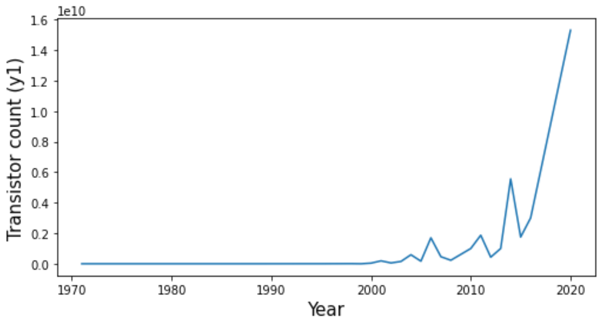
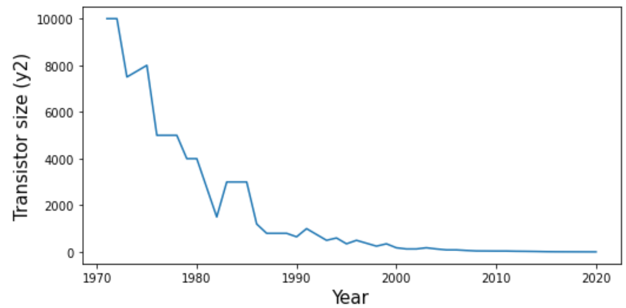
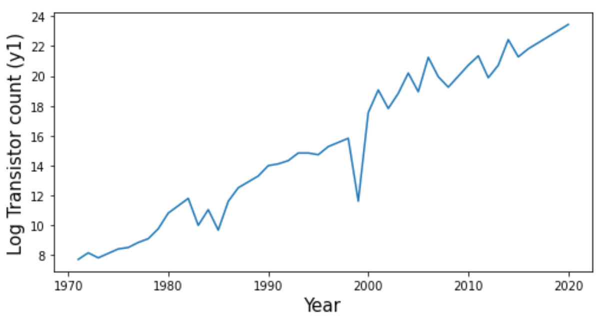
Q1:

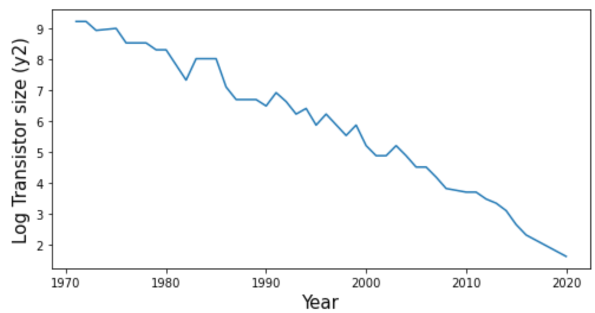
- Plot the actual data for both cases, (xvs.y1, xvs.y2)





- Plot the log value of actual data for both cases (xvs.y1, xvs.y2)



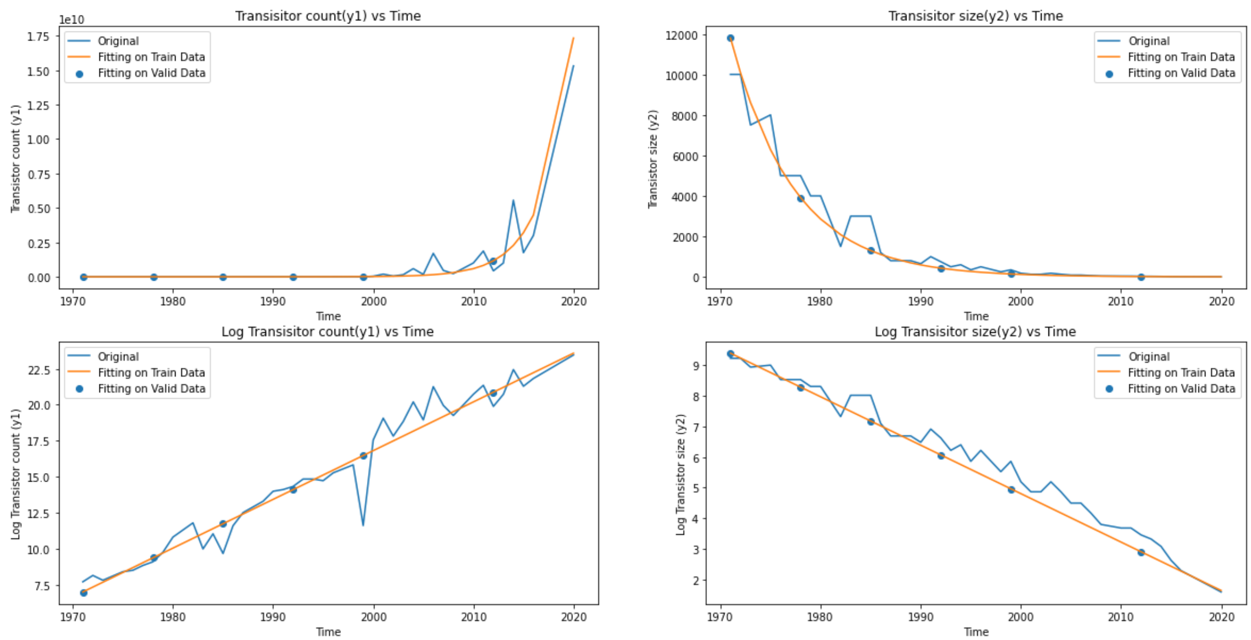


- Provide a brief description of the choice of parameters used for the computation (Learning Rates, selection of validation dataset).
    - Tried various learning rates from 1e-2 to 1e-5.
        - 1e-2 did not converge well
        - 1e-5 was very slow in converging
        - 1e-4 was quick and easy to converge
        - 0.3e-4 gave the minimum loss error.
    - Have kept the tolerance value of 1e-5. With 1e-10, models were overfitting.
    - Data points were chosen at equal intervals from 1970 to 2020, so that validation data is general enough.
    - Initial values of b0 and b1 were selected 1 randomly.

- Provides a brief description of data over-fitting and under-fitting of data. Which of the above categories your current model fits?
    - Overfitting is the scenario, where a model makes a complex decision boundary to learn totally training data while it fails to reduce the validation error. Variance is substantially high.

- Underfitting is the scenario, where the model is unable to learn the training data. Here the training error itself will be large.
- From the graph below, validation and training has very well learnt the actual data distribution. Our model is neither over or under fitted. It is an unbiased model.

➢ Plot your actual data along with the linear regression line for both y1, y2 separately.



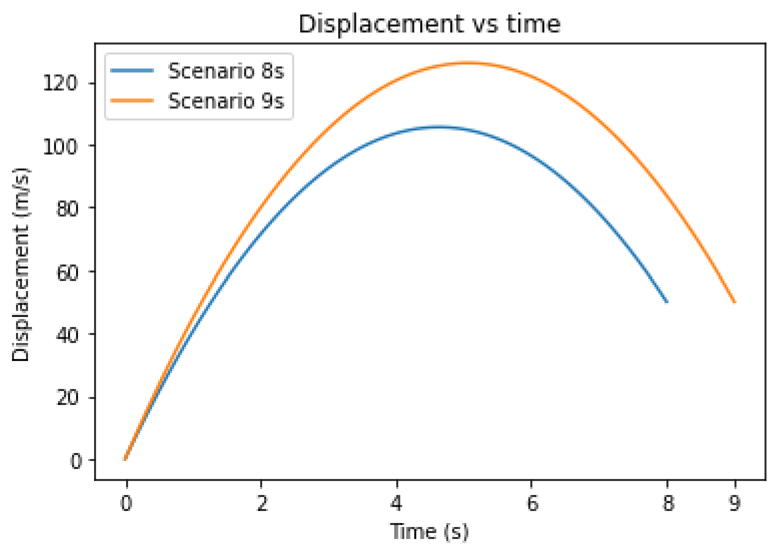
➢ Document your training error and validation error for both y1 and y2 separately.

	Training Error	Validation Error
For Y1	5.73	8.82
For Y2	432585.8	1238420.5

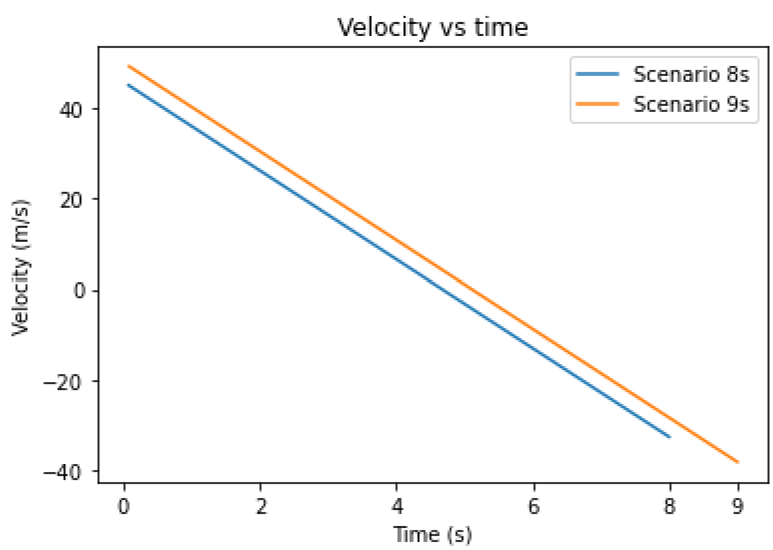
Error is calculated by transforming data into the original exponential format.

Q2:

- ➢ Displacement vs time plot



- ➢ Velocity vs time plot



- ➢ Does the bird have sufficient velocity in the y−direction to break the obstacle?
    - ○ For scenario (8s), the velocity of the bird at the point of target is 32.57 m/s which is less than 35m/s. Hence, It will not be able to break the obstacle.

- ➢ Consider one other scenario, where the bird is flung in such a way that it hits the target at t = 9 s. In this scenario, does the bird have sufficient velocity in the y− direction to break the obstacle ?

- In this scenario, the velocity of the bird at the point of target is 38.14 m/s which is greater than 35m/s. Hence it will be able to break the obstacle.

➢ Matrix coefficient derivation:

$$f(x + h) = f(x) + h f'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f''''(x) + \ldots \qquad \text{- (A)}$$

$$f(x - h) = f(x) - h f'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f''''(x) - \ldots \qquad \text{- (B)}$$

By addition A and B,

$$f(x + h) + f(x - h) = 2 f(x) + h^2 f''(x) + \frac{h^4}{12}f''''(x) + \ldots$$

Rearranging equations,

$$f''(x) = \frac{f(x-h) - 2f(x) + f(x+h)}{h^2} + O(h^2)$$

Thus,

$$f''(1) = \frac{f(0) - 2f(1) + f(2)}{1^2}$$

Similarly for all i = 1 to n

Matrix coefficients =
$$\begin{matrix} [1 & -2 & 1\,0\,0\,0\,0 \,.\,.\,.\,.\,] \\ [0\ 1 & -2\,1\,0\,0\,0 \,.\,.\,.\,.] \\ [0\ 0\ 1 & -2\,1\,0\,0 \,.\,.\,.\,.\,] \\ [.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,] \\ \\ [.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\,1 & -2 & 1\,] \end{matrix}$$

Q3.Derive the five point finite difference formula to find the second derivative at a point (in 1D). Comment on truncation error and accuracy

$$f(x \pm h) = f(x) \pm h f'(x) + \frac{h^2}{2} f''(x) \pm \frac{h^3}{6} f'''(x) + \frac{h^4}{4!} f^{iv}(x) + \dots$$

$$f(x \pm 2h) = f(x) \pm 2h f'(x) + \frac{4h^2}{2!} f''(x) \pm \frac{8h^3}{3!} f'''(x) + \frac{16h^4}{4!} f^{iv}(x) + \dots$$

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2 f''(x) + \frac{32h^4}{4!} f^{iv}(x)$$

$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \frac{2h^4}{4!} f^{iv}(x)$$

$$16\left[f(x+h) + f(x-h)\right] = 32f(x) + 16h^2 f''(x) + \frac{32h^4}{4!} f^{iv}(x) + O(h^6)$$

$$16f(x+h) + 16f(x-h) - f(x+2h) - f(x-2h)$$

$$= 30f(x) + 12h^2 f''(x) + O(h^6)$$

$$\therefore f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2} + O(h^4)$$

Truncation error is of order h^4. for h = 0.1 => Error = 10^{-5}