E1 215-O: Tutorial Questions

Linear and Non-linear Optimization

February 4, 2022

1. Determine whether the following functions are convex.

   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = (x_1 - 4x_2)^4 + e^{(9x_1^2 + 6x_1x_2 + 4x_2^2)}$

   (b) $f : S \to \mathbb{R}$, $f(x) = -\log\left(-\log(\sum_{i=1}^{5} e^{(a_i^T x_i + b_i)})\right)$, where $S := \{x \in \mathbb{R}^5 : \sum_{i=1}^{5} e^{(a_i^T x_i + b_i)} < 1\}$

   (c) $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) = ||Ax - b||_2$

2. For the given function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = x_1^4 + 4x_1x_2 + x_2^4$ find the stationary points and find the global minimum.

3. Find the lipschitz constant for the following functions.

   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{1+x^2}$.

   (b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = e^{-x^2}$

4. Check if the following functions are convex, and give the global optimal.

   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = \sqrt{x_1^2 + x_2^2 + 1}$

   (b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x) = e^{x_1^2 + x_2^2 + 2x_1}$

5. When f is convex, any local minimizer x^* is a global minimizer of f. If differentiability is not assumed prove the above statement.