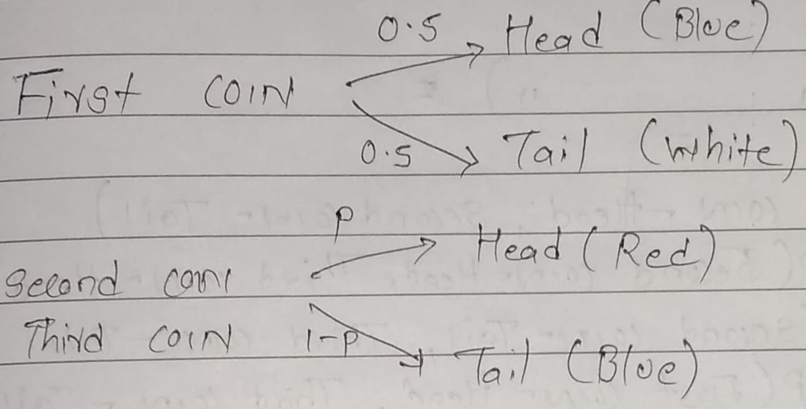
Introduction to data science

Problem set 2

1)



Two coin chosed at random and fliped

Sample space = { (First coin - Head, second coin - Head),
(First coin - Head, second coin - Tail),
(First coin - Tail, second coin - Head),
(First coin - Tail, second coin - Tail),

(Second coin - Head, Third coin - Head)
(Second coin - Head, Third coin - Tail),
(Second coin - Tail, Third coin - Head),
(Second coin - Tail, Third coin - Tail),

(First coin - Head, Third coin - Head),
(First coin - Head, Third coin - Tail),
(First coin - Tail, Third coin - Head),
(First coin - Tail, Third coin - Tail) }

Given,
P(sides that land face up are the same color) = $\frac{29}{96}$

P(sides " ") =

P(First coin - Head, Second coin - Tail)
+ P(Second coin - Head, Third coin - Head) +
P(Second coin - Tail, Third coin - Tail) +
P(First coin - Head, Third coin - Tail)

$= P(\text{First \& Second coin})\, P\left(\frac{HT}{\text{First \& Second}}\right) + \ldots$

$= \frac{1}{3}\frac{1}{2}(1-p) + \frac{1}{3}p.p + \frac{1}{3}(1-p)(1-p) + \frac{1}{3}\frac{1}{2}(1-p)$

$= \frac{1}{3}\left[1-p+p^2+(1-p)^2\right]$

$= \frac{1}{3}\left[1-p+p^2+1-2p+p^2\right]$

$= \frac{1}{3}\left[2p^2-3p+2\right]$

$\therefore\ 2p^2-3p+2 = 3 \times \frac{29}{96}$

$2p^2-3p+2-\frac{29}{32} = 0$

$\therefore\ 2p^2-3p+\frac{35}{32} = 0$

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$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b = -3 \quad a = 2 \quad c = +35/32$

$$p = \frac{3 \pm \sqrt{9 - 4(2)(35/32)}}{2(2)}$$

$$= \frac{3 \pm 0.5}{4} = \frac{3.5}{4} \text{ or } \frac{2.5}{4}$$

$$= 0.875 \text{ or } 0.625.$$

p can take values 0.875 or 0.625.

27)

a)

A : Student has 3 years of Industry Experience (TYI)
B : Student has a degree in EE

Given,

$P(\bar{A} \cap \bar{B}) = 0.3$

$P(\bar{A} \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap \bar{B}) = 0.8$

Find $p(\bar{A} \cap B) + p(A \cap \bar{B})$ i.e [Exactly one of the two]

$\therefore P(\bar{A} \cap B) + p(A \cap \bar{B}) = 0.8 - P(\bar{A} \cap \bar{B})$
$= 0.8 - 0.3$

$\boxed{P(\text{Exactly one of the two}) = 0.5}$

b)

C - registered for DA231

Given Student who had at most one of TYI and EE did not register for DA231 this semester.

~~So student who didn't had any of the TYI and EE register for the program,~~
~~i.e $P(C) = P(A^c \cap B^c)$~~

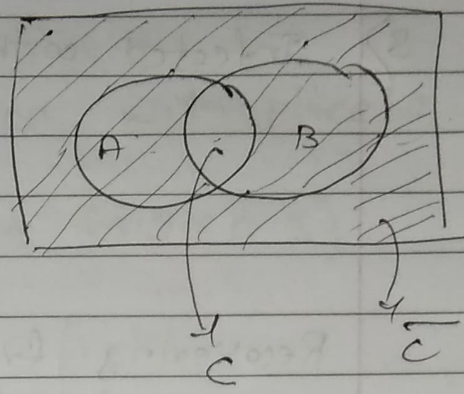
~~Since C is same as $P(A^c \cap B^c)$. C^c will be mutually Exclusive to $A^c \cap B^c$.~~

~~Thus $P(A^c \cap B^c \cap C^c) = 0.x$~~

$P(C^c) = P(A^c \cap B) + P(A \cap B^c) + P(A^c \cap B^c)$

$P(C) = P(A \cap B) = 1 - P(C^c)$
$= 1 - 0.8$
$= 0.2$

From Venn diagram.



(i) $P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - P(A \cup B)$

$= 1 - [P(\bar{A} \cap B) + P(A \cap \bar{B}) + P(A \cap B)]$

$= 1 - [0.5 + 0.2]$

$= \underline{0.3}$.

$$\boxed{P(\bar{A} \cap \bar{B} \cap \bar{C}) = 0.3}$$

(ii) ~~Stude~~

~~P(student of not picked~~

Student picked not registered for DA231

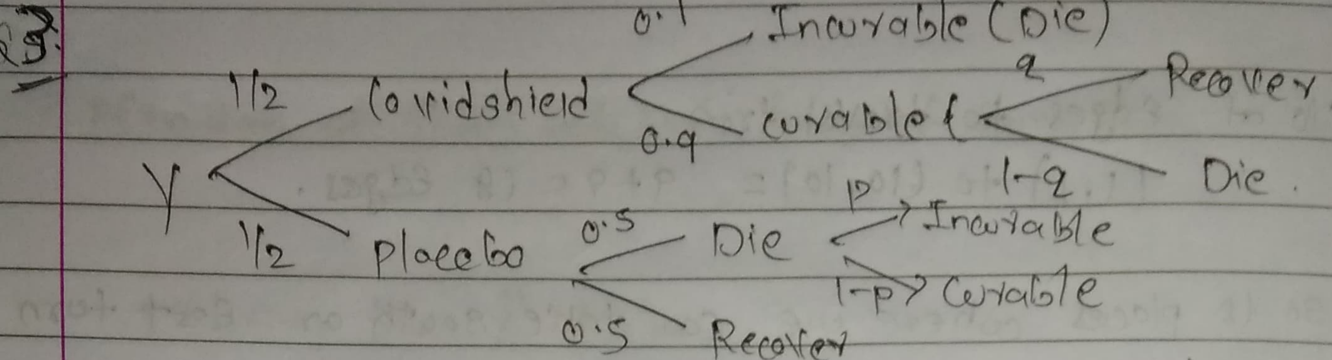
$= P(C^c) = P(A \cap B^c) + P(A^c \cap B) + P(A^c \cap B^c)$

probability (student not registered for DA231 has exactly one of TM or EE) =

$= P(A \cap B^c) + P(A^c \cap B)$

$= \underline{0.5}$.

Q3.



(a) 80% people on Covidshield recover.

i.e $0.9 \times q = 0.8$

$q = \frac{0.8}{0.9} = 0.889$

% of patients with a curable form of covid-19 that are cured by covidshield $= \frac{0.9 \times 0.889}{(0.9 \times 0.889) + (0.9 \times 0.111)}$

$= 0.889 \times 100$

$= 88.9\%$

(b)

P(Incurable people died in placebo)

$0.1 = P\left(\frac{\text{people died}}{\text{placebo group}}\right) P\left(\frac{\text{Incurable people}}{\text{people died}}\right)$

$0.1 = 0.5 \times p$

$p = \frac{0.1}{0.5} = 0.2 = P\left(\frac{\text{Incurable people}}{\text{People died in placebo}}\right)$

$\frac{0.4}{0.9}$

$\frac{4}{9}$

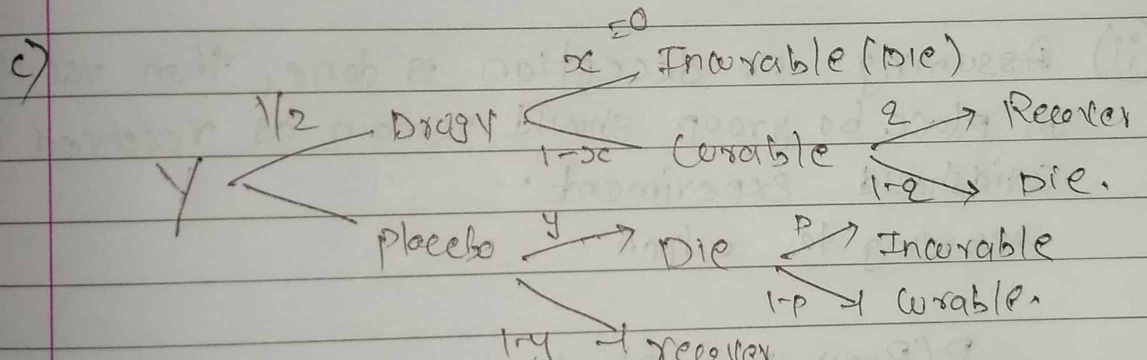
Fraction of patients with curable covid19 die in placebo group $= \frac{P(\text{Curable died in placebo group})}{P(\text{Total curable in placebo group})}$

$$= \frac{0.5\times0.8}{P(\text{curable died in placebo}) + P(\text{curable recovered in placebo})}$$

$$= \frac{0.5\times0.8}{0.5\times0.8 + 0.5}$$

$$= \frac{0.4}{0.9} = \underline{\underline{\frac{4}{9}}}$$ fraction of people die in placebo group which were curable.

c)



Since, Y has knowledge of which patient are incurable. He can place incurable patient in placebo group to maximize recovery chances with curable group with vaccine.

$\therefore\ x = 0$

Given that P(85% people recovered on Drug Y)

$$= P\left(\frac{\text{curable}}{\text{Drug Y}}\right) P\left(\frac{\text{Recovered}}{\text{curable}}\right)$$

$$0.85 = 1 \cdot q$$

$\therefore\ q = 0.85.$

% of patient with curable form of Covid are cured by Y $= \dfrac{P(\text{cured by Y (curable patient)})}{P(\text{Died by Y}_{\text{curable patient}}) + P(\text{cured by Y}_{\text{curable patient}})}$

$= \underline{\underline{85\%}}$

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d)

If there is a deceoth, the Incurable people must have moved to placebo group. indicating that placebo group has more incurable patient then

(i) ~~of ped~~ % patients with curable form that are cured by Covidshield > % patients with curable form that are cured by V. so covidshield is more effective.

(ii) Assuming no deception is done, then results on placebo group should remain as received in covidshield experiment.
According to which

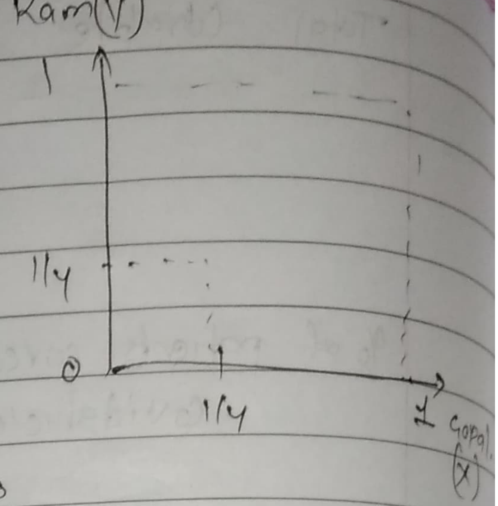
$P(\text{Recovery in placebo group}) = 0.5$

$P\left(\frac{\text{Dead in placebo}}{\text{curable}}\right) = \frac{0.5 \times 0.8}{0.9} = \frac{4}{9}$

$$\boxed{\text{if } P\left(\frac{\text{Dead in placebo}}{\text{curable in V's drug Experiment}}\right) < \frac{4}{9}\text{, then it is deceptive.}}$$

$$\boxed{\text{It is deceptive if } P\left(\frac{\text{Recovery for}}{\text{placebo group in V's drug test}}\right) < \frac{0.5}{2}}$$

4)



A: Magnitude of the difference of their scores is greater than 1/4.

$|X-Y| > 1/4$

B: Atleast one of their scores is greater than 1/4.

[1 - both less than 1/4]

C: Both score Equally

D: Gopal's score is greater than 1/4.

$\rightarrow$ $P(B) = 1 - P(\text{both score less than } 1/4)$

$= 1 - 1/4 \times 1/4$

$$\boxed{P(B) = \frac{15}{16}}$$

$P(C) = P(\text{Both are equal})$

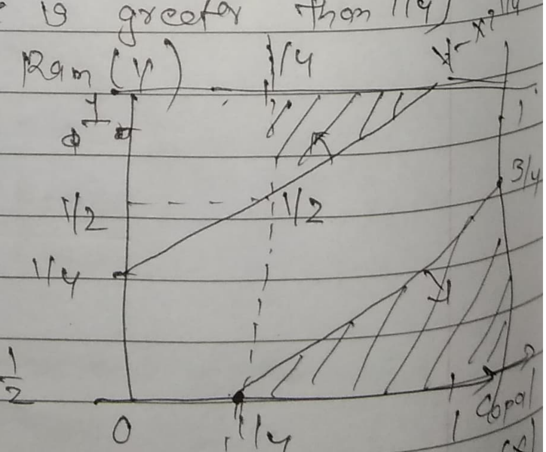
$\boxed{P(C) = 0.}$

$P(A\cap B) = P(\text{Magnitude difference} > 1/4$ and Gopal score is greater than $1/4)$

A : I,

$X - Y > 1/4 \rightarrow$ at $Y=0$, $X > 1/4$, $Y < 3/4$, $X = 1$

$Y - X > 1/4 \rightarrow$ $Y > 1/4$, $X < 3/4$, $X = 0$, $Y = 1$



$P(A\cap B)$

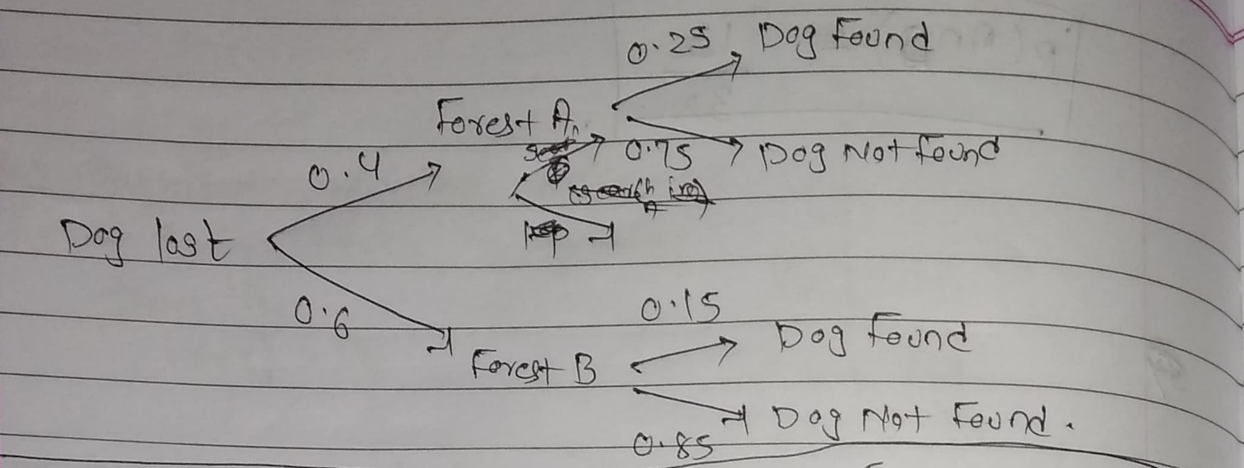
$= 1/2 \times 3/4 \times 3/4 + \frac{1}{2} \times \left(\frac{3}{4} - \frac{1}{4}\right) \times \frac{1}{2}$

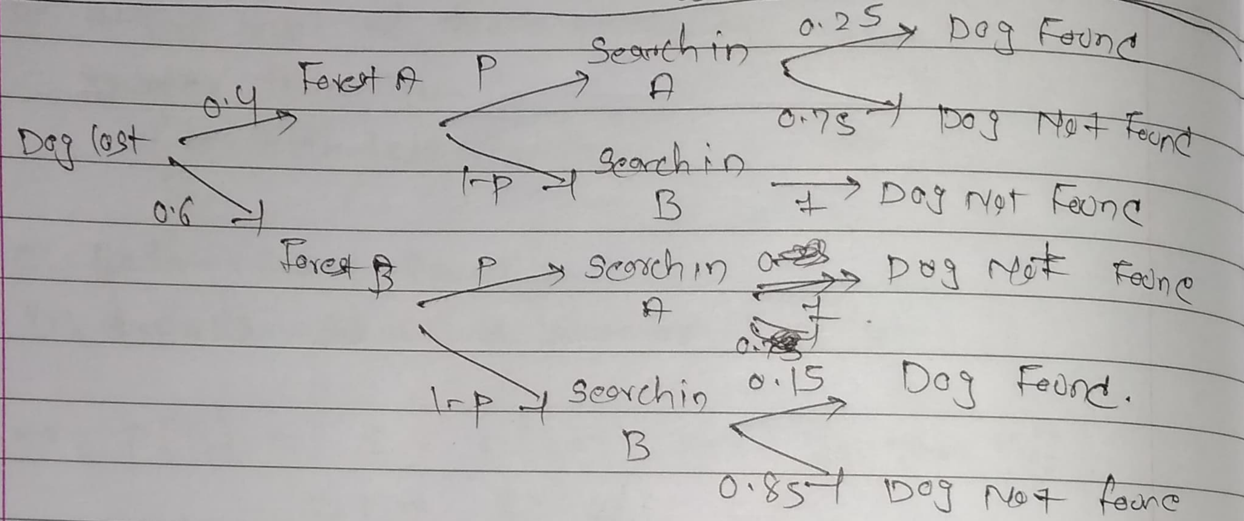
$= \frac{9}{32} + \frac{1}{8}$

$$\boxed{p(A \cap B) = \frac{13}{32}}$$

9)

5)





on 1st day

$P\left(\frac{\cancel{\text{Dog Found}}}{\cancel{\text{Search in A}}}\right)$

(a) P(Dog lost in A ∩ Dog found on 1st day by searching in forest A)

= 0.4 × 0.25 = 0.1 (∵ P = 1)

P(Dog lost in B ∩ searched in forest B)

= 0.6 × 0.15 = 0.09. (∵ P = 0)

$\boxed{\text{Dog found in A > Dog found in B,}}$

So oscar should look into forest A to maximize the probability of finding dog on first day.

b) $P\left(\begin{array}{l}\text{Dog is in A} \\ \hline \text{No Dog found} \\ \text{while searching in A}\end{array}\right) = \dfrac{P(\text{Dog is in A \& not found})}{P(\text{Dog not found while searching in A})}$

$$= \frac{0.4 \times 0.75}{P\left(\begin{array}{l}\text{Dog in A \& not found} \\ \hline \text{search in A}\end{array}\right) + P\left(\begin{array}{l}\text{Dog in B \& not found} \\ \hline \text{search in A}\end{array}\right)}$$

$$= \frac{0.4 \times 0.75}{0.4 \times 0.75 + 0.6 \times 1} \quad (\because P = 1)$$

$$= \frac{0.3}{0.3 + 0.6} = 0.33$$

c) $P = 0.5$.

$$\cancel{P\left(\begin{array}{l}\text{Looked in A} \\ \hline \text{Found Dog on 1st day}\end{array}\right) = P\left(\begin{array}{l}\text{Looked in A} \\ \hline \text{Dog is in A}\end{array}\right)}$$

$$\cancel{= \frac{P(\text{Looked in A} \cap \text{Dog in A})}{P(\text{Dog in A} \cap \text{looked in A}) + P(\text{Dog in A} \cap}}$$

c) $P = 0.5$

$P\left(\dfrac{\text{Looked in A}}{\text{Found Dog on 1st day}}\right)$

$= \dfrac{P(\text{Looked in A} \cap \text{Found Dog on 1st day})}{P(\text{Found Dog on 1st day})}$

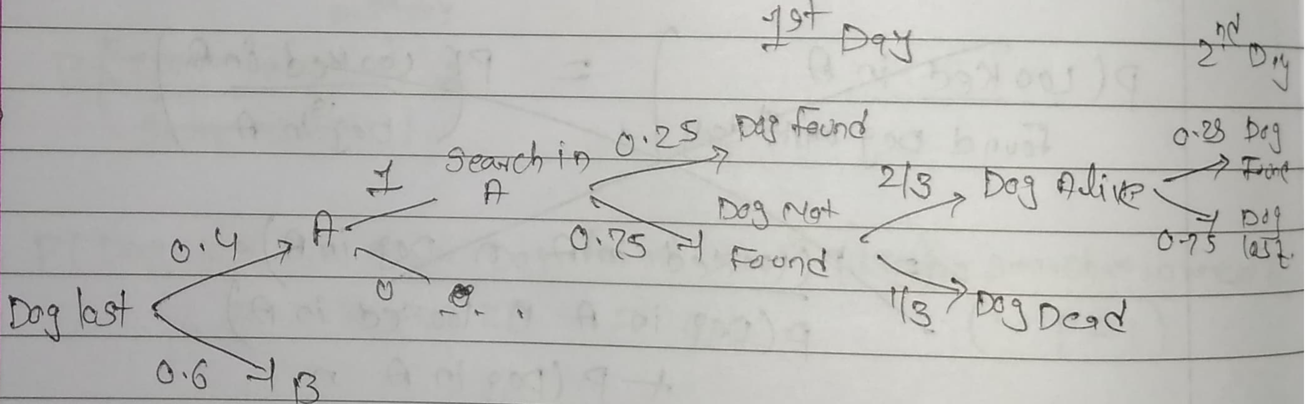
$= \dfrac{P(\text{Looked in A} \cap \text{Dog lost in A})}{P(\text{Looked in A} \cap \text{Dog lost in A}) + P(\text{looked in B} \cap \text{Dog lost in B})}$

$= \dfrac{0.4 \times P \times 0.25}{0.4 P \times 0.25 + 0.6 \times (1-P) \times 0.15}$

$\because P = 0.5$

$= \dfrac{0.05}{0.05 + 0.045} = 0.526$

d)



P(Dog Found on 2nd day)

$= 0.4 \times 1 \times 0.75 \times \frac{2}{3} \times 0.25$

$= 0.05$

6)

a) since there are 4 doors and prize is Equally likely to be behind any door.

$$P(\text{prize is behind door } A) = \frac{1}{4}.$$

b) $P\left(\frac{\text{prize behind door 4}}{\text{you picked 1 \& host opend door 2}}\right)$

P(prize behind door 4 & you picked 1 & host opend door 2)

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{1}{32}$$

(∵ host has only 2 options, Either to open door 2 or door3) ~~given prize is behind~~

P(you picked 1 & host opend door 2)

= P(prize behind 1 ∩ you picked 1 ∩ host opened 2)
+ P(prize behind 2 ∩ you picked 1 ∩ host opened 2)
+ P(prize behind 3 ∩ you picked 1 ∩ host opened 2)
+ P(prize behind 4 ∩ you picked 1 ∩ host opened 2)

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{4} \times 0 + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{1}{16}\left[\frac{1}{3} + \frac{1}{2} + \frac{1}{2}\right] = \frac{1}{12}.$$

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$$P\left(\frac{\text{prize behind door 4}}{\text{you picked 1 \& host opened 2}}\right)$$

$$= \frac{1/32}{1/12}$$

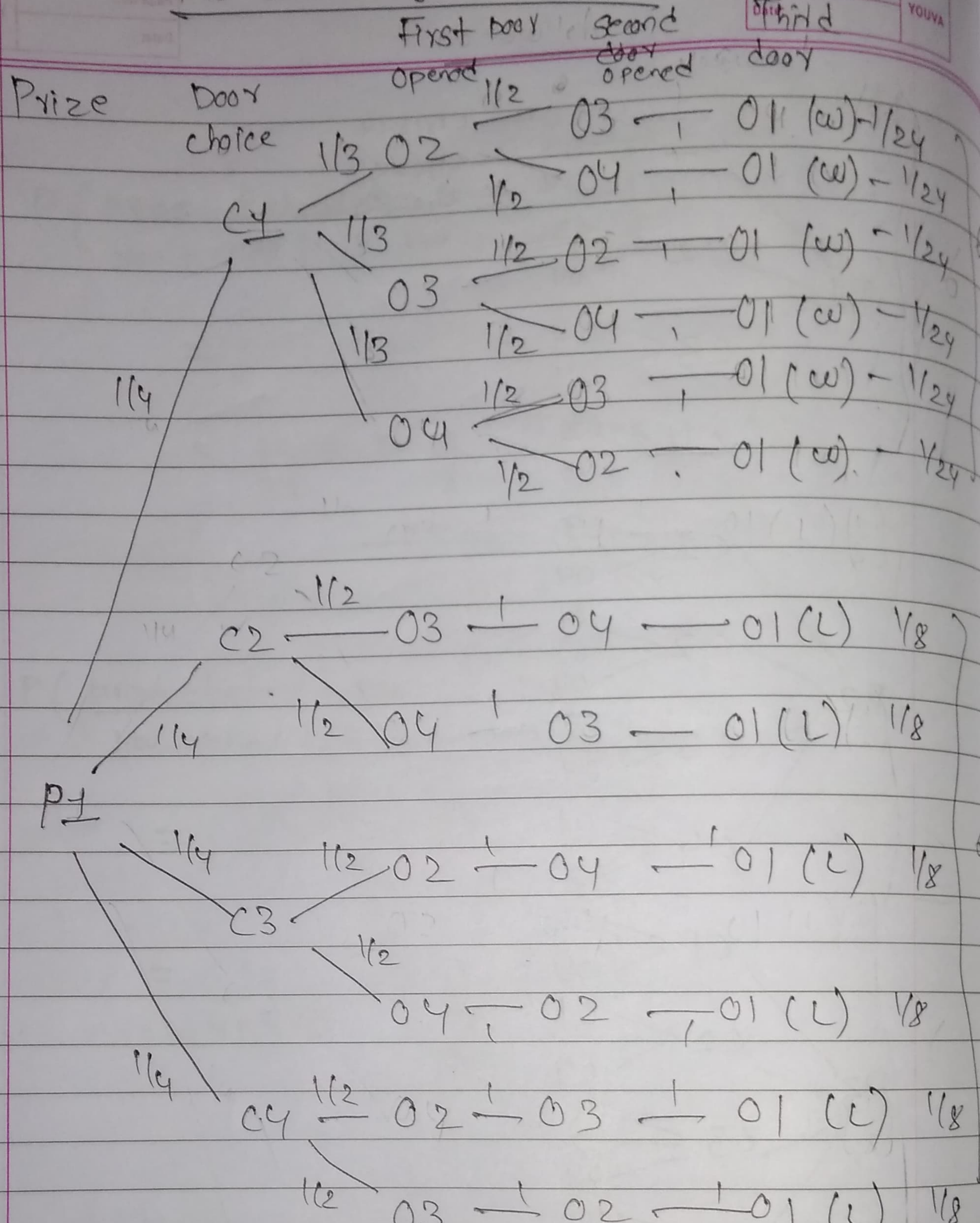
$$= \frac{12}{32} = \frac{3}{8} = 0.375.$$

$$P\left(\frac{\text{prize behind door 1}}{\text{you picked 1 \& host opened 2}}\right)$$

$$= \frac{1/4 \times 1/4 \times 1/3}{1/12}$$

$$= \frac{12}{4 \times 4 \times 3} = \frac{1}{4} = 0.25$$

No switching at all



Same will be repeated if prize at other locations.

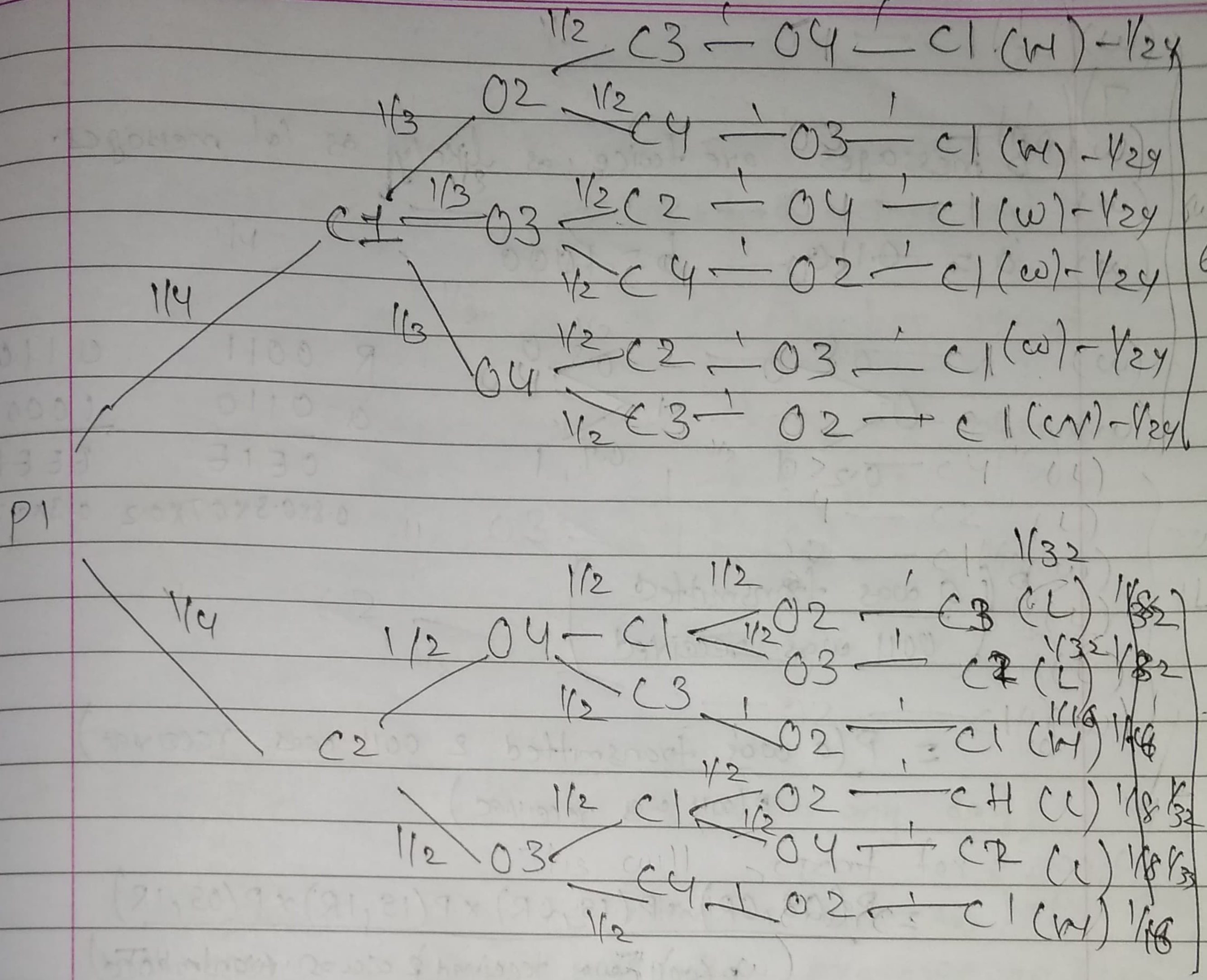
$$P(\text{win}) = \frac{\text{Total win}}{\text{Total win + loss}} = \frac{4(6 \times 1/24)}{4[6 \times 1/24 + 6/8]}$$

without switching.

$$= 1/4 = 0.25$$

[Always switching].

Always switch



This $\frac{C2}{P1}$ is repeated for C3, C4/P1,

Total win = $4\left(\frac{6}{24} + \frac{3\times2}{16}\right)$

Total win + loss = $4\left[\frac{6}{24} + \frac{6}{16}\right] + 4\left[3\times\frac{4}{32}\right]$

P (win) with both time switching. = $\frac{4\left[\frac{6}{24} + \frac{6}{16}\right]}{4\left[\frac{6}{24} + \frac{6}{16} + \frac{12}{32}\right]}$ = $\frac{5/8}{1}$ = 0.625.

By switching, we increase odds of winning. So strategy is to switch.

7) 'b' messages are twice as likely as 'a' messages.

a = 0110   b = 1000

$$0 \begin{cases} \xrightarrow{0.8} 0 \\ \xrightarrow{0.2} 1 \end{cases} \qquad 1 \begin{cases} \xrightarrow{0.3} 0 \\ \xrightarrow{0.7} 1 \end{cases}$$

R	0011	0110
a	0110	1000
	OEIE	EEE0
	$0.8\times0.3\times0.7\times0.2$	$0.3\times0.2\times0.2\times0.8$

$$P\left(\frac{a \text{ was transmitted}}{0011 \text{ was received}}\right)$$

$$= \frac{P(a \text{ was transmitted } \& \text{ } 0011 \text{ was received})}{P(0011 \text{ was received})}$$

$$= \frac{P(OS, OR) \times P(1S, OR) \times P(1S, 1R) \times P(OS, 1R)}{P(0011 \text{ was received } \& \text{ a was transmitted}) + P(0011 \text{ was received } \& \text{ b was transmitted})}$$

$$= \frac{0.8\times0.3\times0.7\times0.2}{0.8\times0.3\times0.7\times0.2 + 0.3\times0.2\times0.2\times0.8}$$

$$= \frac{0.0336}{0.0336 + 0.0096} = 0.778$$

8)

(a) P (session will not prorogue on a bad wether day)

$= P$ (atleast K out of n member attend)

$= 1 - P$(at most K-1 members are absent)

$$= 1 - \sum_{i=0}^{K-1} {}^{n}C_i * (1-q)^i (q)^{n-i}$$

(b) P (session will not prorogue on any day)

= P(session will not prorogue on bad day) × P(bad day) + P(session will not prorogue on good day) P (good day)

$$= \frac{1}{2}\left[1 - \sum_{i=0}^{K-1} {}^{n}C_i * (1-q)^i (q)^{n-i} + 1 - \sum_{i=0}^{K-1} {}^{n}C_i (1-p)^i (p)^{n-i}\right]$$

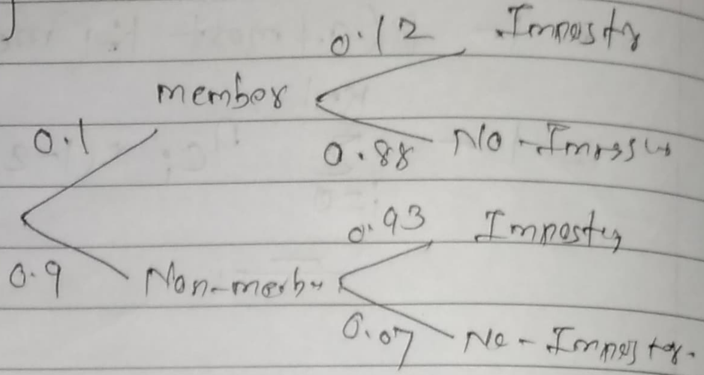
$$= \frac{2}{2}\left[1 - \sum_{i=0}^{K-1} {}^{n}C_i \left[(1-q)^i q^{n-i} + (1-p)^i (p)^{n-i}\right]\right]$$

9)

$P(\text{member}) = 0.1$

$P\left(\frac{\text{imposter}}{\text{member}}\right) = 0.12$

$P\left(\frac{\text{imposter}}{\text{non-member}}\right) = 0.93$

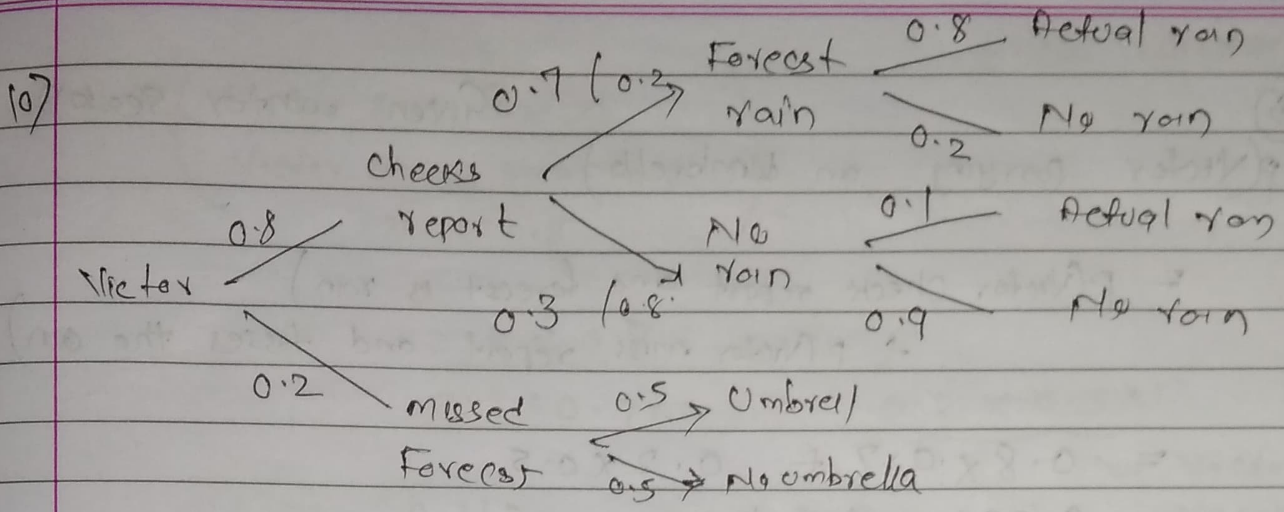


$$P\left(\frac{\text{real member}}{\text{Not-Impostr}}\right) = \frac{P(\text{real member} \cap \text{non-Impostr})}{P(\text{Non-Imposter})}$$

$$= \frac{0.1 \times 0.88}{0.1 \times 0.88 + 0.9 \times 0.07}$$

$$= \frac{0.088}{0.088 + 0.063} = 0.583$$

10)



a) $P\left(\frac{\text{Forecast rain}}{\text{Actual rain during winter}}\right) = \frac{P(FR \cap AR)}{P(\text{Actual rain during winter})}$

$$= \frac{0.7 \times 0.8}{0.7 \times 0.8 + 0.3 \times 0.1}$$

$$= \frac{0.56}{0.56 + 0.03} = 0.949$$

b) $P\left(\frac{\text{Forecast rain}}{\text{Actual rain during summer}}\right) = \frac{0.2 \times 0.8}{0.2 \times 0.8 + 0.8 \times 0.1}$

$$= \frac{0.16}{0.16 + 0.08} = 0.667$$

(b) Given winter Season

p(Victor carrying an umbrella)

= p(Victor check report and forecast is rain)
+ p(Victor miss report and tosses the coin)

= 0.8 x 0.7 + 0.2 x 0.5
= 0.56 + ~~0.1~~ 0.1
= 0.66

P(Forecast is no rain) = 0.2

p(Victor carrying umbrella & Forecast is no rain)
= p(Victor miss report & tosses coin)
= 0.2 x 0.5
= 0.1
≠ 0.66 x 0.2

Thus Victor carrying umbrella and Forecast no rain are not independent.

For summer Season,
p(Victor carrying umbrella) = 0.8 x 0.2 + 0.2 x 0.5
= 0.16 + 0.1 = 0.26.

p(Forecast no rain) = 0.8.

p(Victor carrys umbrella ∩ Forecast no rain) = 0.2 x 0.5
= 0.1
≠ 0.8 x 0.26.

No, answer do not depend on season. Both Events are dependent

Given winter season.

c) $P\left(\frac{\text{Victor saw Forecast}}{\text{Victor carrys umbrella \& not raining}}\right)$

$$= \frac{0.8 \times 0.7 \times 0.2}{0.8 \times 0.7 \times 0.2 + \underbrace{0.2 \times 0.5 \times 1}}$$

→ Victor misses report and tossed coin

$$= \frac{0.112}{0.212} = 0.528$$

Given summer season

$P\left(\frac{\text{Victor saw Forecast}}{\text{Victor carrys umbrella \& not raing}}\right)$

$$= \frac{0.8 \times 0.2 \times 0.2}{0.8 \times 0.2 \times 0.2 + 0.2 \times 0.5}$$

$$= 0.242$$

Yess it depends on Season.

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11) For tina to win game, she needs atleast 6 points in 8 games.

$$P_t = {}^8C_6 (p)^6 (1-p)^2 + {}^8C_7 (p)^7 (1-p) + {}^8C_8 (p)^8.$$

For Vikram to win game, he needs atleast 4 points in 8 games

$$P_v = {}^8C_4\, p^4 (1-p)^4 + {}^8C_5\, p^5 (1-p)^3 + {}^8C_6\, p^6 (1-p)^2 + {}^8C_7\, p^7 (1-p) + {}^8C_8\, p^8.$$

$$\text{Tina gets} = \frac{P_t}{P_t + P_v} \times 1000$$

(2)

a) $P(S_j \text{ is received})$

$$P(S_1 \text{ is received}) = P\left(\frac{S_1 \text{ is received}}{S_1 \text{ is transmit}}\right) \cdot P(S_1 \text{ is transmitt})$$

$$+ P\left(\frac{S_1R}{S_2T}\right) P(S_2T)$$

$$+ P\left(\frac{S_1R}{S_3T}\right) P(S_3T)$$

$$= 0.25 \times \frac{1}{3} + 0.04 \times \frac{1}{3} + 0.8 \times \frac{1}{3}$$

$$= \frac{1}{3}[1.09] = \underline{0.363}$$

$$P(S_2 \text{ is received}) = P\left(\frac{S_2R}{S_1T}\right) P(S_1T) + P\left(\frac{S_2R}{S_2T}\right) P(S_2T)$$

$$+ P\left(\frac{S_2R}{S_3T}\right) P(S_3T)$$

$$= \frac{(0.5 + 0.9 + 0.15)}{3} = \underline{0.516}$$

$$P(S_3 \text{ is received}) = P\left(\frac{S_3R}{S_1T}\right) P(S_1T) + P\left(\frac{S_3R}{S_2T}\right) P(S_2T)$$

$$+ P\left(\frac{S_3R}{S_3T}\right) P(S_3T)$$

$$= \frac{(0.25 + 0.06 + 0.05)}{3} = \underline{0.12}$$

(b)

$P\left(\frac{s_i\ sent}{s_j\ Received}\right)$		Sent s_1	s_2	s_3
Received	s_1	$\frac{0.25\times0.33}{0.363} = 0.227$	$\frac{0.04\times0.33}{0.363} = 0.036$	$\frac{0.8\times0.33}{0.363} = 0.727$
	s_2	$\frac{0.5\times0.33}{0.516} = 0.319$	$\frac{0.9\times0.33}{0.516} = 0.575$	$\frac{0.15\times0.33}{0.516} = 0.096$
	s_3	$\frac{0.25\times0.33}{0.12} = 0.6875$	$\frac{0.06\times0.33}{0.12} = 0.165$	$\frac{0.05\times0.33}{0.12} = 0.1375$

$$P\left(\frac{s_i\ sent}{s_j\ received}\right) = \frac{P(s_i\ sent \cap s_j\ received)}{P(s_j\ received)}$$

$$= P\left(\frac{s_j\ received}{s_i\ sent}\right) P(s_i\ sent)$$

c) One scheme could be to send the same signal twice. If received signal is same for both ~~the~~ same transmitted signal, the error probability can reduce.

If s_1 is transmitted twice

possible outcome =

$s_1 s_1$	$s_2 s_1$	$s_3 s_1$
$s_1 s_2$	$s_2 s_2$	$s_3 s_2$
$s_1 s_3$	$s_2 s_3$	$s_3 s_3$

(a)

3) P(randomly choosen symbol is received correctly)

$= P(0 \text{ was sent } \& \text{ received}) + P(1 \text{ was sent } \& \text{ received})$

$= P(1-e0) + (1-P)(1-e1)$

(b)

P(1011 is received correctly)

$= \cancel{P\left(\frac{1S}{1T}\right)} \times$

$= P(1S \cap 1T) \times P(0S \times 0T) \times P(1S \& 1T) \times P(1S \times 1T)$

$= (1-e1)^3 (1-e0)$

(c) If 0 is transmitted possible outcome ~~do~~ are for correct decoding

$\{000, 001, 010, 100\}$, 101, ~~011, 110, 111~~}

by ~~majority~~ rule → $\frac{4}{8}$ = ~~50~~ % To get 000 – $(1-e0)^3$

001, 010, 100 – $e0(1-e0)^2$

~~P(received 0 / tras 0)~~ = 1/2

= $\boxed{P(\text{correct decoding}) = (1-e_0)^3 + e_0(1-e_0)^2}$

~~d) P(0 was transmitt / 101 was received)~~ ~~= 0~~ =

by ~~majority~~ rule, ~~it will be consider~~ ~~as~~ 1.

(d)

$$P\left(\frac{0}{101}\right) = \frac{P(0)\,P(101/0)}{P(0)\,P(101/0) + P(1)\,P(101/1)}$$

$$= \frac{p\,e_0^2(1-e_0)}{p\,e_0^2(1-e_0) + (1-p)\,e_1(1-e_1)^2}$$

14) 2 five sided dice

a) A - sum of die is 10 $P(A) = \frac{1}{25}$

- {[redacted], {5,5}, [redacted]}

B,

(i) Atleast one of the dice has a 5 showing $P(B) = \frac{9}{25}$.

- {(5,1), (5,2), (5,3), (5,4), (5,5), [redacted]

(1,5), (2,5), (3,5), (4,5) [redacted]}

$P(A \cap B) = \{5,5\} = \frac{1}{25}$

$P(A \cap B) \neq P(A) \cdot P(B)$.

Event A is dependent upon 'atleast one of the dice has a 5 showing'.

(ii) C - atleast one of the dice has a 1 showing $P(C) = \frac{9}{25}$

- {(1,5), (1,1), (1,2), (1,3), (1,4)

(5,1), (2,1), (3,1), (4,1)}.

$A \cap C = 0$ - {from sample space}.

a $P(A \cap C) \neq P(A) \cdot P(C)$

Event A is dependent upon Event C.

(b) Event B → "you roll both the die and total is 8)
→ $\{(3,5), (4,4), (5,3)\} = \frac{3}{25}$

~~P(B~~

B_1 –

(i) Both dice showing same number
– $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\} = \frac{5}{25}$

$P(B \cap B_1) = \{4,4\} = \frac{1}{25}$

$P(B \cap B_1) \neq P(B) \times P(B_1)$
Thus they are dependent Events.

(ii) ~~B~~ –
$P\left(\frac{\text{atleast one has a 3 shows}}{\text{sum is 8}}\right)$

$= \frac{P(\text{atleast one has a 3 \& sum is 8})}{P(\text{sum is 8})}$

$= \frac{2/25}{3/25} = \frac{2}{3}$

(iii) $P\left(\frac{\text{atleast one of dice has 5}}{\text{sum is 8}}\right)$

$= \frac{P(\text{atleast one die is 5 and sum is 8})}{P(\text{sum is 8})}$

$= \frac{2/25}{3/25} = \frac{2}{3}$

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15)

Voice users	Data users
n_1	n_2
P_1	P_2
r_1 bits/sec	r_2 bits/sec

Total capacity = c bits/sec

P(more user want to use system than the system can accomodate)

→ If both voice and Data user are active at same time and if

$c < d_1 r_1 + d_2 r_2$, then system will not be able to accomodate

where $d_1 \to$ no of voice users active
$d_2 \to$ no of data users active.

P(d_1 users are active) = ${}^{n}C_{d_1} (P_1)^{d_1} (1-P_1)^{n_1 - d_1}$

P(d_2 users are active) = ${}^{n}C_{d_2} (P_2)^{d_2} (1-P_2)^{n_2 - d_2}$

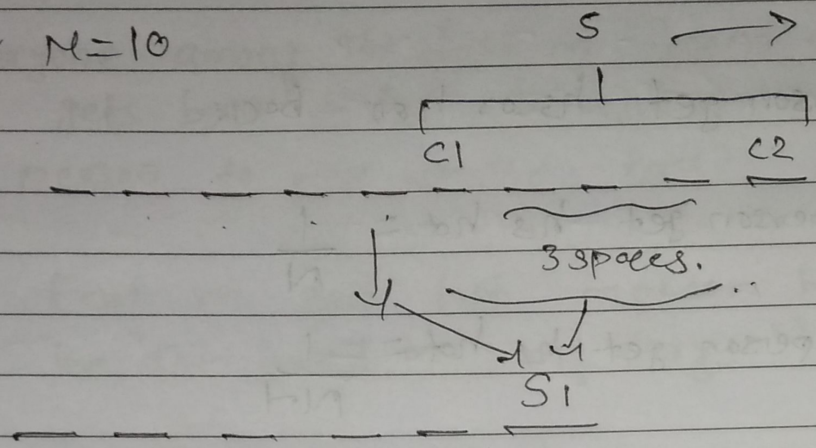
P(d_1 and d_2 users are active)

$$= {}^{n}C_{d_1} (P_1)^{d_1} (1-P_1)^{n_1-d_1} \times {}^{n}C_{d_2} (P_2)^{d_2} (1-P_2)^{n_2-d_2}$$

P(more user want here)

$$= \sum_{\{c < d_1 r_1 + d_2 r_2,\ d_1 \le n_1,\ d_2 \le n_2\}} P(d_1 \text{ are active})\, P(d_2 \text{ are active})$$

16) For $N=10$



This 5 entries can be considerat as 1 unit as Exactly 3 space is needed

S_1 can be placed in $N-4$ places in $\binom{N-4}{1} = N-4$ ways

After getting S_1 position, C_1 and C_2 choices can ~~Total no. of ways in which~~ - interchose

Exactly 3 spaces $\} = (N-4) \times 2$

Total No of ways to select two position for car $\Big\} = {}^{N}C_2$

$\therefore P\left(\begin{array}{l}\text{Exactly 3 spaces} \\ \text{between chose car.}\end{array}\right) = \dfrac{(N-4) \times 2}{{}^{N}C_2}$

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4)

a) Every person get his or her back.

→

1^{st} person get his hat $= \frac{1}{N}$

2^{nd} person get his hat $= \frac{1}{N-1}$

n^{th} person get his hat $= \frac{1}{N-N+1} = 1.$

$$p(\text{Every peson get hats}) = \frac{1}{N \times N-1 \times \cdots 1} = \frac{1}{N!}$$

(b) First m person get their own hat back.

m^{th} person got his hat $= \frac{1}{N-M+1}$

$(m+1)^{th}$ person choose any hat other than his $= \frac{N-M}{N-M}$

⋮

n^{th} person choose other hat $= \frac{N-N+1}{N-N+1} = 1$

p(First m got their hat)

$$= \frac{1}{N(N-1)(N-2)\cdots(N-M+1)} \times \frac{(N-M)(N-M-1)}{(N-M)(N-M-1)} \times 1$$

$$= \frac{(N-M)!}{N!}$$

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(c) Everyone among the first m persons to pick up the hats gets ~~bred~~ a hat belongs to one of the last m person to pick up the hats?

→ First m gets last m person hats - which can be distributed in $m!$ ways.

→ Remaining could be distributed in $(n-m)!$ ways.

$$P(c) = \frac{m!\,(n-m)!}{n!}$$

18) 8 lower :- (L1, L2 ... L8)
10 higher :- (H1, H2 ... H10)

A Valid curriculum - 4 Lower & 3 Higher.

a) No of different curriculum = $^8C_4 \times {}^{10}C_3$

$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} \times \frac{10 \times 9 \times 8}{3 \times 2}$

$= 56 \times 150$

$= 8400$

(b) Suppose {H1 ... H5} have L1 as a prerequisite and {H6 ... H10} have L2 and L3 as prerequisite.

→ ~~Given L1 is present → possible combo = $^7C_3 \times {}^5C_3$~~

~~Only possible Lower~~ or ~~subjects~~ are
= ~~all combinatn~~ - [~~Combinatn w/o L1, L2, L3~~]
= ~~$^8C_4$~~ - $^5C_4$

a) Only L1 and ~~Either of~~ or L2, L3 = $^6C_3 \times {}^5C_3 + {}^6C_3 \times {}^5C_3$
(choose from H1 to H5)

$= 2 \times {}^6C_3 \times {}^5C_3$

~~b) Only L9~~

b) No L1 and both L2 & L3 = $^5C_2 \times {}^5C_3$
(choose from H5 to H10)

c) L1, L2, L3 present = $^5C_1 \times {}^{10}C_3$

No of different curriculum

$= 2 \times {}^6C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_3 + {}^5C_1 \times {}^{10}C_3$

$= \cancel{2} \times \frac{\cancel{6} \times 5 \times 4}{\cancel{3 \times 2}} \times \frac{5 \times 4 \times \cancel{3}}{\cancel{3 \times 2}} + \frac{5 \times \cancel{4}}{\cancel{2}} \times \frac{5 \times 4 \times \cancel{3}}{\cancel{3 \times 2}} + 5 \times \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4}}{3 \times 2}$

$= 400 + 100 + 600$

$= 1100$

Date:

19)

P(no two of them celebrate bday at same day)

P(having bday on x^{th} day) = $\frac{1}{365}$.

First person

P(having bday on any of th 365 days) = $\frac{365}{365} = 1$

Second peson

P(having bday on any day other than 1st person) = $\frac{364}{365}$

Similarly

P(No two person have same bday)

$$= \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \cancel{\frac{(365-n+1)}{365}} \frac{(365-n+1)}{365}$$

$$= \frac{(365!)}{(365)^n (365-n)!}$$

20)

No of Edges to travel from

$(1,1)$ to $(10,10) = 9+9 = 18$ Edges.

So 18 places where we can take south or East turn

→ It means 9 south and 9 East turn.

No. of ways $= {}^{18}C_9$ ~~x~~.

Nandini stall is at $(6,7)$

$(1,1)$ to $(6,7) = 5+6 = 11$ Edges.

It means 5 south and 6 East

– No of ways $= {}^{11}C_5$

$(6,7)$ to $(10,10) = 4+3 = 7$ Edges

– No of ways $= {}^{7}C_3$

$$P = \frac{(1,1) \text{ to } (10,10) \text{ using } (6,7)}{\text{Total no. of ways}} = \frac{{}^{11}C_5 \times {}^{7}C_3}{{}^{18}C_9}$$