Gauss-Jordan Elimination

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BSCS 3-B1

MIDTERM

Background

Gauss-Jordan method is invented by Henry Maltby, Jason Dyer, and Shaurya Singh. Gauss-Jordan method is an algorithm to solve a system of linear equations by representing it as an augmented matrix, reducing it using row operations, and expressing the system in reduced row-echelon form to find the values of the variables. The Gauss Jordan Elimination's main purpose is to use the elementary row operations on an augmented matrix to reduce it into the reduced row echelon form (RREF). A matrix is said to be in reduced row echelon form, also known as row canonical form.

Derivation

The Gauss-Jordan method is similar to the Gaussian elimination process, except that the entries both above and below each pivot are zeroed out. The general solution process is to reduce the A matrix to a form such that the system of equations can be solved directly. With Gauss-Jordan elimination, the A matrix is reduced to the identity matrix. One advantage of Gauss-Jordan is that it will also give you the inverse of the A matrix. Gauss-Jordan, when pivoted, is a very stable algorithm. One disadvantage is that it requires about three times the number of operations of Gaussian elimination or LU decomposition and thus is slower than those methods. For this reason, Gauss-Jordan elimination is less frequently used than Gaussian elimination or LU decomposition.

Steps to perform Gauss-Jordan Elimination:

1. Swap the rows so that all rows with all zero entries are on the bottom
2. Swap the rows so that the row with the largest, leftmost nonzero entry is on top.
3. Multiply the top row by a scalar so that top row's leading entry becomes 1.
4. Add/subtract multiples of the top row to the other rows so that all other entries in the column containing the top row's leading entry are all zero.
5. Repeat steps 2-4 for the next leftmost nonzero entry until all the leading entries are 1.
6. Swap the rows so that the leading entry of each nonzero row is to the right of the leading entry of the row above it.

Example:
Solve the system shown below using the Gauss Jordan Elimination method:

$-x+2y=-6$

$3x-4y=14$

The first step is to write the augmented matrix of the system. We show this below:

$$\begin{bmatrix} -1 & 2 & -6 \\ 3 & -4 & 14 \end{bmatrix}$$

Step 1:
We can multiply the first row by -1 to make the leading entry 1.

$$\begin{bmatrix} 1 & -2 & 6 \\ 3 & -4 & 14 \end{bmatrix}$$

Step 2:
We can now multiply the first row by and subtract it from the second row.

$$\begin{bmatrix} 1 & -2 & 6 \\ 0 & 2 & -4 \end{bmatrix}$$

We have a 0 as the first entry of the second row.

Step 3:

To make the second entry of the second row 1, we can multiply the second row by ½.

$$\begin{bmatrix} 1 & -2 & 6 \\ 0 & 1 & -2 \end{bmatrix}$$

Step 4:

The second entry of the first row should be 0. In order to do that, we multiply the second row by 2 and add it to the first row.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{bmatrix}$$

This is the reduced row echelon form. From the augmented matrix, we can write two equations (solutions):

x+0y=2

0x+y=-2

$$\boxed{\begin{array}{l} x=2 \\ y=-2 \end{array}}$$

Thus, the solution of the system of equations is x=2 and y=-2.

Advantage of the method over other methods

Although the methods of Gauss-Jordan and Gausss elimination can look almost the same, the former requires approximately 50% fewer operations. Therefore, the Gauss-Jordan elimination method is simple for excellence in obtaining exact solutions to simultaneous linear equations. One of the main reasons for including the Gauss-Jordan is to provide direct method for obtaining the inverse matrix.