CS 231 : Types and Programming Languages
Homework #3

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Citations: Questions discussed with Sharavani Senapathy and George Fang but the solutions were written separately.

Question 1

Small Step Operational Semantics

$$\frac{}{\texttt{while true do } \texttt{t}_2 \rightarrow \texttt{t}_2 \texttt{ ; while true do } \texttt{t}_2} \qquad \text{(E-WhileTrue)}$$

$$\frac{}{\texttt{while false do } \texttt{t}_2 \rightarrow \textit{unit}} \qquad \text{(E-WhileFalse)}$$

$$\frac{\texttt{t}_1 \rightarrow \texttt{t}_1{}'}{\texttt{while } \texttt{t}_1 \texttt{ do } \texttt{t}_2 \rightarrow \texttt{while } \texttt{t}_1{}' \texttt{ do } \texttt{t}_2} \qquad \text{(E-While)}$$

Syntactic Sugar

$$\frac{}{\texttt{while } \texttt{t}_1 \texttt{ do } \texttt{t}_2 \rightarrow \texttt{if } \texttt{t}_1 \texttt{ then } (\ \texttt{t}_2 \texttt{ ;while } \texttt{t}_1 \texttt{ do } \texttt{t}_2\ ) \texttt{ else unit}} \qquad \text{(E-While)}$$

Typing Rule

$$\frac{\Gamma \vdash \texttt{t}_1 : \texttt{Bool} \qquad \Gamma \vdash \texttt{t}_2 : \texttt{Unit}}{\Gamma \vdash \texttt{while } \texttt{t}_1 \texttt{ do } \texttt{t}_2 : \texttt{Unit}} \qquad \text{(T-While)}$$

Question 2

Syntactic Sugar: while loop as letrec expression

$$\frac{}{\texttt{while } \texttt{t}_1 \texttt{ do } \texttt{t}_2 \rightarrow \texttt{letrec x } = \texttt{if } \texttt{t}_1 \texttt{ then } (\ \texttt{t}_2 \texttt{ ;} x\ ) \texttt{ else unit in x}} \qquad \text{(E-While2)}$$

Question 3

(a)

Run-time semantics for new judgement $\vdash \mathtt{v}$ ***matches*** $\mathtt{p} \;\Rightarrow \mathtt{E}$

$$\frac{}{\vdash \mathtt{v}\ \textit{matches}\ \mathtt{n} \Rightarrow \emptyset} \quad \text{(V-NUM)}$$

$$\frac{}{\vdash \mathtt{v}\ \textit{matches}\ \mathtt{x} : \mathtt{T} \Rightarrow (\{x, v\})} \quad \text{(V-VAR)}$$

$$\frac{}{\vdash \mathtt{v}\ \textit{matches}\ \_ : \mathtt{T} \Rightarrow \emptyset} \quad \text{(V-WILD)}$$

$$\frac{}{\vdash \mathtt{v}\ \textit{matches}\ (\mathtt{p_1}, \mathtt{p_2}) \;\Rightarrow (\mathtt{fst}\ \mathtt{v}\ \textit{matches}\ \mathtt{p_1}\ ,\ \mathtt{snd}\ \mathtt{v}\ \textit{matches}\ \mathtt{p_2})} \quad \text{(V-MATCH)}$$

(b)

Rules for judgement $\mathtt{t} \to \mathtt{t'}$ **to define the run-time behavior of the** `match` **expression**

$$\frac{\mathtt{t} \to \mathtt{t'}}{\mathtt{match}\ \mathtt{t}\ \mathtt{with}\ \mathtt{p_1} \;\Rightarrow \mathtt{t_1} \mid \mathtt{p_2} \;\Rightarrow \mathtt{t_2} \;\longrightarrow \mathtt{match}\ \mathtt{t'}\ \mathtt{with}\ \mathtt{p_1} \;\Rightarrow \mathtt{t_1} \mid \mathtt{p_2} \;\Rightarrow \mathtt{t_2}} \quad \text{(E-STEP)}$$

$$\frac{\vdash \mathtt{v}\ \textit{matches}\ \mathtt{p_1} \;\Rightarrow \mathtt{E_1}}{\mathtt{match}\ \mathtt{v}\ \mathtt{with}\ \mathtt{p_1} \;\Rightarrow \mathtt{t_1} \mid \mathtt{p_2} \;\Rightarrow \mathtt{t_2} \;\longrightarrow \mathtt{t_1}} \quad \text{(E-COMP1)}$$

$$\frac{\not\vdash \mathtt{v}\ \textit{matches}\ \mathtt{p_1} \qquad \vdash \mathtt{v}\ \textit{matches}\ \mathtt{p_2} \;\Rightarrow \mathtt{E_2}}{\mathtt{match}\ \mathtt{v}\ \mathtt{with}\ \mathtt{p_1} \;\Rightarrow \mathtt{t_1} \mid \mathtt{p_2} \;\Rightarrow \mathtt{t_2} \;\longrightarrow \mathtt{t_2}} \quad \text{(E-COMP2)}$$

(c)

Run-time semantics for new judgement $\vdash \mathtt{p}\ :\ \mathtt{T} \Rightarrow \Gamma$

$$\frac{}{\vdash \mathtt{x} : \mathtt{T} \to \{(\mathtt{x},\ \mathtt{T})\}} \quad \text{(T-VAR)}$$

$$\frac{}{\vdash \_ : \mathtt{T} \to \emptyset} \quad \text{(T-WILD)}$$

$$\frac{}{\vdash \mathtt{n} : \mathtt{T} \to \emptyset} \quad \text{(T-NUM)}$$

$$\frac{}{\vdash \mathtt{p} : \mathtt{T_1}\ \wedge\ \mathtt{T_2} \to \{(\mathtt{fst}\ \mathtt{p} : \mathtt{T_1})\ ,\ (\mathtt{snd}\ \mathtt{p} : \mathtt{T_2})\}} \quad \text{(T-PAIR)}$$

(d)

Rules for judgement $\Gamma \vdash \mathtt{t} : \mathtt{T}$ **to define static type-checking for the** `match` **expression**

$$\frac{\Gamma \vdash \mathtt{p_1}\ :\ \mathtt{T_1} \Rightarrow \Gamma_1 \qquad \Gamma \vdash \mathtt{p_2}\ :\ \mathtt{T_2} \Rightarrow \Gamma_2 \qquad \Gamma \vdash \mathtt{t}\ :\ \mathtt{T_1} \vee \mathtt{T_2} \qquad \Gamma_1 \vdash \mathtt{t_1}\ :\ \mathtt{T} \qquad \Gamma_2 \vdash \mathtt{t_2}\ :\ \mathtt{T}}{\Gamma \vdash \mathtt{match}\ \mathtt{t}\ \mathtt{with}\ \mathtt{p_1} \Rightarrow \mathtt{t_1} \mid \mathtt{p_2} \Rightarrow \mathtt{t_2}\ : \mathtt{T}} \quad \text{(T-MATCH)}$$

Question 4

(a)

Transitivity of Implication: $((A \to B) \land (B \to C)) \to (A \to C)$

Expression

$$\texttt{function (f : A} \to \texttt{B , g : B} \to \texttt{C)} \to \texttt{function x : A} \to \texttt{g (f x)}$$

(b)

Commutativity of Disjunction: $((A \lor B) \to (B \lor A))$

Expression

$$\begin{aligned}\texttt{function f : A} \lor \texttt{B} \to \ & \texttt{match f with} \\ & \texttt{left x } \to \texttt{right x}, \\ & \texttt{right y } \to \texttt{left y}\end{aligned}$$

(c)

A form of Distributivity of Implication over Disjunction: $((A \lor B) \to C) \to ((A \to C) \land (B \to C))$

Expression

$$\texttt{function (f : A} \lor \texttt{B} \to \texttt{C)} \to \texttt{(function x : A} \to \texttt{f (left x) , function y : B} \to \texttt{f (right y))}$$