ESO203 Tutorial-3

Question 1:

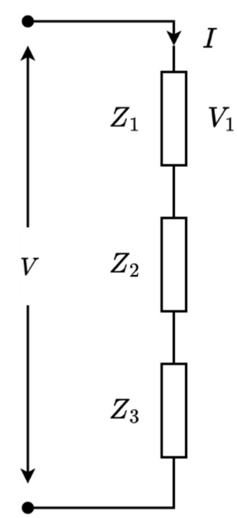
Three impedances are in series: $Z_1 = 3.0\angle 45°\Omega, Z_2 = 10\sqrt{2}\angle 45°\Omega, Z_3 = 5.0\angle -90°\Omega$. Find the applied voltage V, if the voltage across Z_1 is $27.0\angle -10°V$.

Solution:

The current I is calculated by equation.

$$I = \frac{V_1}{Z_1}$$

$$I = \frac{27.0\angle -10°}{3.0\angle 45°} = 9\angle -55°\ A$$



The impedances are in series, so the equivalent impedance is

$$Z_{eq} = Z_1 + Z_2 + Z_3$$

Now, we know that

$$V = IZ_{eq}$$

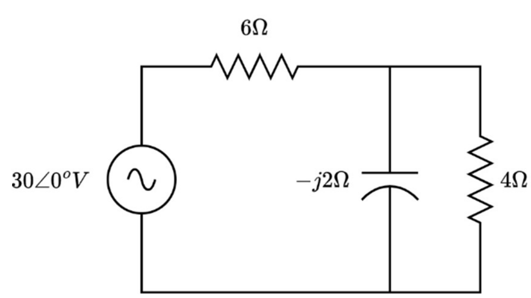
$$V = 9\angle -55°(3.0\angle 45° + 10\sqrt{2}\angle 45° + 5.0\angle -90°)$$

$$V = 9\angle -55°\left[\left(\frac{3}{\sqrt{2}} + 10\right) + j\left(\frac{3}{\sqrt{2}} + 10 - 5\right)\right]$$

$$V = 126.52\angle -24.56°\ volts$$

Question 2:

Determine the power factor of the entire circuit as seen by the source. Calculate the average power and apparent power delivered by the source.



Solution:

Firstly, the equivalent impedance is calculated by

$$Z_{eq} = 4||(-j2) + 6$$

$$Z_{eq} = \frac{(4)(-j2)}{4 - j2} + 6$$

$$Z_{eq} = 6.8 - j1.6\ \Omega$$

The phase of impedance Z_{eq} is

$$\theta = \tan^{-1}\left(\frac{-1.6}{6.8}\right) = -13.24°$$

The power factor leads as θ is leading and calculated by

$$pf = cos\theta = 0.9734\ lead$$

The current magnitude is

$$|I_m| = \frac{|V|}{|Z|} = \frac{30}{\sqrt{6.8^2 + 1.6^2}} = 4.298\ A$$

The average power is given by

$$P_{avg} = \frac{1}{2} I_m^2 R$$

$$P_{avg} = \frac{1}{2}(4.298^2) \times 6.8 = 62.65\ W$$

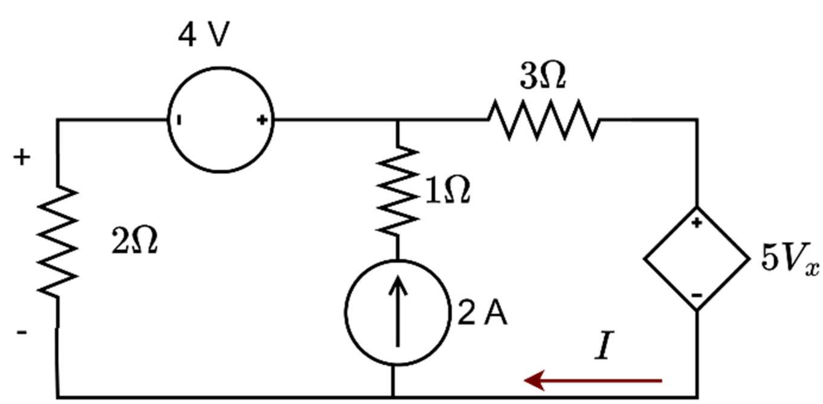
The apparent power is given by

$$S = V_{rms} I_{rms} = \frac{1}{2} V_m I_m$$

$$S = \frac{1}{2}(30)(4.298) = 64.47\ VA$$

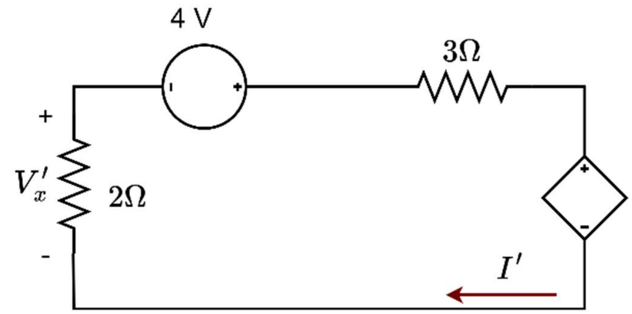
Question 3:

Find the current I in the given figure using the superposition theorem.



Solution:

When the 4-V voltage source is acting alone the circuit can be redrawn as given below.



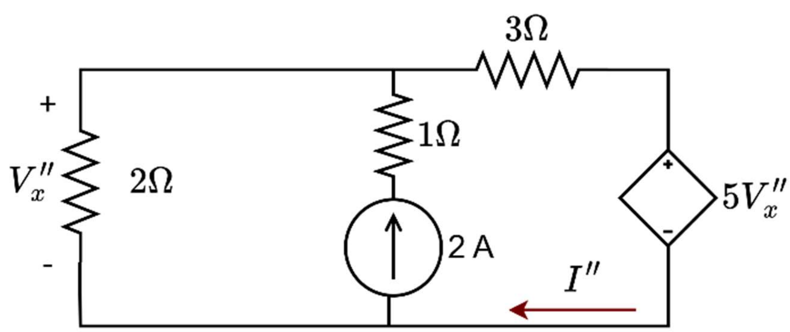
Apply the KVL,

$$-4 + 3I' + 5V_x' - V'_x = 0$$

$$3I' + 4 \times (-2I') = 4 \qquad [\therefore V_x' = -2I']$$

$$I' = -\frac{4}{5}A = -0.8A$$

When the 2A current source is acting alone, the circuit becomes as given below.



Apply the KCL,

$$\frac{V_x''}{2} + \frac{V_x'' - 5V_x''}{3} = 2 \Rightarrow V_x'' = -\frac{12}{5} = -2.4V$$

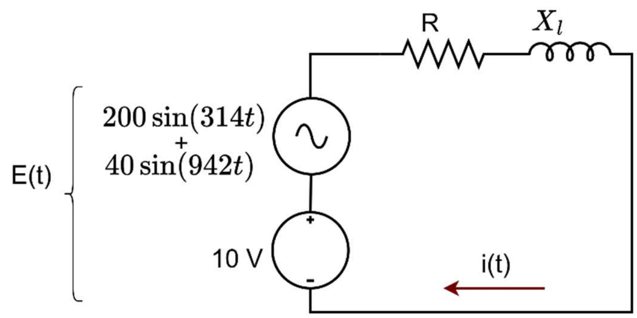
$$\therefore I'' = \frac{V_x'' - 5V_x''}{3} = \frac{-2.4 - 5 \times (-2.4)}{3} = 3.2A$$

When both sources are acting simultaneously, the current by superposition theorem is given as:

$$I = I' + I'' = (-0.8 + 3.2) = 2.4A$$

Question 4:

An RL series circuit with R = 10 ohms, L = 0.0318 H is energized with a voltage source $\mathrm{E(t)} = 200\sin(314\mathrm{t}) + 40\sin(942\mathrm{t} + 30°) + 10\ \mathrm{V}$. Determine the rms value of the current flowing in the circuit. Determine the rms value of the applied voltage source also.



Solution:

Break the applied voltage source into three components depending on the frequency component. Then, $\mathrm{E(t)} = \mathrm{V_1(t)} + \mathrm{V_2(t)} + \mathrm{V_0}$

The current due to the source $\mathrm{V_1(t)}$

$$\mathrm{i_1(t)} = \frac{200\sin(314\mathrm{t})}{10 + \mathrm{j}314 \times 0.0318} = 14.14\sin(314\mathrm{t})\angle - 44.95°$$

The current due to the source $\mathrm{V_2(t)}$

$$\mathrm{i_2(t)} = \frac{40\sin(942\mathrm{t} + 30°)}{10 + \mathrm{j}942 \times 0.0318} = 1.28\sin(942\mathrm{t} + 30°)\angle - 71.54°$$

The current due to the DC source $\mathrm{V_0}$

$$\mathrm{i_0} = \frac{10}{10} = 1$$

The total current flowing in the circuit.

$$\mathrm{i(t)} = 14.14\sin(314\mathrm{t} - 44.95°) + 1.28\sin(942\mathrm{t} - 41.54°) + 1$$

The rms current can be calculated as

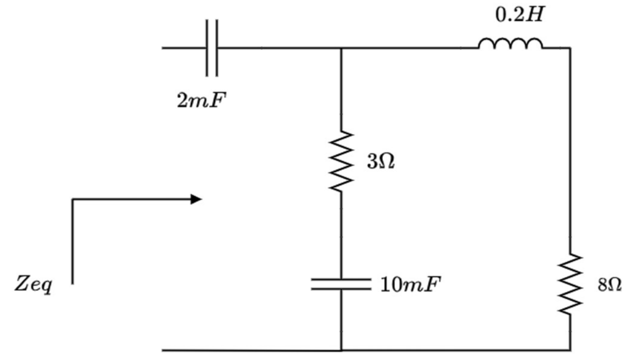
$$\mathrm{Irms} = \sqrt{1^2 + \frac{14.14^2}{2} + \frac{1.28^2}{2}} = 10.089\mathrm{A}$$

RMS value of applied source voltage is

$$\mathrm{V_{rms}} = \sqrt{10^2 + \frac{200^2}{2} + \frac{40^2}{2}} = 144.568\mathrm{A}$$

Question 5:

Find the input impedance and admittance of the circuit below which operates at the frequency, $w = 50$ rad/s.



Solution:

Z_1= impedance of 2mF capacitor

Z_2= impedance of 3Ω resistor in series with 10mF capacitor

Z_3= impedance of the 0.2H inductor in series with 8Ω resistor

$$Z_1 = \frac{1}{jwC} = \frac{1}{j50 * 2 * 10^{-3}} = -j10\Omega$$

$$Z_2 = 3 + \frac{1}{jwC} = 3 + \frac{1}{j50 * 10 * 10^{-3}} = (3 - j2)\Omega$$

$$Z_3 = 8 + jwL = 8 + j50 * 0.2 = (8 + j10)\Omega$$

The equivalent impedance is

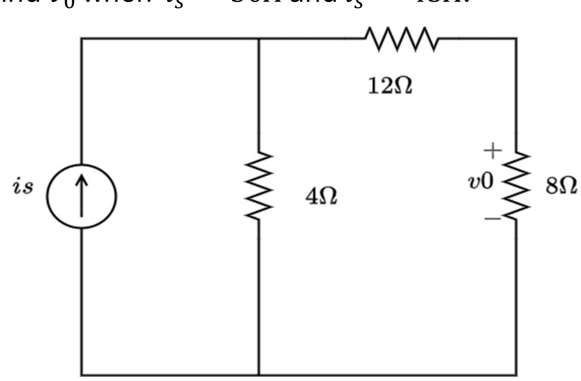
$$Z_{eq} = Z_1 + Z_2||Z_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$Z_{eq} = 3.22 - j11.07\ \Omega$$

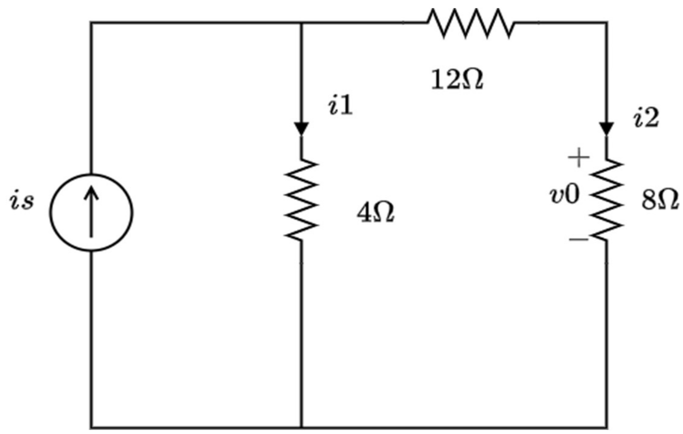
Admittance $Y = \frac{1}{Z_{eq}} = 0.0242 + j0.0832\ S$

Question 6:

For the circuit below find v_0 when $i_s = 30A$ and $i_s = 45A$.



Solution:



By current divide rule,

$$i_1 = i_s * \frac{12+8}{4+12+8}$$

$$i_1 = i_s * \frac{20}{24}$$

Similarly,

$$i_2 = i_s * \frac{4}{4+12+8}$$

$$i_2 = i_s * \frac{4}{24} \quad ; \quad v_0 = 8 * i_2$$

When $i_s = 30A$

$$i_1 = 25A$$

$$i_2 = 5A$$

$$v_0 = 8 * 5 = 40V$$

When $i_s = 45A$

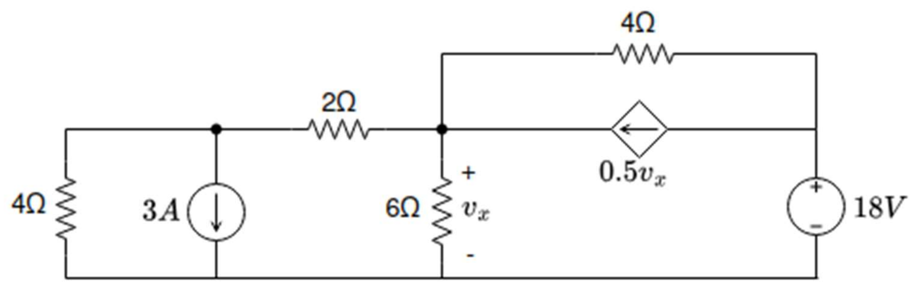
$$i_1 = 37.5A$$

$$i_2 = 7.5A$$

$$v_0 = 8 * 7.5 = 60V$$

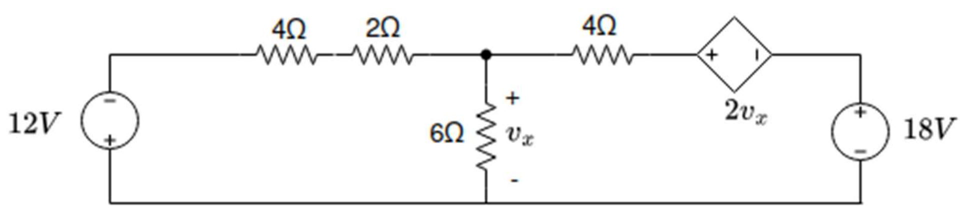
Question 7:

For the given circuit, find the voltage v_x using source transformations?

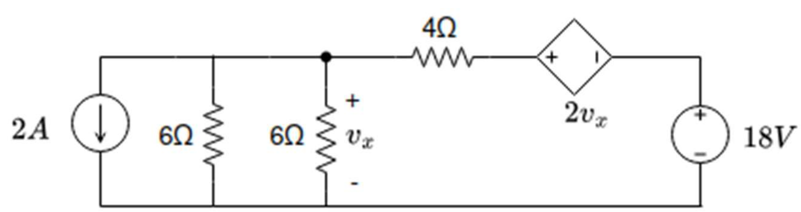


Solution:

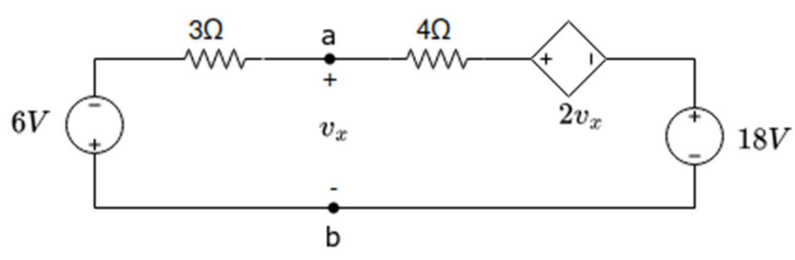
We first transform the current and the voltage dependent current sources to obtaine the circuit shown below.



Further, add the series resistances and transform the 12V source, we get.



Now, simplify the parallel resistance network and then transform the current source.



Apply KVL, we have

$$6 - 7i + 2v_x + 18 = 0$$

$$24 - 7i + 2v_x = 0 \qquad \ldots (1)$$

$$6 - 3i + v_x = 0$$

$$v_x = 3i - 6 \quad \dots(2)$$

Solving equations (1) and (2), we get.

$$i = 12A$$

$$v_x = 30V$$

Question 8:

A single-phase AC load draws an apparent power of 10 kVA at a power factor of 0.8 lagging. Calculate the following:

a) The real power (P) consumed by the load.
b) The reactive power (Q) consumed by the load.
c) The rms value of the current drawn by the load, If the supply voltage is 240 V (rms).

Solution:

Given: Apparent power (S) = 10 kVA = 10,000 VA, Power factor (PF) = 0.8 lagging

(a) Real Power (P):

$$P = S \times PF$$

$$P = 10000 \times 0.8$$

$$P = 8000W$$

(b) Reactive Power (Q):

$$S^2 = P^2 + Q^2,$$

Now,

$$Q = \sqrt{S^2 - P^2}$$

$$Q = 6000\,VAR$$

(c) The RMS Current (I): $S = V \times I$, Given $V = 240\,Volt(rms)$

Now,

$$I = \frac{S}{V} = \frac{10000}{240}$$

$$I = 41.67A$$