2024/01/16

Block Diagrams: Mason's Gain Formula

The gain from U to Y:

$$\frac{Y}{U} = \frac{1}{\Delta}\sum_{i=1}^{N} P_i \Delta_i$$

N – # forwards from U to Y

P_i – i-th forward path gain

Δ_i – the determinant of the portion of the SFG that does not touch the i-th forward path.

Δ – Determinant of the SFG

$$\Delta = 1 - \sum \text{loop gains of loops taken one at a time} + \sum \text{products of loop gains of non-touching loops taken 2 @ time} - \sum \text{products of loop gains of nontouching loops taken 3 at a time} + \sum \cdots$$

Let's attend to 2 terms:

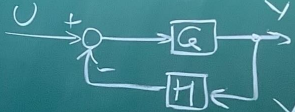
gain and loop gain.

We have defined a transfer function (TF) as the Laplace transform of the output divided by the Laplace transform of the input:

$$\frac{Y(s)}{U(s)} \triangleq G(s)$$

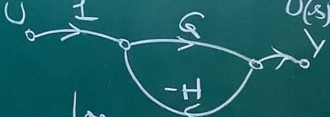
In frequency domain "gain" shall mean the magnitude of $G(j\omega)$.

However, in the discussion of SFGs, gain = TF.



loopgain is GH.

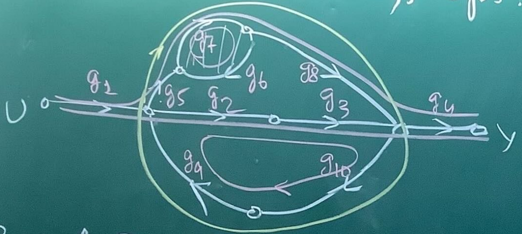
$$\frac{Y(s)}{U(s)} = \frac{G}{1+GH}$$



loopgain is $G(-H)$.

$$\frac{Y(s)}{U(s)} = \frac{G}{1+GH}$$

What are nontouching loops?



2 parts of an SFG are nontouching if they don't have at least one node in common.

2 parts of an SFG are nontouching if they don't have even one any common node.

$$\Delta = 1 - (g_6 g_7 + g_5 g_7 g_8 g_{10} g_9 + g_2 g_3 g_{10} g_9) + (g_6 g_7 \cdot g_2 g_3 g_{10} g_9) - (0)$$

$$P_1 = g_1 g_2 g_3 g_4$$

$$P_2 = g_1 g_5 g_7 g_8 g_4$$

$$\Delta_1 = 1 - g_6 g_7 + 0$$

$$\Delta_2 = 1 -$$

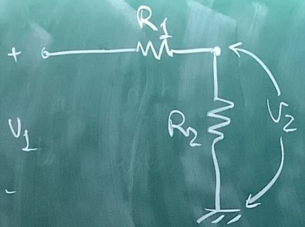
Block Diagrams: Mason's Gain Formula

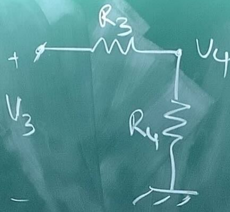
Caution: In a BD, each block is non-loading.

For g_2 to be attached as shown in series with g_1, the signal y_1 should be the same before and after attaching g_2

g_2 does not load g_1

Example





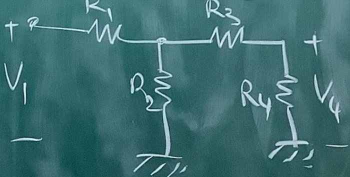
$$\frac{V_2}{V_1} = \frac{R_2}{R_1+R_2} \qquad \frac{V_4}{V_3} = \frac{R_4}{R_3+R_4}$$

$$V_1 \longrightarrow \boxed{\frac{R_2}{R_1+R_2}} \xrightarrow{V_2} \qquad V_3 \longrightarrow \boxed{\frac{R_4}{R_3+R_4}} \xrightarrow{V_4}$$

$$V_1 \longrightarrow \boxed{g_1} \longrightarrow \boxed{g_2} \xrightarrow{V_4} \quad g_1 g_2 = \frac{\quad}{R_1R_3+R_2R_3+R_1R_4+R_2R_4}$$

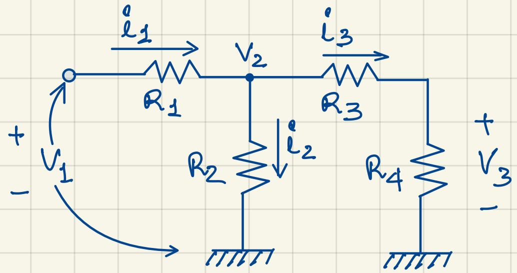
That is from the math!

Let's physically attach:



$$\frac{V_4}{V_1} = ?$$

HW: Draw a block diagram (BD) and a signal flow graph (SFG) for the following circuit. Call them BD1 and SFG1.



Given that blocks need to be nonloading for them to be usable in a BD, are the predictions that BD1 or SFG1 make about the various gains correct?

Justify your answer with reasoning.