RAG Evaluation - NDCG (Normalized Discounted Cumulative Gain)

📚 Given Relevance Scores

Rank	Retrieved Document	Relevance Score
1	The Pacific Ocean is the largest ocean.	3
2	The Atlantic Ocean separates Europe and America.	2
3	The Sahara Desert is the largest hot desert.	0
4	The Indian Ocean is the third largest ocean.	1

🎯 Objective

Compute the following metrics:

1. **DCG@4** (Discounted Cumulative Gain)
2. **IDCG@4** (Ideal DCG)
3. **NDCG@4** (Normalized DCG)

📊 Step 1: Compute DCG@4

Formula:

$$\text{DCG@K} = \sum_{i=1}^{K} \frac{\text{rel}_i}{\log_2(i+1)}$$

Calculation (term by term):

Position (i)	Relevance	$\log_2(i+1)$	rel / $\log_2(i+1)$
1	3	$\log_2(2)$ = 1.000	3.000

Position (i)	Relevance	$\log_2(i+1)$	rel / $\log_2(i+1)$
2	2	$\log_2(3) \approx 1.585$	1.262
3	0	$\log_2(4) = 2.000$	0.000
4	1	$\log_2(5) \approx 2.321$	0.431

Result:

$$\text{DCG@4} = 3.000 + 1.262 + 0.000 + 0.431 = 4.693$$

📈 Step 2: Compute IDCG@4 (Ideal DCG)

First, sort the relevance scores in **descending order** (best possible ranking):

Ideal Order: [3, 2, 1, 0]

Calculation:

Position (i)	Relevance	$\log_2(i+1)$	rel / $\log_2(i+1)$
1	3	$\log_2(2) = 1.000$	3.000
2	2	$\log_2(3) \approx 1.585$	1.262
3	1	$\log_2(4) = 2.000$	0.500
4	0	$\log_2(5) \approx 2.321$	0.000

Result:

$$\text{IDCG@4} = 3.000 + 1.262 + 0.500 + 0.000 = 4.762$$

🎯 Step 3: Compute NDCG@4

Formula:

$$\text{NDCG@4} = \frac{\text{DCG@4}}{\text{IDCG@4}} = \frac{4.693}{4.762} \approx 0.9855$$

✅ Final Results

Metric	Value
DCG@4	4.693
IDCG@4	4.762
NDCG@4	**0.9855**

Interpretation

An NDCG@4 score of **0.9855** (98.55%) indicates excellent ranking quality. The retrieved documents are nearly optimally ordered, with only a minor deviation from the ideal ranking.