Consider the response of a single PMT in the one-ton detector filled with water.

- Let $d_j =$ the sum over data events with the measured number of photoelectrons between e_j and e_{j+1} (in other words, the content of the j^{th} bin),
- Let $m_j =$ the sum over MC events with the simulated number of observed photoelectrons between e_j and e_{j+1} which is equal to the number of generated photons times the probability a that a generated photon creates a photoelectron in the PMT.
- $m_j = \sum_k^j c \times a \times g_k$ where $k =$ event number, $g_k =$ number of generated photons in the k^{th} event and $\sum_k^j$ means the sum over all entries with $c \times a \times g_k$ in the j^{th} bin. $c =$ is the *calibration factor* defined such that the overall probability that a generated photon creates a photoelectron is the same as the data.
- Let $M \equiv \sum_j m_j =$ the total number of MC events
- and $D \equiv \sum_j d_j =$ the total number of data events for a single PMT.

Using the notation of the previous page, for a single PMT, determine the *calibration factor* c by defining the $\chi^2(c)$ as

$$\chi^2(c) \equiv \sum_j \left(\frac{d_j - m_j \frac{D}{M}}{\sigma_j}\right)^2 \tag{1}$$

σ_j can be calculated as follows. Let $y_j = d_j - m_j \frac{D}{M}$, then

$$\begin{aligned}
\sigma_j \equiv \delta y_j^2 &= \left(\frac{\partial y_j}{\partial d_j}\delta d_j\right)^2 + \left(\frac{\partial y_j}{\partial m_j}\delta m_j\right)^2 && (2)\\
&= (\delta d_j)^2 + \left(\frac{D}{M}\delta m_j\right)^2 && (3)\\
&= (\sqrt{d_j})^2 + \left(\frac{D}{M}\sqrt{m_j}\right)^2 && (4)\\
&= d_j + \left(\frac{D}{M}\right)^2 m_j && (5)
\end{aligned}$$

Note that σ_j depends on c, the calibration factor.
Note that a sum must be taken over the number of photoelectrons in the MC events $m_j = \sum_k^j c \times h_k$ to evaluate $\chi^2(c)$, where $h_k \equiv a \times g_k$ and $\sum_k^j$ was defined on the previous page.

Comment on Rong's χ^2 definition:

$$\chi^2 = \sum_i \left(\frac{\sum_j Q_i \times N_{ij}^{data} - N_{ij}^{MC}}{\sigma_{ij}}\right)^2 \tag{6}$$

where Q_i = calibration factor of i^{th} PMT, N_{ij} = number of pe in the j^{th} bin for the i^{th} PMT, $\sigma_{ij} = \sqrt{N_{ij}^{MC}/scalefactor + N_{ij}^{data}}$. I have modified the definition of σ_{ij} to explicitly include the indices i, j.

Eqn. 6 cannot be correct as written because σ_{ij} is outside the sum over j. A modified version of the χ^2 can be

$$\chi^2 = \sum_i \sum_j \left(\frac{Q_i \times N_{ij}^{data} - N_{ij}^{MC}}{\sigma_{ij}}\right)^2 \tag{7}$$

however this is not correct because it simply scales the number of data events in every bin by the factor Q_i. It also changes the total number of events for the i^{th} PMT from $\sum_j N_{ij}^{data}$ to $Q_i \sum_j N_{ij}^{data}$.

The factor Q_i is not equivalent to the *calibration factor c* on the previous pages.