E1 251-O: Tutorial Questions

Linear and Non-linear Optimization

February 18, 2022

1. Find the first 5 iterates of the steepest descent algorithm for minimizing the following functions.
   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ using step-sizes $\alpha \in \{0.01, 0.1, 1\}$, $x^0 = 4$.
   (b) $f : \mathbb{R}_{++} \to \mathbb{R}$, $f(x) = -\log x + e^x$ using step-sizes $\alpha \in \{0.05, 0.05, 5\}$, $x^0 = 2$

2. Consider the function $f : \mathbb{R}^n \to \mathbb{R}$, $\quad f(x) = \frac{x^T A x}{2} - b^T x + c$, A being positive definite.
   (a) Find an expression for the step length iterate (α_k) using the optimal line search method for the steepest descent algorithm. ($x^{k+1} = x^k - \alpha^k d^k$)
   (b) Given, $x^0 = \begin{bmatrix} 2 & 4 \\ 4 & 10 \end{bmatrix}$, $b = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ and $x^0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, find the first step length iterate (α^0) and also the first iterate (x^1).

3. Implement the backtracking algorithm (steepest descent with Armijo's rule) for minimizing the following function with the parameters $\alpha^0 = 15, \beta = 0.85, \sigma = 0.15$ (Compute the first 3 iterates)
   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = 3x_1^2 + 2x_2^2 + 20 \cos x_2 \cos x_1 + 40$, $x^0 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

4. Find the first 3 iterations for the Newton's method with the starting point $x^0 = [3, -1, 0, 1]^T$.
   $f : \mathbb{R}^4 \to \mathbb{R}$, $f(x) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$.

5. Find the first iterate of conjugate gradient method for minimizing the following function. $f : \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = \frac{1}{2} x^T A x - b^T x$$

   where A is positive definite and has distinct eigenvalues. Take the initial point to be $x^0 = \underline{0}$.

Coding

1. Write a code for steepest descent algorithm for minimizing the following functions.
   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = x^T A x - b^T x$ where $A = \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix}$ and $b = \begin{bmatrix} 15.5 \\ -7.4 \end{bmatrix}$. Draw the contour for the above function and try different step-sizes α and comment on the convergence of the algorithm.
   (b) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x) = x^T A x + b^T x$ where $A = \begin{bmatrix} 14 & -2 & 10 \\ 2 & 1 & 2 \\ 4 & 5 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3.14 \\ -2.2 \\ 5.7 \end{bmatrix}$. Try the code for different step-sizes α and comment on the convergence.