Lecture-11

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Thevenin’s Theorem.
- Norton’s Theorem.
- Maximum Power Transfer.

- **Quiz-1:**

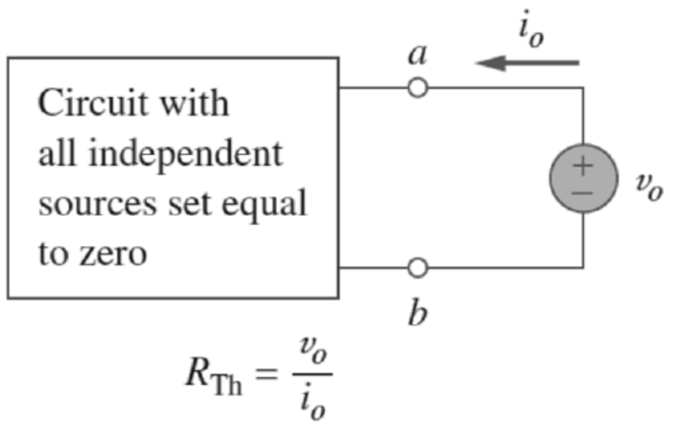
  - **Date:** 5th Feb 2025 (Wednesday).
  - **Time:** 5:00 pm to 6:00 pm.
  - **Venue:** Tutorial block (T101 to T106).
  - **Syllabus:** till 4th Feb 2025.

Thevenin's Theorem (Cont…)

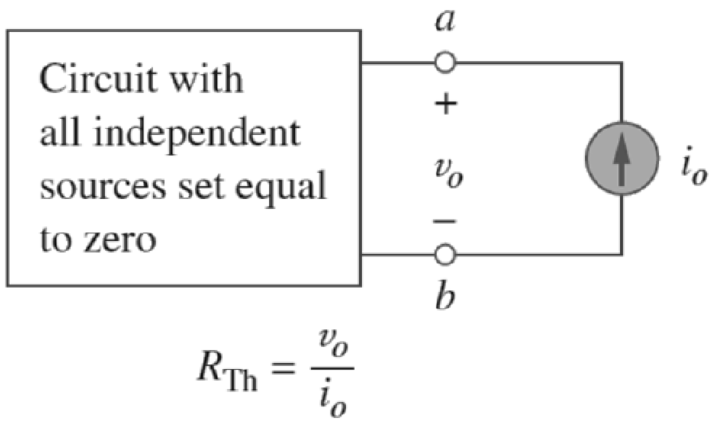
- To find out R_{TH} we need to consider two cases.
- **Case 1:**
  - ❖ If the network has no dependent sources, we turn off all the independent sources.
  - ❖ R_{TH} is the input resistance of the network looking between terminals a and b, as shown in the previous figure.
- **Case 2:**
  - ❖ If the network has dependent sources, we turn off only all independent sources.
  - ❖ The dependent sources cannot be turned off as they are controlled by circuit variables.
  - ❖ We apply a voltage v_0 at terminals a-b and determine the current i_0.

Thevenin's Theorem (Cont…)

- Then, R_{TH} is given by $\rightarrow R_{TH} = {}^{v_0}/_{i_0}$



- Alternatively, R_{TH} can be evaluated by inserting a current source as shown in the figure below.



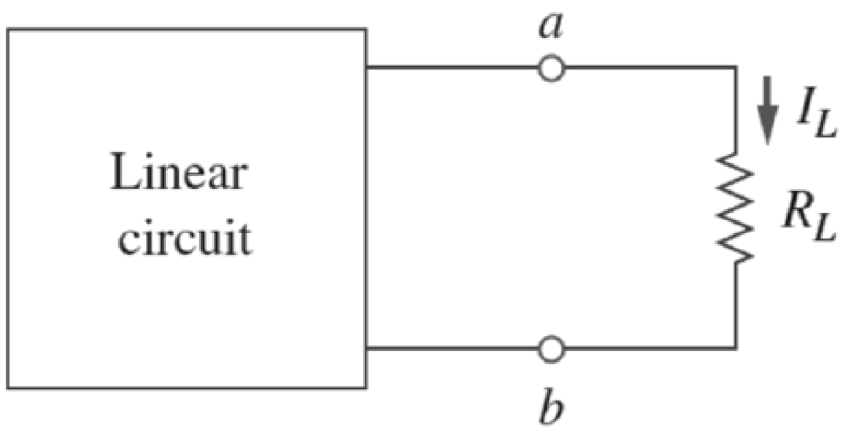
- Again, R_{TH} is ${}^{v_0}/_{i_0}$.

Thevenin’s Theorem (Cont…)

- Both approaches will give the same result.
- In either approach the value of v_0 and i_0 may be assumed to be equal to any value.
- It often occurs that R_{TH} takes a negative value. This implies that the circuit is delivering power.
- Negative value of R_{TH} is also possible in the circuit with dependent sources.
- Thevenin’s theorem is very important in the circuit analysis because helps in simplifying the circuit.
- A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

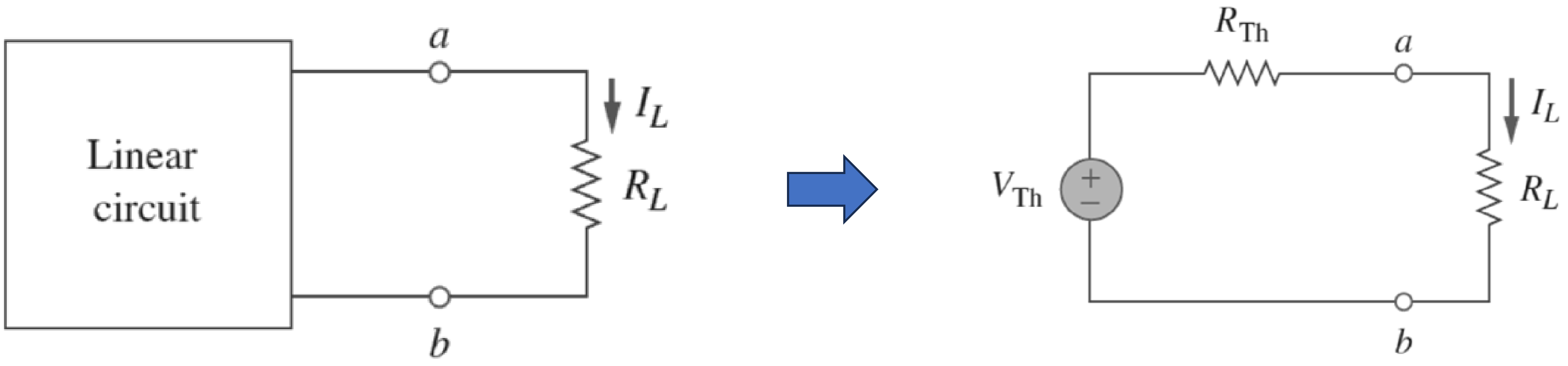
Thevenin's Theorem (Cont…)

- As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.
- The equivalent network behaves the same way externally as the original circuit.
- Let us consider a linear circuit terminated by a load R_L.



Thevenin's Theorem (Cont…)

- The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit are determined.
- The circuit can then be transformed as shown below.



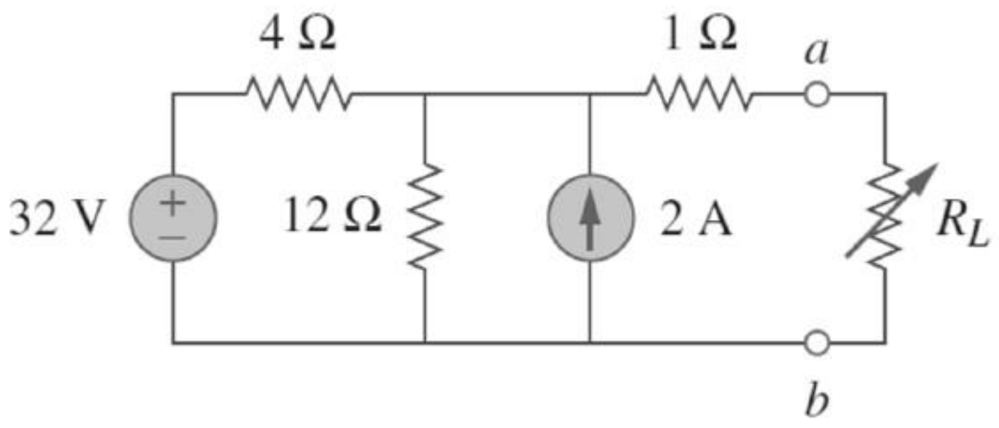
- From the above figure we obtain,

$$I_L = {V_{Th}}/{(R_{Th} + R_L)}$$

$$V_L = I_L R_L = {V_{Th} R_L}/{(R_{Th} + R_L)}$$

❑ Example:

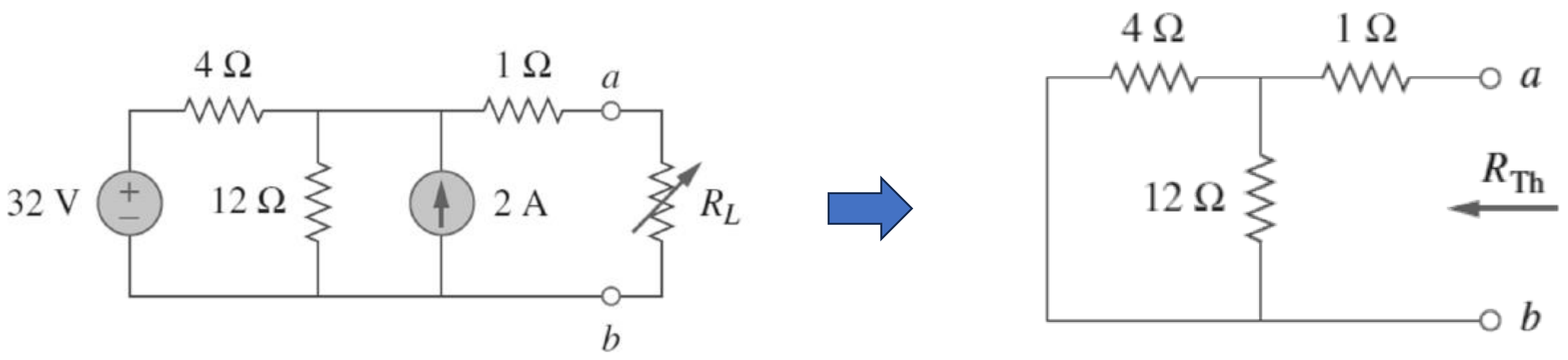
For the below circuit find the Thevenin equivalent to the left of terminals a-b? Also, find current through load when R_{Th} = 6 and 36Ω.



Solution:

We find the Thevenin resistance by turning off the 32V source (replacing it with a short circuit) and the 2A current source (replacing it with an open circuit).

The circuit then becomes as shown in the figure below.

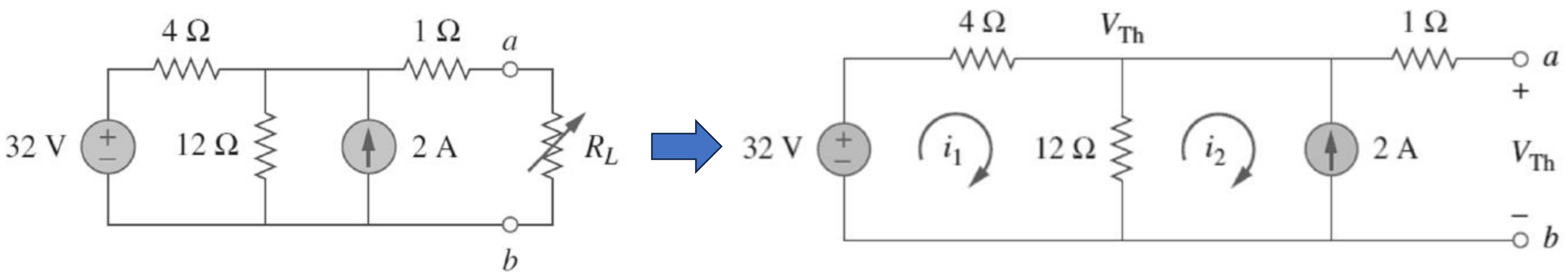


From the above figure R_{Th} can be evaluated as,

$$R_{Th} = 4||12 + 1 = \frac{4 * 12}{4 + 12} + 1 = 4\Omega$$

The next step is to find V_{Th}.

To find V_{Th} we consider the circuit given below.



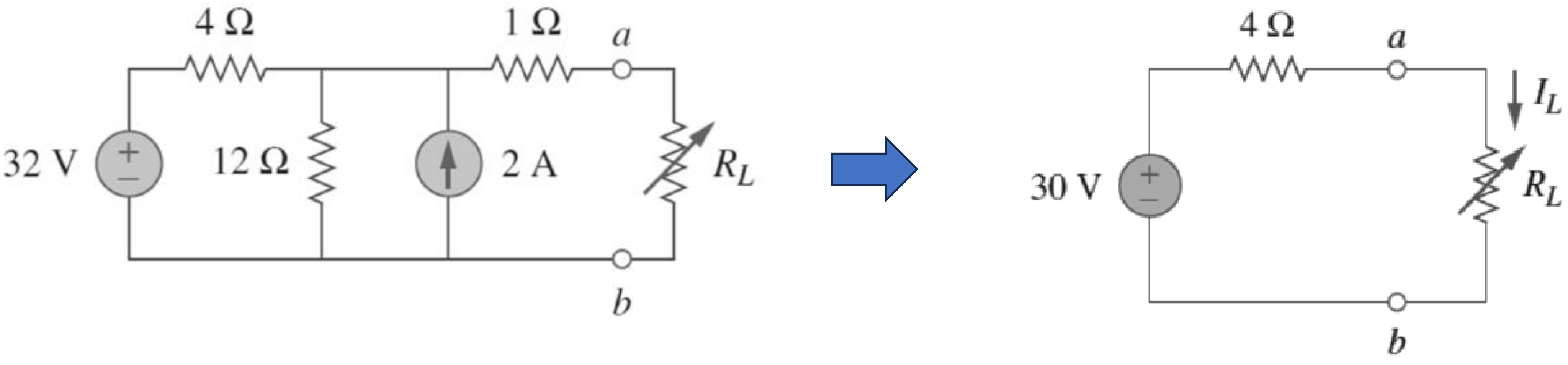
Applying mesh analysis to the two loops,

$$-32 + 4i_1 + 12(i_1 - i_2) = 0$$
$$i_2 = -2A$$

Solving for i_1, we get i_1 = 0.5A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2) = 30V$$

The Thevenin equivalent circuit is as shown in the below figure.



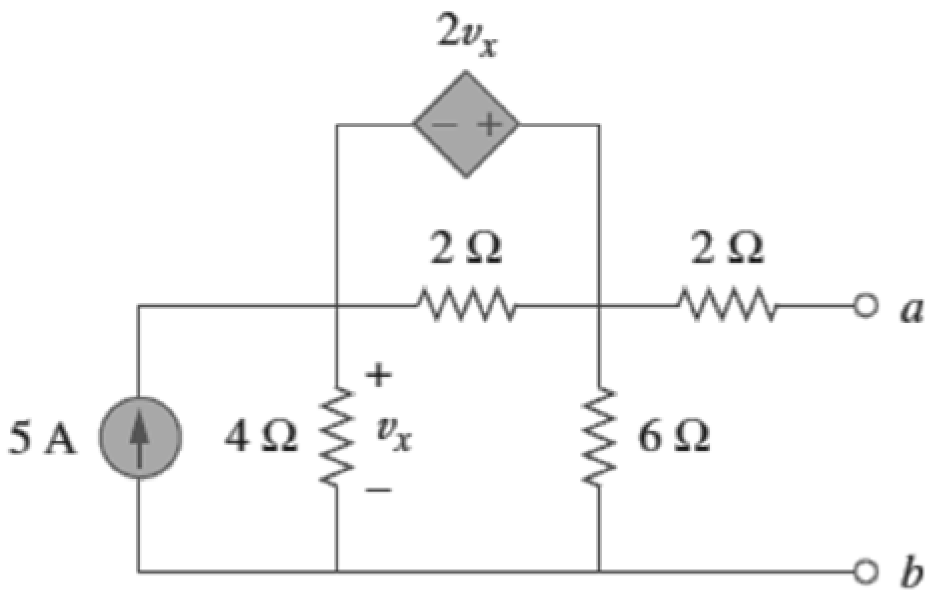
The current through R_L is,

When $R_L = 6\Omega, I_L = \frac{30}{10} = 3A.$

When $R_L = 16\Omega, I_L = \frac{30}{20} = 1.5A.$

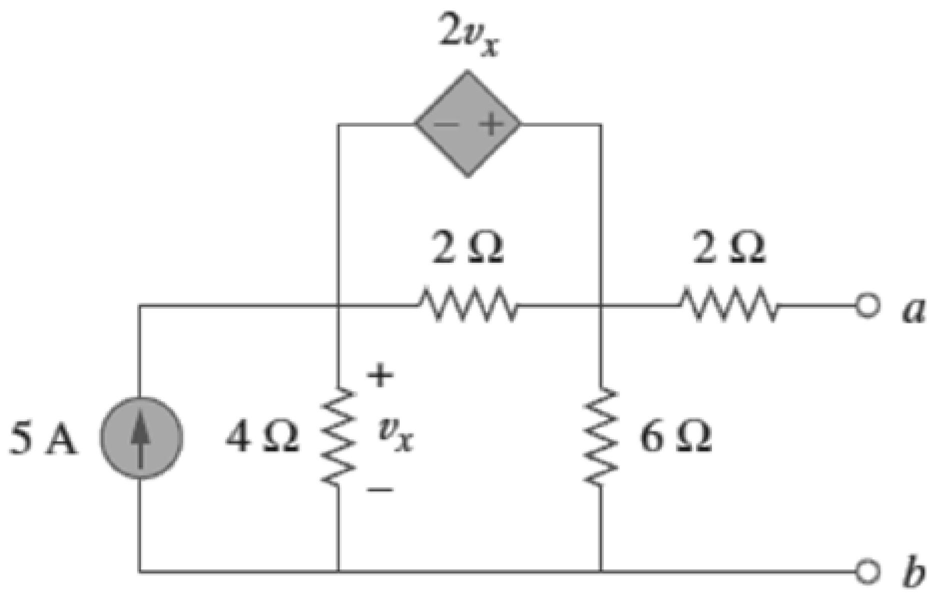
❑ Example:

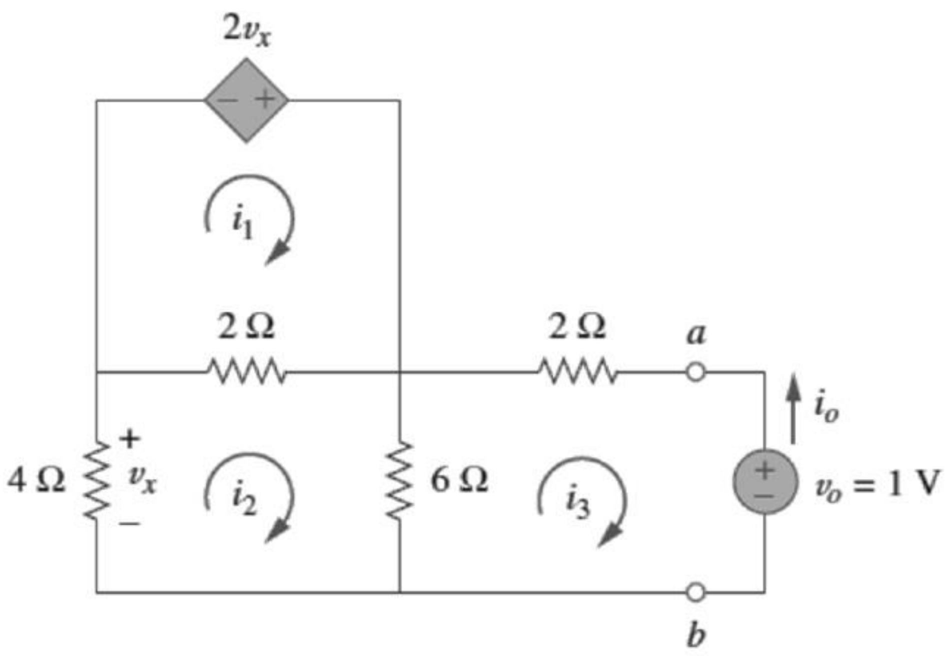
For the below circuit find the Thevenin equivalent to the left of terminals a-b?



Solution: The circuit contains dependent source, unlike the circuit given in the previous example.

To find R_{Th} we set the independent source to zero but leave the dependent source alone. As there is a dependent source, we excite the network with a source voltage v_0 connected to the terminals as shown in the circuit below.





We may set $v_0 = 1\text{V}$ to ease the calculation, since the circuit is linear.

- The next step is to find the current i_0 through the terminals and then evaluate $R_{Th} = \frac{i_0}{1}$.

- Alternatively, insert a 1 A current source, find the corresponding voltage v_0 and obtain $R_{Th} = \frac{v_0}{1}$.
- Applying mesh analysis to loop 1 of the circuit results in.

$$-2v_x + 2(i_1 - i_2) = 0 \Rightarrow v_x = i_1 - i_2$$

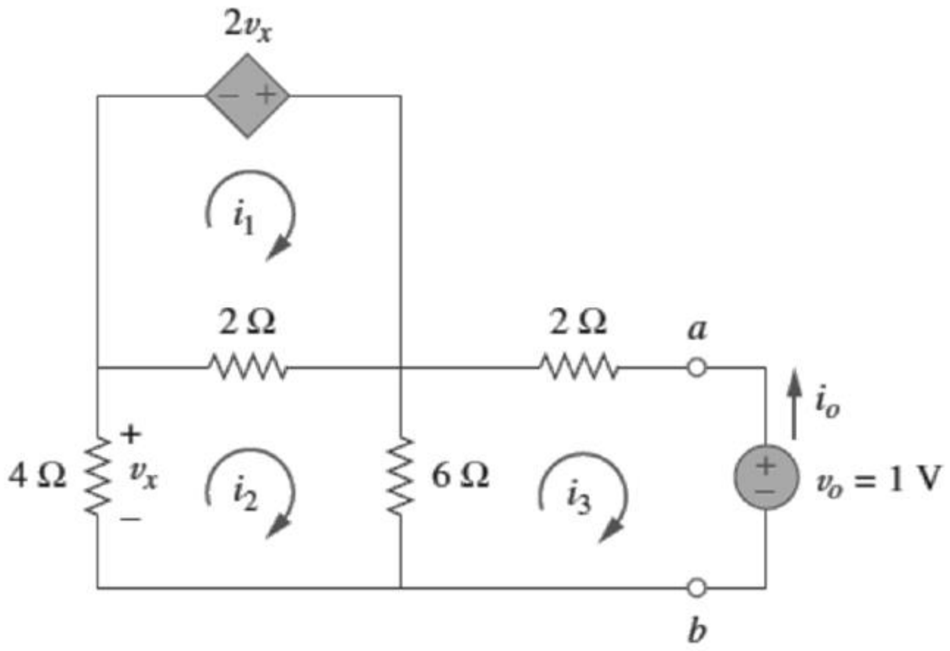
- But -4 i_2 = v_x = $i_1 - i_2$, hence

$$i_1 = -3i_2$$

- Applying KVL to loops 2 and 3 gives,

$$4i_2 + 2(i_1 - i_2) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$



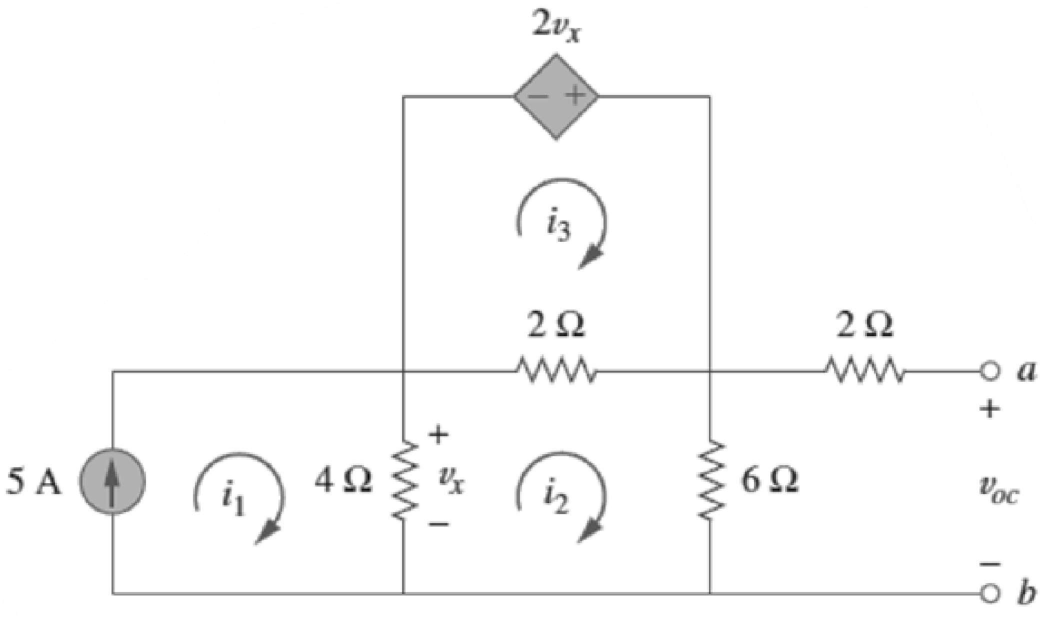
- Solving the above equations we get,

$$i_3 = -\frac{1}{6}A$$

- But i_0 = - i_3 = 1/6 A. Hence,

$$R_{Th} = \frac{1V}{i_0} = 6\Omega$$

- To determine V_{Th}, we find v_{oc} in the circuit shown below.

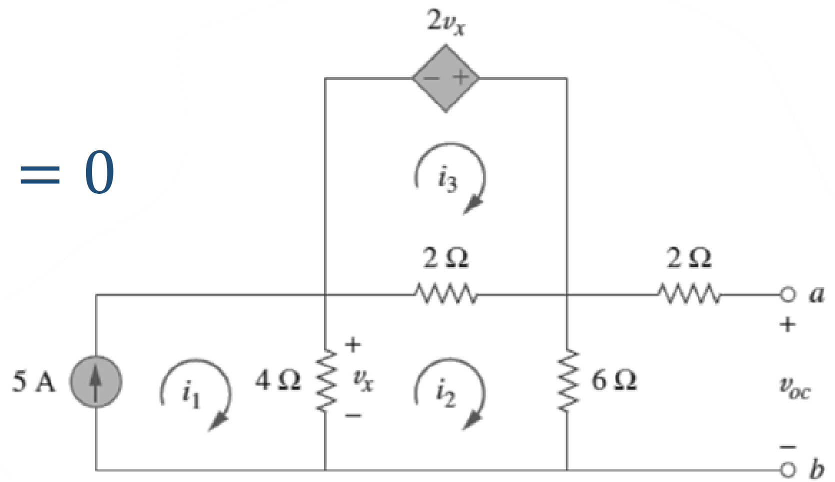


- Applying mesh analysis we obtain,

$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2$$

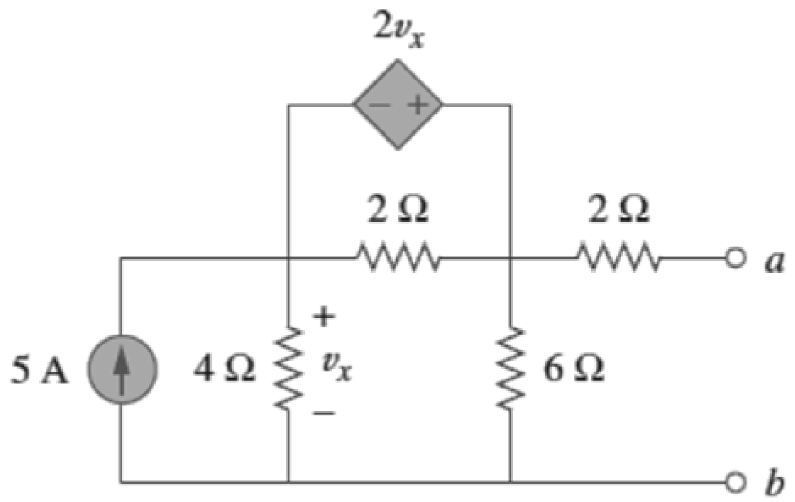
$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_1 - 2i_3 = 0$$

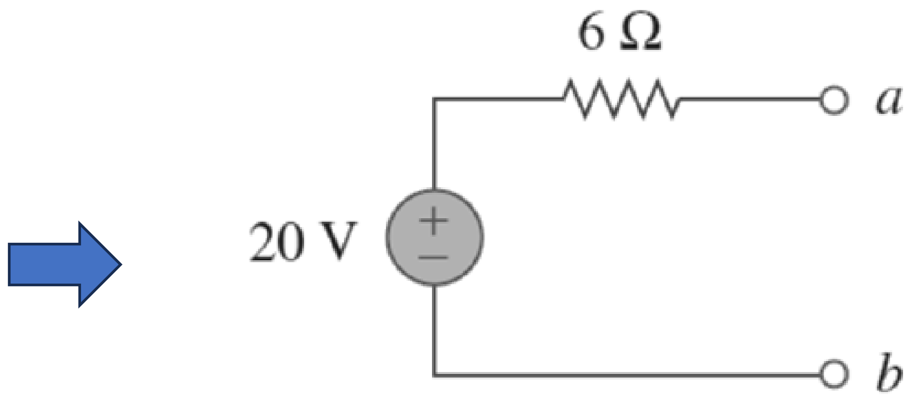


- But $4(i_1 - i_2) = v_x$.
- Solving these equations leads to $i_2 = 10/3$.
- Hence,

$$V_{Th} = v_{oc} = 6i_2 = 20V$$

- The Thevenin equivalent is as shown in the adjacent figure.





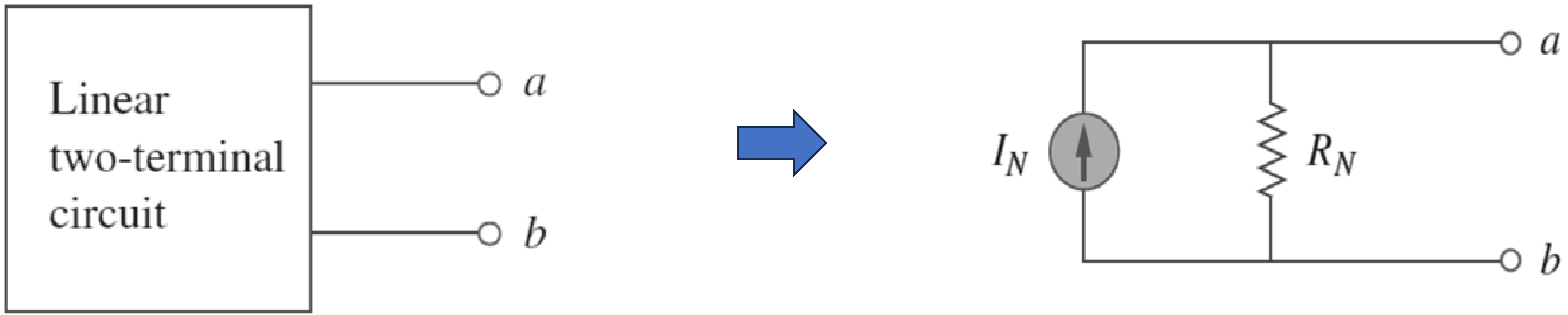
Norton’s Theorem

- In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem, called Norton’s theorem.
- Norton’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Norton's Theorem (Cont…)

- Similar to the Thevenin's theorem, our major concern here is to find out the values of I_N and R_N.
- We find R_N the same way we found R_{Th}.
- In fact, from what we know from source transformation, the Thevenin and Norton resistances are the same, i.e.,

$$R_N = R_{Th}$$

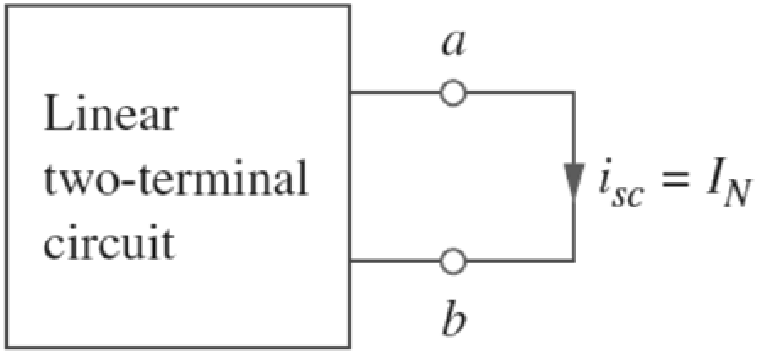


Norton's Theorem (Cont…)

- To find the Norton's equivalent current I_N we find the short circuit current flowing from terminal a to b in both the circuits shown in the previous figure.
- It is evident from the figure that the short circuit is equal to I_N.
- This must be same short circuit current from terminal a to b in the figure on the left in previous slide, since both circuits are equivalent.
- Thus,

$$I_N = i_{sc}$$

as shown in the below figure.



Norton's Theorem (Cont…)

- Dependent and independent sources are treated the same way as in Thevenin's theorem.

❑ Case 1:

- If the network has no dependent sources, we turn off all the independent sources.
- R_N is the input resistance of the network looking between terminals a and b, as shown in the previous figure.

❑ Case 2:

- If the network has dependent sources, we turn off all the independent sources only, leaving dependent sources connected to the circuit.
- As with the Thevenin's theorem, the dependent sources cannot be turned off as they are controlled by circuit variables.
- We apply a voltage v_0 at terminals a-b and determine the current i_0.
- R_N is found the same way as R_{Th}.

Norton's Theorem (Cont…)

- The close relationship between Thevenin circuit and Norton circuit is obvious.
- $R_{Th} = R_N$ and $I_N = V_{Th}/ R_{Th}$.
- This is essentially a source transformation.
- For this reason, the source transformation is often called Thevenin-Norton transformation.
- According to the above relation, V_{Th}, I_N, and R_{Th} are related. Therefore, to determine the Thevenin or Norton equivalent circuit, we find them as follows -
  - The open circuit voltage v_{oc} across the terminals a and b
  - The short circuit current i_{sc} at the terminals a and b
  - The equivalent or the input resistance R_{in} at terminals a and b when all independent sources are turned off.

Norton's Theorem (Cont…)

- We can calculate any two of the three, using the method that takes the least effort, and use them to get the third parameter value using Ohm's law.
- Therefore,

$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

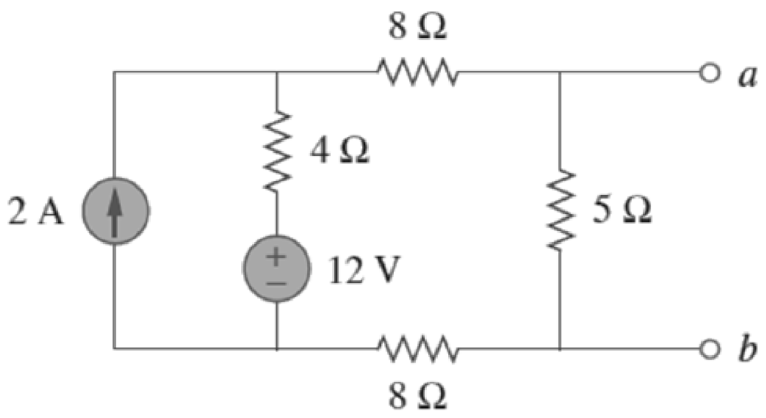
$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

The open and short circuit tests are sufficient to determine any Thevenin or Norton equivalent, of a circuit which contains at least one independent source.

Norton's Theorem (Cont…)

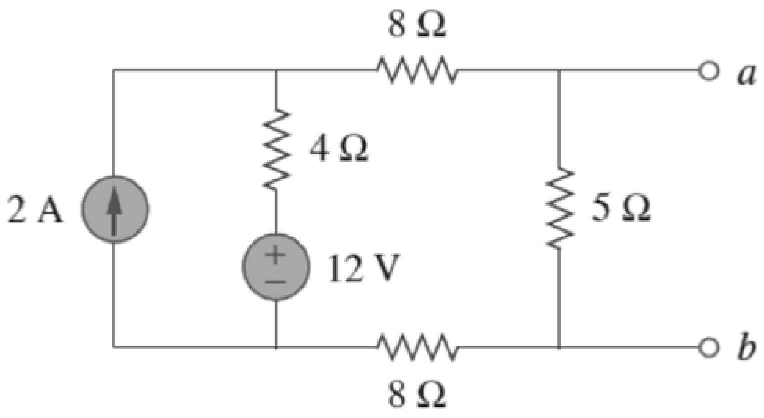
❑ Example:

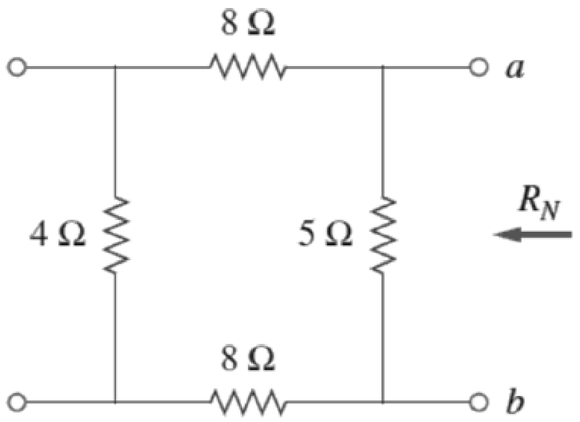
Find the Norton equivalent for the circuit given in the below figure?



Solution: We find the value of Norton resistance the same way we found it for Thevenin equivalent circuit.

Set the independent sources to zero, to obtain the following circuit,





$$R_N = 5||(8+4+8) = 5||20$$

$$= \frac{20 * 5}{25} = 4\Omega$$

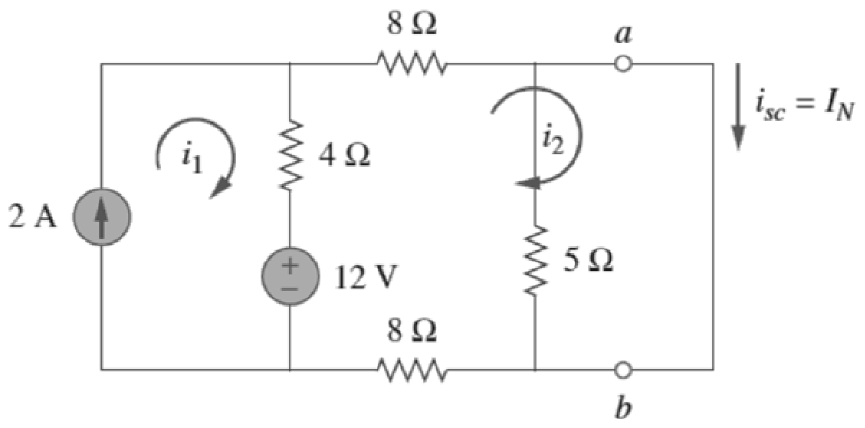
- To find I_N we short the terminals a and b.

- We ignore the 5Ω resistor as it is short circuited.

- Applying mesh analysis we get,

$$i_1 = 2A \qquad\qquad 20i_2 - 4i_1 - 12 = 0$$

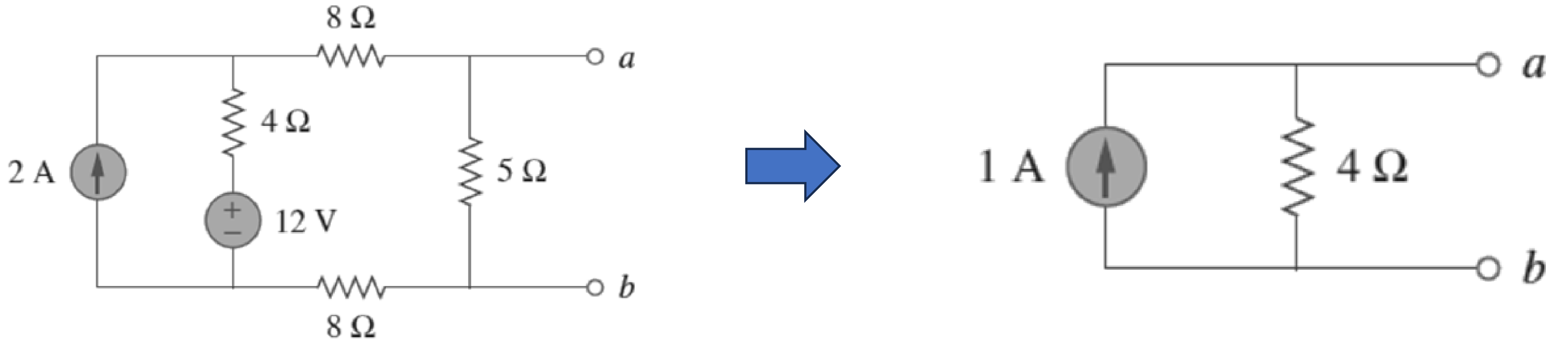
- From these equations we obtain,



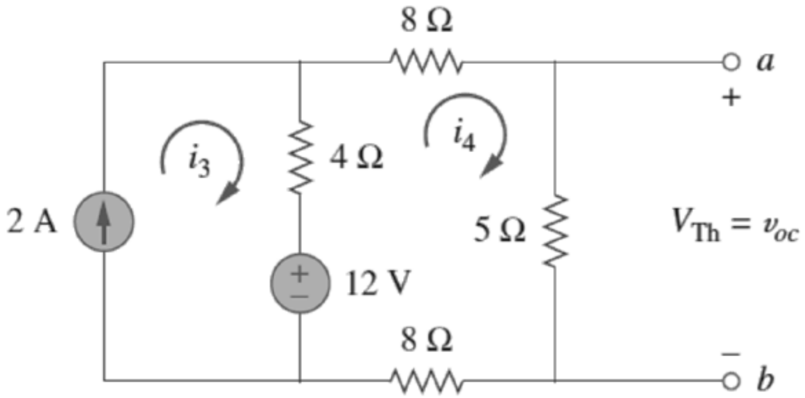
$$i_2 = i_{sc} = I_N = 1A$$

- The Norton circuit can also be obtained as,

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$



- We obtain V_{Th} as the open circuit voltage across terminals a and b in the figure given below:



- Using mesh analysis, we obtain,

$$i_3 = 2A$$

$$25i_4 - 4i_3 - 12 = 0 \Rightarrow i_4 = 0.8A$$

- And,

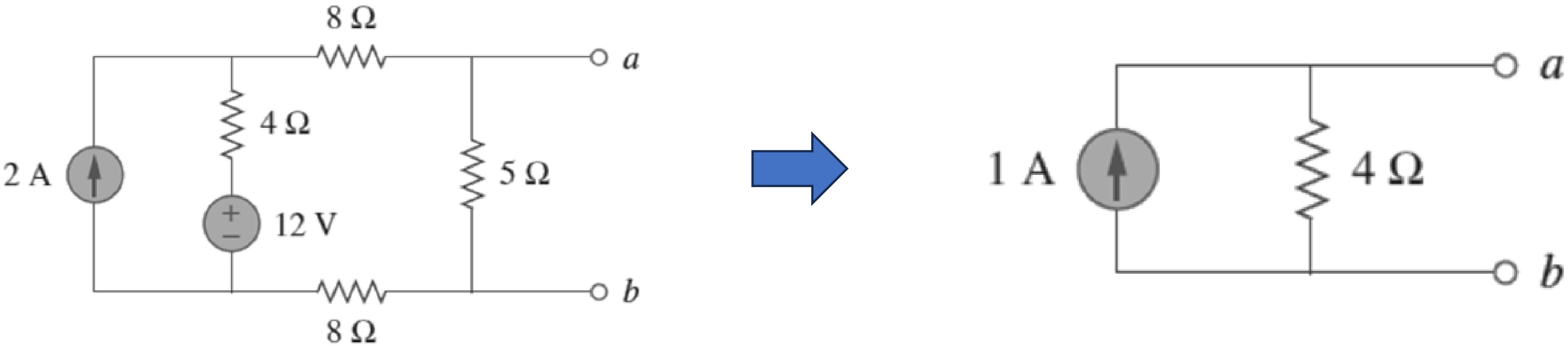
$$v_{oc} = V_{Th} = 5i_4 = 4V$$

• Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1A$$

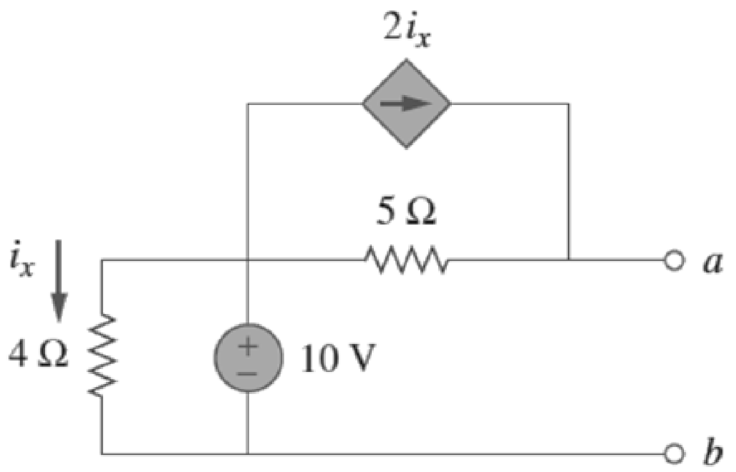
as obtained previously.

Therefore, the Norton's equivalent is -



❑ Example:

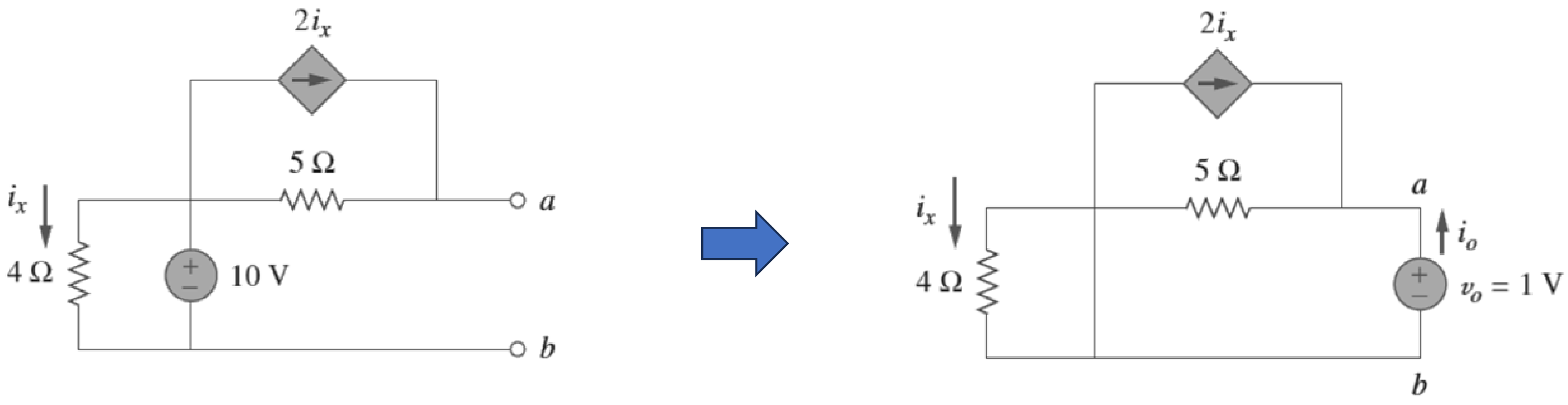
Find the Norton equivalent for the circuit given in the below figure?



Solution: Unlike the previous circuit this circuit has a dependent source.

To find the Norton resistance we set the independent source equal to zero but leave the dependent source as it is.

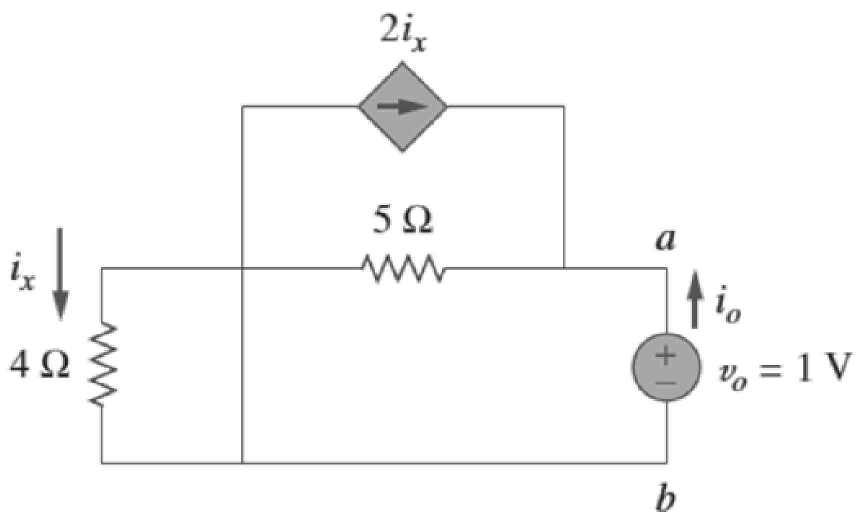
- A voltage source v_0 = 1V (or of any value) is connected across the terminals as shown in the below figure,



- We ignore the 4Ω resistor as it is short circuited.
- Also, due to the short circuit the 5Ω resistor, the voltage source, and the dependent current source are in parallel.

Hence,

$$i_x = 0$$



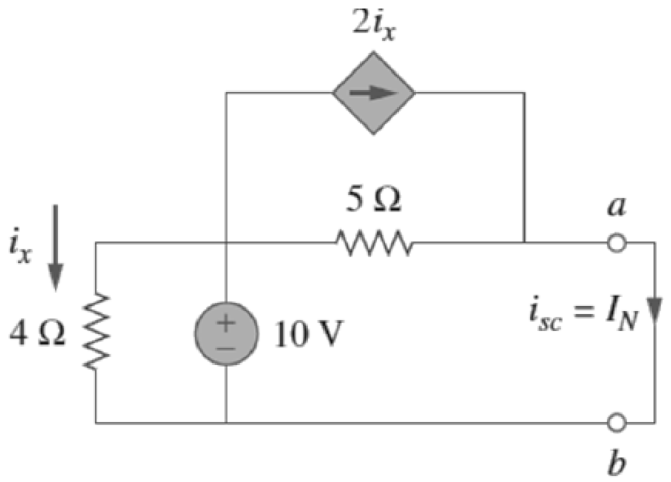
At node a,

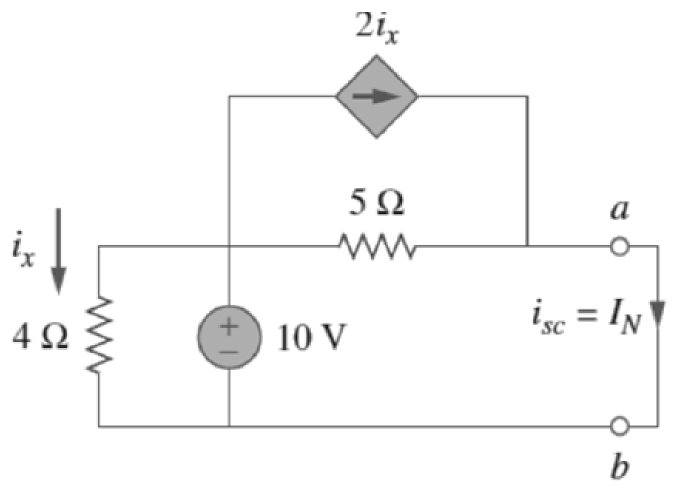
$$i_0 = \frac{1V}{5\Omega} = 0.2A$$

And,

$$R_N = \frac{v_0}{i_0} = \frac{1}{0.2} = 5\Omega$$

- To find I_N, we short circuit the terminals a and b and find the current i_{sc} as indicated in the figure given below:



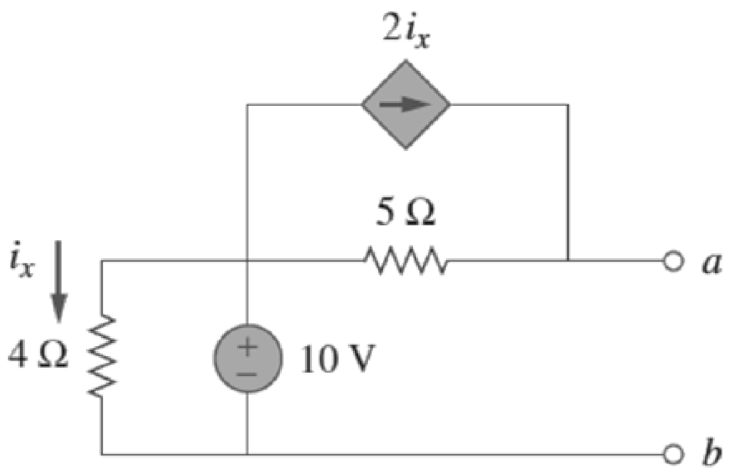


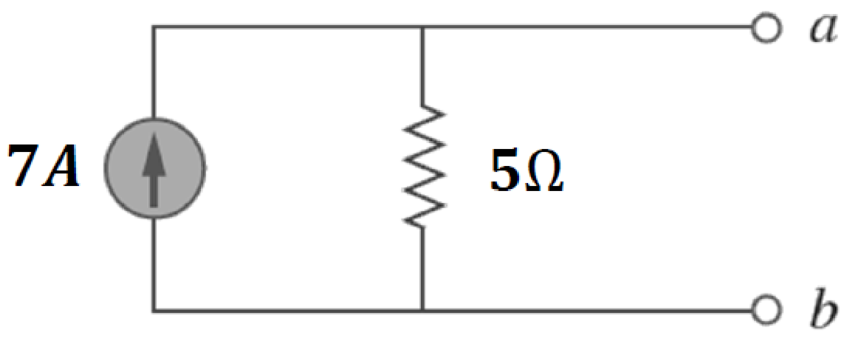
- Note from this figure that the 4Ω resistor, the 10V voltage source, the 5Ω resistor, and the dependent current source are all in parallel.
- Hence,

$$i_x = \frac{10}{4} = 2.5A$$

- At node a KCL gives,

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7A$$


$2i_x$
$5\ \Omega$
i_x
$4\ \Omega$
10 V
a
b

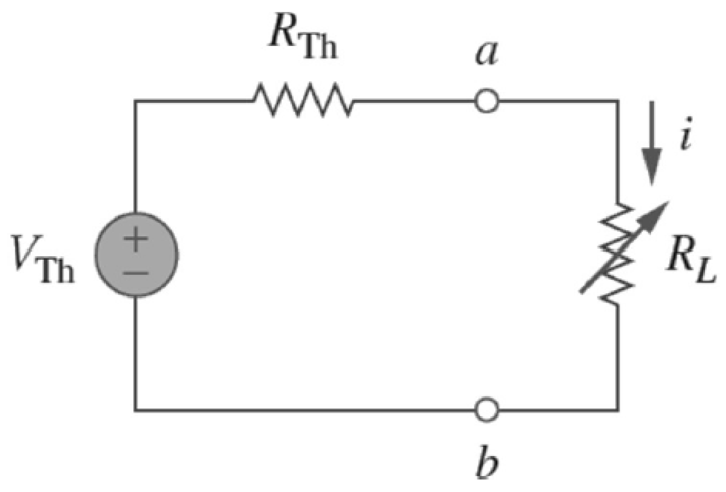

$7A$
5Ω
a
b

Maximum Power Transfer

- In many practical situations, a circuit is designed to provide power to a load.
- There are applications in areas such as communications where it is desirable to maximize the power delivered to a load.
- We need to address the problem of delivering the maximum power to a load when internal losses are known for given system.
- It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

Maximum Power Transfer (cont…)

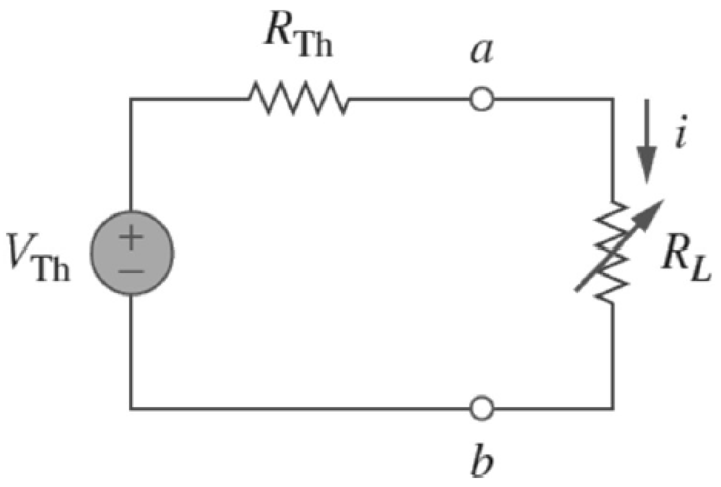
- The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.
- We assume that we can adjust the load resistance R_L.
- Let the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in the figure below.



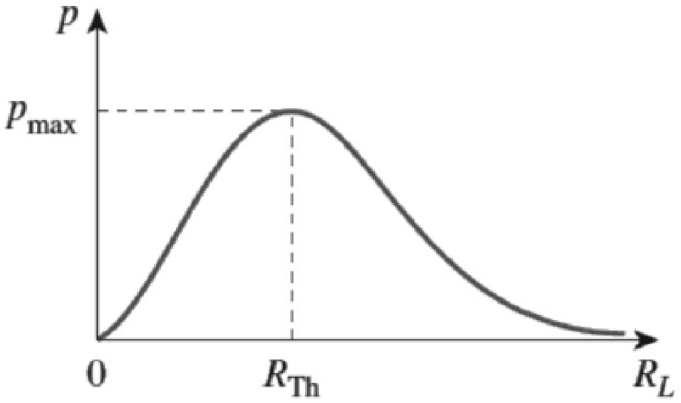
Maximum Power Transfer (Cont…)

- The power delivered to the load in this scenario is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



- For any given circuit the values of R_{Th} and V_{Th} are fixed.
- By varying the load resistance R_L, the power delivered to the load varies as shown in the figure below.



Maximum Power Transfer (Cont…)

- We notice that the power delivered is small for small and large values of R_L but is maximum for some value of R_L in between 0 and ∞.
- We will show that maximum power occurs when the value of $R_L = R_{Th}$.
- This is known as the maximum power transfer theorem.
- Maximum power is transferred to the load when the load resistance equals the Thevenin's resistance as seen from the load.
- To prove maximum power transfer theorem, we differentiate p, as given in the following equation, with respect to R_L and set the result to 0.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Maximum Power Transfer (Cont…)

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

- Differentiating

$$\frac{dp}{dR_L} = \frac{V_{Th}^2[(R_{Th} + R_L) - 2R_L]}{(R_{Th} + R_L)^3}$$

$$= \frac{V_{Th}^2[(R_{Th} + R_L) - 2R_L]}{(R_{Th} + R_L)^3} = 0$$

- Equating the numerator to 0

$$(R_{Th} + R_L) - 2R_L = 0 = R_{Th} - R_L$$

- Therefore,

$$R_{Th} = R_L$$


THANKS
FOR YOUR
ATTENTION