HVCA - Exercise 4

Please submit your answers as a PDF file and be prepared to discuss your answers in the live session. For automatic checking, make sure to also perform the calculation questions in the Moodle.

1. Suppose we have a visual system that uses k sensors.

   (a) Describe conditions under which k primary lights will **not** allow us to match an arbitrary light with a linear combination of the primaries. Give a specific example.

   (b) Show that if we randomly choose the primaries then with probability one we will be able to match an arbitrary light with a linear combination of the primaries (or that with probability zero you wont be able to do so).

   (c) Suppose we are extremely unlucky and chose k primaries that will not enable matching an arbitrary light source. Design an experiment that will tell us that these primaries are a bad choice. Design your experiment so that the number of color matches the subject will need to do is minimal. In the worst case, what is the minimal number of matches?

2. Consider a visual system with k linearly independent sensors. We will perform a color-matching experiment using $k-1$ primary lights.

   (a) Give an explicit formula for the family of light sources l, that we will not be able to match as a linear combination of the primaries.

   (b) Show that if we choose l at random, then with probability one we will not be able to match it as a linear combination of the $k-1$ primaries.

3. An alien arrives from outer space and you are assigned to the team that studies its visual system. In particular, you want to know how many color sensors its visual system has and what are their response properties. You are not allowed to stick any needles or electrodes into the alien but you have a machine that can display arbitrary lights and you can ask the alien to turn knobs that control the lights until the two lights appear to be the same color. Describe a first experiment that you could use to find out how many color sensors the alien has, and a second experiment that you could use to find the subspace spanned by its color sensors. Give the reasoning for your choice of experiments.

4. Suppose that we are discretizing the visible spectrum into 10 bins so that each light source is described by a vector of length 10. Let $l_1 = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)^T$ and $l_2 = (10, 9, 8, 7, 6, 5, 4, 3, 2, 1)^T$. Consider an animal whose visual system has three sensors whose spectral sensitivities are: $s_1 = (1, 2, 3, 2, 1, 0, 0, 0, 0, 0)$, $s_2 = (0, 0, 0, 1, 1, 1, 1, 0, 0, 0)$, $s_3 = (0, 0, 0, 0, 0, 1, 2, 3, 2, 1)$.

   Denote by (y_1, y_2, y_3) the response of the animal to l_1 and (z_1, z_2, z_3) the response to l_2. Calculate (y_1, y_2, y_3), (z_1, z_2, z_3).

5. Consider three color blind versions of the animals. A is missing the first sensor, B is missing the second sensor, C is missing the first and the third. Would any of the three versions think that l_1 and l_2 are metamers?