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Estadistica Aplicada 651

①

a) $E[Y_t] = E[\delta + \varphi_1 Y_{t-1} + \varepsilon_t] = E[\delta] + E[\varphi_1 Y_{t-1}] + \cancel{E[\varepsilon_t]}^{0} = \delta + \varphi_1 \mu$

$\mu = \delta + \varphi_1 \mu$

$\mu - \varphi_1 \mu = \delta$

$\mu = \dfrac{\delta}{1-\varphi_1}$

b) $Var[Y_t] = Var[Y_t - \mu Y_t]^2 = E[Y_t - \mu]^2$

$= Var[\delta + \varphi_1 Y_{t-1} + \varepsilon_t] = \cancel{Var[\delta]}^{0} + \varphi\, Var[Y_{t-1}] + Var[\varepsilon_t]$

$Y_0 = \varphi_1^2 Y_0 + \sigma_\varepsilon^2$

$Y_0 - \varphi_1^2 Y_0 = \sigma_\varepsilon^2$

$Y_0(1-\varphi_1^2) = \sigma_\varepsilon^2$

$Y_0 = \dfrac{\sigma^2 \varepsilon}{1-\varphi_1^2}$

c) $Cov[Y_{t}, Y_{t+1}]$

$Y_1 = E[(Y_t - \mu)(Y_{t-1} - \mu) = E[\tilde{Y}_t \tilde{Y}_{t-1}]$

$= E[(\varphi \tilde{Y}_{t-1} + \varepsilon_t)\tilde{Y}_{t-1}] = E[(\varphi \tilde{Y}^2_{t-1} + \varepsilon \tilde{Y}_{t-1}]$

$= \varphi E[\tilde{Y}^2_{t-2}] + E[\cancel{\varepsilon_t \tilde{Y}_{t-1}}] = \varphi Y_0$

d) $\gamma_k = \operatorname{cov}(Y_t, Y_{t+k}) = E[(Y_t - \mu)(Y_{t+k} - \mu)]$

$= E\left(\sum_{i=0}^{\infty} \phi^i \varepsilon_{t-i}\right)\left(\sum_{i=0}^{\infty} \phi^i \varepsilon_{t+k-i}\right)$

$= E\left(\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi^i \phi^j \varepsilon_{t-i} \varepsilon_{t+k-j}\right)$

Al desarrollar

$= \phi^k \sigma_\varepsilon^2 \left[1 + (\phi^2)^1 + (\phi^2)^2 + (\phi^2)^3 + \dots\right] = \dfrac{\sigma_\varepsilon^2 \phi^k}{1 - \phi^2}$

e) Si es estacionaria

f) Pronostico Y_{t+1}

$\hat{Y}_{t+1} = E(Y_{t+1} \mid Y_t, Y_{t-1}, \dots, Y_1) = E(\phi Y_t + \varepsilon_{t+1})$

$= \phi Y_t$

g) Pronostico Y_{t+2}

$\hat{Y}_{t+2} = E(Y_{t+2} \mid Y_t, Y_{t-1}, \dots, Y_2) = E(\phi Y_{t+1} + \varepsilon_{t+2})$

$E(\phi(\phi Y_t + \varepsilon_{t+1}) + \varepsilon_{t+2}) = E(\phi^2 Y_t + \phi \varepsilon_{t+1} + \varepsilon_{t+2})$

$= \phi^2 E[Y_t] + \phi E[\cancel{\varepsilon_{t+1}}] + E(\cancel{\varepsilon_{t+2}}) = \phi^2 Y_t$

h) $\hat{Y}_{t+T} = \phi^T Y_t \rightarrow \lim_{T \to \infty} \hat{Y}_{t+T} = 0$

$\mu_{Y_t} \Rightarrow \dfrac{\delta}{1-\phi} = \mu_{Y_t}$

i) error 1 periodo

$Y_{t+1} - \hat{Y}_{t+1} = \phi Y_t + \varepsilon_{t+1} - \phi Y_t = \varepsilon_{t+1}$

j) error 2 periodo

$Y_{t+2} - \hat{Y}_{t+2} = \phi Y_{t+1} + \varepsilon_{t+2} - \phi^2 Y_t = \phi(\phi Y_t + \varepsilon_{t+1}) +$

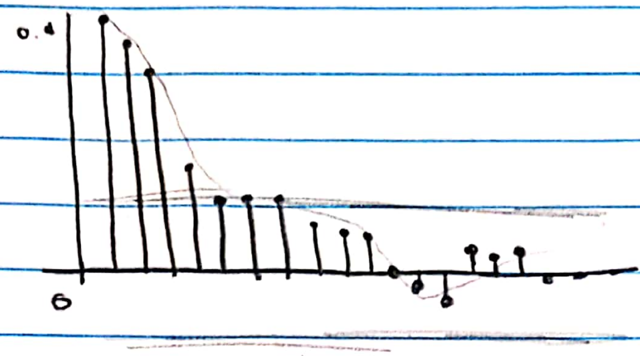
$\varepsilon_{t+2} - \phi^2 Y_t = \phi^2 Y_t + \phi \varepsilon_{t+1} + \varepsilon_{t+2} - \phi^2 Y_t$

$= \phi \varepsilon_{t+2} + \varepsilon_{t+2}$

③ AR(3)

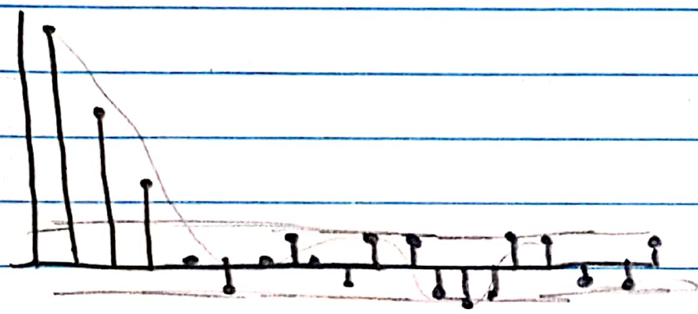
$$Y_t = 0.3\, Y_{t-1} + 0.2\, Y_{t-2} + 0.1\, Y_{t-3} + \varepsilon_t$$

a)



Decrece exponencialmente

b)

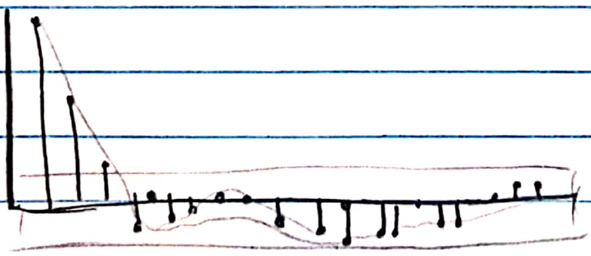


cos-sen

④ MA(3)

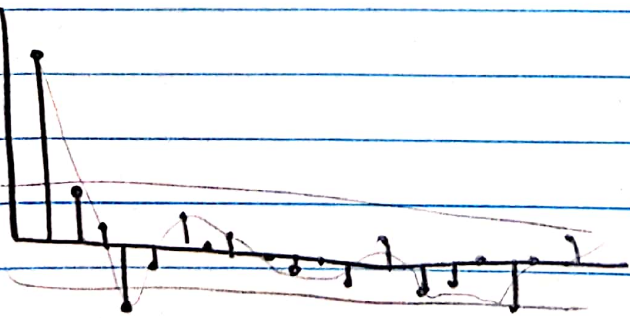
$$Y_t = 0.3\,\varepsilon_{t-1} + 0.2\,\varepsilon_{t-2} + 0.1\,\varepsilon_{t-3} + \varepsilon_t$$

a)



cos-sen

b)



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