

E1 251-O: Tutorial Questions

Linear and Non-linear Optimization

April 12, 2022

1. Find the Sub-differential for the following functions.

(a) $f : \mathbb{R} \mapsto \mathbb{R}$, $f(x) = \max\{0, ax + b\}$ at all x , where $a, b \in \mathbb{R}$

(b) $f : \mathbb{R} \mapsto \mathbb{R}$, $f(x) = |x + 4|$ at $x = -4$

(c) $f : \mathbb{R}^2 \mapsto \mathbb{R}$, $f(x) = |x_1 - 1| + |x_2 + 1|$ at $x = (1, -1)$

2. Find a basic feasible solution to

$$2x_1 + x_2 + 2x_3 = 4$$

$$3x_1 + 3x_2 + x_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

3. Solve the following LP using simplex method.

$$\begin{array}{ll} \max_x & -2x_1 + 4x_2 + 7x_3 + x_4 + 5x_5 \\ \text{sub to} & -x_1 + x_2 + 2x_3 + x_4 + 2x_5 = 7 \\ & -x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 6 \\ & -x_1 + x_2 + x_3 + 2x_4 + x_5 = 4 \\ & x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0. \end{array}$$

4. Solve the following problem using quadratic penalty function.

$$\begin{array}{ll} \max_x & x_1 + x_2 \\ \text{sub to} & x_1^2 + x_2^2 - 2 = 0 \end{array}$$

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5. Solve the following problem using penalty and barrier function.

$$\begin{array}{ll} \max_x & 2(x_1^2 + x_2^2 - 1) - x_1 \\ \text{sub to} & x_1^2 + x_2^2 - 1 = 0 \end{array}$$

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6. Solve the following problem using Barrier method.

$$\begin{array}{ll} \max_x & x^2 + 1 \\ \text{sub to} & (x - 2)(x - 4) \leq 0 \end{array}$$

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7. Find the optimal solution for the following function using the theory of sub-gradients.

$$f(x) = \max\{(-x - 2, x - 5, -1)\}$$

8. Find the first iterate to the following problem using the subgradient projection algorithm.

$$\begin{array}{ll} \min_x & |x_1| + |x_2| + |x_3| \\ \text{sub to} & \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \end{array}$$