STAT 443 Assignment #3 Fall 2017

(due Tuesday, October 31 by 1:00 pm)

Problem 5 will not be graded.

1. Suppose that $\{w_t\}$ is a white noise process with variance σ^2. Determine whether or not the following process is stationary

$$X_t = 2w_{t+1} - w_t, \quad t = 1, 2, \ldots.$$

Explain your answer.

2. Consider an AR(2) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + w_t,$$

where $\{w_t\}$ is a white noise process with variance σ^2, and ϕ_1 and ϕ_2 are such that $E(X_t) = 0$ and

$$E[X_t w_{t+h}] = 0 \quad \text{for each } t \text{ and } \; h > 0.$$

Suppose that we know that $X_1 = x_1$ and $X_2 = x_2$. Using the "method of projection", show that

$$\phi_1 x_2 + \phi_2 x_1$$

is the BLP of X_3. In your solution you may find useful the fact that a projection into a linear space is unique.

3. Suppose that $X_1, X_2, \ldots$ is a stationary time series with mean μ and autocorrelation function $\rho(\cdot)$. Show that the best predictor of X_{n+1} of the form

$$aX_n + b$$

is obtained when $a = \rho(1)$ and $b = \mu(1 - \rho(1))$.

4. Suppose that $\{Z_t\}$ follows a MA(1) process:

$$Z_t = a_t + \delta a_{t-1},$$

where a_t is a white noise with variance σ_a^2. Let v_t be another white noise series with variance σ_v^2 and such that $\{a_t\}$ and $\{v_t\}$ are uncorrelated at all lags:

$$E(a_t v_{t-j}) = 0 \quad \text{for all } j.$$

Let an observed series Y_t represent the sum of the MA(1) and the white noise series:

$$Y_t = Z_t + v_t.$$

Find the autocovariances of the series Y_t and show that they are those of an MA(1).

5. Consider a MA(3) process

$$S_t = (1 + \gamma_1 B + \gamma_2 B^2 + \gamma_3 B^3) a_t$$

where the white noise process a_t has variance σ^2.

(a) Find $\theta_0 = Var(S_t)$ in terms of $\gamma_1, \gamma_2, \gamma_3$ and σ^2.

(b) Show that $\rho_k = Corr(S_t, S_{t+k}) = 0$ for $k \geq 4$.

(c) Derive expressions for ρ_1, ρ_2, ρ_3 in terms of $\gamma_1, \gamma_2, \gamma_3$.

6. The data **gold.y** contains daily gold prices for 177 consecutive business days (from January 3, 1991 to October 18, 1991).

(a) *Discard* the first two week's worth of data (10 business days). We will ignore it because it behaves very differently from the later part of the data set. Also strip off the last two week's data and store it in a vector called **gold.future**. (We will use it later as a testing set for prediction.) The data that remains (after removing the first two and last two weeks) will be the training set in the problem. Plot the data. Do you think that a slowly drifting linear model might be appropriate here? (why or why not)

(b) Perform double exponential smoothing (by using, for example, **HoltWinters** function) to find α_0 that gives the minimum sum of squares (SOS) of 1-step forecast errors within the training set. Specify α_0 to two decimal places. To choose initial starting values, first do a least squares fit to the first 20 observations of the training set, obtaining the intercept and slope estimates $\hat{\beta}_0(0)$ and $\hat{\beta}_1(0)$.

(c) Once you have found the correct α_0, begin forecasting the **gold.future** data one step at a time, updating your model with the observed future data as you go along. Also provide updated 1-step prediction errors as you go along. Provide a time series plot of your predicted values, with upper and lower 95% prediction bounds, superimposed on the actual future data. Discuss how well your predictions did.