

QUIZ - 2A
PROBABILITY AND STATISTICS (MSO-201)

Name (Roll Number):

TIME 20 MINS.

MAXIMUM MARKS: 20

If you do not write your name and roll number two marks will be deducted.
No extra sheet will be collected or provided. You have to encircle the answer clearly, otherwise no marks will be awarded.

1. Suppose the random variable X has the following CDF

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{x+1}{8} & \text{if } -1 \leq x < 1 \\ \frac{x+1}{4} & \text{if } 1 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

- (a) Sketch the graph of $F_X(x)$. (b) Find $P(-2 \leq X < 1)$. (c) Find $P(X = 0)$. (d) Find $P(X = 1)$. (e) Find $P(X > 1)$. [2+2+2+2+2=10]

(a) It is 0 for $x \leq -1$. From -1 to 1- it is a linear function increasing from 0 to 1/4. At 1, it is discontinuous and takes value 1/2. From 1 to 2- it is a linear function increasing from 1/2 to 3/4. It is 1 for $x \geq 2$. (b) $P(-2 \leq X < 1) = F(1-) - F(-2-) = 1/4 - 0 = 1/4$.
(c) $P(X = 0) = F(0) - F(0-) = 0$, (d) $P(X = 1) = F(1) - F(1-) = 1/2 - 1/4 = 1/4$, (e) $P(X > 1) = 1 - F(1) = 1 - 1/2 = 1/2$.

2. Suppose the random variable X has the following PDF

$$f_X(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

Consider a new random variable $Y = X^2$. (a) $P(1 \leq X \leq 2) = \int_1^2 2e^{-2x} dx = e^{-2} - e^{-4}$. (b)

$$P(1 \leq Y \leq 2) = P(1 \leq X \leq \sqrt{2}) = \int_1^{\sqrt{2}} 2e^{-2x} dx = e^{-2} - e^{-2\sqrt{2}}$$