Proof that why unresolved total stellar mass is always less than resolved total stellar mass

By Taylor et al. 2011, we have

$$\log(M_*/L_i) = -0.68 + 0.70(g - i) \tag{1}$$

We rearange to get linear scale

$$M_*/L_i = 10^{-0.68+0.70(g-i)} \tag{2}$$

Now let's say for our galaxy, it acqurie k pixels in image, then for each pixel we have

$$x_n = g_n - i_n \tag{3}$$

for g-i magnitude and L_n for i band luminosity, where $n = 1, 2, \ldots, k$.

And we further express the mass-to-light ratio to be a functin of x, then we have

$$f(x) = 10^{-0.68+0.70x}. \tag{4}$$

Here we also define a weight

$$\omega_n = \frac{L_n}{L_{total}}, \tag{5}$$

which L_{total} is the total luminosity in i band. and it is notmalized:

$$\Sigma_n \omega_n = 1 \tag{6}$$

Then we have the total g-i band magnitude to be

$$x_{total} = \Sigma_n(\omega_n x_n) \tag{7}$$

Now we can express the unresolved total stellar mass as

$$
\begin{aligned}
M_{unresolved} &= L_{total} \cdot f(x_{total}) && (8) \\
&= L_{total} \cdot f(\Sigma_n(\omega_n x_n)) && (9)
\end{aligned}
$$

Also, we can express the unresolved total stellar mass as

$$
\begin{aligned}
M_{resolved} &= \Sigma_n(L_n f(x_n)) && (10) \\
&= \Sigma_n(\omega_n L_{total} f(x_n)) && (11) \\
&= L_{total} \cdot \Sigma_n(\omega_n f(x_n)) && (12)
\end{aligned}
$$

Now, we know that for a real convex function φ, numbers x_1, x_2, ..., x_n is in its domain, and positive weights a_i, Jensen's inequality can be stated as:

$$
\varphi(\frac{\Sigma a_i x_i}{\Sigma a_i}) \leq \frac{\Sigma a_i \varphi(x_i)}{\Sigma a_i} \tag{13}
$$

Here, since we have defined that $\Sigma_n \omega_n = 1$ and $f(x)$ is clearly a convex function, then we must have

$$
f(\Sigma_n(\omega_n x_n)) \leq \Sigma_n(\omega_n f(x_n)) \tag{14}
$$

Therefore, we have

$$
M_{unresolved} = L_{total} \cdot f(\Sigma_n(\omega_n x_n)) \leq L_{total} \cdot \Sigma_n(\omega_n f(x_n)) = M_{resolved} \tag{15}
$$

QED.