pictures of your hand-written solutions are accepted. Please include a front page with your name and date of birth as well as an indication of the total number of pages.

Exercise 1: (Magic states & T-gates (30%)) In the following, we will see how to use so-called Magic-states as a resource to substitute for T-gates in a quantum circuit. The advantage is that these states can be prepared beforehand whereas during the computation only CNOT gates and S gates are required. Hence, suppose we have a qubit in the state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ to which we would like to apply the T-gate - however we assume that this is difficult in our available architecture. Luckily enough, we have a second qubit, which is in the state $\frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\frac{\pi}{4}}|1\rangle\right)$ and we can apply the CNOT and the S-gate.

a) What state do we get if we apply a CNOT to the first and second qubit?

b) Suppose we measure the second qubit in the computational basis. What are the probabilities of outcomes 0 and 1, respectively?

c) Suppose the measurement yields 0. Show how we can get $T|\phi\rangle$ in the first qubit.

d) Suppose the measurement yields 1. Show how we can get $T|\phi\rangle$ in the first qubit, up to an (irrelevant) global phase.

Exercise 2: (Period-finding (40%)) Consider the following function:

$$f(x) = 7^x \mod 10$$

a) What is the periodicity of this function if x takes integer values?

b) Let $d = 128$, and consider a quantum systems with d levels with the computational basis $\{|x\rangle\}_{x=0}^{d-1}$. How many qubits m do we need to represent this system? The period-finding algorithm has the following steps

   i) Take $|0\rangle \otimes |0\rangle$ as an input

ii) Apply QFT_d to the first system.

iii) Apply the oracle corresponding to $f(x)$ to the first qubit

vi) Measure the oracle system

v) Apply QFT_d to the first system and perform a measurement

Assume that the measurement on the oracle system gives result 1. Give the state after each step (as explicitly as possible), and show how this leads to information about the periodicity of $f(x)$.

Exercise 3: (Error-correcting codes (30%)) Consider the following 4-qubit code which allows to detect a bitflip and/or a phaseflip. By this we mean that after the detection procedure we either have the original uncorrupted state back, or we know that an error occurred (though we do not know which one). The logical 0 and 1 are encoded as:

$$|\bar{0}\rangle = \frac{1}{2}(|00\rangle + |11\rangle) \otimes (|00\rangle + |11\rangle) \qquad |\bar{1}\rangle = \frac{1}{2}(|00\rangle - |11\rangle) \otimes (|00\rangle - |11\rangle)$$

a) Give a procedure (either as a circuit or as sufficiently-detailed pseudo-code) that detects a bitflip error on one of the 4 qubits of $\alpha |0\rangle + \beta |1\rangle$.

b) Give a procedure (either as a circuit or as sufficiently-detailed pseudo-code) that detects a phaseflip error on one of the 4 qubits of $\alpha |0\rangle + \beta |1\rangle$.

c) Does that mean that we can now detect any unitary 1-qubit error on one of the 4 qubits? Explain your answer.