# SC205- Discrete Mathematics

Home Work 1

**Week**: January 6, 2020

**Tutorial Discussion Week**: January 13, 2020

(1) Find all irreducible polynomials (which has no root in $\mathbb{Z}_2$) of degree 3 over $\mathbb{Z}_2$.

(2) Construct a field with 16 number of elements using appropriate irreducible polynomial of degree 4 over $\mathbb{Z}_2$. List all the elements as powers of $\alpha$, as a polynomial in $\alpha$ and as binary vectors of length 4. Show that $\alpha^{15} = \alpha^0 = 1$. In this field $GF(16)$, find the inverse of $(1010)$, find square root $(0110)^{\frac{1}{2}}$. Also find the multiplication of 0111 and 1111 and the division $\frac{0111}{1111}$.

(3) Show that $\mathbb{Z}_4$, $\mathbb{Z}_6$ and $\mathbb{Z}_9$ are not fields. Construct their Tables for addition and multiplication and list the elements which do not have inverse.

(4) What is the value of $\frac{1}{4} mod$ 9. Is there an integer equal to $\frac{1}{3} mod$ 9?

(5) Does every equation of the form $a \cdot n \; x = b$, with $a(\neq 0), b \in \mathbb{Z}_n$ have a solution in $\mathbb{Z}_5$? In $\mathbb{Z}_7$? In $\mathbb{Z}_9$? In $\mathbb{Z}_{11}$?