Part 2 – Assignment

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This assignment is worth 50 marks in total. Solutions must be written in **R** markdown and submitted *both* as Rmd-file and compiled pdf-file. Use equation environments (`$...$` or `$$...$$`) if you wish to show analytical calculations, and use embedded **R**-code for numerical calculations.

Warm-up [10 marks]

Numerically evaluate the mean and the skewness of the distance between two random points sampled from a uniform unit sphere. The result should have an absolute error below 0.001.

```
Mean = 1.028, Skewness = -0.049 (unit ball)
```

```
Mean = 1.333, Skewness = -0.566 (unit sphere)
```

Permutations [10 marks]

Consider N types of distinct objects. There is exactly 1 object of the 1^{st} type, there are 2 indistinguishable objects of the 2^{nd}, 3 of the 3^{rd}, etc. Let $f(N)$ be the number of *distinct* ways of arranging these objects in an ordered sequence.

a) Evaluate *all* digits of $f(10)$. [5 marks]

```
1906765806522767212441719098019963758016000000
```

b) Evaluate the smallest integer N such that $f(N)$ exceeds $10^{10^{12.2}}$? [5 marks]

```
740195
```

Galactic disk [10 marks]

Consider a self-gravitating flat exponential disk of mass M and scale radius R. Compute the (mass-weighted) line-of-sight velocity moments $\mu_m = \langle (v_{LOS} - \langle v_{LOS} \rangle)^m \rangle$ for $m = 2$, $m = 3$ and $m = 4$, as measured by an observer looking at the galaxy edge-on. Express the solutions in terms of a numerical constant, M, R and the gravitational constant G. For full marks, numerical values must be correct to six significant digits.

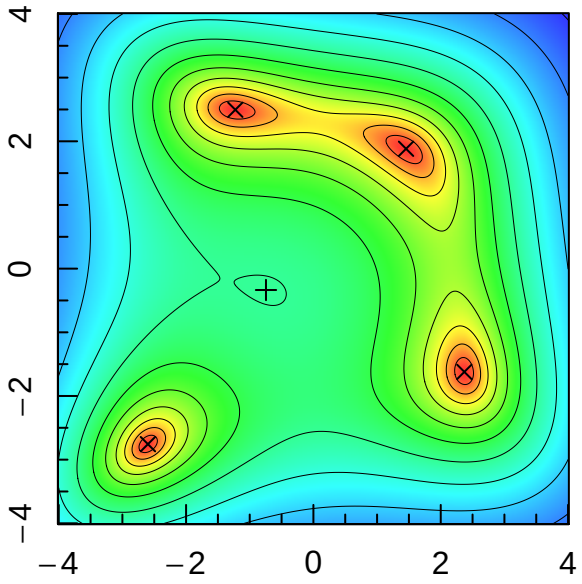
```
Moment(2) = 0.147262 [GM/R]^1.0
Moment(3) = 0.000000 [GM/R]^1.5
Moment(4) = 0.035971 [GM/R]^2.0
```

Himmelblau's function [10 marks]

Numerically evaluate the locations (x, y) of *all* the minima and maxima of the modified Himmelblau's function, $f(x, y) = (x^2 + y - 4)^2 + (x + y^2 - 5)^2$. To get full points, your results must be correct to at least *three* significant digits. Also show a contour plot with all extrema marked by crosses.

```
Minimum 1 at (-2.599,-2.757) with value  0.000
Minimum 2 at (-1.227,+2.495) with value  0.000
Minimum 3 at (+1.455,+1.883) with value  0.000
Minimum 4 at (+2.371,-1.621) with value  0.000
```

```
Maximum 1 at (-0.745,-0.336) with value 46.016
```



Monte Carlo integration [10 marks]

Compute the gravitational binding energy of a solid cube of side length L and mass density $\rho(r) = \rho_0 \exp(-r^3/L^3)$, where r is the distance from the centre of the cube. Express the solution in terms of a numerical constant, L, ρ_0 and G. The numerical part should be correct to three significant digits.

```
U = (-0.7392+-0.0001)*G*L^5*rh0^2
```