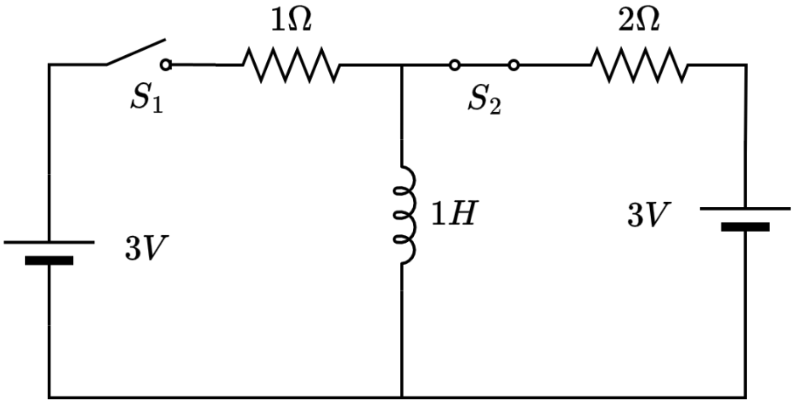
ESO203 Tutorial 4 Solutions

Question 1:

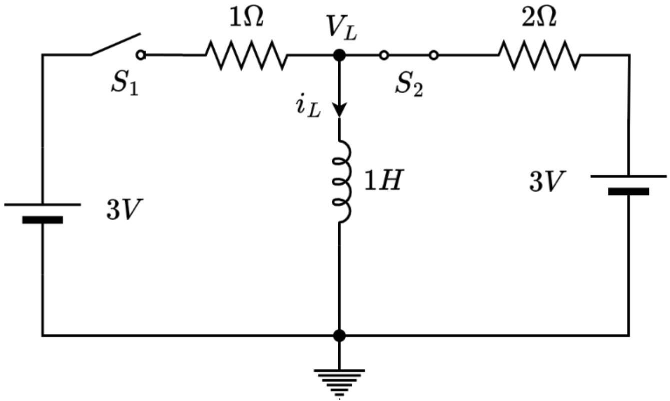
In the circuit shown, switch S_2 has been closed for a long time. At time $t = 0$, S_1 is closed. At $t = 0^+$, the rate of change of current through the inductor, in amperes per second, is ___________.



Solution:

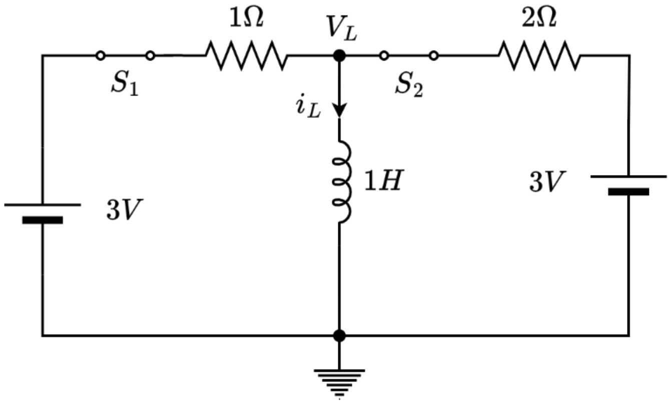
At $t = 0^-$, S_1 is open and S_2 is closed.

Circuit in steady state is



$$i_L(0^-) = \frac{3}{2} = 1.5A$$

For $t > 0$, S_1 is closed and S_2 is closed.



By KCL,

$$\frac{(V_L - 3)}{1} + \frac{(V_L - 3)}{2} + i_L = 0$$

$$2V_L - 6 + V_L - 3 + 2i_L = 0$$

$$V_L = 3 - \frac{2}{3}i_L$$

We know that

$$V_L = L\frac{di}{dt}$$

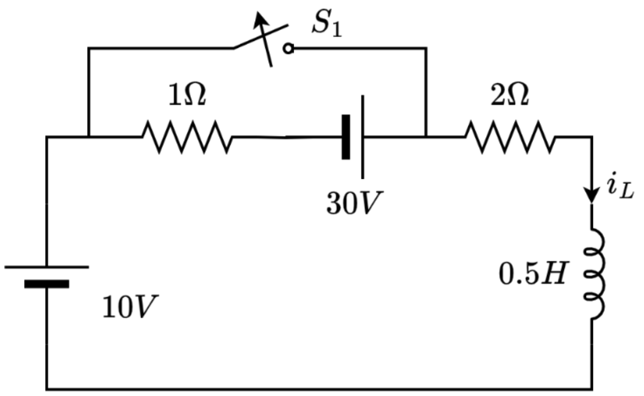
At $t = 0^+$,

$$1\frac{di}{dt} = 3 - \frac{2}{3}i_L(0^+) = 2\ A/s$$

Question 2:

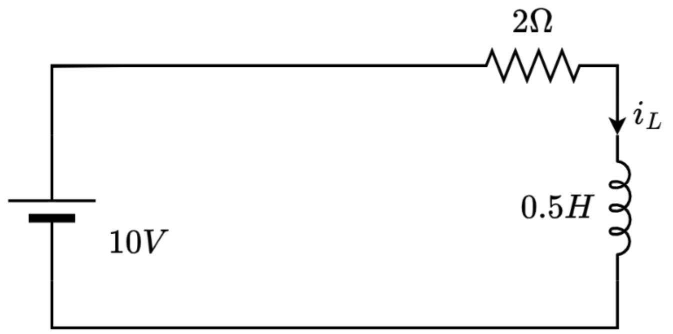
The switch S_1 has been closed for a long time.

(a) Find i_L for $t < 0$.
(b) Find $i_L(t)$ for all t after the switch opens at $t = 0$.



Solution:

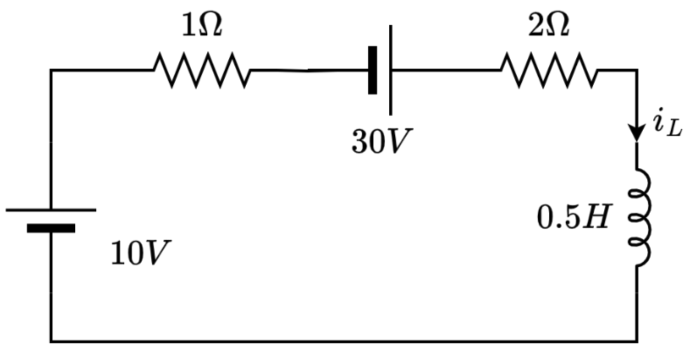
(a) For $t < 0$, S_1 is closed for long time. Inductor act as short circuit.
The circuit in steady state is



$$i_L(0^-) = \frac{10}{2} = 5A$$

$$i_L(0^-) = 5u(-t)\ A$$

(b) For $t > 0$, S_1 is open.
Circuit is shown in figure.



Equivalent resistance is given by

$$R_{eq} = 1 + 2 = 5\Omega$$

The time constant is

$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{3} = 0.167\ sec$$

Steady state current is given by

$$i_L(\infty) = \frac{40}{3} = 13.33\ A$$

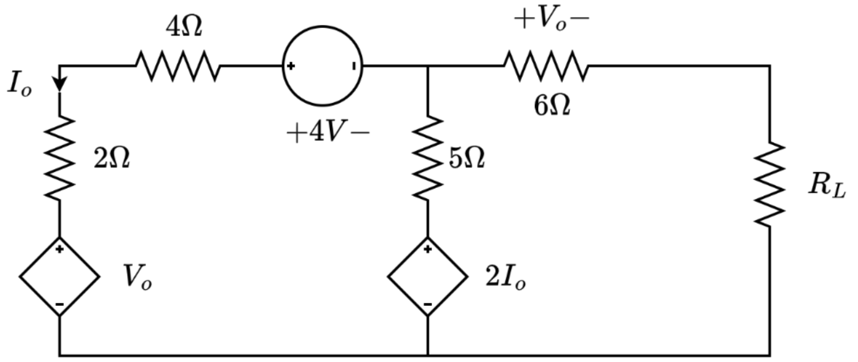
By formula,

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$i_L(t) = 13.33 - 8.33e^{-6t}$$

Question 3:

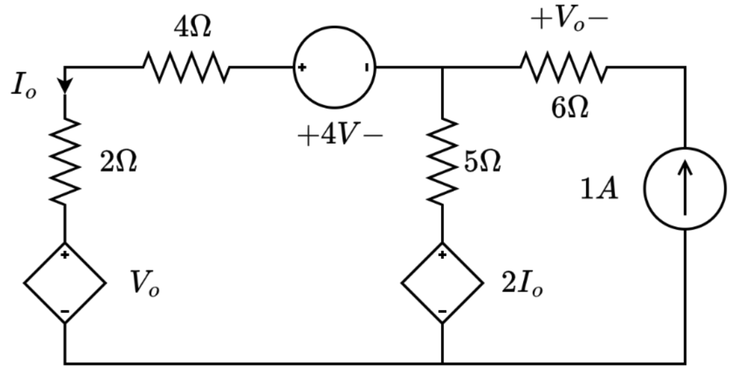
In the circuit shown below if the maximum power is transferred to the R_L, then find R_L.



Solution:

Resistance R_L is kept open and independent source is kept short.

A $1A$ current source replaced the R_L.



$$V_o = -6 \times 1 = -6V$$

Now, by KVL,

$$-2I_o + 5(I_o - 1) + 4I_o + 2I_o + V_o = 0$$

$$-9I_o + 11 = 0$$

$$I_o = \frac{11}{9}A$$

Now, let the voltage across R_L is V_L.

By KVL,

$$-V_L - V_o + 5(1 - I_o) + 2I_o = 0$$

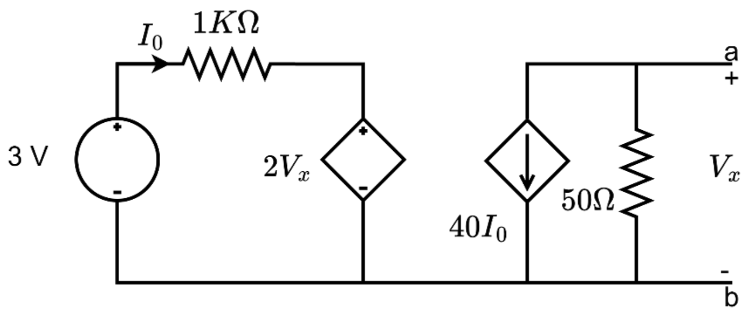
$$V_L = 6 + 5 - 3I_o$$

$$V_L = \frac{22}{3}V$$

$$R_L = R_{th} = \frac{V_L}{1} = \frac{22}{3} = 7.33\Omega$$

Question 4:

Find the Thevenin's equivalent between terminals a and b of the circuit given in figure.



Solution:

Apply the KVL on the right-hand side of the mesh,

$$V_x = V_{oc} = (-40 \times I_0) \times 50 = -2000 I_0 \qquad \dots (1)$$

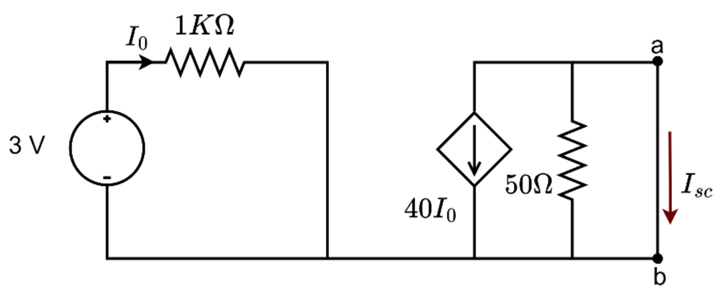
Find out the current I_0 form left-hand side loop,

$$I_0 = \frac{3 - 2V_x}{1000} = \frac{3 - 2V_{oc}}{1000} \qquad \dots (2)$$

From equation (1) and (2), we get,

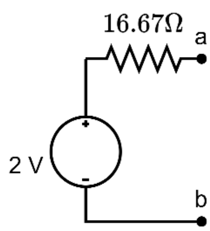
$$V_{oc} = -2000\left(\frac{3 - 2V_{oc}}{1000}\right) \Rightarrow V_{oc} = 2\ V$$

To determine the Thevenin's impedance, we short-circuit the terminals a and b.



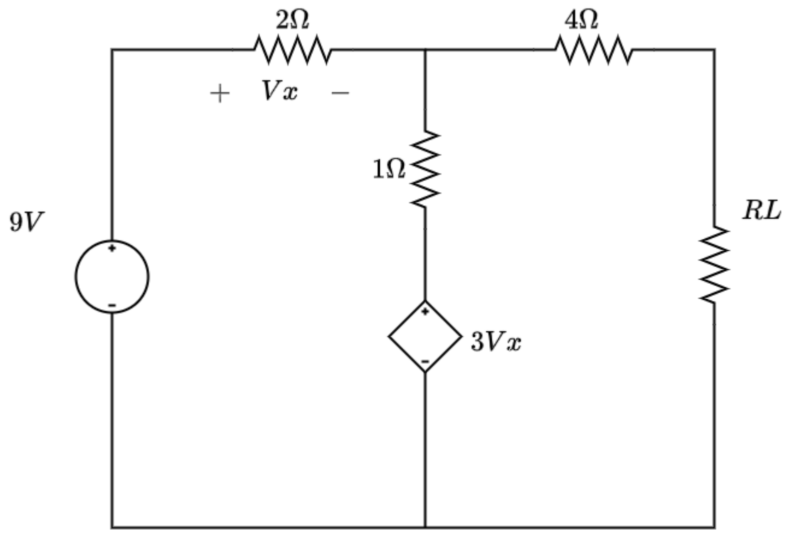
$$I_{sc} = -40 \times \left(\frac{3}{1000}\right) = -0.12$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{0.2}{0.12} = 16.67\Omega$$



Question 5:

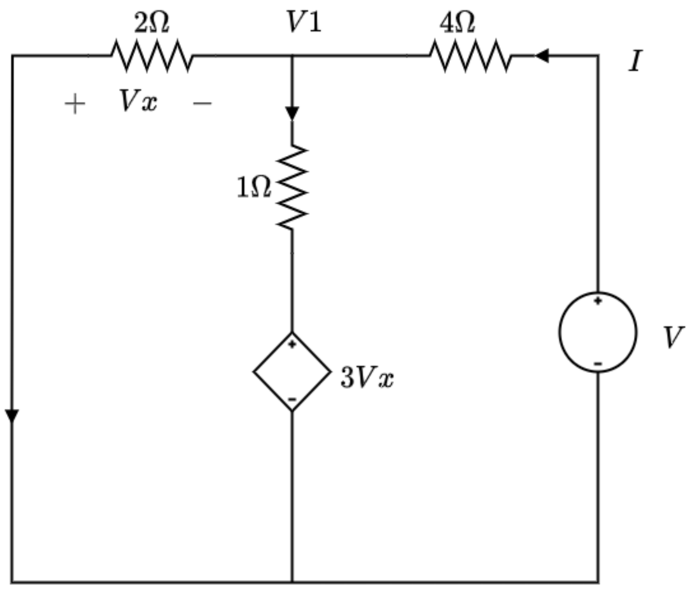
Determine the value of R_L that will draw the maximum power from the circuit. Find the maximum power.



Solution:

We need to find Thevenin resistance R_{th}

$$R_{th} = \frac{V}{I}$$



Applying nodal analysis we get,

$$\frac{V_1}{2} + (V_1 - 3V_x) + \frac{V_1 - V}{4} = 0$$

$$V_x = -V_1$$

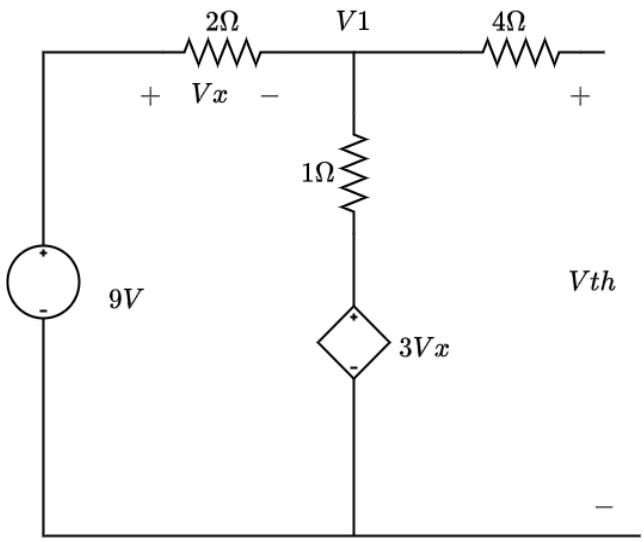
$$\frac{9}{2}V_1 = \frac{V - V_1}{4} = I$$

$$V_1 = \frac{2}{9}I$$

So, putting V_1 in above equation, we get

$$R_{th} = \frac{V}{I} = 4.222\Omega$$

Using nodal analysis to find V_{th}



$$\frac{V_1 - 9}{2} + \frac{V_1 - 3V_x}{1} = 0$$

$$4V_1 = \frac{9 - V_1}{2}$$

$$V_1 = 1V$$

$$V_{th} = V_1 = 1V$$

Maximum power transferred is when

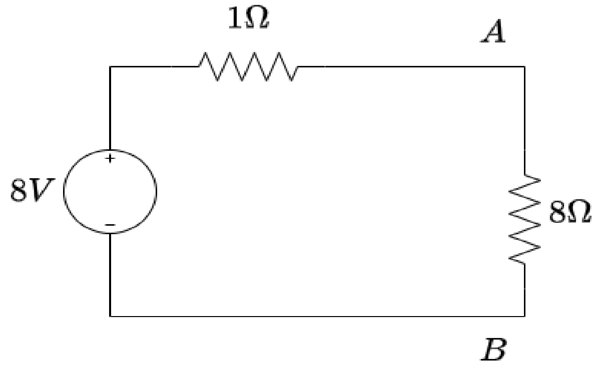
$$R_L = R_{th} = 4.222\Omega$$

Maximum power transferred

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{1^2}{4 * 4.22} = 59.24\ mW$$

Question 6:

The value of resistance between terminals A and B is changed to 6Ω. Find the compensation voltage.



Solution:

Applying the KVL,

$$-8 + 1i + 8i = 0$$

$$9i = 8$$

$$i = \frac{8}{9}A$$

$$\Delta R = 8\Omega - 6\Omega$$

$$\Delta R = 2\Omega$$

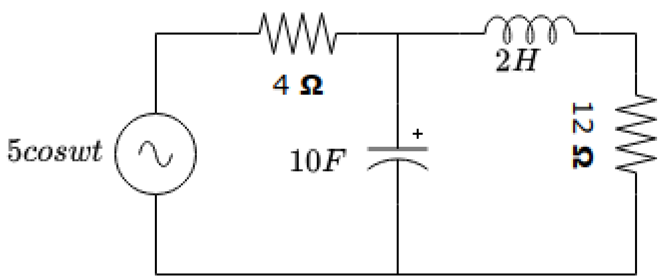
Compensation Voltage

$$V_C = -i(\Delta R)$$

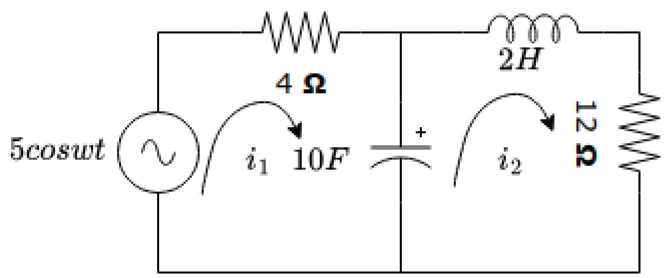
$$V_C = -\frac{8}{9} * 2 = 1.77V$$

Question 7:

For the given circuit, construct an exact dual circuit.



Solution:



Assuming at t = 0, no voltage and no current across the capacitor and inductor.

Mesh equations in loop 1 & 2, we have.

$$5\cos(\omega t) - 4i_1 - \frac{1}{10}\int (i_1 - i_2)dt = 0 \quad (1)$$

$$12i_2 + 2\frac{di_2}{dt} - \frac{1}{10}\int (i_1 - i_2)dt = 0 \quad ...(2)$$

Represent these equations in nodal equations form.

$$5\cos(\omega t) - 4v_1 - \frac{1}{10}\int (v_1 - v_2)dt = 0 \quad ...(3)$$

$$12v_2 + 2\frac{dv_2}{dt} - \frac{1}{10}\int (v_1 - v_2)dt = 0 \quad ...(4)$$

Now, we can construct dual circuit.