STAT 440/840 - CM 761 - Assignment 1 - Spring 2019

Name

Due: Tuesday, June 4 at 9:00am on Crowdmark

Assignment solutions need to be submitted using `R Markdown` from `RStudio` or `LaTeX` if you prefer. **R Markdown is strongly recommended.** This file is an `R Markdown` document.

1. The Weibull distribution with shape parameter $\alpha > 0$ and scale parameter $\sigma > 0$ is given by

$$f(x|\lambda, \alpha) = \begin{cases} \frac{\alpha}{\sigma}\left(\frac{x}{\sigma}\right)^{\alpha-1} e^{-(x/\sigma)^{\alpha}} & x \geq 0, \\ 0 & x < 0, \end{cases}$$

a) Derive the cdf for the Weibull distribution.

b) Derive the quantile function for the exponential distribution. i.e. find $q(p|\alpha, \sigma)$ where $p \in (0, 1)$.

c) For a given α, find an expresion for the MLE, $\widehat{\sigma}$, and derive the observed and fisher information.

d) To estimate the median using ML we use

$$q(p = 0.5|\alpha, \widehat{\sigma})$$

derive the fisher information for this quantity.

e) Using a simulation study with $m = 1000$ replications to compare the MLE and X_{median} as estimators for the median value of the Weibull distribution when $\sigma = 1/2, 1, 2$, $n = 10, 20, 50$ and $\alpha = 3$. Summarize the simulation by

- comparing the bias and standard deviation of the estimators,
- combine bias and standard deviation into a single metric called the square root mean square error,

$$MSE(\widetilde{\theta}) = \sqrt{(\widetilde{\theta} - \theta)^2 + \mathbb{V}\text{ar}(\widetilde{\theta})}$$

- use tables and plots to present the results,
- and provide some comments on the results.

2. The file "eng-monthly-011942-112007.csv" contains monthly weather data from Yellowknife, NT over the years 1959 to 1996. Here are considering the maximum wind gust.

```r
temp = read.csv( "eng-monthly-011942-112007.csv" )
speed = temp$Spd.of.Max.Gust..km.h.
```

The Weibull distribution with shape parameter $k = \alpha > 0$ and scale parameter $\lambda = \sigma > 0$ is given by

$$f(x|\lambda, \alpha) = \begin{cases} \frac{\alpha}{\sigma}\left(\frac{x}{\sigma}\right)^{\alpha-1} e^{-(x/\sigma)^{\alpha}} & x \geq 0, \\ 0 & x < 0, \end{cases}$$

In this question we assume the scale parameter is known and equal to 64, i.e. $\sigma = 64$. We are interested in estimating the unknown shape parameter, α.

a) State and plot the log-likelihood.

b) Derive the Score and Observed information Matrix.

c) Derive a newton and raphson algorithm to find the MLE and using the starting value of $\alpha^{(0)} = 12$ give the next five iterations.

d) Construct a confidence interval for α using the

- the Wald statistic normality of the MLE, and
- the log-likelihood ratio statistc.