Q1.

1.

State: a complete tour of all the cities starting from city A and ending with A. And the tour visits each city exactly once.

Neighbor relation : swapping 2 nodes in order. For example, when the state is A, B, C, D, A the neighbors are ①A, C, B, D, A; ②A, D, B, A;③A, B, D, C, A

Cost function: the sum of the cost of all the edges on the tour.

2.

The required results are in the following tables. We can clearly observe that the average steps and average quality are increasing as the number of city increasing, which mean the model performs worse as the number of cities increase. And the average percentages are around 0, which means the hill climbing can not find the solution as good as NEOS. And another trend is the total length of tour increases, because of the increasing number of cities.

table1: the result of 14 cities

14 instance				
	count-steps	average_length	average quality	average percentage
1	5.51	544.1529693	1.722003067	0%
2	5.54	636.9685076	1.965952184	1%
3	5.58	620.6935072	1.847302105	0%
4	6	465.2858738	1.458576407	0%
5	5.44	543.4642287	1.548331136	0%
6	5.68	515.7394729	1.65832628	0%
7	5.56	491.1359157	1.805646749	0%
8	5.32	536.7758915	1.486913827	0%
9	5.53	398.9749066	1.456112798	0%
10	5.45	487.2424794	1.513175402	0%
average	5.561	524.0433753	1.646233995	0.1%

table2: the result of 15 cities

15 instance				
	count-steps	average_length	average quality	average percentage
1	6.28	522.0592232	1.667920841	0%
2	5.7	557.4160252	1.752880582	0%
3	6.04	504.9109745	1.796836208	0%
4	5.6	587.1969657	1.812336314	0%
5	5.27	603.9540658	1.597762079	1%
6	4.88	568.1708492	1.952477145	0%
7	5.87	630.5131951	1.811819526	0%
8	5.25	420.6749327	1.230043663	0%
9	5.88	537.3902771	1.52235206	0%
10	5.44	563.2759228	1.733156686	0%
average	5.621	549.5562431	1.68775851	0.1%

table3: the result of 16 cities

16 instance				
	count-steps	average_length	average quality	average percentage
1	6.36	596.0252307	1.475309977	0%
2	5.67	720.5226121	2.041140544	0%
3	5.65	620.2958321	1.718271003	0%
4	5.98	637.5572985	1.826811744	0%
5	6.52	658.5030336	1.839393949	0%
6	5.77	557.6310798	1.621020581	0%
7	5.68	659.461465	1.767993204	0%
8	6.48	605.4979839	1.705628124	0%
9	6.26	517.8814908	2.13119955	0%
10	5.91	594.2506151	1.80075944	0%
average	6.028	616.7626642	1.792752811	0%

3.

For the first instance of 14 cities, the average length founded by hill climbing is 544, which is greater than the result calculated by NEOS solver(316.6776082168322). So we can say it is a local optimum.

4.

I will use the length of distance as the criteria to measure the performance of algorithm.

As we can see the results obtained by hill climbing with tabu list, on average the average length of tour calculated by tabu search is smaller than that by simple hill climbing. So the hill climbing algorithm performs better if it allows sideway moves and/or maintains a tabu list.

table4: the result of average length of tour calculated by hill climbing with tabu list

Number of city	14	15	16
Average length	318.5774	329.2234	306.6512

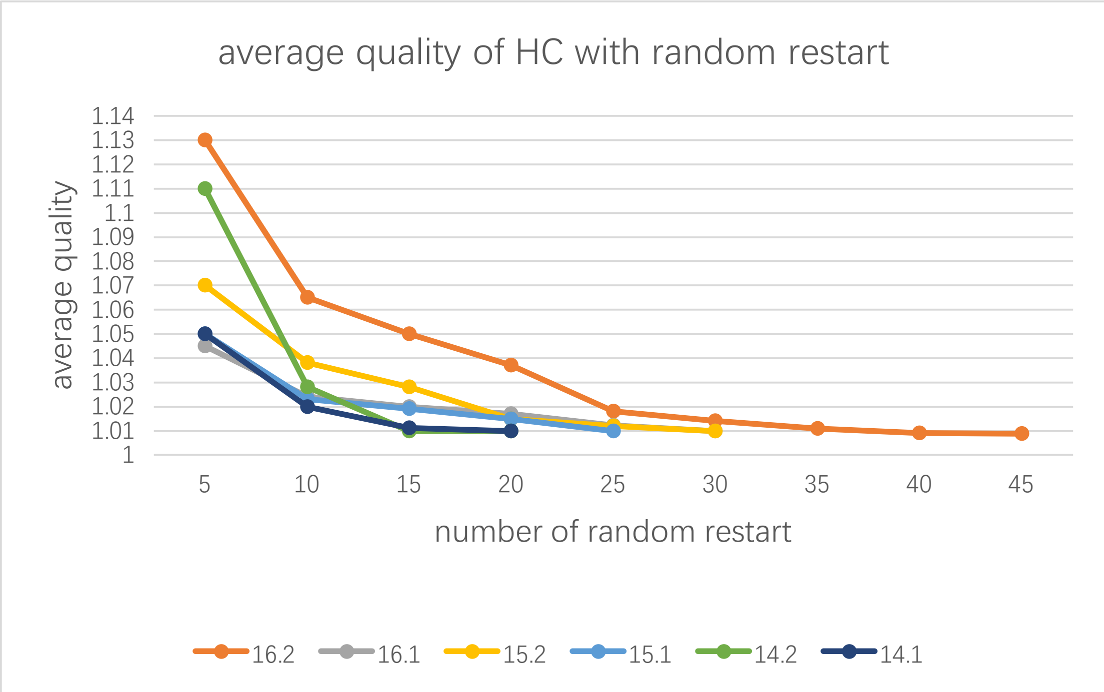
5.

The sufficient number of restarts to ensure that the solution is within 1% of the best solution found by any alternative algorithm.
16 city instance 2: 35
16 city instance 1: 25
15 city instance 2: 30
15 city instance 1: 25
14 city instance 2: 15
14 city instance 1: 15

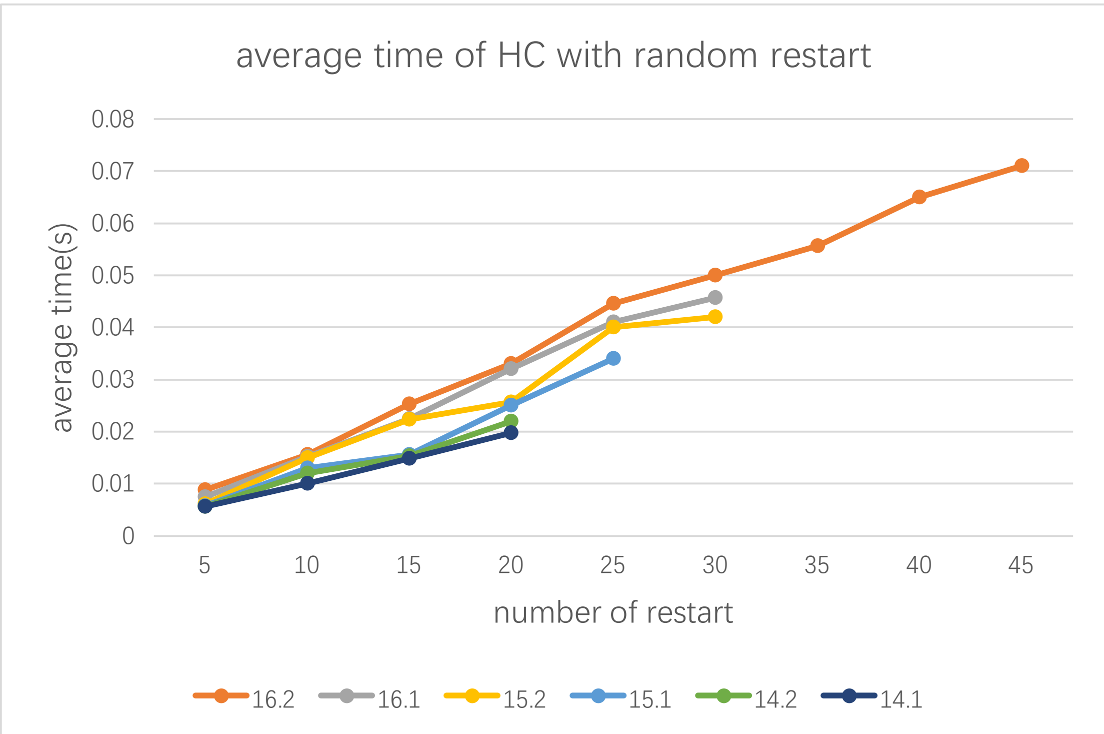
As shown on graph, the average quality is decreasing and average executive time increases as the number of random restart increasing.
So we can conclude that if we want to get a smaller quality we need more time.

The bigger number of random restart will certainly let us find the global optimum, but it will cost too much time to search the state space.
So considering the execution time and quality , I will choose the number of restarts in following way:
16 city instance 2: 40
16 city instance 1: 30
15 city instance 2: 35
15 city instance 1: 30
14 city instance 2: 20

14 city instance 1: 20



graph1: average quality of HC with random restart

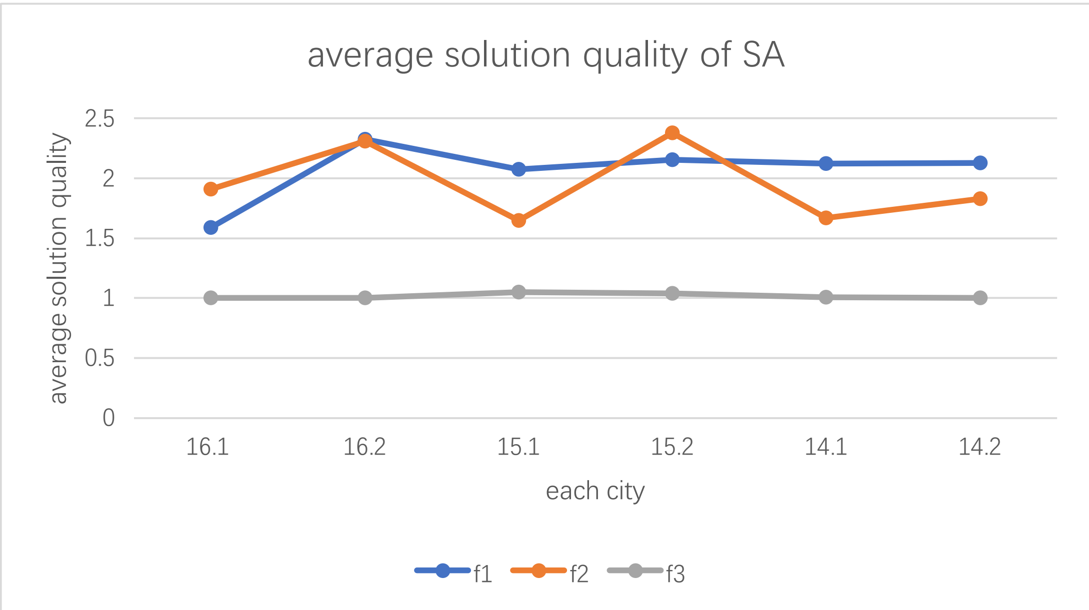


graph2: average time of HC with random restart

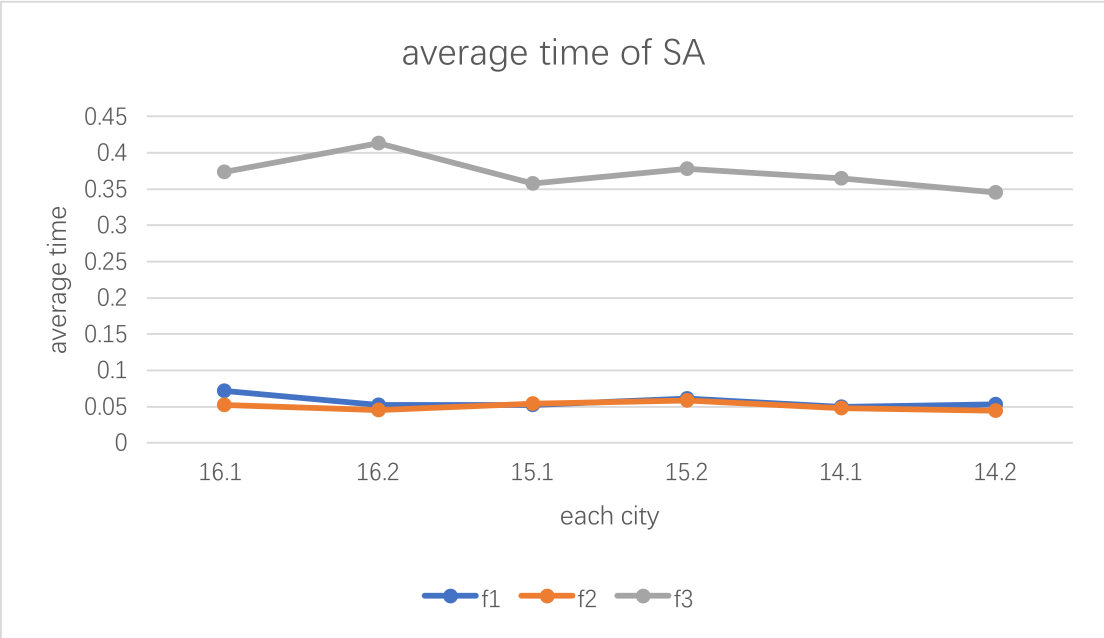
6.
the 3 ways to decrease T
f1: T(t)=T/ln(1+t)
f2: T(t)=T*exp(-t)
f3: T(t)=T*0.9*t

As observed in graph, the function with larger quality(the performance worse) need less time. So there a trade-off between the execution time and the choice of annealing schedule

In fact the f3 will be the choice for 3 instances. Because f3 allows us to find a best solution in very short time. Although other 2 functions take less time their performance are not well(the average qualities of f1 and f2 are almost as twice as f3). So I will take the function with quality near 1 instead of saving 0.3s to get result with quality of 2.



graph3: average solution quality of SA



graph3: average time of SA

7.

simple hill climbing	HC with tabu list	HC with random restart	SA with ln decrease	SA with exponential decrease	SA with linear decrease (decrease rate is 0.9998)
1663.61	1495.16	1758.6	1592.5	1579.7	1321.36
7.2s	1.55m	4.7min	22.79s	0.177s	1.7min

Considering the execution time and performance of algorithm I would choose SA with linear decrease (decrease rate is 0.9998)

Q2

1. $P(A, B, C, D) = P(D|A,B,C) \cdot P(C|A,B) \cdot P(B|A) \cdot P(A)$

$= 0.6 \times 0.7 \times 0.8 \times 0.6$

$= 0.2016.$ (the other possibilities please see in 2nd question -- (2) (below).

2. $P(B,A) = P(B|A) \cdot P(A) = 0.8 \times 0.6 = 0.48$

$P(B,\neg A) = P(B|\neg A) \cdot P(\neg A) = 0.4 \times (1-0.6) = 0.16$

$P(B) = P(B,A) + P(B,\neg A) = 0.64.$

(2) (b) $\boxed{P(C,A,B) = P(C|A,B) \cdot P(B|A) \cdot P(A) = 0.7 \times 0.8 \times 0.6 = 0.336}$

$\boxed{P(C,A,\neg B) = P(C|A,\neg B) \cdot P(\neg B|A) \cdot P(A) = 0.9 \times (1-0.8) \times 0.6 = 0.108}$

$P(C,\neg A,B) = P(C|\neg A,B) \cdot P(B|\neg A) \cdot P(\neg A) = 0.7 \times 0.4 \times (1-0.6) = 0.112.$

$P(C,\neg A,\neg B) = P(C|\neg A,\neg B) \cdot P(\neg B|\neg A) \cdot P(\neg A) = 0.9 \times (1-0.4) \times (1-0.6) = 0.216$

(b) $\boxed{P(C,A) = P(C,A,B) + P(C,A,\neg B) = 0.444}$

$P(C,\neg A) = P(C,\neg A,B) + P(C,\neg A,\neg B) = 0.338$

$P(C) = P(C,A) + P(C,\neg A) = 0.772$

(a) $P(D) = P(D,A) + P(D,\neg A)$

$= P(D,A,B) + P(D,A,\neg B) + P(D,\neg A,B) + P(D,\neg A,\neg B)$

$= P(D,A,B,C) + P(D,A,B,\neg C) + P(D,A,\neg B,C) + P(D,A,\neg B,\neg C) + P(D,\neg A,B,C) +$
$P(D,\neg A,B,\neg C) + P(D,\neg A,\neg B,C) + P(D,\neg A,\neg B,\neg C)$

$P(D,A,B,C) = 0.2016$ (shown in Q1)

$P(D,A,B,\neg C) = P(D|A,B,\neg C) \cdot P(\neg C|A,B) \cdot P(B|A) \cdot P(A)$

$= 0.6 \times (1-0.7) \times 0.8 \times 0.6 = 0.0864.$

$P(D,A,\neg B,C) = P(D|A,\neg B,C) \times P(C|A,\neg B) \cdot P(\neg B|A) \cdot P(A)$

$= 0.2 \times 0.9 \times (1-0.8) \times 0.6 = 0.0216$

$P(D,A,\neg B,\neg C) = P(D|A,\neg B,\neg C) \cdot P(\neg C|A,\neg B) \cdot P(\neg B|A) \cdot P(A)$

$= 0.2 \times (1-0.9) \times (1-0.8) \times 0.6 = 0.0024.$

$P(D,\neg A,B,C) = P(D|\neg A,B,C) \cdot P(C|\neg A,B) \cdot P(B|\neg A) \cdot P(\neg A)$

$= 0.6 \times 0.7 \times 0.4 \times (1-0.6) = 0.0672.$

$P(D,\neg A,B,\neg C) = P(D|\neg A,B,\neg C) \cdot P(\neg C|\neg A,B) \cdot P(B|\neg A) \cdot P(\neg A)$

$= 0.6 \times (1-0.7) \times 0.4 \times (1-0.6) = 0.0288$

$P(D,\neg A,\neg B,C) = P(D|\neg A,\neg B,C) \cdot P(C|\neg A,\neg B) \cdot P(\neg B|\neg A) \cdot P(\neg A)$

$= 0.2 \times 0.9 \times (1-0.4) \times (1-0.6) = 0.0432$

results for Q2-1 $\Leftarrow$

$P(A,B,C,\neg D) = 0.1344$ $P(\neg A,B,\neg C,D) = 0.0288$ $P(\neg A,B,C,\neg D) = 0.0448.$ $P(A,\neg B,C,\neg D) = 0.0864$

$P(\neg A,\neg B,\neg C,D) = 0.0048$ $P(\neg A,\neg B,C,\neg D) = 0.1728$ $P(A,\neg B,\neg C,D) = 0.096$ $P(\neg A,\neg B,\neg C,D) = 0.0192$

$P(D,\neg A\ \neg B\ \neg C) = P(D|\neg A,\neg B,\neg C)\cdot P(\neg C|\neg A\ \neg B)\cdot P(\neg B|\neg A)\cdot P(\neg A)$

$= 0.2\times(1-0.9)\times(1-0.4)\times(1-0.6)$

$= 0.0048 \approx 0.005$

So. $P(D) = 0.456$.

(c) $P(A|\neg B) = \frac{P(A,\neg B)}{P(\neg B)} = \frac{P(A)-P(A,B)}{1-P(B)} = \frac{0.6-0.48}{1-0.64} = \frac{0.12}{0.36} = \frac{1}{3} \approx 0.333$

(d) $P(A|B,\neg C) = \frac{P(A,B,\neg C)}{P(B,\neg C)}$.

$P(B,\neg C) = P(A,B,\neg C) + P(\neg A,B,\neg C)$

$= P(\neg C|A,B)\cdot P(B|A)\cdot P(A) + P(\neg C|\neg A,B)\cdot P(B|\neg A)\cdot P(\neg A)$

$= (1-0.7)\times 0.8\times 0.6 + (1-0.7)\times 0.4\times(1-0.6) =$

$= 0.144 + 0.048 = 0.192$.

$P(A,B,\neg C) = P(A,B) - P(A,B,C) = 0.48 - 0.336 = 0.144$

So $P(A|B,\neg C) = \frac{P(A,B,\neg C)}{P(B,\neg C)} = \frac{0.144}{0.192} = 0.75$.

(e) $P(A,\neg B|C) = \frac{P(A,\neg B,C)}{P(C)} = \frac{0.108}{0.772} = 0.14$.

(f) $P(\neg A\ C|\neg B,\neg D) = \frac{P(\neg A,\neg B,C,\neg D)}{P(\neg B,\neg D)} = \frac{0.1728}{0.3024} = 0.571$

3 (a) $P(A) = 0.6$ $P(C) = 0.772$. $P(A,C) = 0.444$

$P(A)\cdot P(C) \neq P(A,C)$. So A and C are not independent.

(b) A and C are conditionally indepent when B is given

$P(C|A\wedge B) = P(C|\neg A\wedge B)$

(c) $P(D,C) = P(A,B,C,D) + P(\neg A,B,C,D) + P(\neg A,\neg B,C,D) + P(A,\neg B,C,D)$

$= 0.2016 + 0.0672 + 0.0432 + 0.0216 = 0.3336$.

$P(D) = 0.456$ $P(C) = 0.772$ $P(C)\cdot P(D) \neq P(C,D)$

So C and D are not independent

d) $P(D|A.B.C) = P(D|A.B.\neg C)$

$P(D|A.\neg B.C) = P(D|A.\neg B.\neg C)$

$P(D|\neg A.B.C) = P(D|\neg A.B.\neg C)$

$P(D|\neg A.\neg B.C) = P(D|\neg A.\neg B.\neg C)$

So D and C are independent when A and B are given. But when we only

e) ① $P(A)=0.6$ $P(B)=0.64$. $P(A.B) \neq P(A).P(B)$ So they are not indepent

$P(A.B)=0.48$.

$P(A|B.C) = \frac{P(A.B.C)}{P(B.C)} = \frac{0.336}{0.64-0.192} = 0.75$ $\quad (P(B.C) = P(B) - P(B.\neg C))$

$P(A|\neg B.C) = \frac{P(A.\neg B.C)}{P(\neg B.C)} = \frac{P(A.C) - P(A.B.C)}{P(C) - P(B.C)} = \frac{0.444-0.336}{0.772-(0.64-0.192)} = \frac{0.108}{0.324} = 0.333$

Since $P(A|B.C) \neq P(A|\neg B.C)$ So A and B are not conditionally independent when C is given.

② $P(A|B.D) = \frac{P(A.B.D)}{P(B.D)} = \frac{P(A.B.D)}{P(\neg A.B.D)+P(A.B.D)}$ $\quad P(A|\neg B.D) = \frac{P(A.\neg B.D)}{P(\neg B.D)}$

$P(A.B.D) = P(A.B.C.D) + P(A.B.\neg C.D)$
$= 0.216 + 0.0864 = 0.3024$.

$P(\neg A.B.D) = P(\neg A.B.C.D) + P(\neg A.B.\neg C.D)$
$= 0.0672 + 0.0288 = 0.096$.

$P(B.D) = P(\neg A.B.D) + P(A.B.D) = 0.3984$.

So $P(A|B.D) = \frac{P(A.B.D)}{P(B.D)} = \frac{0.3024}{0.3984} = 0.759$

$P(A.\neg B.D) = P(A.\neg B.C.D) + P(A.\neg B.\neg C.D)$
$= 0.0216 + 0.0024 = 0.024$

$P(\neg B.D) = P(D) - P(B.D) = 0.456 - 0.3984 = 0.0576$

$P(A|\neg B.D) = \frac{P(A.\neg B.D)}{P(\neg B.D)} = \frac{0.024}{0.0576} = 0.417$

Since $P(A|B.D) \neq P(A|\neg B.D)$ So A and B are not conditinally indepent when D is given

(iii) $$P(A \mid B.C.D) = \frac{P(A.B.C.D)}{P(A.B.C.D) + P(\neg A.B.C.D)}$$

$$= \frac{0.2016}{0.2016 + 0.0672} = \frac{0.2016}{0.2688} = 0.75.$$

$$P(A \mid \neg B.C.D) = \frac{P(A.\neg B.C.D)}{P(A.\neg B.C.D) + P(\neg A.\neg B.C.D)} = \frac{0.0216}{0.0216 + 0.0432} = \frac{0.0216}{0.0648} = 0.33$$

So. A and B are not conditionally indepent when C and D are given

So. in total A and B are not contionally independent