How to compute the seismogram for a signle-force by QSEIS

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1 Brief description

Given a force vector $\mathbf{f}(\mathbf{r})$, we express it in the local Cartesian coordinate system using Aki's convention, where X is northward, Y is eastward, and Z is downward, as

$$\mathbf{f}(\mathbf{r}) = (f_x, f_y, f_z)^T . \tag{1}$$

All Green's functions computed by QSEIS are provided in the ZRT (vertical, radial, transverse) cylindrical coordinate system for a unit force source. In this system, Z is downward, R is radial (pointing outward from the source), and T is transverse (pointing in the direction of increasing azimuth). The azimuth angle θ is defined as the angle measured clockwise from north to east.

Since the Green's functions account only for source/receiver depths and epicentral distance, the radiation pattern must be incorporated during post-processing using azimuthal weighting factors $\alpha(\theta)$. These azimuthal factors are listed in the *Notes* section of the input file and can also be derived from the expressions below.

With these definitions, the displacement in the ZRT coordinate system at a given station can be obtained by summing the contributions of all three force components in Eq. (1) as follows:

$$u_r(t) = g_r^h(t)\, f_x \cos(\theta) + g_r^h(t)\, f_y \sin(\theta) + g_r^z(t)\, f_z, \tag{2}$$

$$u_t(t) = g_t^h(t)\, f_x \sin(\theta) - g_t^h(t)\, f_y \cos(\theta), \tag{3}$$

$$u_z(t) = g_z^h(t)\, f_x \cos(\theta) + g_z^h(t)\, f_y \sin(\theta) + g_z^z(t)\, f_z, \tag{4}$$

where $g_r^h(t), g_r^z(t), g_t^h(t), g_z^h(t)$, and $g_z^z(t)$ are the time-domain Green's functions provided in the QSEIS output files: 'fh.tr', 'fz.tr', 'fh.tt', 'fh.tz', and 'fz.tz', respectively.

Finally, the displacements in the local NEZ system, denoted by $\mathbf{u} = (u_n, u_e, u_z)^T$, can be obtained from the ZRT components via a standard coordinate rotation:

$$\mathbf{u} = \mathbf{R}(\theta)\,(u_r, u_t, u_z)^T, \tag{5}$$

where $\mathbf{R}(\theta)$ is the rotation matrix from ZRT to NEZ coordinates.

2 Example

As an example, we demonstrate the generation of synthetic seismograms using Green's functions for a single-force source. The results are validated by comparison with those from QSSP, as both the single-force input and resulting displacements can be expressed in the ENZ coordinate system.

For the QSSP simulation, we apply a force vector $\mathbf{f}(\mathbf{r}) = (f_e, f_n, f_z)^T = (2, 1, 0)^T$ at the origin, located at a depth of 5 km. The receiver is placed on the Earth's surface at coordinates $\mathbf{r} = (8°\text{E}, 6°\text{N})$, leading to an azimuth of 52.94°.

In the QSEIS setup, the epicentral distance is set to 1110.645 km, calculated using the program *distaz*. Green's functions for a single-force source are computed accordingly, and the resulting seismograms are synthesized using Eqs. (2)–(5). The comparison of results is presented in Figure 1.

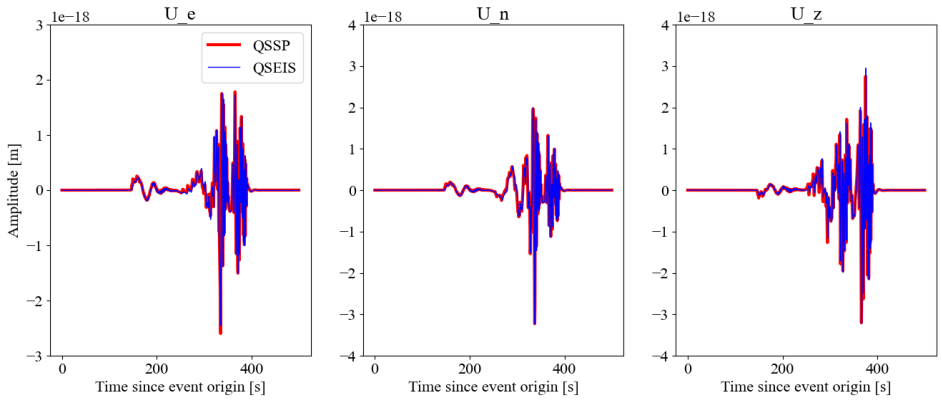

Figure 1: Comparion between seismograms generated by QSSP and QSEIS.