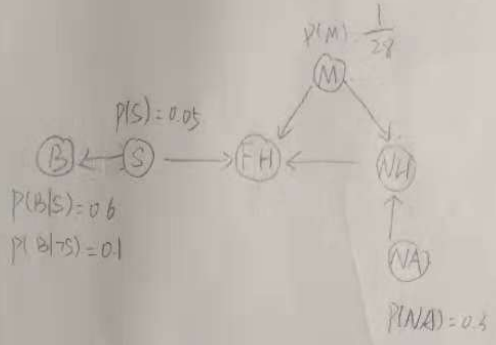


$P(S) = 0.05$
$P(B|S) = 0.6$
$P(B|\neg S) = 0.1$
$P(NA) = 0.3$
$P(M) = \frac{1}{28}$
$P(NH|\neg M, \neg NA) = 0$
$P(NH|\neg M, NA) = 0.5$
$P(NH|M, \neg NA) = 0.4$
$P(NH|M, NA) = 0.8$
$P(FH|S, M, NH) = 0.99$
$P(FH|S, \neg M, \neg NH) = 0$
$P(FH|S, \neg M, NH) = 0.5$
$P(FH|S, M, \neg NH) = 0.75$
$P(FH|\neg S, M, NH) = 0.9$
$P(FH|\neg S, M, \neg NH) = 0.4$
$P(FH|\neg S, \neg M, NH) = 0.2$

(b) $P(FH) = \sum_{b \in \{t,f\}} \sum_{m \in \{t,f\}} \sum_{na \in \{t,f\}} \sum_{nh \in \{t,f\}} \sum_{s \in \{t,f\}} P(B=b, FH=t, M=m, NA=na, NH=nh, S=s)$

$= \sum_{b} \sum_{m} \sum_{na} \sum_{nh} \sum_{s} P(B=b \mid S=s) \cdot P(FH \mid S=s, M=m, NH=nh) \cdot P(M=m) \cdot P(NA=na) \cdot P(NH=nh \mid NA=na, M=m) \cdot P(S=s)$

$= \sum_{s} P(S) \sum_{m} \sum_{na} P(NA=na) \sum_{nh} P(FH \mid S=s, M=m, NH=nh) \cdot P(NH=nh \mid NA=na, M=m) \cdot P(M=m)$

$(\sum_{b} P(B=b \mid S=s) = 1)$

Step 1.

define factor $f_1(NA) = \begin{bmatrix} P(NA) \\ P(\neg NA) \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix}$

define factor $f_2(NH, NA, M)$ to correspond to $P(NH=h \mid NA=na, M=m)$

$f_2(NH, NA, M=t) = \begin{bmatrix} P(NH \mid NA \wedge M) & P(\neg NH \mid NA \wedge M) \\ P(NH \mid \neg NA \wedge M) & P(\neg NH \mid \neg NA \wedge M) \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$

$f_2(NH, NA, M=f) = \begin{bmatrix} P(NH \mid NA \wedge \neg M) & P(\neg NH \mid NA \wedge \neg M) \\ P(NH \mid \neg NA \wedge \neg M) & P(\neg NH \mid \neg NA \wedge \neg M) \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 1 \end{bmatrix}$

define factor $f_3(FH, S, M, NH)$

$f_3(FH, S, M=t, NH=t) = \begin{bmatrix} P(FH \mid S, M, NH) & P(\neg FH \mid S, M, NH) \\ P(FH \mid \neg S, M, NH) & P(\neg FH \mid \neg S, M, NH) \end{bmatrix} = \begin{pmatrix} 0.99 & 0.01 \\ 0.65 & 0.35 \end{pmatrix}$

$f_3(FH, S, M=f, NH=t) = \begin{bmatrix} P(FH \mid S, \neg M, NH) & P(\neg FH \mid S, \neg M, NH) \\ P(FH \mid S, \neg M, NH) & P(\neg FH \mid S, \neg M, NH) \end{bmatrix} = \begin{bmatrix} 0.75 & 0.25 \\ 0.2 & 0.8 \end{bmatrix}$

$f_3(FH, S, M=t, NH=f) = \begin{bmatrix} P(FH \mid S, M, \neg NH) & P(\neg FH \mid S, M, \neg NH) \\ P(FH \mid \neg S, M, \neg NH) & P(\neg FH \mid \neg S, M, \neg NH) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}$

$f_3(FH, S, M=f, NH=f) = \begin{bmatrix} P(FH \mid S, \neg M, \neg NH) & P(\neg FH \mid S, \neg M, \neg NH) \\ P(FH \mid \neg S, \neg M, \neg NH) & P(\neg FH \mid \neg S, \neg M, \neg NH) \end{bmatrix}$

$= \begin{bmatrix} 0.5 & 0.5 \\ 0 & 1 \end{bmatrix}$

define factor $f_2(M)$

$f_2(M) = \begin{bmatrix} P(M=t) \\ P(M=f) \end{bmatrix} = \begin{bmatrix} \frac{1}{28} \\ \frac{27}{28} \end{bmatrix}$

define factor $f_1(S)$

$f_1(S) = \begin{bmatrix} P(S=t) \\ P(S=f) \end{bmatrix} = \begin{pmatrix} 0.05 \\ 0.95 \end{pmatrix}$

$f_{HW}(B \mid S) = \begin{bmatrix} P(B \mid S) & P(\neg B \mid S) \\ P(B \mid \neg S) & P(\neg B \mid \neg S) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{bmatrix}$

step 3 sum out each hidden variable

$f_6(FH, S, M, NH, NA) = f_3(FH, S, M, NH) \, f_4(NH, M, NA) \, f_5(M)$

$f_6(FH, S=t, NH=t, NA=t, M)$

$= \begin{bmatrix} P(FH \mid S=t, NH=t, M=t) P(NH=t \mid NA=t, M=t) P(M=t) & P(\neg FH \mid S=t, NH=t, M=t) P(NH=t \mid NA=t, M=t) \cdot P(M=t) \\ P(FH \mid S=t, NH=t, M=f) P(NH=t \mid NA=t, M=f) P(M=f) & P(\neg FH \mid S=t, NH=t, M=f) P(NH=t \mid NA=t, M=f) \cdot P(M=f) \end{bmatrix}$

$= \begin{bmatrix} 0.0283 & 0.000286 \\ 0.036 & 0.12 \end{bmatrix}$

$f_6(FH, S=f, NH=t, NA=t, M)$

$= \begin{bmatrix} P(FH \mid \neg S, NH, NA, M) & P(\neg FH \mid \neg S, NH, NA, M) \\ P(FH \mid \neg S, NH, NA, \neg M) & P(\neg FH \mid \neg S, NH, NA, \neg M) \end{bmatrix} = \begin{bmatrix} 0.0186 & 0.01 \\ 0.096 & 0.386 \end{bmatrix}$

$f_6(FH, S=t, NH=f, NA=t, M)$

$= \begin{bmatrix} P(FH \mid S, \neg NH, NA, M) & P(\neg FH \mid S, \neg NH, NA, M) \\ P(FH \mid S, \neg NH, NA, \neg M) & P(\neg FH \mid S, \neg NH, NA, \neg M) \end{bmatrix} = \begin{bmatrix} 0.0064 & 0.0007 \\ 0.24 & 0.24 \end{bmatrix}$

$f_6(FH, S=t, NH=t, NA=f, M)$

$$= \begin{bmatrix} P(FH \mid S=t, NH=t, NA=f, M) & P(\neg FH \mid S=t, NH=t, NA=f, M) \\ P(FH \mid S=t, NH=t, NA=f, \neg M) & P(\neg FH \mid S=t, NH=t, NA=f, \neg M) \end{bmatrix} = \begin{bmatrix} 0.014 & 0.00014 \\ 0 & 0 \end{bmatrix}$$

$f_6(FH, S=f, NH=f, NA=t, M)$

$$= \begin{bmatrix} P(FH \mid S=f, NH=f, NA=t, M) & P(\neg FH \mid S=f, NH=f, NA=t, M) \\ P(FH \mid S=f, NH=f, NA=t, \neg M) & P(\neg FH \mid S=f, NH=f, NA=t, \neg M) \end{bmatrix} = \begin{bmatrix} 0.0028 & 0.024 \\ 0 & 0.48 \end{bmatrix}$$

$f_6(FH, S=f, NH=t, NA=f, M)$

$$= \begin{bmatrix} P(FH \mid \neg S, NH, \neg NA, M) & P(\neg FH \mid \neg S, NH, \neg NA, M) \\ P(FH \mid \neg S, NH, \neg NA, \neg M) & P(\neg FH \mid \neg S, NH, \neg NA, \neg M) \end{bmatrix} = \begin{bmatrix} 0.009 & 0.05 \\ 0 & 0 \end{bmatrix}$$

$f_6(FH, S=t, NH=f, NA=f, M)$

$$= \begin{bmatrix} P(FH \mid S, \neg NH, \neg NA, M) & P(\neg FH \mid S, \neg NH, \neg NA, M) \\ P(FH \mid S, \neg NH, \neg NA, \neg M) & P(\neg FH \mid S, \neg NH, \neg NA, \neg M) \end{bmatrix} = \begin{bmatrix} 0.019 & 0.002 \\ 0.482 & 0.482 \end{bmatrix}$$

$f_6(FH, S=f, NH=f, NA=f, M)$

$$= \begin{bmatrix} P(FH \mid \neg S, \neg NH, \neg NA, M) & P(\neg FH \mid \neg S, \neg NH, \neg NA, M) \\ P(FH \mid \neg S, \neg NH, \neg NA, \neg M) & P(\neg FH \mid \neg S, \neg NH, \neg NA, \neg M) \end{bmatrix} = \begin{bmatrix} 0.0086 & 0.093 \\ 0 & 1 \end{bmatrix}$$

$$f_7(FH, S, NH, NA) = \sum_{M \in \{T,F\}} f_6(FH, S, NH, NA, M)$$

$$f_7(FH, S=t, NH=t, NA=t) = \begin{bmatrix} f_7(FH=t, S=t, NH=t, NA=t) \\ f_7(FH=f, S=t, NH=t, NA=t) \end{bmatrix} = \begin{bmatrix} 0.3883 \\ 0.120286 \end{bmatrix}$$

$$f_7(FH, S=f, NH=t, NA=t) = \begin{bmatrix} f_7(FH=t, S=f, NH=t, NA=t) \\ f_7(FH=f, S=f, NH=t, NA=t) \end{bmatrix} = \begin{bmatrix} 0.1146 \\ 0.396 \end{bmatrix}$$

$$f_7(FH, S=t, NH=f, NA=t) = \begin{bmatrix} f_7(FH=t, S=t, NH=f, NA=t) \\ f_7(FH=f, S=f, NH=t, NA=t) \end{bmatrix} = \begin{bmatrix} 0.2464 \\ 0.2407 \end{bmatrix}$$

$$f_7(FH{=}t, S{=}t, MH{=}t, NA{=}f) = \begin{bmatrix} f_7(FH{=}t, S{=}t, MH{=}t, NA{=}f) \\ f_7(FH{=}f, S{=}t, MH{=}t, NA{=}f) \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.00014 \end{bmatrix}$$

$$f_7(FH, S{=}f, MH{=}f, NA{=}t) = \begin{bmatrix} f_7(FH{=}t, S{=}f, MH{=}f, NA{=}t) \\ f_7(FH{=}f, S{=}f, MH{=}f, NA{=}t) \end{bmatrix} = \begin{bmatrix} 0.029 \\ 0.484 \end{bmatrix}$$

$$f_7(FH, S{=}f, MH{=}t, NA{=}f) = \begin{bmatrix} f_7(FH{=}t, S{=}f, MH{=}t, NA{=}f) \\ f_7(FH{=}f, S{=}f, MH{=}t, NA{=}f) \end{bmatrix} = \begin{bmatrix} 0.09 \\ 0.05 \end{bmatrix}$$

$$f_7(FH, S{=}t, MH{=}f, NA{=}f) = \begin{bmatrix} f_7(FH{=}t, S{=}t, MH{=}f, NA{=}f) \\ f_7(FH{=}f, S{=}t, MH{=}f, NA{=}f) \end{bmatrix} = \begin{bmatrix} 0.501 \\ 0.484 \end{bmatrix}$$

$$f_8(FH, S{=}f, MH{=}f, NA{=}f) = \begin{bmatrix} f_8(FH{=}t, S{=}f, MH{=}f, NA{=}f) \\ f_8(FH{=}f, S{=}f, MH{=}f, NA{=}f) \end{bmatrix} = \begin{bmatrix} 0.0036 \\ 1.013 \end{bmatrix}$$

$$f_8(FH, S, MH, NA) = f_7(FH, S, MH, NA) \cdot f_1(NA)$$

$$f_8(FH, S, MH, NA) = \begin{bmatrix} f_8(FH{=}t, S{=}t, MH{=}t, NA{=}t) \\ f_8(FH{=}f, S{=}t, MH{=}t, NA{=}t) \end{bmatrix} = \begin{bmatrix} 0.3883 \times 0.3 \\ 0.120286 \times 0.3 \end{bmatrix} = \begin{bmatrix} 0.11649 \\ 0.036075 \end{bmatrix}$$

$$f_8(FH, S{=}f, MH{=}t, NA{=}t) = \begin{bmatrix} f_8(FH{=}t, S{=}f, MH{=}t, NA{=}t) \\ f_8(FH{=}f, S{=}f, MH{=}t, NA{=}t) \end{bmatrix} = \begin{bmatrix} 0.1146 \times 0.3 \\ 0.386 \times 0.3 \end{bmatrix} = \begin{bmatrix} 0.03438 \\ 0.1158 \end{bmatrix}$$

$$f_8(FH, S{=}t, MH{=}f, NA{=}t) = \begin{bmatrix} f_8(FH{=}t, S{=}t, MH{=}f, NA{=}t) \\ f_8(FH{=}f, S{=}t, MH{=}f, NA{=}t) \end{bmatrix} = \begin{bmatrix} 0.2464 \times 0.3 \\ 0.240 \times 0.3 \end{bmatrix} = \begin{bmatrix} 0.07392 \\ 0.07224 \end{bmatrix}$$

$$f_8(FH, S{=}t, MH{=}t, NA{=}f) = \begin{bmatrix} f_8(FH{=}t, S{=}t, MH{=}t, NA{=}f) \\ f_8(FH{=}f, S{=}t, MH{=}t, NA{=}f) \end{bmatrix} = \begin{bmatrix} 0.014 \times 0.7 \\ 0.00014 \times 0.7 \end{bmatrix} = \begin{bmatrix} 0.0098 \\ 0.000098 \end{bmatrix}$$

$$f_8(FH, S{=}f, MH{=}f, NA{=}t) = \begin{bmatrix} f_8(FH{=}t, S{=}f, MH{=}f, NA{=}t) \\ f_8(FH{=}f, S{=}f, MH{=}f, NA{=}t) \end{bmatrix} = \begin{bmatrix} 0.029 \times 0.3 \\ 0.484 \times 0.3 \end{bmatrix} = \begin{bmatrix} 0.0087 \\ 0.1452 \end{bmatrix}$$

$f_8(FH, S=f, NH=t, NA=f) = \begin{bmatrix} f_8(FH=t, S=f, NH=t, NA=f) \\ f_8(FH=f, S=f, NH=t, NA=f) \end{bmatrix} = \begin{bmatrix} 0.009 \times 0.7 \\ 0.05 \times 0.7 \end{bmatrix} = \begin{bmatrix} 0.0063 \\ 0.035 \end{bmatrix}$

$f_8(FH, S=t, NH=f, NA=f) = \begin{bmatrix} f_8(FH=t, S=t, NH=f, NA=f) \\ f_8(FH=f, S=t, NH=f, NA=f) \end{bmatrix} = \begin{bmatrix} 0.50 \times 0.7 \\ 0.634 \times 0.7 \end{bmatrix} = \begin{bmatrix} 0.350 \\ 0.3388 \end{bmatrix}$

$f_8(FH, S=f, NH=f, NA=f) = \begin{bmatrix} f_8(FH=t, S=f, NH=f, NA=f) \\ f_8(FH=f, S=f, NH=f, NA=f) \end{bmatrix} = \begin{bmatrix} 0.0086 \times 0.7 \\ 1.013 \times 0.7 \end{bmatrix} = \begin{bmatrix} 0.00602 \\ 0.7091 \end{bmatrix}$

$f_9(FH, S, NH) = \sum_{NA} f_8(FH, S, NH, NA)$

$f_9(FH=t, S=t, NH) = \begin{bmatrix} f_9(FH=t, S=t, NH=t) \\ f_9(FH=t, S=t, NH=f) \end{bmatrix} = \begin{bmatrix} 0.1174 \\ 0.4246 \end{bmatrix}$

$f_9(FH=t, S=f, NH) = \begin{bmatrix} f_9(FH=t, S=f, NH=t) \\ f_9(FH=t, S=f, NH=f) \end{bmatrix} = \begin{bmatrix} 0.04068 \\ 0.01472 \end{bmatrix}$

$f_9(FH=f, S=t, NH) = \begin{bmatrix} f_9(FH=f, S=t, NH=t) \\ f_9(FH=f, S=t, NH=f) \end{bmatrix} = \begin{bmatrix} 0.036828 \\ 0.41101 \end{bmatrix}$

$f_9(FH=f, S=f, NH) = \begin{bmatrix} f_9(FH=f, S=f, NH=t) \\ f_9(FH=f, S=f, NH=f) \end{bmatrix} = \begin{bmatrix} 0.1538 \\ 0.3542 \end{bmatrix}$

$f_{10}(FH, S) = \sum_{NH \in \{t,f\}} f_9(FH, S, NH)$

$f_{10}(FH, S) = \begin{bmatrix} f_{10}(FH=t, S=t) \\ f_{10}(FH=t, S=f) \end{bmatrix} = \begin{bmatrix} 0.54109 \\ 0.0554 \end{bmatrix}$

$f_{10}(FH, S) = \begin{bmatrix} f_{10}(FH=f, S=t) \\ f_{10}(FH=f, S=f) \end{bmatrix} = \begin{bmatrix} 0.447838 \\ 1.008 \end{bmatrix}$

$$f_{11}(F,H,S) = f_{10}(F,S) \cdot f_3(S)$$

$$f_{11}(F,H,S) = \begin{bmatrix} f_{10}(F=t,S=t)\, f_3(S=t) \\ f_{10}(F=t,S=f)\, f_3(S=f) \end{bmatrix} = \begin{bmatrix} 0.5420 \times 0.05 \\ 0.0554 \times 0.95 \end{bmatrix} = \begin{bmatrix} 0.027 \\ 0.053 \end{bmatrix}$$

$$f_{11}(F,H,S) = \begin{bmatrix} f_{10}(F=f,S=t)\, f_3(S=t) \\ f_{10}(F=f,S=f)\, f_3(S=f) \end{bmatrix} = \begin{bmatrix} 0.447838 \times 0.05 \\ 1.008 \times 0.95 \end{bmatrix} = \begin{bmatrix} 0.022 \\ 0.9576 \end{bmatrix}$$

$$f_{12} = \sum_{S \in \{t,f\}} f_{10}(F,H,S)$$

$$f_{12}(F,H) = \begin{bmatrix} f_{12}(F=t) \\ f_{12}(F=f) \end{bmatrix} = \begin{bmatrix} 0.07863 \\ 0.98 \end{bmatrix}$$

Normalize the resulting factor

let $f_{13}(F,H) = \dfrac{f_{12}(F,H)}{\sum_{H} f_{12}(F,H)}$

$$f_{13}(F,H) = \begin{bmatrix} f_{13}(F=t) \\ f_{13}(F=f) \end{bmatrix} = \begin{bmatrix} 0.07 \\ 0.93 \end{bmatrix}$$

(c) $P(S|FH, M) = \frac{P(S \wedge FH \wedge M)}{P(FH \wedge M)} = \frac{P(S \wedge FH \wedge M)}{P(S \wedge FH \wedge M) + P(\neg S \wedge FH \wedge M)}$

$P(S, FH, M)$

$= \sum_{b \in \{t,f\}} \sum_{NA \in \{t,f\}} \sum_{NH \in \{t,f\}} P(B=b, FH=t, M=t, NA=na, NH=nh, S)$

$= P(S) \sum_{b \in \{t,f\}} \sum_{NA \in \{t,f\}} \sum_{NH \in \{t,f\}} P(B=b|S=s)\, P(FH=t|S=s, M=t, NH=nh)\, P(M=t)\, P(NA=na)\, P(NH=nh|NA=na, M=t)\, P(M=t)$

$= P(S) \sum_{NH \in \{t,f\}} \sum_{NA \in \{t,f\}} P(NA=na)\, P(FH=t|S=s, M=t, NH=nh)\, P(M=t)\, P(NH=nh|NA=na, M=t)$

$[\sum_b P(B=b|S=s) = 1)$

top1 As denoted in part b (where factors f_1-f_5 are the same as those of part b)

step2 Restricted the factors

$f_6(NH, NA) = f_2(NH, NA, M=t)$

$f_6(NH, NA) = \begin{bmatrix} P(NH|NA, M) & P(\neg NH|NA, M) \\ P(NH|\neg NA, M) & P(\neg NH|\neg NA, M) \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$

$f_7(S, NH) = f_3(FH=t, S, M=t, NH)$

$f_7(S, NH) = \begin{bmatrix} P(FH|S, M, NH) & P(FH|S, M, \neg NH) \\ P(FH|\neg S, M, NH) & P(FH|\neg S, M, \neg NH) \end{bmatrix} = \begin{bmatrix} 0.99 & 0.9 \\ 0.65 & 0.4 \end{bmatrix}$

$f_8(M) = f_4(M=t) = \frac{1}{28}$

So $P(S=s, FH=t, M=t) = f_5(S) \sum_{NH \in \{t,f\}} \sum_{NA \in \{t,f\}} f_1(NA)\, f_7(S, NH)\, f_6(NH, NA)\, f_8(M)$

step3 Sum out hidden variables

$f_9(NA, NH) = f_1(NA) f_6(NH, NA) = \begin{bmatrix} f_6(NH=t, NA=t) f_1(NA=t) & f_6(NH=t, NA=f) f_1(NA=t) \\ f_6(NH=t, NA=f) f_1(NA=f) & f_6(NH=f, NA=f) f_1(NA=f) \end{bmatrix}$

$= \begin{bmatrix} 0.24 & 0.06 \\ 0.28 & 0.42 \end{bmatrix}$

$$f_{10}(M) = \sum_{M \in \{t,f\}} f_9(M, M) = \begin{bmatrix} f_{10}(M=t) \\ f_{10}(M=f) \end{bmatrix}$$

$$= \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix}$$

$$f_{11}(S, M) = f_7(S, M) \cdot f_{10}(M)$$

$$= \begin{bmatrix} f_7(S=t, M=t) \cdot f_{10}(M=t) & f_7(S=t, M=f) \cdot f_{10}(M=f) \\ f_7(S=f, M=t) \cdot f_{10}(M=t) & f_7(S=f, M=f) \cdot f_{10}(M=f) \end{bmatrix}$$

$$= \begin{bmatrix} 0.5148 & 0.432 \\ 0.338 & 0.192 \end{bmatrix}$$

$$f_{12}(S) = \sum_{M \in \{t,f\}} f_{11}(S, M) = \begin{bmatrix} f_{12}(S=t) \\ f_{12}(S=f) \end{bmatrix} = \begin{bmatrix} 0.9468 \\ 0.53 \end{bmatrix}$$

$$f_{14}(S) = p(M=t) f_{13}(S) \cdot f_{13}(S) = \begin{bmatrix} f_{14}(S=t) \\ f_{14}(S=f) \end{bmatrix}$$

$$= \begin{bmatrix} 0.0454/.28 \\ 0.2025/.28 \end{bmatrix}$$

step 4 normalize the resulting factor

$$f_{15}(S) = \frac{f_{14}(S)}{\sum_S f_{14}(S)}$$

$$f_{15}(S) = \begin{bmatrix} f_{15}(S=t) \\ f_{15}(S=f) \end{bmatrix} = \begin{bmatrix} 0.086 \\ 0.914 \end{bmatrix}$$

So $p(S=t \mid F4, M) = 0.086$

1.d) $p(S|FH, M, B) = \frac{p(S, FH, M, B)}{p(FH, M, B)} = \frac{p(S, FH, M, B)}{p(S, FH, M, B) + p(\neg S, FH, M, B)}$

$p(S, FH, M, B)$

$= \sum_{NA \in \{t,f\}} \sum_{NM \in \{t,f\}} p(B=t, FH=t, M=t, NA=na, NM=nk, S)$

$= p(S)\, p(B=t|S) \sum_{NA \in \{t,f\}} \sum_{NM \in \{t,f\}} p(NA=na)\, p(FH=t|S=s, M=t, NM=nk)\, p(M=t)\, P(NM=nk|NA=na, M=t)$

step1 As defined in part b

step2 As calculated in part c

$f_{14}(B, S) = f_{14}(B=t, S) = \begin{bmatrix} p(B|S) \\ p(B|\neg S) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.1 \end{bmatrix}$

(Here the factors from f_1-f_3 are the same as part c)

step3 As calculated in part c

$f_3(S) = \sum_{NM \in \{t,f\}} f_1(S, NM) = \begin{bmatrix} f_{13}(S=t) \\ f_{13}(S=f) \end{bmatrix} = \begin{bmatrix} 0.9468 \\ 0.53 \end{bmatrix}$

$f_{14}(S) = f_{14}(B, S)\, f_{12}(S)\, f_3(S)\, p(M=t)$

$= \begin{bmatrix} f_{14}(S=t) \\ f_{14}(S=f) \end{bmatrix} = \begin{bmatrix} 0.023404/28 \\ 0.05032/28 \end{bmatrix}$

step4 normalize the resulting factor

$f_{15}(S) = \frac{f_{14}(S)}{\sum_{s \in \{t,f\}} f_{14}(S)}$

$f_{15}(S) = \begin{bmatrix} f_{15}(S=t) \\ f_{15}(S=f) \end{bmatrix} = \begin{bmatrix} 0.36 \\ 0.64 \end{bmatrix}$

$p(S|FH, M, B) = 0.36$

(c) $P(s \mid FH, M, B, NA) = \frac{P(s, FH, M, B, NA)}{P(s, FH, M, B, NA) + P(\neg s, FH, M, B, NA)}$

$P(s, FH, M, B, NA)$

$= \sum_{MH \in \{t,f\}} P(B=t, FH=t, M=t, NA=t, MH=mh, s)$

$= P(s)\, P(B=t \mid s) \sum_{NA \in \{t,f\}} P(NA=t)\, P(FH=t \mid S=s, M=t, MH=mh)\, P(MH=mh \mid NA=t, M=t)\, P(M=t)$

step1 As defined in part b (Here factors f_1-f_5 are the same as those of part b)

step2 Restricted the factors

$f_6(MH) = f_2(MH, NA=t, M=t)$

$f_6(MH) = \begin{bmatrix} f_6(MH=t) \\ f_6(MH=f) \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$

$f_7(s, MH) = f_3(FH=t, S, M=t, MH)$

$f_7(s, MH) = \begin{bmatrix} 0.99 & 0.9 \\ 0.65 & 0.4 \end{bmatrix}$

$f_8(NA) = f_1(NA=t) = 0.3$

$f_9(M) = f_4(M=t) = \frac{1}{50}$

So $P(S, FH=t, M=t, B=t, NA=t) = f_5(s) \sum_{MH \in \{t,f\}} f_6(MH) f_7(s, MH) f_8(NA) f_9(M)$

step3 sum out hidden variables

$f_{10}(s, MH) = f_6(MH)\, f_7(s, MH)$

$= \begin{bmatrix} f_{10}(s=t, MH=t) & f_{10}(s=t, MH=f) \\ f_{10}(s=f, MH=t) & f_{10}(s=f, MH=f) \end{bmatrix}$

$= \begin{bmatrix} 0.792 & 0.18 \\ 0.52 & 0.08 \end{bmatrix}$

$f_{11}(s) = \sum_{MH \in \{t,f\}} f_{10}(s, MH) = \begin{bmatrix} f_{11}(s=t) \\ f_{11}(s=f) \end{bmatrix} = \begin{bmatrix} 0.972 \\ 0.6 \end{bmatrix}$

$f_{12}(S) = f_{11}(S) \cdot f_2(S)\ p(M=t)\ p(NA=t)\ f_{10}(B=t, S)$

$= \begin{bmatrix} 0.02916 \times 0.3/28 \\ 0.057 \times 0.3/28 \end{bmatrix}$

step4 normalize resulting factor

$$f_{13}(S) = \frac{f_{12}(S)}{\sum_{S \in \{S\}} f_{12}(S)}$$

$$f_{13}(S) = \begin{bmatrix} f_{12}(S)=t \\ f_{12}(S)=f \end{bmatrix} = \begin{bmatrix} 0.3384 \\ 0.6616 \end{bmatrix}$$

So $P(S \mid FH, M, B, NA) = 0.3384$

2 (1) $P(G|W) = \frac{P(G,W)}{P(W)} = \frac{\sum_A \sum_E \sum_B P(B,E,A,G,W)}{\sum_G \sum_E \sum_B \sum_A P(B,E,A,G,W)}$

$\sum_A \sum_E \sum_B P(B,E,A,G,W) = P(W,G)$

$P(A,B,E,W,G) + P(\neg A,B,E,W,G) + P(A,\neg B,E,W,G) + P(\neg A,\neg B,E,W,G) + P(A,B,\neg E,W,G) + P(\neg A,B,\neg E,W,G) + P(A,\neg B,\neg E,W,G) + P(\neg A,\neg B,\neg E,W,G)$

$= 0.06246$

$\sum_G \sum_A \sum_E \sum_B P(B,E,A,G,W) = P(W)$

$P(A,B,E,W,G) + P(\neg A,B,E,W,G) + P(A,\neg B,E,W,G) + P(\neg A,\neg B,E,W,G) + P(A,B,\neg E,W,G) + P(\neg A,B,\neg E,W,G) + P(A,\neg B,\neg E,W,G) + P(\neg A,\neg B,\neg E,W,G) + P(A,B,E,W,\neg G) + P(\neg A,B,E,W,\neg G) + P(A,\neg B,E,W,\neg G) + P(\neg A,\neg B,E,W,\neg G) + P(A,B,\neg E,W,\neg G) + P(\neg A,B,\neg E,W,\neg G) + P(A,\neg B,\neg E,W,\neg G) + P(\neg A,\neg B,\neg E,W,\neg G)$

$= 0.464$

$\sum_A \sum_E \sum_B P(B,E,A,G,\neg W) = P(A,B,E,\neg W,G) + P(\neg A,B,E,\neg W,G) + P(A,\neg B,E,\neg W,G) + P(\neg A,\neg B,E,\neg W,G) + P(A,B,\neg E,\neg W,G) + P(\neg A,B,\neg E,\neg W,G) + P(A,\neg B,\neg E,\neg W,G) + P(\neg A,\neg B,\neg E,\neg W,G)$

$= 0.03621$

$P(G|W) = \frac{P(W,G)}{P(W)} = \frac{0.06246}{0.464} = 0.134$

$P(G|\neg W) = \frac{P(G,\neg W)}{P(\neg W)} = \frac{0.03621}{1-0.464} = 0.068$

(2) $P(B|W,G,A) = \frac{P(A,B,W,G)}{P(W,G,A)} = \frac{\sum_E P(A,B,W,G,E)}{\sum_E \sum_B P(A,B,W,G,E)}$

$\sum_E P(A, B, E, W, G) = P(A, B, E, W, G) + P(A, B, \neg E, W, G) = 0.02388$

$\sum_E \sum_B P(A, B, E, W, G) = P(A, B, E, W, G) + P(A, \neg B, E, W, G) + P(A, B, \neg E, W, G) + P(A, \neg B, \neg E, W, G)$

$= 0.044$

$P(A, B) = \sum_E \sum_W \sum_G P(A, B, E, W, G)$

$= P(A, B, E, W, G) + P(A, B, \neg E, W, G) + P(A, B, E, \neg W, G) + P(A, B, \neg E, \neg W, G) + P(A, B, E, W, \neg G) + P(A, B, \neg E, W, \neg G) + P(A, B, E, \neg W, \neg G)$

$= 0.09003$

$P(A) = \sum_B \sum_E \sum_W \sum_G P(A, B, E, W, G)$

$= P(A, B, E, W, G) + P(A, \neg B, E, W, G) + P(A, B, \neg E, W, G) + P(A, B, E, \neg W, G) + P(A, \neg B, \neg E, W, G) + P(A, \neg B, E, \neg W, G) + P(A, B, \neg E, \neg W, G) + P(A, \neg B, E, W, \neg G) + P(A, \neg B, \neg E, \neg W, G) + P(A, \neg B, E, \neg W, \neg G) + P(A, B, \neg E, \neg W, \neg G) + P(A, \neg B, \neg E, \neg W, \neg G)$

$= 0.1575$

$P(B \mid W, G, A) = \frac{0.02388}{0.044} = 0.5364$

$P(B \mid A) = \frac{0.09025}{0.1575} = 0.5564 = P(B \mid W, G, A)$

3. $P(B \mid W) = \frac{P(B, W)}{P(W)}$

$P(B, W) = \sum_A \sum_G \sum_E P(A, B, G, W, E)$

$= P(A, B, E, W, G) + P(\neg A, B, E, W, G) + P(A, B, \neg E, W, G) + P(A, B, E, \neg W, G) + P(\neg A, B, \neg E, W, G) + P(A, \neg B, E, W, \neg G) + P(\neg A, B, E, W, \neg G) + P(\neg A, B, \neg E, W, G)$

$= 0.07781$

$P(B \mid W) = \frac{P(B, W)}{P(W)} = \frac{0.07781}{0.464} = 0.1672$

$P(B \mid A, G, W) = 0.5564$ obviously they are not equal

4. $P(E \mid A, B) = \frac{P(E, A, B)}{P(A, B)}$

$P(E, A, B) = \sum_W \sum_{G_1} P(A, B, E, W, G_1)$

$= P(A, B, E, W, G_1) + P(A, B, E, \neg W, G_1) + P(A, B, E, W, \neg G_1) + P(A, B, E, \neg W, \neg G_1)$
$= 0.00475$

$P(A, B) = \sum_E \sum_W \sum_{G_1} P(A, B, E, W, G_1)$

$= P(A, B, E, W, G_1) + P(A, B, \neg E, W, G_1) + P(A, B, E, \neg W, G_1) + P(A, B, E, W, \neg G_1) +$
$P(A, B, \neg E, \neg W, G_1) + P(A, B, \neg E, W, \neg G_1) + P(A, B, E, \neg W, \neg G_1) + P(A, B, \neg E, \neg W, \neg G_1)$
$= 0.09025$

$P(E, A) = \sum_B \sum_W \sum_{G_1} P(A, B, E, W, G_1)$

$= P(A, B, E, W, G_1) + P(A, \neg B, E, W, G_1) + P(A, B, E, \neg W, G_1) + P(A, B, E, W, \neg G_1) +$
$P(A, \neg B, E, \neg W, G_1) + P(A, \neg B, E, W, \neg G_1) + P(A, B, E, \neg W, \neg G_1) + P(A, \neg B, E, \neg W, \neg G_1)$
$= 0.00925$

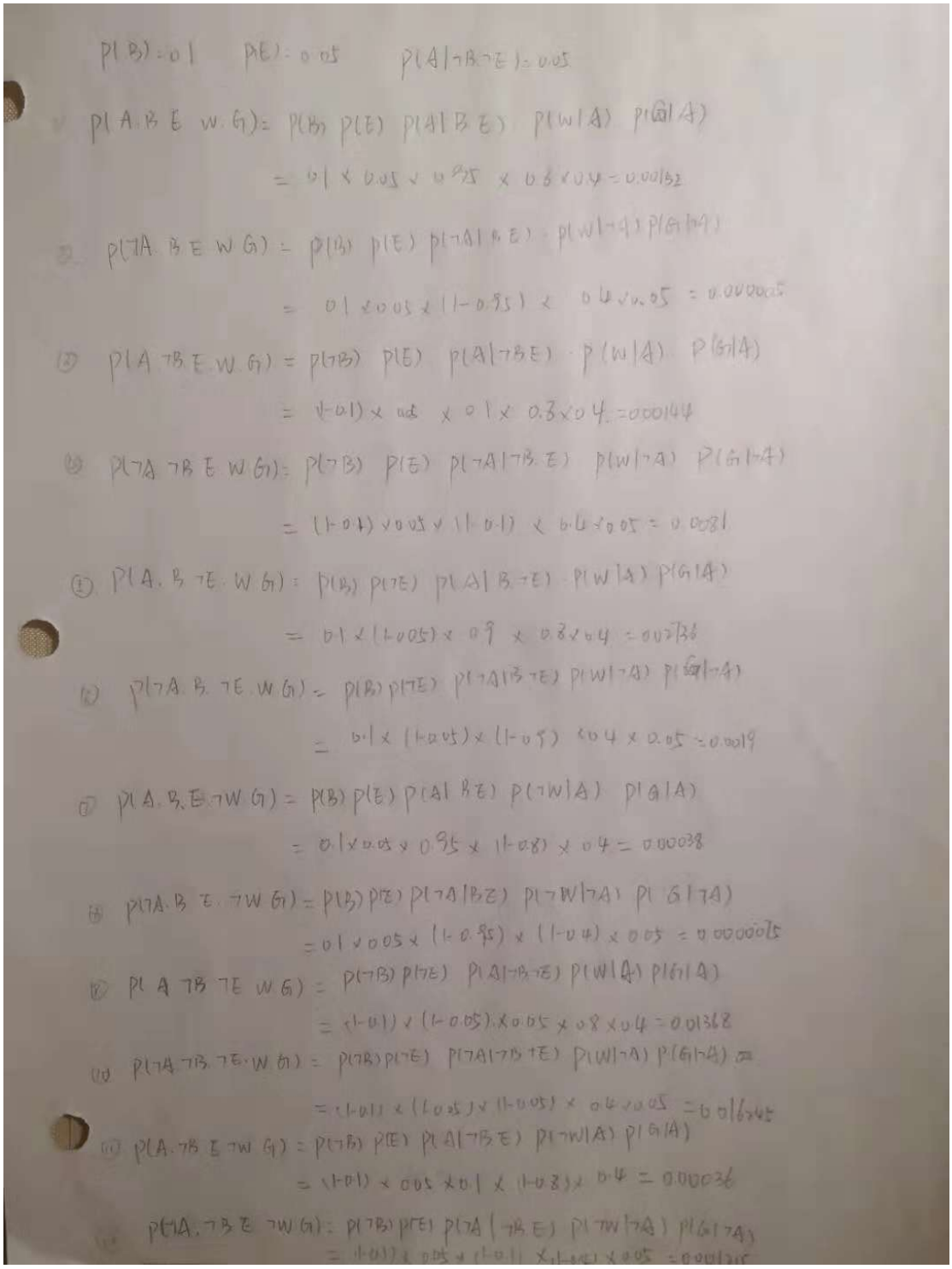
$P(A) = 0.1375$

$P(E \mid A, B) = \frac{P(A, B, E)}{P(A, B)} = \frac{0.00475}{0.09025} = 0.0526$

$P(E \mid A) = \frac{P(A, E)}{P(A)} = \frac{0.00925}{0.1375} = 0.067$

Obviously they are not equal

appendix



⑰ $P(A \neg B, E, W, \neg G) = P(\neg B)\,P(E)\,P(A|\neg B, E)\,P(W|A)\,P(\neg G|A)$

$= (1-0.1) \times 0.05 \times 0.1 \times 0.8 \times (1-0.4) = 0.00216$

⑱ $P(\neg A, \neg B, E, W, \neg G) = P(\neg B)\,P(E)\,P(\neg A|\neg B, E)\,P(W|\neg A)\,P(\neg G|\neg A)$

$= (1-0.1) \times 0.05 \times (1-0.1) \times 0.1 \times (1-0.05) = 0.015219$

⑲ $P(A, B, \neg E, \neg W, G) = P(B)\,P(\neg E)\,P(A|B, \neg E)\,P(\neg W|A)\,P(G|A)$

$= 0.1 \times (1-0.05) \times 0.9 \times (1-0.8) \times 0.4 = 0.006834$

⑳ $P(A, B, \neg E, W, \neg G) = P(A)\,P(\neg E)\,P(A|B, \neg E)\,P(W|A)\,P(\neg G|A)$

$= 0.1 \times (1-0.05) \times 0.9 \times 0.8 \times 0.6 = 0.041904$

㉑ $P(\neg A, B, \neg E, \neg W, G) = P(B)\,P(\neg E)\,P(\neg A|B, \neg E)\,P(\neg W|\neg A)\,P(G|\neg A)$

$= 0.1 \times (1-0.05) \times (1-0.9) \times (1-0.4) \times 0.05 = 0.000285$

㉒ $P(\neg A, B, \neg E, W, \neg G) = P(B)\,P(\neg E)\,P(\neg A|B, \neg E)\,P(W|\neg A)\,P(\neg G|\neg A)$

$= 0.1 \times (1-0.05) \times (1-0.9) \times 0.4 \times (1-0.05) = 0.003611$

㉓ $P(A, B, E, \neg W, \neg G) = P(B) \times P(E) \times P(A|B, E) \times P(\neg W|A)\,P(\neg G|A)$

$= 0.1 \times (1-0.05) \times 0.5 \times (1-0.8) \times (1-0.4) = 0.000057$

㉔ $P(\neg A, B, E, \neg W, \neg G) = P(B)\,P(E)\,P(\neg A|B, E)\,P(\neg W|\neg A)\,P(\neg G|\neg A)$

$= 0.1 \times 0.05 \times (1-0.95) \times (1-0.6) \times (1-0.05) = 0.000142$

㉕ $P(A, \neg B, E, \neg W, \neg G) = P(\neg B)\,P(E)\,P(A|\neg B, E)\,P(\neg W|A)\,P(\neg G|A)$

$= (1-0.1) \times 0.05 \times 0.1 \times (1-0.8) \times (1-0.4) = 0.00054$

㉖ $P(\neg A, B, \neg E, \neg W, G) = P(B)\,P(\neg E)\,P(\neg A|B, \neg E)\,P(\neg W|\neg A)\,P(G|\neg A)$

$= 0.1 \times (1-0.05) \times (1-0.9) \times (1-0.4) \times 0.4 = 0.002166$

㉗ $P(A, \neg B, \neg E, \neg W, \neg G) = P(A)\,P(\neg E)\,P(\neg B|A, \neg E)\,P(\neg W|A)\,P(\neg G|A)$

$= 0.1 \times (1-0.05) \times (1-0.9) \times (1-0.8) \times (1-0.4) = 0.000513$

㉘ $P(\neg A, \neg B, E, \neg W, \neg G) = P(\neg B)\,P(E)\,P(\neg A|\neg B, E)\,P(\neg W|\neg A)\,P(\neg G|\neg A)$

$= (1-0.1) \times 0.05 \times (1-0.1) \times (1-0.4) \times (1-0.05) = 0.023085$

㉙ $P(\neg A, \neg B, \neg E, \neg W, G) = P(\neg B)\,P(\neg E)\,P(\neg A|\neg B, \neg E)\,P(\neg W|\neg A)\,P(G|\neg A)$

$= (1-0.1) \times (1-0.05) \times (1-0.9) \times (1-0.4) \times 0.4 = 0.020577$

(26) $P(A \neg B \neg E \neg W G) = P(\neg E)\,P(\neg B)\,P(A|\neg B \neg E)\,P(\neg W|A)\,P(G|A)$

$= (1-0.05) \times (1-0.1) \times 0.05 \times (1-0.8) \times 0.4 = 0.00342$

(27) $P(A \neg B \neg E W \neg G) = P(\neg B)\,P(\neg E)\,P(A|\neg B \neg E)\,P(W|A)\,P(\neg G|A)$

$= (1-0.1) \times (1-0.05) \times 0.05 \times 0.8 \times (1-0.4) = 0.02052$

(28) $P(A B \neg E \neg W \neg G) = P(B)\,P(\neg E)\,P(A|\neg E B)\,P(\neg W|A)\,P(\neg G|A)$

$= 0.1 \times (1-0.05) \times 0.9 \times (1-0.8) \times (1-0.4) = 0.01026$

(29) $P(A B E W \neg G) = P(B)\,P(E)\,P(A|E B)\,P(W|A)\,P(\neg G|A)$

$= 0.1 \times 0.05 \times 0.8 \times (1-0.4) = 0.00228$

(30) $P(\neg A B E W \neg G) = P(B)\,P(E)\,P(\neg A|E B)\,P(W|\neg A)\,P(\neg G|\neg A)$

$= 0.1 \times 0.05 \times (1-0.95) \times 0.4 \times (1-0.05) = 0.0000285$

(31) $P(\neg A \neg B \neg E W \neg G) = P(\neg B)\,P(\neg E)\,P(\neg A|\neg B \neg E)\,P(W|\neg A)\,P(\neg G|\neg A)$

$= (1-0.1) \times (1-0.05) \times (1-0.05) \times 0.4 \times (1-0.05) = 0.308655$

(32) $P(\neg A \neg B \neg E \neg W \neg G) = P(\neg B)\,P(\neg E)\,P(\neg A|\neg B \neg E)\,P(\neg W|\neg A)\,P(\neg G|\neg A)$

$= (1-0.1) \times (1-0.05) \times (1-0.05) \times (1-0.4) \times (1-0.5)$

$= 0.4629825$