Lecture-15b

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Step Response of RL circuit.
- Second Order Response.

Second Order Response

❑ Introduction

- A second-order circuit is characterized by a second-order differential equation.

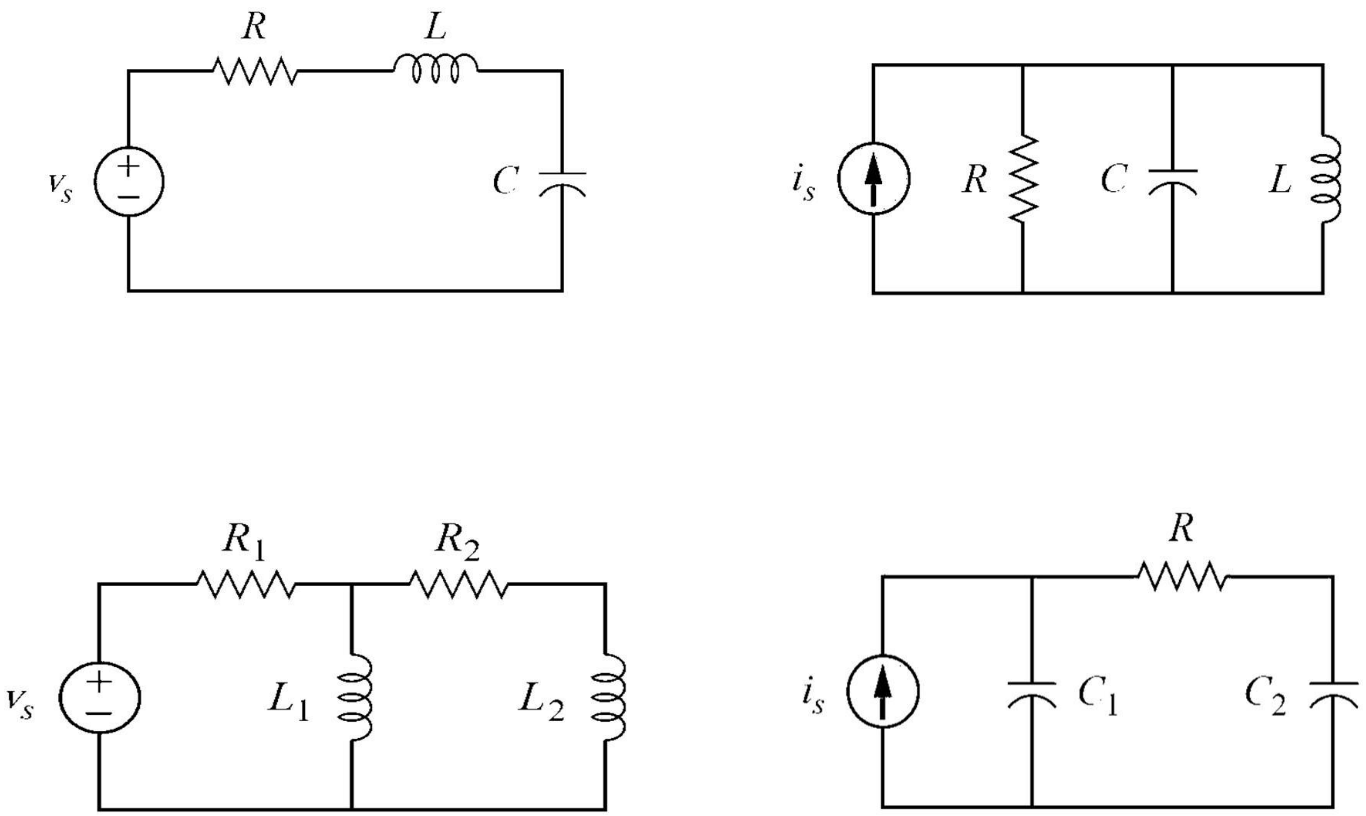
- Therefore, the circuits which contain two storage elements are called *second-order* circuits as their responses include differential equations that contain second order derivatives.

- Second-order circuits are typically called as *RLC* circuits. Here, three kinds of passive elements, i.e. R, L, and C, are present.

- Second-order circuit may have two storage elements of different type or the same type (provided elements of the same type cannot be represented by an equivalent single element)

Second Order Response (Cont…)

- Examples of such circuits are shown in the Figure in next slide.



Second Order Response (Cont…)

❑ Finding Initial and Final Values

- It is easy to get the initial and final values of v and i but we often have difficulty in finding the initial values of their derivatives, i.e. dv/dt and di/dt

❑ Two key points to keep in mind in determining the initial conditions.

- Polarity of voltage $v(t)$ across capacitor and the direction of current $i(t)$ through the inductor.
- Voltage across capacitor can not change abruptly, therefore,

$$v(0^+) = v(0^-)$$

- Similarly, inductor current can not change abruptly, therefore,

$$i(0^+) = i(0^-)$$

- Thus, in finding the initial conditions, we first focus on those variables that cannot change abruptly, i.e. capacitor voltage and inductor current,

The Source-Free Series *RLC* Circuit

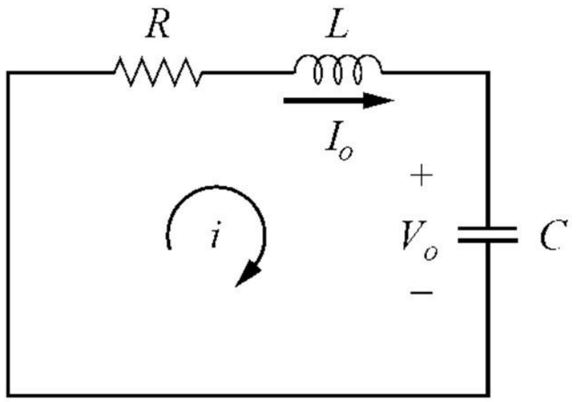
- The circuit shown in figure is being excited by the energy initially stored in the capacitor and inductor. The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0. Thus, at $t = 0$,

$$v(0) = \frac{1}{C}\int_{-\infty}^{0} i dt = V_0 \qquad (1)$$

$$i(0) = I_0 \qquad (2)$$

Applying KVL around the loop,

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i dt = 0 \qquad (3)$$



The Source-Free Series *RLC* Circuit (Cont..)

- To eliminate the integral, we differentiate with respect to t and rearrange terms. We get

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \qquad (4)$$

This is a *second-order differential equation*

- To solve such a second-order differential equation, we require to have two initial conditions, such as the initial value of i and its first derivative or initial values of i and v.
- The initial value of i is given in Eq. (2). We get the initial value of the derivative of i from Eqs. (1) & (3);

$$Ri(0) + L\frac{di(0)}{dt} + V_0 = 0 \qquad or$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) \qquad (5)$$

The Source-Free Series *RLC* Circuit (Cont..)

- With the two initial conditions in Eqs. (2) and (5), we can now solve Eq. (4).
- Our experience in the previous discussions on first-order circuits suggests that the solution is of exponential form. So, we let :

$$i = Ae^{\sigma t} \qquad (6)$$

where A and σ are constants to be determined

Substituting Eq. (6) into Eq. (4) and carrying out the necessary differentiations, we obtain

$$A\sigma^2 e^{\sigma t} + \frac{AR}{L}\sigma e^{\sigma t} + \frac{A}{LC}e^{\sigma t} = 0 \qquad or$$

$$Ae^{\sigma t}\left(\sigma^2 + \frac{R}{L}\sigma + \frac{1}{LC}\right) = 0 \qquad (7)$$

The Source-Free Series RLC Circuit (Cont..)

- Since $i = Ae^{\sigma t}$ is the assumed solution, therefore, only expression in parentheses can be zero -

$$\sigma^2 + \frac{R}{L}\sigma + \frac{1}{LC} = 0 \qquad (8)$$

- This quadratic equation is known as the ***characteristic equation*** of the differential Eq. (4), since the roots of the equation dictate the character of i .

The two roots of Eq. (8) are

$$\sigma_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (9a)$$

$$\sigma_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (9b)$$

The Source-Free Series *RLC* Circuit

- A more compact way of expressing the roots is -

$$\sigma_1 = -\alpha + \sqrt{\alpha^2 - {\omega_0}^2}, \sigma_2 = -\alpha - \sqrt{\alpha^2 - {\omega_0}^2} \qquad (10)$$

where,

$$\alpha = \frac{R}{2L}, \qquad (11)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- The roots σ_1 and σ_2 are called *natural frequencies*, measured in nepers per second (Np/s). ω_0 is known as the *resonant frequency* or the *undamped natural frequency*, expressed in radians per second (rad/s); and α is the *damping factor*, expressed in nepers per second.


THANKS
FOR YOUR
ATTENTION