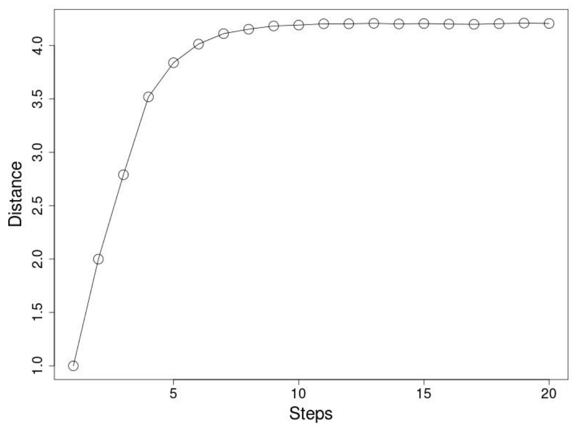
# Solution to Assignment 02

## Problem 1

(b) $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ are shown in the following. The average diameter of the network (1000 nodes and $p = 0.01$) is 5.4.



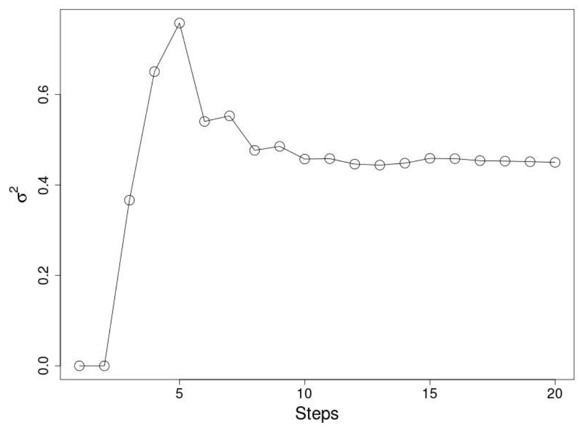




Figure 1: $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$. The results were obtained by averaging over different networks and multiple walkers.

(c) The relations are very different from those of $d$ dimensional space. In the network, all distances are positive, while in $d$ dimensional space $s$ can be negative. In the networks, the random walk is restricted by the diameter of the network.

(d) The distance is upper bounded by the diameter of the network. For networks with 10000 nodes and $p = 0.01$, the plots for $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ are shown in the following

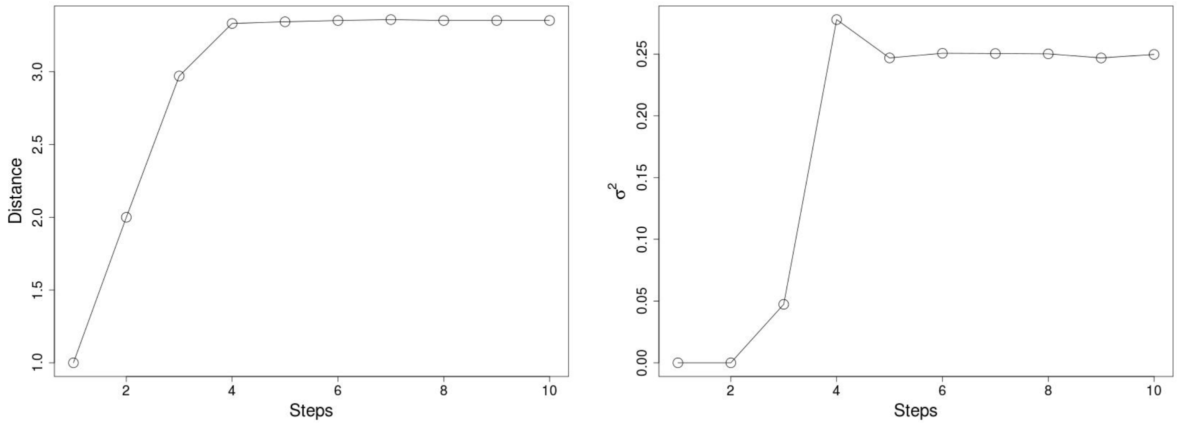



Figure 2: $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ for networks with 10000 nodes and $p = 0.01$. The results were obtained by averaging over different networks and multiple walkers.

In this case, the network is very dense, and the diameter is small. The walkers are restricted around the origin node.

For networks with 100 nodes and $p = 0.01$, we obtained the similar plots in the following. 9.5

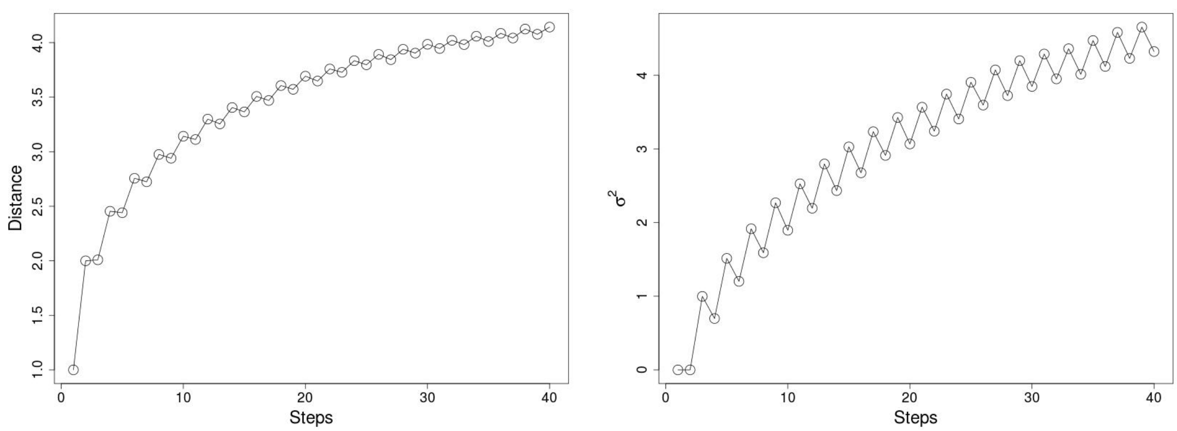



Figure 3: $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ for networks with 100 nodes and $p = 0.01$. The results were obtained by averaging over different networks and multiple walkers.

(e) The resulting degree distribution is

$$F(k) = \frac{kP(k)}{\sum_k kP(k)} \ ,$$

where $P(k)$ is the degree distribution of the networks. The random walk tends to pick out nodes with large degree.

## Problem 2

(b) $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ are shown in the following.

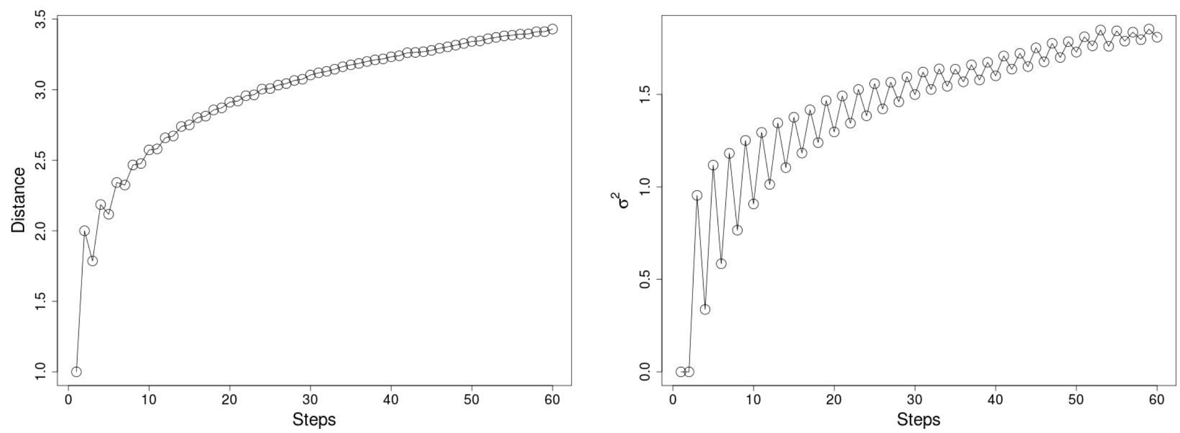



Figure 4: $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ for networks with 100 nodes and $p = 0.01$. The results were obtained by averaging over different networks and multiple walkers.

(d) For networks with 10000 nodes and $p = 0.01$, the plots for $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ are shown in the following

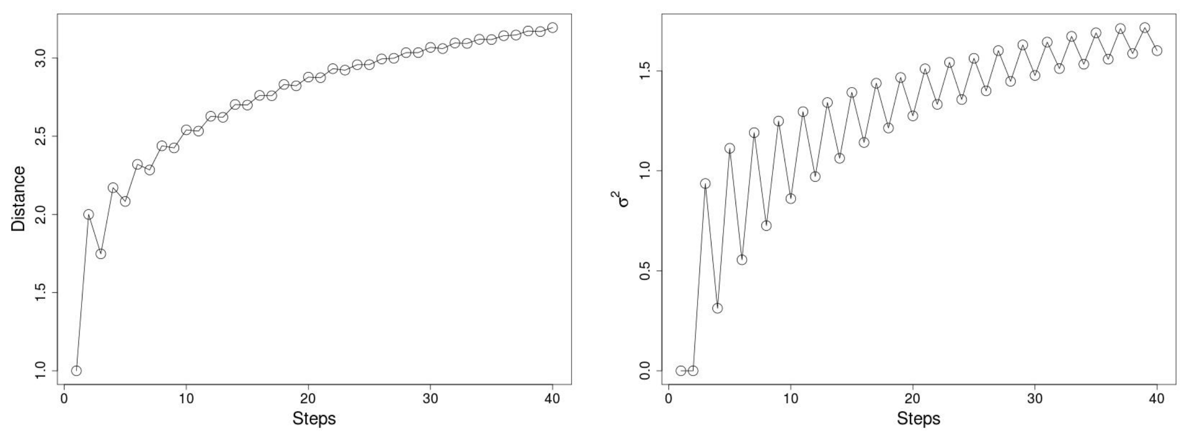



Figure 5: $\langle s(t) \rangle$ and $\langle \sigma^2(t) \rangle$ for networks with 10000 nodes and $p = 0.01$. The results were obtained by averaging over different networks and multiple walkers.

For networks with 100 nodes and $p = 0.01$, we obtained the similar plots in the following.

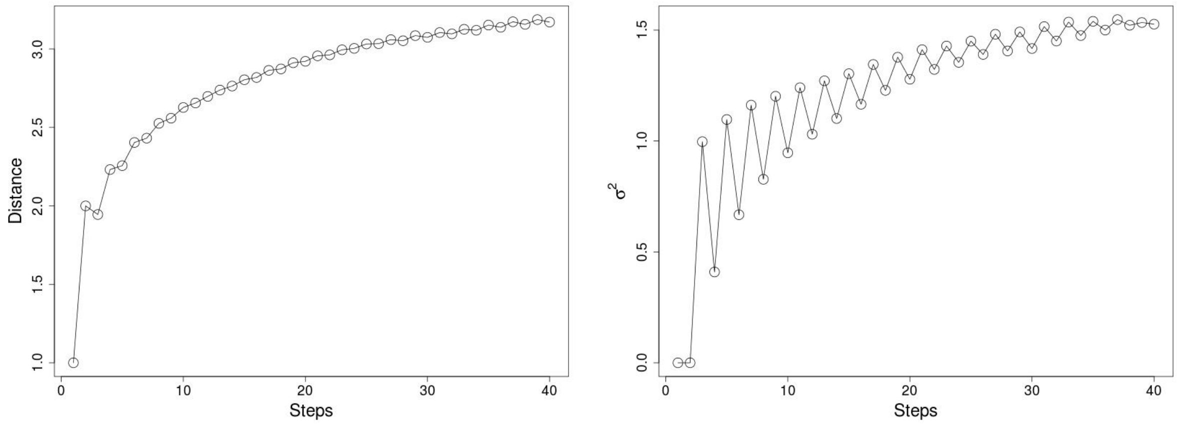



Figure 6: $\langle s(t)\rangle$ and $\langle \sigma^2(t)\rangle$ for networks with 100 nodes and $p = 0.01$. The results were obtained by averaging over different networks and multiple walkers.

(e) Same as Problem 1.

## Problem 3

(a) The visiting probability here is proportional to the degree of the nodes. In the following, the visiting probability vs degree is plotted. A straight line is fit to the data.

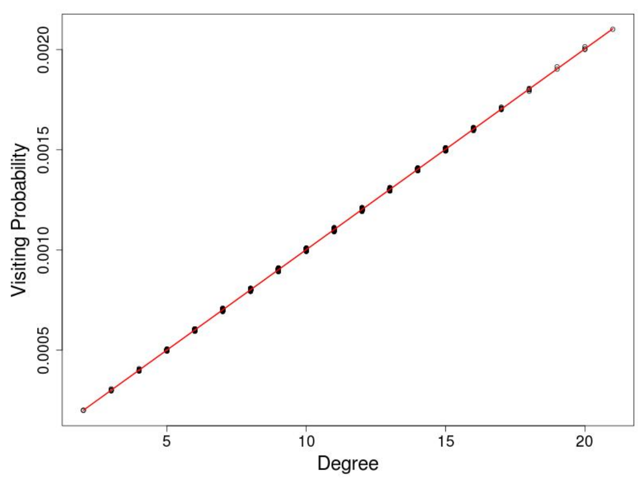



Figure 7: The visiting probability vs degree for undirected networks.

(b) Again the visiting probability vs degree is plotted. There is no simple relation.

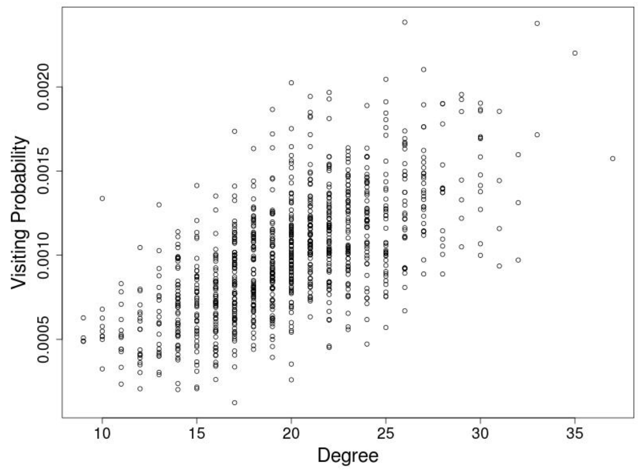



Figure 8: The visiting probability vs degree for directed networks.

(c) The visiting probability vs degree is plotted in the following. The relation is close to linear but there is systematic deviations.

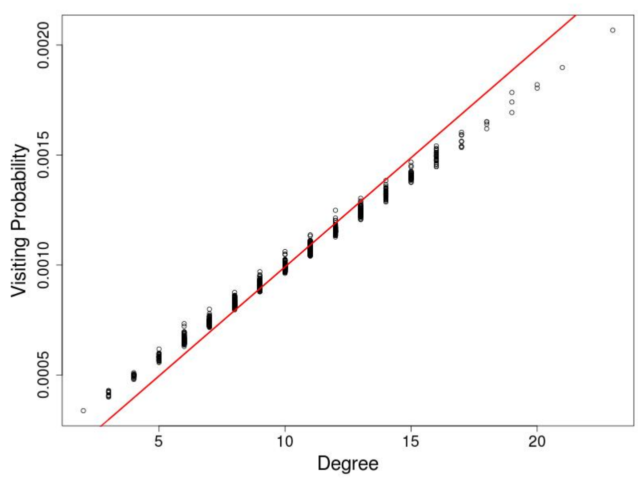



Figure 9: The visiting probability vs degree for undirected networks with damping=0.85.

## Problem 4

(a, b) The PageRanks for (a) and (b) are plotted in the following. The PageRank of (b) has more large values and more small values.

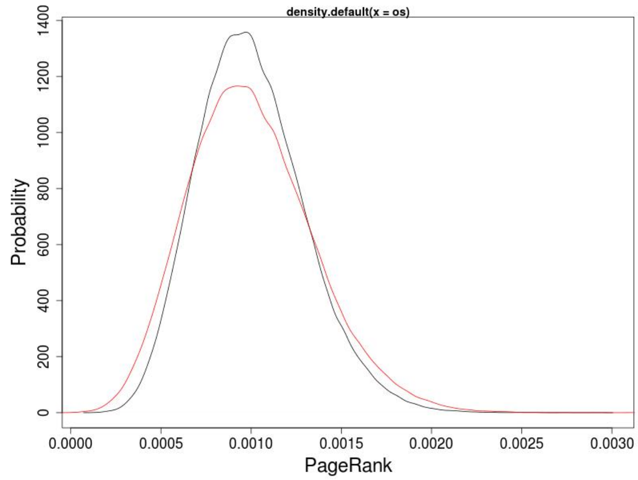



Figure 10: PageRanks for (a) (black) and (b) (red). Averaged over 100 independent networks.

We can also plot PageRank(b) vs PageRank(a), which clearly shows large values in (a) become even larger in (b).

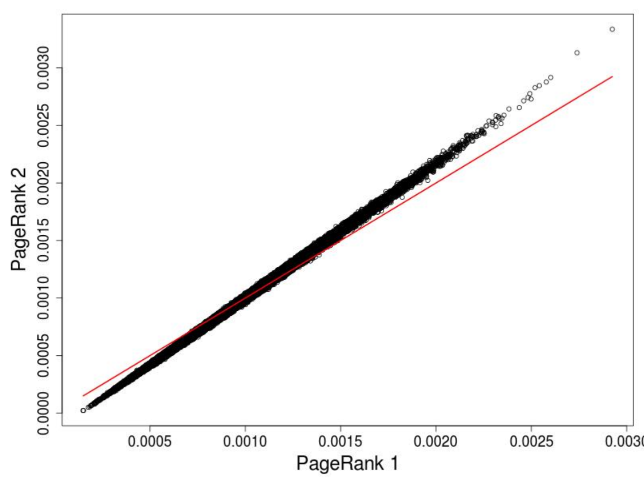



Figure 11: PageRank(b) vs PageRank(a). Averaged over 100 independent networks. The red line is $y = x$

(c) The interest to a node are controlled by the teleportation probability to this node. One can put the PageRank back into the simulation as the teleport.prob parameters recursively. In the following figure, such a solution is shown. The PageRank soon converges to the red curve.

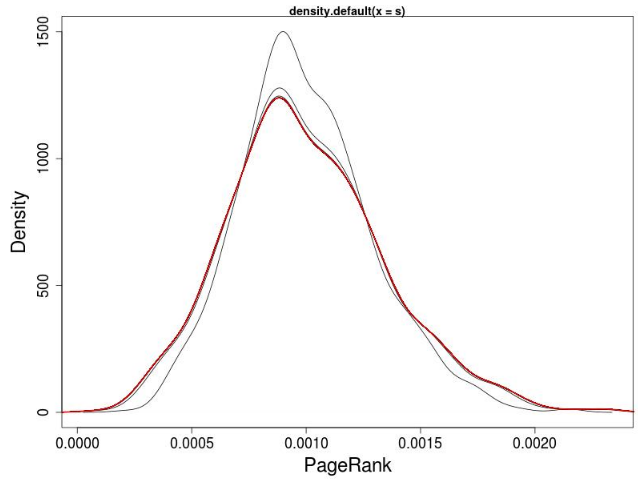



Figure 12: An example solution to the recursive PageRank problem.