

STAT 443 Assignment #4 Fall 2017

(due Thursday, November 16 by 1:00 pm)

1. Let X_t be a causal ARMA(1,1) process

$$\phi_1(B)X_t = \theta_1(B)Z_t,$$

where B is the backshift operator and $\{Z_t\}$ follows $WN(0, \sigma^2)$. Find ψ_1 and ψ_2 in the Yule-Wold representation of X_t .

2. The data set `linear.y.txt` consists of 100 observations from a time series for which we propose the simple causal model

$$Y_t = \beta_0 + \beta_1 t + \epsilon_t$$

(plot the data and see why this is so). Obtain the residuals after fitting this model using `lsfit` and perform a residual analysis. Using the sample `acf` and sample `pacf` (as well as model parsimony) as a guide, select the **ARMA(p, q)** model (or models) that are good candidates for describing the correlation structure of the residuals. You should check the quality of your selections by ensuring that they pass the “white noise” test when run under `arima`; if you have multiple selections compare them using the `arima` diagnostics and AIC statistic (if applicable). Using the selected model, find your forecasts of Y_{101}, \dots, Y_{105} , together with the prediction intervals.

3. The data set `quarter.y.txt` consists of 120 quarterly observations from a seasonal time series for which we propose the simple causal model

$$Y_t = \beta_0 + \sum_{k=1}^3 \beta_k X_{t,k} + \epsilon_t$$

where $X_{t,1}, X_{t,2}, X_{t,3}$ are indicator variables (corresponding, for example, to the first three quarters). Obtain the residuals after fitting this model using `lsfit` and perform a residual analysis. Using the sample `acf` and sample `pacf` (as well as model parsimony) as a guide, select the **ARMA(p, q)** model (or models) that are good candidates for describing the correlation structure of the residuals. You should check the quality of your selections by ensuring that they pass the “white noise” test when run under `arima`; if you have multiple selections compare them using the `arima` diagnostics and AIC statistic (if applicable). Using the selected model, find your forecasts for the next three quarters, together with the prediction intervals.