Homework #2 Cheat Sheet

CS231

1 Language of Booleans and Integers

1.1 Syntax

```
t  ::=  true | false | if t then t else t
     |  n | t + t | t > t
n  ::=  integer constant
v  ::=  true | false | n
T  ::=  Bool | Int
```

1.2 Small-Step Operational Semantics

$$\frac{}{\texttt{if true then } t_2 \texttt{ else } t_3 \longrightarrow t_2} \quad \text{(E-IfTrue)}$$

$$\frac{}{\texttt{if false then } t_2 \texttt{ else } t_3 \longrightarrow t_3} \quad \text{(E-IfFalse)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{if } t_1 \texttt{ then } t_2 \texttt{ else } t_3 \longrightarrow \texttt{if } t_1' \texttt{ then } t_2 \texttt{ else } t_3} \quad \text{(E-If)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ + } t_2 \longrightarrow t_1' \texttt{ + } t_2} \quad \text{(E-Plus1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ + } t_2 \longrightarrow v_1 \texttt{ + } t_2'} \quad \text{(E-Plus2)}$$

$$\frac{n = n_1 \; [[+]] \; n_2}{n_1 \texttt{ + } n_2 \longrightarrow n} \quad \text{(E-PlusRed)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ > } t_2 \longrightarrow t_1' \texttt{ > } t_2} \quad \text{(E-GT1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ > } t_2 \longrightarrow v_1 \texttt{ > } t_2'} \quad \text{(E-GT2)}$$

$$\frac{v = n_1 \; [[>]] \; n_2}{n_1 \texttt{ > } n_2 \longrightarrow v} \quad \text{(E-GTRed)}$$

1.3 Static Type System

$$\frac{}{\texttt{true : Bool}} \quad \text{(T-True)} \qquad \frac{}{\texttt{false : Bool}} \quad \text{(T-False)}$$

$$\frac{\texttt{t}_1 : \texttt{Bool} \quad \texttt{t}_2 : \texttt{T} \quad \texttt{t}_3 : \texttt{T}}{\texttt{if t}_1 \texttt{ then t}_2 \texttt{ else t}_3 : \texttt{T}} \tag{T-IF}$$

$$\frac{}{\texttt{n} : \texttt{Int}} \tag{T-NUM}$$

$$\frac{\texttt{t}_1 : \texttt{Int} \quad \texttt{t}_2 : \texttt{Int}}{\texttt{t}_1 \texttt{ + t}_2 : \texttt{Int}} \tag{T-PLUS}$$

$$\frac{\texttt{t}_1 : \texttt{Int} \quad \texttt{t}_2 : \texttt{Int}}{\texttt{t}_1 \texttt{ > t}_2 : \texttt{Bool}} \tag{T-GT}$$

2 Call-by-Value Lambda Calculus

2.1 Syntax

$\texttt{t} \;::=\; \texttt{x} \mid \texttt{function x} \to \texttt{t} \mid \texttt{t t}$
$\texttt{v} \;::=\; \texttt{function x} \to \texttt{t}$

2.2 Substitution

$[\texttt{x} \mapsto \texttt{v}]\texttt{x} = \texttt{v}$
$[\texttt{x} \mapsto \texttt{v}]\texttt{x}' = \texttt{x}'$, where $\texttt{x} \neq \texttt{x}'$
$[\texttt{x} \mapsto \texttt{v}]\texttt{function x} \to \texttt{t}_0 = \texttt{function x} \to \texttt{t}_0$
$[\texttt{x} \mapsto \texttt{v}]\texttt{function x}_0 \to \texttt{t}_0 = \texttt{function x}_0 \to [\texttt{x} \mapsto \texttt{v}]\texttt{t}_0$, where $\texttt{x} \neq \texttt{x}_0$
$[\texttt{x} \mapsto \texttt{v}]\texttt{t}_1\ \texttt{t}_2 = [\texttt{x} \mapsto \texttt{v}]\texttt{t}_1\ [\texttt{x} \mapsto \texttt{v}]\texttt{t}_2$

2.3 Small-Step Operational Semantics

$$\frac{}{((\texttt{function x} \to \texttt{t})\ \texttt{v}) \longrightarrow [\texttt{x} \mapsto \texttt{v}]\texttt{t}} \tag{E-APPBETA}$$

$$\frac{\texttt{t}_1 \longrightarrow \texttt{t}_1'}{\texttt{t}_1\ \texttt{t}_2 \longrightarrow \texttt{t}_1'\ \texttt{t}_2} \tag{E-APP1}$$

$$\frac{\texttt{t}_2 \longrightarrow \texttt{t}_2'}{\texttt{v}_1\ \texttt{t}_2 \longrightarrow \texttt{v}_1\ \texttt{t}_2'} \tag{E-APP2}$$