E1 215-O: Tutorial Questions

Linear and Non-linear Optimization

January 28, 2022

1. Find the stationary points of the following functions, are they local minima or maxima.
   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = \sin x_1 \cos x_2$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$
   (c) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = x_1 x_2 e^{-(x_1^2+x_2^2)}$
2. Show that the following functions are convex,
   (a) $f : \mathbb{R}_{++} \to \mathbb{R}$, $f(x) = e^x$
   (b) $f : \mathbb{R}_{++} \to \mathbb{R}$, $g(x) = -\log x$
   (c) $f : \mathbb{R}_{++} \to \mathbb{R}$, $af(x) + bg(x)$, when $a, b \in (0, \infty)$
3. Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = ||x||_1$, where $||x||_1 := |x_1| + |x_2|$ is convex.
4. Find the local minima for the following functions,
   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3x^2 + 5x - 4$, in the interval $[-10, 10]$. Is it a global minima?
   (b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3x^4 + 5x^3 - 4x^2 + 2$, in the interval $[0.3, 1]$. Is it a global minima?
5. For each of the following function, determine whether it is convex, strictly convex, strongly convex or none of the above.
   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = (x_1 - 3x_2)^2$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = (x_1 - 3x_2)^2 + (x_1 - 2x_2)^2$.
   (c) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = (x_1 - 3x_2)^2 + (x_1 - 2x_2)^2 + x_1^2$.
6. Find the stationary points for the following functions, are they local minima or maxima.(Optional: Are they global minima or maxima?)
   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = 4x_1^2 + 9x_2^2 + 8x_1 - 36x_2 + 24$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = \frac{1}{3}x_1^3 + x_2^2 + 2x_1x_2 - 6x_1 - 3x_2 + 4$.