



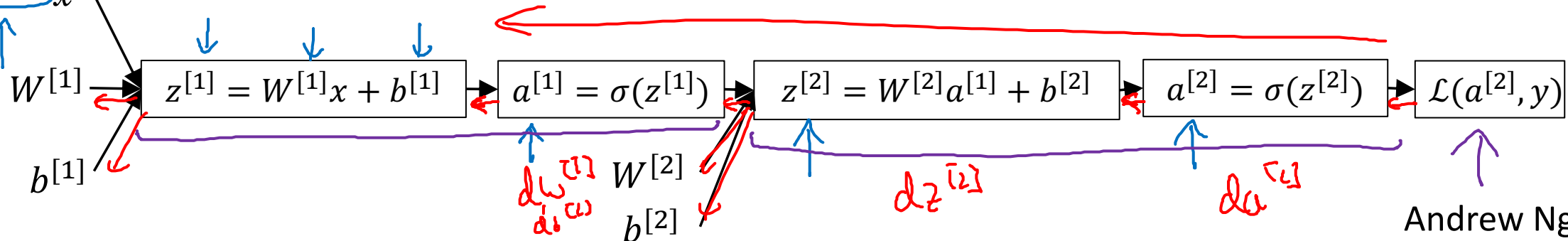
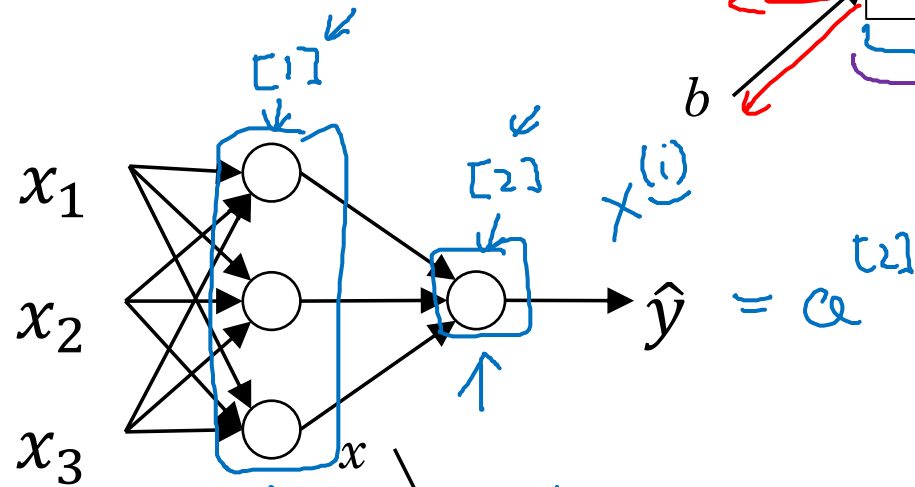
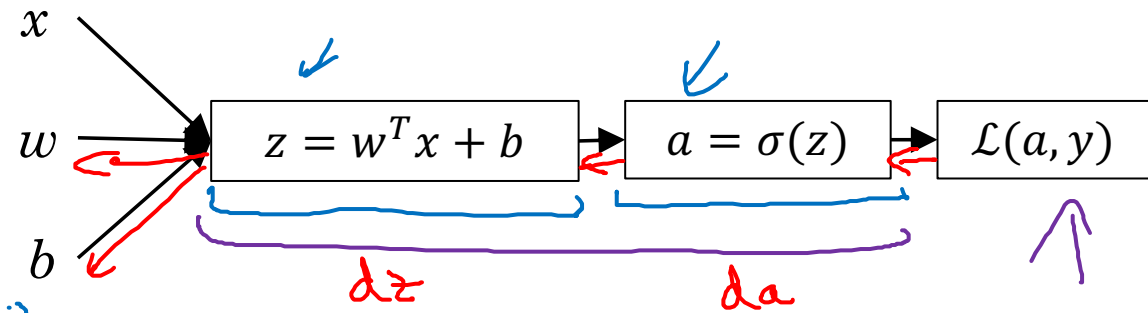
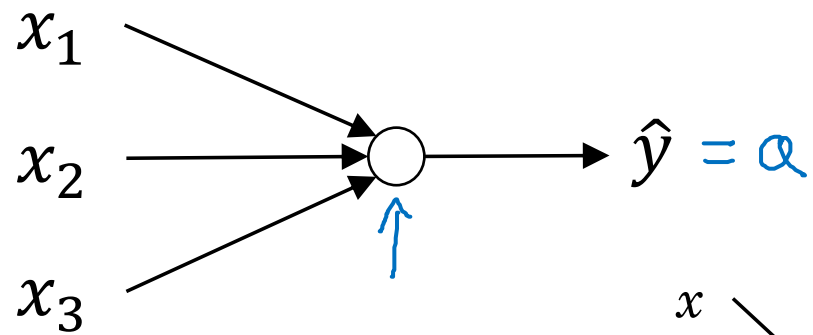
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One hidden layer  
Neural Network

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# Neural Networks Overview

# What is a Neural Network?





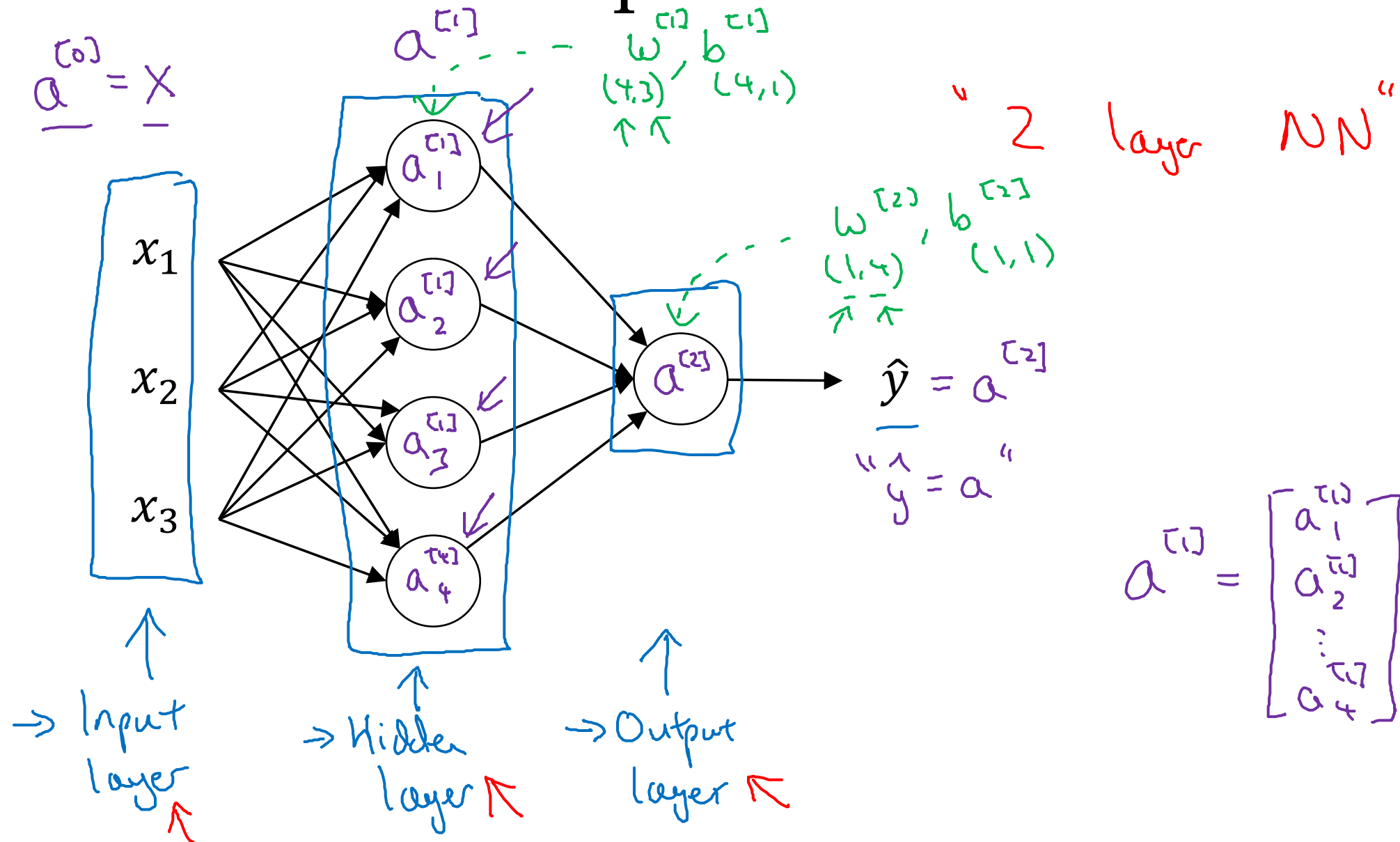
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One hidden layer  
Neural Network

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Neural Network  
Representation

# Neural Network Representation





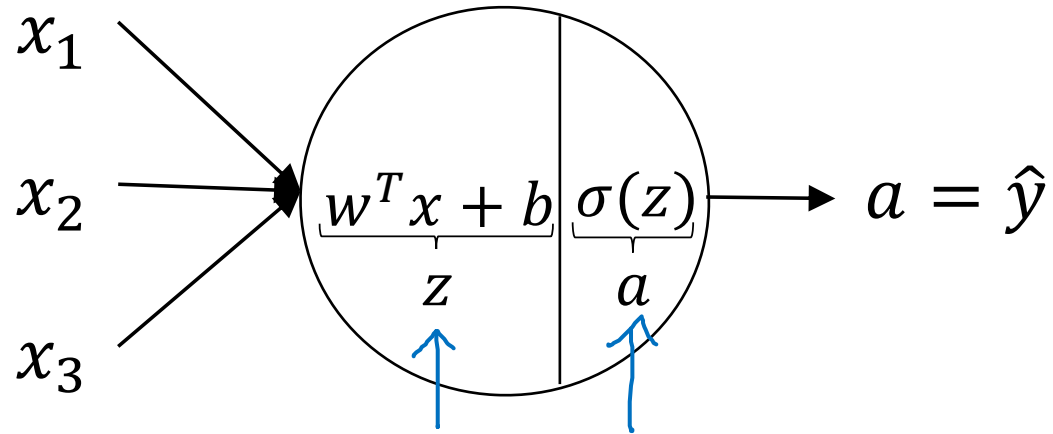
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# One hidden layer Neural Network

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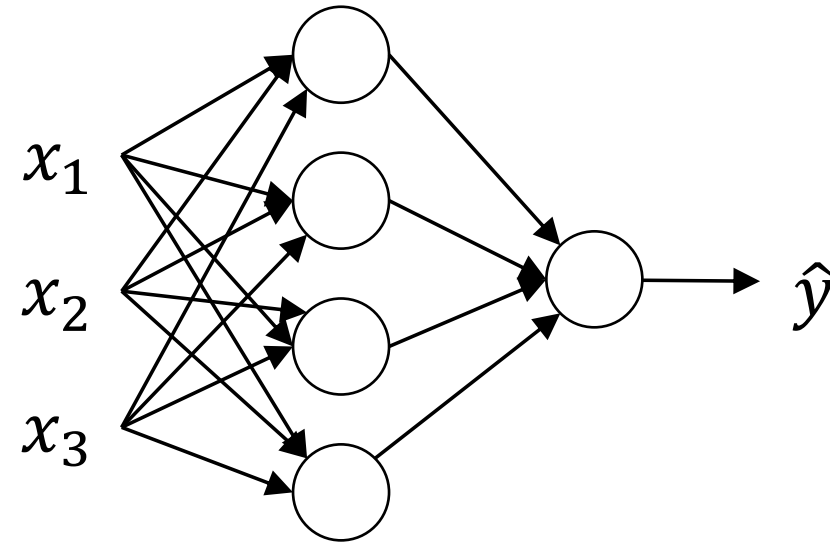
Computing a  
Neural Network's  
Output

# Neural Network Representation

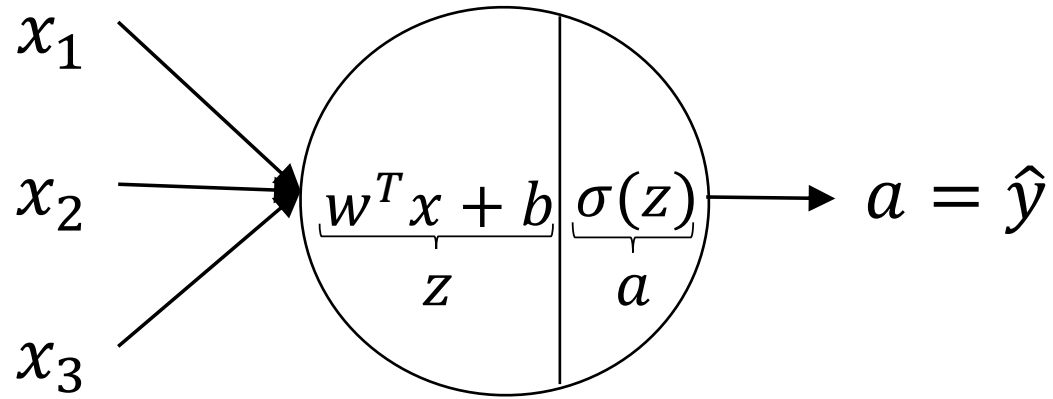


$$z = w^T x + b$$

$$a = \sigma(z)$$

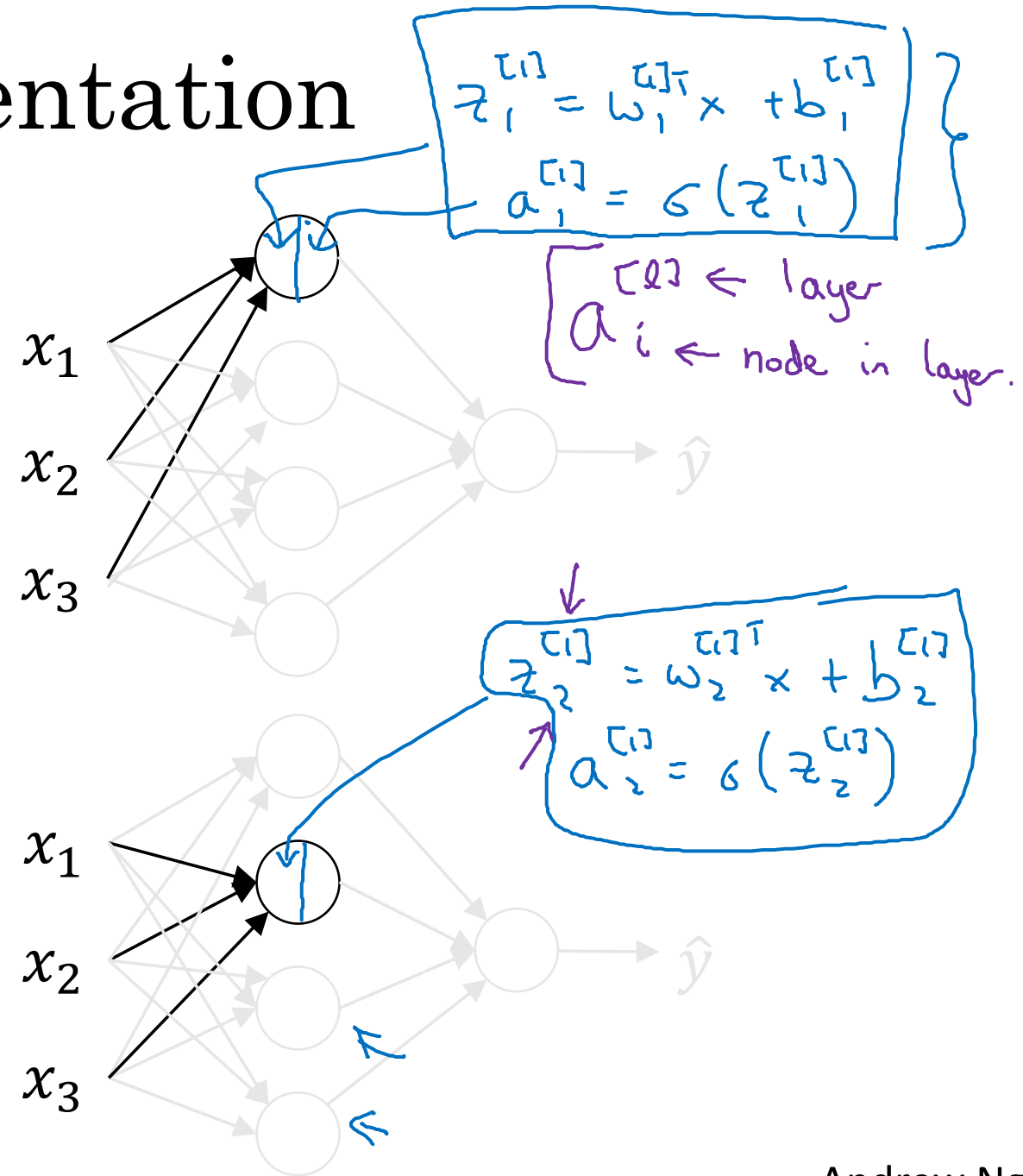


# Neural Network Representation

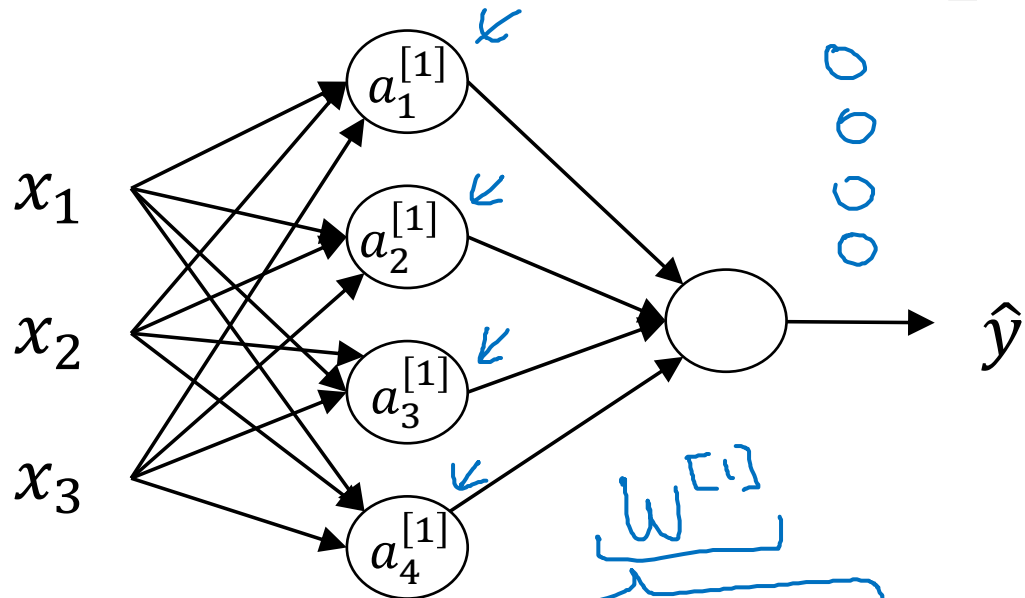


$$z = w^T x + b$$

$$a = \sigma(z)$$



# Neural Network Representation



$$\begin{aligned}
 z_1^{[1]} &= w_1^{[1]T} x + b_1^{[1]} & a_1^{[1]} &= \sigma(z_1^{[1]}) \\
 z_2^{[1]} &= w_2^{[1]T} x + b_2^{[1]} & a_2^{[1]} &= \sigma(z_2^{[1]}) \\
 z_3^{[1]} &= w_3^{[1]T} x + b_3^{[1]} & a_3^{[1]} &= \sigma(z_3^{[1]}) \\
 z_4^{[1]} &= w_4^{[1]T} x + b_4^{[1]} & a_4^{[1]} &= \sigma(z_4^{[1]})
 \end{aligned}$$

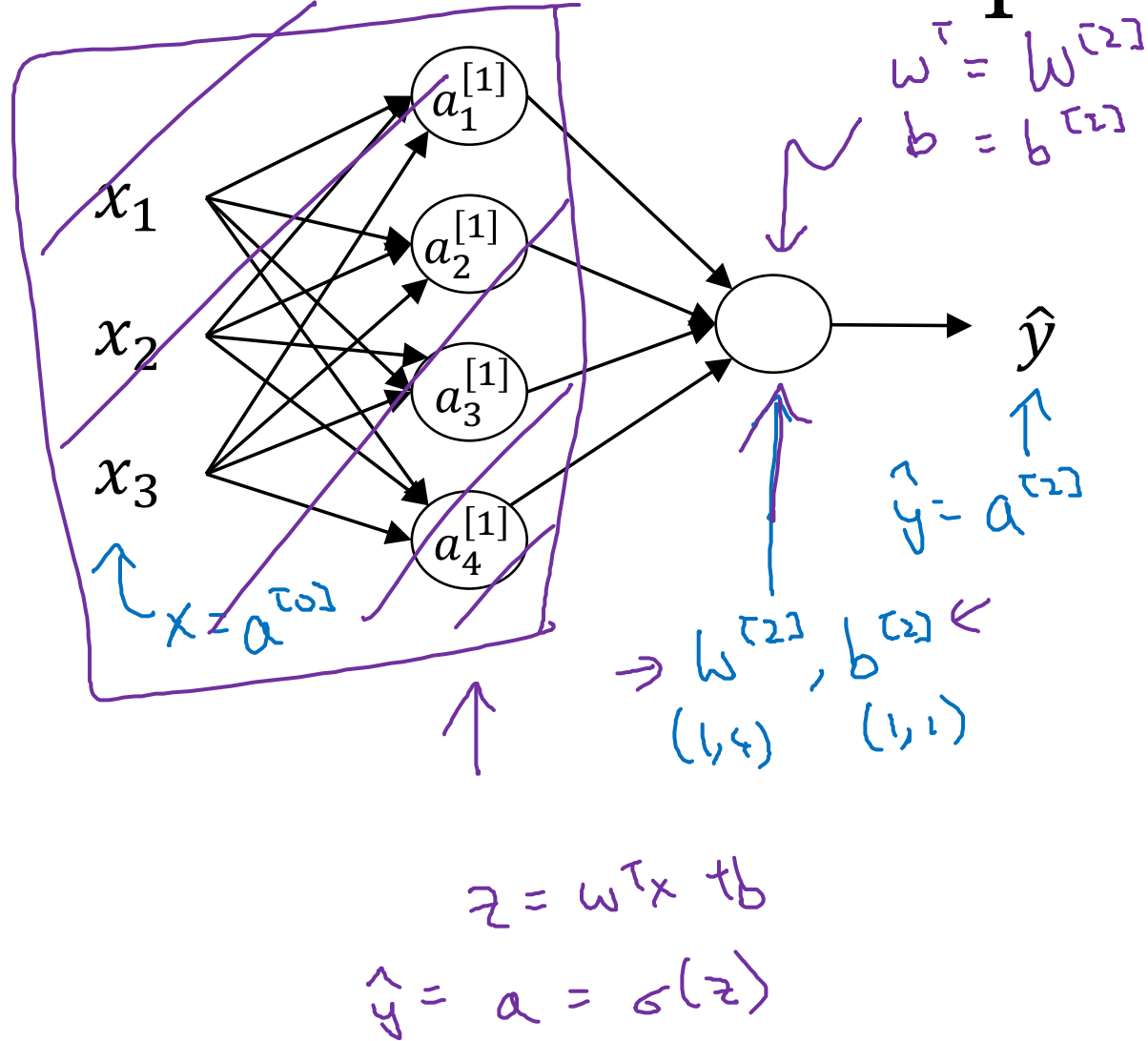
Handwritten notes:  $(w_1^{[1]})^T x$  and  $Q^{[1]}$  are written above the first equation. A red box highlights the activation equations. A blue arrow points from the activation equations down to the vector  $z^{[1]}$ .

$$\begin{aligned}
 &\rightarrow z^{[1]} = \begin{bmatrix} -w_1^{[1]T} \\ -w_2^{[1]T} \\ -w_3^{[1]T} \\ -w_4^{[1]T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix} = \begin{bmatrix} \rightarrow w_1^{[1]T} x + b_1^{[1]} \\ \rightarrow w_2^{[1]T} x + b_2^{[1]} \\ \rightarrow w_3^{[1]T} x + b_3^{[1]} \\ \rightarrow w_4^{[1]T} x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix} \\
 &\rightarrow a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})
 \end{aligned}$$

Handwritten notes:  $(4, 3)$  is written below the weight matrix.  $b^{[1]} (4, 1)$  is written below the bias vector. A red box highlights the activation vector equation.



# Neural Network Representation learning



Given input  $x$ :

$$\begin{aligned} \rightarrow z^{[1]} &= W^{[1]} a^{[0]} + b^{[1]} \\ &\quad (4,1) \quad (4,3) \quad (3,1) \quad (4,1) \\ \rightarrow a^{[1]} &= \sigma(z^{[1]}) \\ &\quad (4,1) \quad (4,1) \\ \rightarrow z^{[2]} &= W^{[2]} a^{[1]} + b^{[2]} \\ &\quad (1,1) \quad (1,4) \quad (4,1) \quad (1,1) \\ \rightarrow a^{[2]} &= \sigma(z^{[2]}) \\ &\quad (1,1) \quad (1,1) \end{aligned}$$



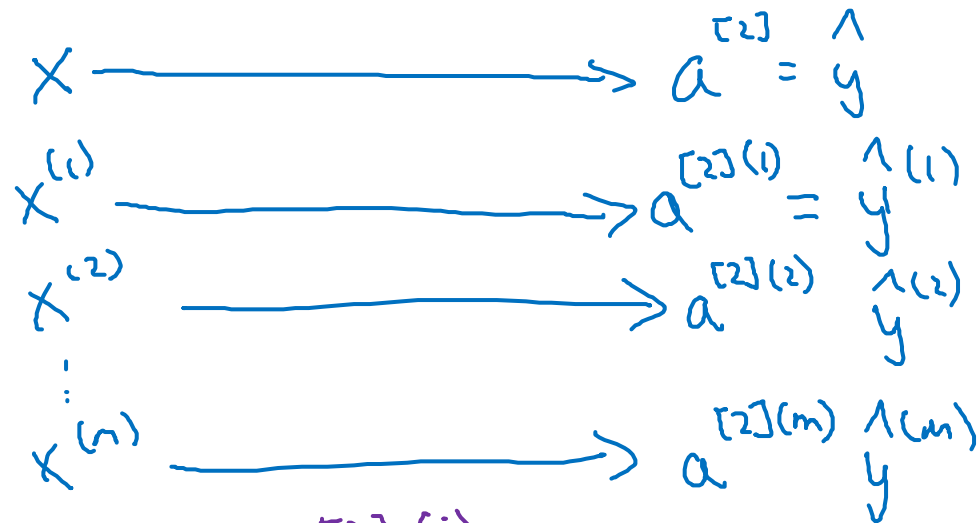
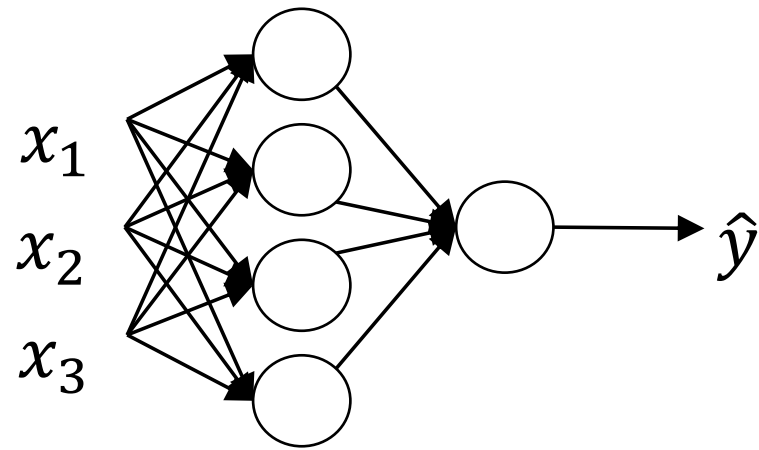
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# One hidden layer Neural Network

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Vectorizing across  
multiple examples

# Vectorizing across multiple examples



$a^{[2](i)}$   
 $\nwarrow$  example  $i$   
 $\swarrow$  layer 2

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \sigma(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma(z^{[2]})$$

for  $i = 1$  to  $m$ ,

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

# Vectorizing across multiple examples

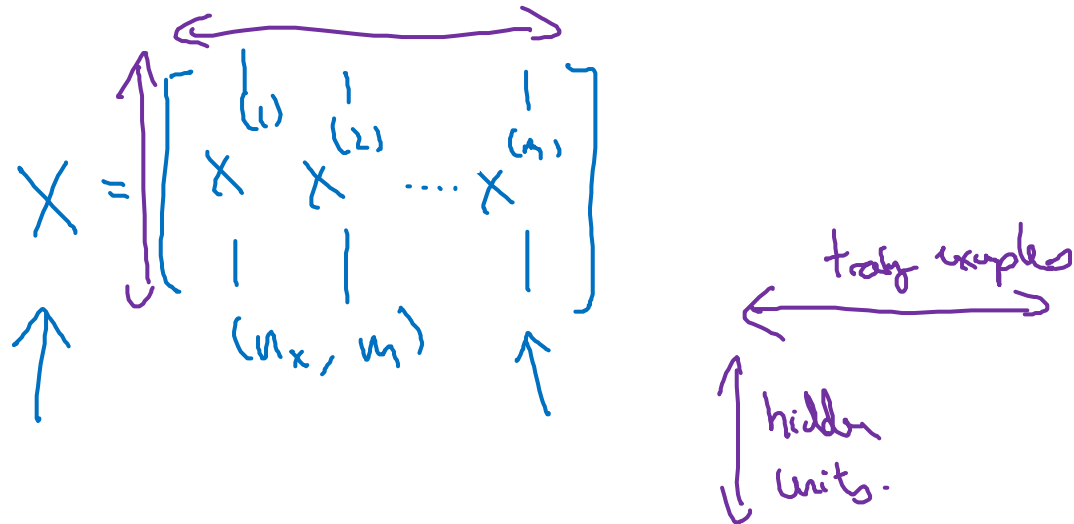
for  $i = 1$  to  $m$ :

$$z^{[1]}(i) = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1]}(i) = \sigma(z^{[1]}(i))$$

$$z^{[2]}(i) = W^{[2]}a^{[1]}(i) + b^{[2]}$$

$$a^{[2]}(i) = \sigma(z^{[2]}(i))$$

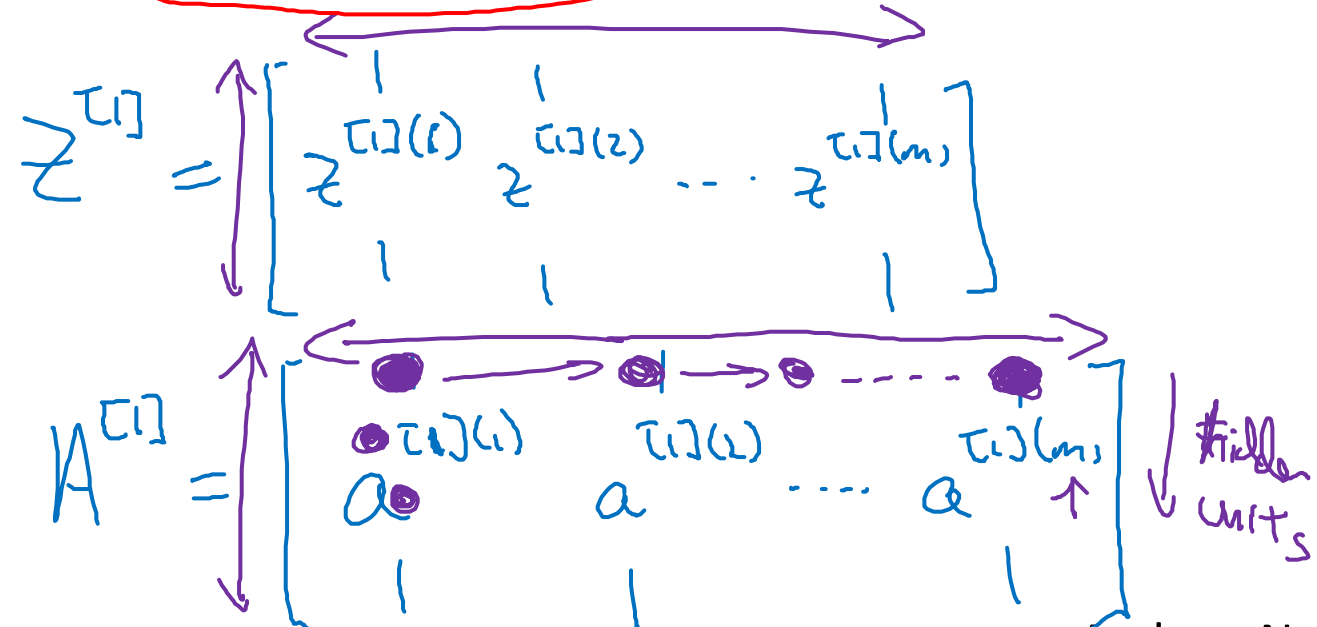


$$z^{[1]} = W^{[1]}X + b^{[1]}$$

$$\rightarrow A^{[1]} = \sigma(z^{[1]})$$

$$\rightarrow z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$\rightarrow A^{[2]} = \sigma(z^{[2]})$$





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# One hidden layer Neural Network

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Explanation  
for vectorized  
implementation

# Justification for vectorized implementation

$$z^{[1](1)} = \underbrace{w^{[1]} \underbrace{x^{(1)}}_{\text{green}}} + \cancel{b^{[1]}} \quad , \quad z^{[1](2)} = \underbrace{w^{[1]} \underbrace{x^{(2)}}_{\text{green}}} + \cancel{b^{[1]}} \quad , \quad z^{[1](3)} = \underbrace{w^{[1]} \underbrace{x^{(3)}}_{\text{yellow}}} + \cancel{b^{[1]}}$$

↑ 0      ↑ 0      ↑ 0

$$w^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \quad w^{[1]} x^{(1)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \quad w^{[1]} x^{(2)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \quad w^{[1]} x^{(3)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix}$$

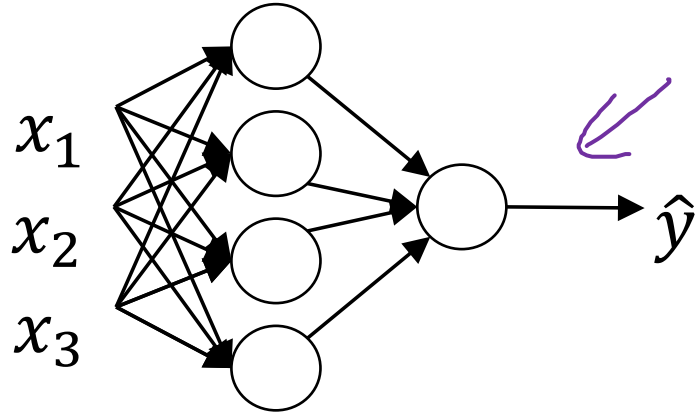
$$w^{[1]} \begin{bmatrix} | & | & | & \dots \\ x^{(1)} & x^{(2)} & x^{(3)} & \dots \\ | & | & | & \dots \end{bmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \end{bmatrix} = \begin{bmatrix} | & | & | & \dots \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \dots \\ | & | & | & \dots \end{bmatrix} = z^{[1]}$$

$\hat{X}$        $w^{[1]} \hat{X} = z^{[1]}$

↑  $+ b^{[1]}$       ↑  $+ b^{[1]}$       ↑  $+ b^{[1]}$

$z^{[1]} = w^{[1]} X + b^{[1]}$

# Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & \dots & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & \dots & | \end{bmatrix}$$

$$\underline{A^{[1]}} = \begin{bmatrix} | & | & \dots & | \\ a^{[1]}(1) & a^{[1]}(2) & \dots & a^{[1]}(m) \\ | & | & \dots & | \end{bmatrix}$$

for  $i = 1$  to  $m$

$$\rightarrow z^{[1]}(i) = W^{[1]}x^{(i)} + b^{[1]}$$

$$\rightarrow a^{[1]}(i) = \sigma(z^{[1]}(i))$$

$$\rightarrow z^{[2]}(i) = W^{[2]}a^{[1]}(i) + b^{[2]}$$

$$\rightarrow a^{[2]}(i) = \sigma(z^{[2]}(i))$$

$$Z^{[1]} = W^{[1]} \underline{X} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

$$x = a^{[0]} \quad x^{(i)} = a^{[0]}(i)$$

$$W^{[1]}A^{[0]} + b^{[1]}$$



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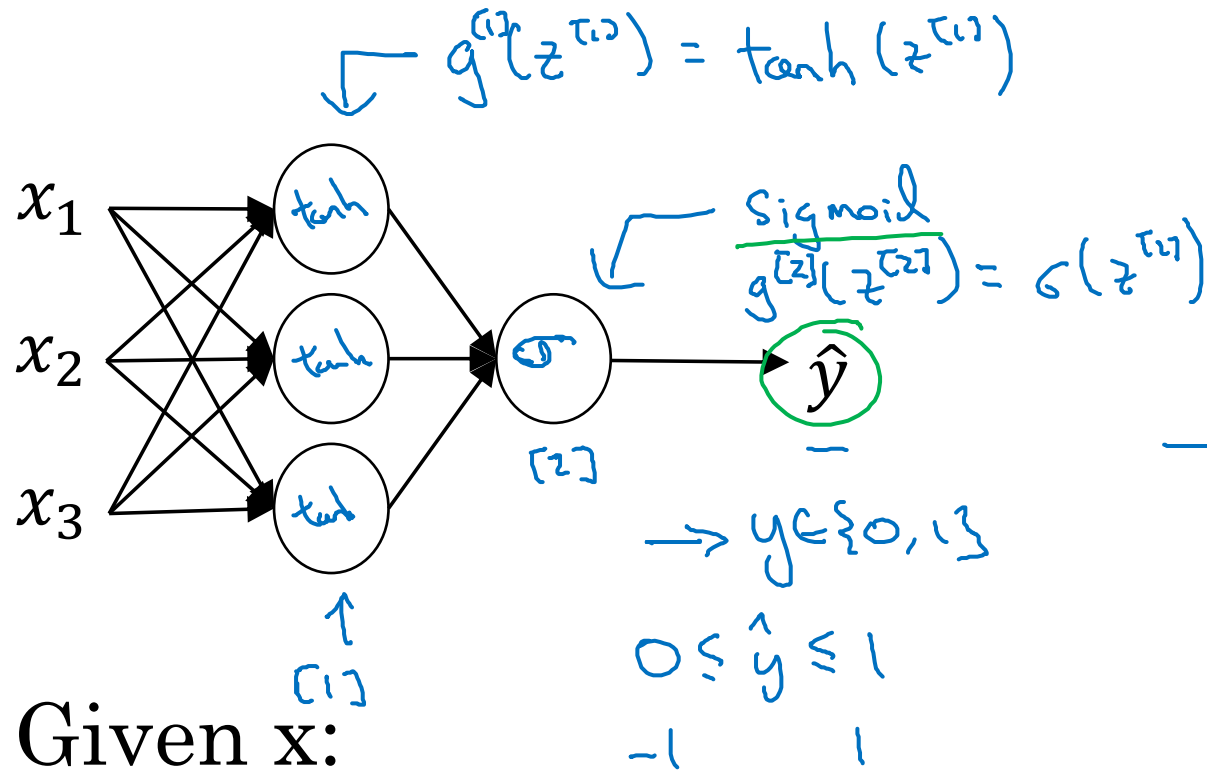
# One hidden layer Neural Network

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## Activation functions



# Activation functions



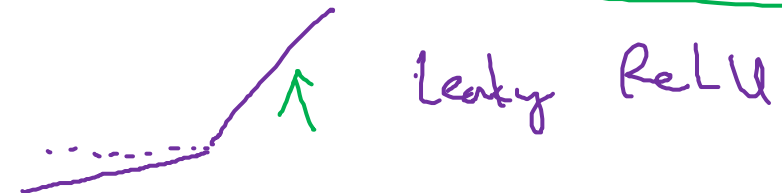
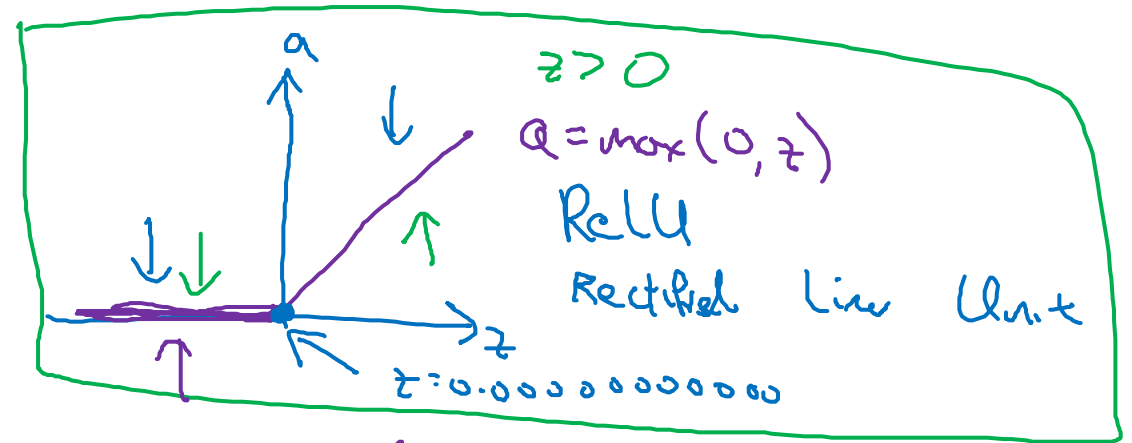
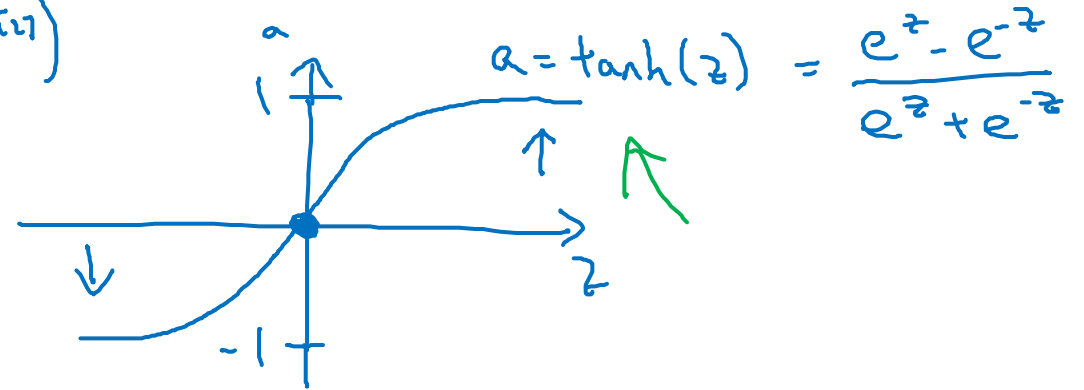
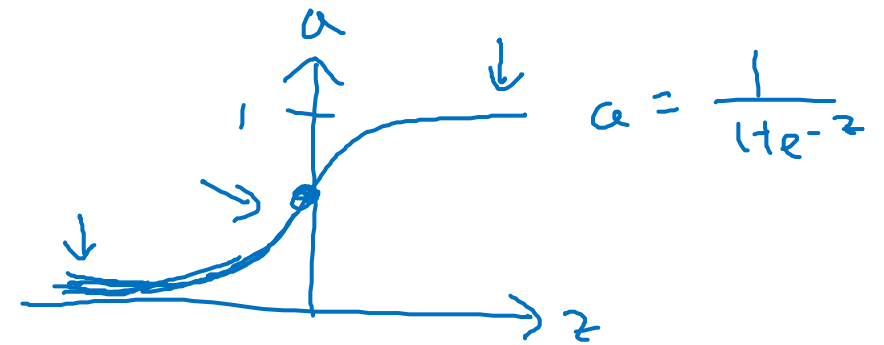
Given  $x$ :

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

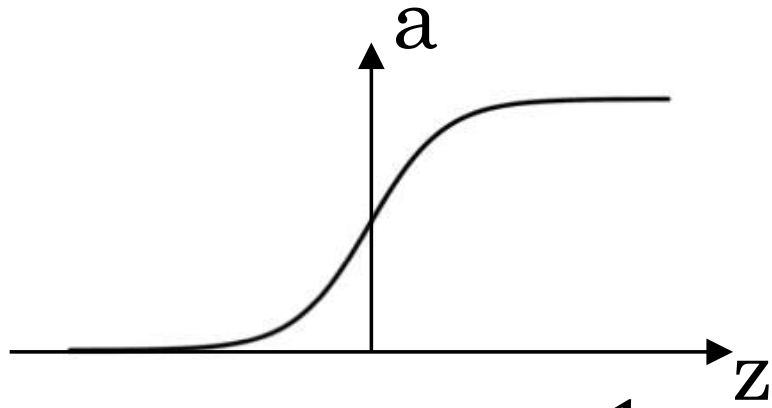
$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})} g^{(1)}(z^{(1)})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

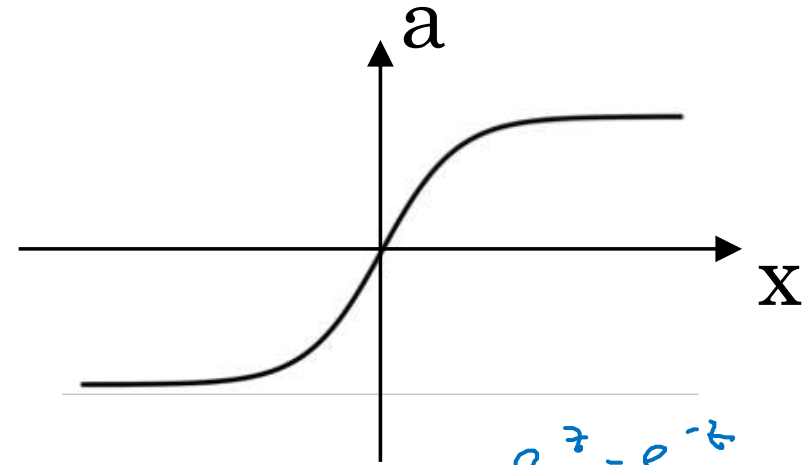
$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})} g^{(2)}(z^{(2)})$$



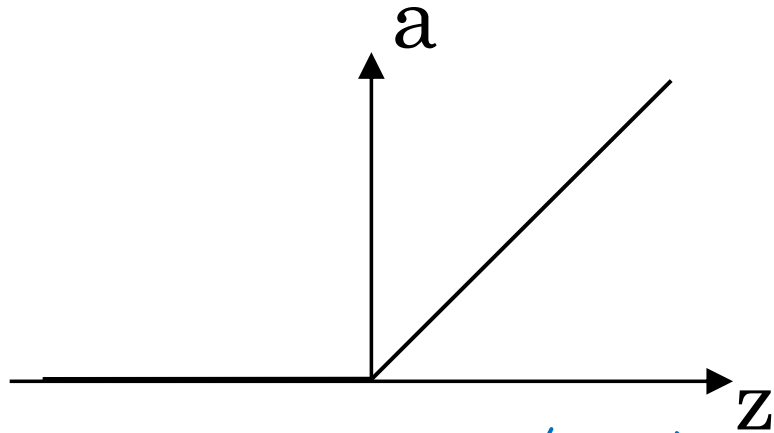
# Pros and cons of activation functions



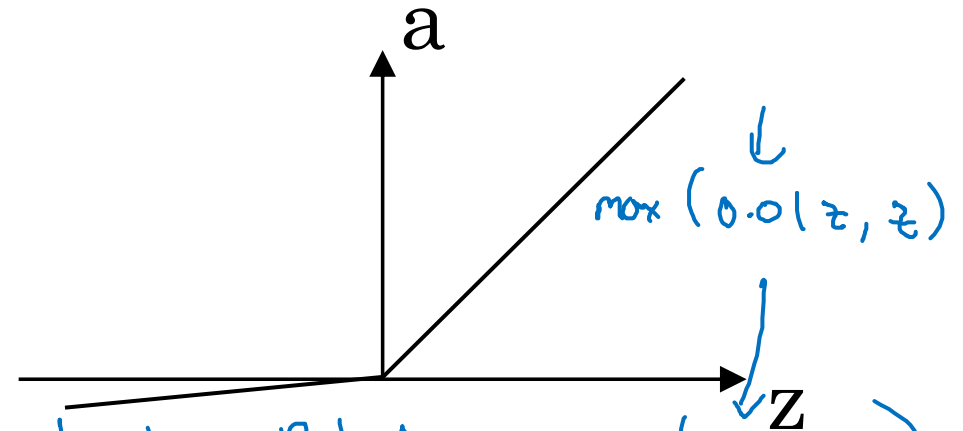
sigmoid:  $a = \frac{1}{1 + e^{-z}}$



tanh:  $a = \frac{e^z - e^{-z}}{e^z + e^{-z}}$



ReLU  $a = \max(0, z)$



Leaky ReLU  $a = \max(0.01z, z)$



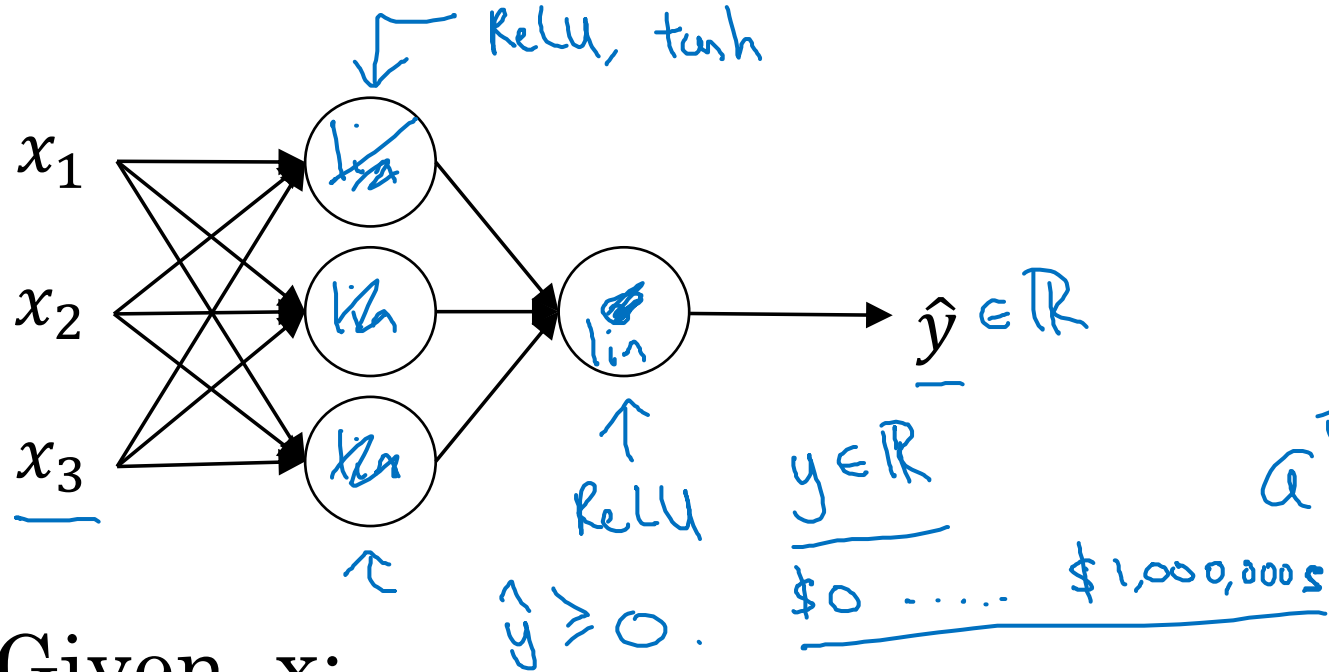
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# One hidden layer Neural Network

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Why do you  
need non-linear  
activation functions?

# Activation function



Given  $x$ :

$$\begin{aligned} \rightarrow z^{[1]} &= W^{[1]}x + b^{[1]} \\ \rightarrow a^{[1]} &= \cancel{g^{[1]}(z^{[1]})} z^{[1]} \\ \rightarrow z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\ \rightarrow a^{[2]} &= \cancel{g^{[2]}(z^{[2]})} z^{[2]} \end{aligned}$$

$g(z) = z$   
"linear activation function"

$$\begin{aligned} a^{[1]} = z^{[1]} &= W^{[1]}x + b^{[1]} \\ a^{[2]} = z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \end{aligned}$$

$$a^{[2]} = W^{[2]} \left( W^{[1]}x + b^{[1]} \right) + b^{[2]}$$

$$\begin{aligned} &= \underbrace{\left( W^{[2]} W^{[1]} \right)}_{w'} x + \underbrace{\left( W^{[2]} b^{[1]} + b^{[2]} \right)}_{b'} \\ &= \underline{w'x + b'} \end{aligned}$$

$$g(z) = z$$



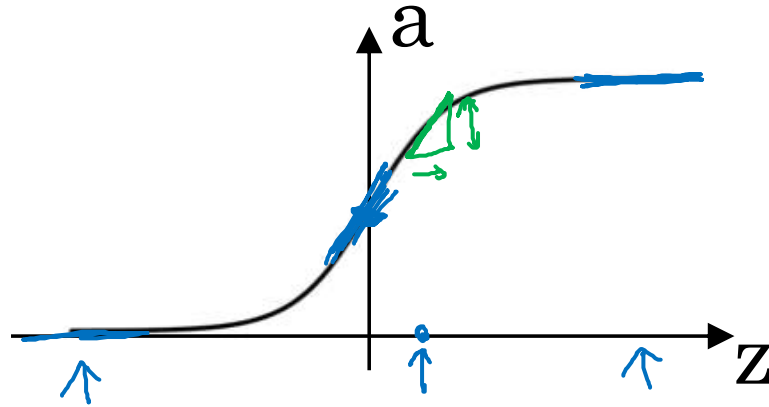
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# One hidden layer Neural Network

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## Derivatives of activation functions

# Sigmoid activation function



$$\underline{g(z) = \frac{1}{1 + e^{-z}}}$$

$$a = g(z) = \frac{1}{1 + e^{-z}}$$

$$\begin{aligned} \boxed{g'(z)} &= \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(z) \text{ at } z \\ &= \frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right) \\ &= g(z) (1 - g(z)) \leftarrow \\ &= \boxed{a(1-a)} \end{aligned}$$

$$g'(z) = a(1-a)$$

↑

$$z = 10. \quad g(z) \approx 1$$

$$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$$

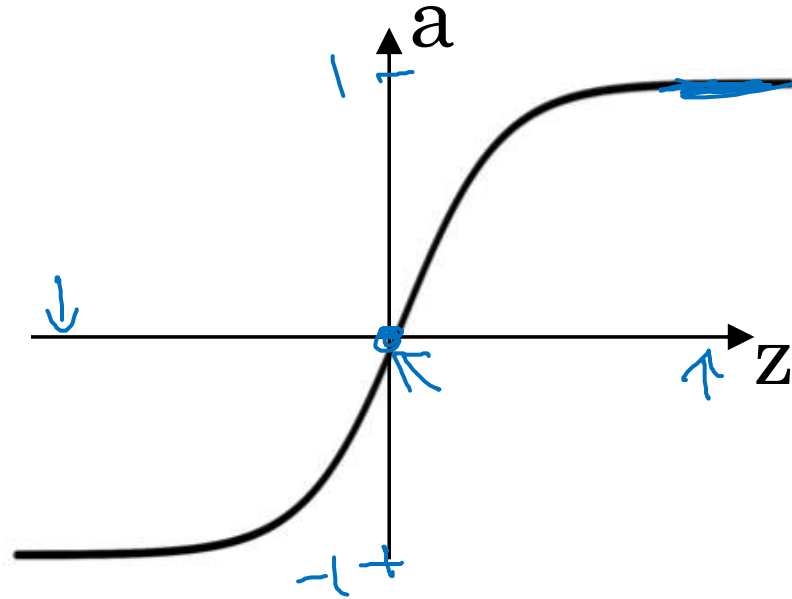
$$z = -10 \quad g(z) \approx 0$$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$$z = 0 \quad g(z) = \frac{1}{2}$$

$$\frac{d}{dz} g(z) = \frac{1}{2} \left( 1 - \frac{1}{2} \right) = \frac{1}{4}$$

# Tanh activation function



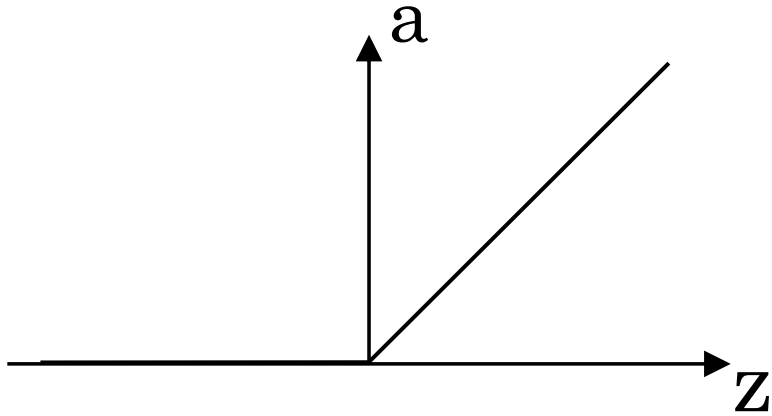
$$g(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z = \frac{1 - (\tanh(z))^2}{1} \leftarrow$$

$$a = g(z), \quad g'(z) = 1 - a^2$$

$$\left| \begin{array}{ll} z=10 & \tanh(z) \approx 1 \\ & g'(z) \approx 0 \\ z=-10 & \tanh(z) \approx -1 \\ & g'(z) \approx 0 \\ z=0 & \tanh(z) = 0 \\ & g'(z) = 1 \end{array} \right.$$

# ReLU and Leaky ReLU



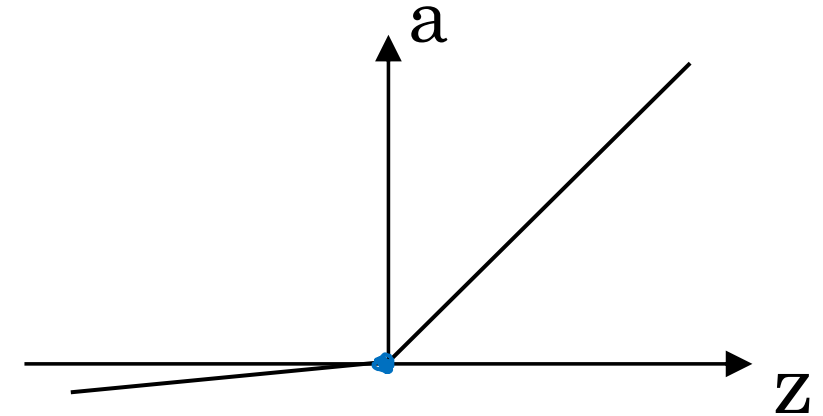
ReLU

$$g(z) = \max(0, z)$$

$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$

~~$undefined$  if  $z = 0$~~

$z = 0.0000...0$



# Leaky ReLU

$$g(z) = \max(0.01z, z)$$
$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$





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One hidden layer  
Neural Network

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Gradient descent for  
neural networks

# Gradient descent for neural networks

Parameters:  $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}$   
 $(n^{[1]}, n^{[0]})$   $(n^{[1]}, 1)$   $(n^{[2]}, n^{[1]})$   $(n^{[2]}, 1)$

$$n_x = n^{[0]}, \quad n^{[1]}, \quad \underline{n^{[2]} = 1}$$

Cost function:  $J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{m} \sum_{i=1}^m \ell(\hat{y}, y)$   
 $\uparrow \quad \uparrow \quad \quad \quad \uparrow a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predictions  $(\hat{y}^{(i)}, i=1, \dots, m)$

$$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \dots$$

$$W^{[1]} := W^{[1]} - \alpha dW^{[1]}$$

$$b^{[1]} := b^{[1]} - \alpha db^{[1]}$$

$$W^{[2]} := \dots \quad b^{[2]} := \dots$$

# Formulas for computing derivatives

Forward propagation:

$$z^{[1]} = w^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]}) \leftarrow$$

$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$$

Back propagation:

$$dz^{[2]} = A^{[2]} - y \leftarrow$$

$$dw^{[2]} = \frac{1}{m} dz^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dz^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$$dz^{[1]} = \underbrace{w^{[2]T} dz^{[2]}}_{(n^{[1]}, m)} \star \underbrace{g^{[1]'}(z^{[1]})}_{\text{element-wise product}} \quad (n^{[1]}, m)$$

$$dw^{[1]} = \frac{1}{m} dz^{[1]} x^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dz^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$\underbrace{\hspace{10em}}_{(n^{[1]}, 1)} \quad (n^{[1]}, ) \quad \text{reshape} \uparrow$

$$Y = [y^{(1)} \ y^{(2)} \ \dots \ y^{(m)}]$$

$$(n^{[2]}) \leftarrow$$

$$\downarrow (n^{[2]}, 1) \leftarrow$$



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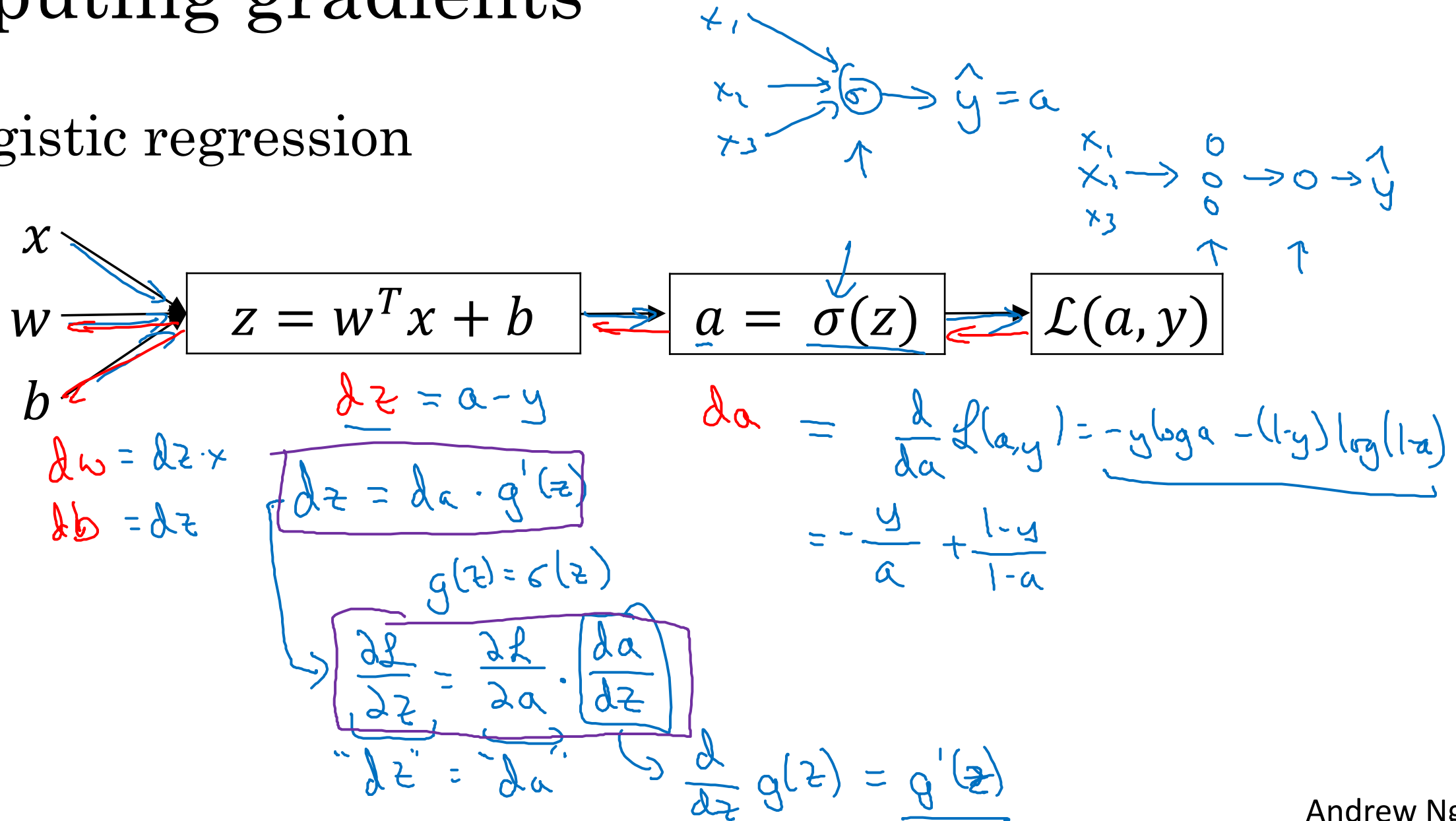
One hidden layer  
Neural Network

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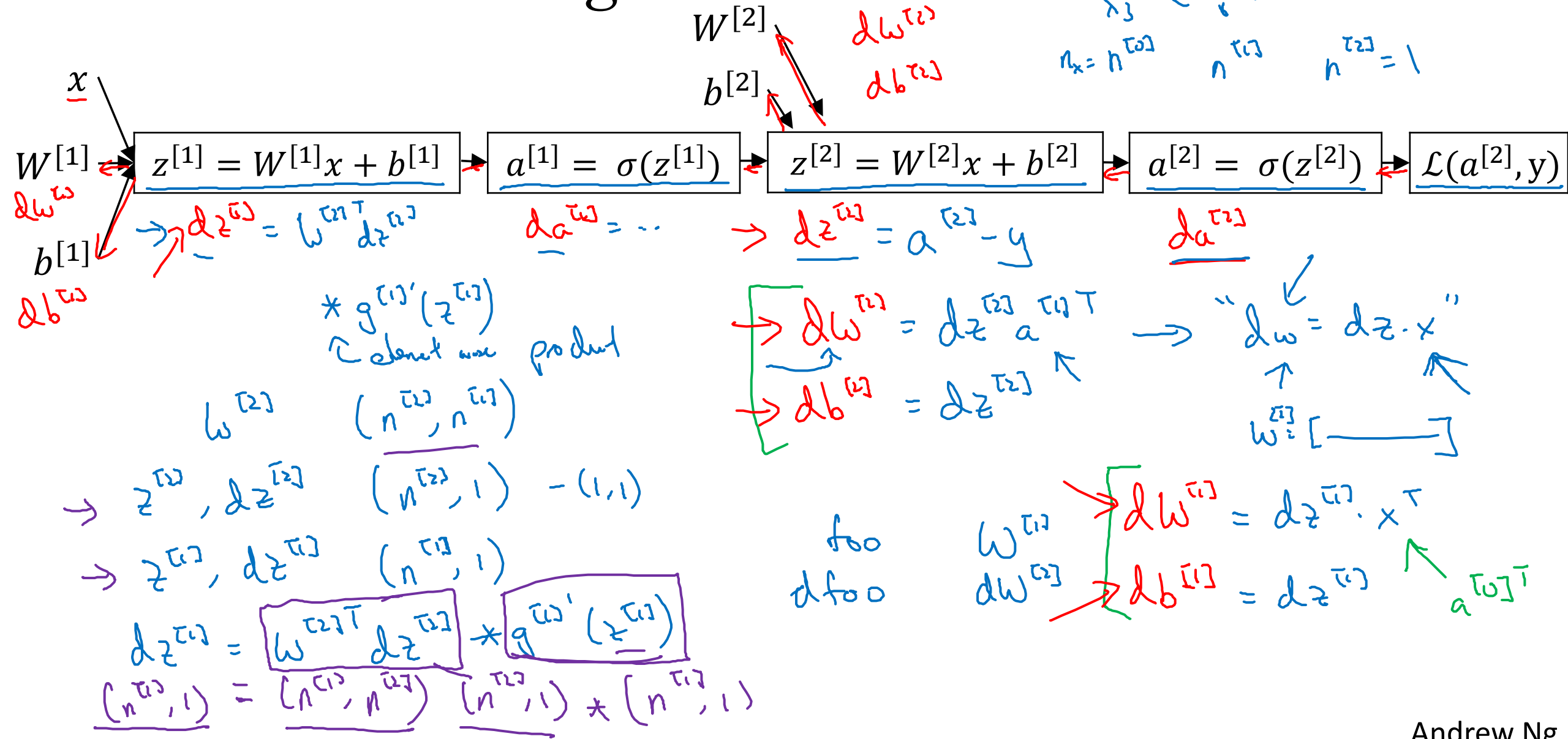
Backpropagation  
intuition (Optional)

# Computing gradients

## Logistic regression



# Neural network gradients



# Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$
$$a^{[1]} = g^{[1]}(z^{[1]})$$
$$z^{[1]} = \begin{bmatrix} z^{[1](1)} \\ z^{[1](2)} \\ \dots \\ z^{[1](n)} \end{bmatrix}$$
$$z^{[1]} = W^{[1]} X + b^{[1]}$$
$$A^{[1]} = g^{[1]}(z^{[1]})$$

# Summary of gradient descent

$$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underbrace{dz^{[1]}}_{(n^{[1]}, 1)} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ^{[2]}} = A^{[2]} - Y$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis} = 1, \text{keepdims} = \text{True})$$

$$\underbrace{dZ^{[1]}}_{(n^{[1]}, m)} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]}, m)}$$

↙ elementwise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis} = 1, \text{keepdims} = \text{True})$$

$$J(\cdot) = \frac{1}{m} \sum_{i=1}^n \mathcal{L}(\hat{y}_i, y_i)$$





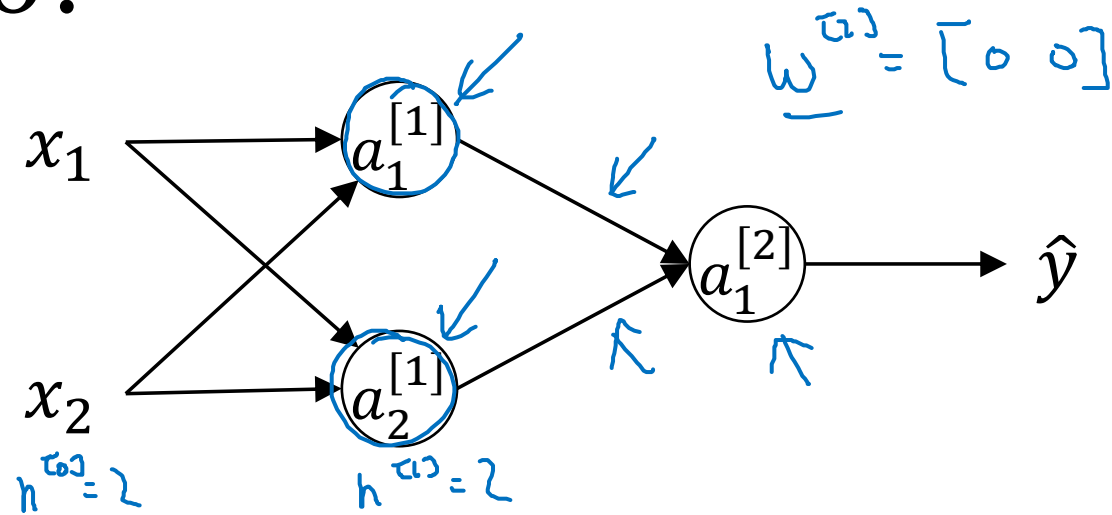
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# One hidden layer Neural Network

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## Random Initialization

# What happens if you initialize weights to zero?



$$w^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]}$$

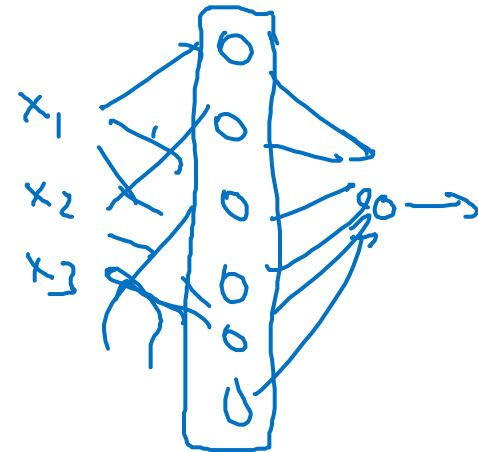
$$\Delta w = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$$

$$b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Delta z_1 = \Delta z_2$$

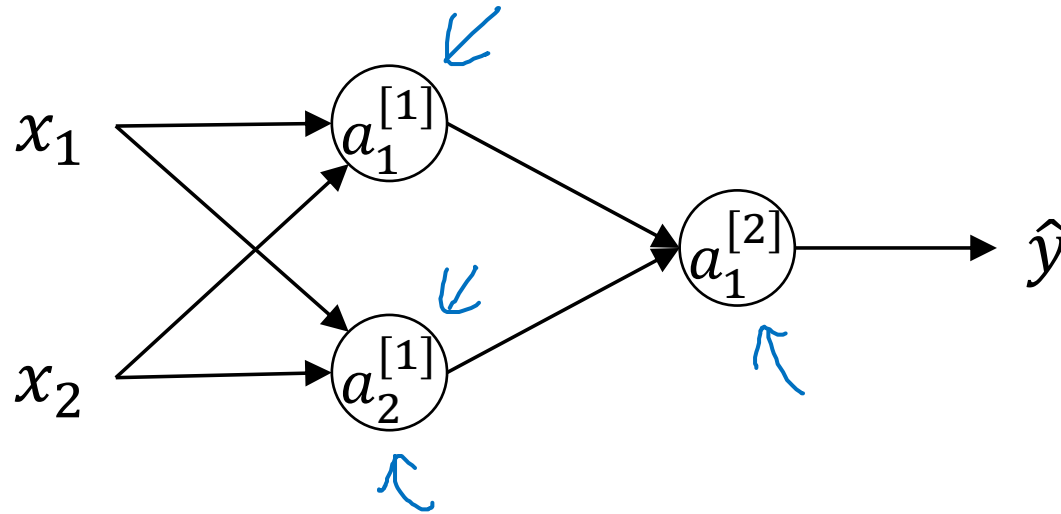
$$w^{[1]} = w^{[1]} - \eta \Delta w$$

Symmetric

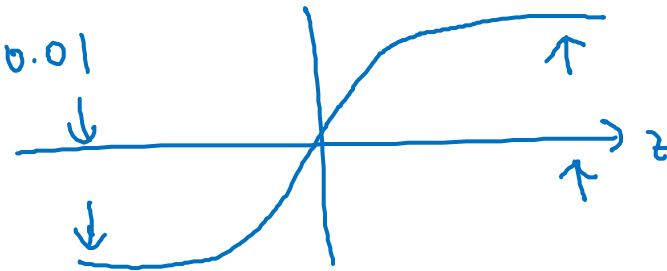


$$w^{[1]} = \begin{bmatrix} \dots & \cdot \\ \dots & \cdot \end{bmatrix}$$

# Random initialization



→  $w^{[1]} = \text{np.random.randn}(2,2) * \frac{0.01}{100?}$   
 $b^{[1]} = \text{np.zeros}(2,1)$   
 $w^{[2]} = \text{np.random.randn}(1,2) * 0.01$   
 $b^{[2]} = 0$



$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = g^{[1]}(z^{[1]})$$