$$\xi = \frac{1}{2|\Omega_t|}\left((z_3-z_1)(r-r_1) - (r_3-r_1)(z-z_1)\right) \qquad \frac{\partial \xi}{\partial r} = \frac{(z_3-z_1)}{2|\Omega_t|} \qquad \frac{\partial \xi}{\partial z} = \frac{(r_1-r_3)}{2|\Omega_t|}$$

$$\eta = \frac{1}{2|\Omega_t|}\left(-(z_2-z_1)(r-r_1) + (r_2-r_1)(z-z_1)\right) \qquad \frac{\partial \eta}{\partial r} = \frac{(z_1-z_2)}{2|\Omega_t|} \qquad \frac{\partial \eta}{\partial z} = \frac{(r_2-r_1)}{2|\Omega_t|}$$

$$r = r_1 + (r_2-r_1)\xi + (r_3-r_1)\eta \qquad \frac{\partial r}{\partial \xi} = (r_2-r_1) \qquad \frac{\partial r}{\partial \eta} = (r_3-r_1)$$

$$z = z_1 + (z_2-z_1)\xi + (z_3-z_1)\eta \qquad \frac{\partial z}{\partial \xi} = (z_2-z_1) \qquad \frac{\partial z}{\partial \eta} = (z_3-z_1)$$

$$\iint_A f(x,y)\,dx\,dy = \iint_B f(u,v)\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$

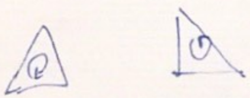
with $\begin{vmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial r}{\partial \eta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} \end{vmatrix} = \begin{vmatrix} (r_2-r_1) & (r_3-r_1) \\ (z_2-z_1) & (z_3-z_1) \end{vmatrix} = 2\,\Omega_t$

$$\Omega_t = (r_2-r_1)(z_3-z_1) - (r_3-r_1)(z_2-z_1)$$

$\psi_1 = \eta$

$$\frac{\partial \psi_1}{\partial r} = \frac{\partial \psi_1}{\partial \xi}\frac{\partial \xi}{\partial r} + \frac{\partial \psi_1}{\partial \eta}\frac{\partial \eta}{\partial r} = \frac{(z_1-z_2)}{2|\Omega_t|} \qquad \text{top in } (r_3, z_3)$$

$$\frac{\partial \psi_1}{\partial z} = \frac{\partial \psi_1}{\partial \xi}\frac{\partial \xi}{\partial z} + \frac{\partial \psi_1}{\partial \eta}\frac{\partial \eta}{\partial z} = \frac{(r_2-r_1)}{2|\Omega_t|}$$



$\psi_2 = \xi$

$$\frac{\partial \psi_2}{\partial r} = \frac{\partial \psi_2}{\partial \xi}\frac{\partial \xi}{\partial r} + \frac{\partial \psi_2}{\partial \eta}\frac{\partial \eta}{\partial r} = \frac{(z_3-z_1)}{2|\Omega_t|} \qquad \text{top in } (r_2, z_2)$$

$$\frac{\partial \psi_2}{\partial z} = \frac{\partial \psi_2}{\partial \xi}\frac{\partial \xi}{\partial z} + \frac{\partial \psi_2}{\partial \eta}\frac{\partial \eta}{\partial z} = \frac{(r_1-r_3)}{2|\Omega_t|}$$

$\psi_3 = 1 - \xi - \eta$

$$\frac{\partial \psi_3}{\partial r} = \frac{\partial \psi_3}{\partial \xi}\frac{\partial \xi}{\partial r} + \frac{\partial \psi_3}{\partial \eta}\frac{\partial \eta}{\partial r} = -\frac{(z_3-z_1)}{2|\Omega_t|} - \frac{(z_1-z_2)}{2|\Omega_t|} = \frac{(z_2-z_3)}{2|\Omega_t|} \qquad \text{top in}$$

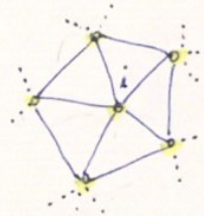
$$\frac{\partial \psi_3}{\partial z} = \frac{\partial \psi_3}{\partial \xi}\frac{\partial \xi}{\partial z} + \frac{\partial \psi_3}{\partial \eta}\frac{\partial \eta}{\partial z} = -\frac{(r_1-r_3)}{2|\Omega_t|} - \frac{(r_2-r_1)}{2|\Omega_t|} = \frac{(r_3-r_2)}{2|\Omega_t|} \qquad (r_1, z_1)$$

$$\int_\Omega q_u \cdot \nabla\varphi_i \, d\Omega \qquad \text{w/ } q_u = r\,S_u \nabla C_u = r\,S_u \sum_j c_j \nabla\varphi_j$$

$$= \int_\Omega \Big(r S_u \sum_j c_j \nabla\varphi_j\Big)\cdot\nabla\varphi_i \, d\Omega$$

$$= \sum_j c_j \int_\Omega r(S_u \nabla\varphi_j)\cdot\nabla\varphi_i \, d\Omega$$

$$= \sum_{j\in J} c_j \Big(\sum_{m\in M}\int_{\Omega_{jm}} r\Big(\sigma_{ur}\frac{\partial\varphi_j}{\partial r}\frac{\partial\varphi_i}{\partial r} + \sigma_{uz}\frac{\partial\varphi_j}{\partial z}\frac{\partial\varphi_i}{\partial z}\Big)dr\,dz$$

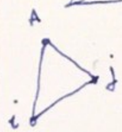


$i \neq j$: 

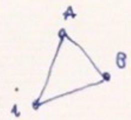
$i = j$: 

over elke Δ wordt 9 keer geloopt.

$$= \boxed{\sum_{j\in J} c_j \Big(\sum_{t\in T} C_{jt}\,(r_{t1} + r_{t2} + r_{t3})\Big)} \qquad \forall\ 1 \le i \le M$$



$$i \neq j: \quad C_{jt} = \sigma_{ur}\frac{(z_i - z_A)(z_A - z_j)}{\cancel{2|\Omega_{jt}|}} + \sigma_{uz}\frac{(r_j - r_A)(r_A - r_i)}{\cancel{2|\Omega_{jt}|}}$$



$$i = j: \quad C_{jt} = \sigma_{ur}\frac{(z_A - z_B)^2}{\cancel{2|\Omega_{jt}|}} + \sigma_{uz}\frac{(r_A - r_B)^2}{2|\Omega_{jt}|}$$

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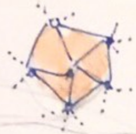
nonlinearity → linear maken in coefficients

$$\int_\Omega r R_u \varphi_i \, d\Omega$$



$$= \sum_{n \in \mathcal{N}} \frac{1}{3} |\Omega_n| \left(r_i \frac{V_{mu} \, c_i}{(K_{mu} + c_i)\left(1 + \frac{C_{m+i}}{K_{mv}}\right)} \right)$$

$$\int_\Omega r R_v \varphi_i \, d\Omega$$

$$= \sum_{n \in \mathcal{N}} \frac{1}{3} |\Omega_n| \left(r_i \left(r_q \frac{V_{mu} \, C_i}{(K_{mu} + C_i)\left(1 + \frac{C_{m+i}}{K_{mv}}\right)} + \frac{V_{mfv}}{1 + \frac{C_i}{K_{mfu}}} \right)\right)$$

⇒ check orde van nauwkeurigheid

als orde van minstens = orde methode

$$R_u^{\text{lineair}} \approx R_u(C_{uAMB}, C_{vAMB}) + \frac{\partial R_u}{\partial C_u}(C_{uAMB}, C_{vAMB})(C_u - C_{uAMB}) + \frac{\partial R_u}{\partial C_v}(C_{uAMB}, C_{vAMB})(C_v - C_{vAMB})$$

$$= \quad V([1, 5, 10])$$

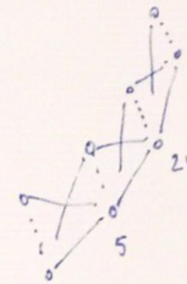
⇒ check orde v/d metho op het einde

→ ds discretizatie ↓ hoe fout ↓

$$\int_\Gamma r \rho_u C_u^* \varphi_i \, d\Gamma$$

$$= \int_{\Gamma_1} r \rho_u C_u^* \varphi_i \, d\Gamma + \int_{\Gamma_2} r \rho_u \sum_j c_j \varphi_j \varphi_i \, d\Gamma - \int_{\Gamma_2} r \rho_u C_{uAMB} \varphi_i \, d\Gamma$$

① ② ③

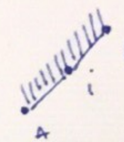


① = 0

$$② = \begin{cases} 0 & i \notin \Gamma_2 \\ \sum_{j \in \mathcal{J}} c_j \, \frac{1}{12} \rho_u |E_{ij}| \, C_j & \end{cases}$$

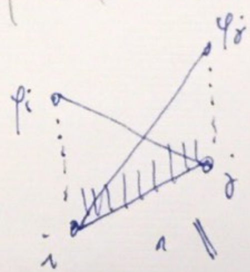
$i \neq j$: $C_j = (r_i + r_j)$

$i = j$: $C_j = (6 r_i + r_A + r_B)$



$$③ = \begin{cases} 0 & i \notin \Gamma_2 \\ \frac{1}{6} \rho_u C_{uAMB} \left(\cancel{r_B^2 - r_A^2 + r_i r_B - r_i r_A}\right) & \end{cases}$$

$$\frac{1}{6} \rho_u C_{uAMB} \left(|E_{Ai}|(2r_i + r_A) + |E_{iB}|(2r_i + r_B) \right)$$



1 ||

2 X

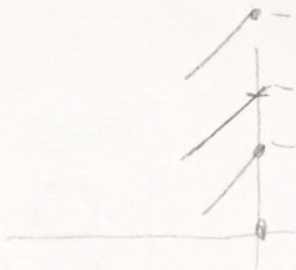
3 /

...men met.

$$C_u \approx C_u^m = \sum c_i \varphi_i$$

$$C_v \approx C_v^m = \sum c_{i+m} \varphi_i$$

12 · 02



$$q_u = r\begin{pmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{pmatrix}\begin{pmatrix} \sum_i c_i \frac{\partial \varphi_i}{\partial r} \\ \sum_i c_i \frac{\partial \varphi_i}{\partial z} \end{pmatrix} = r\begin{pmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{pmatrix}\sum_i c_i \nabla\varphi_i$$

Σ_v

$$q_v = r\begin{pmatrix} \sigma_{vr} & 0 \\ 0 & \sigma_{vz} \end{pmatrix}\begin{pmatrix} \sum_i c_{m+i} \frac{\partial \varphi_i}{\partial r} \\ \sum_i c_{m+i} \frac{\partial \varphi_i}{\partial z} \end{pmatrix} = r\begin{pmatrix} \sigma_{vr} & 0 \\ 0 & \sigma_{vz} \end{pmatrix}\sum_i c_{m+i} \nabla\varphi_i$$

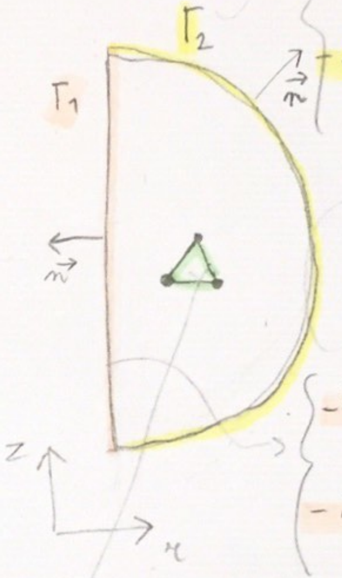
$$u(x) \approx u_m(x) = \varphi_0 + \sum c_m \varphi_m \qquad x \in \Omega$$

$$\left.\begin{aligned} p_u &= \nabla q_u - r R_u \\ p_v &= \nabla q_v + r R_v \end{aligned}\right] \quad \text{eis} \quad \begin{aligned} &(1) \langle p_u, \varphi_i \rangle = 0 \quad \forall i \quad (r,z) \in \Omega \\ &(2) \langle p_v, \varphi_i \rangle = 0 \quad \forall i \quad (r,z) \in \Omega \end{aligned}$$

$$\langle f, g \rangle = \int_\Omega f g \, d\Omega = 0$$

$$\Rightarrow (1) \quad \int_\Omega (\nabla q_u - r R_u)\varphi_i \, d\Omega = 0 \qquad \forall \; 1 \le i \le m$$

$$(2) \quad \int_\Omega (\nabla q_v + r R_v)\varphi_i \, d\Omega = 0$$



$$C_u^*$$

$$\begin{cases} -\vec{n}\left(\begin{pmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{pmatrix}\nabla C_u\right) = \rho_u\left(C_u - C_{u\,amb}\right) \\ -\vec{n}\left(\begin{pmatrix} \sigma_{vr} & 0 \\ 0 & \sigma_{vz} \end{pmatrix}\nabla C_v\right) = \rho_v\left(C_v - C_{v\,amb}\right) \end{cases}$$

C_v^*

convection

$$\begin{cases} -\vec{n}\left(\begin{pmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{pmatrix}\nabla C_u\right) = 0 \\ -\vec{n}\left(\begin{pmatrix} \sigma_{vr} & 0 \\ 0 & \sigma_{vz} \end{pmatrix}\nabla C_v\right) = 0 \end{cases} \Rightarrow \quad \begin{aligned} &\vec{n} \perp \nabla C_u \\ &\vec{n} \perp \nabla C_v \end{aligned}$$

impose null-flux

"steady state solution"

respiration

$$\begin{cases} \nabla\left(r\begin{pmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{pmatrix}\nabla C_u\right) = r \cdot R_u(C_u, C_v) \\ \nabla\left(r\begin{pmatrix} \sigma_{vr} & 0 \\ 0 & \sigma_{vz} \end{pmatrix}\nabla C_v\right) = -r R_v(C_u, C_v) \end{cases} \qquad (r,z) \in \Omega$$

respiration

$$\begin{cases} R_u = \dfrac{V_{mu} C_u}{(K_{mu} + C_u)\left(1 + \dfrac{C_v}{K_{mv}}\right)} \\ R_v = r_q R_u + \dfrac{V_{mfv}}{1 + \dfrac{C_u}{K_{mfu}}} \end{cases}$$

∇C_u w/ $C_u = \sum c_i \varphi_i$ $\quad \nabla C_u = \sum c_i \nabla\varphi_i$

$r S_v \nabla C_v$ $\quad C_v = \sum c_{mv,i} \varphi_i$ $\quad \nabla C_v = \sum c_{mv,i} \nabla\varphi_i$

$$\int_\Omega q_u \cdot \nabla\varphi_i \, d\Omega + \int_\Omega r R_u \varphi_i \, d\Omega + \int_\Gamma r \rho_u C_u^* \varphi_i \, d\Gamma = 0$$

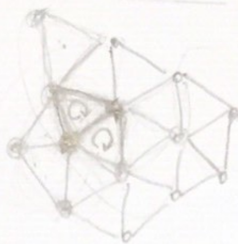
$$\int_\Omega q_u \cdot \nabla\varphi_i \, d\Omega = \int_\Omega r S_u \nabla C_u \cdot \nabla\varphi_i \, d\Omega$$

$$= \int_\Omega r S_u \sum c_j \nabla\varphi_j \cdot \nabla\varphi_i \, d\Omega$$

$S_u = \begin{bmatrix} \sigma_{ur} & 0 \\ 0 & \sigma_{uz} \end{bmatrix}$

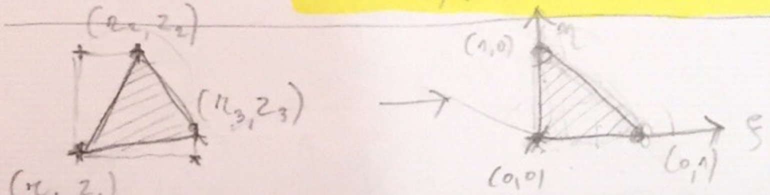
$$= \sum c_j \int_\Omega r S_u \nabla\varphi_j \cdot \nabla\varphi_i \, d\Omega$$

$\boxed{\begin{array}{l} i = j : \text{max } 2\ \Omega_t \\ i \neq j : \text{alle } \Omega_t \end{array}}$



$$= \sum_{j \in J} c_j \left(\int_{\Omega_{j1}} r \left(\sigma_{ur} \frac{\partial\varphi_j}{\partial r} \cdot \frac{\partial\varphi_i}{\partial r} + \sigma_{uz} \frac{\partial\varphi_j}{\partial z} \cdot \frac{\partial\varphi_i}{\partial z} \right) d\Omega + \int_{\Omega_{j2}} r \left(\sigma_{ur} \frac{\partial\varphi_j}{\partial r} \cdot \frac{\partial\varphi_i}{\partial r} + \sigma_{uz} \frac{\partial\varphi_j}{\partial z} \cdot \frac{\partial\varphi_i}{\partial z} \right) d\Omega \right)$$

max 2 Δ, maar kan ook enkel Ω_{j1}



$$\frac{\partial\xi}{\partial r} = \frac{(z_3 - z_1)}{2|\Omega_t|} \qquad \frac{\partial\xi}{\partial z} = \frac{-(r_3 - r_1)}{2|\Omega_t|} \qquad \frac{\partial\eta}{\partial r} = \frac{-(z_2 - z_1)}{2|\Omega_t|} \qquad \frac{\partial\eta}{\partial z} = \frac{(r_2 - r_1)}{2|\Omega_t|}$$

$$\xi = \frac{1}{2|\Omega_t|} \left((z_3 - z_1)(r - r_1) - (r_3 - r_1)(z - z_1) \right)$$

$$\eta = \frac{1}{2|\Omega_t|} \left(-(z_2 - z_1)(r - r_1) + (r_2 - r_1)(z - z_1) \right)$$

$\psi_1 = \eta$

$\psi_2 = \xi$

$\psi_3 = 1 - \xi - \eta$

$$\varphi_i = \psi_3 = 1 - \xi - \eta = 1 - \xi(r,z) - \eta(r,z)$$

$$\frac{\partial\varphi_i}{\partial r} = \frac{\partial\varphi_i}{\partial\xi} \frac{\partial\xi}{\partial r} + \frac{\partial\varphi_i}{\partial\eta} \cdot \frac{\partial\eta}{\partial r} = -\frac{(z_3 - z_1)}{2|\Omega_t|} + \frac{(z_2 - z_1)}{2|\Omega_t|} = \frac{(z_2 - z_3)}{2|\Omega_t|}$$

$$\frac{\partial\varphi_i}{\partial z} = \frac{\partial\varphi_i}{\partial\xi} \cdot \frac{\partial\xi}{\partial z} + \frac{\partial\varphi_i}{\partial\eta} \cdot \frac{\partial\eta}{\partial z} = +\frac{(r_3 - r_1)}{2|\Omega_t|} - \frac{(r_2 - r_1)}{2|\Omega_t|} = \frac{(r_3 - r_2)}{2|\Omega_t|}$$

$$\int_{\Omega_1} r \left(\sigma_{ur} \frac{\partial\varphi_j}{\partial r} \cdot \frac{\partial\varphi_i}{\partial r} + \sigma_{uz} \frac{\partial\varphi_j}{\partial z} \cdot \frac{\partial\varphi_i}{\partial z} \right) d\Omega$$

$$= \int_{r_1}^{r_2} \int_{z_1}^{z_2} r \left(\sigma_{ur} \frac{\partial\varphi_j}{\partial r} \cdot \frac{\partial\varphi_i}{\partial r} + \sigma_{uz} \frac{\partial\varphi_j}{\partial z} \cdot \frac{\partial\varphi_i}{\partial z} \right) dr \, dz$$

$\varphi_i = \psi_3$

$\varphi_j = \psi_1$

$$= \int_0^1 \int_0^{1-\xi} r \left(\sigma_{ur} \frac{(z_1 - z_2)}{2|\Omega_t|} \cdot \frac{(z_2 - z_3)}{2|\Omega_t|} + \sigma_{uz} \frac{(r_2 - r_1)}{2|\Omega_t|} \cdot \frac{(r_3 - r_2)}{2|\Omega_t|} \right) dr \, dz$$

$$\xi + \frac{(z_3 - z_1)}{(z_2 - z_1)} \eta = \frac{1}{2|\Omega_t|} \left(-(r_3 - r_1)(z - z_1) + \frac{(z_3 - z_1)}{(z_2 - z_1)} (r_2 - r_1)(z - z_1) \right)$$

$$(z_2 - z_1)\xi + (z_3 - z_1)\eta = \frac{1}{2|\Omega_t|} \left((z_3 - z_1)(r_2 - r_1) - (z_2 - z_1)(r_3 - r_1) \right)(z - z_1)$$

$$\Rightarrow z = z_1 + \frac{2|\Omega_t| \left((z_2 - z_1)\xi + (z_3 - z_1)\eta \right)}{(z_3 - z_1)(r_2 - r_1) - (z_2 - z_1)(r_3 - r_1)} \qquad \frac{\partial z}{\partial\xi} = \frac{2|\Omega_t|(z_2 - z_1)}{(z_3 - z_1)(r_2 - r_1) - (z_2 - z_1)(r_3 - r_1)}$$

$$\xi + \frac{(r_3 - r_1)}{(r_2 - r_1)} \eta = \frac{1}{2|\Omega_t|} \left((z_3 - z_1)(r - r_1) - \frac{(r_3 - r_1)}{(r_2 - r_1)} (z_2 - z_1)(r - r_1) \right)$$

$$(r_2 - r_1)\xi + (r_3 - r_1)\eta = \frac{1}{2|\Omega_t|} \left((r_2 - r_1)(z_3 - z_1) - (r_3 - r_1)(z_2 - z_1) \right)(r - r_1)$$

$$\Rightarrow r = r_1 + \frac{2|\Omega_t| \left((r_2 - r_1)\xi + (r_3 - r_1)\eta \right)}{(r_2 - r_1)(z_3 - z_1) - (r_3 - r_1)(z_2 - z_1)}$$

$\varphi_j = \psi_1 = \eta$

$$\frac{\partial\varphi_j}{\partial r} = \frac{\partial\varphi_j}{\partial\xi} \cdot \frac{\partial\xi}{\partial r} + \frac{\partial\varphi_j}{\partial\eta} \cdot \frac{\partial\eta}{\partial r} = \frac{(z_1 - z_2)}{2|\Omega_t|}$$

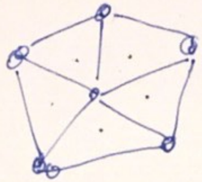
$$\frac{\partial\varphi_j}{\partial z} = \frac{\partial\varphi_j}{\partial\xi} \frac{\partial\xi}{\partial z} + \frac{\partial\varphi_j}{\partial\eta} \frac{\partial\eta}{\partial z} = \frac{(r_2 - r_1)}{2|\Omega_t|}$$

$\varphi_j = \psi_2 = \xi$

$$\frac{\partial\varphi_j}{\partial r} = \frac{\partial\varphi_j}{\partial\xi} \frac{\partial\xi}{\partial r} + \frac{\partial\varphi_j}{\partial\eta} \cdot \frac{\partial\eta}{\partial r} = \frac{(z_3 - z_1)}{2|\Omega_t|}$$

$$\frac{\partial\varphi_j}{\partial z} = \frac{\partial\varphi_j}{\partial\xi} \frac{\partial\xi}{\partial z} + \frac{\partial\varphi_j}{\partial\eta} \frac{\partial\eta}{\partial z} = \frac{(r_1 - r_3)}{2|\Omega_t|}$$

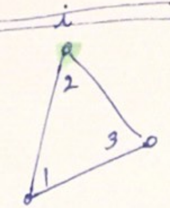
$\int_\Omega q_u \cdot \nabla\varphi_i \, d\Omega$ for $(i=j)$



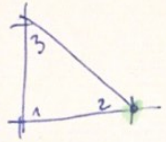
$i=j$

$$\sum_m \int_{\Omega_{im}} r\left(\sigma_{vr} \frac{\partial\varphi_i}{\partial r}\frac{\partial\varphi_i}{\partial r} + \sigma_{vz}\frac{\partial\varphi_i}{\partial z}\frac{\partial\varphi_i}{\partial z}\right) dr\,dz$$

$\varphi_i = \varphi_2$

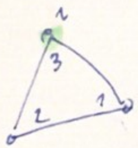


$$\int_0^1\int_0^{1-\xi} \left(r_1 + (r_2-r_1)\xi + (r_3-r_1)\eta\right) C_m \, d\xi\, d\eta$$

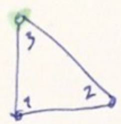


$$C_m = \sigma_{vz}\frac{(z_3-z_1)^2}{2|\Omega_t|} + \sigma_{vz}\frac{(r_1-r_3)^2}{2|\Omega_t|}$$

$\varphi_i = \varphi_1$

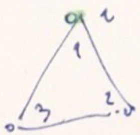


$$\int_0^1\int_0^{1-\xi} \left(r_1+(r_2-r_1)\xi+(r_3-r_1)\eta\right) C_m \, d\xi\, d\eta$$

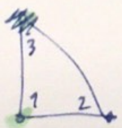


$$C_m = \sigma_{vr}\frac{(z_1-z_2)^2}{2|\Omega_t|} + \sigma_{vz}\frac{(r_2-r_1)^2}{2|\Omega_t|}$$

$\varphi_i = \varphi_3$

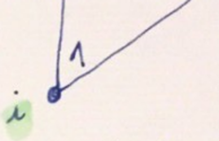


$$\int_0^1\int_0^{1-\xi} \left(r_1+(r_2-r_1)\xi+(r_3-r_1)\eta\right) C_m \, d\xi\, d\eta$$



$$C_m = \sigma_{vz}\frac{(z_2-z_3)^2}{2|\Omega_t|} + \sigma_{vz}\frac{(r_3-r_2)^2}{2|\Omega_t|}$$

$i=i \Rightarrow \int_\Omega = \sum_m C_m(r_1+r_2+r_3)$



$$\int_{\Omega_{j1}} r\left(\sigma_{vz}\frac{\partial\psi_1}{\partial r}\frac{\partial\psi_3}{\partial z} + \sigma_{vz}\frac{\partial\psi_1}{\partial z}\frac{\partial\psi_3}{\partial z}\right) dr\,dz$$

$$= \int_0^1\int_0^{1-\xi}\left(r_1+(r_2-r_1)\xi+(r_3-r_1)\eta\right) C_1 \, d\xi\, d\eta \qquad -\frac{1}{6}+\frac{1}{2}-\frac{1}{6}=$$

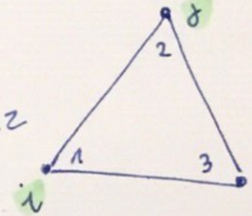
$$w \quad \boxed{C_1 = \left(\sigma_{vz}\frac{(z_1-z_2)}{2|\Omega_t|}\frac{(z_2-z_3)}{2|\cancel{\Omega_t}|} + \sigma_{vz}\frac{(r_2-r_1)}{2|\Omega_t|}\frac{(r_3-r_2)}{2|\cancel{\Omega_t}|}\right) 2\cancel{\Omega_t}}$$

$$= \int_0^1 C_1 r_1(1-\xi) + C_1(r_2-r_1)(\xi-\xi^2) + \frac{C_1}{2}(r_3-r_1)(1-\xi)^2 \, d\xi$$

$$= -C_1 r_1 \frac{(1-\xi)^2}{2} + C_1(r_2-r_1)\left(\frac{\xi^2}{2}-\frac{\xi^3}{3}\right) - \frac{C_1}{6}(r_3-r_1)(1-\xi)^3 \Big]_0^1$$

$$= \frac{1}{6} C_1(r_2-r_1) + \frac{1}{2} C_1 r_1 + \frac{1}{6}(r_3-r_1) = \frac{C_1}{6}(r_2+r_3+r_1)$$

over Ω_{j2}: $\varphi_i = \varphi_3$ $\quad \varphi_j = \varphi_2$



$$\int_{\Omega_{j2}} r\left(\sigma_{vr}\frac{\partial\psi_2}{\partial r}\frac{\partial\psi_3}{\partial r} + \sigma_{vz}\frac{\partial\psi_2}{\partial z}\frac{\partial\psi_3}{\partial z}\right) dr\,dz$$

$$= \int_0^1\int_0^{1-\xi}\left(r_1+(r_2-r_1)\xi+(r_3-r_1)\eta\right) C_2 \, d\xi\, d\eta$$

$$w \quad \boxed{C_2 = \left(\sigma_{vr}\frac{(z_3-z_1)}{2|\Omega_t|}\frac{(z_2-z_3)}{2|\cancel{\Omega_t}|} + \sigma_{vz}\frac{(r_1-r_3)}{2|\Omega_t|}\frac{(r_3-r_2)}{2|\cancel{\Omega_t}|}\right) 2\cancel{\Omega_t}}$$

$$(z_i-z_A)(z_A-z_j)$$

$$= \frac{C_2}{6}(r_1+r_2+r_3)$$

$$= -(z_A-z_i)(z_A-z_j) = (z_j-z_A)(z_A-z_i)$$

$\frac{\partial z}{\partial} = 2|\Omega_t|(z_2-z_A)$