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Portfolio Optimization with Prediction-Based Return Using Long Short-Term Memory Neural Networks: Testing on Upward and Downward European Markets

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Abstract

In recent years, artificial intelligence has helped to improve processes and performance in many different areas: in the field of portfolio optimization, the inputs play a crucial role, and the use of machine learning algorithms can improve the estimation of the inputs to create robust portfolios able to generate returns consistently. This paper combines classical mean–variance optimization and machine learning techniques, concretely long short-term memory neural networks to provide more accurate predicted returns and generate profitable portfolios for 10 holding periods that present different financial contexts. The proposed algorithm is trained and tested with historical EURO STOXX 50® Index data from January 2015 to December 2020, and from January 2021 to June 2022, respectively. Empirical results show that our LSTM neural networks are able to achieve minor predictive errors since the average of the MSE of the 10 holding periods is 0.00047, the average of the MAE is 0.01634, and predict the direction of returns with an average accuracy over the 10 investment periods of 95.8%. Our prediction-based portfolios consistently beat the EURO STOXX 50® Index, achieving superior positive results even during bear markets.

Keywords Portfolio optimization · Return prediction · Asset allocation · Deep learning · Neural networks

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1 Introduction

The prediction of the financial market behavior and optimal budget allocation to specific stocks is one of the main research topics in the financial field. Various factors including financial or monetary policies, exchange rates, inflation, or interest rates, influence financial markets (Hamdani et al., 2020). The complexity and multitude of factors impacting financial markets have made the selection of assets in a portfolio a challenging problem that has been studied by numerous authors.

Since 1952, when Harry Markowitz presented the mean–variance (MV) portfolio selection model (Markowitz, 1952, 1959), different approaches have been applied by researchers to address the topic of portfolio optimization. Markowitz's approach is the cornerstone of the modern portfolio theory (MPT). Subsequently, numerous other authors, including Tobin, who published his work on the risk aversion liquidity preference theory (Tobin, 1958), or Sharpe who extended Markowitz's ideas (Sharpe, 1963), contributed to the development of the field. Since then, many other academics and practitioners have published their studies related to asset pricing (Fabozzi, 1999; Fama, 1996; Sharpe, 1964).

One limitation of Markowitz's model is its sensitivity to the inputs, where the allocation of the weights for portfolio assets varies based on estimated returns, variance, and covariance (Kolm et al., 2014; Michaud & Michaud, 2008). Consequently, inaccurate estimations of expected returns can present a poor performance out of the sample, missing the ability to generalize with unknown data. This underscores the necessity for new methods that can robustly handle the estimation and provide a more stable performance.

Additionally, some studies have shown that optimized portfolios using the mean–variance model have been outperformed by equally weighted portfolios (Jorion, 1985; Korkie & Jobson, 1981). These sub-optimal weights are often attributed to estimation errors in expected returns (Chopra & Ziemba, 1993), further highlighting the need for approaches that can mitigate the estimation of these errors.

There have been advances in the estimation of the parameters such as the Black-Litterman model (Black & Litterman, 1992), Bayes estimator (Jorion, 1986), and robust estimators (DeMiguel & Nogales, 2009). Besides, artificial intelligence has proven its ability to enhance expected return estimation, employing machine learning techniques for improved accuracy in predicting the expected returns (Ban et al., 2018; Chen et al., 2021; Ma et al., 2021) and covariance (De Prado, 2016).

The main objective of this research is to propose an alternative solution to one of the major limitations of portfolio optimization, which is the estimation of input parameters, by applying machine learning algorithms. Specifically, we develop long short-term memory (LSTM) recurrent neural networks (RNN) to predict the expected returns to perform prediction-based portfolio allocations. Therefore, considering the gaps in the current literature, our contribution and the distinctiveness of our paper with respect to the existing literature can be characterized as threefold.

First, by developing sliding window-based LSTM RNN we improve the prediction of future returns. Consequently, more accurate expected returns would improve the allocation of weights in the construction of optimal portfolios. In our study, we treat each stock's prediction independently as a univariate time series regression problem, given the index comprises companies from various Eurozone countries with differing trading days.

Second, by combining our predicted future returns and classic mean–variance portfolio optimization, we are able to construct optimal portfolios for several short and medium-term investment periods that consistently beat the main stock index of the Eurozone, based on a free-float market cap, and the equally weighted portfolio over the analyzed periods, demonstrating that active portfolio management based on the output of our algorithm achieves superior returns compared to passive management.

Third, this paper focuses on the European market to construct optimal prediction-based portfolios to obtain superior returns. We evaluate our investment strategies over two very different scenarios. On the one hand, we use 2021, a period in which the market shows an upward trend and consistent growth, showing that our model performs better than the benchmark in favorable market conditions. On the other hand, we also use the first half of 2022 to evaluate our model, a period that presents a downward trend, with prices going down due to the war in Ukraine, growing inflation, interest rates increased by central banks, and recession concerns among others. Thus, testing our machine learning model and investment strategies during this period allows us to analyze the performance during bear market conditions. This aspect sets our work apart from other papers, as machine learning algorithms are typically not evaluated under adverse market conditions. This study demonstrates the ability of our LSTM to predict negative growth and create investment strategies that beat the market in this context.

The remainder of this paper is structured as follows. Section 2 reviews previous studies directly related to this paper, summarizing the different methodologies followed and briefly mentioning the results obtained empirically. Section 3 presents theoretical and practical knowledge about LSTM and portfolio optimization and describes the methodology employed, including the data source, the treatment of the data, the LSTM architecture, and states the portfolio optimization problem. Section 4 provides the experimental results. Section 5 explores the significance of the results of the work and draws a conclusion.

2 Literature Review

The prediction of the inputs used in portfolio optimization represents one of the main challenges in the field of portfolio management. The optimal allocation of the assets that make up the portfolio depends on the estimation of the expected return and the variance–covariance matrix. As an estimation of the future may be uncertain, the returns and the variance–covariance matrix could be inaccurately estimated, giving place to poor out-of-sample performance (Basile & Ferrari, 2016). In addition, the sensitivity of portfolio weights to changes in the means of the assets is considerably high (Best & Grauer, 1991).

Many studies use conventional models to predict the price of stocks like autoregressive integrated moving average (ARIMA) (Adebiyi et al., 2014; Mondal et al., 2014) or generalized autoregressive conditional heteroskedasticity (GARCH) (Herwartz, 2017). However, it has been shown that machine learning and deep learning algorithms, such as neural networks, achieve better accuracy than conventional methods in the prediction of time series. Models like ARIMA or GARCH are able to capture linear relations in the data. Nevertheless, considering the inherent assumption of linearity in these models, they fall short in capturing complex non-linear relations, particularly in longer forecasting horizons (Adebiyi et al., 2014; Ghiassi et al., 2005; Rius et al., 1998). Moreover, one of the significant advantages of using artificial intelligence techniques, such as LSTM networks, in stock price prediction is their ability to model the data without the need to assume the normality of the distribution (Hansen & Nelson, 2002).

With the aim of predicting stock prices, machine learning models have been applied by many researchers. Lin et al. (2006) published their dynamic portfolio selection model, where they simulated the dynamic behavior of securities by using a recurrent neural network (RNN), the Elman network. The results are compared to the vector autoregressive (VAR) model, which was outperformed by the RNN. Freitas et al., (2009) developed a method called autoregressive moving reference neural network to optimize a portfolio based on the predicted values of Brazilian stocks, obtaining better results than the MV model and outperforming the IBOVESPA, Brazilian market index. This conclusion is based on several evaluation metrics presented by the authors, which are Mean Error (ME), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and Hit Rates.

Alizadeh et al. (2011) used an adaptive neuro-fuzzy inference system to predict the return. Portfolio optimization based on predicted returns shows a better performance than Markowitz's model, a multiple regression, a neural network, and the Sugeno-Yasukawa method in terms of minimum RMSE. Huang (2012) developed a hybrid methodology that used support vector regression (SVR) and genetic algorithms (GAs) for stock selection to obtain higher returns than the proposed benchmark. Ticknor (2013) proposed a Bayesian regularized artificial neural network to predict the closing price of stocks on the following day using MAPE as a performance metric. The results obtained by the model are comparable to the fusion model of HMM and the ARIMA model proposed by Hassan et al. (2007).

Patel et al. (2015) applied several machine learning techniques to predict two Indian stock market indices. The authors combined SVR with artificial neural networks (ANN), random forest (RF), and SVR itself. The results are compared to the non-hybrid versions of these algorithms, being the hybrid models the ones achieving better performance in terms of MAPE, MAE (Mean Absolute Error), relative RMSE, and MSE (Mean Squared Error). Wang and Wang (2015) predicted financial time series using principal component analysis and a stochastic time-effective neural network (PCA-STNN). The proposed model outperformed a traditional backpropagation neural network (BPNN), principal component analysis combined with BPNN, and the stochastic time-effective neural network. In order to assess the performance of the models, the authors used MAE, RMSE, and MAPE as metrics.

Baek and Kim (2018) developed a model, ModAugNet, based on data augmentation that using LSTM prevented overfitting and predicted the stock market index. ModAugNet outperformed a model that did not consider overfitting prevention in MSE, MAPE, and MAE. Kim and Won (2018) developed a hybrid model combining LSTM with GARCH models, which performed better than existing models such as GARCH, exponential GARCH, or LSTM. They used MAE, MSE, heteroscedasticity-adjusted MAE, and heteroscedasticity-adjusted MSE to compare the performance of the models. In their comparative study, Lee and Yoo (2020) showed that LSTM predictions present a better result than RNN and gated recurrent unit evaluating the predictive ability of the models by using the Hit Ratio. Rezaei et al. (2021) proposed a hybrid deep learning model to predict the stock price and then optimized the portfolio using prediction-based inputs using the Black-Litterman model. The hybrid model, which consists of a combination of complete ensemble empirical mode decomposition, convolutional neural network, and LSTM performed better than the MV portfolio, the Black-Litterman portfolio, and the equally weighted portfolio, in terms of MSE, MAE, and normalized MSE. Collectively, these studies underscore the potential of LSTM-based models as a superior method in financial forecasting.

Ma et al. (2021) combined several machine learning and deep learning models with mean–variance and omega portfolio optimization for daily trading investment in the China Securities Index 100. The results show that the combination of Random Forest (RF) and mean–variance optimization is the one that performed better based on several metrics such as expected return, standard deviation, information ratio, or turnover rate. Also, considering only stock return prediction, RF presented a lower MSE and MAE than the other models.

Du (2022) predicted the return of CSI 300 and S&P 500 with SVM, random forest, and attention-based LSTM, being the last one, the machine learning technique with the best results compared to the others. Predicted returns were evaluated using MSE, MAE, and Hit Ratios, achieving an accuracy superior to 90% for both analyzed markets. This high level of accuracy underscores the effectiveness of attention-based LSTMs in forecasting financial market movements.

All this literature shows the growing importance of artificial intelligence and machine learning algorithms in financial markets, concretely in the prediction of stock prices and returns. Thus, this paper aims to complete the research on the topic by obtaining more accurate price predictions and combining them with mean–variance optimization creating optimal portfolios that generate superior returns in the European market for different investment horizons, including both favorable and unfavorable market conditions.

3 Material and Methods

3.1 Dataset and Data Treatment

This research has exploited historical closing price data of the components of the EURO STOXX 50® Index from January 1, 2015, to June 30, 2022, on a trading day basis, covering a total of 1903 trading days. The EURO STOXX 50® Index is composed of the 50 largest companies in the Eurozone based on a free-float market cap. The data is obtained from Yahoo! Finance.

As we adopt the technical approach, we believe that despite the importance of the macroeconomic situation, news, and fundamentals, prices fully reflect all the available information and facts that impact financial markets (Mok et al., 2004). In addition, using daily prices instead of weekly or monthly improves the training process of the neural network, as machine learning algorithms' performance increases exponentially with the increase in the amount of data. Also, other studies use daily information, which makes it easier to compare the results of our research (Chen et al., 2021; Du, 2022; Ma et al., 2021; Weng et al., 2018).

In this study, we approach the task as a univariate time series regression problem, where each stock's prediction is handled independently. This approach is particularly relevant because we are dealing with an index comprising companies from various Eurozone countries, which often have differing trading days. Additionally, not all the companies were included in the index on the same date. The missing values are dropped out of the dataset.

The data is normalized by using Min–Max Scaler before training the model. The estimator scales and transforms the values into a given range, in this case between [0, 1] (Pedregosa et al., 2011). The following Eq. (1) presents the mathematical formulation of the Min–Max scaler:

$$x_{t,i_{scaled}} = \frac{x_{t,i} - \min(x_i)}{\max(x_i) - \min(x_i)} \tag{1}$$

where $x_{t,i_{scaled}}$ is the normalized value of $x_{t,i}$, which is the price of the stock i at a due date t. Being $\min(x_i)$ and $\max(x_i)$ the minimum and maximum of x_i, respectively. x_i represents the vector of prices of the stock i for the considered period.

We use a sliding window to generate overlapping sequences of consecutive trading days with a size of 42, corresponding to approximately two months of trading. Thus, the next consecutive price is predicted based on 42 closing stock prices, creating input–output data that will be used to train our long-short term memory. Table 1 illustrates the autoregressive sequence pattern (Jansen, 2020).

Table 1 Sliding window sequence representation: This table illustrates the sliding window sequence used in predictive modeling. It displays 42 consecutive trading days as the input and the subsequent trading day as the output. The table demonstrates how the prices over these 42 days are used to predict the price for the next day

Input	Output
$\langle x1, x2, \ldots, x41, x42 \rangle$	$\langle x43 \rangle$
$\langle x2, x3, \ldots, x42, x43 \rangle$	$\langle x44 \rangle$
$\vdots$	$\vdots$
$\langle xT-42, xT-41, \ldots, xT-2, xT-1 \rangle$	$\langle xT \rangle$

We select a sliding window of 42, since, after testing several options (displayed in Table 3), it provided better results.

The scaled dataset is split into two datasets. We use data from 2015 to 2020, both included, to train the model and data corresponding to 2021 and the first half of 2022 to test it. 25% of the training dataset is used to validate the model's performance while tuning the hyperparameters. Usually, between 70 and 80% of the training set is used to train, and the remaining 30–20% is used to validate the model. For instance, Ma et al. (2021) used the first four years to train and the following year to validate, representing an 80–20% approach. Using different data to train, validate and test allows us to evaluate the ability of the model to generalize. A summary of the data split is shown in Table 2:

3.2 Methodology

This study's methodology can be divided into two parts. Firstly, the stock price of all the components of the EURO STOXX 50® Index is predicted by using long short-term memory neural networks after creating overlapping sequences employing rolling windows. Secondly, the prediction-based portfolio optimization uses the outputs of the LSTM to find the optimal portfolio with the highest Sharpe ratio and evaluate whether obtained portfolios outperform the benchmarks for the different investment periods considered.

3.2.1 LSTM Prediction

Recurrent neural networks are a type of artificial neural network that can learn patterns by using sequential information or time-series data as input. RNNs keep a hidden state that acts as internal memory, in this way the output depends on the input and the previous hidden state. However, RNNs present some challenges. When errors are backpropagated many time steps through a large sequence, it is possible to experience vanishing or exploding gradients. In addition, RNNs are difficult to train because when gradients vanish, the influence of short-term dependencies is predominant in the weights of gradients, and they could be inefficient to learn long-term dependencies (Bengio et al., 1994; Hochreiter, 1998; Hochreiter et al., 2001).

Long short-term memory is a variant network architecture of RNNs. LSTM arises in 1997 as a solution or alternative method to solve the problems of traditional RNN.

Table 2 Data subsets split: This table illustrate the division of the dataset into various subsets for model training and evaluation

Date	Dataset	Percentage of data points with respect to the date (%)
Jan. 1st, 2015-Dec. 31st, 2020	Training	75
Jan. 1st, 2015-Dec. 31st, 2020	Validation	25
Jan. 1st, 2021-Jun. 30th, 2022	Test	100

LSTM networks are faster and able to solve complex problems that were not solved by preceding recurrent neural networks (Hochreiter & Schmidhuber, 1997). This type of architecture addresses the problem of long-range dependencies and allows for tracking dependencies between the elements of the sequence. LSTM presents an additional internal state called "cell state" which contains one input gate i_t, one forget gate f_t, and one output gate o_t that controls the new information, manages the information that should be voided from the memory of the LSTM, and controls when the information should be processed, respectively (Gers et al., 2002; Jansen, 2020). The following formulas show the calculations associated with each mentioned gate, the cell state, and the hidden state:

$$i_t = \sigma\left(W_i x_t + Y_i h_{t-1} + b_i\right) \tag{2}$$

$$f_t = \sigma\left(W_f x_t + Y_f h_{t-1} + b_f\right) \tag{3}$$

$$o_t = \sigma\left(W_o x_t + Y_o h_{t-1} + b_o\right) \tag{4}$$

$$c_t = c_{t-1} * f_t + \eta_t * i_t \tag{5}$$

$$h_t = tanh\left(c_t\right) * o_t \tag{6}$$

where W_i, W_f, W_o, Y_i, Y_f and Y_o represent weight matrices, b_i, b_f, b_o are bias vectors, c_t is the cell state at time t, η_t corresponds to the input candidate at time t, which is regulated by the input gate, and h_t is the hidden state at time t and it is updated by using hyperbolic tangent activation. In the calculation of the input, forget, and output gate sigmoid activation is used, represented as σ, and computed as $\sigma(x) = \frac{1}{1+e^{-x}}$.

Table 3 Parameters and values considered during the LSTM's training: This table provides an overview of the hyperparameters and the respective values that were explored during the training of the Long Short-Term Memory model

Parameter	Value	Tested values
Window size	42	5, 7, 14, 21, 42, 63
Layers	2	2, 3, 4
Hidden units	40	10, 15, …,80
Dense units	1	
Activation function	Tanh	ReLU
Recurrent activation function	Sigmoid	
Loss function	MSE	
Optimizer	RMSprop	RMSprop, Adam, SGD
Batch size	50	20, 30, 40, 50
Learning rate	$1 * 10^{-3}$	$1*10^{-2}$, $1*10^{-3}$, $1*10^{-4}$
Epochs	500	
Patience	10	10, 20, 30

The sigmoid function acts as a filter of information, allowing information to enter based on the output value that lies between [0,1] (Baek & Kim, 2018).

The hyperparameters that have been considered in the LSTM and the values used to fine-tune the model are shown in Table 3. After training the model and fine-tuning the different values to find the optimal hyperparameters of the LSTM neural network, based on commonly used values in related literature (Jansen, 2020). The topology incorporates two layers, a long short-term memory and a regular densely connected layer containing 40 and 1 unit or nodes, respectively. We defined several topologies for the neural networks. However, the results did not improve significantly, and the complexity of the model was higher. Thus, we decided on the values based on a trade-off between complexity and performance.

As explained above and represented in Eqs. (2) to (6), the activation and recurrent activation functions are hyperbolic tangent (tanh) and sigmoid, respectively. Both functions are relevant to overcome the problem of vanishing gradients. We also explored rectified linear unit (ReLU) as activation function, but we finally use tanh due to considerations related to the available runtime and performance optimization (Chollet, 2015).

MSE is used as a loss function due to its simplicity and for being the most common loss function for regression problems (Hastie et al., 2009). The model will seek to minimize the MSE during the training. After training the model and comparing the results for RMSprop, Adam, and Stochastic Gradient Descent (SGD), we observe that RMSprop provides better results and helps to avoid vanishing and exploding gradients by using a moving average of squared gradients (Hinton et al., 2012).

The LSTM is trained using early stopping to reduce overfitting during a maximum of 500 epochs to allow the model to iterate as much as needed, using patience of 10. This stops the training if the results do not improve continuously during 10 epochs. We do not use dropout or L1, or L2 regularization since overfitting is already prevented using early stopping. Lastly, the learning rate and the batch size are 50 and 0.001, respectively. This was selected based on a trade-off between the model's performance and the training time. In addition, the third column of Table 3 shows the values by parameter that have been tested to find the model that provides better performance without increasing its complexity in excess.

3.2.2 Portfolio Optimization

Classical portfolio optimization is based on the mean–variance model proposed by Markowitz (1952). Since then, most models have used the mean of historical returns to define the expected returns and the covariance. Based on Markowitz's portfolio selection model, we propose to optimize the portfolio using returns calculated with predicted share prices similar to the work of Du (2022), Ma et al. (2021), or Freitas et al. (2009).

3.2.2.1 Expected Risk and Return of a Stock The expected return of each stock is calculated using predicted stock prices. The outputs of the LSTM correspond to the predicted prices of each stock for every day of the year 2021. The following formula shows how the return is computed. Being $\hat{r}_t$ the predicted return at time t, $\hat{P}_t$ the predicted price at time t and $\hat{P}_{t0}$ the predicted price at time $t0$, which represent the moment of the sell and buy.

$$\hat{r}_t = \frac{\hat{P}_t - \hat{P}_{t0}}{\hat{P}_{t0}} * 100 \tag{7}$$

The expected risk of one stock is measured by using the standard deviation. It measures the dispersion of the price with respect to its mean and is represented in the following equation:

$$\hat{V}_t = \sqrt{\frac{\sum_{i=1}^{t} (\hat{r}_i - \hat{r})^2}{t - 1}} \tag{8}$$

where the $\hat{r}_i$ is the predicted return, the $\hat{r}$ is the average of the predicted returns, and t corresponds to the number of days included in the calculation.

3.2.2.2 Expected Risk and Return of a Portfolio The portfolio is made up of N stocks selected by the investor. The expected return is the weighted average of the predicted return of each portfolio. The expected return of the portfolio $\hat{r}_p$ is shown in the following equation:

$$\hat{r}_p = \sum_{i=1}^{N} \hat{r}_i \times W_i \tag{9}$$

where $\hat{r}_i$ is the predicted return of stock i and the weight is the proportion of the budget allocated to every stock, being $\sum_{i=1}^{N} W_i = 1$. In the current optimization problem, we do consider the possibility of short selling, as some asset types cannot be sold short (Pfaff, 2016). Therefore, for simplicity, we do not allow short selling, and the weights are always positive ($0 \leq W_i \leq 1$). This non-negativity condition is included as a constraint in the optimization of the portfolio.

On the other hand, the risk of the portfolio is measured using the standard deviation, which is the square root of the variance, and it is calculated as follows:

$$\hat{V}_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \hat{\gamma}_{\mathrm{ij}}} \tag{10}$$

where W_i and W_j represent the weights allocated in the stocks and $\hat{\gamma}_{\mathrm{ij}}$ is the covariance, which serves as a measure of how the stocks vary in relation to each other. This model assumes a fixed covariance structure for each of the holding periods and does not account for time-varying covariances within the same holding period. The calculation is shown in the following equation, where $\hat{\mathrm{r}}_{\mathrm{i,t}}$ and $\hat{\mathrm{r}}_{\mathrm{j,t}}$ are the predicted return of the given stocks i and j, while $\hat{\mathrm{r}}_{\mathrm{i}}$ and $\hat{\mathrm{r}}_{\mathrm{j}}$ represent their means, respectively and N stands the sample size:

$$\widehat{\gamma}_{\mathrm{ij}} = \frac{\sum_{t=1}^{N}\left(\widehat{\mathrm{r}}_{\mathrm{i,t}} - \bar{\widehat{\mathrm{r}}}_{\mathrm{i}}\right) * (\widehat{\mathrm{r}}_{\mathrm{i,t}} - \bar{\widehat{\mathrm{r}}}_{j})}{\mathrm{N}-1} \tag{11}$$

The variance and covariance are calculated considering all the predicted prices throughout the entire investment period considered, from the initial point of purchase to the final point of sale. These measures incorporate the predicted prices as various dates, reflecting the changing value of the assets over the duration of the investment. In contrast, the returns are calculated with the price at the beginning and the end of the investment period, essentially the purchase and sale prices.

3.2.2.3 Portfolio Optimization—Mean–Variance with Forecasting (MVF) Model The portfolio optimization model is built based on the previously defined measures. There are many different approaches, such as minimizing the volatility for a certain level of return or maximizing the return for a given target risk or volatility. In this case, we aim to maximize the Sharpe Ratio (Sharpe, 1994), which reflects the reward to volatility. It is represented in the following formula:

$$SR = \frac{r_p - r_f}{\sigma_p} \tag{12}$$

where r_p is the return of the portfolio, r_f is the Risk-free rate which is assumed to be 0.01 based on the value of the 3-month US Treasury bill according to the Federal Reserve Bank of St. Louis at the end of May 2022, and σ_p is the standard deviation of the portfolio.

The proposed model for portfolio optimization can be formulated as

$$\text{Maximize } \widehat{S} = \frac{\widehat{r}_p - r_f}{\widehat{V}_p} \tag{13}$$

$$\text{Subject to} \sum_{i=0}^{N} \widehat{r}_i \times W_i \geq r_f \tag{14}$$

$$\sum_{i=1}^{N} W_i = 1 \tag{15}$$

$$W_i \geq 0, i = 1, 2, \cdots, N \tag{16}$$

Equation (13) is the objective function that we attempt to maximize. As mentioned before, is the prediction-based Sharpe ratio; Eq. (14) is an inequality constraint function to ensure that the portfolio's returns are higher than the risk-free rate. Otherwise, it would make sense to select a risk-free investment. Equation (15) is an equality constraint function that ensures that all the resources are allocated, whereas Eq. (16) is an inequality constraint function that guarantees non-negative weights in the portfolio.

Maximizing the Sharpe ratio, it is possible to get the optimal portfolio based on risk-adjusted return, showing the expected return in excess of the risk-free rate achieved by the portfolio per unit of risk. To solve the problem is necessary to analyze the set of efficient portfolios, that are the ones that belong to the efficient frontier. These are the portfolios with the highest expected return for each level of risk or the lowest risk for each level of expected return. The selection of one or another portfolio will depend on the risk aversion of the investor.

4 Experimental Results

4.1 Stock Price Prediction

The prediction of stock prices is the cornerstone of the current paper. The predicted price is crucial to obtain the predicted return and volatility of the portfolios. It directly affects the optimal weights and the performance of the optimal portfolio. In the following subsections, we present the evaluation metrics used to assess the robustness of the predictions and a concise interpretation of the results obtained as the output of the proposed models.

4.1.1 Evaluation Metrics

The selected evaluation metrics used to evaluate the performance of the LSTM in forecasting the price of stocks are based, among others, on Freitas et al. (2009), Ma et al. (2021), and Du (2022). Specifically, we used MSE, MAE, and the classification metrics to understand the ability to predict the direction of the return that the model has. These metrics can be defined as follows:

$$MSE = \frac{1}{n}\sum\nolimits_{t=1}^{n}(r_t - \hat{r}_t)^2 \tag{17}$$

$$MAE = \frac{1}{n}\sum\nolimits_{t=1}^{n}\left|(r_t - \hat{r}_t)\right| \tag{18}$$

$$accuracy = \frac{TP + TN}{TP + TN + FP + FN} \tag{19}$$

$$precision = \frac{TP}{TP + FP} \tag{20}$$

$$recall = \frac{TP}{TP + FN} \tag{21}$$

$$f1 = 2 * \frac{precision * recall}{precision + recall} \tag{22}$$

where n is the number of predicted prices or trading days, and r_t and $\hat{r}_t$ are the realized and predicted returns at time t, respectively. *TP* refers to true positive values, *TN* to true negative, *FP* to false positive, and *FN* to false negative.

Although the current study is formulated as a regression problem, calculating classification metrics provides an alternative perspective on model performance. It serves as both a reference for assessing the model's classification-like behavior within the regression context and a potential starting point for future work.

Despite that we calculate the MSE and MAE for every analyzed stock, we use the average MSE and average MAE as global measures of overall prediction

Table 4 Predictive performance for the year 2021 and the first half of 2022: This table offers a comprehensive summary of the performance metrics used to evaluate the predictive capabilities for EURO STOXX 50 stocks across different holding periods

Holding period (days)		20	63	125	191	255
Year 2021						
MSE	Mean	0.000282	0.000315	0.000567	0.000506	0.000607
	Std	0.000356	0.000974	0.001320	0.000804	0.002265
MAE	Mean	0.014035	0.011952	0.016857	0.017717	0.016256
	Std	0.009309	0.013271	0.017002	0.013989	0.018709
Accuracy (%)	Total	92	96	94	96	98
Precision (%)	Up	79	98	98	95	97
	Down	97	86	71	100	100
Recall (%)	Up	92	98	95	100	100
	Down	92	86	83	82	92
F1 (%)	Up	85	98	97	97	99
	Down	95	86	77	90	96
Holding period (days)		25	50	75	100	127
First half of 2022						
MSE	Mean	0.000733	0.000274	0.000296	0.000539	0.000541
	Std	0.000818	0.000349	0.000484	0.000944	0.000632
MAE	Mean	0.023118	0.013162	0.013619	0.018037	0.018692
	Std	0.014228	0.010157	0.010605	0.014749	0.013994
Accuracy (%)	Total	90	100	98	94	100
Precision (%)	Up	79	100	100	85	100
	Down	100	100	97	97	100
Recall (%)	Up	100	100	92	92	100
	Down	84	100	100	95	100
F1 (%)	Up	88	100	96	88	100
	Down	91	100	99	96	100

performance. These measures are compared to other studies (Du, 2022; Ma et al., 2021; Sadaei et al., 2016; Wang et al., 2020; Weng et al., 2018).

4.1.2 Prediction Results

The results obtained by the LSTM are presented in Table 4. They summarize the performance of the recurrent neural network across the 50 components of the EURO STOXX 50® Index by showing the mean and the standard deviation (std) for the two scenarios considered. Table 4 presents the results for 2021, a year with continued growth, and the results for the first half of 2022, during which the market experienced a decline. The results presented correspond to the model that performed best after fine-tuning the hyperparameters for several holding periods. This allows us to evaluate the robustness of the model across different holding days and to be able to consider several investment strategies in terms of the forecast horizon.

Each calculation of the evaluation metrics considers that investors buy on the first day of the year and sell on the day of the selected time horizon. Therefore, the returns that investors will obtain are predicted and analyzed for different holding periods, considering investment strategies from 20 days to 1 year in 2021, which correspond to 1, 3, 6, 9, and 12 months. For the first half of 2022, since the total amount of trading days is 127, we consider 5 investment periods of 25 days, except the last one, which is 27 days. This allows us to have 5 holding periods for each evaluated year.

The results show that the model predicts future returns with minor predictive errors since the average of the MSE of the 10 holding periods is 0.00047, and the average of the MAE is 0.01634.

In 2021, for all the holding periods, the results show small MSE and MAE. Generally, both MSE and MAE increase with time. Also, in order to evaluate how the model predicts the direction of the returns of the different stocks, we employ several classification metrics. For all the analyzed holding horizons, the accuracy is above 90%. Besides, the model can accurately predict both upward and downward movements.

For the period that covers the first half of 2022, the model performs better for the holding periods of 50 and 75 trading days, which present similar results in terms of predictive errors. It shows higher errors for the shortest investment period considered. Similar to the year 2021, the classification metrics show that the model can predict the direction of returns, achieving an accuracy of 100% for two of the analyzed periods.

A comparison between predicted and real returns is shown for every considered holding period in Fig. 1. It is observable that predicted returns are close to real returns and the direction of the returns is well predicted in the vast majority of cases, as can be observed in the mentioned figure, and as it was shown in the classification metrics in Table 4.

Our results in terms of return prediction are comparable to the current literature. We obtain similar or superior results to other studies such as Du (2022), Ma et al. (2021), Sadaei et al. (2016), Wang et al. (2020), or Weng et al. (2018). Generally, our results present predictive errors with smaller mean errors. Nevertheless, it

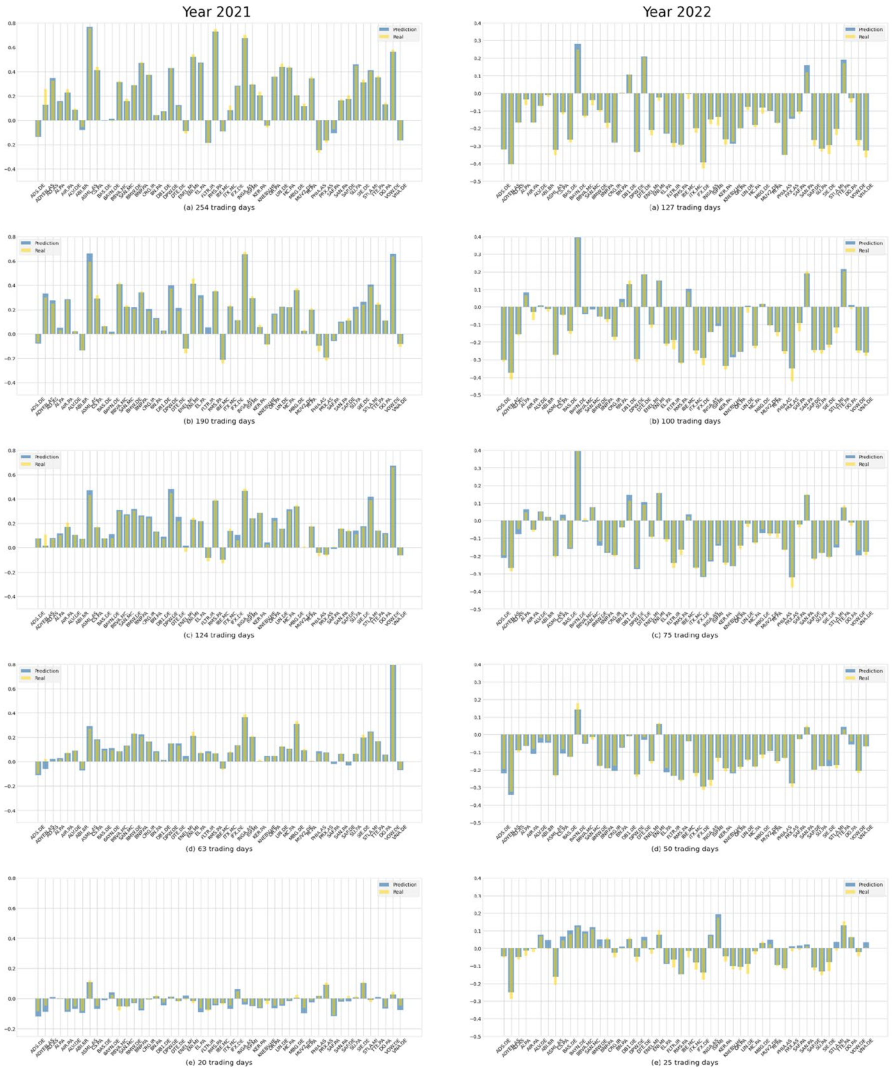

Fig. 1 Comparison of predicted and real return per holding period: It illustrates a comprehensive comparison of predicted and actual returns of EURO STOXX 50 stocks, evaluating their performance across multiple distinct holding periods

should be considered that we aim to predict returns in different markets, in different contexts, and considering different holding periods. For instance, Ma et al. (2021) focused on the components of the China Securities Index 100, testing the model with data spanning from 2012 to 2015. Meanwhile, Du (2022) encompassed the China Securities Index 300 and the S&P 500, evaluating the model's performance using data covering the period from 2018 to 2020. The consistency of our results with the existing literature, despite considering different time periods and markets, underscores the ability of LSTM models to adapt and accurately predict across different financial environments.

4.1.3 Prediction Benchmark Validation

In order to validate the superiority and effectiveness of our proposed model, we have conducted an extensive comparative analysis of our LSTM-based stock price prediction approach. We comprehensively evaluate the results of the LSTM by comparing them to other established machine learning models. Our selection of benchmark models encompasses different machine learning techniques, including decision trees, random forest, artificial neural networks, and support vector machine (SVM).

We used the evaluation metrics from Table 4 (detailed in Sect. 4.1.1) for both regression and classification. Our assessment considers the average performance of these metrics across the different holding periods, providing us with a comprehensive overview of the algorithm's performance. Importantly, we conduct this analysis separately for different years, enabling us to gain insights into how our model performs under both bullish and bearish market conditions. This approach allows for a more comprehensive and robust validation of our model. It is essential to emphasize that our primary objective is not to delve into algorithmic details, but rather to validate the superior performance of the LSTM methodology.

The results of the comparison are displayed in Table 5. It provides a comprehensive comparison of the LSTM model with various benchmark models across both the specified scenarios with the explained evaluation metrics for each algorithm. The results show that the LSTM model consistently outperforms the selected benchmark models across both the specified scenarios, namely, the year 2021 and the first

Table 5 Comparative Predictive Performance of LSTM and Benchmark Models for the Year 2021 and the First Half of 2022: This table provides a comprehensive comparison of the LSTM model with various benchmark models across both the specified scenarios

Metric	LSTM	Decision tree	Random forest	SVM	ANN
Year 2021					
MSE	0.000455	0.027642	0.0266164	0.0836882	0.0032534
MAE	0.015363	0.100684	0.0909884	0.179945	0.0410784
Accuracy (%)	95.2	83.2	86.6	61	91.4
Precision (%)	92.1	71.2	75.2	62	90
Recall (%)	92	83.4	89.2	72.6	91
F1 (%)	92	73.8	78.8	55.4	87.8
First half of 2022					
MSE	0.000477	0.015379	0.0143448	0.0424244	0.0045154
MAE	0.017326	0.091939	0.0842554	0.1500896	0.0548612
Accuracy (%)	96.4	64	64.4	48.8	88.8
Precision (%)	95.8	64	60.4	49.4	83.2
Recall (%)	96.3	72.4	67	49.2	91.8
F1 (%)	95.8	58.8	57.6	43.8	84.4

half of 2022. This observation holds true across various evaluation metrics, including Mean Squared Error, Mean Absolute Error, Accuracy, Precision, Recall, and F1-score. These results are consistent with the existent literature (Wang et al., 2020). This demonstrates the LSTM's proficiency in handling sequential data while successfully identifying and capturing long-range dependencies and non-linear patterns.

The LSTM is followed by the non-LSTM artificial neural network which ranks as the second-best performer in predicting outcomes for both scenarios. Subsequently, the random forest model secures the third position, trailed by the decision tree model, with SVM presenting the least effective predictive performance among the algorithms considered.

In conclusion, while the primary objective of this section is not an exhaustive examination of the factors contributing to one algorithm's superior performance over another, the aim is to validate the LSTM model concerning other employed models, encompassing both bullish and bearish market conditions. The results consistently indicate that the LSTM stands as the most proficient algorithm for the given task.

4.2 Prediction-Based Portfolio Optimization

The main reason for predicting the returns is to construct optimized prediction-based portfolios to solve the main drawback of classical portfolio optimization, the sensitivity to estimated inputs. In this section, the experimental results of the portfolio optimization are presented, and the different analyzed scenarios are compared. In addition, the results of the portfolios are benchmarked with the performance of the index. This is crucial, as it will show if the returns of the portfolios outperform the benchmark and, therefore, if it is worth actively managing the portfolio. Otherwise, passive management would be a better option.

4.2.1 Portfolio Construction

We construct portfolios for the same holding periods described in the previous section. The expected return (ER), the volatility (vol), and the Sharpe Ratio (SR) of each portfolio are the metrics employed to evaluate the different portfolios and to compare the results to other studies (Du, 2022; Ma et al., 2021; Sadaei et al., 2016; Wang et al., 2020 or Weng et al., 2018). These results are shown in Table 6.

First, it is observable for the year 2021 that the ER for the combination of the LSTM and the MVF model increases with time, showing that the longer the holding period, the higher the expected return. This is consistent with the increase of the ER of the index since the return of investing in the EURO STOXX 50® Index increases for the analyzed holding periods, as summarized in Table 7. The reason is that financial markets show an upward trend during this period, growing consistently, which is accurately predicted by our model. We do not annualize the expected returns since we calculate the return investors would obtain for that specific investment horizon. Thus, selecting one holding period or another would depend on investor preferences and needs. We do not intend to compare the different holding periods.

Table 6 Portfolio performance for the year 2021 and the first half of 2022: It presents a detailed examination of portfolio performance using predicted data for both years, considering a range of holding periods. It provides a comparative analysis of two strategies: LSTM + MVF and LSTM + 1/N, including their respective performance metrics

Holding period (days)		20	63	125	191	255
Year 2021						
LSTM + MVF	ER (%)	8.4	23.4	30.0	42.8	53.8
	Vol (%)	6.9	7.6	7.5	9.1	9.8
	SR	1.06	2.95	3.85	4.6	5.37
LSTM + 1/N	ER (%)	− 2.7	11.1	17.6	17.7	21.6
	Vol (%)	3.1	5.3	7.19	9.4	11.6
	SR	− 1.21	1.89	2.3	1.78	1.76
Holding period (days)		25	50	75	100	127
First half of 2022						
LSTM + MVF	ER (%)	16.4	14.3	43.2	36.5	23.0
	Vol (%)	15.6	24.6	21.8	20.8	18.4
	SR	0.99	0.54	1.93	1.71	1.19
LSTM + 1/N	ER (%)	− 0.86	− 13.3	− 9.33	− 10.8	− 15.1
	Vol (%)	5.5	11.6	12.9	13.9	15.1
	SR	− 0.35	− 1.24	− 0.81	− 0.86	− 1.07

Table 7 Portfolio and index return for the year 2021 and the first half of 2022. It provides the returns of the portfolios using real returns and compares them to the index proposed as benchmark across various holding periods

Holding period (days)		20	63	125	191	255
Year 2021						
Portfolio Return	(%)	9.02	24.19	28.21	40.6	54.34
Benchmark	(%)	− 0.94	10.15	14.03	13.57	20.81
Holding period (days)		25	50	75	100	127
First half of 2022						
Portfolio Return	(%)	14.85	17.81	41.32	34.91	20.38
Benchmark	(%)	− 4.88	− 13.64	− 10.04	− 15.11	− 20.24

Parallelly, as expected, volatility levels enlarge with the increase in the return, except for the holding period of 63 trading days. This represents 3 months and covers January, February, and March. The higher level of expected volatility could be explained by the previous months of February and March 2020. These months are used to train the LSTM, and from mid-February until the end of March, the EURO STOXX 50® Volatility (VSTOXX®) recorded its highest increase and level since 2008 due to COVID-19.

In the first half of 2022, overall markets decreased, being the market's worst first half in 50 years. As it is observable in Table 7, the EURO STOXX 50® Index represented as the benchmark, which is based on a free-float market cap, presents negative returns for all the analyzed periods, showing a decrease of more than 20% at the end of the sixth month. Despite that, our model achieves positive returns based on predicted data for the five holding periods considered in 2022.

Upon conducting a more comprehensive comparative analysis of our algorithm's performance under distinct market conditions, a notable distinction becomes evident between the two markets under examination: growth and bear markets. This differentiation primarily pertains to the composition of the portfolio with optimized weights. In both scenarios, the portfolios are constructed by selecting a subset of the 50 components belonging to the EURO STOXX 50® Index. These components are selected through the optimization method detailed in the preceding section, with the primary objective being to achieve the highest attainable Sharpe ratio based on predictions generated by the LSTM neural network. Thus, the optimized portfolios hold the components of the EURO STOXX 50® Index but with optimized weights that maximize the Sharpe ratio, exhibiting weightings that deviate from those solely based on free-float market capitalization.

During the period of market growth, the number of stocks with predicted positive returns tends to be higher compared to the first half of 2022, a period marked by substantial market declines. As a result, in the first half of 2022, the number of stocks with predicted positive returns is reduced, leading to a smaller set of stocks comprising the portfolio compared to the previous year.

Furthermore, the portfolios exhibit a superior relative performance in terms of the Sharpe ratio for the year 2021 compared to 2022. The higher Sharpe ratios in 2021 can be attributed to a larger number of companies yielding positive results, as illustrated in Fig. 1. Conversely, in 2022, a reduced number of companies with positive results implies lower diversification, increased risk, and consequently, a smaller Sharpe ratio.

Second, the performance of the LSTM + MVF model is compared to the LSTM + 1/N (see Table 6). The LSTM + 1/N corresponds to the equally weighted portfolio based on the predicted returns. In this case, the weight of each component of the index is 1/50, i.e. 2%. The results show that the LSTM + MVF model outperforms equally weighted portfolios for both years, as it obtains higher ER and SR for all the analyzed holding periods. Even in cases when the equally weighted portfolio shows a negative Sharpe ratio, the combination of our algorithm and the mean–variance model achieves high levels of predicted returns.

Third, the performance of LSTM + MVF portfolios is tested using historical data and compared to the return of the index. For the first comparison, we use the optimized weights of the LSTM + MVF for each holding period with the real return of the 50 components of the index. This way, we see what would have been the real return of the portfolios and we analyze if the proposed investment strategies are profitable or not in reality. By looking at the returns, which are shown in Table 7, it is possible to observe that in the year 2021, having held the investment for 1 month (20 trading days) would have generated a real return of 9.02%. However, if the holding period is higher than 3 months, the real returns oscillate between 24.19 and

54.34%. In 2022, despite the negative results of the index, our portfolios are profitable, and our investment strategies achieve a return of 14.85% in the first month and from 17.81 to 41.32% for longer holding periods.

Moreover, when comparing the relative performance of both analyzed years, the difference between the portfolio returns and the benchmark is more pronounced during the bear market. This difference can be explained by the differences in portfolio composition during growth and bear market periods. During the market growth period, portfolios consist of a larger number of stocks, which is the reason that there are more profitable stocks among the 50 components available in the EURO STOXX 50® Index, which are the ones our model can select. In this scenario, the algorithm has more options to choose from, increasing the likelihood of a greater similarity between the portfolio components and the benchmark. In contrast, during the bear market, the level of diversification is lower due to the lack of stocks with positive returns. Additionally, during this period, stocks with higher market capitalization tend to have lower performance and are therefore less likely to be selected by the algorithm. These factors contribute to a greater difference between the benchmark and the optimized portfolios during the bear market. It is worth noting that the unequal length of both years results from data unavailability during the research period.

Lastly, as it is represented in Fig. 2, the real return obtained by using the weights of optimized portfolios based on predicted returns outperforms the benchmark. In the year 2021, the EURO STOXX 50® Index return is 20.8%. This implies a difference of more than 30% compared to the 54.34% obtained at the end of the year by the portfolio managed by the LSTM + MVF. Even during the first month, in which the EURO STOXX 50® Index went down, the optimized portfolio was able to obtain positive results. In the period that corresponds to 2022, the index went down more than 20% and is consistently outperformed by optimal portfolios. It is important to clarify that the returns presented in this study do not account for transaction costs or other expenses incurred during due to the trading activity. Our research is primarily oriented towards forecasting and planning investment strategies, considering only one-time buy and sell transactions. We do not delve into the creation of automated trading bots, high-frequency strategies, or continuous portfolio rebalancing. However, there are different ways to treat transaction costs. For instance, Ledoit

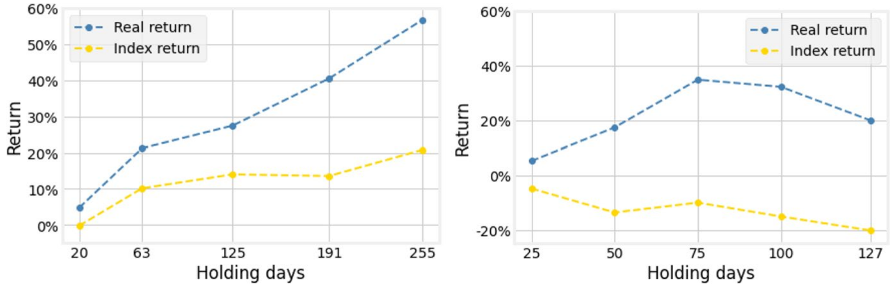

Fig. 2 Comparison of real returns of portfolio and index per holding period for the years 2021 (left) and 2022 (right). It illustrates the performance of the portfolios in terms of returns and compares them to the returns obtained by the EURO STOXX 50® Index

and Wolf (2022) propose a method to integrate transaction costs into the portfolio selection phase in a realistic way, and when they are properly considered, enhances the Sharpe ratio.

If we compare the performance of our created portfolios to the existing literature, our results are in line with it (Du, 2022; Ma et al., 2021; Wang et al., 2020; or Freitas et al., 2009). Our Sharpe ratio is higher or lower depending on the holding period analyzed. Since the research of other authors covers different markets and investment strategies is difficult to compare one to one. However, if we look at the returns, our portfolios usually outperform the expected return achieved by the other studies, being the lower Sharpe ratio driven by volatility.

5 Discussion and Conclusion

5.1 Discussion of key Findings

This paper extends the existing literature by creating profitable investment strategies that clearly and consistently beat the market over two very different scenarios, one in which the market shows consistent growth and another in which is considered a bear market. Our LSTM neural networks can accurately predict the price of European stocks used to create portfolios that achieve superior returns for both mentioned scenarios. Our deep learning algorithms are trained with data from January 2, 2015, to December 30, 2020, and tested by predicting prices for 2021 and the first half of 2022, considering several investment horizons. Our research focuses on the EURO STOXX 50® Index, for which we calculate the return of the 50 components based on predicted prices. Then, we combine calculated returns with mean–variance optimization to generate optimized portfolios that generate returns. By using this approach, we reduce or fully eliminate the subjective human factor that affects the selection of stocks and trading actions.

First, this study presents how we are able to overcome one of the main drawbacks or limitations of portfolio optimization since our rolling-window-based LSTM networks generate predicted prices to calculate returns with minor predictive errors that are used as inputs for the optimization of portfolios. We apply six different metrics that allow us to understand the performance of the model's prediction from a regression problem point of view and consider the prediction a binary classification problem. With these two approaches, we can see how accurate our predicted returns are and whether our model correctly predicts the direction of stock returns. These evaluation metrics fully reflect the performance of the recurrent neural network. The results are compared to the existing literature, showing similar or improved performance. This confirms that our LSTM can address the problem of long-range dependencies and allows us to track dependencies between the elements of the sequence. Adding some economic context, financial markets plummeted in March 2020 due to Covid-19, and the value of the EURO STOXX 50® Index went down from 3.840 on February 14, 2020, to 2.548 on March 20, 2020. Despite that and considering the uncertainty around the global political and economic situation, with many countries applying several measures due to COVID-19. Also, as aforementioned, during 2022 we have experienced the worst market's first half of the past 50 years, and we are experiencing the highest inflation levels since 1981. Despite the adverse economic context, our model is able to overcome this uncertain environment and generate accurate predictions.

Second, we combine our predicted future returns and MV portfolio optimization, defining several holding periods during 2021 and the first half of 2022. Our empirical results show that the created investment strategies consistently beat the EURO STOXX 50® Index, proposed as the benchmark, and the equally weighted portfolio for all the investment horizons considered. We take advantage of the accurate predicted returns to improve the allocation of weights in the construction of optimal portfolios. The portfolios not only beat the benchmarks but also generate positive returns even when the index and overall markets plummet under the conditions mentioned before. In addition, we validate our selected portfolios by calculating the real return by combining historical data and the weights allocated to each stock that makes up the optimal portfolio for each period. The results show that, in reality, our portfolios beat the index for every investment horizon by far.

5.2 Theoretical Implications

This paper enriches the theoretical research on prediction-based portfolio optimization and portfolio management. First, the proposed LSTM neural networks predict future returns with minor predictive errors and overcome the problem of long-range dependencies. Second, using MV optimization, the selection of the portfolios is more precise, due to more accurate predicted returns. This allows us to define several investment strategies that outperform the European market tested using real data for two periods with very different economic and social contexts and are able to consistently generate remarkable returns for investors, which shows the robustness and reliability of our approach.

5.3 Practical Implications

From a practical point of view, this study proposes the application of deep learning techniques to improve the selection of portfolios. For asset and portfolio managers, it can help to make investment decisions, create investment strategies, or complement their current market research and investment processes. For individual investors, it can help to invest without having specific knowledge of the companies and investments. For both, it can reduce the time necessary to study or deep dive into company details, automate their investments and fully isolate emotions that affect the selection of stocks.

5.4 Limitations and Future Work

Despite the results achieved, this research also has limitations. The prediction is based only on historical data; therefore, we do not consider news, economic indicators, technical or fundamental indicators. We adopt a purist technical perspective, considering that prices fully reflect the available information. However, further research can try to include other inputs to complement the historical data. Also, there can be other ways to estimate the risk, such as the accuracy or the errors of the prediction, which in this case would consider the certainty of the predictions of the model, creating the portfolio based on the trade-off between predicted return and the confidence level of the model in that predicted return. In addition, considering a multivariate LSTM model and comparing it to the univariate approach could be interesting to assess its effectiveness in capturing potential dependencies and correlations among the different stocks. Lastly, mean–variance optimization has been used for many years and portfolios could be optimized using other technics like deep reinforcement learning or quantum-inspired algorithms.

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Declarations

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References

Adebiyi, A. A., Adewumi, A. O., & Ayo, C. K. (2014). Comparison of ARIMA and artificial neural networks models for stock price prediction. *Journal of Applied Mathematics*. https://doi.org/10.1155/2014/614342

Alizadeh, M., Rada, R., Jolai, F., & Fotoohi, E. (2011). An adaptive neuro-fuzzy system for stock portfolio analysis. *International Journal of Intelligent Systems, 26*(2), 99–114. https://doi.org/10.1002/int.20456

Baek, Y., & Kim, H. (2018). ModAugNet: A new forecasting framework for stock market index value with an overfitting prevention LSTM module and a prediction LSTM module. *Expert Systems with Applications, 113*, 457–480. https://doi.org/10.1016/j.eswa.2018.07.019

Ban, G.-Y., El Karoui, N., & Lim, A. E. (2018). Machine learning and portfolio optimization. *Management Science, 64*(3), 1136–1154. https://doi.org/10.1287/mnsc.2016.2644

Basile, I., & Ferrari, P. (2016). *Asset management and institutional investors*. Springer. https://doi.org/10.1007/978-3-319-32796-9

Bengio, Y., Simard, P., & Frasconi, P. (1994). Learning long-term dependencies with gradient descent is difficult. *IEEE Transactions on Neural Networks, 5*(2), 157–166. https://doi.org/10.1109/72.279181
Best, M. J., & Grauer, R. R. (1991). On the sensitivity of mean-variance-efficient portfolios to changes in asset means: Some analytical and computational results. *Review of Financial Studies, 4*(2), 315–342. https://doi.org/10.1093/rfs/4.2.315
Black, F., & Litterman, R. (1992). Global portfolio optimization. *Financial Analysts' Journal, 48*(5), 28–43. https://doi.org/10.2469/faj.v48.n5.28
Chen, W., Zhang, H., Mehlawat, M. K., & Jia, L. (2021). Mean–variance portfolio optimization using machine learning-based stock price prediction. *Applied Soft Computing, 100*, 106943. https://doi.org/10.1016/j.asoc.2020.106943
Chollet, F (2015). Keras. https://keras.io/api/layers/recurrent_layers/lstm/
Chopra, V. K., & Ziemba, W. T. (1993). The effect of errors in means, variances, and covariances on optimal portfolio choice. *Journal of Portfolio Management, 19*(2), 6–12. https://doi.org/10.1142/9789814417358_0021
De Prado, M. L. (2016). Building diversified portfolios that outperform out of sample. *Journal of Portfolio Management, 42*(4), 59–69. https://doi.org/10.3905/jpm.2016.42.4.059
DeMiguel, V., & Nogales, F. J. (2009). Portfolio selection with robust estimation. *Operations Research, 57*(3), 560–577. https://doi.org/10.1287/opre.1080.0566
Du, J. (2022). Mean–variance portfolio optimization with deep learning based-forecasts for cointegrated stocks. *Expert Systems with Applications, 201*, 117005. https://doi.org/10.1016/j.eswa.2022.117005
Fabozzi, F. J. (1999). *Investment management.* New Jersey: Prentice Hall (2nd ed.).
Fama, E. F. (1996). Multifactor portfolio efficiency and multifactor asset pricing. *Journal of Financial and Quantitative Analysis, 31*(4), 441–465. https://doi.org/10.2307/2331355
Freitas, F. D., De Souza, A. F., & De Almeida, A. R. (2009). Prediction-based portfolio optimization model using neural networks. *Neurocomputing, 72*(10–12), 2155–2170. https://doi.org/10.1016/j.neucom.2008.08.019
Gers, F. A., Schraudolph, N. N., & Schmidhuber, J. (2002). Learning precise timing with LSTM recurrent networks. *Journal of Machine Learning Research, 3*, 115–143.
Ghiassi, M., Saidane, H., & Zimbra, D. K. (2005). A dynamic artificial neural network model for forecasting time series events. *International Journal of Forecasting, 21*(2), 341–362. https://doi.org/10.1016/j.ijforecast.2004.10.008
Hamdani, Z., Dif, A., Zairi, B., Benziane, A., Hamdani, M. (2020). Factors Affecting the Behavior of Financial Markets in the Light of Financial Risks. *Advances in Human Factors, Business Management and Leadership.* AHFE 2020. *Advances in Intelligent Systems and Computing*, vol 1209. Springer, Cham. https://doi.org/10.1007/978-3-030-50791-6_24
Hansen, J. V., & Nelson, R. D. (2002). Data mining of time series using stacked generalizers. *Neurocomputing, 43*(1–4), 173–184. https://doi.org/10.1016/S0925-2312(00)00364-7
Hassan, R., Nath, B., & Kirley, M. (2007). A fusion model of HMM, ANN, and GA for stock market forecasting. *Expert Systems with Applications, 33*(1), 171–180. https://doi.org/10.1016/j.eswa.2006.04.007
Hastie, T., Tibshirani, R., & Friedman, J. (2009). *The elements of statistical learning: data mining, inference, and prediction* (Vol. 2, pp. 1–758). New York: Springer.
Herwartz, H. (2017). Stock return prediction under GARCH—An empirical assessment. *International Journal of Forecasting, 33*(3), 569–580. https://doi.org/10.1016/j.ijforecast.2017.01.002
Hinton, G., Srivastava, N., & Swersky, K. (2012). *Neural networks for machine learning lecture 6a overview of mini-batch gradient descent.*
Hochreiter, S., Bengio, Y., Frasconi, P., & Schmidhuber, J. (2001). Gradient flow in recurrent nets: the difficulty of learning long-term dependencies. *A field guide to dynamical recurrent neural networks*. https://www.researchgate.net/publication/2839938
Hochreiter, S. (1998). The vanishing gradient problem during learning recurrent neural nets and problem solutions. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 6*(02), 107–116. https://doi.org/10.1142/S0218488598000094
Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural Computation, 9*(8), 1735–1780. https://doi.org/10.1162/neco.1997.9.8.1735
Huang, C. F. (2012). A hybrid stock selection model using genetic algorithms and support vector regression. *Applied Soft Computing, 12*(2), 807–818. https://doi.org/10.1016/j.asoc.2011.10.009

Jansen, S. (2020). RNNs for Multivariate Time Series and Sentiment Analysis. In S. Jansen, *Machine Learning for Algorithmic Trading: Predictive models to extract signals from market and alternative data for systematic trading strategies with Python* (pp. 591–624). Packt Publishing Ltd.

Jorion, P. (1985). International portfolio diversification with estimation risk. *The Journal of Business, 58*(3), 259–278.

Jorion, P. (1986). Bayes-Stein Estimation for Portfolio Analysis. *Journal of Financial and Quantitative Analysis, 21*(3), 279–292. https://doi.org/10.2307/2331042

Kim, H. Y., & Won, C. H. (2018). Forecasting the volatility of stock price index: A hybrid model integrating LSTM with multiple GARCH-type models. *Expert Systems with Applications, 103*, 25–37. https://doi.org/10.1016/j.eswa.2018.03.002

Kolm, P. N., Tütüncü, R., & Fabozzi, F. (2014). 60 Years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research, 234*(2), 356–371. https://doi.org/10.1016/j.ejor.2013.10.060

Korkie, R. M., & Jobson, J. (1981). Putting Markowitz theory to work. *The Journal of Portfolio Management, 7*(4), 70–74. https://doi.org/10.3905/jpm.1981.408816

Ledoit, O., & Wolf, M. (2022). Markowitz portfolios under transaction costs. *Working paper series/ Department of Economics*, (420). https://ideas.repec.org/p/zur/econwp/420.html

Lee, S. I., & Yoo, S. J. (2020). Threshold-based portfolio: The role of the threshold and its applications. *The Journal of Supercomputing, 76*(10), 8040–8057. https://doi.org/10.1007/s11227-018-2577-1

Lin, C.-M., Huang, J. J., Gen, M., & Tzeng, G.-H. (2006). Recurrent neural network for dynamic portfolio selection. *Applied Mathematics and Computation, 175*(2), 1139–1146. https://doi.org/10.1016/j.amc.2005.08.031

Ma, Y., Han, R., & Wang, W. (2021). Portfolio optimization with return prediction using deep learning and machine learning. *Expert Systems with Applications, 165*, 113973. https://doi.org/10.1016/j.eswa.2020.113973

Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. Yale University Press.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77–91. https://doi.org/10.2307/2975974

Michaud, R. O., & Michaud, R. O. (2008). *Efficient asset management: A practical guide to stock portfolio optimization and asset allocation*. Oxford University Press.

Mok, P., Lam, K., & Ng, H. (2004). An ica design of intraday stock prediction models with automatic variable selection. *2004 IEEE international joint conference on Neural Networks* (pp. 2135–2140). https://doi.org/10.1109/IJCNN.2004.1380947

Mondal, P., Shit, L., & Goswam, S. (2014). Study of effectiveness of time series modeling (ARIMA) in forecasting stock prices. *International Journal of Computer Science, Engineering and Applications*. https://doi.org/10.5121/ijcsea.2014.4202

Patel, J., Shah, S., Thakkar, P., & Kotecha, K. (2015). Predicting stock market index using fusion of machine learning techniques. *Expert Systems with Applications, 42*(4), 2162–2172. https://doi.org/10.1016/j.eswa.2014.10.031

Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V., & Vanderplas, J. (2011). Scikit-learn: Machine learning in python. *Journal of Machine Learning Research, 12*, 2825–2830.

Pfaff, B. (2016). *Financial risk modelling and portfolio optimization with R*. John Wiley & Sons.

Rezaei, H., Faaljou, H., & Mansourfar, G. (2021). Stock price prediction using deep learning and frequency decomposition. *Expert Systems with Applications, 169*, 114332. https://doi.org/10.1016/j.eswa.2020.114332

Rius, A., Ruisanchez, I., Callao, M. P., & Rius, F. X. (1998). Reliability of analytical systems: Use of control charts, time series models and recurrent neural networks (RNN). *Chemometrics and Intelligent Laboratory Systems, 40*(1), 1–18. https://doi.org/10.1016/S0169-7439(97)00085-3

Sadaei, H. J., Enayatifar, R., Lee, M. H., & Mahmud, M. (2016). A hybrid model based on differential fuzzy logic relationships and imperialist competitive algorithm for stock market forecasting. *Applied Soft Computing, 40*, 132–149. https://doi.org/10.1016/j.asoc.2015.11.026

Sharpe, W. F. (1994). The sharpe ratio. *The Journal of Portfolio Management, 21*(1), 49–58. https://doi.org/10.3905/jpm.1994.409501

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management Science, 9*(2), 277–293. https://doi.org/10.1287/mnsc.9.2.277

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance, 19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x
Ticknor, J. L. (2013). A Bayesian regularized artificial neural network for stock market forecasting. *Expert Systems with Applications*. https://doi.org/10.1016/j.eswa.2013.04.013
Tobin, J. (1958). Liquidity preference as behavior towards risk. *The Review of Economic Studies, 25*(2), 65–86. https://doi.org/10.2307/2296205
Wang, J., & Wang, J. (2015). Forecasting stock market indexes using principle component analysis and stochastic time effective neural networks. *Neurocomputing, 156*, 68–78. https://doi.org/10.1016/j.neucom.2014.12.084
Wang, W., Li, W., Zhang, N., & Liu, K. (2020). Portfolio formation with preselection using deep learning from long-term financial data. *Expert Systems with Applications, 143*, 113042. https://doi.org/10.1016/j.eswa.2019.113042
Weng, B., Lu, L., Wang, X., Megahed, F. M., & Martinez, W. (2018). Predicting short-term stock prices using ensemble methods and online data sources. *Expert Systems with Applications, 112*, 258–273. https://doi.org/10.1016/j.eswa.2018.06.016

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Stock Price Forecasting with Deep Learning: A Comparative Study

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Abstract: The long short-term memory (LSTM) and gated recurrent unit (GRU) models are popular deep-learning architectures for stock market forecasting. Various studies have speculated that incorporating financial news sentiment in forecasting could produce a better performance than using stock features alone. This study carried a normalized comparison on the performances of LSTM and GRU for stock market forecasting under the same conditions and objectively assessed the significance of incorporating the financial news sentiments in stock market forecasting. This comparative study is conducted on the cooperative deep-learning architecture proposed by us. Our experiments show that: (1) both LSTM and GRU are circumstantial in stock forecasting if only the stock market features are used; (2) the performance of LSTM and GRU for stock price forecasting can be significantly improved by incorporating the financial news sentiments with the stock features as the input; (3) both the LSTM-News and GRU-News models are able to produce better forecasting in stock price equally; (4) the cooperative deep-learning architecture proposed in this study could be modified as an expert system incorporating both the LSTM-News and GRU-News models to recommend the best possible forecasting whichever model can produce dynamically.

Keywords: deep learning; long short-term memory (LSTM); gated recurrent unit (GRU); financial news sentiments; stock market forecasting

1. Introduction

Stock market forecasting is an important task in stock market exchange in the world. Usually stock market forecasting is concerned with accurate prediction of either/both the trend or/and the price of a stock so as to gain a higher profit through trading. However, obtaining an accurate prediction of the stock trend and/or price has been a challenging and difficult task due to the nonlinear and volatile nature of stock exchange. Traditionally, some people who believe the efficient market hypothesis theory [1] argue that the future stock price is predictable based on the historical stock data. Others who trust the random walk theory believe that the future stock price does not depend on the historical stock data, and hence no useful patterns could be found in the historical data to reflect the pattern of the upcoming stock sequences [2].

As an intuitive choice, many statistical models were developed to estimate the stock price using the past and present data, such as autoregressive integrated moving average (ARIMA) models [3]. These statistical models map linear relationships well but are not practically useful in stock market forecasting due to the nonlinear nature of stock market exchange. With the emergence of computational intelligence in the past three decades, more nonlinear models empowered by artificial neural networks

(ANNs), fuzzy–neural systems, genetic algorithms, evolutionary and/or particle swarm techniques have been proposed by many researchers for stock market forecasting [4–9].

Researchers also noticed that the financial news, along with the social media gossips through various social media platforms in the recent decade, could impose on further volatility on the stock market. The sentiment of such news must be considered in any expert system for stock market forecasting. Understandably, it is impossible to manually arrange such news sentiment for the purpose of dynamic stock market forecasting. Accordingly, algorithms of natural language processing and media sentiment analysis have been proposed to align the news sentiment and stock market indices automatically to increase the accuracy of stock market prediction [10].

Many studies by far indicated deep learning that employs multiple layers of ANNs has shown some promising results in stock market forecasting with or without considering the news sentiment measures [11–17]. The most popular deep-learning architecture for stock market forecasting seems the long short-term memory (LSTM) model or its hybridization [11–15]. The other popular deep-learning architecture for stock market forecasting is the gated recurrent unit (GRU) model or its hybridization [16–19]. Both models seemingly boast about producing the better performance in every new publication. However, these studies ware based on different designs, assumptions, implementations—and applying to different stocks in different countries—it is hardly to make any objective comparison between the performances of these two popular models in the same conditions. Furthermore, even for the same model, outcomes from various studies were far from reaching a certainty on whether incorporating financial news sentiments in the stock forecasting could lead to a better performance than without considering this factor due to the diverse conditions in different studies.

This study aims at first, conducting a normalized comparison on the performances of the LSTM and GRU models for stock market forecasting under the same conditions, and second, objectively assessing the significance of incorporating the financial news sentiments in stock market forecasting with respect to that if only the stock features are used. To achieve these goals, we design a cooperative deep-learning architecture that can treat both the LSTM and GRU models equally with the same inputs consisting of relevant features from past stock data and the news sentiment score for stock prediction. To ensure an objective comparison, the implementation of this cooperative deep-learning architecture has been made by using the existing algorithms or tools used in previously published studies by other researchers. This removes potential bias that may be introduced in any untried algorithm proposed by us. Given the high diversity in assessing the level of news sentiment for stock market exhibited by numerous media outlets in developed countries, it is logical to choose the stock market data from a country where the media outlets are consistent and accessible over a period of time long enough to support the purpose of this study. Hence, the Nepali stock datasets and financial news datasets are chosen for this study to explore the influence of the financial news sentiment on stock market forecasting.

The contributions of this study include: (a) Answering the key question: in the same conditions, which deep-learning model, LSTM or GRU, would be a better choice for achieving the best possible stock forecasting? (b) Confirming whether the financial news sentiment has a causal influence on stock market forecasting? (c) Establishing a cooperative deep-learning architecture that can treat both the LSTM and GRU models equally with the same inputs consisting of relevant features from past stock data and the news sentiment scores for stock prediction. This cooperative architecture could be modified as a platform for conducting objective comparative studies in many other disciplines, and more important, as an expert system incorporating both the LSTM-News and GRU-News models to recommend the best possible forecasting whichever model can produce dynamically.

The remainder of this paper is organized as follows: related work is reviewed in Section 2; design and methodologies are presented in Section 3; experiment settings and results are shown in Section 4; discussions on the results and implications are made in Section 5, followed by conclusion and future work in Section 6.

2. Related Work

In recent years, various deep-learning techniques have been applied in stock market prediction in different stock markets around the world. The LSTM model was used by Chen et al. [11] to predict China's stock market in Shanghai and Shenzhen Exchanges (SSE). This model contains a single input layer, followed by multiple LSTM layers, a dense layer and a single output layer with several neurons. Multiple stock features such as high price, low price, open price, close price, were experimented with six different methods to predict stock prices. This study indicated that the normalized features and SSE indices could increase the accuracy of forecasting. Financial news sentiment was not considered in this study.

Samarawickrama and Fernando [14] selected three companies from the Colombo Stock Exchange (CSE) to predict the stock prices using multilayer perceptron (MLP), simple recurrent neural network (SRNN), LSTM and GRU architectures. The experiment used closing, high and low prices of the past two days as the input variables without considering financial news sentiments. What is more interesting about the research output was that the MLP model produced the best result when predicting the closing price for the next day. This finding may be attributed to the fact that only the stock features of the past two days were considered for these models, which restricted the ability of the deep-learning models to explore more potential clues.

Althelaya et al. [12] evaluated and compared the bidirectional LSTM (BLSTM) and stacked LSTM (SLSTM) models for stock price prediction. In BLSTM, preceding and succeeding input sequences were used to exploit all input data in the learning process. In SLSTM, several LSTM layers were stacked to perform deep learning. Data from Standard and Poor 500 Index (S&P500) were used for training, and the closing price at the end of every trading day was the predicting target. Overall, the BLSTM model performed well for both long and short-term predictions whereas the SLSTM model produced a better performance for predicting short-term prices only. However, this study did not consider any financial sentiment influence.

Li et al. [15] incorporated stock indicators with investor sentiments to predict the CSI300 index values based on the LSTM model that contained four layers with 30 nodes. The investor sentiments were analyzed with a naïve Bayes classifier. This study showed that this model outperformed the support vector machine (SVM) methods in prediction accuracy. However, it fell short in making comparison with the performance of the LSTM model if only using the stock indicators without considering the investor sentiments.

Jiawei and Murata [13] attempted to identify the factors influencing stock market trend prediction with a LSTM model with a preprocessing algorithm to reduce the dimension of stock features and a sentiment analyzer to render financial news for stock trend prediction. The result showed that the 'market emotion' was a very important factor influencing the stock market and could help improving prediction accuracy. However, it stopped exploring further on assessing the extent of such influence with respect to that if only the stock features were used as the input.

Li. et al. [20] proposed a LSTM model using the stock features with the sentiment polarity from financial news as inputs for stock trend prediction. This LSTM model with the basic news sentiment produced better performance compared with the baseline model consisting of SVM and multiple-kernel learning (MKL). A comparison with the performance of the LSTM model if only used the stock indicators as the input would have been more useful in assessing the influence of the news sentiments in stock forecasting.

GRU introduced by Cho et al. [21] is a deep-learning architecture to overcome the problems of vanishing and explosion of gradients in traditional recurrent neural networks (RNNs) when learning long-term dependencies. Hence, similar to LSTM, GRU and GRU-related models have been used in financial investment prediction recently, for example, Bitcoin price prediction [22].

Shen et al. [16] applied two GRU models for predicting trading signals of stock indices of the Hang Seng Indexes (HSI), the Deutscher Aktienindex (DAX) and the S&P 500 Index from 1991 to 2017 and compared the performance of this GRU-based model with that of SVM and other models.

The results showed that the two GRU models produced a higher prediction accuracy than other models. It should be noted than this study was focused on predicting trading signals of stock indices by using stock data without considering financial news sentiments.

Rahman et al. [17] applied the GRU model to predict the future prices of stock markets using stock datasets collected from Yahoo finance. The authors claimed that the proposed method predicted the future prices with good accuracy. Obviously, financial news sentiment was not considered, and no comparison was made with the performance of other models in this study.

Saud and Shakya [18] compared the performances of stock price prediction from three deep-learning models Vanilla RNN, LSTM and GRU using the stock market features extracted from the Nepal Stock Exchange (NEPSE). This study found that GRU was the most successful model in stock price prediction among the three models. Financial news sentiment was not considered in this study.

Dang et al. [19] proposed a two-stream GRU model that incorporated the financial news sentiments with stock features as inputs to forecast S&P 500 index trends and prices. Results showed that the two-stream GRU model outperformed other models, including both the LSTM model and the original GRU model. The authors also pointed out that the two-stream GRU model requires long time for training and huge computational resources because of the complexity of the enlarged GRU model.

The review of literature echoes the aims of this study in the previous section, i.e., to conduct a normalized comparison on the performances of the LSTM and GRU models for stock market forecasting under the same conditions and to objectively assess the significance of incorporating the financial news sentiments in stock market forecasting.

3. Methods and Design

The proposed deep-learning architecture for stock price prediction is shown in Figure 1. The historical stock data and the sentiment scores of financial news headlines are combined as a single vector to generate a time series sequence as the input. Two deep-learning models, LSTM and GRU, are then trained in supervised learning settings with the mean absolute error (MAE) chosen as the main loss function. Finally, these models are evaluated based on metrices discussed in Section 3.4 later.

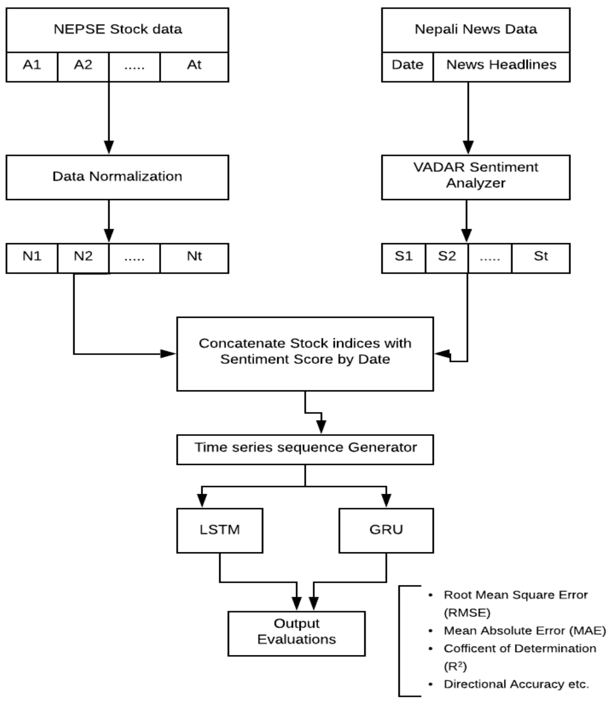


Figure 1. Cooperative deep learning architecture for stock price prediction framework.

3.1. Data Preparations

Historical stock data and financial news headlines are scraped from website (www.sharesansar.com) using a web crawler. This website is a popular stock trading source in Nepal which contains a plethora of information about the Nepalese stock market. It lists daily trading data of individual stocks listed in the Nepal Stock Exchange (NEPSE), Initial Public Offering (IPO), Further Public Offering (FPO) and financial news. Historical stock data of agriculture development bank (ADBL) on daily basis are scrapped with the following attributes: open price, last traded price (LTP), high price, low price and quantity of share from 20 March 2011 to 14 November 2019. The daily financial news of a total of 42,110 samples are collected for the same period. The statistics of data collected is shown in Table 1.

Table 1. Description of datasets.

Dataset	Data Attributes	Number of Samples
Historical stock price	Date, Open, LTP, High, Low, and Quantity	1996 (days)
Financial news	Date, News headline, News body	42,110 (news)

3.2. Data Alignment and Analysis

News headlines obtained from web scraping are processed to remove those unnecessary texts in the HTML tags and escape sequences. As punctuations like exclamation marks and multiple question marks reflect the emotion and strength in the headlines, such punctuation marks are not removed. Instead of creating our own algorithm that may lack of credibility, we choose the published algorithm VADER [23,24] to render the financial news headlines to sentiment scores. Note that regardless of how the sentiment scores are converted from the news headlines, they are fed to both the LSTM and GRU models equally without bias to one or the other. The sentiment scores are placed into a new field named 'Score' and saved into a CSV file. The 'date' field in the news datasets is converted to the format mm/dd/yyyy to match the date format with the stock datasets. Commas in the 'quantity' field in the stock datasets representing the numeric positions are removed to make the numbers as pure numeric values. Examples of the cleaned daily data are shown in Table 2. The source codes used in this study are available at https://github.com/tejshahi/StockPricePrediction-NEPSE-.

Table 2. Samples of combined datasets.

Date	LTP	Open	High	Low	Quantity	Score
11/14/2019	417	417	417	415	18,775	0.4108
11/13/2019	416	414	417	413	23,563	0.3468
11/12/2019	417	425	425	417	12,550	0.1764
11/10/2019	421	414	421	414	3581	0.5236

Since the Nepal stock exchange runs from Sunday to Thursday, but financial news headlines are published seven days in a week, we assert that news on weekend days have an impact on the stock price on Sunday (the first trading day of next week). Therefore, we aligned the sentiment score to stock data daily as shown in Figure 2.

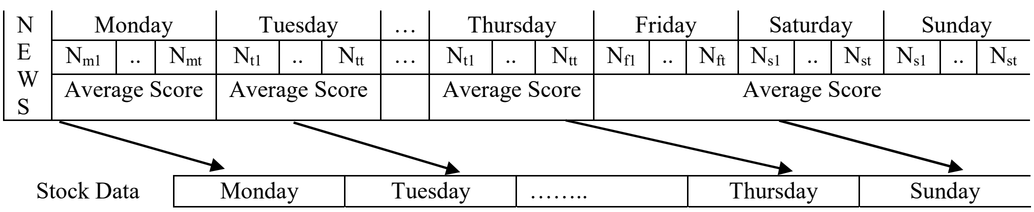


Figure 2. Mapping between financial news sentiment scores and stock data.

The min–max normalization [25] technique is used to scale each feature to range [0, 1]. The min–max transformation is achieved by using Formula (1).

$$x_{norm} = (x - x_{min}) / (x_{max} - x_{min}) \tag{1}$$

A total of 1991 datasets are available after merging the sentiment scores to the stock data. Among these 1991 datasets, there are 1786 positive, 204 negative and 1 neutral news headlines. Applying regression to all the corelated variables produces the correlation matrix presented in Figure 3. There are significant correlations between the sentiment score and each of the chosen stock indices.

Figure 3. Correlation matrix between all features.

The fields open, high, low, quantity and score are combined as the input to the models and the target variable is LTP. Two input sets are created for different purposes: Set I with only the stock attributes and Set II with both the stock attributes and the news sentiment score, both using n-days lag (i.e., look-back value).

3.3. Deep-Learning Models: GRU and LSTM

The structure of a GRU unit consists of an update gate (z_t), reset gate (r_t), and a current memory content ($\hat{h}_t$) whereas the output (h_t) is stored in the final memory of the GRU [8]. The update gate decides how much the input (x_t) and previous output (h_{t-1}) to be passed to the next cell, which is controlled by the weight (W_z). The reset gate is used to determine how much of the past information to forget. The current memory content ensures that only the relevant information to be passed to the next iteration, which is determined by the weight W. The main operations in GRU are governed by the following formulae:

$$z_t = \sigma(W_z \cdot [h_{t-1}, x_t]) \tag{2}$$

$$r_t = \sigma(W_z \cdot [h_{t-1}, x_t]) \tag{3}$$

$$\widetilde{h_t} = \tanh(W[r_t * h_{t-1}, x_t]) \tag{4}$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * h_t \tag{5}$$

where z_t and r_t are intermediate values obtained from the update and reset gates, respectively; tanh is hyperbolic tangent function; σ is the sigmoid function.

Compared with the GRU model, the LSTM architecture includes one more gate i.e., the output gate, in addition to the update (or input) and reset (or forget) gates in GRU. LSTM also has more operations with respect to each of the gates briefed as follows:

Forget gate

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right) \tag{6}$$

$$C_t = C_t \otimes f_t \tag{7}$$

Input gate

$$\widetilde{C_t} = \tanh(W_i[h_{t-1}, x_t] + b_c) \tag{8}$$

$$i_t = \sigma(W_t[h_{t-1}, x_t] + b_i) \tag{9}$$

$$C_t = f_t * C_{t-1} + i_t * \widetilde{C_t} \tag{10}$$

Output gate

$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o) \tag{11}$$

$$h_t = o_t * \tanh(C_t) \tag{12}$$

where f_t is the value from the forget gate; b is the bias; C_t is the value from the input gate; i_t is the intermediate output.

3.4. Model Assessment Metrices

The daily stock price and trend prediction are assessed through various metrices. The root mean square error (RMSE) and mean absolute error (MAE) are used to assess the prediction error while coefficient of determination (R^2) is used to measure the goodness of fittings between the actual and predicted values. The accuracy of prediction trend is assessed with directional accuracy (DA). It measures the accuracy of stock trend prediction by comparing the predicted price with actual price using Formula (15). All the measures are formulated below.

$$RMSE = \sqrt{\left(\frac{1}{N}\sum_{d=1}^{N}(a_d - p_d)\right)} \tag{13}$$

$$MAE = \frac{1}{N}\sum_{d=1}^{N}\left|a_d - p_d\right| \tag{14}$$

$$DA = \frac{1}{N}\sum_{d=1}^{N} D_d,\ D_d = \begin{cases} 1, (a_{d+1} - a_d) \times (p_{d+1} - a_d) \\ 0,\ otherwise \end{cases} \tag{15}$$

$$R^2 = 1 - \frac{\sum_{d=1}^{N}(a_d - p_d)^2}{\sum_{d=1}^{N}(a_d - \overline{a})^2} \tag{16}$$

where $\overline{a} = \frac{1}{N}\sum_{d=1}^{N} a_d$; p_d is the predicted price and a_d is the actual price of stock at day d.

4. Experiment Settings and Results

4.1. Experimental Setup

In the experiment, both LSTM and GRU have the same number of layers, activation function and inputs to make the comparison consistent. Each model is composed of one input layer, followed by a LSTM/GRU layer, followed by a dropout layer and finally a dense output layer. The input layer contains the number of memory units equal to the number of input features. The LSTM/GRU layers consist of 120 memory units. The activation function used in each LSTM/GRU layer is the hyperbolic tangent. The full specification of parameters used in these models is listed in Table 3.

In order to prevent overfit and underfit of training due to too many or too few epochs, the early stopping method is being implemented in these models. Early stopping is a method that allows us to specify a large arbitrary number of training epochs and stop training once the performance of the model has stopped improving on the validation dataset.

Table 3. Specification of parameters for training.

Parameters	Values
Number of nodes in input layer	Number of input features × lookback value
Number of epochs	100 with early stopping criteria of 10 epoch delay
Batch size	30
Hidden layer	1 LSTM/GRU layer with 120 units
Activation function	tanh
Lock back value (lag in number of days)	10,12,14,16,18,20
Dropout layer	1 with (0.2 dropout rate)
Output layer	1

The total data are divided into three subsets: training, validation and test sets as shown in Table 4. The validation set is used during the training process to validate the value of the loss function in each epoch. The loss function used in the model is MAE. MAE is calculated at the end of each epoch. The training process is stopped if there is no further improvement or no change in the loss function value, after a certain number of epoch delay. The first sign of no further improvement may not be the optimal time to stop training as the model may coast into a plateau of no improvement or even get slightly worse before getting better. To account for this situation, a delay to the trigger in terms of the number of epochs on which there is no improvement is set.

Table 4. Training and test data distribution.

Data	Dates (mm/dd/year)	No. of Samples
Total data	03/20/2011 to 11/14/2019	1991
Training	03/20/2011 to 05/30/2016	1186
Validation	06/01/2016 to 09/09/2017	292
Test samples:	10/09/2017 to 11/14/2019	513

4.2. Experimental Results

The designed LSTM and GRU models are executed for several look-back values with an epoch value of 100 and batch size 30. The experiment has two input sets. Set I only uses the historical stock attributes whereas Set II contains both the historical stock attributes and the news sentiment scores. These inputs are used to train the LSTM/GRU model separately. During the experiments, different look-back values are set. The MAE and RMSE at different look-back values are plotted in Figure 4 whereas the coefficients of determination (R^2) for the four models are plotted in Figure 5. The detailed statistical figures are tabulated in Table 5.

All indicators (except training time) show that the LSTM model with the news sentiment is the best performer, followed by the GRU model with news sentiment. This order is almost completely inversed for LSTM and GRU if the news sentiment is not considered. However, both LSTM and GRU perform far inferior than they do by incorporating the news sentiment with stock feature in the input. In other words, the impact of the financial news sentiments on stock market prediction is very influential. The difference in DA among the four models is the smallest; hence all four models can predict the stock trend with a similar level of accuracy.

Table 5. Summary of results.

Evaluation Metrics	LSTM-Only	LSTM-News	GRU-Only	GRU-News
MAE	48.47	17.689	42.81	24.472
RMSE	52.933	23.070	47.31	29.153
R^2	0.867	0.979	0.879	0.967
DA	0.58	0.60	0.55	0.59
CPU time (second)	8.61	21.34	11.9	13.64

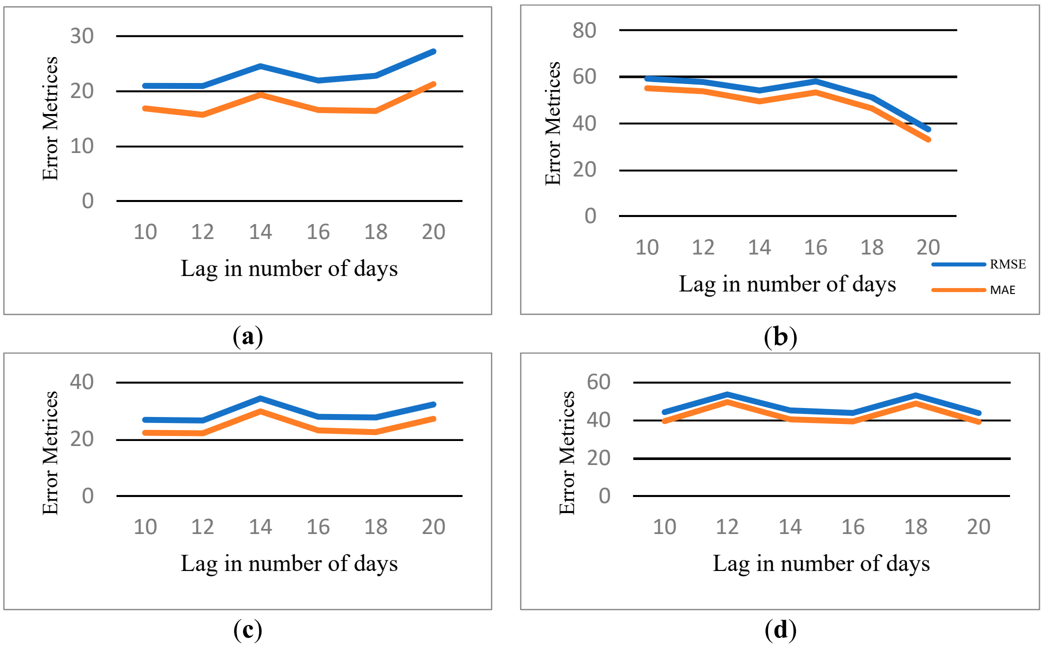


Figure 4. Mean absolute error (MAE) and root mean square error (RMSE) plots at different look-back values: (**a**) long short-term memory (LSTM) with news sentiment; (**b**) LSTM without news sentiment; (**c**) gated recurrent unit (GRU) with news sentiment; and (**d**) GRU without news sentiment.

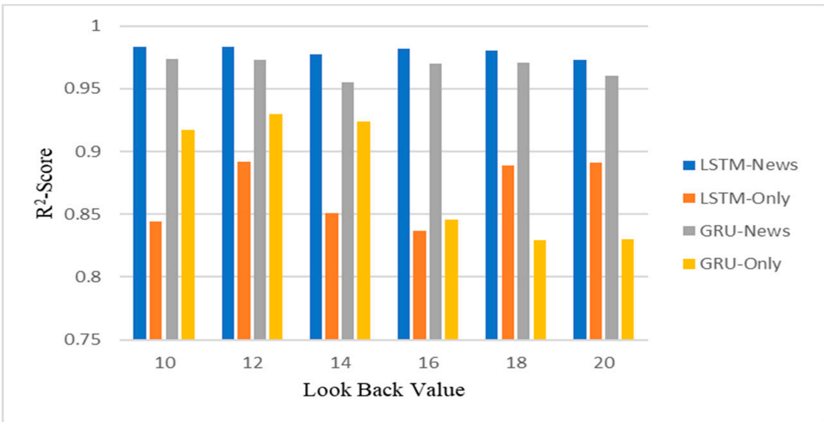


Figure 5. Chart of R^2 scores of the four models at different look-back values.

4.3. Statistical Test

Although some indicators point to the existence of significant differences between the outcomes with or without considering the financial news sentiments as part of the input to the models, such indicative fact must be evaluated through statistical analysis. We use the Diebold–Mariano (DM) test [26] to achieve this goal. Let a_t be the actual time series and p_t be the predicted time series; then $e_{i,t}$ is the forecast error of ith model. In the DM test, the null hypothesis assumes that two models have equal levels of prediction accuracy, i.e., $E(d_t) = 0$, where $dt = f(e_{1,t}) - f(e_{2,t})$ which refers to a loss differential with given loss function $f(.)$. The formula for DM statistics is given in Formula (17).

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \dot{f}_d(0)}{N}}} \tag{17}$$

where $\overline{d} = \frac{1}{N}\sum_{t=1}^{N} d_t$ and $\dot{f}_d(0)$ are consistent estimate of spectral density of the loss differential at frequency 0. We conducted the DM test on the loss function MAE as it is the simplest indicator with the least distortion caused by nonlinear operations like the square root. The statistical results are shown in Table 6.

Table 6. Diebold–Mariano (DM) tests on Mean absolute error (MAE) between forecasting models for Nepal Stock Exchange (NEPSE).

DM Stat./*p*-Value	GRU-Only	GRU-News	LSTM-Only	LSTM-News
GRU-Only		0.0373	0.295	0.000
GRU-News	2.087		0.012	0.001
LSTM-Only	−1.046	2.520		0.000
LSTM-News	3.546	3.181	3.597	

In Table 6, the values below the diagonal indicate the DM-statistics and the values above the diagonal represent the *p*-value for DM-test. We should reject the null hypothesis, i.e., the two model has no significant difference at 95% confidence level if DM-statistics is outside of the range −1.96 to 1.96 or *p*-value is less than 0.05. Otherwise, we cannot reject the null hypothesis. The *p*-value less than 0.05 (or the DM value outside of the range −1.96 to 1.96) is *italicized* in Table 6. Statistically the LSTM-News model distinguishes itself from other models in terms of MAE, so does the GRU-News model. There is no statistical difference between the LSTM only and GRU only models.

5. Discussion

It is difficult to compare our results with those from previous studies by other authors due to differences in stock datasets, news channels and sentiment rendering, model design and implementation, evaluation measures, computing environments, etc. The purpose of this study is to conduct a normalized comparison on the performances of the LSTM and GRU models for stock market forecasting under the same conditions and to objectively assess the significance of incorporating the financial news sentiments in stock market forecasting with respect to that if only the stock features are used. Hence, our results are sufficient for us to achieve these preset goals of this study. Our discussions are focused on the issues associated with these preset goals.

5.1. Performances of LSTM and GRU with Only Stock Features

In [18], the authors found that GRU outperformed LSTM in stock price prediction. In another comparative study involved more than ten different models [8], the authors tabulated the outcomes from applying these models to various stocks in different countries. It showed that the GRU and LSTM models had mixed performances, i.e., on some cases LSTM was better than GRU whereas on other occasions GRU outperformed LSTM.

Under the same conditions with MAE as the measure, our results show that the performances of LSTM and GRU are statistically different from each other, and GRU is the better performer with a MAE of 42.8 compared with a MAE of 48.5 for LSTM (Table 5). This seems to support the conclusion in [18]. By carefully inspecting the test plots of these two models in Figure 6, however, LSTM seems outperformed GRU in most period except from April 2019 to September 2019. It seems the overall average of MAE misrepresents the entire picture of forecasting to some extent. It is more logical to suggest that both LSTM and GRU are circumstantial in stock forecasting with only the stock market features as the input. This finding seems consistent with the outcomes from [8].

(**a**)

(**b**)

Figure 6. The actual and predicted price of agriculture development bank (ADBL) from 12/09/2017 to 14/11/2019. (**a**) LSTM; (**b**) GRU.

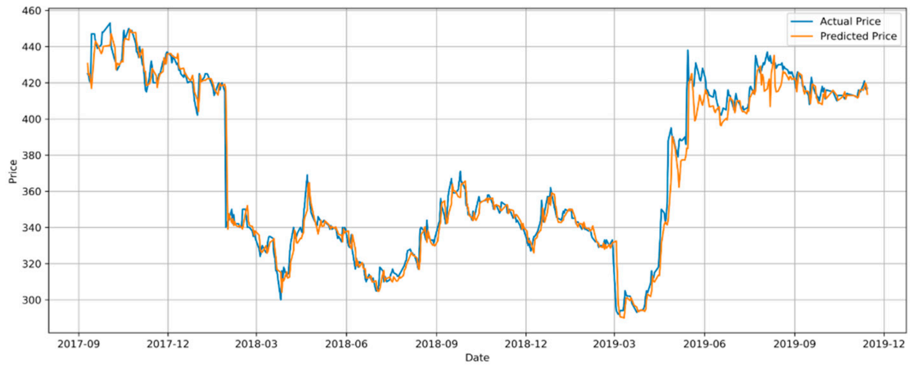
5.2. Performances of LSTM with and without Financial News Sentiments

In [15] and [20], the authors reported that LSTM with financial sentiment indicators showed improved performances over other traditional models, but they did not conduct any comparison on the performances of LSTM with and without considering any financial sentiment indicator in stock prediction. The result from [13] showed that the 'market emotion' was a very important factor influencing the stock market and could help prediction accuracy but failed to present any statistical support to this finding.

Under the same conditions with MAE as the measure, our results show that the performances of LSTM with and without the financial news sentiments are statistically different from each other, and the LSTM-News model has a far better performance with a MAE of 17.7 compared with a MAE of 48.5 for the LSTM only model (Table 5). The much stronger coefficient of determination (R^2) of 0.979 for the LSTM-News model indicates a consistent improvement in stock price prediction over the whole period, which can be seen by contrasting Figures 6a and 7a. The large discrepancies from April 2019 to September 2019 in Figure 6a for the LSTM only model are significantly reduced in Figure 7a for the LSTM-News model whereas the good fits in other sections are maintained. We can draw a conclusion with statistical confidence that the performance of LSTM for stock price forecasting can be significantly improved by incorporating the financial news sentiments with the stock features as the input.

However, it must be noted that the financial news sentiments must be carefully examined for relevance and rendered for appropriate representation. It is also noticeable that the processing time for the LSTM-News model is much longer compared with if only using the stock features (Table 5).

(**a**)



(**b**)

Figure 7. The actual and predicted price of ADBL from 12/09/2017 to 14/11/2019: (**a**) LSTM with News; (**b**) GRU with News.

5.3. Performances of GRU with and without Financial News Sentiments

Similar to the LSTM-News model, under the same conditions with MAE as the measure, our results show that the performances of GRU with and without the financial news sentiments are statistically different from each other, and the GRU-News model has a far better performance with a MAE of 24.5 compared with a MAE of 42.8 for the GRU only model (Table 5). The strong coefficient of determination (R^2) of 0.967 for the GRU-News model indicates a consistent improvement in stock price prediction over the whole period (Figure 7b). We can also draw a conclusion with statistical confidence that the performance of GRU for stock price forecasting can be significantly improved by incorporating the financial news sentiments with the stock features as the input. Once again, the financial news sentiments must be carefully examined for relevance and rendered for appropriate representation.

5.4. Performances of LSTM and GRU with Financial News Sentiments

Our results and previous discussions have confirmed that both LSTM and GRU can produce far better performance in stock forecasting by incorporating the financial news sentiments with the stock features as the input. The other suggestion is that both LSTM and GRU are circumstantial in stock forecasting if without considering the financial news sentiments. The following discussion is focused on assessing if both LSTM and GRU are still circumstantial in stock forecasting if considering both the financial news sentiments and the stock market features as the input.

Under the same conditions with MAE as the measure, our results show that the performances of LSTM-News and GRU-News are statistically different from each other (p = 0.001), and the LSTM-News model has a better performance with a MAE of 17.7 compared with a MAE of 24.5 for the GRU-News (Table 5). However, there is no statistical difference in the coefficient of determination (R^2) between LSTM-News (0.979) and GRU-News (0.967). This means that the direct correlation between the actual and predicted prices is almost the same for both the LSTM-News and GRU-News models. Hence, the averaging nature of MAE may smooth the pikes of errors in some sections.

By carefully inspecting the test plots of these two models in Figure 7, GRU seems performed better than LSTM over the whole period virtually. However, statistically in terms of MAE, LSTM seems a better performer than GRU. Given these contradictory indications, it is logical to suggest that both the LSTM-News and GRU-News models are able to produce better forecasting in stock price, equally, not one over the other. This point implies that the cooperative deep-learning architecture proposed in this study could be modified as an expert system incorporating both the LSTM-News and GRU-News models to recommend the best possible forecasting whichever model can produce dynamically.

6. Conclusions and Further Work

By utilizing the cooperative deep-learning architecture proposed in this study, we have achieved the aims set for this study, i.e., a normalized comparison on the performances of the LSTM and GRU models for stock market forecasting under the same conditions and an objective assessment on the significance of incorporating the financial news sentiments with the stock features as the input in stock market forecasting. Further discussion and statistical analysis on the experiment results have led to the following conclusion—or suggestion or implication—under the same conditions with MAE as the measure,

- It is suggested that both LSTM and GRU are circumstantial in stock forecasting with only the stock market features as the input, not one better than the other;
- It is concluded with statistical confidence that the performance of LSTM for stock price forecasting can be significantly improved by incorporating the financial news sentiments with the stock features as the input;
- It is concluded with statistical confidence that the performance of GRU for stock price forecasting can be significantly improved by incorporating the financial news sentiments with the stock features as the input;
- It is suggested that both the LSTM-News and GRU-News models are able to produce better forecasting in stock price, equally, not one over the other. However, both models require more computation power or take longer time to complete the process;
- It is implied that the cooperative deep-learning architecture proposed in this study could be modified as an expert system incorporating both the LSTM-News and GRU-News models to recommend the best possible forecasting dynamically.

This work can be extended in some ways. First, we only considered the NEPSE-ADBL stock for the experiments over a certain period. An extended period with varieties of stock data and financial news from different countries should make our findings solider. Second, more effort should be made on how the financial news sentiments can be better rendered and represented to ensure a high level of relevance in stock market forecasting. In a broader sense, other media sentiments regarding particular stocks, in addition to the financial news, may be considered in future studies. Of course, it would be more useful if the cooperative deep-learning architecture proposed in this study can be modified as an expert system incorporating both the LSTM-News and GRU-News models to recommend the best possible forecasting dynamically from either or both in the future.

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References

1. Malkiel, B.G.; Fama, E.F. Efficient capital markets: A review of theory and empirical work. *J. Financ.* **1970**, *25*, 383–417. [CrossRef]
2. Fama, E.F. Random walks in stock market prices. *Financ. Anal. J.* **1995**, *51*, 75–80. [CrossRef]
3. Pai, P.-F.; Lin, C.-S. A hybrid ARIMA and support vector machines model in stock price forecasting. *Omega* **2005**, *33*, 497–505. [CrossRef]
4. Araújo, R.d.A. A quantum-inspired evolutionary hybrid intelligent approach for stock market prediction. *Int. J. Intell. Comput. Cybern.* **2010**, *3*, 24–54. [CrossRef]
5. Vui, C.S.; Soon, G.K.; On, C.K.; Alfred, R.; Anthony, P. A review of stock market prediction with Artificial neural network (ANN). In Proceedings of the 2013 IEEE international conference on control system, computing and engineering, Mindeb, Malaysia, 29 November–1 December 2013; pp. 477–482.
6. Shi, C.; Zhuang, X. A study concerning soft computing approaches for stock price forecasting. *Axioms* **2019**, *8*, 116. [CrossRef]
7. Nti, I.K.; Adekoya, A.F.; Weyori, B.A. A systematic review of fundamental and technical analysis of stock market predictions. *Artif. Intell. Rev.* **2019**, *53*, 3007–3057. [CrossRef]
8. Lv, D.; Huang, Z.; Li, M.; Xiang, Y. Selection of the optimal trading model for stock investment in different industries. *PLoS ONE* **2019**, *14*, e0212137. [CrossRef] [PubMed]
9. Atsalakis, G.S.; Valavanis, K.P. Surveying stock market forecasting techniques–Part II: Soft computing methods. *Expert Syst. Appl.* **2009**, *36*, 5932–5941. [CrossRef]
10. Seng, J.-L.; Yang, H.-F. The association between stock price volatility and financial news–a sentiment analysis approach. *Kybernetes* **2017**, *46*, 1341–1365. [CrossRef]
11. Chen, K.; Zhou, Y.; Dai, F. A LSTM-based method for stock returns prediction: A case study of China stock market. In Proceedings of the 2015 IEEE International Conference on Big Data, Santa Clara, CA, USA, 29 October–1 November 2015; pp. 2823–2824.
12. Althelaya, K.A.; El-Alfy, E.-S.M.; Mohammed, S. Evaluation of bidirectional LSTM for short-and long-term stock market prediction. In Proceedings of the 2018 9th international conference on information and communication systems (ICICS), Irbid, Jordan, 3–5 April 2018; pp. 151–156.
13. Jiawei, X.; Murata, T. Stock market trend prediction with sentiment analysis based on LSTM neural network. In Proceedings of the International Multi-Conference of Engineers and Computer Scientists (IMECS), Hong Kong, China, 13–15 March 2019; pp. 13–15.
14. Samarawickrama, A.; Fernando, T. A recurrent neural network approach in predicting daily stock prices an application to the Sri Lankan stock market. In Proceedings of the 2017 IEEE International Conference on Industrial and Information Systems (ICIIS), Peradeniya, Sri Lanka, 15–16 December 2017; pp. 1–6.
15. Li, J.; Bu, H.; Wu, J. Sentiment-aware stock market prediction: A deep learning method. In Proceedings of the 2017 International Conference on Service Systems and Service Management, Dalian, China, 16–18 June 2017; pp. 1–6.
16. Shen, G.; Tan, Q.; Zhang, H.; Zeng, P.; Xu, J. Deep learning with gated recurrent unit networks for financial sequence predictions. *Procedia Comput. Sci.* **2018**, *131*, 895–903. [CrossRef]
17. Rahman, M.O.; Hossain, M.S.; Junaid, T.-S.; Forhad, M.S.A.; Hossen, M.K. Predicting prices of stock market using gated recurrent units (GRUs) neural networks. *Int. J. Comput. Sci. Netw. Secur.* **2019**, *19*, 213–222.
18. Saud, A.S.; Shakya, S. Analysis of look back period for stock price prediction with RNN variants: A case study on banking sector of NEPSE. *Procedia Comput. Sci.* **2020**, *167*, 788–798. [CrossRef]
19. Dang, M.L.; Sadeghi-Niaraki, A.; Huynh, H.D.; Min, K.; Moon, H. Deep learning approach for short-term stock trends prediction based on two-stream gated recurrent unit network. *IEEE Access* **2018**, *6*, 55392–55404.
20. Li, X.; Wu, P.; Wang, W. Incorporating stock prices and news sentiments for stock market prediction: A case of Hong Kong. *Inf. Process. Manag.* **2020**, *57*, 102212. [CrossRef]
21. Cho, K.; van Merrienboer, B.; Gulcehre, C.; Bahdanau, D.; Fethi, B.; Holger, S.; Bengio, Y. Learning phrase representations using RNN encoder–decoder for statistical machine translation. *arXiv* **2014**, arXiv:1406.1078.
22. Dutta, A.; Kumar, S.; Basu, M. A gated recurrent unit approach to bitcoin price prediction. *J. Risk Financ. Manag.* **2020**, *13*, 23. [CrossRef]
23. Chaithra, V. Hybrid approach: Naive Bayes and sentiment VADER for analyzing sentiment of mobile unboxing video comments. *Int. J. Electr. Comput. Eng.* **2019**, *5*, 4452–4459.

24. Hutto, C.J.; Gilbert, E. Vader: A parsimonious rule-based model for sentiment analysis of social media text. In Proceedings of the 8th International AAAI Conference on Weblogs and Social Media, Ann Arbor, MI, USA, 1–4 June 2014.
25. Jayalakshmi, T.; Santhakumaran, A. Statistical normalization and back propagation for classification. *Int. J. Comput. Theory Eng.* **2011**, *3*, 1793–8201.
26. Diebold, F.X.; Mariano, R.S. Comparing predictive accuracy. *J. Bus. Econ. Stat.* **2002**, *20*, 134–144. [CrossRef]