STAT 443 Assignment #5 Fall 2017

(due Monday, December 4, by 1:00 pm)

1. Consider a causal AR(2) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t,$$

where $\{Z_t\}$ is a Gaussian white noise with variance σ^2. Suppose we have observed $X_1, \ldots, X_n$, where n is large enough so that the estimates $\hat{\phi}_1$ and $\hat{\phi}_2$ are close to ϕ_1 and ϕ_2, that is

$$\hat{\phi}_1 \approx \phi_1 \quad \text{and} \quad \hat{\phi}_2 \approx \phi_2.$$

Using the method presented in class for an AR(1) process, find the variance of the forecasting error for X_{n+2} in terms of the model parameters.

2. Let $\{X_t\}$ be an ARIMA(2,1,0) process with $\{Z_t\}$ being a Gaussian white noise with variance σ^2. Suppose we have observed $X_1, \ldots, X_n$, where n is large enough so that the estimates $\hat{\phi}_1$ and $\hat{\phi}_2$ are close to ϕ_1 and ϕ_2. Using the method presented in class for an ARIMA(1,1,0) process, find the the variance of the forecasting error for X_{n+2} in terms of the model parameters.

3. Obtain the time series `knit_y`, which consists of 95 weekly observations. Strip off the last 5 observations and store them separately in `knit.future` to be used later as a testing set. Using only the training set (that is, the first 90 observations) develop two different forecasting models for this time series:

   (a) adopt a slowly-drifting mean model $\boldsymbol{Y}_t = \beta_0(t) + \epsilon_t$ where $\epsilon_t \sim OLSA$, and use the training set to find the optimal smoothing parameter α_0 for applying the simple exponential smoothing forecasting method.

   (b) adopt a constant mean model $\boldsymbol{Y}_t = \beta_0 + \epsilon_t$ with stationary correlated residuals and use the training set to select an optimal `ARMA(p, q)` model that best in your opinion describes the correlation structure of the residuals.

   Then compare your two models by forecasting for the next 5 weeks (use continuous updating in both cases) and computing the SOS. Which model performs best?

4. The data set `sales2` consists of 114 daily observations. By plotting the data, and using the sample `ACF` and `PACF`, select the `ARMA(p, d, q)` models that you believe are good candidates for making forecasts. You should check the quality of your selections by ensuring that they pass the "white noise" test after fitting an ARIMA model. If you have multiple selections compare them using the `arima` diagnostics and `AIC` statistic (if applicable). Choose one model that you believe is the best, and then use it to obtain your forecasts of $Y_{115}, \ldots, Y_{120}$, together with the prediction intervals. Produce a graph that shows the forecasted values together with the original time series.