Homework 2

Ronak Mehta
SJSUID: 014505387

---

4. Entropy (Play Tennis?) $= -\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)$

$= 1$

<u>Hot?</u>

$S_{yes} \leftarrow [1+, 1-]$ $\quad S_{No} \leftarrow [1+, 1-]$

$E(S_{yes}) = E(S_{No}) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 1$

Gain (Play Tennis?, Hot?) $= 1 - \frac{2}{4}\times 1 - \frac{2}{4}\times 1 = 0$

<u>Air Quality Good?</u>

$(S_{yes}) \leftarrow [1+, 1-]$ $\quad S_{No} \leftarrow [1+, 1-]$

$E(S_{yes}) = E(S_{No}) = 1$

Gain (Play Tennis?, Air Quality Good?) $= 1 - \frac{2}{4} - \frac{2}{4} = 0$

<u>Windy?</u>

$S_{yes} \leftarrow [2+, 1-]$ $\quad S_{No} \leftarrow [0+, 1-]$

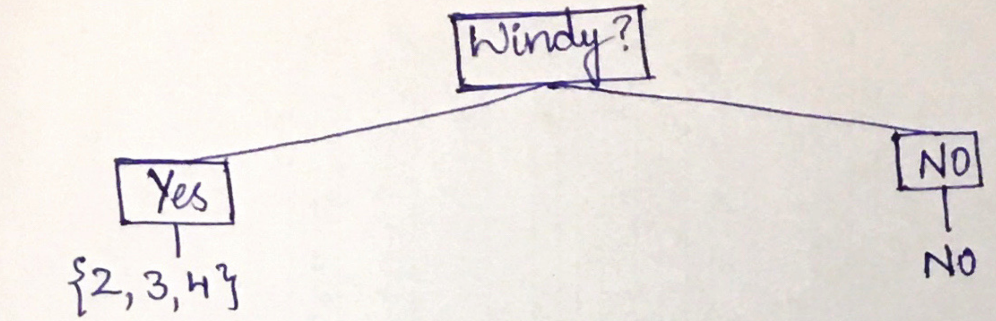
Entropy $(S_{yes}) = -\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right) = 0.9182958341$

$E(S_{No}) = 0$

Gain (Play Tennis?, Windy?) $= 1 - 0.9182958341 \times \frac{3}{4} - 0$

$= 0.3112781245$

$\because$ Gain of Windy? is highest among all other features.
Windy? is the root.



Windy?	Air Quality Good?	Hot?	Play Tennis?
Yes	No	Yes	Yes
Yes	Yes	No	Yes
Yes	Yes	Yes	No

$E(\text{Windy?}) = 0.9182958341$

Air Quality Good?

$S_{yes} \leftarrow [1+, 1-]$ $\quad S_{NO} \leftarrow [1+, 0-]$

$E(S_{yes}) = 1$

$E(S_{NO}) = 0$

$$\text{Gain}(\text{Windy?}, \text{Air Quality Good?}) = 0.9182958341 - \frac{2}{3} \times 1 - 0$$
$$= 0.2516291674$$

Hot?

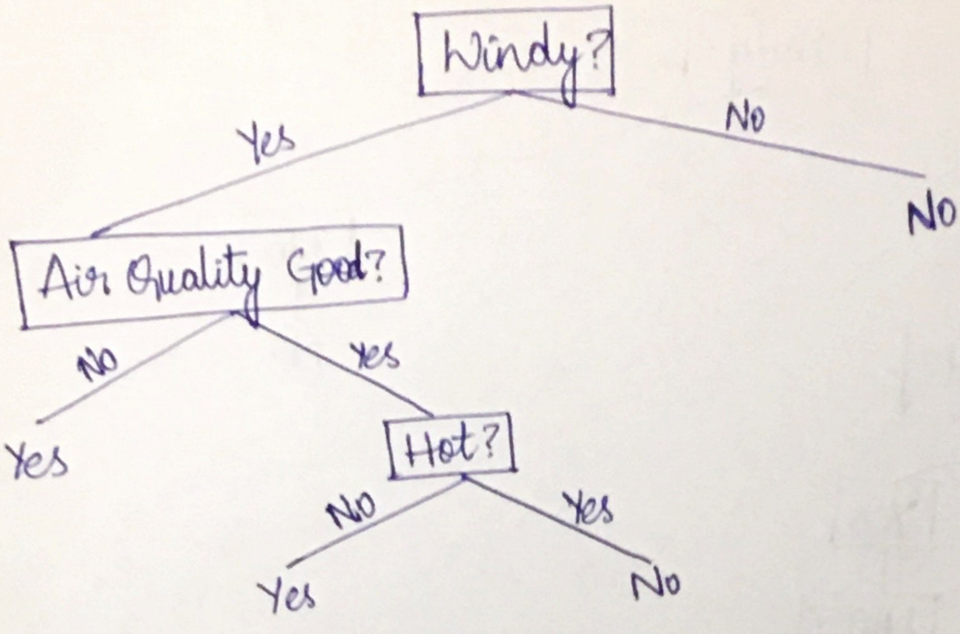
$S_{yes} \leftarrow [1+, 1-]$ $\quad S_{NO} \leftarrow [1+, 0]$

$E(S_{yes}) = 1$

$E(S_{NO}) = 0$

$$\text{Gain}(\text{Windy?}, \text{Hot?}) = 0.9182958341 - \frac{2}{3} \times 1 - 0$$
$$= 0.2516291674$$

Since gain is Same for "Air Quality Good?" and "Hot?"


Windy?
Yes
No
No
Air Quality Good?
No
Yes
Yes
Hot?
No
Yes
Yes
No

2, a)

$$\text{Accuracy} = \frac{TP + TN}{P+N}$$

$$\text{Specificity} = \frac{TN}{N}$$

$$\text{Sensitivity} = \frac{TP}{P}$$

$$\text{Accuracy} = \frac{TP}{P+N} + \frac{TN}{P+N}$$

$$TP = \text{Sensitivity} \times P$$

$$TN = \text{Specificity} \times N$$

$$\text{Accuracy} = \frac{\text{Sensitivity} \times P}{P+N} + \frac{\text{Specificity} \times N}{P+N}$$

$$\text{Prevalence} = \frac{P}{P+N}$$

$$1 - \text{prevalence} = 1 - \frac{P}{P+N}$$

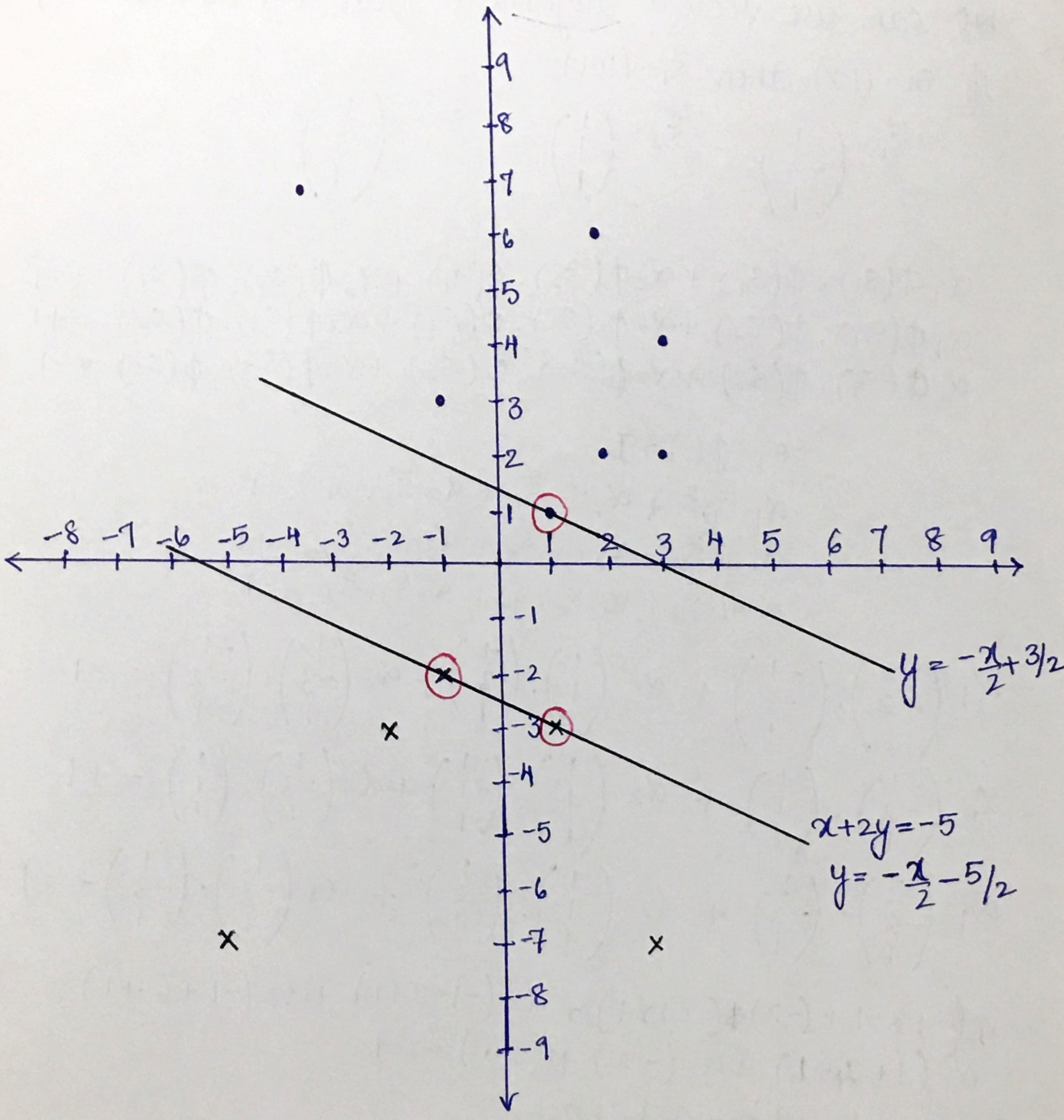
$$\frac{P+N-P}{P+N} = \frac{N}{P+N}$$

$$\therefore \text{Accuracy} = \text{Sensitivity} \times \text{Prevalence} + \text{Specificity} \times (1 - \text{prevalence})$$

5. a)

C1: [2 6], [1 1] [3 4], [3 2] [-4 8] [-1 3] [2 2]   {+1}

C2: [-5 -7], [-2 -3], [-1 -2], [3, -7], [1 -3]   {-1}



i) 1 vector [1 1] from class C1
& ii) 2 vector [-1 -2], [1 -3] from class C_2
} are the support vectors

Since the classes C1 and C2 are linearly seperable

By Inspection the support vectors are:

$$s_1 = \begin{pmatrix}-1\\-2\end{pmatrix}, \quad s_2 = \begin{pmatrix}1\\1\end{pmatrix}, \quad s_3 = \begin{pmatrix}1\\-3\end{pmatrix}$$

we can use vectors augmented with a 1 as a bias input

if $s_1 = (10)$ then $\tilde{s}_1 = (101)$

$$\tilde{s}_1 = \begin{pmatrix}-1\\-2\\1\end{pmatrix} \quad \tilde{s}_2 = \begin{pmatrix}1\\1\\1\end{pmatrix} \quad \tilde{s}_3 = \begin{pmatrix}1\\-3\\1\end{pmatrix}$$

$$\alpha_1 \Phi(s_1)\cdot\Phi(s_1) + \alpha_2\Phi(s_2)\cdot\Phi(s_1) + \alpha_3\Phi(s_3)\cdot\Phi(s_1) = -1$$
$$\alpha_1 \Phi(s_1)\cdot\Phi(s_2) + \alpha_2\Phi(s_2)\cdot\Phi(s_2) + \alpha_3\Phi(s_3)\cdot\Phi(s_2) = +1$$
$$\alpha_1 \Phi(s_1)\cdot\Phi(s_3) + \alpha_2\Phi(s_2)\cdot\Phi(s_3) + \alpha_3\Phi(s_3)\cdot\Phi(s_3) = -1$$

So, $\Phi(\,) = I$

$$\alpha_1 \tilde{s}_1\cdot\tilde{s}_1 + \alpha_2\tilde{s}_2\cdot\tilde{s}_1 + \alpha_3\tilde{s}_3\cdot\tilde{s}_1 = -1$$
$$\alpha_1 \tilde{s}_1\cdot\tilde{s}_2 + \alpha_2\tilde{s}_2\cdot\tilde{s}_2 + \alpha_3\tilde{s}_3\cdot\tilde{s}_2 = +1$$
$$\alpha \tilde{s}_1\cdot\tilde{s}_3 + \alpha_2\tilde{s}_2\cdot\tilde{s}_3 + \alpha_3\tilde{s}_3\cdot\tilde{s}_3 = -1$$

$$\alpha_1\begin{pmatrix}-1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\-2\\1\end{pmatrix} + \alpha_2\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\-2\\1\end{pmatrix} + \alpha_3\begin{pmatrix}1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\-2\\1\end{pmatrix} = -1$$

$$\alpha_1\begin{pmatrix}-1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} + \alpha_2\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} + \alpha_3\begin{pmatrix}1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} = +1$$

$$\alpha_1\begin{pmatrix}-1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-3\\1\end{pmatrix} + \alpha_2\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-3\\1\end{pmatrix} + \alpha_3\begin{pmatrix}1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-3\\1\end{pmatrix} = -1$$

$$\alpha_1(-1\times-1 + (-2)*(-2)*1) + \alpha_2(-1-2+1) + \alpha_3(-1+6+1) = -1$$
$$\alpha_1(1+4+1) + \alpha_2(-2) + \alpha_3(+6) = -1$$

Since $a.b = a_1b_1 + a_2b_2 + a_3b_3$

where $a = a_1, a_2, a_3$

$b = b_1, b_2, b_3$

$$6\alpha_1 - 2\alpha_2 + 6\alpha_3 = -1 \quad \text{—— } ①$$

$$\alpha_1(-1+(-2)+1) + \alpha_2(1+1+1) + \alpha_3(1-3+1) = +1$$

$$-2\alpha_1 + 3\alpha_2 - \alpha_3 = +1 \quad \text{—— } ②$$

$$\alpha_1(-1+6+1) + \alpha_2(1-3+1) + \alpha_3(1+9+1) = -1$$

$$6\alpha_1 - \alpha_2 + 11\alpha_3 = -1 \quad \text{—— } ③$$

$$\begin{array}{r} 6\alpha_1 - 2\alpha_2 + 6\alpha_3 = -1 \\ -6\alpha_1 + \alpha_2 - 11\alpha_3 = +1 \\ \hline -\alpha_2 - 5\alpha_3 = 0 \end{array}$$

$$\cancel{-2\alpha} \quad \begin{array}{r} -6\alpha_1 + 9\alpha_2 - 3\alpha_3 = +3 \\ 6\alpha_1 - \alpha_2 + 11\alpha_3 = -1 \\ \hline 8\alpha_2 + 8\alpha_3 = 2 \\ 4\alpha_2 + 4\alpha_3 = 1 \end{array}$$

$$\cancel{\begin{array}{r} -8\alpha_2 - 40\alpha_3 = 0 \\ 8\alpha_2 + 8\alpha_3 = 0 \\ \hline 32\alpha_3 = 0 \\ \alpha_3 = 0 \end{array}}$$

$$\begin{array}{r} 4\alpha_2 + 4\alpha_3 = 1 \\ -4\alpha_2 - 20\alpha_3 = 0 \\ \hline -16\alpha_3 = 1 \end{array}$$

$$\alpha_3 = -1/16$$

$$\boxed{\alpha_3 = -0.0625}$$

$$-\alpha_2 - 5(-0.0625) = 0$$

$$\alpha_2 = 0.3125$$

$$6\alpha_1 - 2(0.3125) + 6(-0.0625) = -1$$

$$6\alpha_1 = -1 + 1$$
$$\alpha_1 = 0$$

$$\therefore \alpha_1 = 0$$
$$\alpha_2 = 0.3125$$
$$\alpha_3 = -0.0625$$

$$\tilde{w} = \sum_i \alpha_i \tilde{s}_i$$

$$= 0\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} + 0.3125\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (-0.0625)\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3125 \\ 0.3125 \\ 0.3125 \end{pmatrix} + \begin{pmatrix} -0.0625 \\ 0.1875 \\ -0.0625 \end{pmatrix}$$

$$= \begin{pmatrix} 0.25 \\ 0.5 \\ 0.25 \end{pmatrix}$$

hyperplane equation $y = \overset{w}{\cancel{m}}x + b$

$$w = \begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix} \qquad b = 0.25$$

iii) & v) $\therefore y = \begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix} x + 0.25$

iv) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad (x_1, y_1)\ (x_2, y_2)$
$\qquad\qquad\qquad\qquad (-1, -2)\ \ (1, -3)$

$$= \frac{-3+2}{1+1} = \frac{-1}{2}$$

m x slope of maximum ~~las~~ margin line = -1

$$\text{slope} = \frac{-1}{m} = \frac{-1}{-1/2} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -\frac{1}{2}(x+1)$$

$$= 2y + 4 = -x - 1$$

$$x + 2y = -5$$

$$2y = -5 - x$$

$$= -\frac{5}{2} - \frac{x}{2}$$

Equation ~~slope~~ of line of ~~eq~~ 2

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$2y - 2 = -x + 1$$

$$2y = -x + 1 + 2$$

$$y = -x + 3$$

$$y = -\frac{x}{2} + \frac{3}{2}$$

$$\text{distance}(ax + by + c = 0, (x_0, y_0)) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

iv) $\therefore d(x + 2y + 5 = 0, (1,1)) = \dfrac{1 \times 1 + 2 \times 1 + 5}{\sqrt{1^2 + 2^2}} = \dfrac{5+2+1}{\sqrt{5}}$

$$= \underline{\underline{8/\sqrt{5}}}$$