Solution 01

1. Create random networks

   (a) See the following figure. As it can be seen the degrees follow a Binomial distribution, which is also very much like samples taken from a continuous Poisson distribution.

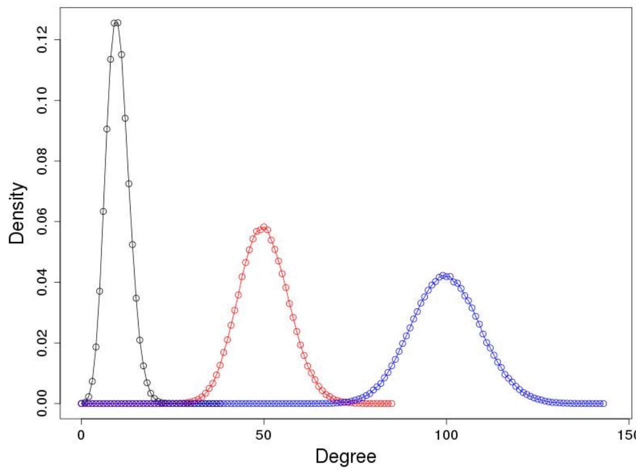

Figure 1: Degree distributions for the random graphs with $p = 0.01$, 0.05 and 0.1. Averaged over 100 samples.

   (b) The probabilities for the network to be connected: more than 95%, 100%, 100% for $p = 0.01, 0.05, 0.1$ respectively. Average diameters: 5.38, 3, 3. This is in accord with the theoretical result that asymptotically almost surely(a.a.s.) $\mathcal{G}(n,p)$ is connected iff $p \geq \frac{\ln n}{n}$. Moreover, the diameter of a $\mathcal{G}(n,p)$ is in the order of $\frac{\ln n}{\ln np}$ a.a.s., when $np \geq 1$.

   (c) By sweeping over different values of p, one can identify $p_c = (7.27 \pm 0.05) \times 10^{-3}$ which is very close to $\frac{\ln n}{n}$.

2. Create a network with a fat tailed degree distribution

   (a) The degree distribution is shown in the following figure

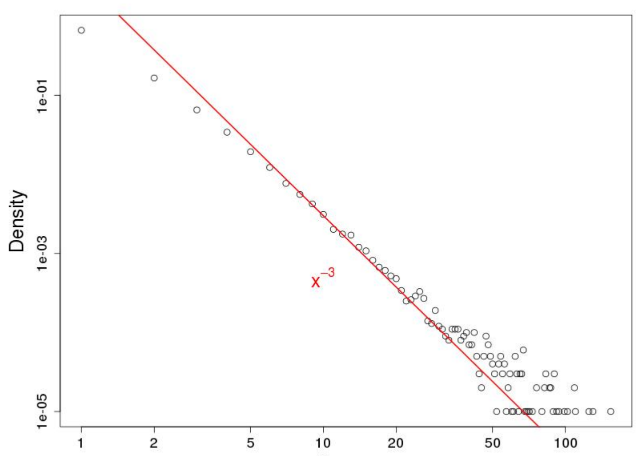

Figure 2: The degree distribution of the fat tailed network. The network is generated using default parameters, where power=1.

   The average diameter is 17.3

(b) The network is 100% connected, and the modularity is around 0.92. The community size distribution is shown in the following figure. The preferential attachment of nodes tends to form clusters around nodes with high degrees, and therefore the modularity is very high.

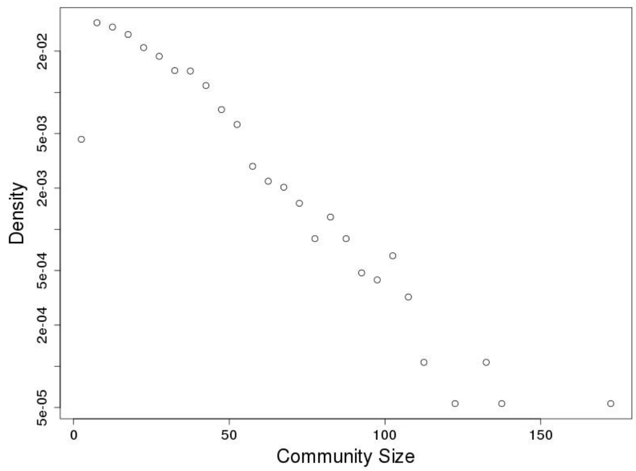

Figure 3: The community size distribution. It is an exponential function. Averaged over 100 different networks.

(c) Generally, networks with smaller edge density demonstrate a higher modularity, as is the case in power law networks; and the modularity increases with the size of the network.

This phenomenon can be attributed to the advent of a larger number of communities in which low degree nodes make connections among themselves and therefore make a significant contribution to the modularity. Also in the larger community many of the low degree nodes are connected to the high degree nodes either directly or through a very short path(the diameter is of order $\ln \ln n$)

(d) What the question really asks is the following: Given a network, you randomly pick nodes from the network, and you get a sample of the nodes. The sample should be uniform. Now, you pick a random node's random neighbor, and you get a second sample of the nodes. Then, are these two samples equivalent?

The answer is no. In the second sample, nodes with larger degrees have larger probability to be picked. The degree distribution for this fat-tailed network has a power law tail of -2.

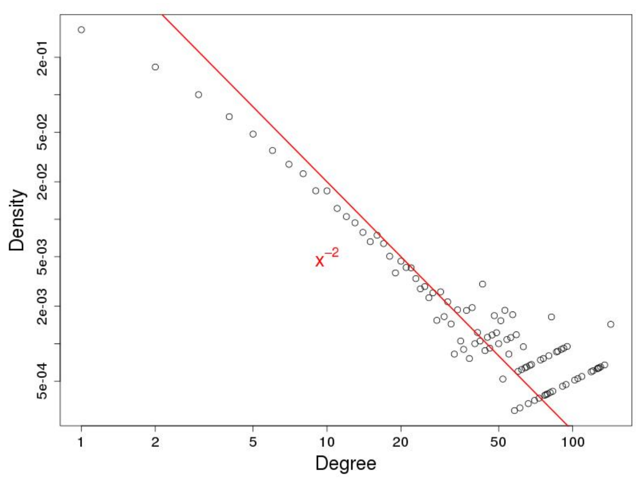

Figure 4: The degree distribution of a random node's random neighbor.

3. Creates a random graph by simulating its evolution

(a) The degree distribution for parameters pa.exp=1, aging.exp=-1 is shown in the following

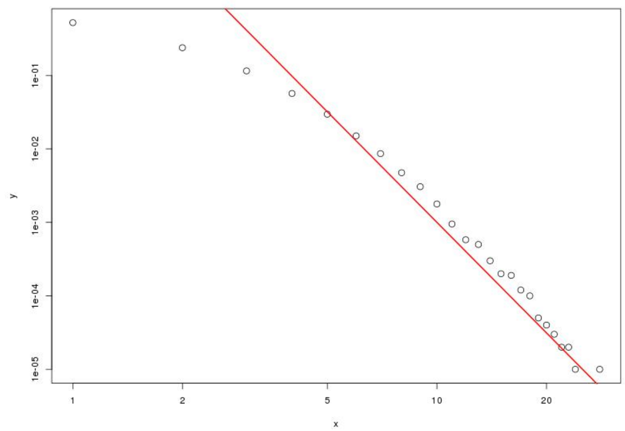

Figure 5: Degree distribution. The power law is −5. Averaged over 100 different networks.

(b) The modularity is 0.935. The community size distribution is shown in the following figure.

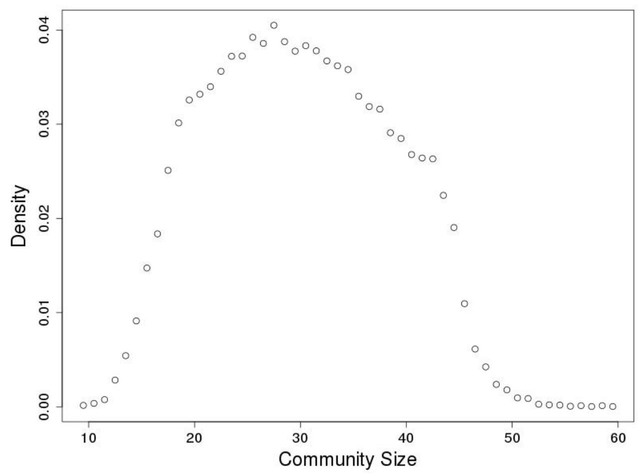

Figure 6: The community size distribution. Averaged over 100 different networks.

4. Use the forest fire model to create a directed network

   (a) The distribution depends on the parameters you choose. I will not plot here. However, the in-edge and out-edge degree distributions are different. The out-degree distribution is close to a power law; while the in-edge distribution is close to an exponential law.

   (b) The diameter v.s. the parameter fw.prob

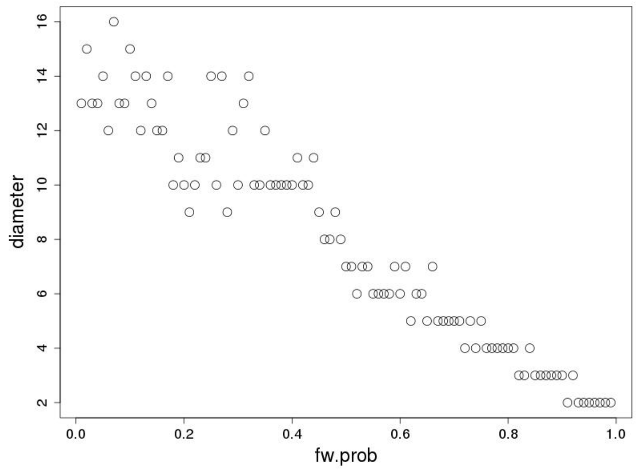

Figure 7: The diameter

(c) The modularity v.s. the parameter fw.prob. For the fw.prob you have used, you can find modularity here.

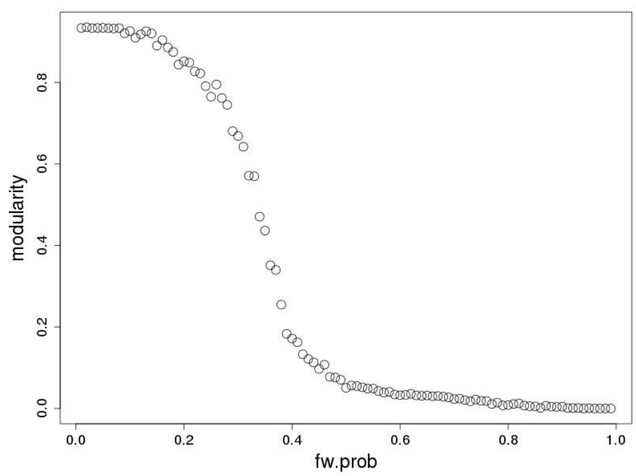

Figure 8: The modularity