Lecture-33

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Induction Machine.

❑ Example:

A 208 V, 10hp, 4-pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent.

(a) What is the synchronous speed of this motor?

(b) What is the rotor speed of this motor at the rated load?

(c) What is the rotor frequency of this motor at the rated load?

❑ Solution:

(a) Synchronous speed of the motor is:

$$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

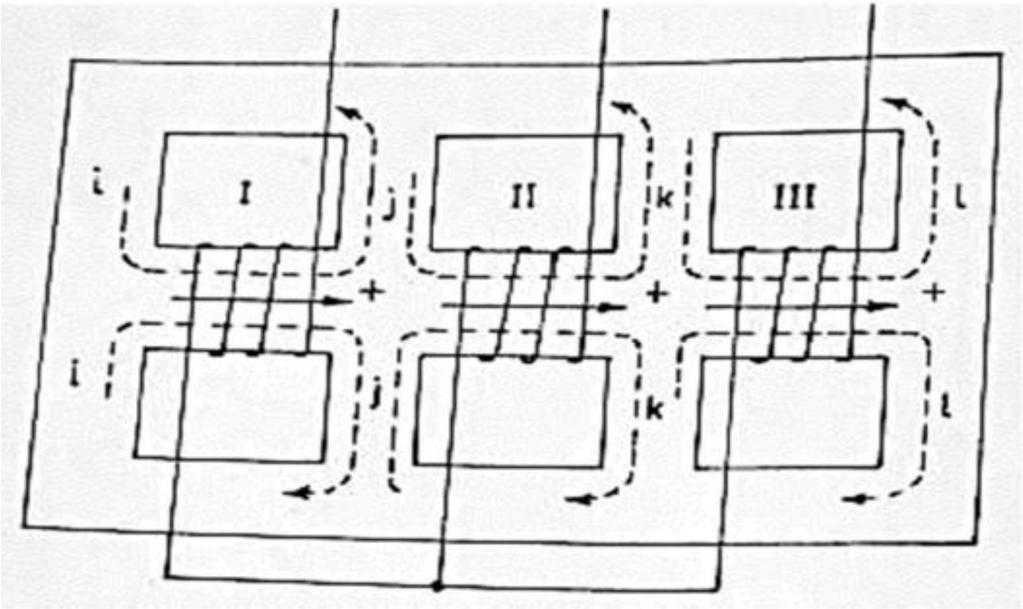
(b) Rotor speed of the motor is given by:

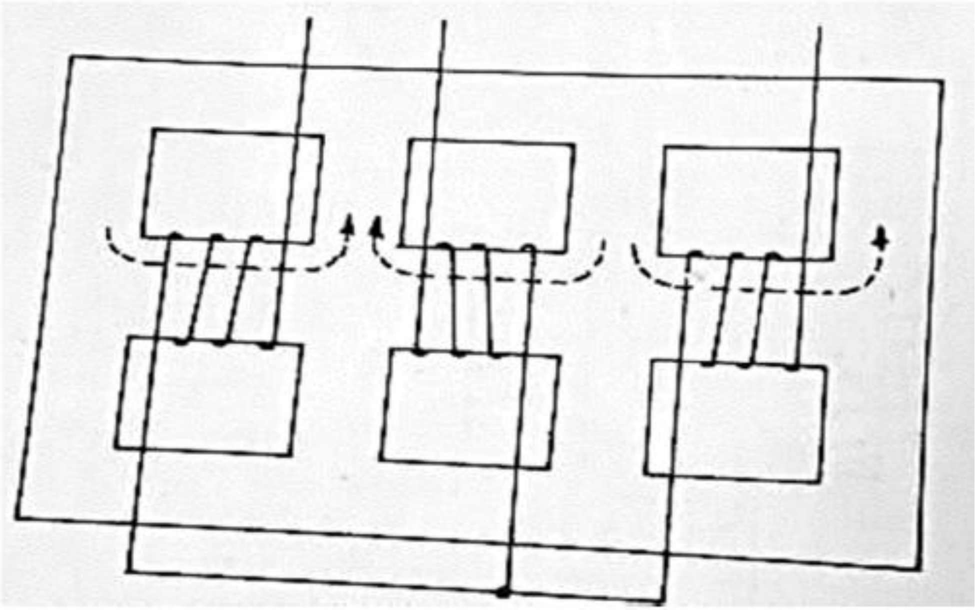
$$n_m = (1 - s) n_{sync}$$

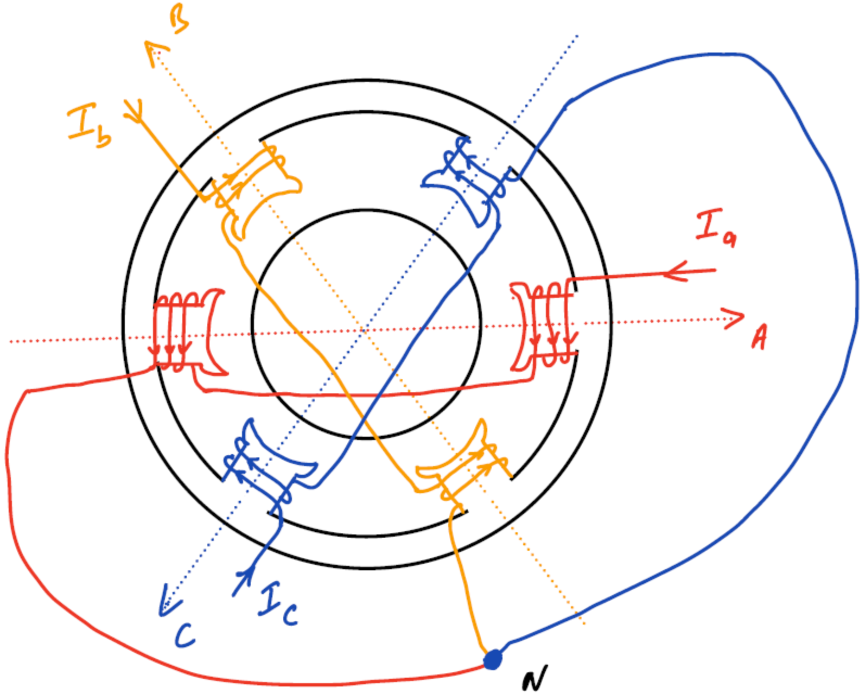
(c) Rotor frequency is given by:

$$f_r = s f_e = 0.05 \times 60 = 3\ Hz$$

Induction Motor Equivalent Circuit (Cont…)







❑ Frequency of the Rotor E.M.F and Current:

- For a three-phase winding with p pairs of poles supplied at a frequency of f hertz, the speed of the rotating flux is given by n_s revolutions per second,

$$f = n_s p$$

- The speed at which the rotor conductors are being cut by the rotating flux is $(n_s - n_r)$ revolutions per second.

$$The\ frequency\ of\ the\ rotor\ e.m.f = (n_s - n_r)p$$

$$\Rightarrow f_r = s n_s p = sf$$

$$where\ s = per - unit\ or\ fractional\ slip = \frac{n_s - n_r}{n_s}$$

- Polyphase currents in the stator winding produce a resultant magnetic field, the axis of which rotates at synchronous speed, n_s revolutions per second, relative to the stator.

❑ Frequency of the Rotor E.M.F and Current:

- Polyphase currents in the rotor winding produce a resultant magnetic field, the axis of which rotates at a speed sn_s revolutions per second relative to the rotor surface, in the direction of rotation of the rotor.
- Rotor is revolving at a speed n_r revolutions per second relative to the stator core.

❑ Frequency of the Rotor E.M.F and Current:

- Polyphase currents in the stator winding produce a resultant magnetic field, the axis of which rotates at synchronous speed, n_s revolutions per second, relative to the stator.
- Polyphase currents in the rotor winding produce a resultant magnetic field, the axis of which rotates at a speed sn_s revolutions per second relative to the rotor surface, in the direction of rotation of the rotor.
- Rotor is revolving at a speed n_r revolutions per second relative to the stator core. The speed of the resultant rotor magnetic field relative to the stator core is

$$sn_s + n_r = (n_s - n_r) + n_r = n_s \ \ revolutions\ per\ second$$

- Axis of the resultant rotor field m.m.f. is travelling at the same speed as that of the resultant stator field m.m.f., so that they are stationary relative to each other.
- Equivalent to a transformer having an airgap separating the steel portions of the magnetic circuit carrying the primary and secondary windings.

Induction Motor Equivalent Circuit

- When the ‘primary’ winding (the stator) of an induction motor is energized from an AC source, a magnetic field results that induces e.m.f.s in the ‘secondary’ winding (the rotor). Secondary currents flow, the magnitude and phase angle of which depend on the rotor speed. A primary current results, of opposite polarity and of magnitude proportional to the secondary current to preserve the m.m.f. balance in the iron core. The primary current drawn determines the energy supplied to drive the rotor and mechanical load.

- At standstill, however, the machine acts as a simple transformer with an air gap and a short-circuited secondary winding. The frequency of the rotor-induced emf is the same as the stator frequency at standstill.

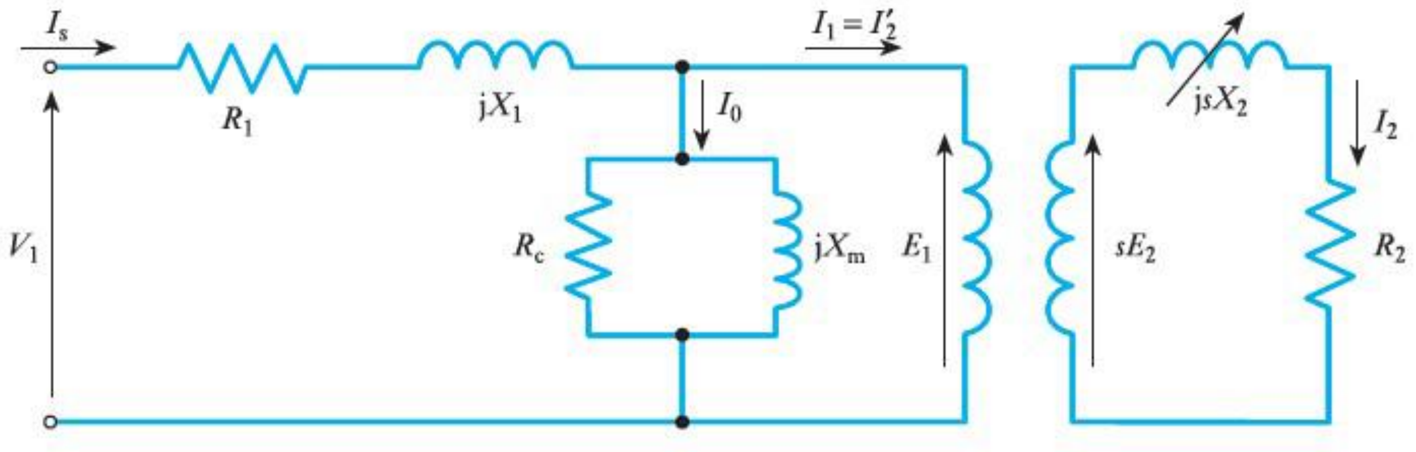
- The resultant air-gap flux is produced by the combined mmfs of the stator and rotor currents.

- For the sake of conceptual and analytical convenience, the total flux is divided into a mutual flux (linking both the stator and the rotor) and leakage fluxes, represented by appropriate reactances.

Induction Motor Equivalent Circuit (Cont…)

- At standstill, $\omega_m = 0,\ \boldsymbol{S} = 1, f_r = f_s$
- Machine acts as a simple transformer with an air gap and a short-circuited secondary winding.
- To produce a steady unidirectional torque, the rotating fields of stator and rotor must be traveling at the same speed.
- A steady starting torque is produced: the polyphase induction motor is self-starting.
- At synchronous speed $\omega_m = \omega_s, \boldsymbol{S} = 0, f_r = 0$
- No induction takes place because there is no relative motion between flux and rotor conductors. Secondary m.m.f is zero, no torque is produced: the induction motor cannot run at synchronous speed.
- The no-load per-unit slip is about 0.005, and the full-load per-unit slip is on the order of 0.05.
- The polyphase induction motor is effectively a constant-speed machine.

Induction Motor Equivalent Circuit (Cont…)



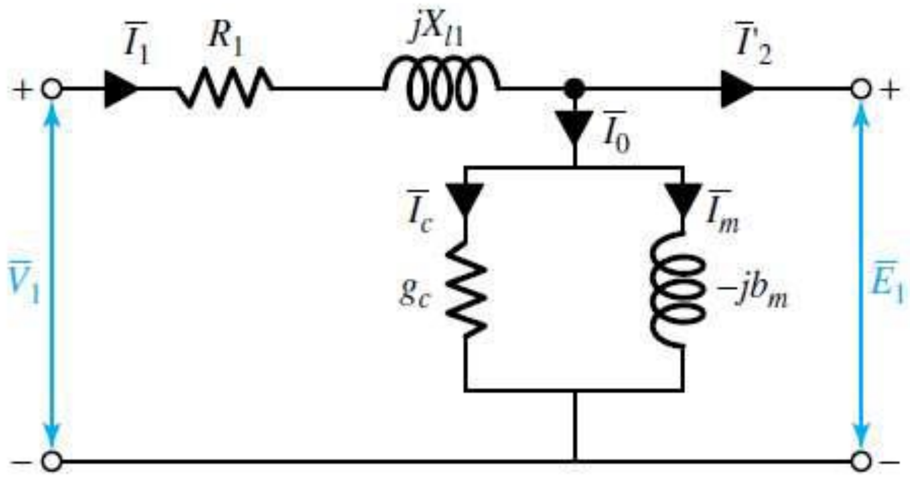
X_m: magnetizing reactance of the induction motor.

R_c: the hysteresis and eddy-current losses i.e. the 'iron loss.

- The establishment of a travelling flux wave → m.m.f. to cross both the airgap of the machine and the iron paths of the magnetic circuit.

- As the motor speeds up, the slip is reduced and the effective reactance of the rotor cage is reduced.

Induction Motor Equivalent Circuit (Cont…)



$$\bar{V}_1 = \bar{E}_1 + \bar{I}_1(R_1 + jX_{l1})$$

$\bar{V}_1$: stator terminal voltage

$\bar{E}_1$: counter emf generated by the resultant air-gap flux

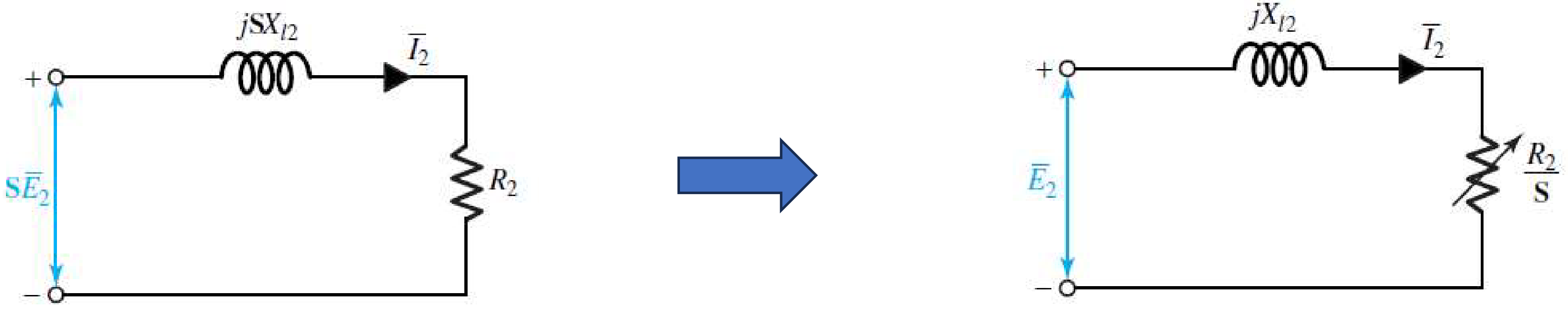
$\bar{I}_1$: the stator current

R_1: stator effective resistance

X_{l1}: stator-leakage reactance

- The total flux is divided into a mutual flux (linking both the stator and the rotor) and leakage fluxes, represented by appropriate reactances.
- The synchronously rotating air-gap wave generates balanced polyphase counter emfs in the phases of the stator.
- The value of the magnetizing reactance tends to be relatively low compared to that of a transformer. The leakage reactance is larger in proportion to the magnetizing reactance than it is in transformers.

Induction Motor Equivalent Circuit (Cont…)



$$I_1 = I_2 = \frac{sE_2}{Z_2}$$

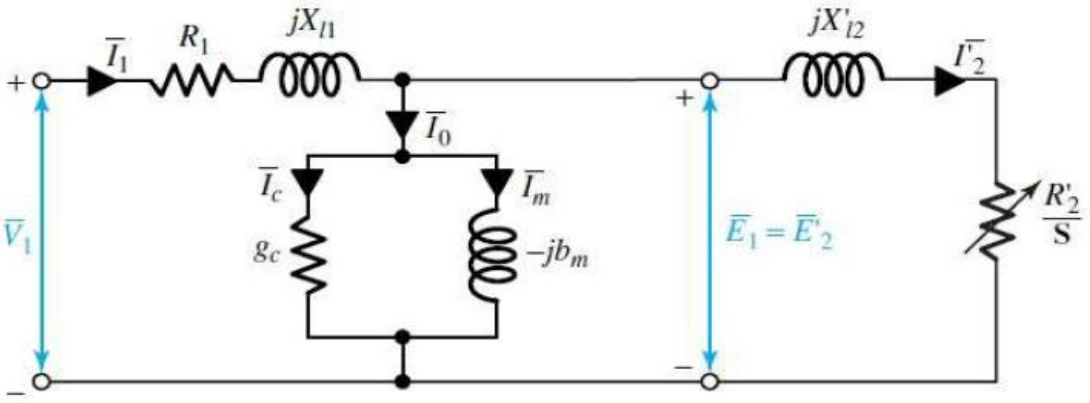
$$I_2 = \frac{\boldsymbol{S}E_2}{\sqrt{R_2^2 + (\boldsymbol{S}X_{l2})^2}}$$

$$\Rightarrow I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{\boldsymbol{S}}\right)^2 + X_{l2}^2}}$$

- Because the frequency of the rotor voltages and currents is the slip frequency, the magnitude of the voltage induced in the rotor circuit is proportional to the slip.
- In terms of the standstill per-phase rotor leakage reactance X_{l2}, the leakage reactance at a slip $\boldsymbol{S}$ is given by sX_{l2}.

- All rotor electrical phenomena, when viewed from the stator, become stator-frequency phenomena because the stator winding sees the m.m.f and flux waves traveling at synchronous speed.

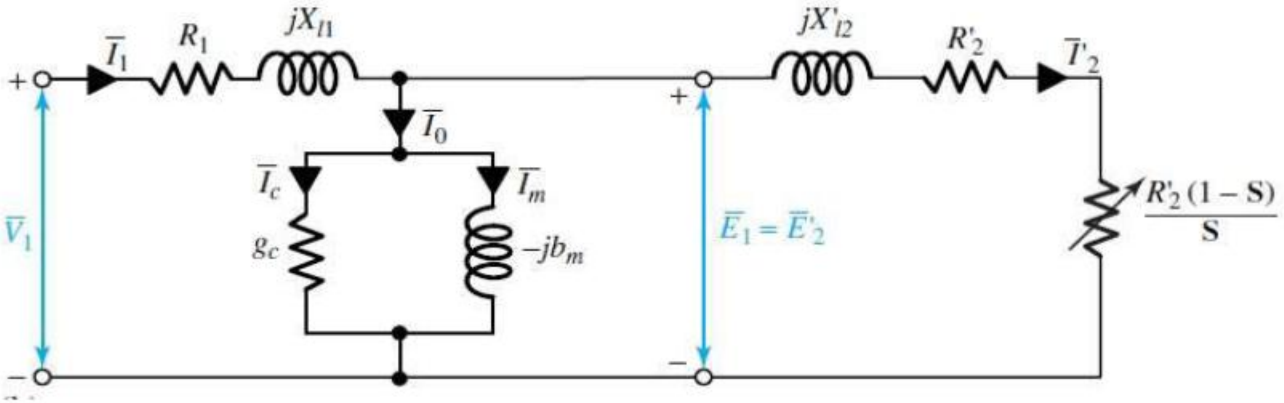
Induction Motor Equivalent Circuit (Cont…)



per-phase standstill rotor resistance referred to the stator

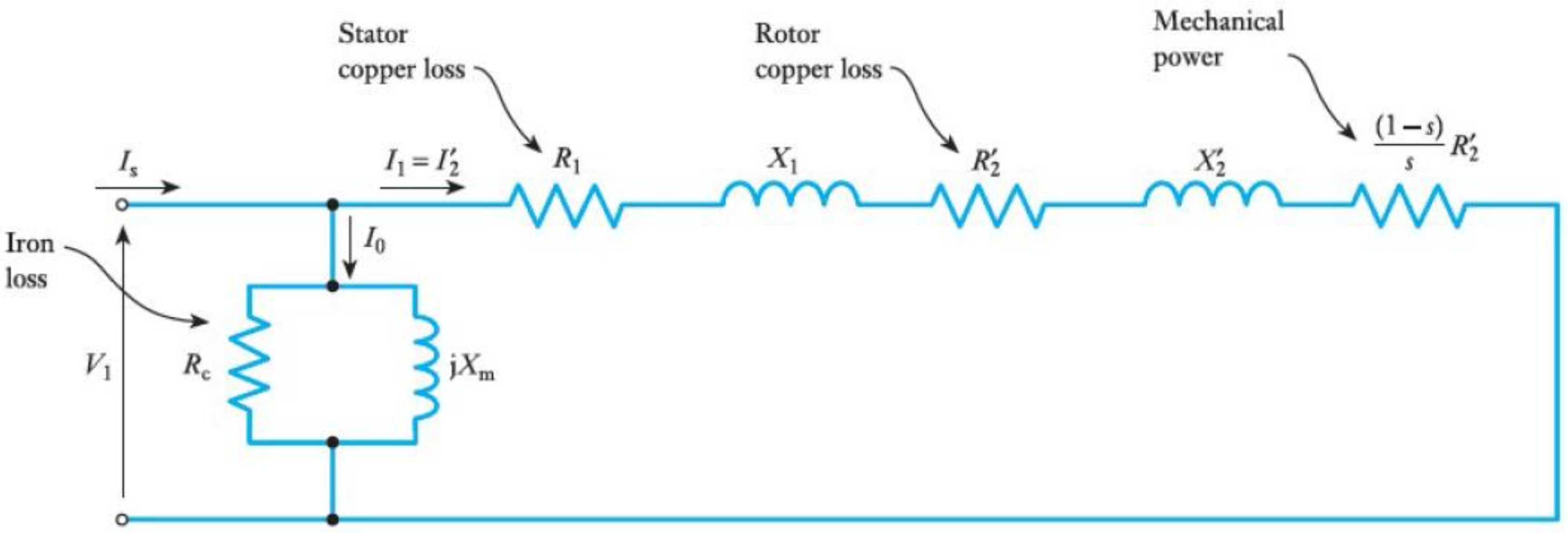
$\frac{R'_2}{S}$: Combined effect of the shaft load and the rotor resistance

$$\frac{R_2'}{S} = R_2' + \frac{R_2'(1-S)}{S}$$



dynamic resistance that depends on the rotor speed and corresponds to the load on the motor

Induction Motor Equivalent Circuit (Cont…)



Induction Motor Equivalent Circuit (Cont…)

- The slope of the induction motor's mmf-flux curve is much shallower than the curve of a good transformer. This is because there must be an air gap in an induction motor, which greatly increases the reluctance of the flux path.

- The higher reluctance caused by the air gap means that a higher magnetizing current is required to obtain a given flux level. Therefore, the magnetizing reactance X_m in the equivalent circuit will have a much smaller value than it would in a transformer.

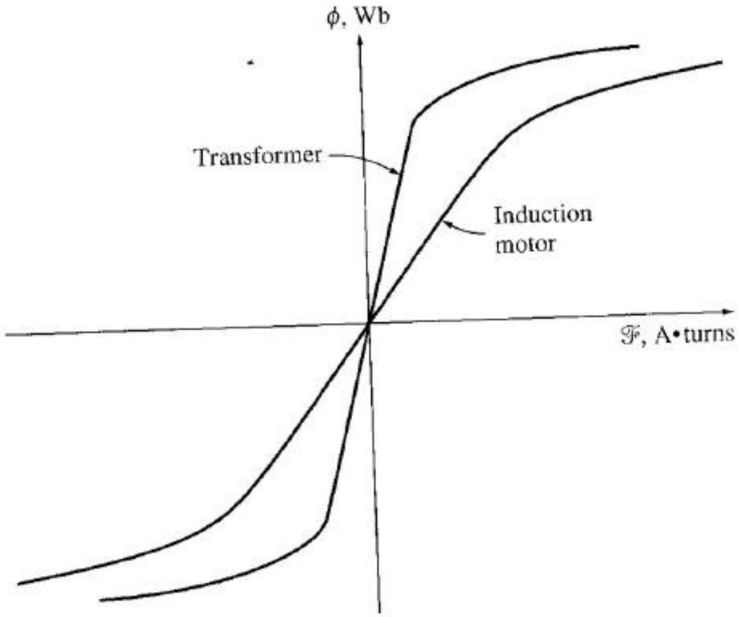
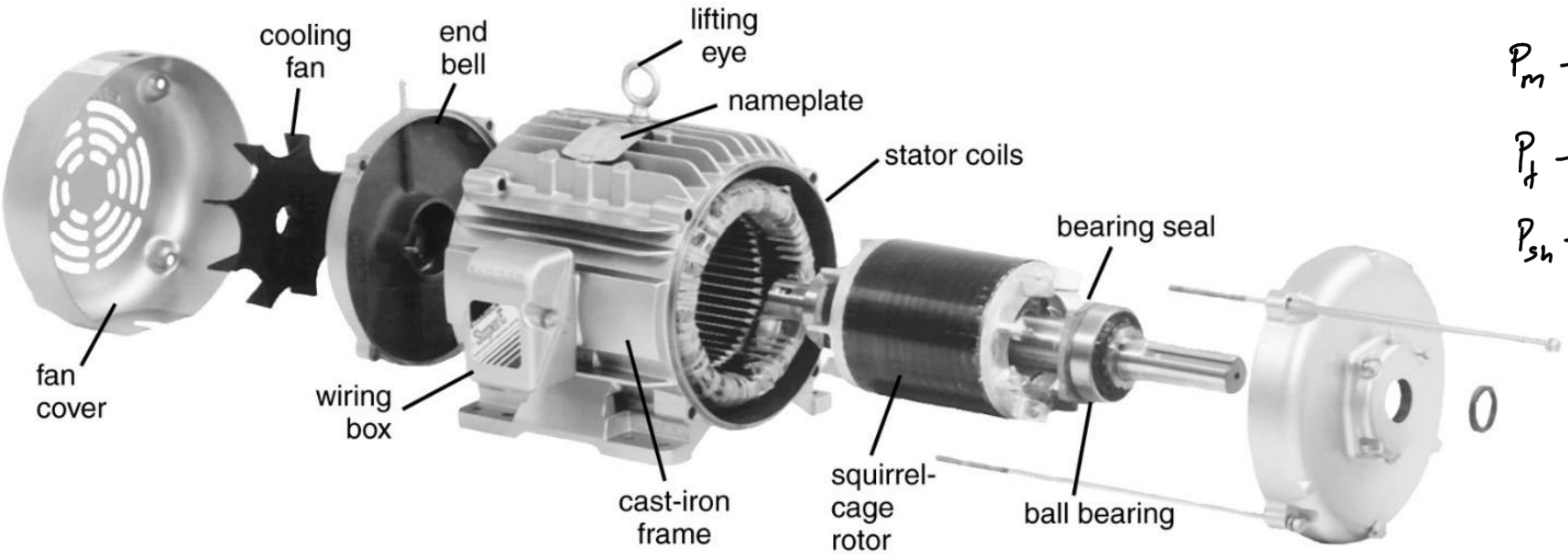


Fig. Magnetization curve of induction motor compared to that of transformer

Induction Motors (Cont…)

❑Power flow diagram of an induction motor:

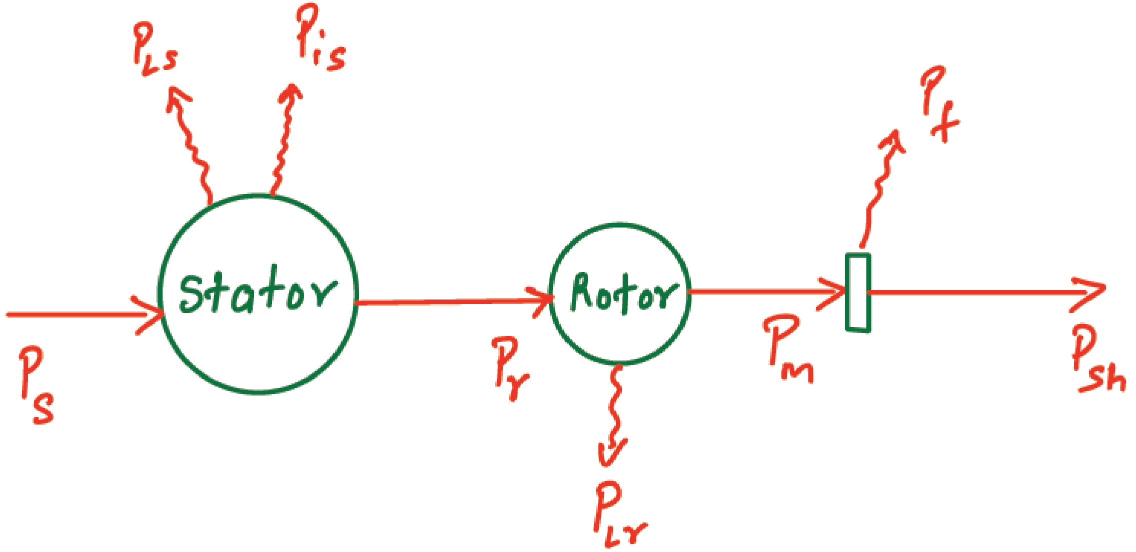


$P_{Lr} \rightarrow$ loss due to rotor winding resistance.

$P_m \rightarrow$ Mechanical power.

$P_f \rightarrow$ friction and windage.

$P_{sh} \rightarrow$ shaft power OR actual mechanical o/p power.



where:-

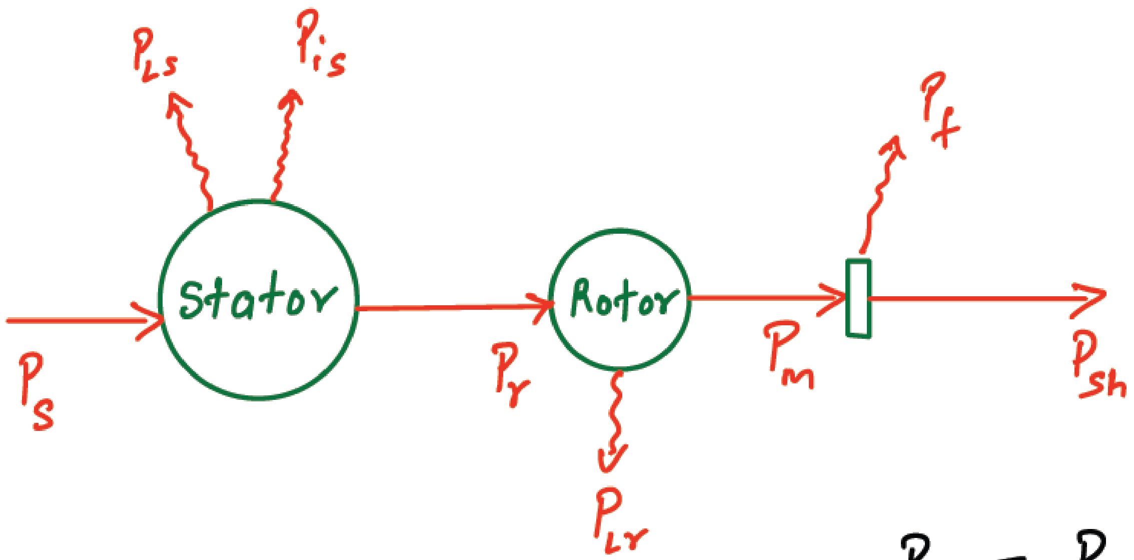
$P_S \rightarrow$ Input power to stator.

$P_{LS} \rightarrow$ loss due to stator winding resistance.

$P_{is} \rightarrow$ loss in stator core.

$P_r \rightarrow$ Input power to rotor OR airgap power.

❑Power flow diagram of an induction motor:



$$P_{Lr} = P_r - P_m$$
$$= T\omega_s - T\omega_m$$
$$= T\omega_s\left(1 - \frac{\omega_m}{\omega_s}\right)$$
$$= sP_r \qquad \left[\because P_r = T\omega_s\right]$$

$$\boxed{P_{Lr} = sP_r}$$

$P_{Lr} \rightarrow$ loss due to rotor winding resistance.

$P_m \rightarrow$ Mechanical power.

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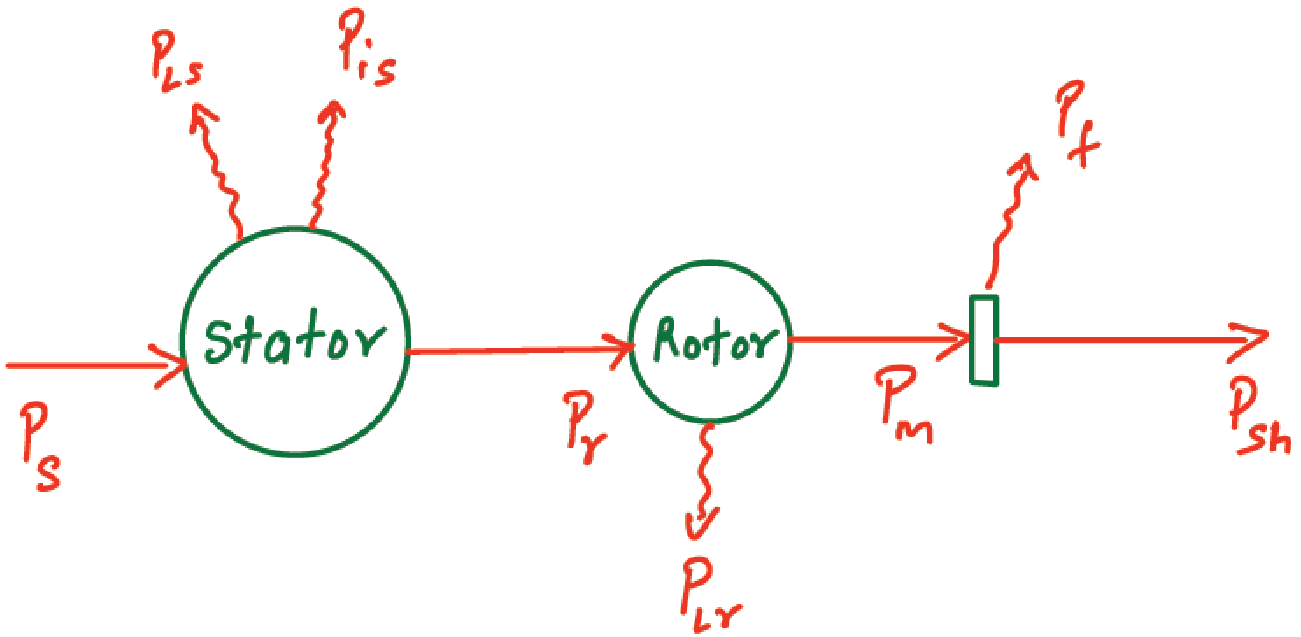
- Power flow diagram of an induction motor:



$$\boxed{\text{efficiency } (\eta) = \frac{P_{sh}}{P_s}}$$

$$P_m = P_r - P_{Lr}$$

$$= P_r - sP_r$$

$$\boxed{P_m = (1-s)P_r}$$

Induction Motor Equivalent Circuit (Cont…)

❑ Power and Torque:

- The relationship between the input electric power and the output mechanical power of this motor is shown below:

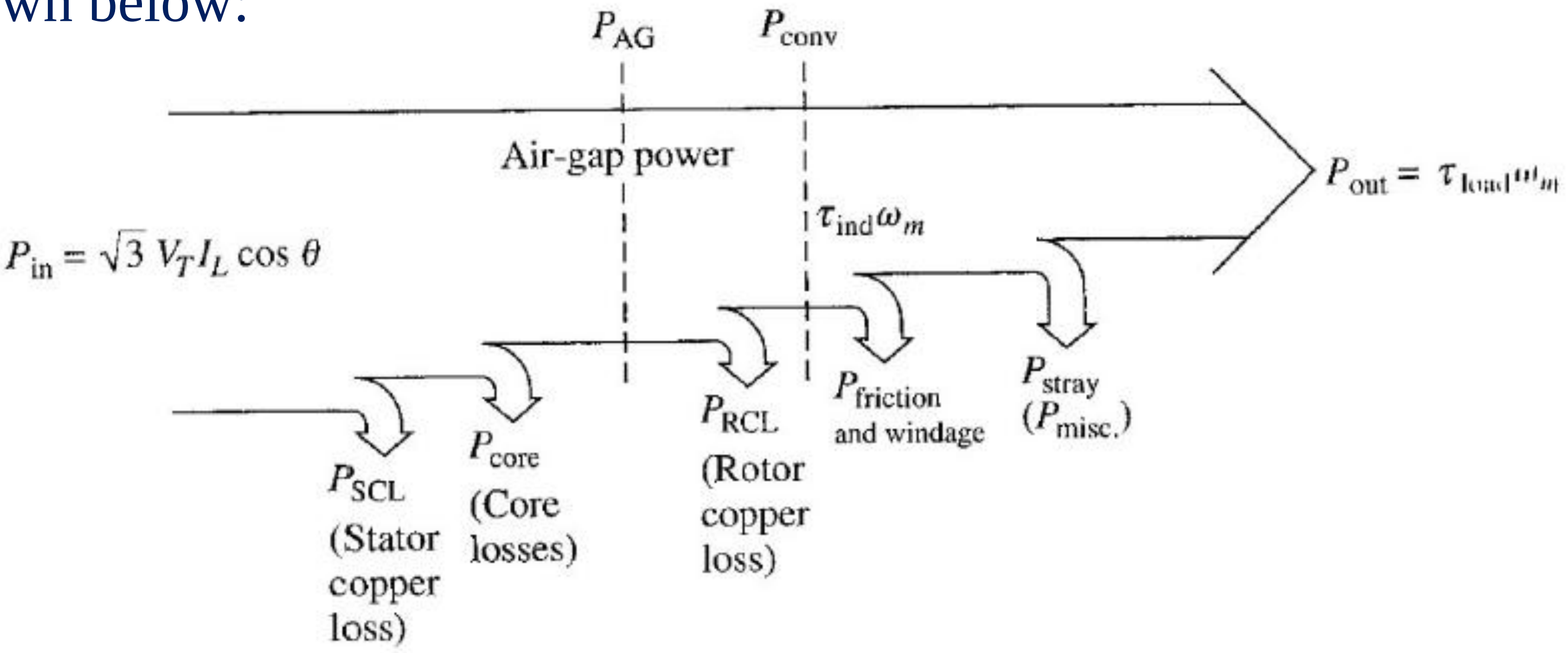
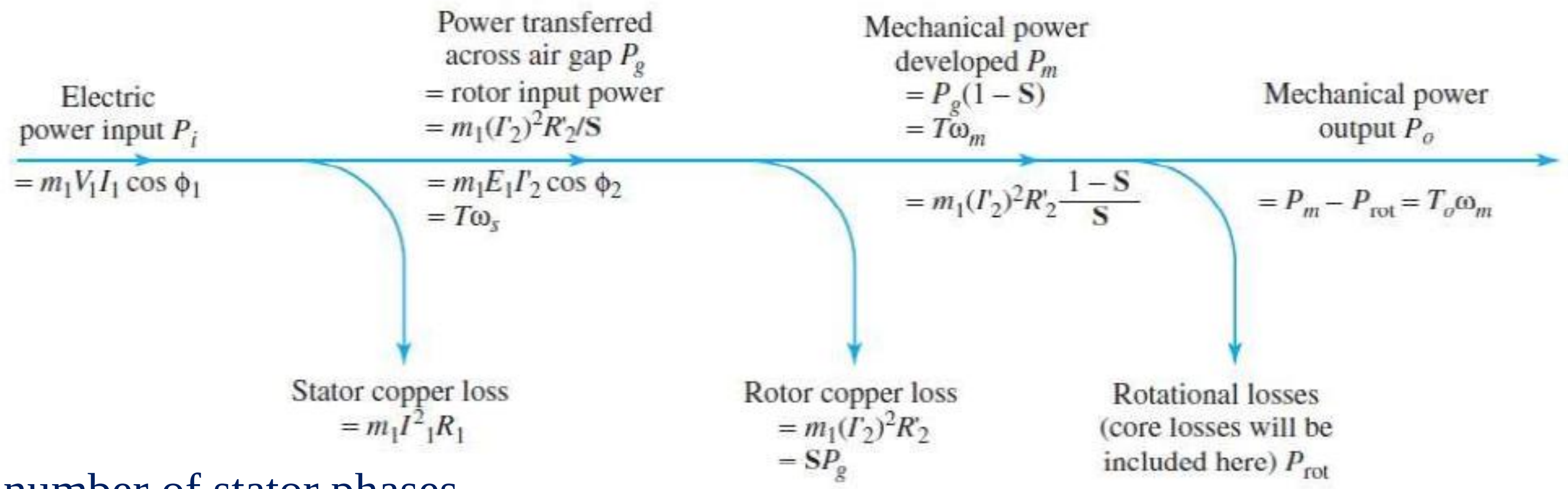


Fig. Power flow diagram of an induction motor

- Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses.

Induction Motor Equivalent Circuit (Cont…)

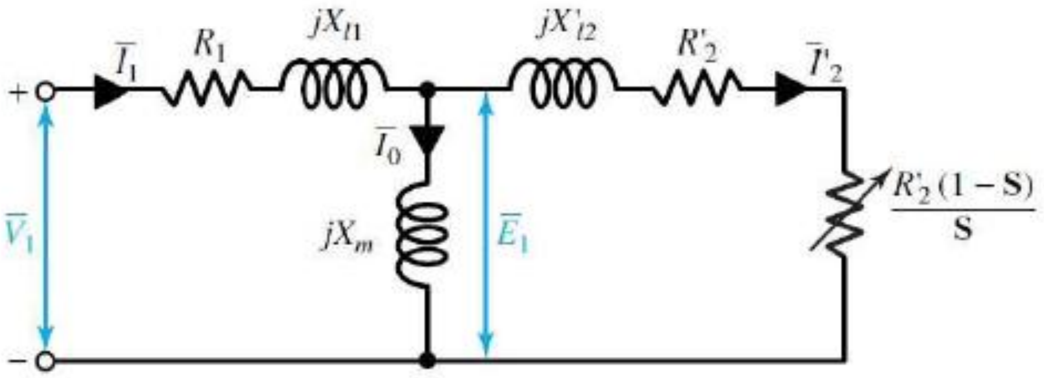


- m_1 is the number of stator phases

$$P_g = m_1 (I_2')^2 \frac{R_2'}{S} = T\omega_s \qquad\qquad P_m = P_g(1 - S) = T\omega_m = m_1 (I_2')^2 \frac{R_2'(1-S)}{S}$$

- Of the total power delivered to the rotor, the fraction **1-S** is converted to mechanical power and the fraction **S** is dissipated as rotor copper loss. An induction motor operating at high slip values will be inefficient.

The per-unit efficiency $\eta = \frac{P_o}{P_i}$



Induction Motor Equivalent Circuit (Cont…)

- If the power losses in the iron core of the Stator are neglected

$$P_{in} = R_{eq}|I_1|^2 = \left(R_1 + \frac{R_2'}{s}\right)|I_1|^2 \; W/ph$$

- The power dissipated in the resistance of stator and rotor windings is

$$P_{\Omega} = (R_1 + R_2')|I_1|^2 \;\; W/ph$$

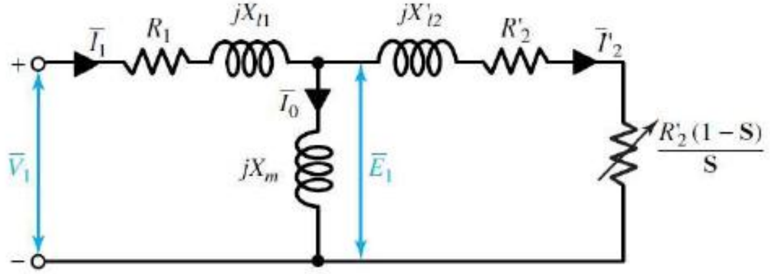
- The mechanical shaft power $P_m = P_{in} - P_{\Omega}$

$$= \left(R_1 + \frac{R_2'}{s}\right)|I_1|^2 - (R_1 + R_2')|I_1|^2$$

$$= |I_1|^2 \left(R_1 + \frac{R_2'}{s} - (R_1 + R_2')\right)$$

$$= |I_1|^2 \left(\frac{R_2'}{s} - R_2'\right)$$

$$= \left(\frac{1 - s}{s}\right) R_2' |I_1|^2$$



Induction Motor Equivalent Circuit (Cont…)

$$P_{3\phi m} = 3\left(\frac{1-s}{s}\right)R_2'|I_1|^2 \ \ W$$

$$T = \frac{P_m}{\omega_m} \quad and \ \ \omega_m = \frac{\pi}{30}(1-s)n_s \, rad/s$$

$$T = \frac{3\left(\frac{1-s}{s}\right)R_2'|I_1|^2}{\frac{\pi}{30}(1-s)n_s} = \frac{90}{\pi n_s} \times \frac{R_2'}{s} \times |I_1|^2 \ N\,m$$

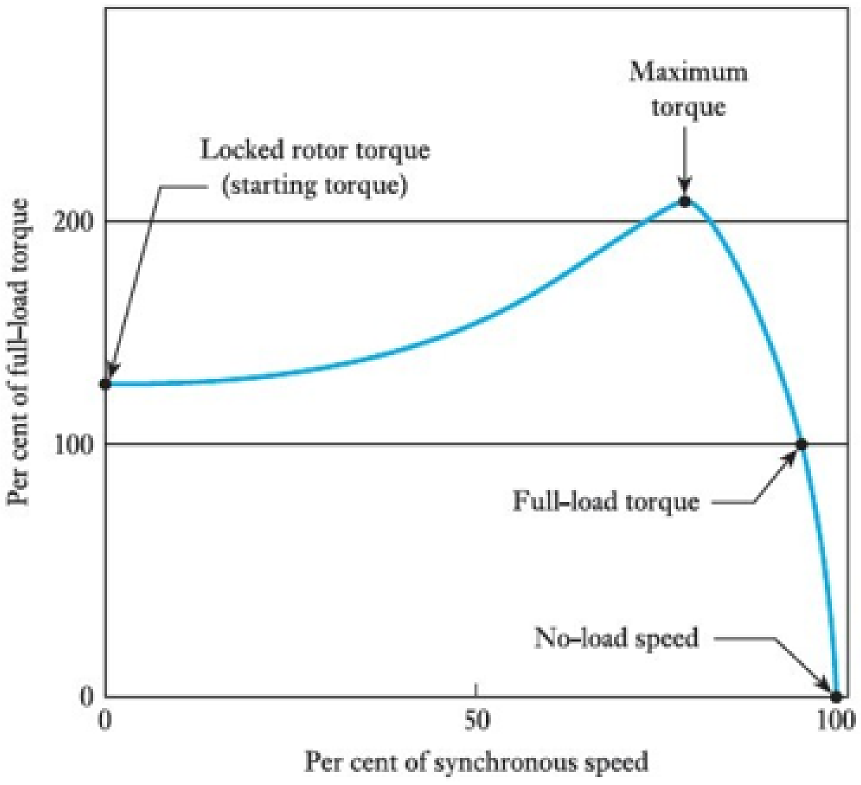
$$|I_1| = \frac{|V_1|}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}}$$

neglecting the magnetzing component of the current

Torque-Slip Characteristics

❑ When a motor is started with full voltage applied:

- The motor is initially stationary and develops locked-rotor torque.
- Some motors produce a slight dip in torque as these accelerate, the lowest point being called the pull-in or pull-up torque.
- As the speed increases further, the torque reaches the highest point on the curve called the pull-out or breakdown torque.
- When the motor is loaded to its full-load torque, the motor speed stabilizes.
- If the motor is not driving anything, the speed increases to the no-load or synchronous speed.



$$T = \frac{90}{\pi n_s} \times \frac{R_2'}{s} \times \frac{|V_1|^2}{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}$$


THANKS
FOR YOUR
ATTENTION