E1 251-O: Tutorial Questions (Coding)

Linear and Non-linear Optimization

April 16, 2022

1. Write codes to solve the following optimization problem using Barrier or penalty function methods with appropriate sequences π.

   (a)
   $$\begin{aligned} \min \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 = 1 \end{aligned}$$

   (b)
   $$\begin{aligned} \min_x \quad & -14x_1 + x_1^2 - 6x_2 + x_2^2 - 7 \\ \text{sub to} \quad & x_1 + x_2 \leq 2 \\ & x_1 + 2x_2 \leq 3 \end{aligned}$$

   (c)
   $$\begin{aligned} \max_x \quad & 5 - (x_1 - 2)^2 - 2(x_2 - 1)^2 \\ \text{sub to} \quad & \\ & x_1 + 4x_2 = 3 \end{aligned}$$

2. (a) Implement the conjugate gradient method to solve the following minimization problem,

   $\min_x f(x)$, where $f(x) = \dfrac{x^T A x}{2} - b^T x + c$, A is a symmetric positive definite matrix.

   (b) Using the above code solve the following problem, Where $A = \begin{bmatrix} 6 & 4 \\ 4 & 10 \end{bmatrix}$ and $b = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

   (c) Write a code for Optimal line search to solve the problem in $2(b)$

3. Implement Armijo's backtracking algorithm for the following minimization problem and report the first 10 iterates starting with initial point $x^0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \alpha = 1, \beta > 0.8, \sigma = 0.6.$

   $$\min_{x \in \mathbb{R}^2} \; 4 - x_1^2 - x_2^2 + x_1^4 + x_2^4$$

4. (a) Implement the projection gradient descent algorithm on the following minimization problem with initial point $x^0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, report the first 3 iterates.

   $$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & e^{-x_1^2 + 3x_2} + 5x_2^2 + x_1 - x_2 + x_1 x_2 \\ \text{sub to} \quad & x_1 + 3x_2 - 5 = 0 \end{aligned}$$

(b) Implement the projection gradient descent algorithm on the following minimization problem with initial point $x^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and report the first 3 iterates.

$$\begin{aligned} \min_{x \in \mathbb{R}^3} \quad & e^{-x_1^2+x_2^2+5x_3^2} + x_1x_2 + x_2x_3 \\ \text{sub to} \quad & 2\,x_1 + x_2 \;\; = 2 \\ & 5x_1 \; + 4x_2 + x_3 \; = 4 \end{aligned}$$