Final Exam
EE 250 (Control Systems Analysis) Spring 2010 *

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

Instructions

1. Before you turn in your answer books, please verify that your answer booklet contains one semilog graph paper.
2. You may use a pencil for trial and error, but your final drawing should use a pen.
3. Show all your assumptions.
4. You will need the following items: pen, pencil, ruler, eraser.

$$t_s = \frac{4}{\zeta\omega_n}\left(\begin{array}{c}2\%\text{ tube,}\\ 0<\zeta<0.8\end{array}\right), \quad M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}.$$

$\mathrm{PM}^\circ \approx 100 \times \zeta$,

PM	25°	40°	60°	70°
M_p	45%	25%	10%	5%

Controller	k_p	T_i	T_D
P	$0.5k_{cr}$	∞	0
PI	$0.45k_{cr}$	$(1/1.2)P_{cr}$	0
PID	$0.6k_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Useful information

Rule 1: The RL has n branches.
Rule 2: The RL starts at $K = 0$ at the poles and ends at $K = \infty$ on the m zeros, and $n - m$ of its branches end at ∞.
Rule 3: On the real axis, the RL exists only on those segments to the right of which the sum of the number of poles and the number of zeros is odd.
Rule 4: The RL is symmetric about the real axis.
Rule 5: As $K \to \infty$, $n - m$ branches of the RL go asymptotically along rays that emanate from the *centroid*

$$s = -\sigma = -\frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m}$$

Rule 6: The angles that these rays form with the real axis are

$$\frac{\pm 180^\circ(2q+1)}{n-m}, q = 0, 1, 2, \ldots$$

Rule 7: The roots of multiplicity greater than one occur on the RL where s satisfies $dG/ds = 0$, or, equivalently, $dK/ds = 0$.
Rule 8: In order to determine the angle of departure from a pole and the angle of arrival at a zero, use the phase criterion (PC) along with the idea of a point $s = \hat{s}$ that is infinitesimally close to the pole (zero) the angle of departure (arrival) from (to) which needs to be calculated.
Rule 9: In order to determine the $j\omega$-axis intercepts of the RL, use the Routh-Hurwitz method.
Rule 10: If necessary, calculate a few additional suitably located points using the PC.

$\Phi_{\max}$ ($\Phi_{\min}$) needed	90°(−90°)	60°(−60°)	45°(−45°)	30°(−30°)
DD between corner freq-s of BP in decades	∞	1.1439 ≈ 1.144	0.76555 ≈ 0.766	0.47712 ≈ 0.477

$$t_r = \frac{\pi - \arccos\zeta}{\omega_n\sqrt{1-\zeta^2}}, \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}},$$

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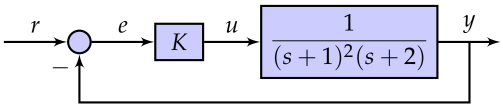
Problems

1. Suppose you wish to use the ultimate gain method to tune a PID controller for a plant that has the following exact transfer function in the open-loop: $\frac{1}{(s+1)^2(s+2)}$. Suppose you discover that this system displays sustained oscillations at $K = 18$ in the following feedback configuration when $r = 1(t)$:



   1.1. **[2 points]** Determine the period of these oscillations.

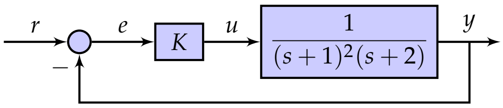
   1.2. **[1 points]** What is the critical gain of this system?

   1.3. **[2 points]** About which value do these oscillations happen — about 1 or 0 or about some other value? Show your calculations.

   1.4. **[1 points]** Using the second Ziegler Nichols method, determine the parameters of a PID controller that can be used in the place of K.

   1.5. **[1 points]** Write the transfer function of this PID controller.

2. For the following control system



   2.1. **[2 points]** Calculate the breakaway and breakin points if any.

   2.2. **[2 points]** If breakaway points exist, then calculate the angle of departure from them.

   2.3. **[1 points]** Calculate the centroid, if it exists.

   2.4. **[1 points]** Calculate the angles of the asymptotes, if they exist.

   2.5. **[2 points]** Sketch the root locus.

   2.6. **[1 points]** What is the value of K for which this root locus intersects the $j\omega$-axis?

   2.7. **[2 points]** Is this system conditionally stable? Explain.

3. For the state space equation

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} x$$

   we wish to design a tracker of the form $u = -Kx + Ny_{\text{ref}}$, such that the unit step response of the closed-loop system has an approximately second-order behavior with an overshoot of approximately 25%, and a settling time of approximately 2 s.

   [5 points] For this purpose, determine the location of the desired closed-loop eigenvalues neglecting the effect of closed-loop zeros.

4. Given the open-loop transfer function, $G(s) = \frac{K}{s^2+\omega_0{}^2}$, we wish to use "Bode plot to polar plot" method to sketch the Nyquist plot (NP) of $G(s)$ for $K \in [0, \infty)$.

   4.1. **[2 points]** Sketch the Bode plot (BP).

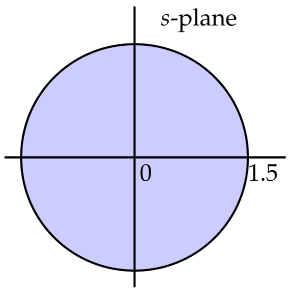
   4.2. **[1 points]** Sketch the polar plot (PP) section of the NP. Show this section by thick solid lines.

   4.3. **[2 points]** Work out a few points on the s-plane contour and the $G(s)$-plane contour that will help complete the NP.

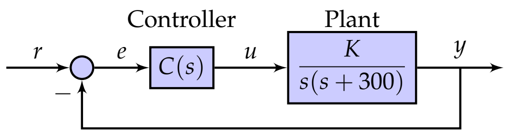
   4.4. **[2 points]** Sketch the NP. Label the sections of the s-plane contour $C1, C2, \cdots$ and the corresponding sections of the NP $C1', C2', \cdots$. Label a few points on the s-plane contour $1, 2, \cdots$, and the corresponding points on the NP $1', 2', \cdots$.

5. **[2 points]** For one clockwise traversal of the following s-plane contour, determine the number of times the $G(s)$-plane contour encircles the origin of the $G(s)$-plane in the clockwise direction, given that

$$G(s) = \frac{(s+1)(s+2)}{(s^2+s+1)}.$$



6. Consider an aircraft attitude control system shown in the following figure.



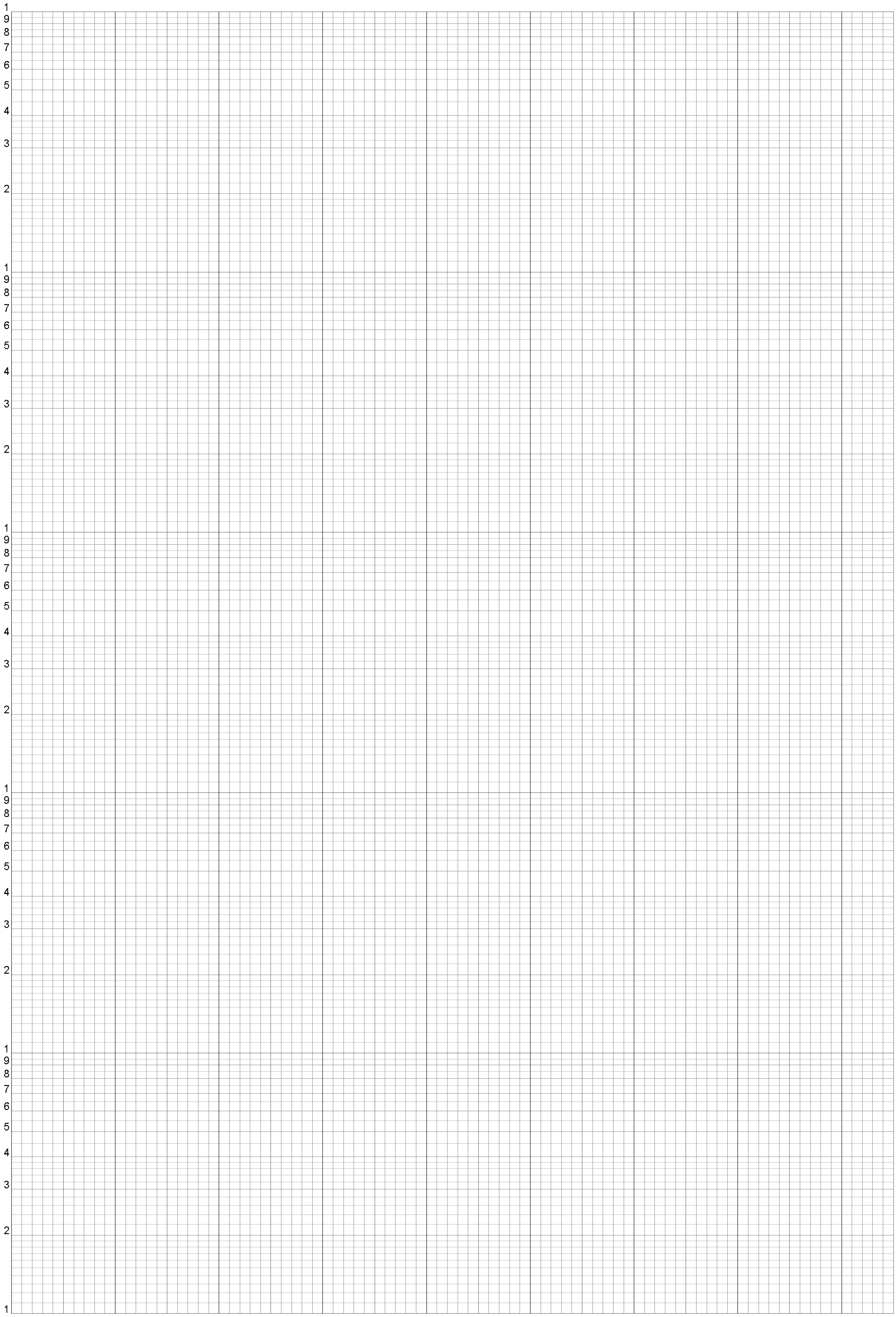
   We wish to determine $C(s)$ and K such that the following performance specifications are satisfied:

   - Steady-state error due to unit-ramp input ≤ 0.000443,
   - Maximum overshoot $10\% - 45\%$,
   - Settling time $t_s \leq 0.005$ s,
   - The controller must be first order.

   6.1. **[3×2 points]** Explain how you will satisfy the first 3 specifications.

   6.2. **[3 points]** On the semilog graph paper provided design the loopgain using loopshaping to satisfy these specifications.

   6.3. **[2 points]** Explain why your design will satisfy the specification on overshoot and controller order.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