Detumbling of Spacecraft with electromagnets and geomagnetic field after deployment from launch vehicles

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Inputs:

The following inputs are required:-

1)Inertia Matrix(3 * 3)
2)Time in YMDhms format
3)Initial Angular Velocity and Quaternion
4)Keplerian orbital elements:

- a)Semi-Major Axis
- b)Eccentricity(e)
- c)The inclination of the orbit(*in degrees*)(i)
- d)Longitude of ascending node(*in degrees*)(Ω)
- e)Argument of Perigee(*in degrees*)(ω)
- f)Mean anomaly of the epoch(*in degrees*)(M)

Procedure:

STEP 1: Calculating Eccentric anomaly(E)

```
big_e=big_m;
delta_e=10;eps=1e-15;
max_iter=100;count=1;

while abs(delta_e) > eps
  delta_e=(big_m-(big_e-ecc*sin(big_e)))/(1-ecc*cos(big_e));
  big_e=big_e+delta_e;
  count = count + 1;
  if count == max_iter, break, disp(' Maximum Number of Iterations Achieved'), end
end
```

Initializing, E_0 = M (Initial Guess)
Using Newtons Method of approximation

$$E_{i+1} = E_i - \frac{f(E_i)}{f'(E_i)}$$

Where

$$f(E) = E - e\sin E - M$$

STEP 2:Calculating initial position (**r**) and velocity(**v**=**r**-dot)

$$\begin{aligned} \boldsymbol{r} &= \hat{x}\,\boldsymbol{P} + \hat{y}\,\boldsymbol{Q} \\ &= r\cos\nu\,\boldsymbol{P} + r\sin\nu\,\boldsymbol{Q} \\ &= a(\cos E - e)\,\boldsymbol{P} + a\sqrt{1-e^2}\sin E\,\boldsymbol{Q} \end{aligned}$$

And the velocity(**v**) by

$$\begin{aligned} \dot{\boldsymbol{r}} &= \dot{\hat{x}}\,\boldsymbol{P} + \dot{\hat{y}}\,\boldsymbol{Q} \\ &= \frac{\sqrt{GM_{\oplus}a}}{r}\left(-\sin E\,\boldsymbol{P} + \sqrt{1-e^2}\cos E\,\boldsymbol{Q}\right) \quad , \end{aligned}$$

Now,

$$\boldsymbol{P} = \begin{pmatrix} +\cos\omega\cos\Omega - \sin\omega\cos i\sin\Omega \\ +\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega \\ +\sin\omega\sin i \end{pmatrix}$$

And

$$\boldsymbol{Q} = \begin{pmatrix} -\sin\omega\cos\Omega - \cos\omega\cos i\sin\Omega \\ -\sin\omega\sin\Omega + \cos\omega\cos i\cos\Omega \\ +\cos\omega\sin i \end{pmatrix}$$

```
rmag=a*(1-ecc*cos(big_e));%2.31
rp=a*(cos(big_e)-ecc);%2.43 magnitude to be multiplied with unit vector P
rq=a*sqrt(1-ecc^2)*sin(big_e);%2.43 magnitude to be multiplied with unit vector Q
vp=-sqrt(mu*a)/rmag*sin(big_e);%2.44
vq=sqrt(mu*a*(1-ecc^2))/rmag*cos(big_e);%2.44
c11=cos(big_omega)*cos(w)-sin(big_omega)*sin(w)*cos(inc);%P1
c21=sin(big_omega)*cos(w)+cos(big_omega)*sin(w)*cos(inc);%P2
c31=sin(w)*sin(inc);%P3
c12=-cos(big_omega)*sin(w)-sin(big_omega)*cos(w)*cos(inc);%Q1
c22=-sin(big_omega)*sin(w)+cos(big_omega)*cos(w)*cos(inc);%Q2
c32=cos(w)*sin(inc);%q3
r1=c11*rp+c12*rq;r2=c21*rp+c22*rq;r3=c31*rp+c32*rq;%2.43
v1=c11*vp+c12*vq;v2=c21*vp+c22*vq;v3=c31*vp+c32*vq;%2.44
r0=[r1;r2;r3];
v0=[v1;v2;v3];
```

STEP 4:

Using Runge Kutta approximation we find the coordinates and velocity of the whole orbit such that

$$dy/dt = f(t, y)$$

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

Where, (dt = h)

$$\begin{aligned}
k_1 &= h\ f(t_n, y_n), \\
k_2 &= h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \\
k_3 &= h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right), \\
k_4 &= h\ f\left(t_n + h, y_n + k_3\right).
\end{aligned}$$

STEP 5:

We find the magnetic field for the whole orbit using WMM (World Magnetic Model)

STEP 6: Main Loop

Part A) Change the frame of the magnetic field from eci to body frame using

$$B_{body} = AB_{eci}$$

Where, A is Attitude Matrix formed using quaternion such that

$$A(\mathbf{q}) = \left(q_4^2 - \|\mathbf{q}_{1:3}\|^2\right) I_3 - 2q_4[\mathbf{q}_{1:3}\times] + 2\,\mathbf{q}_{1:3}\,\mathbf{q}_{1:3}^T$$
$$= \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_2q_1 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_3q_1 + q_2q_4) & 2(q_3q_2 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

$$L = -k(I_3 - b_{body}b^T{}_{body})\omega$$

$$m = (+k/||B||)\omega * b_{body}$$

Calculating next quaternion and angular velocity using Runge Kutta after d*t* time.;o9[