

q3

```
library('invgamma')
set.seed(440)
x = rweibull(24,0.5,64)
loglik <- function(shape = NULL, scale = NULL, x = NULL){
  val = matrix(0, nrow = length(shape), ncol = length(scale))
  for (i in 1:length(shape)){
    for (j in 1:length(scale)){
      val[i,j] = sum(dweibull(x, shape = shape[i], scale = scale[j], log = TRUE))
    }
  }
  return (val)
}

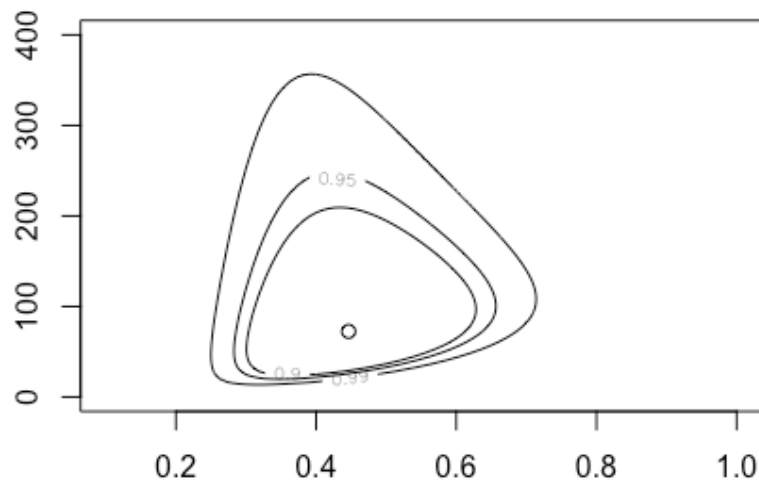
loglik2 <- function(p = NULL, datax = NULL){
  val = sum(dweibull(datax, shape = p[1], scale = p[2], log = TRUE))
  return (val)}

temp = optim(par = c(1,1), loglik2, datax = x, control = list(fnscale = -1))
## Warning in dweibull(datax, shape = p[1], scale = p[2], log = TRUE): NaNs
## produced

## Warning in dweibull(datax, shape = p[1], scale = p[2], log = TRUE): NaNs
## produced
mle = temp$value
mle_shape = temp$par[1]
mle_scale = temp$par[2]
s1 = seq(0.1,1,length.out = 100)
s2 = seq(0.1,400,length.out = 100)
r = loglik(shape = s1, scale = s2, x = x)

contour(x = s1, y = s2, z = -2*(r-mle), levels = qchisq(c(0.9,0.95,0.99), df = 2), labels =
c('0.9','0.95','0.99' ))

points(x = mle_shape, y = mle_scale)
```



```
posterior <- function(shape = NULL, scale = NULL, datax = NULL){
  if (shape <= 0 | scale <= 0){r = 0}
  else {l = length(datax)
    r = shape^l / scale^l * prod(datax^shape - 1) / scale^l * (shape - 1)^l * exp(-
```

```

1 = shape / scale - 1 prod(datax / (shape-1)) / scale - 1 (shape-1)) exp(-
sum(datax^shape)/scale^shape)}

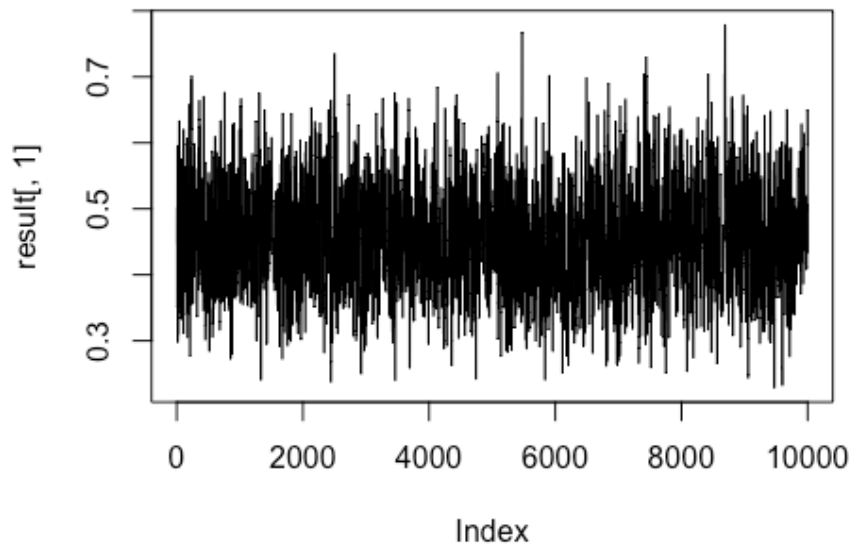
return(r)
}

random_walk <- function(n = NULL, xx=c(0,0), sigma = c(0,0)){
  m = matrix(0, nrow = n, ncol = 2)
  m[1,] = xx
  for (i in 2:n){
    val = rnorm(2, m[i-1,], sd = sigma)
    val2 = runif(1)
    accept = posterior(shape = val[1], scale = val[2], datax = x)/posterior(shape = m[i-1,1], scale =
m[i-1,2], datax = x)

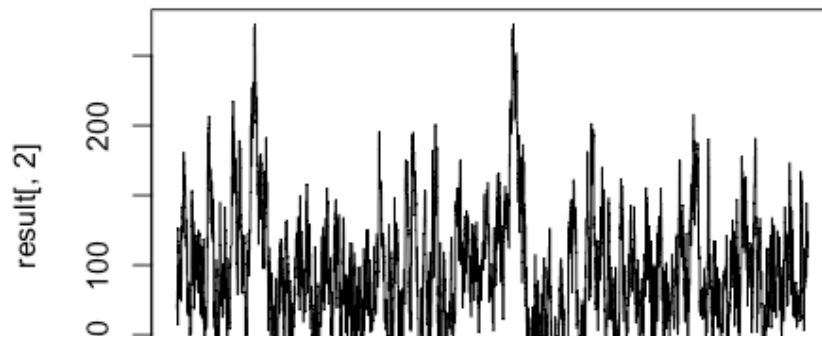
    if (val2 < accept){m[i,] = val}
    else {
      m[i,] = m[i-1,]
    }
  }
  return (m)
}

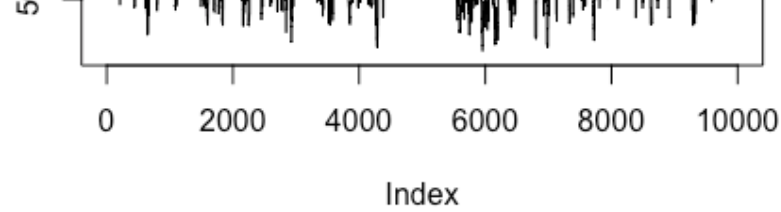
set.seed(440)
n = 10^4
result = random_walk(n = n, xx = c(0.446,72.439),sigma = c(0.2,15))
mean(result[-1,1] != result[-n,1])
## [1] 0.3862386
mean(result[-1,2] != result[-n,2])
## [1] 0.3862386
plot(result[,1], type = 'l')

```



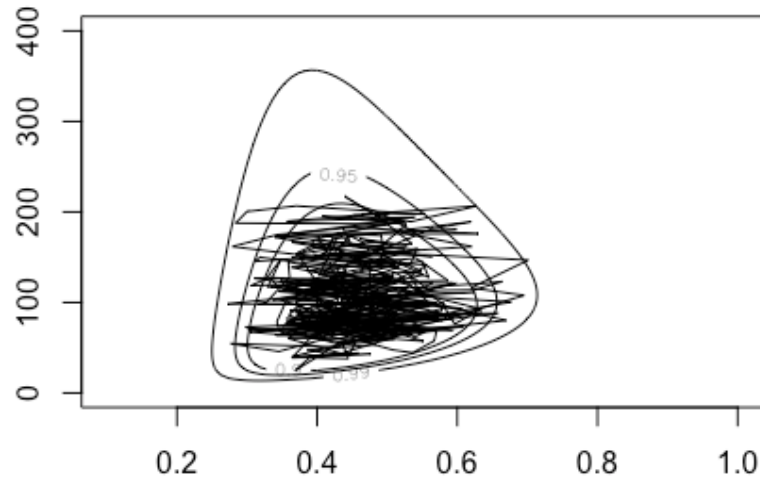
```
plot(result[,2], type = 'l')
```





```
contour(x = s1, y = s2, z = -2*(r-mle), levels = qchisq(c(0.9, 0.95, 0.99), df = 2), labels = c('0.9',
'0.95', '0.99'))
```

```
lines(result[1:1000,],type = 'l')
```



Since $p(\cdot | x) \propto \text{inv}(n-1, x_i^\alpha)$ we have $p(l, x) \propto$ which follows $\text{inv}(n-1, x_i^\alpha)$ w.r.t x_i^α

```
alpha_est = 0.44029
```

```
prob <- function(alpha = NULL, datax = NULL){
  if (alpha <= 0 )
    return(-Inf)
  else{ n = length(datax)
    result = n*log(alpha)+(alpha-1)*sum(log(data))-n*alpha*log(64)-sum(data^alpha)/(64^alpha)

    return (result)}
}

f <- function(n=NULL, xx = c(0,0)){
  m = matrix(0, nrow = n, ncol = 2)
  m[1,] = xx

  for (i in 2:n){
    if (i %% 2 == 0){
      val = rnorm(1, mean = alpha_est, sd = sqrt(1/(24/alpha_est^2+sum((x/64)^alpha_est*
(log(x/64))^2))))
      val2 = runif(1)

      accept = posterior(shape = val, scale = m[i-1,2], datax = x)/posterior(shape = m[i-1,], scale =
m[i-1,2], datax = x)*dnorm(m[i-1,], mean = alpha_est, sd =
sqrt(1/(24/alpha_est^2+sum((x/64)^alpha_est*(log(x/64))^2))))/dnorm(val, mean = alpha_est, sd
= sqrt(1/(24/alpha_est^2+sum((x/64)^alpha_est*(log(x/64))^2))))

      if (val2 < accept) m[i,] = c(val, m[i-1,2])
      else m[i,] = m[i-1,]
    }

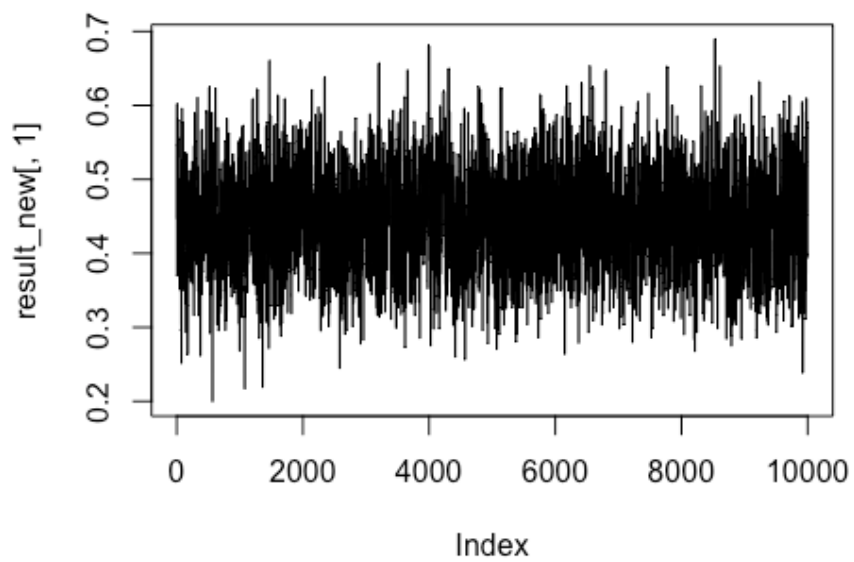
    else {
      val_new = (gamma(1, shape=23, rate=sum(x^alpha_i-1,1))^(1/m[i-1,1]))
```

```

val_new = (rmvgamma(1, shape=25, rate=sum(x ~ m[i-1,1]))) * (1/m[i-1,1])
val2_new = runif(1)
accept = 1
if (val2_new < accept){
  m[i,] = c(m[i-1,1], val_new)}
else m[i,] = m[i-1,]

}
}
return (m)
}
set.seed(440)
num = 10000
result_new = f(n = num, xx = c(0.446, 72.439))
mean(result_new[-1,1] != result_new[-num, 1])
## [1] 0.4168417
mean(result_new[-1,2] != result_new[-num, 2])
## [1] 0.49995
plot(result_new[,1], type = 'l')

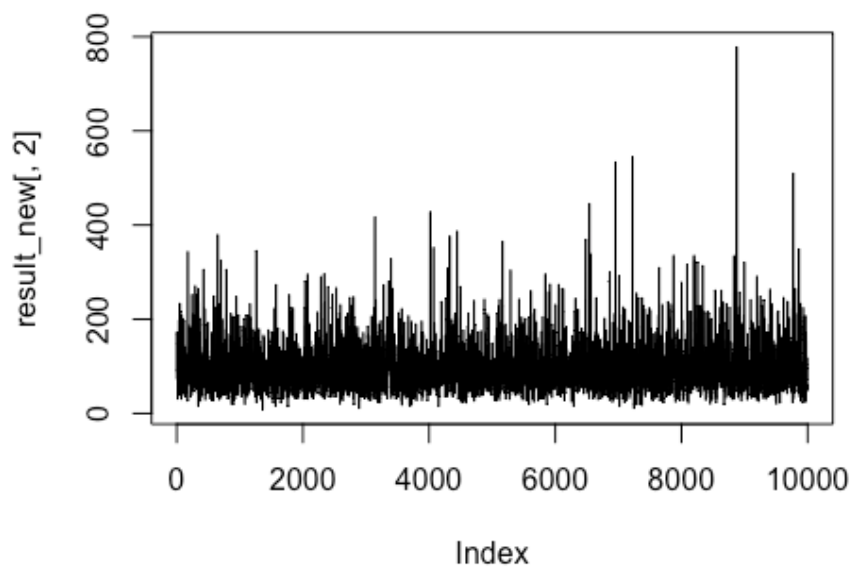
```



```

plot(result_new[,2], type = 'l')

```



```

contour(x = s1, y = s2, z = 2*(x - m1), levels = c(0.0, 0.05, 0.99), df=2), labels = c("0.0",

```

```
contour(x = s1, y = s2, z = -2*(1-line), levels = qchisq(c(0.9,0.95,0.99), df=2), labels = c('0.9',  
'0.95', '0.99'))
```

```
lines(result_new[1:1000,], type = 'l')
```

