Homework #3 Cheat Sheet

CS231

1 Language of Booleans and Integers

1.1 Syntax

```
t  ::=  true | false | if t then t else t
     |  n | t + t | t > t
n  ::=  integer constant
v  ::=  true | false | n
T  ::=  Bool | Int
```

1.2 Small-Step Operational Semantics

$$\frac{}{\texttt{if true then } t_2 \texttt{ else } t_3 \longrightarrow t_2} \quad \text{(E-IfTrue)}$$

$$\frac{}{\texttt{if false then } t_2 \texttt{ else } t_3 \longrightarrow t_3} \quad \text{(E-IfFalse)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{if } t_1 \texttt{ then } t_2 \texttt{ else } t_3 \longrightarrow \texttt{if } t_1' \texttt{ then } t_2 \texttt{ else } t_3} \quad \text{(E-If)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ + } t_2 \longrightarrow t_1' \texttt{ + } t_2} \quad \text{(E-Plus1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ + } t_2 \longrightarrow v_1 \texttt{ + } t_2'} \quad \text{(E-Plus2)}$$

$$\frac{n = n_1 \; [[+]] \; n_2}{n_1 \texttt{ + } n_2 \longrightarrow n} \quad \text{(E-PlusRed)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ > } t_2 \longrightarrow t_1' \texttt{ > } t_2} \quad \text{(E-GT1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ > } t_2 \longrightarrow v_1 \texttt{ > } t_2'} \quad \text{(E-GT2)}$$

$$\frac{v = n_1 \; [[>]] \; n_2}{n_1 \texttt{ > } n_2 \longrightarrow v} \quad \text{(E-GTRed)}$$

1.3 Static Type System

$$\frac{}{\texttt{true : Bool}} \quad \text{(T-True)} \qquad \frac{}{\texttt{false : Bool}} \quad \text{(T-False)}$$

$$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T} \tag{T-IF}$$

$$\frac{}{n : \text{Int}} \tag{T-NUM}$$

$$\frac{t_1 : \text{Int} \quad t_2 : \text{Int}}{t_1 + t_2 : \text{Int}} \tag{T-PLUS}$$

$$\frac{t_1 : \text{Int} \quad t_2 : \text{Int}}{t_1 > t_2 : \text{Bool}} \tag{T-GT}$$

2 Simply-Typed Lambda Calculus

2.1 Syntax

$$\begin{array}{lll} t & ::= & x \mid \text{function } x{:}T \to t \mid t\ t \\ v & ::= & \text{function } x{:}T \to t \\ T & ::= & T_1 \to T_2 \end{array}$$

2.2 Substitution

$[x \mapsto v]x = v$
$[x \mapsto v]x' = x'$, where $x \neq x'$
$[x \mapsto v]\text{function } x{:}T \to t_0 = \text{function } x{:}T \to t_0$
$[x \mapsto v]\text{function } x_0{:}T \to t_0 = \text{function } x_0{:}T \to [x \mapsto v]t_0$, where $x \neq x_0$
$[x \mapsto v]t_1\ t_2 = [x \mapsto v]t_1\ [x \mapsto v]t_2$

2.3 Small-Step Operational Semantics

$$\frac{}{((\text{function } x{:}T \to t)\ v) \longrightarrow [x \mapsto v]t} \tag{E-APPBETA}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t_2 \longrightarrow t_1'\ t_2} \tag{E-APP1}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1\ t_2 \longrightarrow v_1\ t_2'} \tag{E-APP2}$$

2.4 Static Type System

Γ is a finite function from variable names to types.

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \tag{T-VAR}$$

$$\frac{\Gamma, x{:}T_1 \vdash t : T_2}{\Gamma \vdash \text{function } x{:}T_1 \text{ -> } t : T_1 \to T_2} \tag{T-FUN}$$

$$\frac{\Gamma \vdash t_1 : T_2 \to T \qquad \Gamma \vdash t_2 : T_2}{\Gamma \vdash t_1\ t_2 : T} \tag{T-APP}$$

3 Extensions

We augment our language with a unit value, pairs, tagged unions, `let`, `letrec`.

3.1 Syntax

```
t  ::=  () | (t,t) | fst t | snd t
      | left t | right t | (match t with left x -> t | right x -> t)
      | let x=t in t | letrec x=v in t
v  ::=  () | (v,v) | left v | right v
T  ::=  Unit | T ∧ T | T ∨ T
```

3.2 Small-Step Operational Semantics

$$\frac{t_1 \longrightarrow t_1'}{(t_1, t_2) \longrightarrow (t_1', t_2)} \text{ (E-PAIR1)} \qquad \frac{t_2 \longrightarrow t_2'}{(v_1, t_2) \longrightarrow (v_1, t_2')} \text{ (E-PAIR2)}$$

$$\frac{t \longrightarrow t'}{\texttt{fst } t \longrightarrow \texttt{fst } t'} \text{ (E-FST)} \qquad \frac{}{\texttt{fst } (v_1, v_2) \longrightarrow v_1} \text{ (E-FSTRED)}$$

$$\frac{t \longrightarrow t'}{\texttt{snd } t \longrightarrow \texttt{snd } t'} \text{ (E-SND)} \qquad \frac{}{\texttt{snd } (v_1, v_2) \longrightarrow v_2} \text{ (E-SNDRED)}$$

$$\frac{t \longrightarrow t'}{\texttt{left } t \longrightarrow \texttt{left } t'} \text{ (E-LEFT)} \qquad \frac{t \longrightarrow t'}{\texttt{right } t \longrightarrow \texttt{right } t'} \text{ (E-RIGHT)}$$

$$\frac{t \longrightarrow t'}{\texttt{match } t \texttt{ with left } x_1 \texttt{ -> } t_1 \texttt{ | right } x_2 \texttt{ -> } t_2 \longrightarrow \texttt{match } t' \texttt{ with left } x_1 \texttt{ -> } t_1 \texttt{ | right } x_2 \texttt{ -> } t_2} \text{ (E-MATCH)}$$

$$\frac{}{\texttt{match left } v \texttt{ with left } x_1 \texttt{ -> } t_1 \texttt{ | right } x_2 \texttt{ -> } t_2 \longrightarrow [x_1 \mapsto v]t_1} \text{ (E-MATCHLEFT)}$$

$$\frac{}{\texttt{match right } v \texttt{ with left } x_1 \texttt{ -> } t_1 \texttt{ | right } x_2 \texttt{ -> } t_2 \longrightarrow [x_2 \mapsto v]t_2} \text{ (E-MATCHRIGHT)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{let } x=t_1 \texttt{ in } t_2 \longrightarrow \texttt{let } x=t_1' \texttt{ in } t_2} \text{ (E-LET)} \qquad \frac{}{\texttt{let } x=v \texttt{ in } t \longrightarrow [x \mapsto v]t} \text{ (E-LETRED)}$$

$$\frac{}{\texttt{letrec } x=v \texttt{ in } t \longrightarrow \texttt{let } x=[x \mapsto \texttt{letrec } x=v \texttt{ in } x]v \texttt{ in } t} \text{ (E-LETREC)}$$

3.3 Static Type System

$$\frac{}{\Gamma \vdash () : \texttt{Unit}} \text{ (T-UNIT)}$$

$$\frac{\Gamma \vdash t_1 : T_1 \qquad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \wedge T_2} \text{ (T-PAIR)}$$

$$\frac{\Gamma \vdash t : T_1 \wedge T_2}{\Gamma \vdash \texttt{fst } t : T_1} \text{ (T-FST)} \qquad \frac{\Gamma \vdash t : T_1 \wedge T_2}{\Gamma \vdash \texttt{snd } t : T_2} \text{ (T-SND)}$$

$$\frac{\Gamma \vdash \texttt{t} : \texttt{T}_1}{\Gamma \vdash \texttt{left t} : \texttt{T}_1 \vee \texttt{T}_2}\ \text{(T-LEFT)} \qquad \frac{\Gamma \vdash \texttt{t} : \texttt{T}_2}{\Gamma \vdash \texttt{right t} : \texttt{T}_1 \vee \texttt{T}_2}\ \text{(T-RIGHT)}$$

$$\frac{\Gamma \vdash \texttt{t} : \texttt{T}_1 \vee \texttt{T}_2 \qquad \Gamma, \texttt{x}_1 : \texttt{T}_1 \vdash \texttt{t}_1 : \texttt{T} \qquad \Gamma, \texttt{x}_2 : \texttt{T}_2 \vdash \texttt{t}_2 : \texttt{T}}{\Gamma \vdash \texttt{match t with left } \texttt{x}_1 \texttt{ -> } \texttt{t}_1 \texttt{ | right } \texttt{x}_2 \texttt{ -> } \texttt{t}_2 : \texttt{T}}\ \text{(T-MATCH)}$$

$$\frac{\Gamma \vdash \texttt{t}_1 : \texttt{T}_1 \qquad \Gamma, \texttt{x} : \texttt{T}_1 \vdash \texttt{t}_2 : \texttt{T}}{\Gamma \vdash \texttt{let x=}\texttt{t}_1 \texttt{ in } \texttt{t}_2 : \texttt{T}}\ \text{(T-LET)} \qquad \frac{\Gamma, \texttt{x} : \texttt{T}_1 \vdash \texttt{v}_1 : \texttt{T}_1 \qquad \Gamma, \texttt{x} : \texttt{T}_1 \vdash \texttt{t}_2 : \texttt{T}}{\Gamma \vdash \texttt{letrec x=}\texttt{v}_1 \texttt{ in } \texttt{t}_2 : \texttt{T}}\ \text{(T-LETREC)}$$