

# Notational Convention

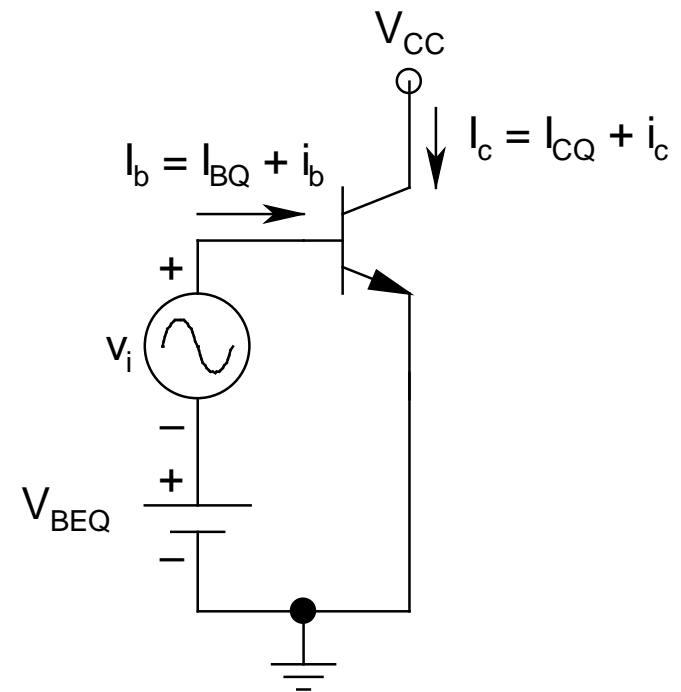
- *Pure DC quantities:*
  - *Capital letter with capital subscript* (e.g.,  $V_{BE}$ )
- *Pure ac quantities:*
  - *Small-case letter with small-case subscript* (e.g.,  $v_{be}$ )
- *Instantaneous (DC + ac) quantities:*
  - Either *capital letter with small-case subscript* (e.g.,  $V_{be}$ ) or *small-case letter with capital subscript* (e.g.,  $v_{BE}$ )

# Small-Signal Model

- The *electrical equivalent* of the BJT at the *DC bias point*
- Basically an *electrical network*, having *passive and active elements*
- To obtain this model, *DC analysis* is needed, since the *information* regarding the *Q-point* ( $I_C$ ,  $V_{CE}$ ) is necessary
- *This model for npn and pnp BJT is same*

# Validity of the Small-Signal Model

- Basically *linearization* of the *operating region* around the *Q-point*
- *This linearization should not contain any higher-order terms*



- To start with, assume  $V_A \rightarrow \infty$

- $I_{CQ} = I_S \exp(V_{BEQ}/V_T)$

- Thus:

$$\begin{aligned} I_c &= I_S \exp\left(\frac{V_{be}}{V_T}\right) = I_S \exp\left(\frac{V_{BEQ} + v_i}{V_T}\right) \\ &= I_S \exp\left(\frac{V_{BEQ}}{V_T}\right) \exp\left(\frac{v_i}{V_T}\right) = I_{CQ} \exp\left(\frac{v_i}{V_T}\right) \end{aligned}$$

- Expand the *exponential term* in series:

$$\blacktriangleright I_c = I_{CQ} \left[ 1 + \frac{v_i}{V_T} + \frac{1}{2!} \left( \frac{v_i}{V_T} \right)^2 + \frac{1}{3!} \left( \frac{v_i}{V_T} \right)^3 + \dots \right]$$

- Thus:

$$\blacktriangleright i_c = I_c - I_{CQ} = I_{CQ} \left[ \frac{v_i}{V_T} + \frac{1}{2!} \left( \frac{v_i}{V_T} \right)^2 + \frac{1}{3!} \left( \frac{v_i}{V_T} \right)^3 + \dots \right]$$

- *True linearization* of  $i_c$ - $v_i$  relation *can be achieved* only if *all higher-order terms* can be *neglected*  $\Rightarrow v_i$  should be  $\ll V_T$

# Small-Signal Model Parameters

- *Incremental Emitter Resistance* ( $r_E$ ):

$$r_E = \left( \frac{i_e}{v_i} \right)^{-1} = \left( \frac{\Delta I_E}{\Delta V_{BE}} \right)^{-1} \equiv \left( \frac{dI_E}{dV_{BE}} \right)^{-1} \bigg|_{V_{CE} \text{ constant}} = \frac{V_T}{I_E}$$

- *Transconductance* ( $g_m$ ):

$$g_m = \frac{i_c}{v_i} = \frac{\Delta I_C}{\Delta V_{BE}} \equiv \frac{dI_C}{dV_{BE}} \bigg|_{V_{CE} \text{ constant}} = \frac{I_C}{V_T}$$

- Thus,  $g_m r_E = I_C / I_E = \alpha \approx 1$
- *A frequently used approximation:*
  - $g_m = 1/r_E$
- For  $I_C = 1 \text{ mA}$ :
  - $r_E = 26 \Omega$  and  $g_m = 1/26 \text{ A/V}$
- As  $I_C \uparrow$ :
  - $g_m \uparrow$  and  $r_E \downarrow$
  - Also  $P_D \uparrow$
- Gain =  $f(g_m)$ 
  - $\Rightarrow$  For *higher gain*, the circuit has to be fed *more power*

- **Base-Emitter Resistance** ( $r_\pi$ ):

$$r_\pi = \frac{v_i}{i_b} = \frac{\Delta V_{BE}}{\Delta I_B} \equiv \left. \frac{dV_{BE}}{dI_C} \frac{dI_C}{dI_B} \right|_{V_{CE} \text{ constant}} = \frac{\beta}{g_m} \simeq \beta r_E$$

➤ For  $I_C = 1 \text{ mA}$  and  $\beta = 100$ :  $r_\pi = 2.6 \text{ k}\Omega$

- **Output Resistance** ( $r_o$ ):

$$r_o = \frac{v_{ce}}{i_c} = \left[ \frac{dI_C}{dV_{CE}} \right]^{-1} \bigg|_{V_{BE} \text{ constant}} = \frac{V_A}{I_C} = \frac{V_A}{V_T} \frac{V_T}{I_C} = \frac{1}{\eta g_m}$$

- For  $I_C = 1 \text{ mA}$ ,  $V_{AN} = 130 \text{ V}$ , and  $V_{AP} = 52 \text{ V}$ :  
 $r_o(\text{nnp}) = 130 \text{ k}\Omega$  and  $r_o(\text{pnp}) = 52 \text{ k}\Omega$
- $\eta (= V_T/V_A)$ :  $2 \times 10^{-4}$  (nnp) and  $5 \times 10^{-4}$  (pnp)
- $g_m r_o = \eta^{-1}$

- **Collector-Base Resistance** ( $r_\mu$ ):

$$r_\mu = \frac{V_{ce}}{i_b} = \frac{\Delta V_{CE}}{\Delta I_B} \bigg|_{V_{BE} \text{ constant}} = \frac{dV_{CE}}{dI_C} \frac{dI_C}{dI_B} = \beta r_o$$

- **Oversimplification** – *actual value much higher* ( $\sim 5\text{-}10\beta r_o$ )  $> 100\text{s of M}\Omega$

- ***Emitter-Base Capacitance*** ( $C_\pi$ ):

$$C_\pi = C_{je} + C_b$$

- $C_{je}$ : ***Emitter-base depletion capacitance***

$$\approx 2C_{je0}$$

- $C_{je0}$ : ***Emitter-base depletion capacitance at zero bias***

- $C_b$ : ***Emitter-base diffusion capacitance***  
(known as ***base charging capacitance***)

$$= \tau_F g_m \quad (>> C_{je})$$

- $\tau_F$ : ***Base transit time***

- $C_\pi \uparrow$  as  $g_m \uparrow$  (***Problem!***)