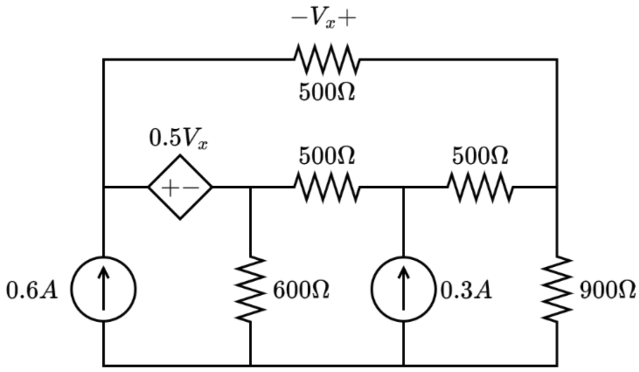
ESO203 Tutorial 2 SOLUTIONS

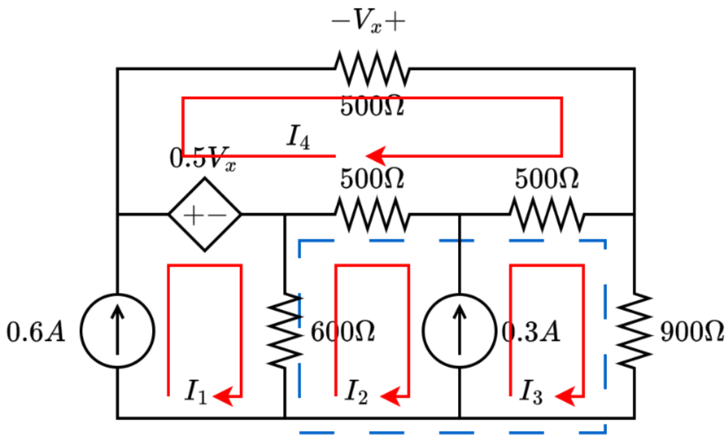
Q. 1: Find the value of V_x in the following figure.



Solution:

Let us consider 4 meshes with mesh current I_1, I_2, I_3, I_4 .

Mesh 2 and mesh 3 have a current source in common. So, they will form a supermesh.



Now, from mesh 1, we get

$$I_1 = 0.6\ A \qquad \ldots(i)$$

Also,

$$I_2 - I_3 = -0.3\ A \qquad \ldots(ii)$$

From supermesh,

$$600(I_2 - I_1) + 500(I_2 - I_4) + 500(I_3 - I_4) + 900I_3 = 0$$

$$-600I_1 + 1100I_2 + 1400I_3 - 1000I_4 = 0 \qquad \ldots(iii)$$

From mesh 4,

$$-0.5V_x + 500I_4 + 500(I_4 - I_3) + 500(I_4 - I_2) = 0$$

$$-0.5V_x - 500I_2 - 500I_3 + 1500I_4 = 0 \qquad \ldots(iv)$$

Also,

$$V_x = -500I_4 \quad \ldots(v)$$

Putting equation (i), (ii), (v) in equation (iii), (iv), we get

$$2500I_3 - 1000I_4 = 690 \qquad \ldots(vi)$$

$$-1000I_3 + 1750I_4 = -150 \qquad \ldots(vii)$$

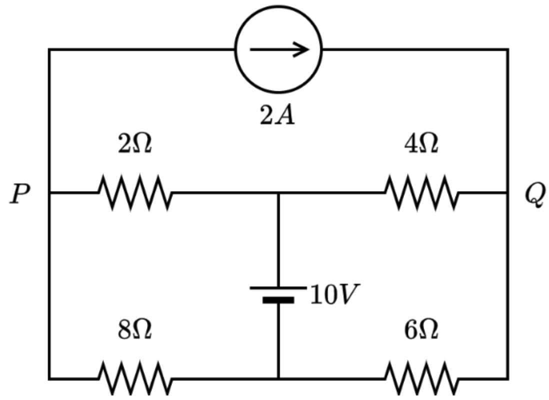
From equation (vi) and (vii), we get

$$I_3 = 0.3133A, \qquad I_4 = 0.0933A$$

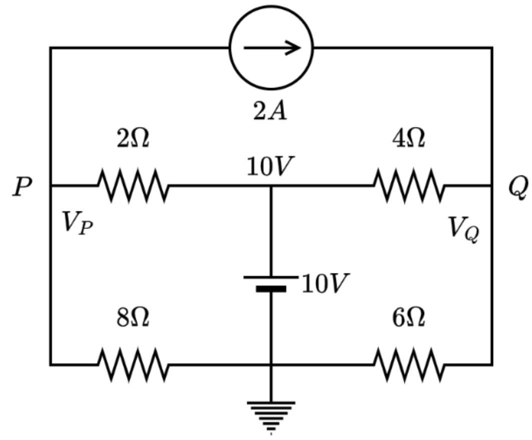
So,

$$V_x = -46.65V$$

Q. 2: Determine the potential difference between P and Q.



Solution:



Let a reference node. Consider the reference node at the ground.

Using nodal analysis at node P,

$$\frac{V_P - 0}{8} + \frac{V_P - 10}{2} + 2 = 0$$

$$V_P = 4.8V$$

Using nodal analysis at node Q,

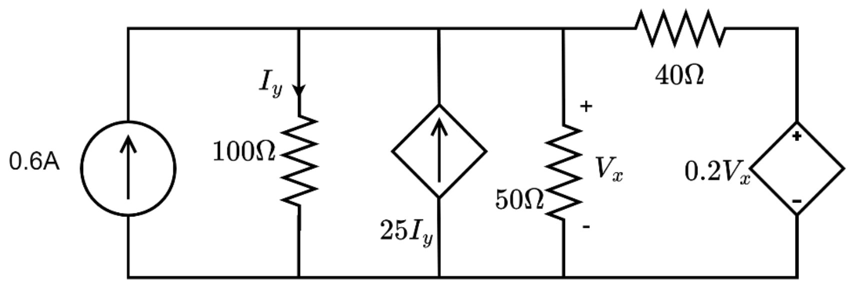
$$\frac{V_Q - 0}{6} + \frac{V_Q - 10}{4} = 2$$

$$V_Q = 10.8V$$

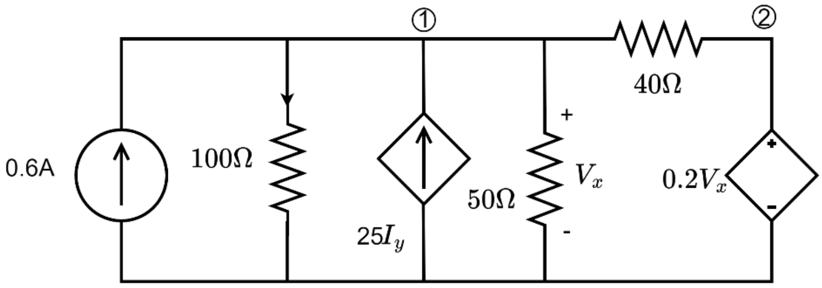
So, the potential difference between P and Q is

$$V_{PQ} = -6V$$

Q. 3: For the circuit of the given figure, find the voltage V_x using the nodal analysis.



Solution:



By applying KCL at node (1)

$$-0.6 + I_y + \frac{V_x}{50} - 25I_y + \frac{V_1 - V_2}{40} = 0 \qquad \dots\dots\dots (1)$$

By KCL at node (2),

$$V_2 = 0.2V_x \qquad \dots\dots\dots (2)$$

And other constraint equations,

$$I_y = \frac{V_x}{100} \text{ and } V_1 = V_x$$

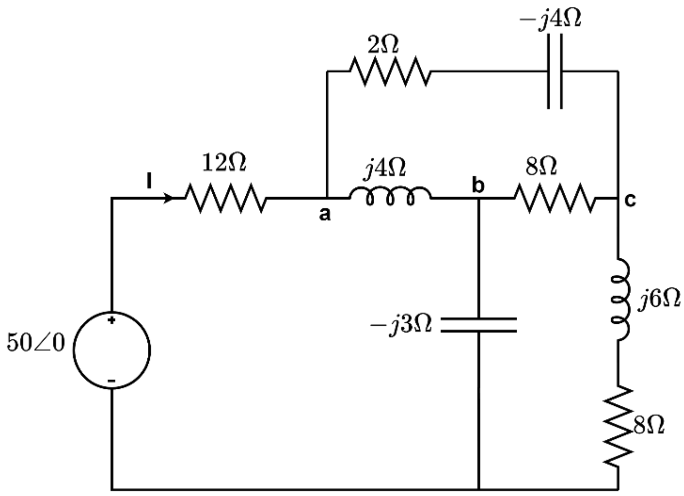
Putting value of $I_y, V_1, and\ V_2$ in terms of V_x in the equation (1),

$$-0.6 + \frac{V_x}{100} + \frac{V_x}{50} - 25\frac{V_x}{100} + \frac{(V_x - 0.2V_x)}{40} = 0$$

$$\Rightarrow -120 + 2V_x + 4V_x - 50V_x + 5V_x - 5 \times 0.2V_x = 0$$

$$\Rightarrow V_x = \frac{120}{-40} = -3V$$

Q. 4: Find the current I in the given circuit.



Solution:

The delta network connected to nodes a, b, and c can be converted to the Y network of the given figure. We obtain the Y impedances as:

$$Z_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\Omega$$

$$Z_{bn} = \frac{j4(8)}{10} = j3.2\Omega$$

$$Z_{cn} = \frac{8(2-j4)}{10} = (1.6 - j3.2)$$

The total impedance at the source terminal is

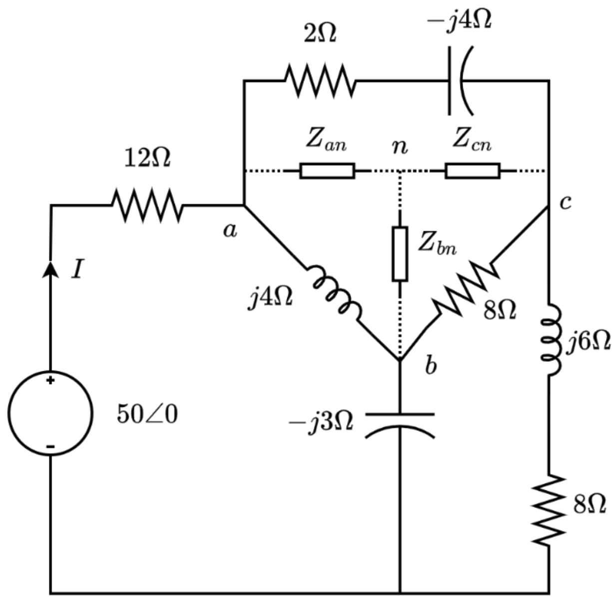
$$Z = 12 + Z_{an} + (Z_{bn} - j3) \parallel (Z_{cn} + j6 + 8)$$

$$= 12 + 1.6 + j0.8 + (j0.2)||(9.6 + j2.8)$$

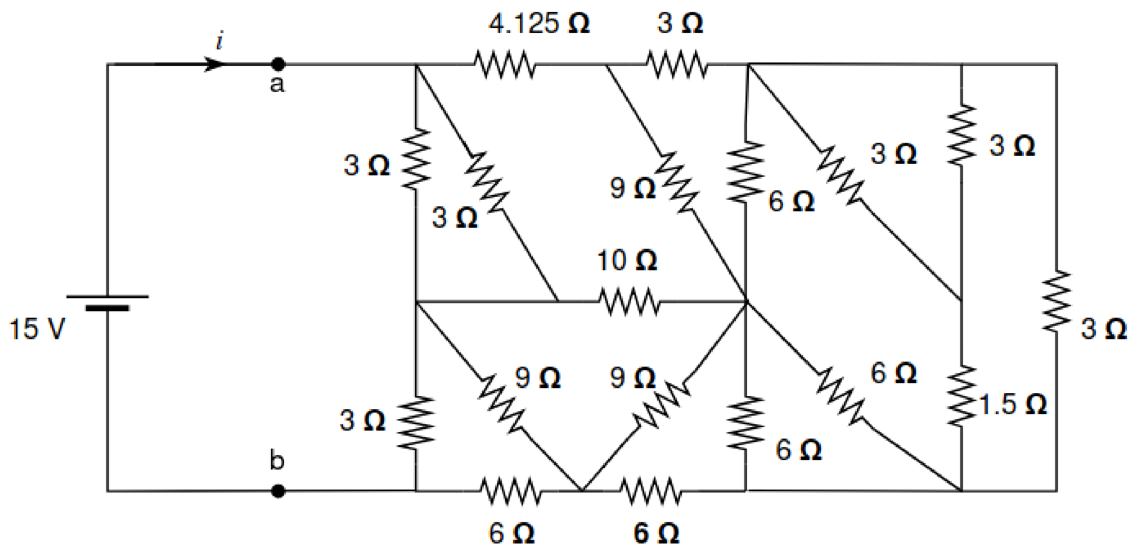
$$= 13.64\angle 4.204°\Omega$$

The current is

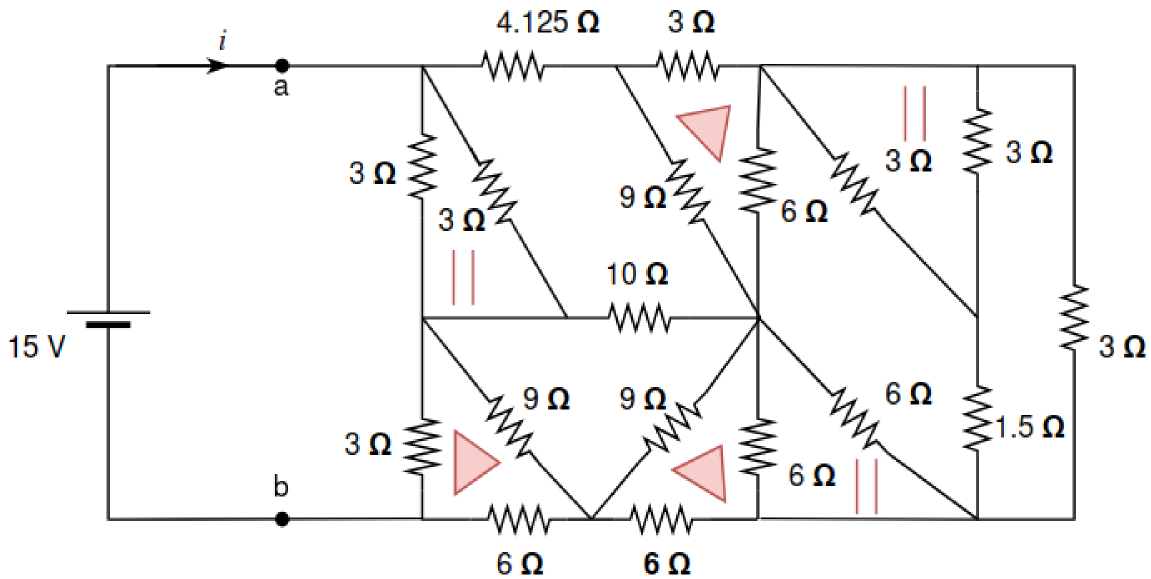
$$I = \frac{V}{Z} = \frac{50\angle 0°}{13.64\angle 4.204°} = 3.666\angle - 4.204°\text{ A}$$



Q. 5: Obtain the equivalent resistance R_{ab} for the circuit shown in the figure and find the current "i".

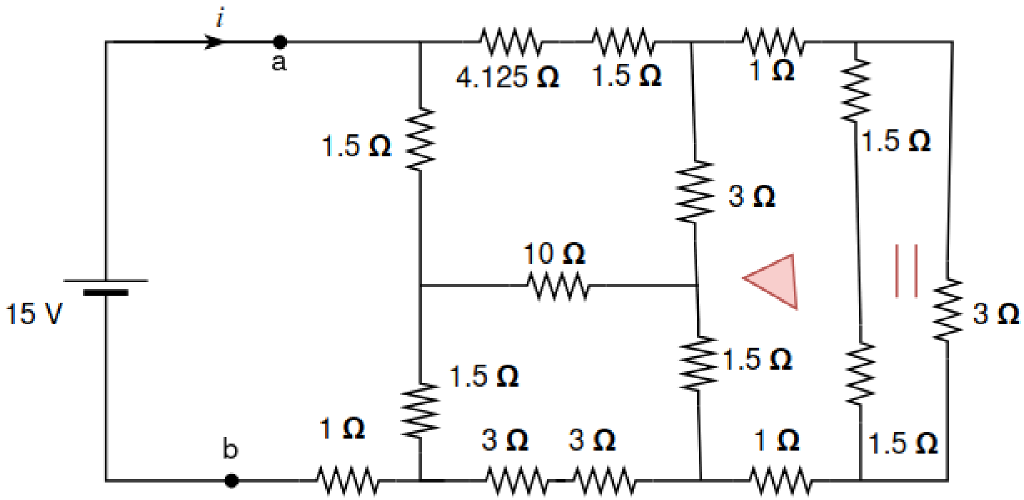


Solution:

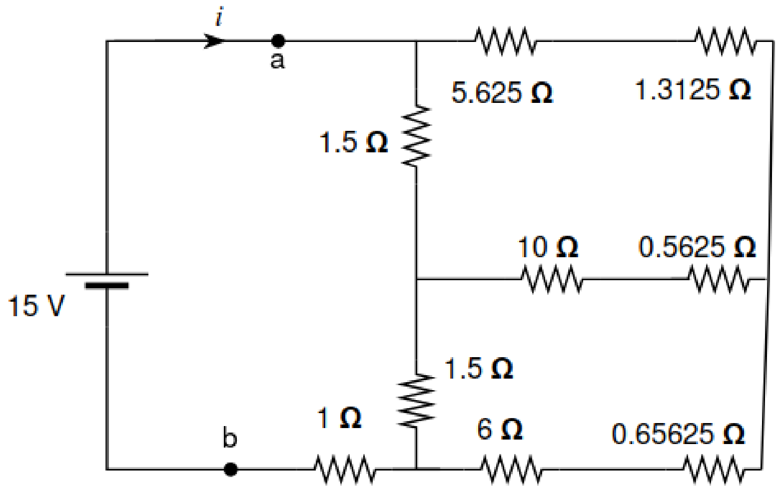


Using star-delta conversion, the above deltas shown in the figure is converted into stars,

Now,



Further simplification,



Taking approximation value to deduce the circuit (6.65625 Ω ≈ 6.9375 Ω), and applying wheatstone bridge.

The 10 Ω and 0.5625 Ω can be removed.

Now 3 Ω and 14 Ω is parallel.

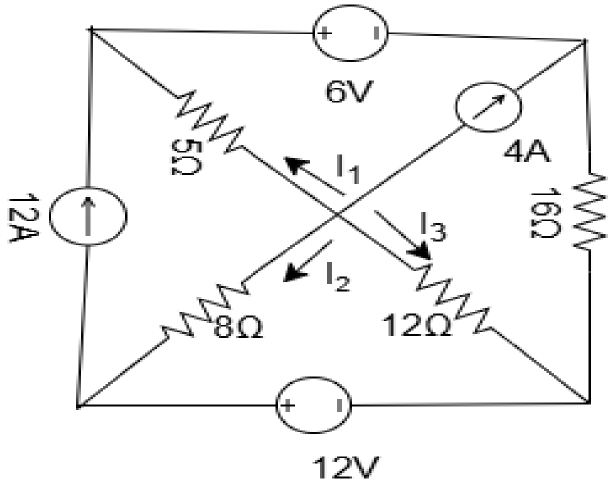
We can calculate the equivalent resistance between a and b by

$$R_{ab} = \frac{14 \times 3}{17} + 1\Omega, \qquad R_{ab} = 3.47\Omega$$

and

$$i = \frac{15}{3.47} A, \qquad i = 4.32A$$

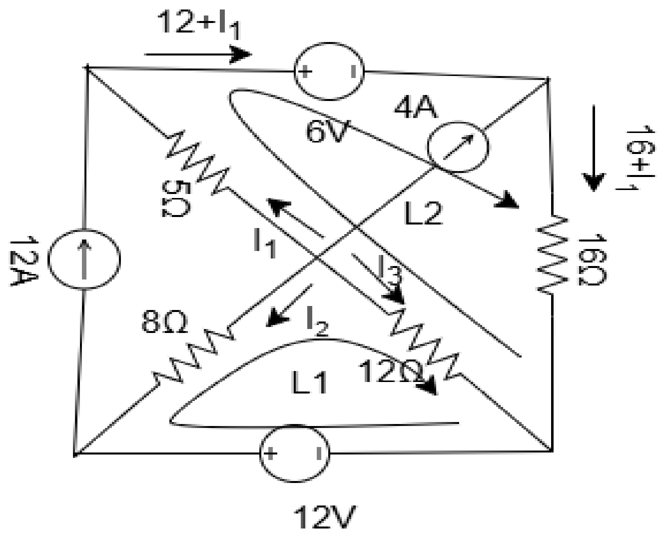
Q. 6: In the given circuit, find the current I_1, I_2, and I_3 using nodal and mesh analysis.



Solution:

Using KCL, the current is 6V battery is $12 + I_1$.

Similarly, current in 16 Ω resistance is $16 + I_1$.



Apply KCL at the central node,

$$I_1 + I_2 + I_3 + 4 = 0 \quad \dots (i)$$

Apply KVL in Bottom mesh L1,

$$8 * I_2 - 12 * I_3 + 12 = 0 \quad \dots (ii)$$

Apply KVL in Right top supermesh L2:

$$-12 * I_3 + 5 * I_1 + 6 + 16 * (16 + I_1) = 0$$

$$12 * I_3 - 21 * I_1 - 262 = 0 \quad \dots (iii)$$

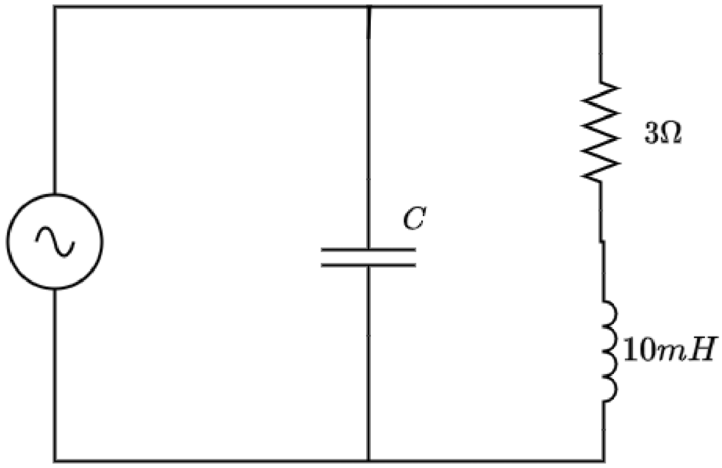
By solving above equation, we get:

$$I_1 = -10.62A$$

$$I_2 = 3.372A$$

$$I_3 = 3.248A$$

Q. 7: A source $w = 314\ rad/sec$ is connected to a load Z_L as shown in the figure. Find the value of the capacitance for the load Z_L to be completely resistive. Z_L is the impedance seen from the source.



Solution:

The equivalent impedance Z_L can be calculated by parallel combination of capacitance and series R-L.

$$Z_L = \left(-\frac{j}{wC}\right) || (3 + jwL)$$

$$Z_L = \frac{(3 + jwL)\left(-\frac{j}{wC}\right)}{3 + jwL - (\frac{j}{wC})}$$

$$Z_L = \frac{\left(3 - j\left(wL - \left(\frac{1}{wC}\right)\right)\right) * \left(\left(\frac{L}{C}\right) - (\frac{j3}{wC})\right)}{9 + \left(wL - \left(\frac{1}{wC}\right)\right)^2}$$

$$Z_L = \frac{3\left(\frac{L}{C}\right) - \left(wL - \left(\frac{1}{wC}\right)\right)\left(\frac{3}{wC}\right) - j\left(wL - \left(\frac{1}{wC}\right)\right)\left(\frac{L}{C}\right) - j\left(\frac{9}{wC}\right)}{9 + \left(wL - \left(\frac{1}{wC}\right)\right)^2}$$

Z_L is purely resistive if imaginary part of Z_L is zero.

Equate the imaginary part to 0

$$-\left(wL - \left(\frac{1}{wC}\right)\right)\left(\frac{L}{C}\right) - \left(\frac{9}{wC}\right) = 0$$

$$\frac{L}{wC^2} = \frac{wL^2}{C} + \left(\frac{9}{wC}\right)$$

$$L = w^2L^2C + 9C \;\;\Rightarrow\;\; C = \frac{L}{w^2L^2 + 9}$$

$$C = 53mF$$