CS 231 : Types and Programming Languages
Homework #4

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Question 1

Solution - Union Inference

$$\frac{\Gamma \vdash t : T_1 \mid C \quad \text{T fresh} \quad \text{X fresh}}{\Gamma \vdash \text{left } t : T \mid C \cup \{T = T_1 \vee X\}} \quad \text{(Inf-Left)}$$

$$\frac{\Gamma \vdash t : T_2 \mid C \quad \text{T fresh} \quad \text{X fresh}}{\Gamma \vdash \text{right } t : T \mid C \cup \{T = X \vee T_2\}} \quad \text{(Inf-Right)}$$

$$\frac{\Gamma \vdash t_1 : T_1 \mid C_1 \quad \Gamma, x : X \vdash t_2 : T_2 \mid C_2 \quad \Gamma, y : Y \vdash t_3 : T_3 \mid C_3 \quad \text{T fresh} \quad \text{X fresh} \quad \text{Y fresh}}{\Gamma \vdash \text{match } t_1 \text{with left } x \to t_2 \mid \text{right } y \to t_3 : T \mid C_1 \cup C_2 \cup C_3 \cup \{T_1 = X \vee Y, T_2 = T_3 = T\}} \quad \text{(Inf-Match)}$$

Question 2

Solution

(a) $\Gamma = \emptyset$
t = (function X → function x:X → x) Bool true
T = Bool

(b) $\Gamma = \emptyset$
t = function x: ∀X.X→X → ((function y:Unit → (x Bool true)) (x Unit ()))
T = (∀X.X→X)→Bool

(c) $\Gamma = \emptyset$
t = function x: ∀X.X→Bool → ((function y:Bool → (x Bool true)) (x Unit ()))
T = (∀X.X→Bool)→Bool

(d) Γ = {app:∀X.∀X'.(X → X') → X → X'}
t = app Unit Bool (function x:Unit → true)
T = Unit → Bool

(e) **No** possible rewritten term and type.

(f) Γ = {app:∀X.∀X'.(X → X') → X → X'}
t = app A → A' A → A' (app A A')
T = (A → A') → A → A'

Question 3

Part a.

```
let r = ref 41 in
let x  = r := 42
in !r
```

Part b.

```
let r = ref 41 in
let x = ((function r:Ref Int → (r:=41 ; 500)) (r:=42 ; ref 42))
in !r
```

Part c.

```
let f =
let counter = ref 5 in
function n:Unit → (counter := !counter + 1; !counter) in
(f ()) * (f ())
```

Question 4

Part a.

Eventually stuck term

Term: `let l = ref 42 in (free l ; !l)`
Typing : $\emptyset$; $\emptyset \vdash$ `let l = ref 42 in (free l ; !l : Int)`

Evaluation

`let l = ref 42 in (free l ; !l)`
$\Rightarrow$ `val l : int ref = {contents = 42}` $\Rightarrow$ `(free l ; !l)`|`{l,42}`
`(free l ; !l)`
$\Rightarrow$ `val l : int ref =` $\emptyset \Rightarrow$ `!l` | $\emptyset$
`!l`
$\Rightarrow$ `Eventually Stuck`

Part b.

Progress Theorem The *modified progress theorem* does **Hold**.

Part c.

Preservation Theorem The *modified preservation theorem* does **Not Hold**.
Counterexample

Σ: `{(l, Int)}`
`t: free l; !l`
`T: Int`
μ: `{(l,42)}`
`t': !l`
Σ': `{(l, Int)}`)
μ': $\emptyset$
$\text{dom}(\Sigma') = \text{dom}(\mu')$ is violated