Agricultural Trade and Industrial Development

Angus C. Chu Yuichi Furukawa Pietro F. Peretto Rongxin Xu

November 20, 2024

Abstract

Is agricultural productivity conducive to economic development? We develop a two-country open-economy Schumpeterian growth model with endogenous takeoff. With agricultural trade and a subsistence requirement, higher domestic agricultural productivity has ambiguous effects on the economy's takeoff and its transitional growth rate if domestic and imported agricultural goods are substitutes. Without the subsistence requirement, higher domestic agricultural productivity delays industrialization and lowers transitional growth by increasing domestic demand for agricultural labor. This specialization force works in the opposite direction of the change in domestic consumption pattern governed by the subsistence requirement, which tends to release labor from agriculture. Without agricultural trade, the specialization force is absent and the subsistence requirement on agricultural consumption implies that higher domestic agricultural productivity reallocates labor from agriculture to industry, hastening industrialization and raising transitional growth. Using cross-country panel-data, we find that agricultural productivity has a direct positive effect on economic growth but this positive effect weakens and even becomes negative when reliance on agricultural imports is sufficiently high. Simulating the calibrated model, we find that improvement in domestic agricultural productivity accounts for about one-third of the changes in TFP growth in China and Japan, respectively, and more so for their main trading partner, the US.

JEL classification: O30, O40, F43
Keywords: international trade, agricultural productivity, innovation, endogenous takeoff

Chu: anguscccc@gmail.com. Department of Economics, University of Macau, Macau.
Furukawa: you.furukawa@gmail.com. Faculty of Economics, Chuo University, Tokyo, Japan.
Peretto: peretto@econ.duke.edu. Department of Economics, Duke University, Durham, United States.
Xu: rongxin.xu@connect.um.edu.mo. Department of Economics, University of Macau, Macau.

1 Introduction

Is high agricultural productivity conducive to the industrial development of an economy? Early studies by Nurkse (1953) and Schultz (1953) argue that an improvement in agricultural productivity hastens the process of industrialization because it reallocates labor from farm to factory by changing the consumption pattern of households.[1] Subsequent studies by Mokyr (1976), Field (1978) and Wright (1979) argue that high agricultural productivity causes the economy to specialize in agricultural production and thus delays industrialization because it reallocates labor from factory to farm.[2] Both of these theoretical predictions have received empirical support; see for example, Foster and Rosenzweig (2004, 2008) and Bustos *et al.* (2016, 2020). Thus far, the literature has not been able to answer unambiguously our starting question because of the tension between two forces: reallocation from farm to factory due the changing domestic consumption pattern and reallocation from factory to farm due to the changing specialization in international trade.

To make progress, in this study we develop an open-economy Schumpeterian growth model with endogenous takeoff that allows us to explore the different effects of agricultural productivity on the entire transition, from pre-industrial stagnation to modern innovation-driven growth, of economies engaged in international trade. In particular, the model has two countries, Home and Foreign, that trade both industrial and agricultural goods. Trade is asymmetric, however, in that Foreign does not import the Home agricultural good. Moreover, Home has a subsistence requirement for consumption of its agricultural good whereas Foreign does not have such a constraint. This realistic asymmetric structure allows us to capture cleanly the competition between the two forces identified by the literature.[3]

Our main finding is that, if the elasticity of substitution between domestic and foreign agricultural goods is less than one, higher Home agricultural productivity hastens Home industrialization and raises its transitional growth rate while it delays Foreign industrialization and lowers its transitional growth rate. On the other hand, higher Foreign agricultural productivity delays Home industrialization and lowers its transitional growth rate while it hastens Foreign industrialization and raises its transitional growth rate. When the elasticity of substitution between domestic and foreign agricultural goods is greater than one instead, higher Home agricultural productivity has ambiguous effects on Home while it hastens Foreign industrialization and raises its transitional growth rate. The effects of higher Foreign agricultural productivity are the opposite of those in the low elasticity of substitution case.

An important aspect of our analysis is that the economy's steady-state growth rate is always independent of the level of agricultural productivity due to the scale-invariance of our Schumpeterian growth model with endogenous market structure. This property highlights the importance of considering the entire transition dynamics of our trading economies: to see the growth effects of agricultural productivity, we must look at the transitional growth rate because the steady-state growth rate does not respond to factors that operate through the scale

[1] See Chu, Peretto and Wang (2022) for a recent study and a discussion of earlier studies in this literature.

[2] See Matsuyama (1992) for a theoretical formalization of this idea.

[3] For example, if we let Foreign be the US and Home be either China or Japan, as we do in our quantitative analysis, this characterization is reasonable because the US imports a negligible amount of agricultural products from China and Japan, while China and Japan are among the largest importers of US agricultural goods. Similarly, ruling out the agricultural subsistence requirement is reasonable for a rich country like the US since it no longer affects household behavior.

of economic activity.

The mechanism driving the ambiguous results on industrialization and transitional growth is the competition between the change in the Home consumption pattern, which is governed by the subsistence requirement for its agricultural good and the change in the degree of specialization due to international trade. The former always reallocates labor from farm to factory. The latter reallocates labor in a direction that depends on whether domestic and foreign agricultural goods are complements or substitutes. In the case where domestic and foreign agricultural goods are complements, the demand for the agricultural good is inelastic. In this case, as Home agricultural productivity rises and the price of its agricultural good falls, the quantity sold rises less than one for one. In the case where domestic and foreign agricultural goods are substitutes, the demand for the agricultural good is elastic. In this case, as Home agricultural productivity rises and the price of its agricultural good falls, the quantity sold rises more than one for one. The end result is that the strength of the specialization force depends on the elasticity of the demand for the good whose supply rises due to higher productivity. Consequently, when demand is inelastic (i.e., domestic and foreign agricultural goods are complements), an improvement in the Home agricultural productivity causes the consumption pattern force and the specialization force to push in the same direction in Home, leading Home to reallocate labor from farm to factory, while Foreign reallocates labor from factory to farm. When demand is elastic (i.e., domestic and foreign agricultural goods are substitutes), an improvement in the Home agricultural productivity causes the consumption pattern force and the specialization force to push in opposite directions in Home, producing an ambiguous reallocation effect, while Foreign unambiguously reallocates labor from farm to factory. To test this insight, we look at two special cases that isolate the two forces.

In the first special case, we shut down the Home subsistence requirement for consumption of its agricultural good. The effects are as in the general case with the key difference that the effects of higher Home agricultural productivity are no longer ambiguous. When the elasticity of substitution between domestic and foreign agricultural goods is greater than one (i.e., domestic and imported agricultural goods are substitutes) as the data suggests, an improvement in agricultural productivity delays industrial development because the economy specializes even further in agricultural production and thus reallocates labor from factory to farm. This scenario is consistent with Mokyr (1976), Field (1978) and Wright (1979). Furthermore, an improvement in the agricultural productivity of its trading partner has the opposite effects on the domestic economy: it hastens domestic industrial development and raises domestic transitional growth rate. All the effects above are reversed when the elasticity is less than one (i.e., domestic and imported agricultural goods are complements). The insight here is that the assumption of no Home subsistence requirement for its agricultural good shuts down one of the two forces highlighted above, the change in the consumption pattern of the Home households, which in this scenario plays no role.

In the second special case, we shut down international agricultural trade. Equivalently, we retain agricultural trade but set the elasticity of substitution between domestic and foreign agricultural goods equal to one. In this scenario, higher Home agricultural productivity hastens Home industrialization and raises its transitional growth rate. The reason is that the dominant force is the change in the consumption pattern of the Home households, which results in a reallocation of labor from farm to factory, because the specialization force is mitigated when there is no agricultural trade or the elasticity of substitution between Home and Foreign agricultural goods is equal to one.[4] This scenario is consistent with Nurkse (1953) and Schultz (1953). Furthermore, this property yields that the Foreign agricultural productivity has no effects on industrialization in Home and Foreign.

To set the stage for our theoretical analysis, we examine the literature's two theoretical predictions using cross-country panel data and find that agricultural productivity has a direct positive growth effect but also an indirect negative growth effect via agricultural trade. Therefore, the overall growth effect of agricultural productivity is ambiguous and becomes negative as the country's reliance on agricultural imports becomes sufficiently high, where we measure reliance as the ratio between the country's net agricultural imports and its domestic agricultural production. In other words, when the country's agricultural imports are small (large) relative to its own agricultural production, higher domestic agricultural productivity stimulates (stifles) industrial development and economic growth as our theory predicts.

To shed further light on the mechanism driving the theory, we calibrate our model to data for the China-US and Japan-US pairs. In the China-US case, agricultural productivity in China has a positive effect on its economic growth due to its relatively low reliance on agricultural imports. In particular, because agricultural consumption relies mostly on domestic agricultural production, rising agricultural productivity enables the Chinese economy to release labor from agricultural production to industrial production by moving Chinese consumers away from the subsistence constraint on consumption of the domestic agricultural good. In the Japan-US case, in contrast, the agricultural productivity of Japan has a negative effect on its economic growth due to its relatively high reliance on agricultural imports. Specifically, higher agricultural productivity causes the Japanese economy to engage in agricultural import substitution and thereby reallocate labor from industrial production to agricultural production. Quantitatively, changes in domestic agricultural productivity explain about one-third of the changes in the growth rate of technology in both China and Japan. Similarly, changes in agricultural productivity in China or Japan also contribute significantly to the increase in the growth rate of technology in the US.

This study contributes to the literature on innovation-led economic growth. Romer (1990) develops the seminal R&D-based growth model of variety expansion. Aghion and Howitt (1992) develop the creative-destruction Schumpeterian growth model of quality improvement.[5] Subsequent studies develop the Schumpeterian growth model with endogenous market structure, which incorporates both variety-expanding and quality-improving innovation; see Peretto (1994, 1998, 1999), Smulders (1994), Smulder and van de Klundert (1995), Dinopoulos and Thompson (1998) and Howitt (1999).[6] Many of these models feature firms that do in-house R&D to fuel incremental innovation (i.e., creative accumulation); the others feature firms that do not do in-house R&D and wait to be replaced by outside challengers (i.e., creative destruction). Garcia-Macia *et al.* (2019) provide the most recent empirical evidence that economic growth comes mostly from creative accumulation rather than creative destruction. Therefore, in this study we contribute to this literature by developing an open-economy creative-accumulation Schumpeterian growth model with an agricultural sector that produces tractable transitional dynamics

[4] Chu, Peretto and Wang (2022) obtains this effect in a closed-economy Schumpeterian growth model. Our contribution here is to examine when this positive effect also shows up in an open-economy setting.

[5] See also the early studies by Grossman and Helpman (1991) and Segerstrom *et al.* (1990).

[6] See Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008) and Ang and Madsen (2011) for empirical evidence that supports the Schumpeterian growth model with both dimensions of innovation.

featuring an endogenous takeoff. We then use the model to explore the effects of agricultural productivity on the endogenous transition from pre-industrial stagnation to innovation-driven growth of economies that engage in industrial and agricultural trade.

This study also contributes to the literature on agricultural productivity, industrialization and economic development. Early studies by Nurkse (1953), Schultz (1953) and Rostow (1959) argue that agricultural productivity growth releases labor from agriculture to industry and serves as an essential engine of industrialization and economic development.[7] Johnston and Mellor (1961), Mellor (1995) and Johnson (1997) echo this view. Subsequent studies formalize it; see for example, Ranis and Fei (1961) for an extended Lewis model with an institutional wage, Murphy *et al.* (1989), Kogel and Prskawetz (2001) and Restuccia *et al.* (2008) for a two-sector general equilibrium model, and Gollin *et al.* (2002, 2007) for a neoclassical growth model with an agricultural sector.[8] Empirical studies supportive of these theoretical developments are Tiffin and Irz (2006), McArthura and McCord (2007), Ravallion and Chen (2007) and Cao and Birchenall (2013). As mentioned, Mokyr (1976), Field (1978) and Wright (1979) stress the importance of international trade and, in contrast to the view just discussed, argue that high agricultural productivity gives rise to specialization in agriculture and delays industrialization. Subsequent studies by Matsuyama (1992), Duranton (1998) and Chesnokova (2007) formalize this idea and find that higher agricultural productivity triggers early industrialization in a closed economy but delays industrialization in an open economy. Foster and Rosenzweig (2004, 2008) provide empirical evidence for this negative relationship between agricultural productivity and economic growth;[9] see also Gollin (2010) for a thorough review of both theoretical and empirical studies in this literature. Despite the richness of the theoretical literature that studies the role of agricultural productivity in structural transformation and economic development driven by capital accumulation, relatively few studies examine its effects on innovation-driven growth. We contribute to this vast literature by developing an open-economy Schumpeterian growth model that allows us to explore the effects of agricultural productivity on innovation-driven growth in the presence of international trade in agricultural goods. The goal is to sort out the relative contribution of the contrasting forces at play.

Finally, this study contributes to the literature on endogenous takeoff and economic growth. The seminal study by Galor and Weil (2000) develops Unified Growth Theory (UGT), which captures the process of transformation from a Malthusian agricultural economy to a modern industrial economy in a single analytical framework. Subsequent studies by Galor and Moav (2002), Galor and Mountford (2008), Galor *et al.* (2009) and Ashraf and Galor (2011) examine the role of different prehistorical and historical characteristics and provide supportive empirical evidence for UGT.[10] In a related literature, Peretto (2015) develops a closed-economy

[7]In the seminal study by Lewis (1955), the agricultural sector is characterized by labor surplus and disguised unemployment. Also, Krugman (1987) and Lucas (1988) argue that the manufacturing sector is characterized by economies of scale and human capital accumulation.

[8]Echevarria (1995, 1997), Kongsamut *et al.* (2001), Lucas (2004) and Ngai and Pissarides (2007) also incorporate an agriculture sector into growth models to explore the structural transformation from agriculture to industry, but they do not consider the role of agricultural productivity on the takeoff of the economy.

[9]Bravo-Ortega and Lederman (2005) find a positive effect of agricultural productivity on growth in non-agricultural sectors in developing countries, but this effect is negative in developed countries. See also Bustos *et al.* (2022) who show that high agricultural productivity causes structural transformation but not innovation in Brazil.

[10]See Galor (2005, 2011) for a comprehensive review of UGT.

Schumpeterian growth model with endogenous takeoff to capture the endogenous transition from pre-industrial stagnation to innovation-driven growth.[11] Chu, Peretto and Wang (2022) develops a Schumpeterian growth model with an agricultural sector to explore how agricultural productivity affects the transition of an economy from pre-industrial stagnation to innovation-driven growth in a closed economy. Chu, Peretto and Xu (2023) develops a small open economy version of the Schumpeterian growth model in Peretto (2015) to explore export-led takeoff and innovation-driven growth. We contribute to this literature by extending the Schumpeterian growth model with endogenous takeoff to the case of a world general equilibrium featuring two countries that trade industrial and agricultural goods.

The rest of this study is organized as follows. Section 2 documents some stylized facts using cross-country panel data. Section 3 presents our open-economy Schumpeterian growth model with an agricultural sector. Section 4 characterizes the effects of agricultural productivity improvement. Section 5 calibrates the model and investigates the quantitative effects of changes in agricultural productivity. Section 6 concludes.

2 Stylized facts

In this section, we use cross-country data to establish some key facts about the relationship between agricultural productivity, agricultural trade and economic growth. We use the following specification:

$$y_{jt} = \kappa_1 A_{jt} + \kappa_2 A_{jt} \times trade_{jt} + \kappa_3 trade_{jt} + \Gamma\Phi_{jt} + \zeta_j + \zeta_t + \varepsilon_{jt},$$

where y_{jt} is a proxy for industrialization or economic growth in country j at time t for which we use the log level of non-agricultural real GDP per capita, or the log level of real GDP per capita, or total factor productivity (TFP). A_{jt} is agricultural productivity in country j at time t measured by an agricultural TFP index. $trade_{jt}$ is the ratio of net agricultural imports to domestic agricultural production in country j at time t,[12] which is our measure of reliance on agricultural imports. We use the initial value of this ratio at time t, because changes in agricultural productivity may affect the agricultural trade pattern. Given that the cyclical fluctuations in annual data may bias the estimation, we consider five years as a period.[13] Our theory predicts that $\kappa_1 > 0$ and $\kappa_2 < 0$. In other words, high agricultural productivity has a positive effect on industrialization and economic growth, but this positive effect weakens and may become negative when an economy relies heavily on agricultural imports. Φ_{jt} denotes the following set of control variables: the log level of capital stock, government spending as a share of GDP, a human capital index, and the depreciation rate of capital stock. The variables ζ_j and ζ_t denote country and time fixed effects, respectively. Finally, ε_{jt} is the error term.

After merging data from the Food and Agricultural Organization (FAO), the Penn World Tables, the U.S. Department of Agriculture and the World Bank Data, we have a sample of

[11] See also the subsequent studies by Iacopetta and Peretto (2021), Chu, Fan and Wang (2020), Chu, Kou and Wang (2020) and Chu, Furukawa and Wang (2022) for different mechanisms that trigger endogenous takeoff in this framework.

[12] This proxy is calculated based on monetary values due to data limitations on agricultural production volume at the aggregate level. The estimated coefficients have the same sign and mostly remain significant at least at the 10% level when using gross agricultural imports.

[13] Our results remain robust when we consider three years as a period. Results are available upon request.

up to 772 observations covering 148 countries for 1991-2020. Table A1 in Appendix A provides the summary statistics. Table 1 reports the estimation results. The dependent variable is the log of non-agricultural real GDP per capita in columns (1)-(2), the log of real GDP per capita in columns (3)-(4), and the TFP index in columns (5)-(6). In all columns, the coefficient κ_1 on agricultural productivity is significantly positive and the coefficient κ_2 on the ratio of agricultural net imports to domestic agricultural production is significantly negative. Therefore, the positive growth effect of domestic agricultural productivity weakens and may even become negative as a country relies more heavily on agricultural imports.

Table 1: Relationship between agricultural productivity, trade and economic growth

	non-agri GDP per capita		GDP per capita		TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{jt}	0.228**	0.336***	0.253***	0.358***	0.194**	0.200***
	(0.102)	(0.086)	(0.095)	(0.087)	(0.082)	(0.075)
$A_{jt} \times trade_{jt}$	-0.104***	-0.167***	-0.087***	-0.144***	-0.099***	-0.116***
	(0.016)	(0.020)	(0.016)	(0.019)	(0.014)	(0.018)
$trade_{jt}$	0.070***	0.102***	0.058***	0.088***	0.014*	0.048***
	(0.016)	(0.018)	(0.017)	(0.017)	(0.008)	(0.014)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓
Observations	657	569	667	578	497	497
R^2	0.9900	0.9950	0.9899	0.9944	0.7333	0.7570

*Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Robust standard errors in parentheses. Standard errors are clustered at the country level.*

Our measure of reliance on imported agricultural goods can be influenced by factors like market prices, trade polices and subsidies because it is based on monetary values. For example, a country exporting a high-value agricultural product while heavily relying on imports for other agricultural products might appear less reliant than it truly is. To address this limitation and ensure the robustness of our findings, we employ another measure based on volume. Given that cereals are the major part of dietary energy supply worldwide,[14] the cereal import dependency ratio serves as a common measure of agricultural import reliance; see for example, Clapp (2017) and FAO (2022).[15] In addition, the cereal import dependency ratio is also a key indicator of food security, which is closely linked to import substitution. Countries prioritize substituting cereal imports over other agricultural products due to their crucial role in food security. Therefore, we follow McArthura and McCord (2007) to use cereal yields per hectare as an alternative proxy for agricultural productivity. Table 2 reports the estimation results.[16]

[14] From the FAO's definition, cereals include rice, wheat, maize, barley, oats, millet, and sorghum, etc. In addition, cereals serve as the primary source of calories and plant protein in global diet; see Poutanen *et al.* (2022).

[15] According to FAO (2012), import dependency ratio is defined as $(imports)/(production+imports-exports)$.

[16] We also consider the productivity and import dependency ratio of other agricultural products, such as root

Across all columns in Table 2, the coefficient κ_1 on cereal productivity is significantly positive, while the coefficient κ_2 on the interaction with cereal import dependency is significantly negative. Taking column (6) as an example, the coefficient on agricultural productivity is 0.237 and the coefficient on cereal import dependency is -0.437, both of which are statistically significant at the 1% level. Specifically, for an economy with the minimal cereal import dependency ratio, increasing agricultural productivity by 100% increases the TFP index by 0.237 (i.e., $0.237 - 0.437 \times 0.0$), which is statistically significant at the 1% level. For an economy with the average cereal import dependency ratio, increasing agricultural productivity by 100% increases the TFP index by 0.09 (i.e., $0.237 - 0.437 \times 0.327$), which is also statistically significant at the 1% level. For an economy with the maximal cereal import dependency ratio, increasing agricultural productivity by 100% *decreases* the TFP index by 0.20 (i.e., $0.237 - 0.437 \times 1.0$), which is statistically significant at the 1% level. These results show that the positive effect of agricultural productivity improvement in cereals diminishes and eventually becomes negative as its import dependency ratio increases.[17]

Table 2: Relationship between cereal productivity, trade and economic growth

	non-agri GDP per capita		GDP per capita		TFP	
	(1)	(2)	(3)	(4)	(5)	(6)
A_{jt}	0.465***	0.260***	0.449***	0.253***	0.192***	0.237***
	(0.095)	(0.086)	(0.085)	(0.082)	(0.072)	(0.069)
$A_{jt} \times trade_{jt}$	-0.559***	-0.438***	-0.475***	-0.359***	-0.419***	-0.437***
	(0.115)	(0.137)	(0.105)	(0.128)	(0.133)	(0.125)
$trade_{jt}$	5.437***	4.225***	4.656***	3.496***	4.035***	4.280***
	(1.138)	(1.352)	(1.039)	(1.262)	(1.277)	(1.232)
Control variables		✓		✓		✓
Country fixed effects	✓	✓	✓	✓	✓	✓
Period fixed effects	✓	✓	✓	✓	✓	✓
Observations	759	627	772	637	523	523
R^2	0.9890	0.9936	0.9887	0.9932	0.6911	0.7344

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *Robust standard errors in parentheses. Standard errors are clustered at the country level.*

3 A Schumpeterian model with agricultural trade

In this section, we develop a two-country Schumpeterian growth model to explore the role of agricultural productivity in driving the endogenous takeoff of the economy and its convergence to scale-invariant steady-state growth driven by both variety expansion and quality improvement. The model is based on Peretto (2015) but is also inspired by Peretto and Valente

and tuber crops. The estimated coefficients have the same sign and mostly remain significant at least at the 10% level. Results are available upon request.

[17] Our empirical result also indicates that the overall effect of agricultural productivity on TFP is significantly positive in China but significantly negative in Japan, which is consistent with our quantitative analysis.

(2011), who develop the first two-country, world general equilibrium model of endogenous innovation with asymmetric trade due to different endowments of natural resources. Chu, Peretto and Wang (2022) introduce an agricultural sector to the model in Peretto (2015), obtaining a mechanism through which agricultural productivity affects endogenous takeoff in a closed economy. By converting the closed-economy model into a two-country model, we shed light on the relationship between agricultural productivity, international trade in both agricultural and industrial goods, and innovation-driven growth.

3.1 Households

There are two countries: Home, denoted h, and Foreign, denoted f. To ensure the existence of a balanced-growth path in our two-country world-economy model, we assume that the two countries have the same population growth rate, denoted as $\lambda > 0$. With the same population growth rate in the two countries, the population ratio remains constant at the value $L_t^h/L_t^f = L_0^h/L_0^f$, where L_0^h and L_0^f are the initial populations of Home and Foreign, respectively.

The representative household in country $j \in \{h, f\}$ has preferences

$$U^j = \int_0^\infty e^{-(\rho^j-\lambda)t} \left\{ \ln c_t^j + \psi^j \ln \iota_t^j + \gamma^j \ln \left[\delta^j (q_t^j - \eta^j)^{\frac{\epsilon-1}{\epsilon}} + (1-\delta^j)(m_t^j)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \right\} dt, \tag{1}$$

where c_t^j, ι_t^j, q_t^j and m_t^j denote, respectively, consumption per capita of the domestic industrial good, of the imported industrial good, of the domestic agricultural good and of the imported agricultural good. The parameter $\rho^j > \lambda$ is the subjective discount rate. The parameters $\psi^j > 0$ and $\gamma^j > 0$ regulate the contribution to flow utility of the imported industrial good and of the agricultural goods. The parameter $\delta^j \in (0, 1]$ regulates the importance of the domestic agricultural good relative to the imported agricultural good and $\eta^j > 0$ is the subsistence requirement for consumption of the domestic agricultural good. Finally, $\epsilon \in (0, \infty)$ is the elasticity of substitution between domestic and foreign agricultural goods.

The asset-accumulation equation in country j is given by

$$\dot{a}_t^j = (r_t^j - \lambda)a_t^j + w_t^j - p_{Y,t}^j c_t^j - p_{Y,t}^{-j} \iota_t^j - p_{A,t}^j q_t^j - p_{A,t}^{-j} m_t^j, \tag{2}$$

where the superscript $-j$ denotes a country other than country j. a_t^j is the value of asset per capita and r_t^j is the interest rate. Each household member supplies inelastically one unit of labor to earn the wage rate w_t^j. In addition, $p_{Y,t}^j$ and $p_{Y,t}^{-j}$ are, respectively, the price of domestic industrial good and of imported industrial good. Similarly, the prices of domestic and imported agricultural goods are denoted by $p_{A,t}^j$ and $p_{A,t}^{-j}$, respectively.

The household's dynamic optimization in country j yields the consumption Euler equation

$$\frac{\dot{c}_t^j}{c_t^j} = r_t^j - \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j} - \rho^j \tag{3}$$

and the expenditure on the imported industrial good

$$p_{Y,t}^{-j} \iota_t^j = \psi^j p_{Y,t}^j c_t^j. \tag{4}$$

Up to this point, the model treats the two countries symmetrically.

To obtain a sharp characterization of the role of agricultural productivity, we set up an asymmetric agricultural trade structure. Specifically, the Home representative household consumes both domestic and foreign agricultural goods. The Foreign representative household, instead, consumes only the domestic agricultural good.[18] Technically, we set $\delta^f = 1$, which yields $m_t^f = 0$. Moreover, because the model does not have a balanced growth path if we allow for a Foreign subsistence requirement for its own agricultural good, we set $\eta^f = 0$.

With this structure, the Home household's dynamic optimization yields the expenditure functions for domestic and foreign agricultural goods:

$$p_{A,t}^h(q_t^h - \eta^h) = \frac{\delta^h \gamma^h p_{Y,t}^h c_t^h}{\delta^h + (1-\delta^h)/\left(\frac{q_t^h - \eta^h}{m_t^h}\right)^{\frac{\epsilon-1}{\epsilon}}}; \tag{5}$$

$$p_{A,t}^f m_t^h = \frac{(1-\delta^h)\gamma^h p_{Y,t}^h c_t^h}{\delta^h \left(\frac{q_t^h - \eta^h}{m_t^h}\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}. \tag{6}$$

Taking the ratio of these expressions, we obtain

$$b_t^h \equiv \frac{q_t^h - \eta^h}{m_t^h} = \left(\frac{\delta^h}{1-\delta^h} \frac{p_{A,t}^f}{p_{A,t}^h}\right)^{\epsilon}. \tag{7}$$

This variable b_t^h, which captures the ratio of domestic agricultural consumption to imported agricultural consumption, plays a crucial role in our analysis. Therefore, we give it a name and a symbol: we call b_t^h the *agricultural consumption ratio*. The dynamic optimization of the Foreign representative household yields the expenditure function for its own agricultural good

$$p_{A,t}^f q_t^f = \gamma^f p_{Y,t}^f c_t^f. \tag{8}$$

The detailed derivation of these relations is in Appendix C.

3.2 Agriculture

We follow Lagakos and Waugh (2013) and assume that in country j, the agricultural sector is perfectly competitive, producing with the linear technology

$$Q_t^j = A^j L_{A,t}^j = A^j l_{A,t}^j L_t^j, \tag{9}$$

where Q_t^j is agricultural output, $L_{A,t}^j$ and $l_{A,t}^j$ are, respectively, the agricultural labor input and the agricultural labor share, and the parameter $A^j > 0$ denotes agricultural productivity. We set $A^h > \eta^h$ to ensure that the Home economy is viable in the sense that its satisfies its agricultural subsistence constraint.

Profit maximization yields the price of the agricultural good

$$p_{A,t}^j = \frac{w_t^j}{A^j}. \tag{10}$$

[18] See the discussion in footnote 3.

The agricultural market-clearing condition in country j is

$$Q_t^j = q_t^j L_t^j + m_t^{-j} L_t^{-j}, \tag{11}$$

where the superscript $-j$ denotes a country other than country j. In interpreting this condition, recall that by construction $m_t^f = 0$.

3.3 Industrial good

In country j, competitive firms produce the industrial good with the technology

$$Y_t^j = \int_0^{N_t^j} [X_t^j(i)]^{\theta^j} \left\{ [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} \right\}^{1-\theta^j} di, \tag{12}$$

where $\theta^j \in (0,1)$ determines labor intensity, $1-\theta^j$, in industrial production, N_t^j is the variety of intermediate goods, and $X_t^j(i)$ is the quantity of intermediate good i. Intermediate good i has quality $Z_t^j(i)$. The average quality across intermediate goods is $Z_t^j \equiv \int_0^{N_t^j} Z_t^j(i) di / N_t^j$. The parameter $\alpha^j \in (0,1)$ regulates the importance of own quality relative to technology spillovers in determining how good i augments the industrial labor input $L_{Y,t}^j = l_{Y,t}^j L_t^j$, where $l_{Y,t}^j$ is the industrial labor share. The parameter $\sigma^j \in (0,1)$ measures the degree of love of variety in industrial production.

Let $p_{X,t}^j(i)$ be the price of intermediate good i. Profit maximization yields the demand function for intermediate goods

$$X_t^j(i) = \left[\frac{\theta^j}{p_{X,t}^j(i)/p_{Y,t}^j} \right]^{\frac{1}{1-\theta^j}} [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} \quad \text{for } i \in \left[0, N_t^j\right] \tag{13}$$

and the expenditure rules:

$$w_t^j L_{Y,t}^j = (1-\theta^j) p_{Y,t}^j Y_t^j; \tag{14}$$

$$\int_0^{N_t^j} p_{X,t}^j(i) X_t^j(i) di = \theta^j p_{Y,t}^j Y_t^j. \tag{15}$$

The second equation yields our measure of the size of the market for intermediate goods.

3.4 Intermediate goods and in-house R&D

In country j, the typical monopolistic firm uses $X_t^j(i)$ units of the domestic industrial good to produce $X^j(i)$ units of intermediate good i and also uses $\phi^j [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j}$ units of the domestic industrial good as a fixed operating cost. The profit before R&D is

$$\Pi_t^j(i) = p_{X,t}^j(i) X_t^j(i) - p_{Y,t}^j X_t^j(i) - p_{Y,t}^j \phi^j [Z_t^j(i)]^{\alpha^j} (Z_t^j)^{1-\alpha^j}. \tag{16}$$

The firm also invests $R_t^j(i)$ units of the domestic industrial good to obtain quality improvement

$$\dot{Z}_t^j(i) = R_t^j(i). \tag{17}$$

Given initial condition $Z_0^j(i)$, the firm maximizes its value,

$$V_t^j(i) = \int_t^\infty \exp\left(-\int_t^s r_u du\right)[\Pi_s^j(i) - p_{Y,s}^j R_s^j(i)]ds, \tag{18}$$

subject to the demand function (13), the profit equation (16), the R&D technology (17); see Appendix C for the solution to this dynamic optimization problem.

We assume that firms start with identical initial conditions, i.e., $Z_0^j(i) = Z_0^j$ for $i \in [0, N_t^j]$. It follows that firms solve identical problems and thus charge identical prices and invest at the same rate. This yields a symmetric equilibrium where $p_{X,t}^j(i) = p_{X,t}^j$, $X_t^j(i) = X_t^j$, $\Pi_t^j(i) = \Pi_t^j$ and $Z_t^j(i) = Z_t^j$ for $i \in [0, N_t^j]$. In particular, firm i sets

$$p_{X,t}^j(i) = \min\left\{\mu^j p_{Y,t}^j, \frac{1}{\theta^j} p_{Y,t}^j\right\} = \mu^j p_{Y,t}^j, \tag{19}$$

where $\mu^j \in (1, 1/\theta^j)$ is the number of units of domestic industrial good required by fringe competitive firms to produce one unit of intermediate good of the same quality as firm i. Therefore, the firm i sets the monopolistic price as $\mu^j p_{Y,t}^j$ to drive such fringe firms out of the market. Moreover, the R&D decision of firm i yields the rate of return to quality improvement

$$r_t^j = \alpha^j\left[(\mu^j - 1)\frac{X_t^j}{Z_t^j} - \phi^j\right] + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{20}$$

where X_t^j / Z_t^j is the quality-adjusted size of the firm, defined as units sold per unit of quality. Substituting the price $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$ into (13) yields

$$\frac{X_t^j}{Z_t^j} = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_t^j l_{Y,t}^j}{(N_t^j)^{1-\sigma^j}}. \tag{21}$$

This expression contains the two key state variables of the model, namely, the endogenous mass of firms, N_t^j, and the exogenous population, L_t^j.

To characterize the dynamics of the model analytically, we define the composite state variable

$$x_t^j \equiv \left(\frac{\theta^j}{\mu^j}\right)^{\frac{1}{1-\theta^j}} \frac{L_t^j}{(N_t^j)^{1-\sigma^j}}. \tag{22}$$

In this notation, the rate of return to quality-improving innovation is

$$r_t^j = \alpha^j[(\mu^j - 1)x_t^j l_{Y,t}^j - \phi^j] + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{23}$$

where $x_t^j l_{Y,t}^j$ is the quality-adjusted size of the firm. We shall use the shorthand *firm size* for this variable when confusion does not arise.

3.5 Entrants

In pursuit of monopolistic profit, new firms have incentives to enter the market, providing new differentiated intermediate goods of average quality. Entering the market requires payment of a sunk entry cost (for setting up equipment and plant). In country j, entry is positive when the free-entry condition

$$V_t^j = \beta^j p_{Y,t}^j X_t^j \tag{24}$$

holds, where $\beta^j > 0$ is an entry-cost parameter.

We now recall that the intermediate industry equilibrium is symmetric and differentiate the firm-value equation (18) with respect to time to obtain

$$r_t^j = \frac{\Pi_t^j - p_{Y,t}^j R_t^j}{V_t^j} + \frac{\dot{V}_t^j}{V_t^j}. \tag{25}$$

This is the standard asset pricing equation defining the rate of return to owning equity in a firm. Substituting the profit equation (16), the R&D technology (17), the price (19), the expression for quality-adjusted firm size (21), the definition of x_t^j (22) and the free entry condition (24) into the asset pricing equation (25) yields the rate of return to entry or, equivalently, firm ownership

$$r_t^j = \frac{(\mu^j - 1)x_t^j l_{Y,t}^j - \phi^j - z_t^j}{\beta^j x_t^j l_{Y,t}^j} + \frac{\dot{x}_t^j}{x_t^j} + \frac{\dot{l}_{Y,t}^j}{l_{Y,t}^j} + z_t^j + \frac{\dot{p}_{Y,t}^j}{p_{Y,t}^j}, \tag{26}$$

where $z_t^j \equiv \dot{Z}_t^j / Z_t^j$ is the growth rate of quality.

3.6 International trade

In our model, the Home representative household consumes domestic and imported agricultural goods as well as domestic and imported industrial goods. The Foreign representative household also consumes domestic and imported industrial goods but consumes only its domestic agricultural good. Therefore, the balanced-trade condition is

$$p_{Y,t}^h \iota_t^f L_t^f = p_{Y,t}^f \iota_t^h L_t^h + p_{A,t}^f m_t^h L_t^h. \tag{27}$$

3.7 Equilibrium

The equilibrium is a time path of allocations $\{c_t^j, \iota_t^j, q_t^j, m_t^j, l_{Y,t}^j, l_{A,t}^j, X_t^j(i), R_t^j(i)\}$ and a time path of prices $\{w_t^j, r_t^j, p_{Y,t}^j, p_{Y,t}^{-j}, p_{X,t}^j(i), p_{A,t}^j, p_{A,t}^{-j}, V_t^j(i)\}$ in country j such that:

- households choose $\{c_t^j, \iota_t^j, q_t^j, m_t^j\}$ to maximize utility taking $\{w_t^j, r_t^j, p_{Y,t}^j, p_{Y,t}^{-j}, p_{A,t}^j, p_{A,t}^{-j}\}$ as given;
- competitive agricultural firms choose agricultural labor input $L_{A,t}^j$ to maximize profit taking $\{w_t^j, p_{A,t}^j\}$ as given;
- competitive industrial firms choose factor inputs $\{L_{Y,t}^j, X_t^j(i)\}$ to maximize profit taking $\{w_t^j, p_{Y,t}^j, p_{X,t}^j(i)\}$ as given;

- monopolistic intermediate firms choose $\{p^j_{X,t}(i), R^j_t(i)\}$ to maximize their value $V^j_t(i)$ taking $\{r^j_t, p^j_{Y,t}\}$ as given;
- entrants make entry decisions taking $\{V^j_t, p^j_{Y,t}\}$ as given;
- the value of household assets is equal to the value of the monopolistic firms, $a^j_t L^j_t = N^j_t V^j_t$;
- the agricultural good market clears, $Q^j_t = q^j_t L^j_t + m^{-j}_t L^{-j}_t$ (recall that $m^f_t = 0$);
- the labor market clears, $L^j_t = L^j_{Y,t} + L^j_{A,t} = l^j_{Y,t} L^j_t + l^j_{A,t} L^j_t$;
- the industrial good market clears, $Y^j_t = c^j_t L^j_t + N^j_t (X^j_t + \phi^j Z^j_t + R^j_t) + \dot{N}^j_t \beta^j X^j_t + \iota^{-j}_t L^{-j}_t$;
- the balanced-trade condition holds, $p^h_{Y,t} \iota^f_t L^f_t = p^f_{Y,t} \iota^h_t L^h_t + p^f_{A,t} m^h_t L^h_t$.

3.8 Aggregation

We substitute the monopolistic quantity (13) and price (19) in the industrial production function (12) to obtain the equilibrium reduced-form production function in country j

$$Y^j_t = \left(\frac{\theta^j}{\mu^j}\right)^{\frac{\theta^j}{1-\theta^j}} \left(N^j_t\right)^{\sigma^j} Z^j_t L^j_t l^j_{Y,t}. \tag{28}$$

The growth rate of industrial output per capita, $y^j_t \equiv Y^j_t / L^j_t$, then is

$$g^j_t \equiv \frac{\dot{y}^j_t}{y^j_t} = \sigma^j n^j_t + z^j_t + \frac{\dot{l}^j_{Y,t}}{l^j_{Y,t}}, \tag{29}$$

where $n^j_t \equiv \dot{N}^j_t / N^j_t$ and z^j_t are the rates of variety growth and quality growth, respectively.

3.9 Dynamics

Given the definition in equation (22), the law of motion of the state variable x^j_t in country j is

$$\frac{\dot{x}^j_t}{x^j_t} = \lambda - (1 - \sigma^j) n^j_t. \tag{30}$$

We show below that the growth rate of product variety, n^j_t, is a monotonically increasing function of x^j_t. Accordingly, x^j_t grows over time and converges to the unique steady state $(x^j)^*$. We also show that there exist two thresholds of the state variable, denoted as x^j_N and x^j_Z, respectively. For $x^j_t < x^j_N$, agents are not willing to finance variety-expanding innovation (i.e., entry) because firm size is too small. Likewise, for $x^j_t < x^j_Z$, agents are not willing to finance quality-improving innovation (i.e., in-house R&D) because firm size is too small.

We choose parameters such that $x^j_N < x^j_Z$ to obtain a sequence of events that replicates the historical experience of the advanced economies. In particular, the economy goes through three phases: the pre-industrial era, characterized by the absence of innovation; the first phase of the industrial era, characterized by variety expansion but no quality improvement; the second phase of the industrial era, characterized by both variety-expanding and quality-improving innovation. The mechanism generating this pattern is as follows. In the pre-industrial era, firm size is insufficiently large to generate positive monopolistic profit and by implication it is insufficiently large to trigger innovation of any kind. As firm size grows due to the exogenous growth of the population, it crosses the threshold for variety-expanding innovation and the economy enters the industrial era. The first phase of the industrial era features the emergence of monopolistic firms taking over existing intermediate goods lines, and the variety-expanding innovation activity of entrants who invest to capture a share of the market for intermediate goods. As firm size continues to grow, it crosses the threshold for quality-improving innovation. When this happens, the economy enters the second phase of the industrial era that features both variety-expanding and quality-improving innovation. Eventually, firm size converges to its steady-state value and the economy settles into its balanced growth path.

To ensure that in steady state firm size, the growth rate of variety, and the growth rate of quality are positive, we impose the condition

$$\beta^j \phi^j > \frac{1}{\alpha^j}\left[\mu^j - 1 - \beta^j\left(\rho^j + \frac{\sigma^j \lambda}{1-\sigma^j}\right)\right] > \mu^j - 1. \tag{31}$$

The following lemmas describe the key dynamic property of the model, whereby there is a set of intratemporal relations, determining the fast adjusting endogenous variables $b_t^h = (q_t^h - \eta^h)/m_t^h$, c_t^j/y_t^j, $l_{Y,t}^j$ and $l_{A,t}^j = 1 - l_{Y,t}^j$ as functions of the model's parameters. Given the constant equilibrium values of these variables, the transitional dynamics of the model are governed by the law of motion of the slow adjusting state variable x_t^j characterized in equation (30) above. Under condition (31), this process eventually converges to the steady state $(x^j)^*$.

Lemma 1 *(Intratemporal equilibrium) At any time t, the agricultural consumption ratio b_t^h and the consumption-output ratios $\{c_t^h/y_t^h, c_t^f/y_t^f\}$ jump to the unique and stable steady-state values. In particular, the steady-state values of the agricultural consumption ratio and consumption-output ratios are:*[19]

$$b_t^h = \left(b^h\right)^* = \underset{b_t^h}{\arg\text{solve}}\left\{F(b_t^h;\cdot) = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}\right\},$$

$$\frac{c_t^h}{y_t^h} = \left(\frac{c^h}{y^h}\right)^* = \begin{cases} \dfrac{1-\theta^h}{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}} & 0 \le x_t^h \le x_N^h \\ \dfrac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}} & x_N^h < x_t^h < \infty \end{cases},$$

$$\frac{c_t^f}{y_t^f} = \left(\frac{c^f}{y^f}\right)^* = \begin{cases} \dfrac{1-\theta^f}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & 0 \le x_t^f \le x_N^f \\ \dfrac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & x_N^f < x_t^f < \infty \end{cases}.$$

where x_N^j is the threshold of firm size for variety-expanding innovation in country j.

[19] The function $F(b_t^h;\cdot)$ is defined in (B.5) in Appendix B.

Proof. See Appendix B. ■

Lemma 1 shows that the steady-state Home consumption-output ratio is increasing (decreasing) in the steady-state agricultural consumption ratio, while the steady-state Foreign consumption-output ratio is decreasing (increasing) if the elasticity ϵ of substitution between domestic and foreign agricultural goods is greater (smaller) than one. Also, with the expressions of c_t^h/y_t^h and c_t^f/y_t^f, we derive the industrial labor shares in Lemma 2.

Lemma 2 *(Industrial labor shares) At any time t, the steady-state values of the industrial labor shares $l_{Y,t}^h$ and $l_{Y,t}^f$ are:*

$$l_{Y,t}^h = (l_Y^h)^* = \begin{cases} \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}\left(1-\dfrac{\eta}{A^h}\right) & 0 \leq x_t^h \leq x_N^h \\ \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h+\dfrac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}\left(1-\dfrac{\eta}{A^h}\right) & x_N^h < x_t^h < \infty \end{cases}; \tag{32}$$

$$l_{Y,t}^f = (l_Y^f)^* = \begin{cases} \dfrac{1+\psi^f-\dfrac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\gamma^f} & 0 \leq x_t^f \leq x_N^f \\ \dfrac{1+\psi^f-\dfrac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\dfrac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\dfrac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\dfrac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} & x_N^f < x_t^f < \infty \end{cases}. \tag{33}$$

Proof. See Appendix C. ■

If the elasticity of substitution between domestic and foreign agricultural goods is greater (less) than one, the Home industrial labor share is decreasing (increasing) in the agricultural consumption ratio, whereas the Foreign industrial labor share is increasing (decreasing) in it.

Lemma 3 *(Comparative statics of agricultural consumption ratio with respect to agricultural productivity) The steady-state value of $\left(b^h\right)^*$ is always increasing in the Home agricultural productivity A^h and decreasing in the Foreign agricultural productivity A^f.*

Proof. See Appendix C. ■

According to Lemma 2 and Lemma 3, we then have the following patterns. If the elasticity of substitution between domestic and foreign agricultural goods is less than one, higher Home agricultural productivity yields a larger Home industrial labor share and a smaller Foreign industrial labor share. Similarly, higher Foreign agricultural productivity yields a larger Foreign industrial labor share and a smaller Home industrial labor share.

Things get a bit more complicated if the elasticity of substitution between domestic and foreign agricultural goods is greater than one. Higher Foreign agricultural productivity lowers the Foreign industrial labor share and raises the Home industrial labor share. However, while the effect of the Home agricultural productivity on the Foreign industrial labor share is positive, its effect on the Home industrial labor share is ambiguous. The reason is that it facilitates meeting the agricultural subsistence requirement, i.e., it reduces η^h/A^h, while it raises the agricultural consumption ratio, i.e., it raises $\left(b^h\right)^*$. These are the two competing forces — household consumption pattern and international trade specialization — discussed in the Introduction. Furthermore, the change in the household consumption pattern operates only in Home, and thereby only in Home we have the ambiguous effect of higher domestic agricultural productivity on domestic industrial labor share. The ambiguous result explains the competing perspectives discussed in the Introduction: higher agricultural productivity fostering industrial development versus higher agricultural productivity hindering industrial development. The former downplays the role of international trade whereas the latter privileges it, reaching the opposite conclusion. Below we shed further light on this aspect of our analysis by looking at two special cases that capture the essence of these competing perspectives.

4 Agricultural productivity and industrial takeoff

In this section, we discuss how agricultural productivity affects the transition of the economy from pre-industrial stagnation to modern innovation-driven growth. Additionally, we demonstrate the significant role of international agricultural trade in shaping this process.

4.1 The pre-industrial era

In the pre-industrial era, spending on innovation yields negative profits as firm size is not sufficiently large. Hence, in country j, we have $n_t^j = z_t^j = 0$. Furthermore, the industrial labor share is constant by Lemma 2. Therefore, the growth rate of output per capita is $g_t^j = \sigma^j n_t^j + z_t^j = 0$. The law of motion of the state variable, equation (30), yields

$$\frac{\dot{x}_t^j}{x_t^j} = \lambda - (1-\sigma^j)n_t^j = \lambda, \tag{34}$$

which shows that firm size grows exponentially at the constant rate λ and crosses the finite threshold x_N^j for variety-expanding innovation at the finite time $T_N^j = \frac{1}{\lambda}\log\left(x_N^j/x_0^j\right)$ for given initial condition x_0^j. Note that despite the common growth rate of the two populations, the takeoff time is country-specific via the threshold x_N^j.

4.2 The first phase of the industrial era

In the first phase of the industrial era, there is variety-expanding innovation but no quality-improving innovation. Specifically, in country j we have $n_t^j > 0$ but $z_t^j = 0$. Also, the industrial labor share is constant by Lemma 2. Therefore, the growth rate of output per capita is $g_t^j = \sigma^j n_t^j$. Using the Euler equation (3), the rate of return to entry (26) and the fact that $\dot{c}_t^j/c_t^j = \dot{y}_t^j/y_t^j = g_t^j$, we obtain

$$n_t^j = n_1(x_t^j) = \frac{1}{\beta^j}\left[\mu^j - 1 - \frac{\phi^j}{x_t^j\left(l_Y^j\right)^*}\right] + \lambda - \rho^j, \tag{35}$$

where $n_1(x_t^j)$ denotes the growth rate of variety in the first phase of the industrial era. $n_1(x_t^j)$ is positive if and only if

$$x_t^j > x_N^j \equiv \frac{\phi^j}{\left[\mu^j - 1 - \beta^j(\rho^j - \lambda)\right]\left(l_Y^j\right)^*}, \tag{36}$$

which shows that the threshold x_N^j for variety-expanding innovation is decreasing in the domestic industrial labor share $(l_Y^j)^*$. According to Lemma 2 and Lemma 3, for $\epsilon \in (0,1)$, we have $dx_N^h/dA^h < 0$, $dx_N^h/dA^f > 0$, $dx_N^f/dA^h > 0$ and $dx_N^f/dA^f < 0$. In words, in both Home and Foreign, an improvement in the country's own agricultural productivity reduces the threshold for variety-expanding innovation whereas an improvement in the agricultural productivity of the other country increases it. For $\epsilon \in (1,\infty)$, instead, we have dx_N^h/dA^h ambiguous, $dx_N^h/dA^f < 0$, $dx_N^f/dA^h < 0$ and $dx_N^f/dA^f > 0$. In words, higher Foreign agricultural productivity increases the Foreign threshold for variety-expanding innovation and decreases the Home threshold for variety-expanding innovation. However, the effects of higher Home agricultural productivity are ambiguous: while it reduces the Foreign threshold for variety-expanding innovation, it may either increase or decrease the Home threshold for variety-expanding innovation due to its ambiguous effects on the Home industrial labor share.

Equations (30) and (35) yield

$$\frac{\dot{x}_t^j}{x_t^j} = \frac{1-\sigma^j}{\beta^j}\left\{\frac{\phi^j}{x_t^j\left(l_Y^j\right)^*} - \left[\mu^j - 1 - \beta^j\left(\frac{\sigma^j\lambda}{1-\sigma^j} + \rho^j\right)\right]\right\}. \tag{37}$$

Under condition (31), x_t^j grows throughout the first phase of the industrial era and eventually crosses the threshold x_Z^j for quality-improving innovation. Using $g_t^j = \sigma^j n_t^j$ and equation (35) yields

$$g_t^j = \frac{\sigma^j}{\beta^j}\left[\mu^j - 1 - \frac{\phi^j}{x_t^j\left(l_Y^j\right)^*}\right] - \sigma^j(\rho^j - \lambda), \tag{38}$$

which shows that a larger industrial labor share causes a higher transitional growth rate. According to Lemma 2 and Lemma 3, for $\epsilon \in (0,1)$, an improvement in the country's own agricultural productivity causes a higher transitional growth rate whereas an improvement in the agricultural productivity of the other country causes a lower transitional growth rate. For $\epsilon \in (1,\infty)$, agricultural productivity has the opposite effects, with the exception of the ambiguous effect of the Home agricultural productivity on the Home transitional growth rate. We summarize these results in Proposition 1.

Proposition 1 *(Effects of agricultural productivity in the first phase of the industrial era) If the elasticity of substitution between domestic and foreign agricultural goods is less than one, we have: (i) higher Home agricultural productivity hastens the Home takeoff and raises its post takeoff transitional growth rate; (ii) higher Home agricultural productivity delays the Foreign takeoff and lowers its post takeoff transitional growth rate; (iii) higher Foreign agricultural productivity delays the Home takeoff and lowers its post takeoff transitional growth rate; and (iv) higher Foreign agricultural productivity hastens the Foreign takeoff and raises its post takeoff* *transitional growth rate. If the elasticity of substitution between domestic and foreign agricultural goods is greater than one instead, we have: (v) higher Home agricultural productivity has an ambiguous effect on the Home takeoff and its post takeoff transitional growth rate; (vi) higher Home agricultural productivity hastens the Foreign takeoff and raises its post takeoff transitional growth rate; (vii) higher Foreign agricultural productivity hastens the Home takeoff and raises its post takeoff transitional growth rate; and (viii) higher Foreign agricultural productivity delays the Foreign takeoff and lowers its post takeoff transitional growth rate.*

Proof. Proved in the text. ■

4.3 The second phase of the industrial era

As x_t^j grows over time and eventually crosses the threshold x_Z^j for quality-improving innovation, the economy enters the second phase of the industrial era. In this phase, the growth rate of industrial output per capita is $g_t^j = \sigma_t^j n_t^j + z_t^j$ since the industrial labor share is once again constant. We use the Euler equation (3), the rates of return to quality improvement (23) and variety expansion (26), and the fact that $\dot{c}_t^j / c_t^j = \dot{y}_t^j / y_t^j = g_t^j$ to derive the transitional growth rate

$$g_t^j = \alpha^j[(\mu^j - 1)x_t^j \left(l_Y^j\right)^* - \phi^j] - \rho^j, \tag{39}$$

which shows that a larger industrial labor share leads to a higher transitional growth rate. The two components of this growth rate are the innovation rates:

$$n_t^j = n_2(x_t^j) = \frac{(1-\alpha^j)(\mu^j - 1) - \beta^j(\rho^j - \lambda) - [(1-\alpha^j)\phi^j - \rho^j]\frac{1}{x_t^j\left(l_Y^j\right)^*}}{\beta^j - \frac{\sigma^j}{x_t^j\left(l_Y^j\right)^*}}, \tag{40}$$

$$z_t^j = z_2(x_t^j) = \frac{\beta^j\left\{\left[\alpha^j - \frac{\sigma^j}{\beta^j x_t^j\left(l_Y^j\right)^*}\right][(\mu^j - 1)x_t^j\left(l_Y^j\right)^* - \phi^j] - [(1-\sigma^j)\rho^j + \sigma^j\lambda]\right\}}{\beta^j - \frac{\sigma^j}{x_t^j\left(l_Y^j\right)^*}}, \tag{41}$$

where $n_2(x_t^j)$ and $z_2(x_t^j)$ are, respectively, the growth rate of variety and quality in the second phase of the industrial era. Equation (41) says that $z_2(x_t^j) > 0$ if and only if

$$x_t^j > x_Z^j \equiv \underset{x_t^j}{\arg\text{solve}}\left\{\frac{(1-\sigma^j)\rho^j + \sigma^j\lambda}{(\mu^j - 1)x_t^j\left(l_Y^j\right)^* - \phi^j} = \alpha^j - \frac{\sigma^j}{\beta^j x_t^j\left(l_Y^j\right)^*}\right\}, \tag{42}$$

which shows that the threshold x_Z^j for quality-improving innovation is decreasing in the domestic industrial labor share $\left(l_Y^j\right)^*$.

Lemma 2 and Lemma 3 then say that for $\epsilon \in (0,1)$ we have $dx_Z^h/dA^h < 0$, $dx_Z^h/dA^f > 0$, $dx_Z^f/dA^h > 0$ and $dx_Z^f/dA^f < 0$. In words, if the elasticity of substitution between domestic and foreign agricultural goods is less than one, a country's threshold for quality-improving innovation is decreasing in the country's own agricultural productivity and increasing in the other country's agricultural productivity. For $\epsilon \in (1,\infty)$, instead, we have dx_Z^h/dA^h ambiguous, $dx_Z^h/dA^f < 0$, $dx_Z^f/dA^h < 0$ and $dx_Z^f/dA^f > 0$. In words, if the elasticity of substitution between domestic and foreign agricultural goods is greater than one, an improvement in the Home agricultural productivity has an ambiguous effect on the Home threshold for quality-improving innovation while it lowers the Foreign threshold for quality-improving innovation. On the other hand, an improvement in the Foreign agricultural productivity lowers the Home threshold for quality-improving innovation and raises the Foreign threshold for quality-improving innovation.

Equations (30), (40) and (41) yield the dynamics of x_t^j in country j as

$$\frac{\dot{x}_t^j}{x_t^j} = \frac{\frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j}{1-\sigma^j}\lambda\right)}{x_t^j \left(l_Y^j\right)^*} - \left[(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j}{1-\sigma^j}\lambda\right)\right]}{\left[\beta^j - \frac{\sigma^j}{x_t^j \left(l_Y^j\right)^*}\right] \frac{1}{1-\sigma^j}}, \tag{43}$$

which says that under condition (31), x_t^j grows over time and eventually converges to

$$(x^j)^* = \frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j\lambda}{1-\sigma^j}\right)}{(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j\lambda}{1-\sigma^j}\right)} \frac{1}{\left(l_Y^j\right)^*}. \tag{44}$$

The growth rate of industrial output per capita is

$$(g^j)^* = \alpha^j \left[(\mu^j - 1)\frac{(1-\alpha^j)\phi^j - \left(\rho^j + \frac{\sigma^j\lambda}{1-\sigma^j}\right)}{(1-\alpha^j)(\mu^j - 1) - \beta^j \left(\rho^j + \frac{\sigma^j\lambda}{1-\sigma^j}\right)} - \phi^j\right] - \rho^j. \tag{45}$$

This growth rate is independent of the industrial labor share and of agricultural productivity—indeed of any factor determining the scale of economic activity. The intuition for this scale invariance property follows from equation (44), which says that in the steady state, firm size $x_t^j l_{Y,t}^j = (x^j)^* (l_Y^j)^*$ is independent of any parameter related to scale. Agricultural productivity, therefore, has no effect on firm size and thereby has no effect on the steady-state growth rate. We summarize the effects of agricultural productivity in the second phase of the industrial era in Proposition 2.

Proposition 2 *(Effects of agricultural productivity in the second phase of the industrial era) If the elasticity of substitution between domestic and foreign agricultural goods is less than one, we have: (i) higher Home agricultural productivity lowers the Home threshold for quality-improving innovation and increases the Home transitional growth rate; (ii) higher Home agricultural productivity raises the Foreign threshold for quality-improving innovation and lowers the Foreign transitional growth rate; (iii) higher Foreign agricultural productivity raises the Home threshold for quality-improving innovation and lowers the Home transitional growth rate; and (iv) higher Foreign agricultural productivity lowers the Foreign threshold for quality-improving innovation and raises the Foreign transitional growth rate. If the elasticity of substitution between domestic and foreign agricultural goods is greater than one instead, we have: (v) higher Home agricultural productivity has an ambiguous effect on the Home threshold for quality-improving innovation and the Home transitional growth rate; (vi) higher Home agricultural productivity lowers the Foreign threshold for quality-improving innovation and raises the Foreign transitional growth rate; (vii) higher Foreign agricultural productivity lowers the Home threshold for quality-improving innovation and raises the Home transitional growth rate; and (viii) higher*

Foreign agricultural productivity raises the Foreign threshold for quality-improving innovation and lowers the Foreign transitional growth rate. In both Home and Foreign, the steady-state growth rate is affected by neither Home nor Foreign agricultural productivity.

Proof. Proved in the text. ■

4.4 Two special cases

To illuminate further the role of agricultural trade, we now consider two special cases. First, we shut down the Home subsistence requirement for agricultural goods (i.e., $\eta^h = 0$). Second, we consider a unitary elasticity of substitution between domestic and foreign agricultural goods (i.e., $\epsilon = 1$). This case produces the qualitative results that we obtain in the even more special case of no agricultural trade between the two countries when Home has no preference for the imported agricultural good (i.e., $\delta^h = 1$).

4.4.1 No subsistence requirement ($\eta^h = 0$)

To see the role of this modification, we need to only track the equations in Home. When we shut down the Home subsistence requirement for agricultural goods ($\eta^h = 0$), the term $1 - \eta^h/A^h$ in equation (32) becomes 1 and the term $b_t^h = \left(q_t^h - \eta^h\right)/m_t^h$ becomes $b_t^h = q_t^h/m_t^h$. The key change is the first because it makes the industrial labor share monotonic in the agricultural consumption ratio $\left(b^h\right)^*$. Specifically, we have

$$(l_Y^h)^* = \begin{cases} \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]} & 0 \leq x_t^h \leq x_N^h \\ \dfrac{(1+\psi^h)\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h+\dfrac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(\left(b^h\right)^*\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)} & x_N^h < x_t^h < \infty \end{cases}. \tag{46}$$

Lemma 3 changes accordingly but still says that the agricultural consumption ratio is increasing in the Home agricultural productivity for any $\epsilon \in (0, \infty)$. It then follows that all of the effects that we study are unambiguous. In particular, the signs of the effects of A^h and A^f for $\epsilon \in (0, 1)$ are the same as in the general case. The key changes are for $\epsilon \in (1, \infty)$, where in the general case the effects of the Home agricultural productivity are ambiguous. Therefore, under $\eta^h = 0$, the Home agricultural productivity unambiguously decreases the Home industrial labor share and thereby decreases the transitional growth rate. Also, in the general case, we have the two ambiguous effects, $dx_N^h/dA^h \lesseqgtr 0$ and $dx_Z^h/dA^h \lesseqgtr 0$, that under $\eta^h = 0$ become $dx_N^h/dA^h > 0$ and $dx_Z^h/dA^h > 0$.

Thus, eliminating the Home subsistence requirement, we find that the effect of the Home agricultural productivity is no longer ambiguous even when the elasticity of substitution between domestic and foreign agricultural products is greater than one. The intuition is that the Home preferences become homothetic and thus the model no longer features a channel for the changing consumption pattern of the Home representative household. Consequently, the only force driving the reallocation of labor in each country is specialization due to international trade. We summarize the result in Proposition 3.

Proposition 3 *(The effects of agricultural productivity without subsistence requirement) With no subsistence requirement, the effects of agricultural productivity are as in the general case with the only change that higher Home agricultural productivity unambiguously delays the Home take-off and lowers the transitional growth rate when the elasticity of substitution between domestic and foreign agricultural products is greater than one.*

Proof. Proved in the text. ■

4.4.2 Unitary elasticity of substitution ($\epsilon = 1$) or no agricultural trade ($\delta^h = 1$)

We now consider the case in which the elasticity of substitution between domestic and foreign agricultural goods is equal to one. The key implication of this restriction is that the Home expenditure shares on agricultural goods are no longer functions of the prices of agricultural goods; see equations (5) and (6). In this case, we have simply $\left(b_t^h\right)^{(\epsilon-1)/\epsilon} = 1$. The Home industrial labor share becomes

$$(l_Y^h)^* = \begin{cases} \frac{1+\psi^h+(1-\delta^h)\gamma^h}{1+\psi^h+\gamma^h}\left(1-\frac{\eta}{A^h}\right) & 0 \leq x_t^h \leq x_N^h \\ \frac{1+\psi^h+(1-\delta^h)\gamma^h}{1+\psi^h+\gamma^h+\frac{\delta^h}{1-\theta^h}\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)\gamma^h}\left(1-\frac{\eta}{A^h}\right) & x_N^h < x_t^h < \infty \end{cases}. \tag{47}$$

All effects running through prices have washed out. Consequently, the Home industrial labor share is unambiguously increasing in its own agricultural productivity, A^h, and is independent of the Foreign agricultural productivity, A^f. Moreover, the Foreign industrial labor share becomes

$$(l_Y^f)^* = \begin{cases} \frac{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h+\gamma^h(1-\delta^h)}}{1+\psi^f+\gamma^f} & 0 \leq x_t^f \leq x_N^f \\ \frac{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h+\gamma^h(1-\delta^h)}}{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h+\gamma^h(1-\delta^h)}} & x_N^f < x_t^f < \infty \end{cases}, \tag{48}$$

which is independent of both its own agricultural productivity, A^f, and the Home agricultural productivity, A^h. The reason is that Foreign has no subsistence requirement, which is the only channel left operating in equation (47) for the potential transmission of the effect of agricultural productivity.

The equations describing the thresholds for variety-expanding and quality-improving innovation are the same as in the general case, see equations (36) and (42). Therefore, an improvement in the Home agricultural productivity reduces the Home thresholds for variety-expanding and quality-improving innovation, hastening the takeoff and raising the transitional growth rate. However, differently from the general case, now the improvement in the Home agricultural productivity has no effect on Foreign. Furthermore, the Foreign agricultural productivity has no effect on either Home or Foreign. In the even more special case $\delta^h = 1$, there is no agricultural trade, so we obtain these same results. The general intuition behind these two special cases is that they mitigate the force of specialization due to international trade leaving only the changing consumption pattern of the Home household to drive the reallocation of labor within each country. We summarize the results of this section in Proposition 4.

Proposition 4 *(The effects of agricultural productivity with a unitary elasticity of substitution or without agricultural trade) If the elasticity of substitution between domestic and foreign* *agricultural goods is equal to one (or there is no agricultural trade), then: (i) higher Home agricultural productivity hastens the Home takeoff and raises its transitional growth rate while it has no effects on Foreign; and (ii) Foreign agricultural productivity has no effect on Home and Foreign.*

Proof. Proved in text. ■

5 Quantitative analysis

In this subsection, we calibrate the general model to data to perform a quantitative analysis. Given that the analytical results on the effects of the Home agricultural productivity on its economy are ambiguous, we conduct numerical experiments to examine these effects and see how they differ across countries. We designate the US as Foreign and designate, respectively, China and Japan, as Home in two separate numerical experiments. This setup is reasonable because China and Japan are among the largest importers of US agricultural products while they export a very small amount of agricultural products to the US, aligning with the assumption of our theoretical model.[20]

The model features the following parameters $\{\lambda, \epsilon, \theta^h, \theta^f, \alpha^h, \alpha^f, \sigma^h, \sigma^f, \mu^h, \mu^f, \rho^h, \rho^f, \psi^h, \psi^f, \phi^h, \phi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, L_0^h/L_0^f, \eta/A^h, A^h/A^f\}$. We set the average population growth rate to $\lambda = 0.01$. We follow Iacopetta and Peretto (2021) to set the subjective discount rate to a conventional value of 0.03. The labor share of output is set to 0.67 in the US and Japan, and 0.55 in China.[21] We set the markup ratio to 1.3 for China, 1.4 for Japan, and 1.5 for the US.[22] According to Feenstra *et al.* (2018) and Bajzik *et al.* (2020), we set the elasticity ϵ of substitution between domestic and foreign agricultural goods to 3. We follow Iacopetta *et al.* (2019) to set the degree of technology spillovers to $1 - \alpha^h = 1 - \alpha^f = 0.833$ and the social return of variety to $\sigma^h = \sigma^f = 0.25$. Next, we calibrate the remaining parameters $\{\psi^h, \psi^f, \phi^h, \phi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, L_0^h/L_0^f, \eta/A^h, A^h/A^f\}$ by matching moments in the US and China or Japan. After that, we conduct simulations to explore how agricultural productivity affects economic growth.

5.1 China and the US

We first designate China as Home and United States as Foreign. We calibrate the parameters $\{\psi^h, \psi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, A_{aver}^h/A_{aver}^f\}$ by matching the following moments for the period 1990-2019: 1.0% for the agricultural consumption share of GDP in the US,[23] 42% for the consumption share of GDP in China, 62% for the consumption share of GDP in the US, 12.5% for the import share of GDP in China, 10.5% for the import share of GDP in the US, 66.5% for the non-agricultural labor share in China,[24] 98.2% for the non-agricultural labor share in the US,

[20] China, Mexico, Canada and Japan are the top four importers of the US agricultural goods. However, Canada and Mexico do not fit our model as they are not only major importers but also major exporters in agriculture to the US.

[21] See Song *et al.* (2011), Backus *et al.* (2017) and Grossman and Oberfield (2021).

[22] See empirical estimates of the markup ratios in Fan *et al.* (2018), De Loecke *et al.* (2020), Lu and Yu (2015) and Morrison (1992).

[23] Data source: Food and Agriculture Organization Data.

[24] Here we assume that the subsistence requirement is negligible in recent decades; i.e., $\eta/A^h \to 0$.

and 0.03 for the ratio of average agricultural output per agricultural worker between China and the US.[25] Furthermore, we calibrate L_0^h/L_0^f to 4.49 by using the ratio of the average population in China and the US. We use the long-run TFP growth rates, which are 0.90% in China and 0.58% in the US, to calibrate the remaining parameters $\{\phi^h, \phi^f\}$.[26] Finally, we calibrate the initial value of η/A_{1978}^h by using the agricultural consumption share of GDP in China in 1978, which is 29.2%.[27] Table 3 summarizes the parameter values.

Table 3: Calibrated parameters (China and the US)

θ^h	α^h	σ^h	μ^h	ρ^h	ψ^h	ϕ^h	γ^h	β^h	λ	δ^h	η/A_{1978}^h
0.45	0.167	0.25	1.3	0.03	0.377	0.343	0.675	3.358	0.01	0.94	0.14
θ^f	α^f	σ^f	μ^f	ρ^f	ψ^f	ϕ^f	γ^f	β^f	ϵ	L_0^h/L_0^f	A_{aver}^h/A_{aver}^f
0.33	0.167	0.25	1.5	0.03	0.171	0.044	0.016	12.303	3.00	4.49	0.03

Figure 1 plots the historical value of agricultural output per agricultural worker in China, which increases from 871 in 1978 to 6,510 in 2019.[28] Agricultural productivity in China rises steadily before the 1990s and then accelerates significantly from the early 1990s onward. Figure 2 plots the historical value of agricultural output per agricultural worker in the US, which increases from 64,147 in 1978 to 133,916 in 2019. As agricultural productivity changes over time in both China and the US, we calibrate the time-varying ratios A^h/A^f and η/A^h from 1978 to 2019 (see Figure 3 and Figure 4).[29] These changes can be considered as unanticipated permanent shocks in our model. Then, we simulate the path of the technology growth rate driven by simultaneous changes in agricultural productivity in both China and the US. As a counterfactual comparison, we also simulate another technology growth path, for which agricultural productivity A^h in China remains at its initial value in 1978.

Figure 5 shows the simulated paths of the technology growth rate in China. With improvement in China's agricultural productivity, the simulated growth rate in China gradually rises from 1978 to early 1990s. Then, from the early 1990s onward, there is an acceleration in the rise of China's technology growth, as agricultural productivity in China experiences a rapid surge. This rapid rise in agricultural productivity since the early 1990s triggers China's earlier entry into the era of quality-driven growth, leading to a higher growth rate thereafter. The simulated growth rate rises from 0.13% in 1978 to 1.01% in 2019. Without improvement in agricultural productivity, China would not enter the era of quality-driven growth until the early 2000s. Furthermore, in the absence of agricultural productivity improvement in China, the simulated growth rate rises from 0.13% to only 0.67% in 2019. Comparing these two cases, agricultural productivity improvement in China is responsible for an additional increase in the growth rate of 0.34%. In the data, the TFP growth rate in China increased from 0.24% in 1978-1999 to 1.19% in 2000-2019; therefore, our model with agricultural productivity improvement in China accounts for over one-third of the increase in China's TFP growth in this period.

[25] Data source: Food and Agriculture Organization Data.

[26] Data source: Federal Reserve Bank of St. Louis.

[27] Data source: Food and Agriculture Organization Data.

[28] In the quantitative analysis, we measure agricultural productivity in China, Japan and the US using the value of agricultural output per agricultural worker, in constant 2015 US dollars. Data source: Food and Agriculture Organization Data.

[29] Here we use $(\eta/A_{1978}^h)(A_{1978}^h/A^h)$ to calibrate the time path of η/A^h.

Figure 6 shows the simulated paths of the technology growth rate in the US. With the improvement in agricultural productivity in China, the simulated technology growth rate in the US slightly rises from 0.56% in 1978 to 0.62% in 2019. Without the improvement in agricultural productivity in China, the simulated technology growth rate in the US decreases from 0.56% in 1978 to 0.47% in 2019. In the data, the TFP growth rate in US increases from 0.53% to 0.62% during the same period. Therefore, the simulated US technology growth rate, incorporating improvements in China's agricultural productivity, aligns more closely with the data. This also suggests that improvement in agricultural productivity in China contributes positively to economic growth in the US, which is also consistent with our theoretical prediction.

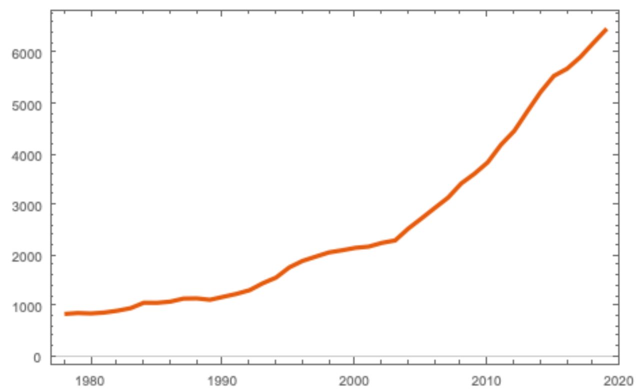

Figure 1: Agricultural productivity in China.

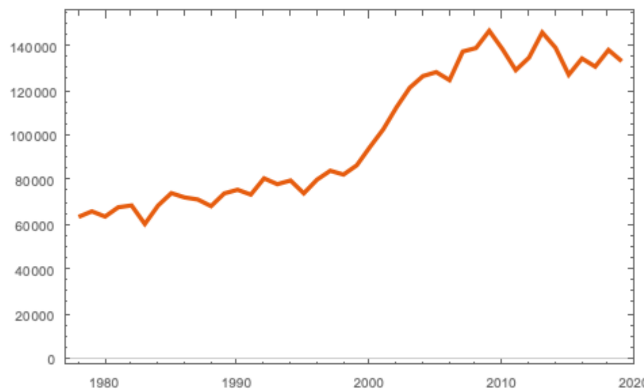

Figure 2: Agricultural productivity in the US.

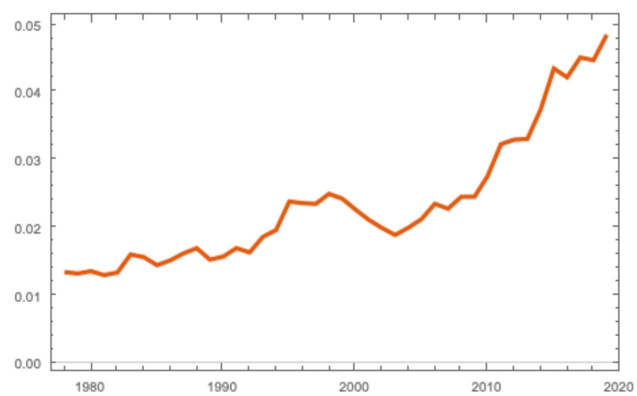

Figure 3: Calibrated path of A^h/A^f.

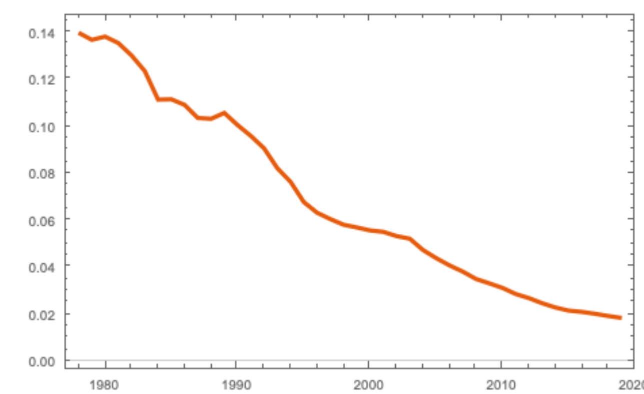

Figure 4: Calibrated path of η/A^h.

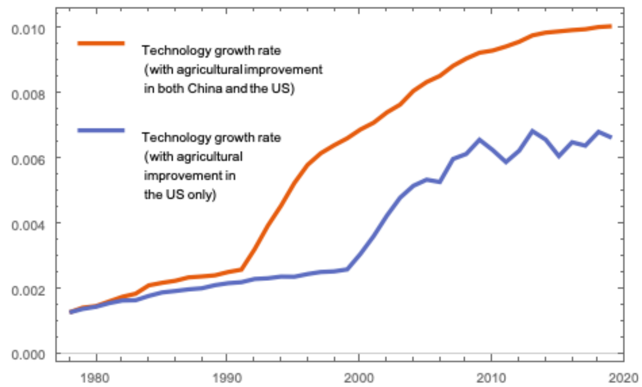

Figure 5: Simulated growth rate in China.

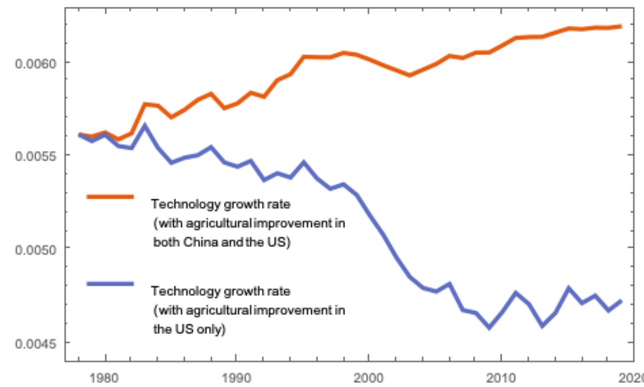

Figure 6: Simulated growth rate in the US.

5.2 Japan and the US

We now designate Japan as Home and the US as Foreign. We calibrate the parameters $\{\psi^h, \psi^f, \gamma^h, \gamma^f, \beta^h, \beta^f, \delta^h, A^h_{aver}/A^f_{aver}\}$ by matching the following moments in the period 1990-2019: 1.0% for the agricultural consumption share of GDP in the US, 53% for the consumption share of GDP in Japan, 62% for the consumption share of GDP in the US, 17.1% for the import share of GDP in Japan, 10.5% for the import share of GDP in the US, 95.0% for the non-agricultural labor share in Japan,[30] 98.2% for the non-agricultural labor share in the US, and 0.29 for the ratio of average agricultural output per agricultural worker between Japan and the US.[31] Furthermore, we calibrate L^h_0/L^f_0 to the ratio of average population in Japan and the US of 0.45. We use the long-run TFP growth rates, which are 0.53% in Japan and 0.58% in the US, to calibrate $\{\phi^h, \phi^f\}$.[32] Finally, we calibrate the initial value of η/A^h_{1978} by using the agricultural consumption share of GDP in Japan in 1978, which is 4.1%.[33] Table 4 summarizes the parameter values.

Table 4: Calibrated parameters (Japan and the US)

θ^h	α^h	σ^h	μ^h	ρ^h	ψ^h	ϕ^h	γ^h	β^h	λ	δ^h	η/A^h_{1978}
0.33	0.167	0.25	1.4	0.03	0.328	0.106	0.076	7.909	0.01	0.86	0.003
θ^f	α^f	σ^f	μ^f	ρ^f	ψ^f	ϕ^f	γ^f	β^f	ϵ	L^h_0/L^f_0	A^h_{aver}/A^f_{aver}
0.33	0.167	0.25	1.5	0.03	0.171	0.044	0.016	12.303	3.00	0.45	0.29

In Figure 7, we plot the path of agricultural productivity in Japan, which increases from 16,283 in 1978 to 39,600 in 2019.[34] The path of agricultural productivity in the US is given in

[30] As before, we assume that the subsistence requirement is negligible in recent decades; i.e., $\eta/A^h \to 0$.

[31] Data source: Food and Agriculture Organization Data.

[32] Data source: Federal Reserve Bank of St. Louis.

[33] Data source: Food and Agriculture Organization Data.

[34] It is measured by the value of agricultural output per agricultural worker, in constant 2015 US dollars. Data source: Food and Agriculture Organization Data.

Figure 2. We calibrate the time-varying ratios A^h/A^f and η/A^h from 1978 to 2019 (see Figure 8 and Figure 9) and then treat them as a sequence of unanticipated permanent shocks to simulate the technology growth path. We also conduct a counterfactual experiment in which there is no improvement in agricultural productivity in Japan by holding A^h constant at its initial level in 1978.

Figure 10 shows the simulated paths of the technology growth rate in Japan. In contrast to the Chinese case, improvement in domestic agricultural productivity exhibits negative effects on the simulated growth rate in Japan. The simulated technology growth rate in Japan decreases as its domestic agricultural productivity increases over time. With the improvement in Japan's agricultural productivity, the simulated growth rate declines from 0.60% in 1978 to 0.58% in 2019. Conversely, without the improvement in Japan's agricultural productivity, the simulated growth rate increases from 0.60% in 1978 to 0.64% in 2019. Comparing these two cases, agricultural productivity improvement in Japan is responsible for an additional decrease in the growth rate of 0.06%. The average TFP growth rate in Japan was 0.64% in 1978-1999, and it declined to 0.46% in 2000-2019. Therefore, our model with agricultural productivity improvement in Japan accounts for about one-third of the decline in TFP growth in Japan.

Figure 11 plots the simulated paths of the technology growth rate in the US. The improvement in Japan's agricultural productivity results in a slight increase in the simulated growth rate in the US, from 0.56% in 1978 to 0.57% in 2019. In the absence of improvement in Japan's agricultural productivity, the simulated growth rate in the US declines from 0.56% in 1978 to 0.53% in 2019. As mentioned before, in the data, the TFP growth rate in the US increased from 0.53% to 0.62% during this period. Therefore, the simulated US growth rate, when accounting for the improvement in Japan's agricultural productivity, aligns more closely with the data. This also suggests that improvement in agricultural productivity in Japan contributes positively to economic growth in the US.

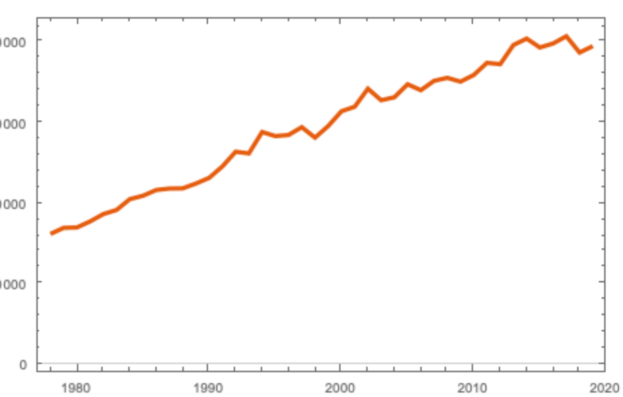

Figure 7: Agricultural productivity in Japan.

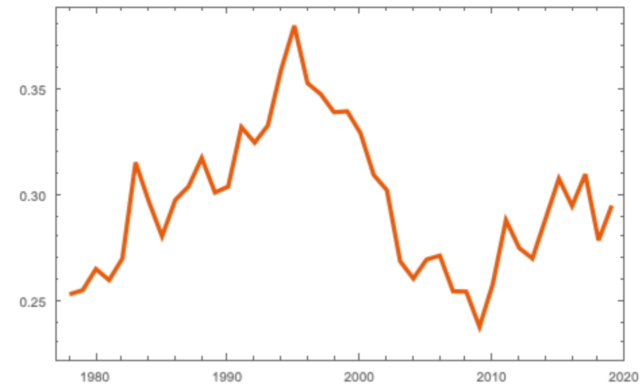

Figure 8: Calibrated path of A^h/A^f.

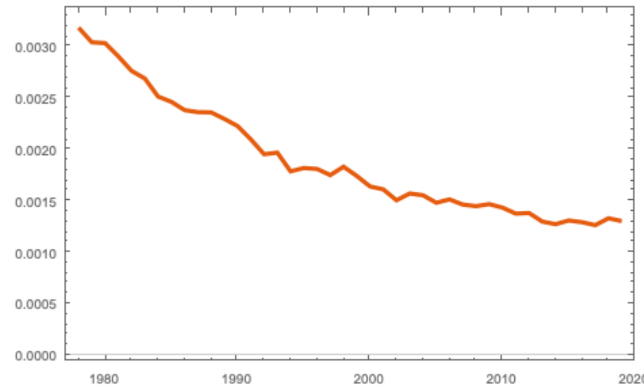

Figure 9: Calibrated path of η/A^h.

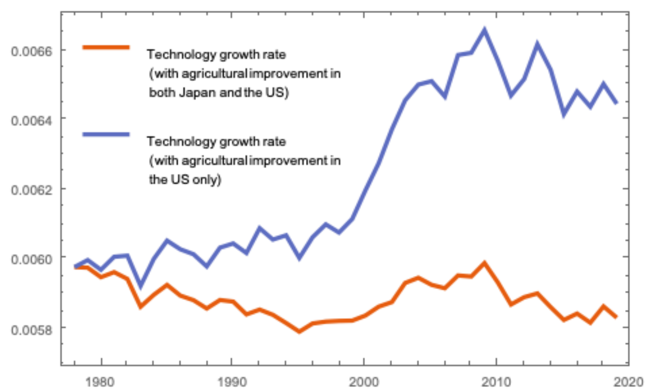

Figure 10: Simulated growth rate in Japan.

Figure 11: Simulated growth rate in the US.

5.3 Discussion

In this section, we have calibrated our model to data in the US and its two major agricultural importers: China and Japan. Then, we have performed quantitative analyses to explore the overall growth effects of agricultural productivity in China/Japan on economic growth in China/Japan and the US. Our counterfactual exercises reveal that improvements in agricultural productivity in both Japan and China positively affect US growth, aligning with our theoretical predictions.

Moreover, we find that agricultural productivity improvement in China and Japan can explain about one-third of the changes in domestic TFP growth. However, the quantitative effects of agricultural productivity on domestic technology growth differ between China and Japan. Specifically, in China, improving agricultural productivity increases its domestic technology growth, whereas this effect is negative in Japan.[35] These drastically different implications can be explained as follows.

First of all, it is useful to note that the calibrated values of δ^h for China and Japan are 0.94 and 0.86, respectively. Suppose we have the absence of agricultural trade (i.e., $\delta^h = 1$). Then, an increase in domestic agricultural productivity does not affect import substitution because household only consumes domestic agricultural good. In this case, the presence of subsistence agricultural requirement implies that an improvement in agricultural productivity releases labor from the agricultural sector to the industrial sector, leading to a positive effect on technology growth. This situation applies to China given its high calibrated value of $\delta^h = 0.94$.

In the case of Japan, its calibrated value of $\delta^h = 0.86$ is relatively low. In this case, the higher level of agricultural imports in Japan implies that it has a stronger incentive for import substitution due to the substitutability between domestic and imported agricultural products. In this case, an increase in domestic agricultural productivity gives rise to agricultural import substitution and leads to a reallocation of labor from the industrial sector to the agricultural sector, leading to a negative effect on technology growth. Our quantitative results illustrate that the degree of reliance on agricultural imports influences the effects of agricultural productivity on economic growth.

6 Conclusion

In this study, we developed a two-country open-economy Schumpeterian growth model with an agricultural sector to explore the role of agricultural productivity in the endogenous takeoff of the economy and the subsequent path of economic growth. We find that agricultural trade plays an important role in shaping the effects of agricultural productivity on innovation-driven growth. Our theoretical results can be summarized as follows.

With agricultural trade and a subsistence requirement, higher domestic agricultural productivity has ambiguous effects on the economy's takeoff time and its transitional growth rate if the elasticity of substitution between domestic and foreign agricultural goods is greater than one. With no subsistence requirement, the ambiguity goes away: higher domestic agricultural productivity delays the economy's industrialization and lowers its transitional growth rate. The reason is that higher domestic agricultural productivity increases the demand for domestic agricultural goods and thereby increases the demand for agricultural labor. When domestic and imported agricultural goods are highly substitutable, this specialization force pushes in the opposite direction of the change in the pattern of domestic consumption, governed by the subsistence requirement, which tends to release labor from agricultural production. The tension between these two forces explains the ambiguous result that we obtain in the general case and why shutting down the subsistence requirement resolves the ambiguity: the latter force no longer operates. In the absence of agricultural trade, the subsistence requirement on agricultural consumption implies that an improvement in domestic agricultural productivity reallocates labor from agricultural production to industrial production, hastening the economy's takeoff and raising the transitional growth rate. This is because in this scenario the specialization force does not operate.

[35] These contrasting quantitative results are consistent with our empirical estimates for China and Japan in Section 2.

We investigated this mechanism empirically and quantitatively. In a cross-country panel-data exercise, we find that agricultural productivity has a direct positive effect on economic growth but this positive effect is weaker and can even become negative when reliance on agricultural imports is sufficiently high. In quantitative counterfactual exercises with the calibrated model, we find that improvement in domestic agricultural productivity accounts for the rise and fall of TFP growth in China and Japan, respectively, and also contributes to the rise of TFP growth in their main trading partner, the US.

References

[1] Aghion, P., and Howitt, P., 1992. A Model of Growth Through Creative Destruction. *Econometrica*, 60, 323-351.

[2] Ang, J., and Madsen, J., 2011. Can second-generation endogenous growth models explain the productivity trends and knowledge production in the Asian miracle economies?. *Review of Economics and Statistics*, 93, 1360-1373.

[3] Ashraf, Q., and Galor, O., 2011. Dynamics and stagnation in the Malthusian epoch. *American Economic Review*, 101, 2003-2041.

[4] Backus, D., Henriksen, E., and Storesletten, K., 2008. Taxes and the global allocation of capital. *Journal of Monetary Economics*, 55, 48-61.

[5] Bajzik, J., Havranek, T., Irsova, Z., and Schwarz, J., 2020. Estimating the Armington elasticity: The importance of study design and publication bias. *Journal of International Economics*, 127, 103383.

[6] Bravo-Ortega, C., and Lederman, D., 2005. Agriculture and national welfare around the world: causality and international heterogeneity since 1960. World Bank Publications.

[7] Bustos, P., Caprettini, B., and Ponticelli, J., 2016. Agricultural productivity and structural transformation: Evidence from Brazil. *American Economic Review*, 106, 1320-1365.

[8] Bustos, P., Castro-Vincenzi, J., Monras, J., and Ponticelli, J., 2022. Industrialization without innovation. NBER Working Paper 25871.

[9] Bustos, P., Garber, G., and Ponticelli, J., 2020. Capital accumulation and structural transformation. *Quarterly Journal of Economics*, 135, 1037-1094.

[10] Cao, K., and Birchenall, J., 2013. Agricultural productivity, structural change, and economic growth in post-reform China. *Journal of Development Economics*, 104, 165-180.

[11] Chesnokova, T., 2007. Immiserizing deindustrialization: A dynamic trade model with credit constraints. *Journal of International Economics*, 73, 407-420.

[12] Chu, A., Fan, H., and Wang, X., 2020. Status-seeking culture and development of capitalism. *Journal of Economic Behavior & Organization*. 180, 275-290.

[13] Chu, A., Furukawa, Y., and Wang, X., 2022. Rent-seeking government and endogenous takeoff in a Schumpeterian economy. *Journal of Macroeconomics*. 72, 103399.

[14] Chu, A., Kou, Z., and Wang, X., 2020. Effects of patents on the transition from stagnation to growth. *Journal of Population Economics*. 33, 395-411.

[15] Chu, A., Peretto, P., and Wang, X., 2022. Agricultural revolution and industrialization. *Journal of Development Economics*, 158, 102887.

[16] Chu, A., Peretto, P., and Xu, R., 2023. Export-led takeoff in a Schumpeterian economy. *Journal of International Economics*, 145, 103798.

[17] Clapp, J., 2017. Food self-sufficiency: Making sense of it, and when it makes sense. *Food Policy*, 66, 88-96.

[18] De Loecker, J., Eeckhout, J., and Unger, G., 2020. The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics*, 135, 561-644.

[19] Dinopoulos, E., and Thompson, P., 1998. Schumpeterian growth without scale effects. *Journal of Economic Growth*, 3, 313-335.

[20] Duranton, G., 1998. Agricultural productivity, trade, and industrialisation. *Oxford Economic Papers*, 50, 220-236.

[21] Echevarria, C., 1995. Agricultural development vs. industrialization: Effects of trade. *Canadian Journal of Economics*, 28, 631-647.

[22] Echevarria, C., 1997. Changes in sectoral composition associated with economic growth. *International Economic Review*, 38, 431-452.

[23] Fan, H., Gao, X., Li, Y., and Luong, A., 2018. Trade liberalization and markups: Micro evidence from China. *Journal of Comparative Economics*, 46, 103-130.

[24] FAO., 2012. *World food and agriculture*. Food and Agriculture Organization of the United Nations, Rome.

[25] FAO., 2022. *World food and agriculture*. Food and Agriculture Organization of the United Nations, Rome.

[26] Feenstra, R., Luck, P., Obstfeld, M., and Russ, K., 2018. In search of the Armington elasticity. *Review of Economics and Statistics*, 100, 135-150.

[27] Field, A., 1978. Sectoral shifts in Antebellum Massachusetts: A reconsideration. *Exploration in Economic History*, 15, 146-171.

[28] Foster, A., and Rosenzweig, M., 2004. Agricultural productivity growth, rural economic diversity, and economic reforms: India, 1970–2000. *Economic Development and Cultural Change*, 52, 509-542.

[29] Foster, A., and Rosenzweig, M., 2008. Economic development and the decline of agricultural employment. *Handbook of Development Economics*, 4, 3051-3083.

[30] Galor, O., 2005. From stagnation to growth: unified growth theory. *Handbook of Economic Growth*, 1, 171-293.

[31] Galor, O., 2011. Unified Growth Theory. Princeton University Press.

[32] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, 117, 1133-1191.

[33] Galor, O., and Mountford, A., 2008. Trading population for productivity: theory and evidence. *Review of Economic Studies*, 75, 1143-1179.

[34] Galor, O., Moav, O., and Vollrath, D., 2009. Inequality in landownership, the emergence of human-capital promoting institutions, and the great divergence. *Review of Economic Studies*, 76, 143-179.

[35] Galor, O., and Weil, D., 2000. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90, 806-828.

[36] Garcia-Macia, D., Hsieh, C., and Klenow, P., 2019. How destructive is innovation?. *Econometrica*, 87, 1507-1541.

[37] Gollin, D., 2010. Agricultural productivity and economic growth. *Handbook of Agricultural Economics*, 4, 3825-3866.

[38] Gollin, D., Parente, S., and Rogerson, R., 2002. The role of agriculture in development. *American Economic Review*, 92, 160-164.

[39] Gollin, D., Parente, S., and Rogerson, R., 2007. The food problem and the evolution of international income levels. *Journal of Monetary Economics*, 54, 1230-1255.

[40] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43-61.

[41] Grossman, G., and Oberfield, E., 2021. The Elusive Explanation for the Declining Labor Share. NBER Working Paper 29165.

[42] Ha, J., and Howitt, P., 2007. Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory. *Journal of Money, Credit and Banking*, 39, 733-774.

[43] Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy*, 107, 715-730.

[44] Iacopetta, M., Minetti, R., and Peretto, P., 2019. Financial markets, industry dynamics and growth. *Economic Journal*, 129, 2192-2215.

[45] Iacopetta, M., and Peretto, P., 2021. Corporate governance and industrialization. *European Economic Review*, 135, 103718.

[46] Johnson, D., 1997. Agriculture and the Wealth of Nations. *American Economic Review*, 87, 1-12.

[47] Johnston, B., and Mellor, J., 1961. The role of agriculture in economic development. *American Economic Review*, 51, 566-593.

[48] Kogel, T., and Prskawetz, A., 2001. Agricultural productivity growth and escape from the Malthusian trap. *Journal of Economic Growth*, 6, 337-357.

[49] Kongsamut, P., Rebelo, S., and Xie, D., 2001. Beyond balanced growth. *Review of Economic Studies*, 68, 869-882.

[50] Krugman, P., 1987. The narrow moving band, the Dutch disease, and the competitive consequences of Mrs. Thatcher: Notes on trade in the presence of dynamic scale economies. *Journal of Development Economics*, 27, 41-55.

[51] Lagakos, D., and Waugh, M., 2013. Selection, agriculture, and cross-country productivity differences. *American Economic Review*, 103, 948-980.

[52] Laincz, C., and Peretto, P., 2006. Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth*, 11, 263-288.

[53] Lewis, W., 1955. *The Theory of Economic Growth*. London: George Allen & Unwin.

[54] Lu, Y., and Yu, L., 2015. Trade liberalization and markup dispersion: evidence from China's WTO accession. *American Economic Journal: Applied Economics*, 7, 221-253.

[55] Lucas R., 1988. On the mechanics of economic development. *Journal of Monetary Economics*, 22, 3-42.

[56] Lucas, R., 2004. Life earnings and rural-urban migration. *Journal of Political Economy*, 112, S29-S59.

[57] Madsen, J., 2008. Semi-endogenous versus Schumpeterian growth models: testing the knowledge production function using international data. *Journal of Economic Growth*, 13, 1-26.

[58] Matsuyama, K., 1992. Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory*, 58, 317-334.

[59] McArthur, J., and McCord, G., 2017. Fertilizing growth: Agricultural inputs and their effects in economic development. *Journal of Development Economics*, 127, 133-152.

[60] Mellor, J., 1995. *Agriculture on the road to industrialization*. Baltimore: Johns Hopkins University Press.

[61] Mokyr, J., 1976. *Industrialization in the Low Countries, 1795-1850*. New Haven, CT: Yale University Press.

[62] Morrison, C., 1992. Unraveling the productivity growth slowdown in the United States, Canada and Japan: The effects of subequilibrium, scale economies and markups. *Review of Economics and Statistics*, 74, 381-393.

[63] Murphy, K., Shleifer, A., and Vishny, R., 1989. Income distribution, market size, and industrialization. *Quarterly Journal of Economics*, 104, 537-564.

[64] Ngai, L., and Pissarides, C., 2007. Structural change in a multisector model of growth. *American Economic Review*, 97, 429-443.

[65] Nurkse, R., 1953. *Problems of Capital Formation in Underdeveloped Countries*. New York: Oxford University Press.

[66] Peretto, P., 1994. Essays on Market Structure and Economic Growth (Ph.D. dissertation). Yale University.

[67] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth*, 3, 283-311.

[68] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics*, 43, 173-195.

[69] Peretto, P., and Valente, S., 2011. Resources, innovation and growth in the global economy. *Journal of Monetary Economics*, 58, 387-399.

[70] Peretto, P., 2015. From Smith to Schumpeter: A theory of take-off and convergence to sustained growth. *European Economic Review*, 78, 1-26.

[71] Poutanen, K., Karlund, A., Gomez-Gallego, C., Johansson, D., Scheers, N., Marklinder, I., Eriksen, A., Silventoinen, P., Nordlund, E., Sozer, N., and Hanhineva, K., 2022. Grains - a major source of sustainable protein for health. *Nutrition Reviews*, 80, 1648-1663.

[72] Ranis, G., and Fei, J., 1961. A theory of economic development. *American Economic Review*, 51, 533-565.

[73] Ravallion, M., and Chen, S., 2007. China's (uneven) progress against poverty. *Journal of Development Economics*, 82, 1-42.

[74] Restuccia, D., Yang, D., and Zhu, X., 2008. Agriculture and aggregate productivity: A quantitative cross-country analysis. *Journal of Monetary Economics*, 55, 234-250.

[75] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.

[76] Rostow, W., 1959. The stages of economic growth. *Economic History Review*, 12, 1-16.

[77] Schultz, T., 1953. *The Economic Organization of Agriculture*. New York: McGraw-Hill.

[78] Segerstrom, P., Anant, T., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review*, 80, 1077-1091.

[79] Smulders, S., 1994. Growth, Market Structure and the Environment: Essays on the Theory of Endogenous Economic Growth (Ph.D. dissertation). Tilburg University.

[80] Smulders, S., and van de Klundert, T., 1995. Imperfect competition, concentration and growth with firm-specific R&D. *European Economic Review*. 39, 139-160.

[81] Song, Z., Storesletten, K., and Zilibotti, F., 2011. Growing like China. *American Economic Review*, 101, 196-233.

[82] Tiffin, R., and Irz, X., 2006. Is agriculture the engine of growth?. *Agricultural Economics*, 35, 79-89.

[83] Wright, G., 1979. Cheap labor and southern textiles before 1880. *Journal of Economic History*, 39, 655-680.

A Data

Table A1: Summary statistics

Variables	Observations	Mean	Sd	Min	Max
Log non-agricultural real GDP per capita	759	8.240	1.561	4.847	11.586
Log real GDP per capita	772	8.373	1.439	5.240	11.588
TFP index	523	0.984	0.186	0.334	2.184
Agricultural TFP index	667	0.937	0.189	0.309	2.238
Agricultural net import to production ratio	686	0.388	1.674	-0.978	20.482
Log cereal yields per hectare	772	10.062	0.718	7.458	11.910
Cereal import dependency ratio	772	0.351	0.327	0.000	1.000
Log capital stock	732	12.540	2.100	7.409	18.288
Government expenditure share of GDP	738	0.183	0.073	0.007	0.616
Capital depreciation rate	732	0.045	0.012	0.013	0.100
Human capital index	643	2.432	0.685	1.041	3.828

Data source: Food and Agricultural Organization Data for the ratio of agricultural net import to production, cereal yields per hectare and cereal import dependency. U.S. Department of Agriculture for agricultural TFP. World Bank Data for real GDP per capita and non-agricultural real GDP per capita. Penn World Table for other variables.

B Intratemporal equilibrium

Substituting (10) into (7) yields

$$b_t^h = \left(\frac{\delta^h}{1-\delta^h} \frac{w_t^f}{w_t^h} \frac{A^h}{A^f} \right)^{\epsilon} = \left(\frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{p_{Y,t}^f y_t^f}{p_{Y,t}^h y_t^h} \frac{l_{Y,t}^h}{l_{Y,t}^f} \frac{A^h}{A^f} \right)^{\epsilon}, \tag{B.1}$$

where the second equality uses (14). We rearrange (B.1) as

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{p_{Y,t}^h c_t^h}{p_{Y,t}^f c_t^f} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{A^h}{A^f}. \tag{B.2}$$

We use (4), (6) and (27) to obtain

$$\frac{p_{Y,t}^h c_t^h}{p_{Y,t}^f c_t^f} = \frac{\psi^f}{\psi^h + \frac{(1-\delta^h)\gamma^h}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}} \frac{L_t^f}{L_t^h} = \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h \right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h} \tag{B.3}$$

Then, we substitute (B.3) into (B.2) to obtain

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h \right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{A^h}{A^f}, \tag{B.4}$$

which holds at any time t. For brevity, we define the left-hand side of (B.4) as the following function:

$$F(\cdot) \equiv \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{L_t^f}{L_t^h} \frac{c_t^f / y_t^f}{c_t^h / y_t^h} \frac{l_{Y,t}^f}{l_{Y,t}^h}. \tag{B.5}$$

We substitute (5), (9), (10), (11) and $l_{A,t}^h + l_{Y,t}^h = 1$ into (14) to derive the industrial labor share in Home as

$$l_{Y,t}^h = \left[1 + \frac{\gamma^h}{1-\theta^h} \frac{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h} \frac{c_t^h}{y_t^h}\right]^{-1} \left(1 - \frac{\eta}{A^h}\right). \tag{B.6}$$

We substitute (6), (8), (9), (10), (11), (B.3) and $l_{A,t}^f + l_{Y,t}^f = 1$ into (14) to derive

$$l_{Y,t}^f = \left\{1 + \frac{1}{1-\theta^f} \left[\gamma^f + \frac{\gamma^h (1-\delta^h) \psi^f}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right] \frac{c_t^f}{y_t^f}\right\}^{-1}. \tag{B.7}$$

Furthermore, we use (9), (10) and (11) to derive

$$w_t^h l_{A,t}^h = p_{A,t}^h q_t^h, \tag{B.8}$$

$$w_t^f l_{A,t}^f = p_{A,t}^f \left(q_t^f + m_t^h \frac{L_t^h}{L_t^f}\right). \tag{B.9}$$

Substituting (B.8), (B.9) and $l_{A,t}^j + l_{Y,t}^j = 1$ into (2), the asset-accumulation equations in Home and Foreign can be expressed as

$$\dot{a}_t^h = (r_t^h - \lambda) a_t^h + w_t^h l_{Y,t}^h - p_{Y,t}^h c_t^h - p_{Y,t}^f \iota_t^h - p_{A,t}^f m_t^h, \tag{B.10}$$

$$\dot{a}_t^f = (r_t^f - \lambda) a_t^f + w_t^f l_{Y,t}^f - p_{Y,t}^f c_t^f - p_{Y,t}^h \iota_t^f + p_{A,t}^f m_t^h \frac{L_t^h}{L_t^f}. \tag{B.11}$$

B.1 Intratemporal equilibrium: both Home and Foreign are in the pre-industrial era

In the pre-industrial era, both $\dot{a}_t^h$ and $\dot{a}_t^f$ are zero due to the absence of asset accumulation during this period. Therefore, (B.10) and (B.11) can be rearranged as

$$p_{Y,t}^h c_t^h = w_t^h l_{Y,t}^h - p_{Y,t}^f \iota_t^h - p_{A,t}^f m_t^h, \tag{B.12}$$

$$p_{Y,t}^f c_t^f = w_t^f l_{Y,t}^f - p_{Y,t}^h \iota_t^f + p_{A,t}^f m_t^h \frac{L_t^h}{L_t^f}. \tag{B.13}$$

Then, we substitute (4), (6) and (14) into (B.12) to derive

$$\frac{c_t^h}{y_t^h} = \frac{1-\theta^h}{1 + \psi^h + \frac{(1-\delta^h)\gamma^h}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}}, \tag{B.14}$$

which shows that c_t^h/y_t^h is a function of b_t^h. Then, we substitute (4), (6), (14) and (B.3) into (B.13) to derive

$$\frac{c_t^f}{y_t^f} = \frac{1-\theta^f}{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}, \tag{B.15}$$

which shows that c_t^f/y_t^f is also a function of b_t^h. Substituting (B.6), (B.7), (B.14) and (B.15) into (B.5), we re-express $F(\cdot)$ as

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)} \frac{1}{1-\frac{\eta}{A^h}} \frac{L_0^f}{L_0^h} \frac{1-\theta^f}{1-\theta^h} \frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}, \tag{B.16}$$

in which b_t^h is the only endogenous variable. We can further verify that $F(\cdot)$ is monotonically increasing in b_t^h for any $\epsilon \in (0,\infty)$.[36] Since (B.4) holds at any time t, b_t^h must be equal to a unique value $(b^h)^*$ and remain constant. Consequently, according to (B.14) and (B.15), the consumption-output ratios c_t^h/y_t^h and c_t^f/y_t^f jump to their unique stationary levels.

B.2 Intratemporal equilibrium: Home is in the pre-industrial era while Foreign is in the industrial era

As Home is still in the pre-industrial era, the expression for the steady-state c_t^h/y_t^h is identical to (B.14). When Foreign enters the industrial era (i.e., $x_t^f > x_N^f$), variety-expanding innovation takes place and the free-entry condition holds. Using $a_t^f = N_t^f V_t^f/L_t^f$, $\theta^f p_{Y,t}^f Y_t^f = \mu^f N_t^f X_t^f$ and the free-entry condition (24) yields

$$a_t^f = \frac{\beta^f\theta^f}{\mu^f} p_{Y,t}^f y_t^f. \tag{B.17}$$

Then, we have

$$\frac{\dot{a}_t^f}{a_t^f} = \frac{\dot{y}_t^f}{y_t^f} + \frac{\dot{p}_{Y,t}^f}{p_{Y,t}^f} \tag{B.18}$$

We use (4), (6), (14), (B.3), (B.11), (B.17) and (B.18) to derive

$$\frac{\dot{y}_t^f}{y_t^f} = (r_t^f-\lambda) + \frac{\mu^f}{\beta^f\theta^f}(1-\theta^f) - \frac{\mu^f}{\beta^f\theta^f}\left[1+\psi^f - \frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]\frac{c_t^f}{y_t^f} - \frac{\dot{p}_{Y,t}^f}{p_{Y,t}^f}. \tag{B.19}$$

We combine (3) and (B.19) to obtain

$$\frac{\dot{c}_t^f}{c_t^f} - \frac{\dot{y}_t^f}{y_t^f} = -(\rho^f-\lambda) - \frac{\mu^f}{\beta^f\theta^f}(1-\theta^f) + \frac{\mu^f}{\beta^f\theta^f}\left[1+\psi^f - \frac{\psi^f\gamma^h(1-\delta^h)}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\right]\frac{c_t^f}{y_t^f}. \tag{B.20}$$

[36]See Appendix C for a detailed derivation.

To simplify the expressions of the dynamic system, we define $\varsigma_t^h \equiv y_t^h/c_t^h$ and $\varsigma_t^f \equiv y_t^f/c_t^f$. Then we rewrite (B.14) as

$$\varsigma_t^h = \left[1 + \psi^h + \frac{(1-\delta^h)\gamma^h}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}\right] \frac{1}{1-\theta^h}. \tag{B.21}$$

The dynamic system in (B.20) can be re-expressed as

$$\dot{\varsigma}_t^f = \underbrace{\left[(\rho^f - \lambda) + \frac{\mu^f}{\beta^f \theta^f}(1-\theta^f)\right] \varsigma_t^f - \frac{\mu^f}{\beta^f \theta^f} \left[1 + \psi^f - \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]}_{\equiv \varsigma(\varsigma_t^f, b_t^h)}, \tag{B.22}$$

where we define the right-hand side of (B.22) as $\varsigma(\varsigma_t^f, b_t^h)$. We combine (B.4), (B.6) and (B.7) to obtain

$$\begin{aligned} &\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{\varsigma_t^h + \frac{\gamma^h}{1-\theta^h} \frac{\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h}}{\varsigma_t^f + \frac{1}{1-\theta^f}\left[\gamma^f + \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]} \\ =& \underbrace{\frac{\delta^h}{1-\delta^h} \frac{1-\theta^f}{1-\theta^h} \frac{A^h}{A^f} \frac{L_0^h}{L_0^f} \left(1 - \frac{\eta}{A^h}\right)}_{\equiv \Gamma}. \end{aligned} \tag{B.23}$$

For brevity, we define the right-hand side of (B.23) as Γ. Substituting (B.21) into (B.23) yields

$$\left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{1 + \psi^h + \gamma^h}{\varsigma_t^f + \frac{1}{1-\theta^f}\left[\gamma^f + \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]} = \Gamma. \tag{B.24}$$

For any $\epsilon \in (0, \infty)$, the left-hand side of (B.24) increases from 0 to ∞ as b_t^h increases from 0 to ∞, while it decreases from a finite value to 0 as ς_t^f increases from 0 to ∞. Therefore, there uniquely exists a function, $\vartheta(\cdot)$, such that $b_t^h \equiv \vartheta(\varsigma_t^f)$, which is increasing in ς_t^f with $\vartheta(0) > 0$ and $\lim_{\varsigma_t^f \to \infty} \vartheta(\varsigma_t^f) \to \infty$.

For $\epsilon \in (0, 1)$, given that $\partial \vartheta(\varsigma_t^f)/\partial \varsigma_t^f > 0$, it is straightforward to show that the right-hand side of (B.22) is monotonically increasing in ς_t^f (i.e., $\partial \varsigma(\varsigma_t^f, \vartheta(\varsigma_t^f))/\partial \varsigma_t^f > 0$). For $\epsilon \in [1, \infty)$, we first solve (B.24) for ς_t^f to obtain

$$\varsigma_t^f = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{1+\psi^h+\gamma^h}{\Gamma} - \left[\frac{\gamma^f}{1-\theta^f} + \frac{1}{1-\theta^f} \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}\right]. \tag{B.25}$$

Then, substituting (B.25) into (B.22) yields

$$\begin{aligned} \dot{\varsigma}_t^f =& \frac{\psi^f \left(1 + \psi^h + \gamma^h\right) \left[(\rho^f - \lambda) + \frac{\mu^f}{\beta^f \theta^f}(1-\theta^f)\right]}{\Gamma} \frac{\left(b_t^h\right)^{\frac{1}{\epsilon}} \left[\delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h (b_t^h)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \\ &- \frac{\rho^f - \lambda}{1-\theta^f} \frac{\psi^f \gamma^h (1-\delta^h)}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} - \frac{\gamma^f(\rho^f - \lambda)}{1-\theta^f} - \frac{\mu^f \left(1 + \psi^f + \gamma^f\right)}{\beta^f \theta^f}, \end{aligned} \tag{B.26}$$

which shows that $\dot{\varsigma}_t^f$ is monotonically increasing in b_t^h. Given that $b_t^h = \vartheta(\varsigma_t^f)$ is increasing in ς_t^f, $\dot{\varsigma}_t^f$ is monotonically increasing in ς_t^f when $\epsilon \in [1, \infty)$.

Given that $\dot{\varsigma}_t^f$ is increasing in ς_t^f for any $\epsilon \in (0, \infty)$, ς_t^f jumps to its unique level and remains constant if $\dot{\varsigma}_t^f < 0$ when $\varsigma_t^f = 0$ and $\dot{\varsigma}_t^f > 0$ when $\varsigma_t^f \to \infty$. According to (B.22), $\dot{\varsigma}_t^f$ is strictly negative when $\varsigma_t^f = 0$, as the following inequality always holds:

$$1 + \psi^f - \frac{\psi^f \gamma^h (1 - \delta^h)}{\psi^h \delta^h \vartheta(0)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1 - \delta^h)} > 0. \tag{B.27}$$

In addition, as $\rho^f > \lambda$, the following inequality

$$(\rho^f - \lambda) + \frac{\mu^f}{\beta^f \theta^f}(1 - \theta^f) > 0, \tag{B.28}$$

always holds, which suggests that $\dot{\varsigma}_t^f$ is strictly positive when $\varsigma_t^f \to \infty$. Therefore, ς_t^f jumps to its unique level and remains constant. Since $b_t^h = \vartheta(\varsigma_t^f)$ is an increasing function of ς_t^f, b_t^h must equal a unique value when ς_t^f is constant. Furthermore, ς_t^h jumps to a unique level when b_t^h is constant. Then we complete the proof that the dynamic system is stable. From $c_t^h/y_t^h = 1/\varsigma_t^h$, $c_t^f/y_t^f = 1/\varsigma_t^f$ and $b_t^h = \vartheta(\varsigma_t^f)$, the endogenous variables c_t^h/y_t^h, c_t^f/y_t^f and b_t^h reach their own unique level and remain constant. Specifically, when Home is the pre-industrial era and Foreign is the industrial era,[37] the expression for steady-state c_t^h/y_t^h is identical to (B.14), and the expression for steady-state c_t^f/y_t^f in the industrial era is given by

$$\frac{c_t^f}{y_t^f} = \frac{1 - \theta^f + \frac{\beta^f \theta^f}{\mu^f}(\rho^f - \lambda)}{1 + \psi^f - \frac{\psi^f (1-\delta^h)\gamma^h}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)}}. \tag{B.29}$$

C Proofs

Dynamic optimization of the Home representative household. The current-value Hamiltonian of the Home representative household is

$$H_t^h = \ln c_t^h + \psi^h \ln \iota_t^h + \gamma^h \ln \left[\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1 - \delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} + \xi_t^h \dot{a}_t^h, \tag{C.1}$$

where ξ_t^h is co-state variable on $\dot{a}_t^h$. Substituting (2) into (C.1) to derive:

$$\frac{\partial H_t^h}{\partial q_t^h} = 0 \Rightarrow p_{A,t}^h (q_t^h - \eta^h) = \frac{\delta^h \gamma^h p_{Y,t}^h c_t^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1 - \delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}}}, \tag{C.2}$$

$$\frac{\partial H_t^h}{\partial m_t^h} = 0 \Rightarrow p_{A,t}^f m_t^h = \frac{(1 - \delta^h)\gamma^h p_{Y,t}^h c_t^h (m_t^h)^{\frac{\epsilon-1}{\epsilon}}}{\delta^h (q_t^h - \eta^h)^{\frac{\epsilon-1}{\epsilon}} + (1 - \delta^h)(m_t^h)^{\frac{\epsilon-1}{\epsilon}}}. \tag{C.3}$$

[37] The proof for the case in which both countries are in the industrial era is relegated to an online appendix available upon request.

Then, we rewrite (C.2) as (5) and rewrite (C.3) as (6). In addition, we use (C.2) and (C.3) to obtain (7). ■

Dynamic optimization of monopolistic firms. The current-value Hamiltonian of monopolistic firms is

$$H_t^j(i) = \Pi_t^j(i) - p_{Y,t}^j R_t^j(i) + \varkappa_t^j(i)\dot{Z}_t^j(i) + \zeta_t^j(i)\left[\mu^j p_{Y,t}^j - p_{X,t}^j(i)\right] \tag{C.4}$$

where $\varkappa_t^j(i)$ is co-state variable on $\dot{Z}_t^j$ and $\zeta_t^j(i)$ is the multiplier on the markup price $p_{X,t}^j(i) \leq \mu^j p_{Y,t}^j$. We substitute (13), (16) and (17) into (C.4) and take derivative to derive

$$\frac{\partial H_t^j(i)}{\partial p_{X,t}^j(i)} = \frac{\partial \Pi_t^j(i)}{\partial p_{X,t}^j(i)} - \zeta_t^j(i) = 0, \tag{C.5}$$

$$\frac{\partial H_t^j(i)}{\partial R_t^j(i)} = -p_{Y,t}^j + \varkappa_t^j(i) = 0 \Rightarrow \varkappa_t^j(i) = p_{Y,t}^j, \tag{C.6}$$

$$\frac{\partial H_t^j(i)}{\partial Z_t^j(i)} = \alpha^j \left\{ \left[p_{X,t}^j(i) - p_{Y,t}^j\right] \left[\frac{\theta^j}{p_{X,t}^j(i)/p_{Y,t}^j}\right]^{\frac{1}{1-\theta^j}} \frac{L_{Y,t}^j}{(N_t^j)^{1-\sigma^j}} - p_{Y,t}^j \phi^j \right\} \left[\frac{Z_t^j}{Z_t^j(i)}\right]^{1-\alpha^j} = r_t^j \varkappa_t^j(i) - \dot{\varkappa}_t^j(i). \tag{C.7}$$

If $p_{X,t}^j(i) < \mu^j p_{Y,t}^j$, then we have $\zeta_t^j(i) = 0$ and $p_{X,t}^j(i) = p_{Y,t}^j/\theta^j$. If the constraint on $p_{X,t}^j(i)$ is binding, we have $\zeta_t^j(i) > 0$ and $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$. As we employ the assumption that $\mu^j \in (1, 1/\theta^j)$, the price of intermediate good $X_t^j(i)$ is given by $p_{X,t}^j(i) = \min\{\mu^j p_{Y,t}^j, p_{Y,t}^j/\theta^j\} = \mu^j p_{Y,t}^j$. Substituting (13), (C.6) and $p_{X,t}^j(i) = \mu^j p_{Y,t}^j$ into (C.7) and imposing symmetry yield (20). ■

Industrial labor shares. Appendix B shows that endogenous variables c_t^h/y_t^h, c_t^f/y_t^f and b_t^h jump to their own unique level and remain constant. Also, it provides the expressions for the industrial labor shares in Home and Foreign, as shown in (B.6) and (B.7). These expressions include the consumption-output ratio c_t^j/y_t^j. We further derive more specific expressions of the industrial labor shares across different eras in country j.

Substituting the pre-industrial consumption-output ratio c_t^h/y_t^h (B.14) into (B.6) yields the Home industrial labor share in the pre-industrial era:

$$l_{Y,t}^h = \frac{(1+\psi^h)\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h+\gamma^h)\left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]} \left(1 - \frac{\eta}{A^h}\right); \tag{C.8}$$

substituting consumption-output ratio c_t^h/y_t^h in the industrial era into (B.6) yields the Home industrial production labor share in the industrial era:

$$l_{Y,t}^h = \frac{(1+\psi^h)\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)}{\left[1+\psi^h + \frac{1-\theta^h + \frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (1+\psi^h+\gamma^h)(1-\delta^h)} \left(1 - \frac{\eta}{A^h}\right). \tag{C.9}$$

In addition, we substitute the pre-industrial consumption-output ratio c_t^f/y_t^f (B.15) into (B.7) to derive the Foreign industrial labor share in the pre-industrial era:

$$l_{Y,t}^f = \frac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\gamma^f}; \tag{C.10}$$

and substitute consumption-output ratio c_t^f/y_t^f in the industrial era (B.29) into (B.7) to derive the Foreign industrial labor share in industrial era:

$$l_{Y,t}^f = \frac{1+\psi^f - \frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}. \tag{C.11}$$

■

Comparative statics of b_t^h with respect to A^h and A^f.

From (B.4) and (B.5), the following equality holds at any time t

$$F(\cdot) \equiv \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}\frac{L_t^f}{L_t^h}\frac{c_t^f/y_t^f}{c_t^h/y_t^h}\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}. \tag{C.12}$$

The expression of $F(\cdot)$ varies depending on the era in which country j is situated, due to the differentiated stationary values of consumption-output ratio between the pre-industrial era and the industrial era. In the following, we prove that $F(\cdot)$ is determined by b_t^h. Then we discuss the monotonicity of $F(\cdot)$ with respect to b_t^h. There are three cases: (1) both Home and Foreign are in the pre-industrial era; (2) Home is in the pre-industrial era while Foreign is in the industrial era; and (3) both Home and Foreign are in the industrial era.

Case 1: Both Home and Foreign are in the pre-industrial era. When Home and Foreign are both in the pre-industrial era, we use c_t^h/y_t^h in (B.14) and c_t^f/y_t^f in (B.15) to derive

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}\frac{1-\theta^f}{1-\theta^h}. \tag{C.13}$$

Then we use (C.8) and (C.10) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta}{A^h}}\frac{\frac{(1+\psi^h+\gamma^h)\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\gamma^f}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.14}$$

Substituting (C.13) and (C.14) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f \left[\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + 1 - \delta^h\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{1}{1-\frac{\eta}{A^h}} \frac{L_0^f}{L_0^h} \frac{1-\theta^f}{1-\theta^h} \frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f \left[\delta^h b_t^h + (1-\delta^h)\left(b_t^h\right)^{\frac{1}{\epsilon}}\right]}{\psi^h \delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}} + (\psi^h + \gamma^h)(1-\delta^h)} \frac{1}{1-\frac{\eta}{A^h}} \frac{L_0^f}{L_0^h} \frac{1-\theta^f}{1-\theta^h} \frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}; \tag{C.15}$$

and

$$F(\cdot) = \frac{\psi^f \left(b_t^h\right)^{\frac{1}{\epsilon}}}{\psi^h + \frac{\gamma^h(1-\delta^h)}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}} \frac{1}{1-\frac{\eta}{A^h}} \frac{L_0^f}{L_0^h} \frac{1-\theta^f}{1-\theta^h} \frac{1+\psi^h+\gamma^h}{1+\psi^f+\gamma^f}. \tag{C.16}$$

(C.15) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in (0,1)$, and (C.16) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in [1,\infty)$. Therefore, we complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (0,\infty)$ when both Home and Foreign are in the pre-industrial era.

Case 2: Home is in the pre-industrial era and Foreign is in the industrial era. When Home is in the pre-industrial era and Foreign is in the industrial era, combining c_t^h/y_t^h in (B.14) and c_t^f/y_t^f in (B.29) yields

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} \frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}. \tag{C.17}$$

Then we use (C.8) and (C.11) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta}{A^h}} \frac{\frac{(1+\psi^h+\gamma^h)\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right]}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.18}$$

Substituting (C.17) and (C.18) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left[\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h\right](1+\psi^h+\gamma^h)\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}\frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f\left[\delta^h b_t^h+(1-\delta^h)\left(b_t^h\right)^{\frac{1}{\epsilon}}\right](1+\psi^h+\gamma^h)\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h}\frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h} \tag{C.19}$$

and

$$F(\cdot) = \frac{\psi^f \left(b_t^h\right)^{\frac{1}{\epsilon}} (1+\psi^h+\gamma^h) \frac{L_0^f}{L_0^h} \frac{1-\theta^f + \frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h} \frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h + \frac{\gamma^h(1-\delta^h)}{\delta^h \left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right] + \frac{\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}. \tag{C.20}$$

(C.19) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in (0,1)$, and (C.20) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in [1,\infty)$. Then we complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (0,\infty)$ when Home is in the pre-industrial era and Foreign is in the industrial era.

Case 3: Both Home and Foreign are in the industrial era. When both Home and Foreign are in the industrial era, combining industrial-era consumption-output ratios c_t^h/y_t^h and c_t^f/y_t^f in Lemma 1 yields

$$\frac{c_t^f/y_t^f}{c_t^h/y_t^h} = \frac{1+\psi^h+\frac{(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}} \frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}. \tag{C.21}$$

Then we use (C.9) and (C.11) to obtain

$$\frac{l_{Y,t}^f}{l_{Y,t}^h} = \frac{1}{1-\frac{\eta}{A^h}} \frac{\frac{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}{(1+\psi^h)\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)}}{\frac{1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}{1+\psi^f-\frac{\psi^f(1-\delta^h)\gamma^h}{\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)}}}. \tag{C.22}$$

Substituting (C.21) and (C.22) into (B.5) yields

$$F(\cdot) = \left(b_t^h\right)^{\frac{1}{\epsilon}} \frac{\psi^f\left\{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(1+\psi^h+\gamma^h)(1-\delta^h)\right\}\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}\frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h},$$

which can be rewritten as

$$F(\cdot) = \frac{\psi^f\left\{\left[1+\psi^h+\frac{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\right]\delta^h b_t^h+(1+\psi^h+\gamma^h)(1-\delta^h)\left(b_t^h\right)^{\frac{1}{\epsilon}}\right\}\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}\frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+(\psi^h+\gamma^h)(1-\delta^h)\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h}; \tag{C.23}$$

and

$$F(\cdot) = \frac{\psi^f\left(b_t^h\right)^{\frac{1}{\epsilon}}\left[1+\psi^h+\gamma^h+\frac{\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}{1-\theta^h}\gamma^h\frac{\delta^h}{\delta^h+(1-\delta^h)/\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}}\right]\frac{L_0^f}{L_0^h}\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^h+\frac{\beta^h\theta^h}{\mu^h}(\rho^h-\lambda)}\frac{1}{1-\frac{\eta}{A^h}}}{\left[\psi^h+\frac{\gamma^h(1-\delta^h)}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}\right]\left[1+\psi^f+\frac{1-\theta^f+\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\gamma^f\right]+\frac{\frac{\frac{\beta^f\theta^f}{\mu^f}(\rho^f-\lambda)}{1-\theta^f}\psi^f(1-\delta^h)\gamma^h}{\delta^h\left(b_t^h\right)^{\frac{\epsilon-1}{\epsilon}}+1-\delta^h}}. \tag{C.24}$$

(C.23) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in (0,1)$, and (C.24) shows that $F(\cdot)$ is increasing in b_t^h for $\epsilon \in [1,\infty)$. We complete the proof that $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (0,\infty)$ when both Home and Foreign are in the industrial era.

The above analysis shows that, regardless of the phase in which Home and Foreign are, $F(\cdot)$ can be considered as a function of b_t^h. Furthermore, $F(\cdot)$ is increasing in b_t^h for any $\epsilon \in (0,\infty)$ (i.e., $F(\underset{+}{b_t^h})$). Recall that (C.12) holds at any time t:

$$F(\cdot) = \frac{\delta^h}{1-\delta^h}\frac{1-\theta^f}{1-\theta^h}\frac{A^h}{A^f}.$$

Therefore, a higher level of the agricultural productivity A^h in Home (A^f in Foreign) implies that the value of $F(\cdot)$ is larger (smaller). Consequently, a higher level of agricultural productivity A^h in Home (A^f in Foreign) leads to a larger (smaller) value of b_t^h. ■