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Another notes

Notes

1 Trivial case for Anyonic Hamiltonian

From our Hamiltonian for anyons:

$$\hat{H} = -J \sum_{<i,j>} \left(\hat{b}_i^\dagger \hat{b}_j e^{i\theta \hat{n}_j} + e^{-i\theta \hat{n}_j} \hat{b}_j^\dagger \hat{b}_i \right) + \frac{U}{2} \sum_{i=1}^{M} \hat{n}_i \left(\hat{n}_i - 1 \right) \tag{1}$$

Starting from a small lattice to illustrate the problem for 2 particles and 2 sites, and then we can find the ground state by hand.
All the possible configurations are:

$$|2, 0\rangle \tag{2}$$
$$|1, 1\rangle \tag{3}$$
$$|0, 2\rangle \tag{4}$$

Writing explicitly the Hamiltonian:

$$\hat{H} = -J \left(\hat{b}_1^\dagger \hat{b}_2 e^{i\theta \hat{n}_2} + e^{-i\theta \hat{n}_2} \hat{b}_2^\dagger \hat{b}_1 \right) + \frac{U}{2} \left[\hat{n}_1 \left(\hat{n}_1 - 1 \right) + \hat{n}_2 \left(\hat{n}_2 - 1 \right) \right] \tag{5}$$

applying the Hamiltonian:

$$\hat{H}|2, 0\rangle = J \left(0 + e^{-i\theta \hat{n}_2} \hat{b}_2^\dagger \sqrt{2} |1, 0\rangle \right) + \frac{U}{2} (2(2-1) + 0) \tag{6}$$

$$\hat{H}|2, 0\rangle = -\sqrt{2} J e^{-i\theta} |1, 1\rangle + U|2, 0\rangle \tag{7}$$

$$\hat{H}|1, 1\rangle = -J \left(\hat{b}_1^\dagger \hat{b}_2 e^{i\theta} |1, 1\rangle + e^{-i\theta \hat{n}_2} \hat{b}_2^\dagger |0, 1\rangle \right) + 0 \tag{8}$$

$$\hat{H}|1, 1\rangle = -J \left(\sqrt{2} e^{i\theta} |2, 0\rangle + \sqrt{2} e^{-2i\theta} |0, 2\rangle \right) \tag{9}$$

$$\hat{H}|0, 2\rangle = -J \left(\sqrt{2} e^{2i\theta} |1, 1\rangle \right) + U|0, 2\rangle \tag{10}$$

Writing the Hamiltonian as a matrix:

$$H = \begin{bmatrix} U & -\sqrt{2}Je^{-i\theta} & 0 \\ -\sqrt{2}Je^{i\theta} & 0 & -\sqrt{2}Je^{-2i\theta} \\ 0 & -\sqrt{2}Je^{2i\theta} & U \end{bmatrix} \tag{11}$$

We can evaluate the eigensystem directly by Mathematica:

Listing 1: Mathematica Code

```
H = {{U, -Sqrt[2] J Exp[-I * \[Theta]], 0},
     {-Sqrt[2] Exp[I * \[Theta]] J, 0, -Sqrt[2] Exp[-2 I * \[Theta]] J},
     {0, -Sqrt[2] J Exp[2 I * \[Theta]], U}};
FullSimplify@Eigensystem@H
```

With the results:

$$E_1 = \frac{U - \sqrt{16J^2 + U^2}}{2}, \qquad \psi_1 = \begin{bmatrix} 2\sqrt{2}\, Je^{-3i\theta} \\ e^{-2i\theta}\left(U + \sqrt{16J^2 + U^2}\right) \\ 2\sqrt{2}\, J \end{bmatrix} \tag{12}$$

$$E_2 = U, \qquad \psi_2 = \begin{bmatrix} e^{-3i\theta} \\ 0 \\ -1 \end{bmatrix} \tag{13}$$

$$E_3 = \frac{U + \sqrt{16J^2 + U^2}}{2}, \qquad \psi_3 = \begin{bmatrix} 2\sqrt{2}\, Je^{-3i\theta} \\ e^{-2i\theta}\left(U - \sqrt{16J^2 + U^2}\right) \\ 2\sqrt{2}\, J \end{bmatrix} \tag{14}$$

Where E_1 is the ground state energy, and the state could be written:

$$|\psi_{\text{ground}} >= C\left(2\sqrt{2}\, J\left(e^{-3i\theta}|2,\, 0\rangle + |0,\, 2\rangle\right) + e^{-2i\theta}\left(U + \sqrt{16J^2 + U^2}\right)|1,\, 1\rangle\right) \tag{15}$$

with C as a normalization constant