Q1.

X_t is ARMA(1,1) process:

$\phi_1(B)X_t = \theta_1(B)Z_t$

$X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$

$X_t = Z_t + \theta_1 Z_{t-1} + \phi_1 X_{t-1}$

$X_t = Z_t + \theta_1 Z_{t-1} + \phi_1(Z_{t-1} + \theta_1 Z_{t-2} + \phi_1 X_{t-2})$

$X_t = Z_t + \theta_1 Z_{t-1} + \phi_1 Z_{t-1} + \phi_1\theta_1 Z_{t-2} + \phi_1\phi_1 X_{t-2}$

$X_t = Z_t + (\theta_1 + \phi_1)Z_{t-1} + \phi_1\theta_1 Z_{t-2} + \phi_1\phi_1 X_{t-2}$

$X_t = Z_t + (\theta_1 + \phi_1)Z_{t-1} + \phi_1\theta_1 Z_{t-2} + \phi_1\phi_1(Z_{t-2} + \theta_1 Z_{t-3} + \phi_1 X_{t-3})$

$X_t = Z_t + (\theta_1 + \phi_1)Z_{t-1} + \phi_1\theta_1 Z_{t-2} + \phi_1\phi_1 Z_{t-2} + \phi_1\phi_1\theta_1 Z_{t-3} + \phi_1\phi_1\phi_1 X_{t-3}$

$X_t = Z_t + (\theta_1 + \phi_1)Z_{t-1} + \phi_1(\theta_1 + \phi_1)Z_{t-2} + \phi_1\phi_1\theta_1 Z_{t-3} + \phi_1\phi_1\phi_1 X_{t-3}$

$\varphi_1 = \theta_1 + \phi_1$

$\varphi_2 = \phi_1(\theta_1 + \phi_1)$

Another approach:

$\frac{1+\theta_1 Z}{1-\phi_1 Z} = 1 + \varphi_1 Z + \varphi_2 Z^2 + ...$

$(1 + \theta_1 Z) = (1 - \phi_1 Z)(1 + \varphi_1 Z + \varphi_2 Z^2 + ...)$

Coefficient for Z:
$\theta_1 = \varphi_1 - \phi_1$
$\varphi_1 = \theta_1 + \phi_1$

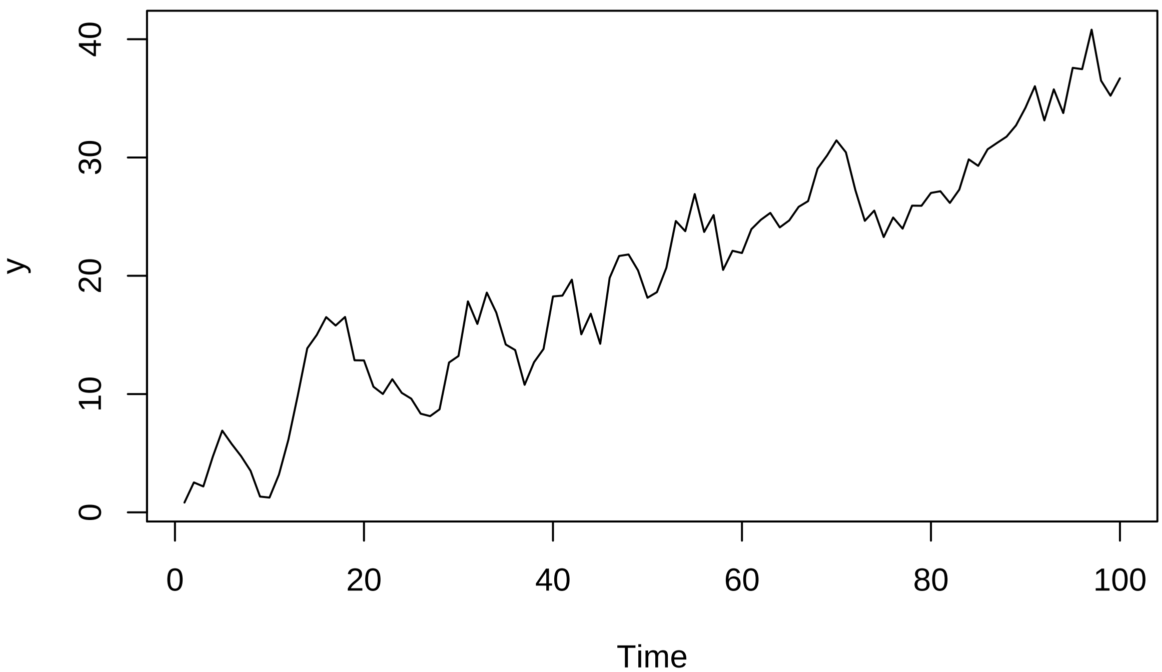
Coefficient for Z^2 :
$0 = \varphi_2 - \phi_1\varphi_1$
$\varphi_2 = \phi_1(\theta_1 + \phi_1)$

A4_q2.R

y563li

Mon Nov 20 09:40:06 2017

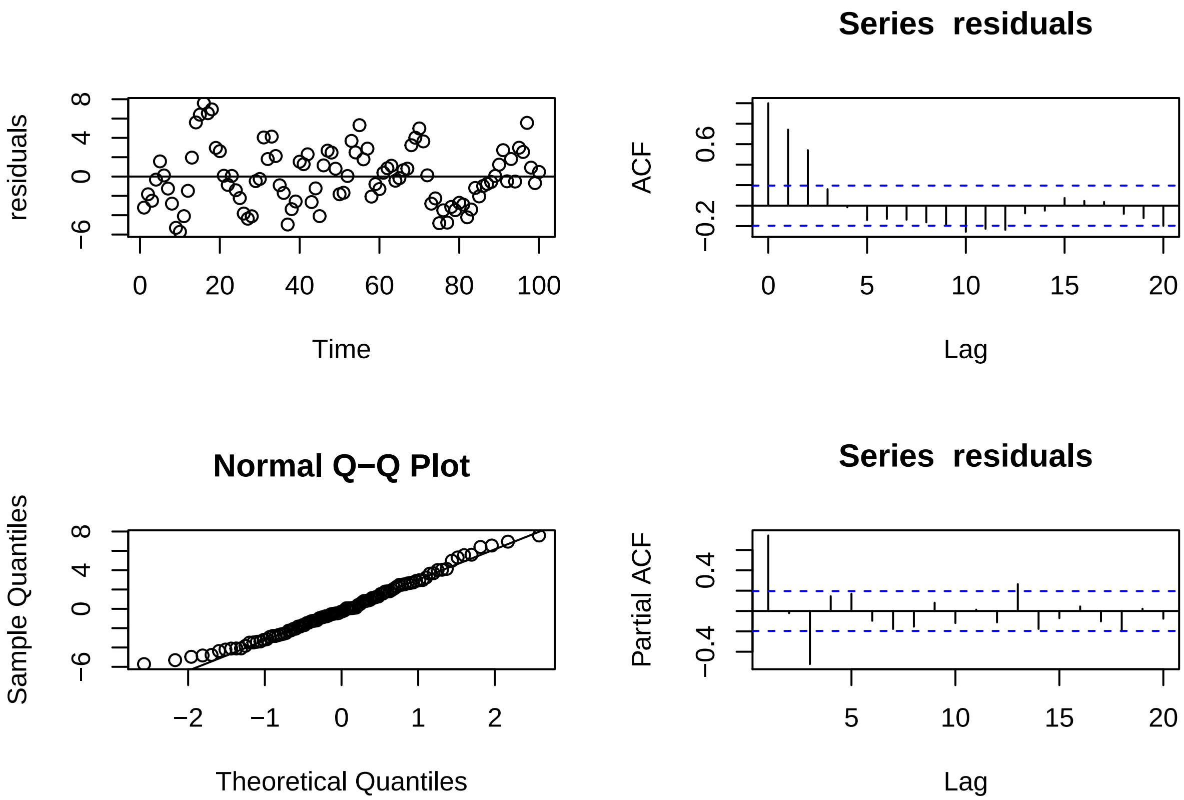
```
#Q2
y<-scan("/Users/y563li/Downloads/linear_y.txt")
plot(y,type='l',xlab="Time")
```




```
x<-c(1:length(y))
model1=lsfit(x,y)

residuals=model1$residuals
par(mfcol=c(2,2))
plot(residuals,xlab="Time")
abline(a=0,b=0)
qqnorm(residuals)
qqline(residuals)

acf(residuals)
pacf(residuals)
```




```r
#The residuals seem resonably normal and identically distributed.In
#ACF plot, there is some correlation and it kind of looks like AR(1).
#but there are spikes at t=1 and t=2 in ACF, I would like to try ARMA(1,2).

#ACF
ar_1=arima(y,order=c(1,0,0),xreg=x,method="ML")
ar_1
```

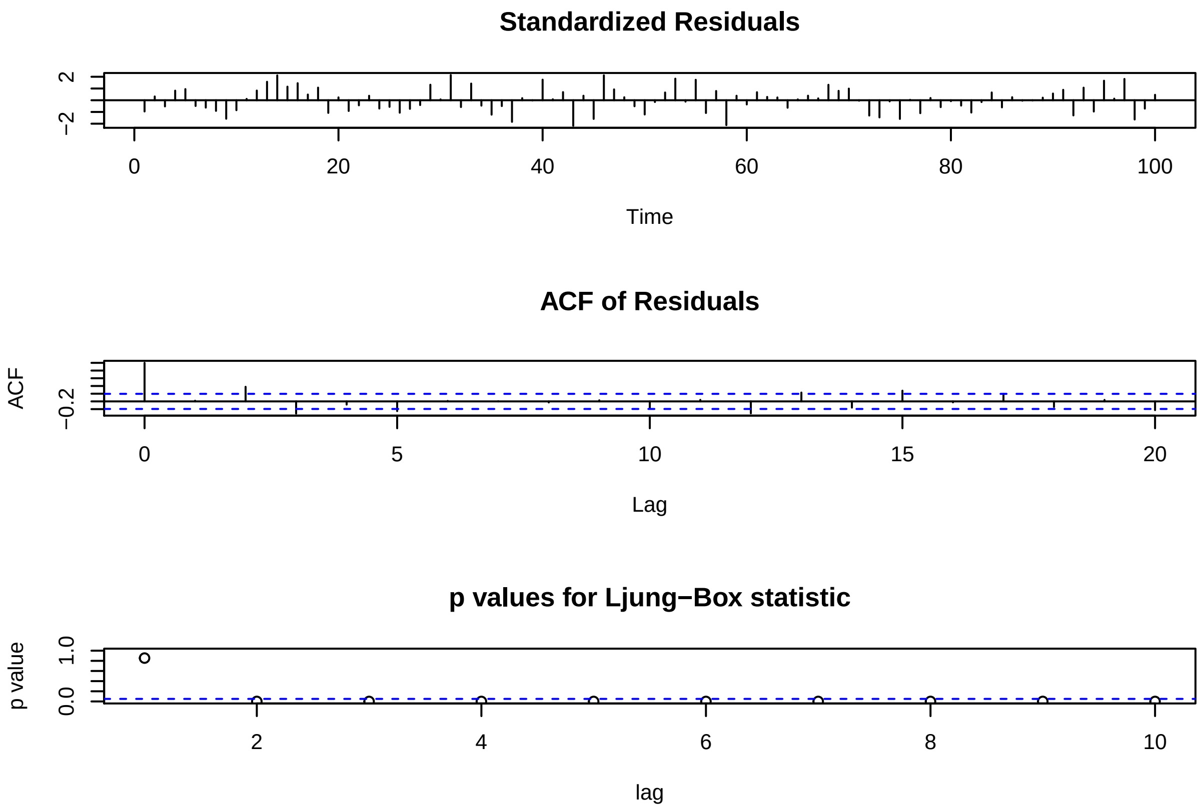
```
##
## Call:
## arima(x = y, order = c(1, 0, 0), xreg = x, method = "ML")
##
## Coefficients:
##          ar1  intercept       x
##       0.7441     3.3476  0.3309
## s.e.  0.0657     1.4651  0.0248
##
## sigma^2 estimated as 3.959:  log likelihood = -211.1,  aic = 430.19
```

```r
arma_1_2=arima(y,order=c(1,0,2),xreg=x,method="ML")
arma_1_2
```

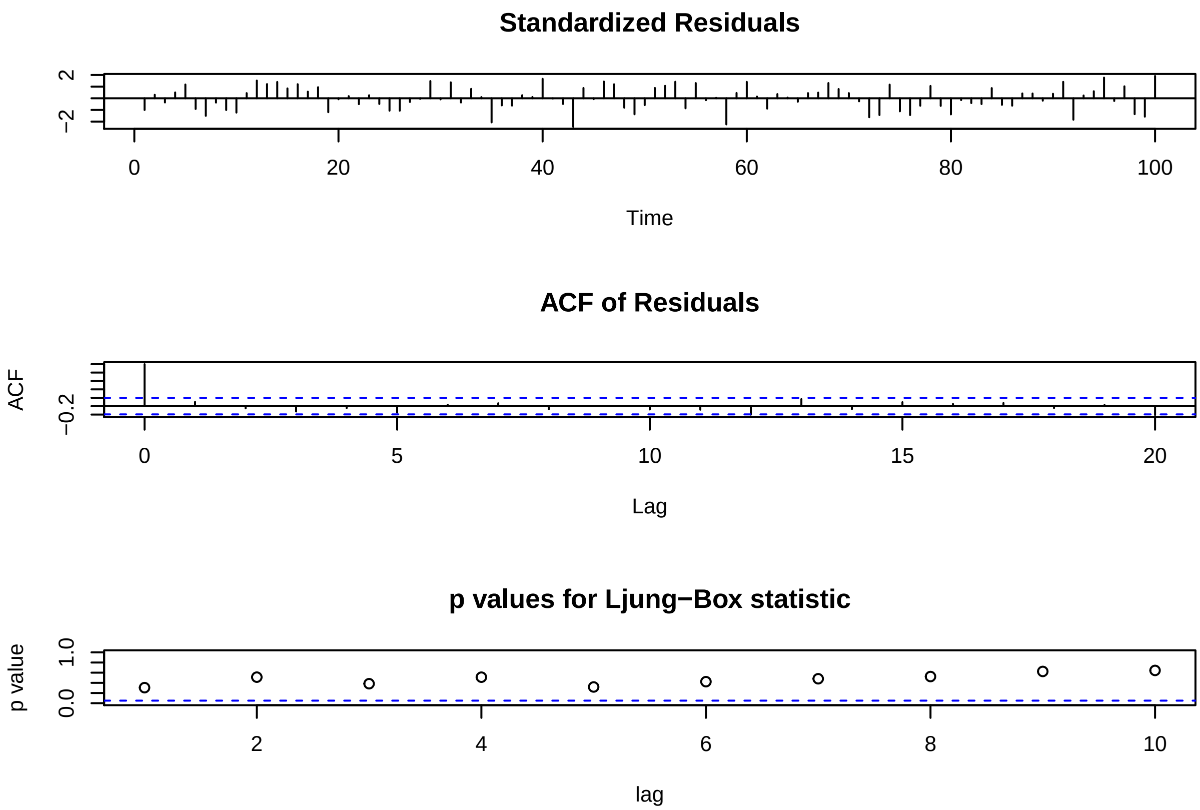
```
##
## Call:
## arima(x = y, order = c(1, 0, 2), xreg = x, method = "ML")
##
## Coefficients:
##          ar1     ma1     ma2  intercept       x
##       0.5664  0.1109  0.8996      3.595  0.3277
## s.e.  0.0848  0.0595  0.0576      1.385  0.0236
```

```
##
## sigma^2 estimated as 2.448:  log likelihood = -188.81,  aic = 389.62
```

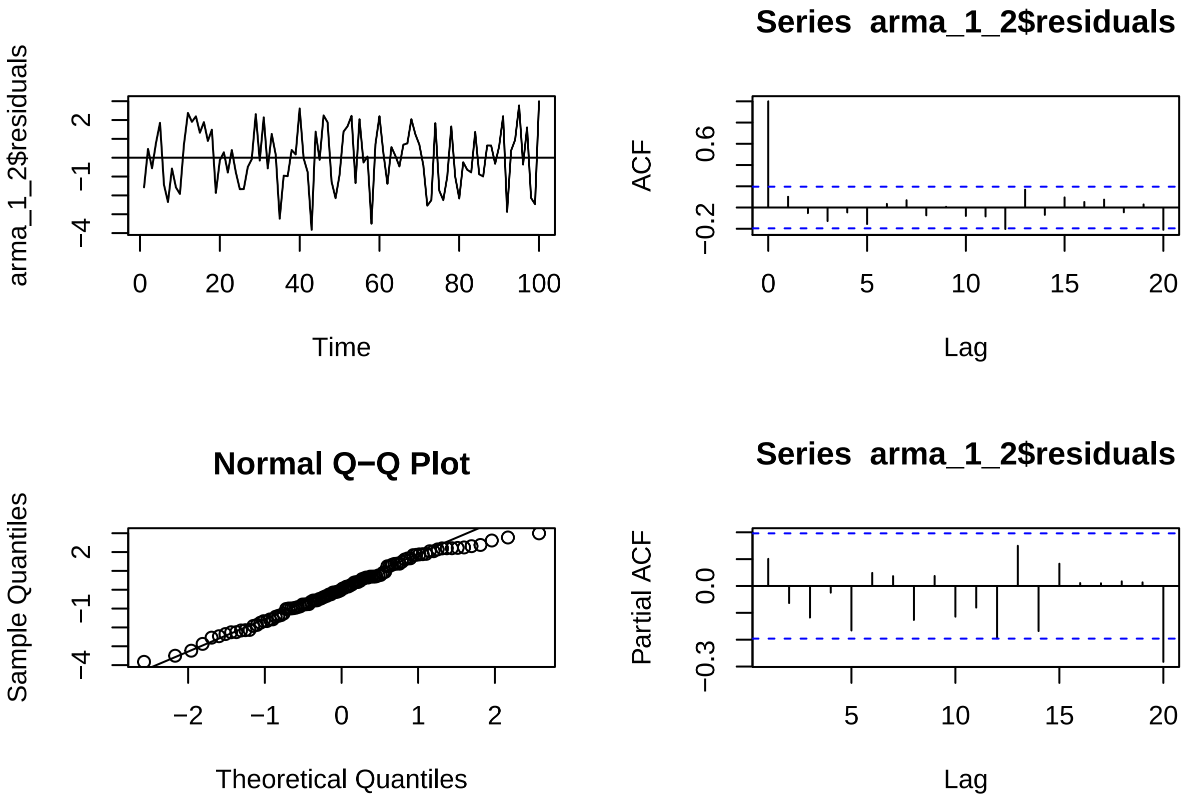
```r
tsdiag(ar_1)
```




```r
tsdiag(arma_1_2)
```

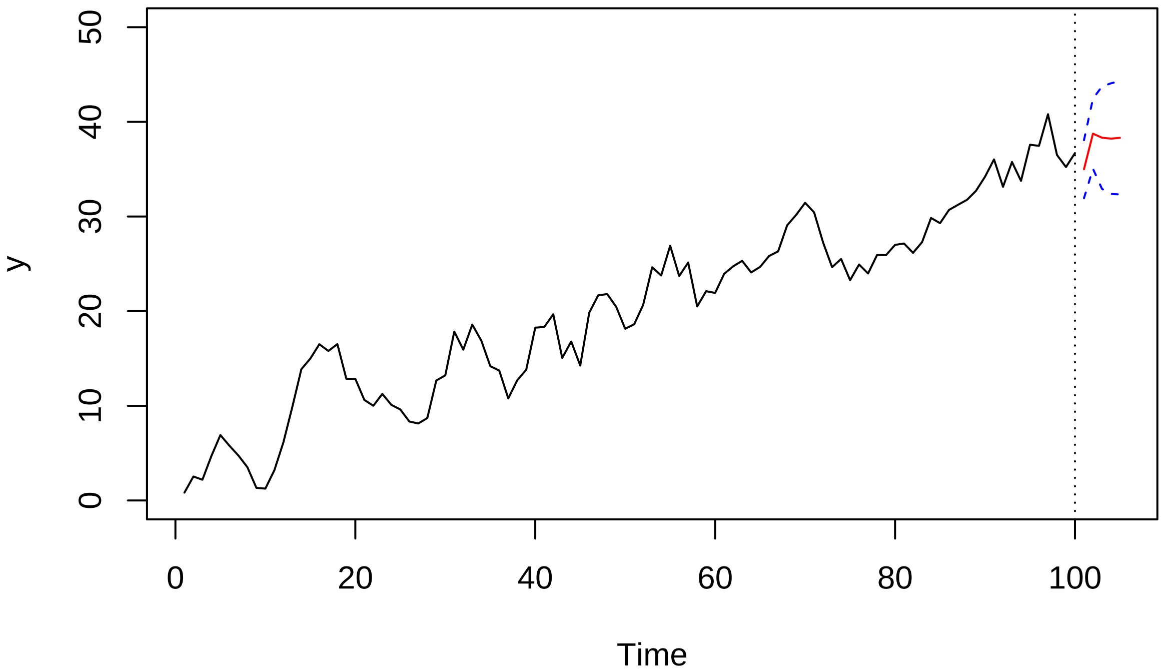



```
#ARMA(1,2) is a better model with smaller AIC and better residuals performance.
par(mfcol=c(2,2))
plot(arma_1_2$residuals)
abline(a=0,b=0)
qqnorm(arma_1_2$residuals)
qqline(arma_1_2$residuals)
acf(arma_1_2$residuals)
pacf(arma_1_2$residuals)
```

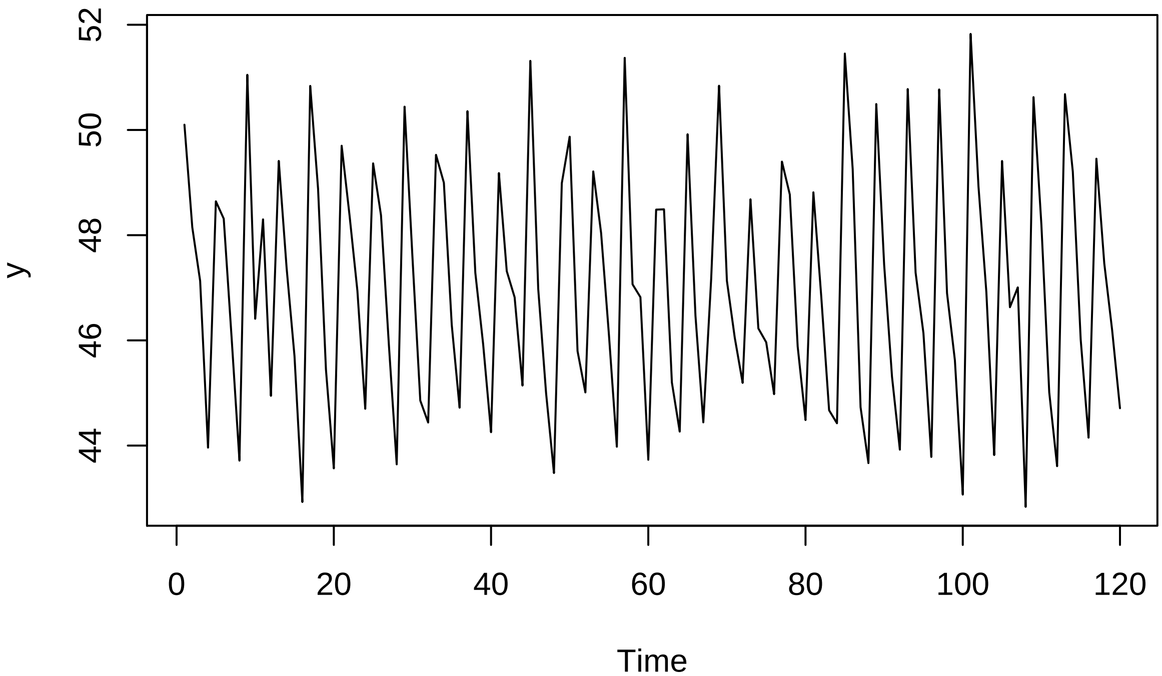



```
#The residuals look like normal and identically distributed and
#no correlation seems to be present.
n_x<-101:105
pr<-predict(arma_1_2,n.ahead=5,newxreg=n_x,se.fit=TRUE)
u<-pr$pred+1.96*pr$se
l<-pr$pred-1.96*pr$se

par(mfcol=c(1,1))
plot(x,y,type='l',xlim=c(1,105),ylim=c(0,50),xlab="Time")
lines(pr$pred,col='red')
lines(u,col='blue',lty='dashed')
lines(l,col='blue',lty='dashed')
abline(v=100,lty='dotted')
```

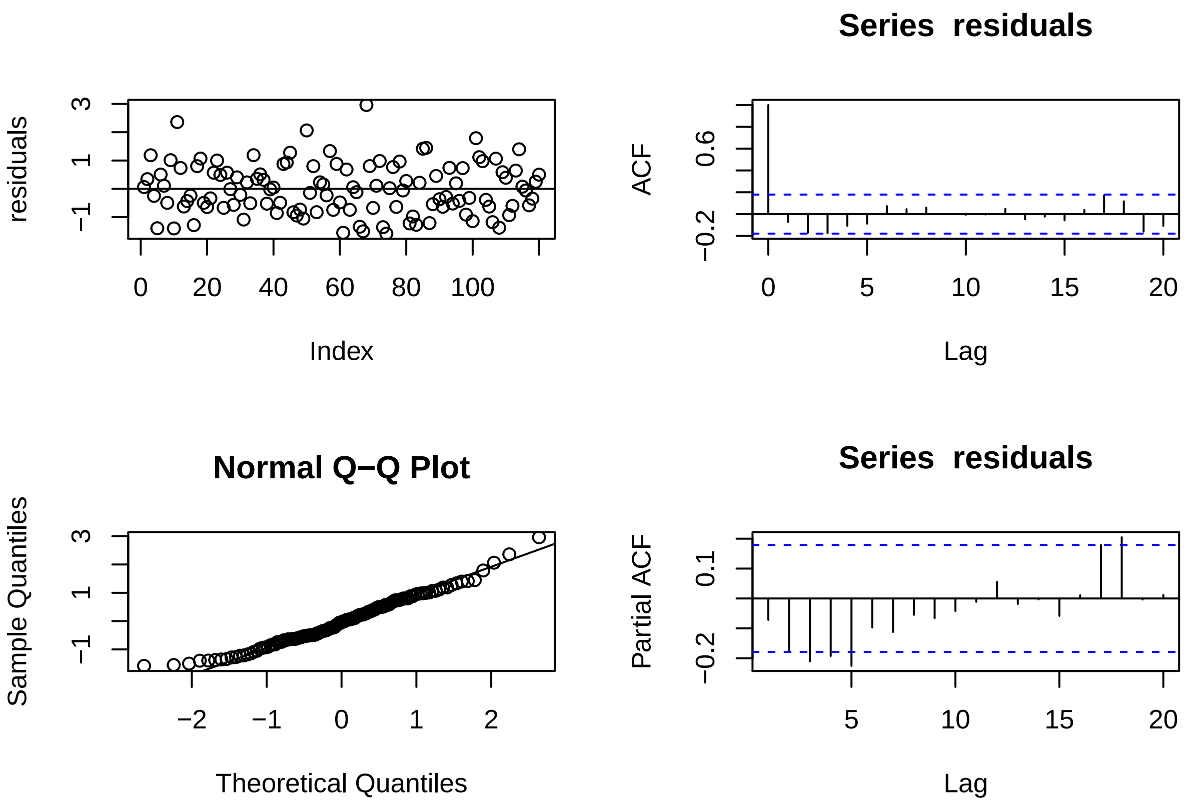



```
#Q3
y<-scan("/Users/y563li/Downloads/quarter_y.txt")
plot(y,type='l',xlab="Time")
```




```
x1<-c(rep(c(1,0,0,0),30))
x2<-c(rep(c(0,1,0,0),30))
x3<-c(rep(c(0,0,1,0),30))
x1.f<-factor(x1)
x2.f<-factor(x2)
x3.f<-factor(x3)
```

```
x<-matrix( c(x1.f,x2.f,x3.f), nrow=120,ncol=3)
model1=lsfit(x,y)
residuals=model1$residuals
par(mfcol=c(2,2))
plot(residuals)
abline(a=0,b=0)
qqnorm(residuals)
qqline(residuals)
acf(residuals)
pacf(residuals)
```



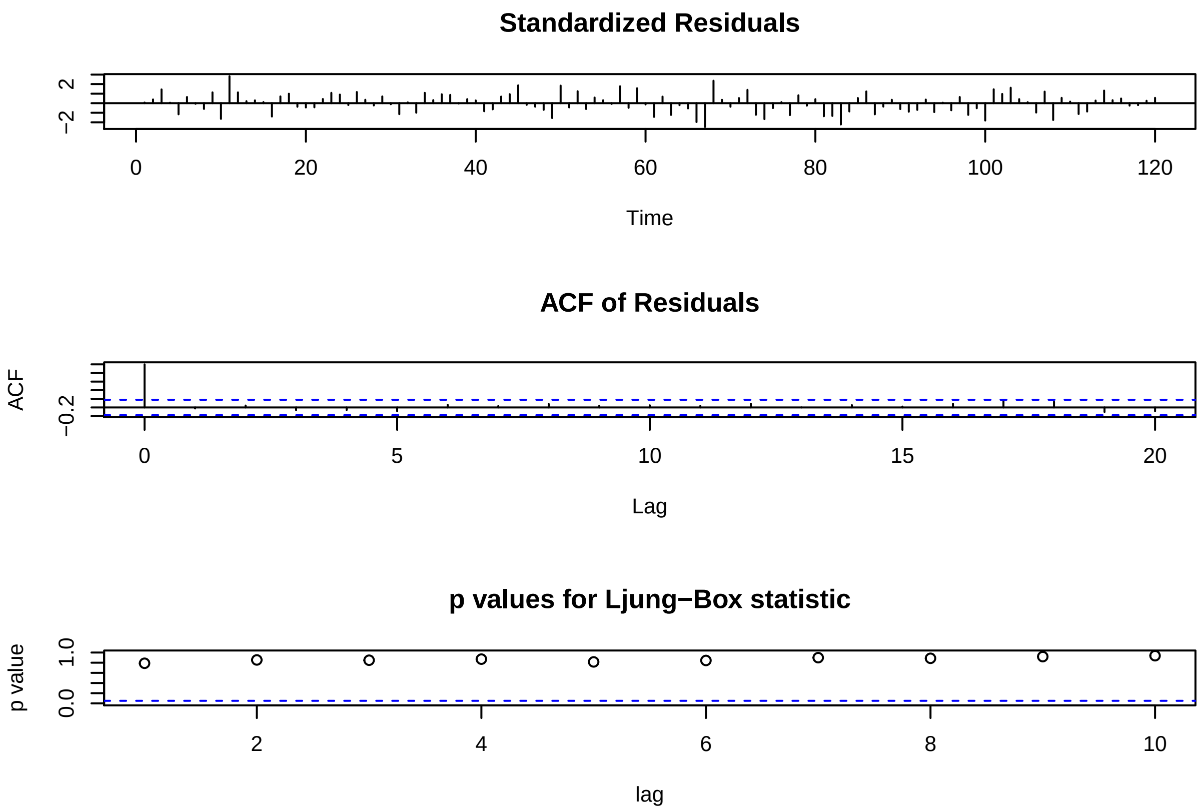

```
#The residuals seem resonably normal and identically distributed.In both ACF plot and PACF plot,
#there is some correlation and it kind of looks like ARMA(2,1) and ARMA(2,2).
arma_2_1=arima(y,order=c(2,0,1),xreg=x,method="ML")
arma_2_1
```

```
##
## Call:
## arima(x = y, order = c(2, 0, 1), xreg = x, method = "ML")
##
## Coefficients:
##          ar1      ar2      ma1  intercept      x1      x2      x3
##       0.6008  -0.2064  -0.8180    33.0251  5.8358  3.6030  1.7343
## s.e.  0.1036   0.0932   0.0612     0.7801  0.2448  0.2733  0.2449
##
## sigma^2 estimated as 0.6693:  log likelihood = -146.48,  aic = 308.97
```

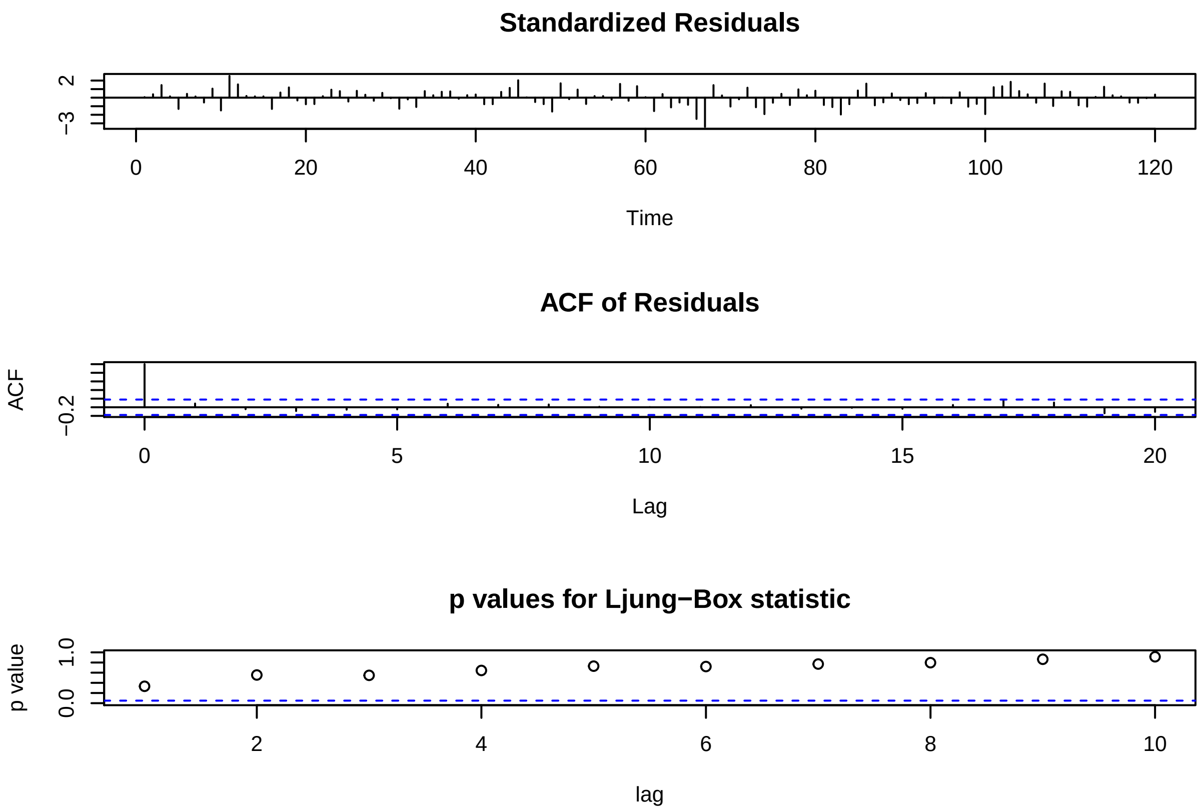
```
arma_2_2=arima(y,order=c(2,0,2),xreg=x,method="ML")
arma_2_2
```

```
##
## Call:
## arima(x = y, order = c(2, 0, 2), xreg = x, method = "ML")
##
## Coefficients:
##          ar1      ar2      ma1     ma2  intercept      x1      x2      x3
##       1.5231  -0.6105  -1.9618  1.0000    33.0296  5.8357  3.6058  1.7367
## s.e.  0.0739   0.0791   0.0573  0.0579     0.7666  0.2503  0.2493  0.2513
##
## sigma^2 estimated as 0.6012:  log likelihood = -142.42,  aic = 302.85
```

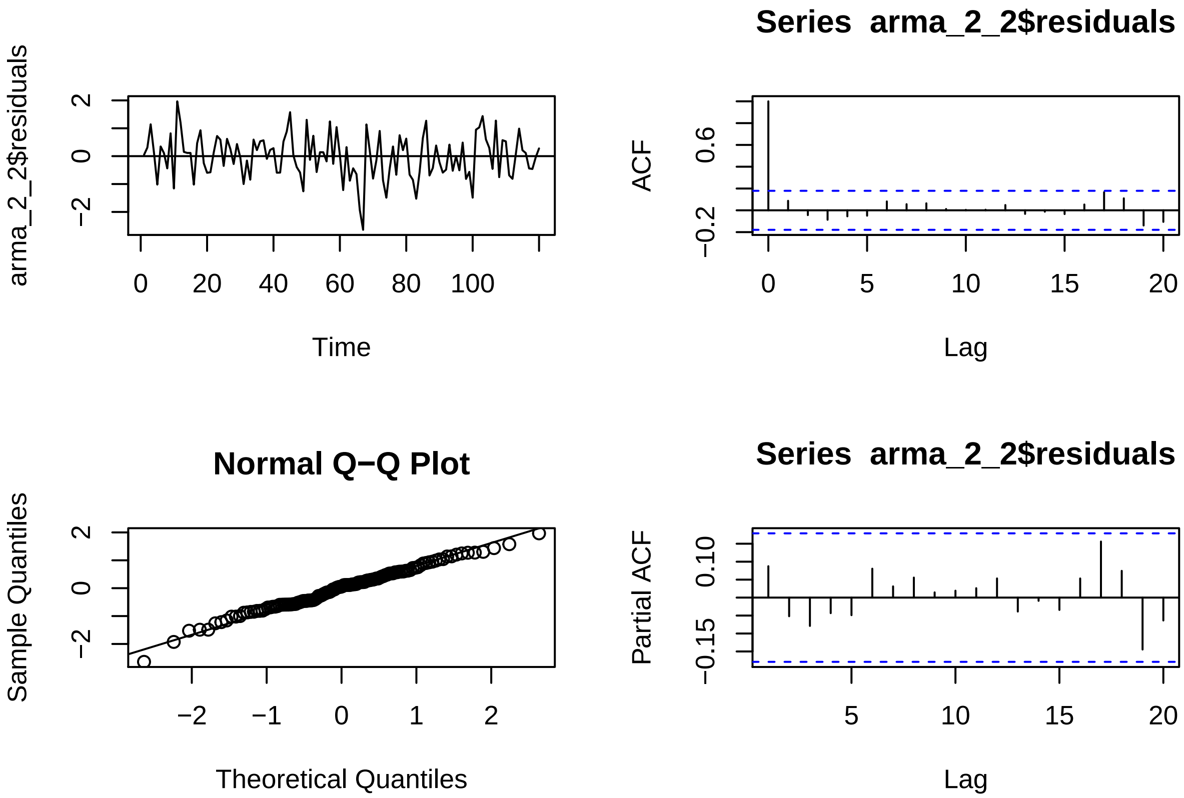
```r
tsdiag(arma_2_1)
```



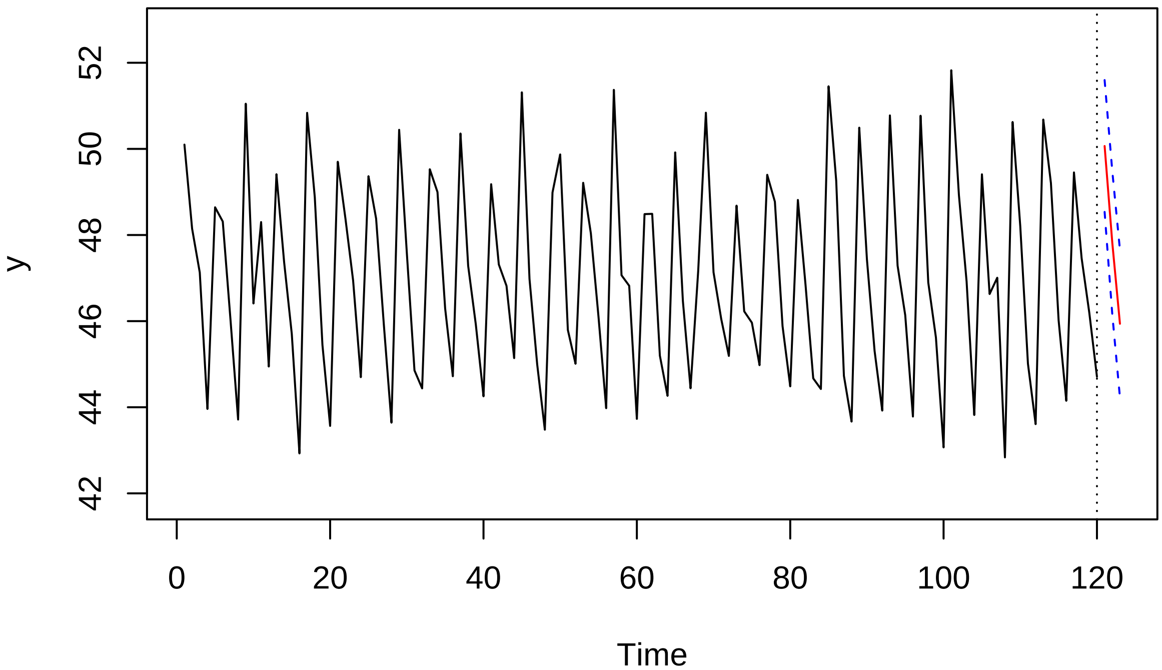

```r
tsdiag(arma_2_2)
```




```
#ARMA(2,2) is a better model with smaller AIC.
par(mfcol=c(2,2))
plot(arma_2_2$residuals)
abline(a=0,b=0)
qqnorm(arma_2_2$residuals)
qqline(arma_2_2$residuals)
acf(arma_2_2$residuals)
pacf(arma_2_2$residuals)
```




```
#The residuals look like normal and identically distributed and
#no correlation seems to be present.
x1_n=c(1,0,0)
x2_n=c(0,1,0)
x3_n=c(0,0,1)
x1_n.f<-factor(x1_n)
x2_n.f<-factor(x2_n)
x3_n.f<-factor(x3_n)
x_n<-matrix( c(x1_n.f,x2_n.f,x3_n.f), nrow=3,ncol=3)
pr<-predict(arma_2_2,n.ahead=3,newxreg=x_n,se.fit=TRUE)
u<-pr$pred+1.96*pr$se
l<-pr$pred-1.96*pr$se
par(mfcol=c(1,1))
plot(y,type='l',xlim=c(1,123),ylim=c(min(y)-1,max(y)+1),xlab="Time")
lines(pr$pred,col='red')
lines(u,col='blue',lty='dashed')
lines(l,col='blue',lty='dashed')
abline(v=120,lty='dotted')
```



y
42
44
46
48
50
52
0
20
40
60
80
100
120
Time