Simulating Dark Matter Halos in the Early Universe

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1 Background

In the early universe, dark matter was distributed more uniformly than it is thought to be in today's universe. Small perturbations formed clumps of dark matter on the order of 1 Mpc across. Current theories of dark matter accretion hypothesize that these smaller clumps grew into the large-scale filament structure of today by combining through gravitational attraction. To that end, we simulated a cube of side length 50 Mpc, with some dark matter clumps scattered within. On these scales, we should see the smaller clumps accrete into larger ones over the course of Gyr timescales. We are interested to discover firstly if this actually occurs, and further how much of the dark matter accretes onto larger-scale structure as opposed to being ejected, and how many larger clumps are produced.

2 Methods

2.1 Coding the Hubble Constant

On the scales of our investigation the, expansion of the universe – often referred to as the "Hubble Flow" – exerts a significant influence on the kinematics of the system. This effect increases the separations between all points and must be modeled on top of gravitationally driven kinetics. While schisms in the observational measurements of this flow are infamous, we use a value of 67 km/s/Mpc in these simulations, consistent with one of two supported values.

To model this computationally, we apply a term to the $\frac{d\vec{r}}{dt}$ value proportional to the position of each particle. Note that while this kick is applied to the derivative of the particle positions, it is not included as a velocity, as this would

2.2 Generating Initial Conditions

The backbone of the initial conditions module were two random number generators. Namely, a continuous uniform distribution and normal distribution. The normalized probability density function (PDF) for a normal distribution is of the form:

$$n(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{1}$$

While the uniform distribution as a PDF of:

$$u(x) = \begin{cases} \frac{1}{b-a} & (a \geq x \leq b) \\ 0 & (x < a \text{ or } x > b) \end{cases} \tag{2}$$

We start out by randomly shooting 10,000 particles into a box with dimensions of 50Mpc x 50Mpc x 50Mpc. There are 100 subhalos that the particles will be distributed amongst. Half the particles were randomly assigned a central position at the center of the box via a normal distribution. The other half of the particles were randomly assigned to the remaining sub-halos. Each of these sub-halos has a central mass where the particles are randomly distributed around the central mass according to a normal distribution. The initial conditions are shown below where the color-bar shows the distribution of velocities.

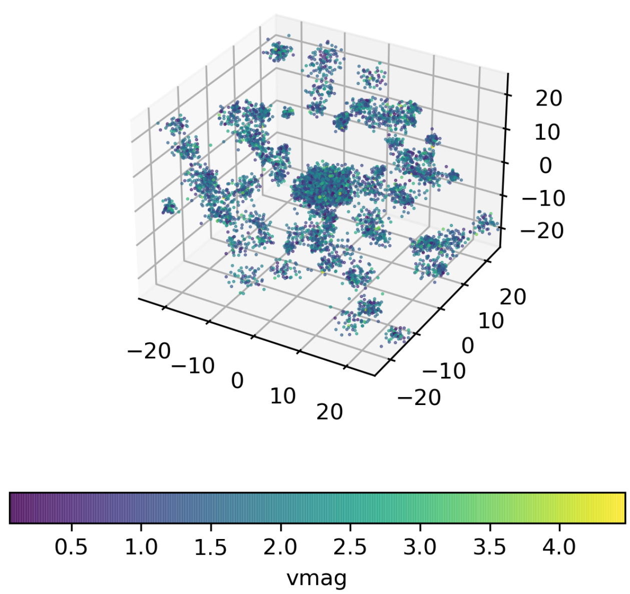

Figure 1: Initial Conditions

The total mass of the particles is a function of the critical mass where the mass of each of the particles is the total mass divided by the total number of particles. Each of the particles has an equal mass. The velocities of the each of the particles is defined by a normal distribution around a central velocity of 0.

3 Results

The plot shown below describes the evolution of the system over the course of 9 Gyr. The colorbar represents the surface density of dark matter

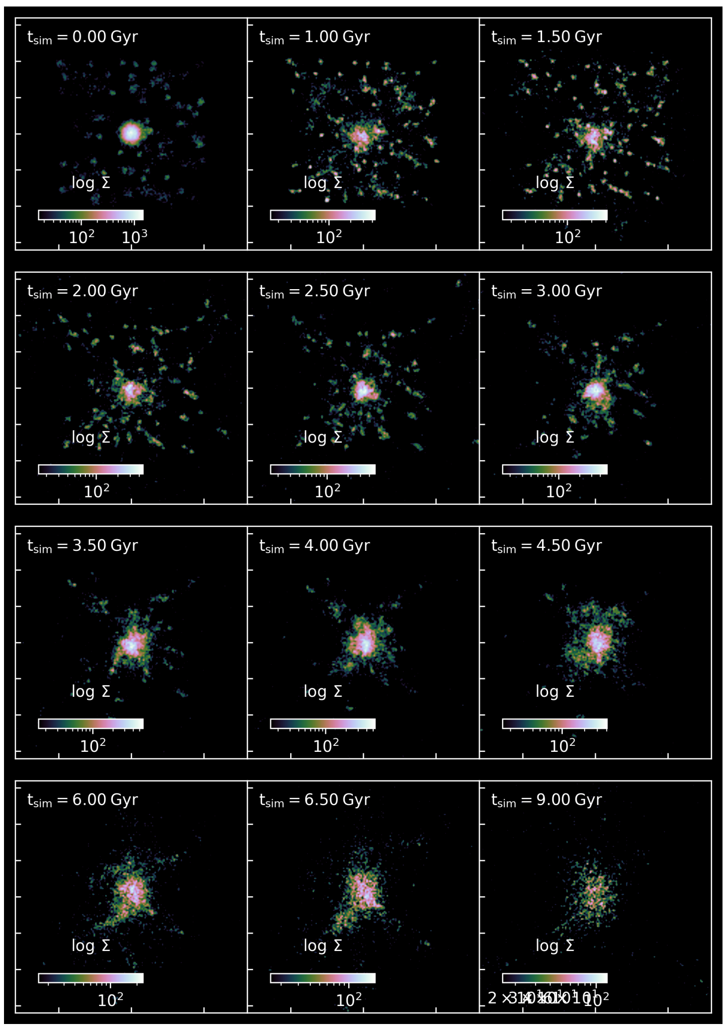

Figure 2: Distribution of Dark Matter

As the Figure above shows, the dark matter initially spreads out from the central mass present in the initial conditions, but over Gyr timescales it begins to gravitationally accrete despite the expansion of the universe, eventually settling into a final central halo at 9 Gyr.
We also ran a simulation in which each side of the cube was twice as large as in the simulation shown above, leading to a density value of $\frac{1}{8}$ what was used above. This, as expected, led to a domination of the simulation by the Hubble flow, with most of the dark matter flowing out.
In order to verify more quantitatively our simulation, we used the Navarro-Frenk-White (NFW) profile to test the density distribution of our dark matter halo.[1] The NFW profile is given below in Eq.3.

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}(1+\frac{r}{R_s})^2} \tag{3}$$

We then plotted this distribution against our data, and acquired Figure 3 below.

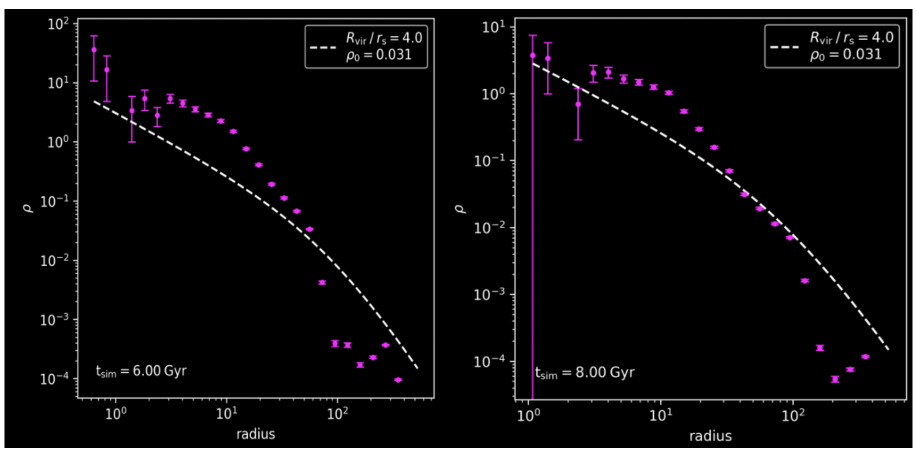

Figure 3: Density of Dark Matter Halo

As the plot shows, our simulation is quite close to the NFW profile for a dark matter halo. We thus take this as a verification of the parameters of our simulation.

4 Conclusion

The dark matter in our simulation did eventually accrete into a more densely packed central mass. This matches the prediction given by current cosmological theories. The overall behavior of the dark matter as well as the density profile are decently consistent with theory, especially given we were only able to simulate 10,000 particles.
The main improvement that could be made to our simulation is to have more

computational power. With a greater ability to simulate more particles in a shorter time, we could both simulate the current conditions and size of region with more precision and extend our region to a greater side length, perhaps thus being able to see some filament structures begin to form as well.

5 Appendix

For a copy of the code, please visit: https://github.com/Ronan-Hix/NBody-415

References

[1] Julio Navarro, Carlos Frenk, and Simon White. "The Structure of Cold Dark Matter Halos". In: *The Astrophysical Journal* 462 (1996), pp. 563–575.