

**ASSIGNMENT 7**  
**MSO-201: PROBABILITY AND STATISTICS**

1. Suppose the PDF of a random variable  $X$  is  $f_X(x) = e^{-x}$ , for  $0 < x < \infty$ , and zero elsewhere. Find the PDF of  $\frac{X}{X+1}$ .
2. Suppose the PDF of a random variable  $X$  is  $f_X(x) = 6x(1-x)$ , for  $0 < x < 1$  and zero elsewhere. Find the mean and variance of  $X$ .
3. If  $\mu$  and  $\sigma^2$  denote the mean and variance of  $X$  find  $P(\mu - 2\sigma < X < \mu + 2\sigma)$  for the problem # 2.
4. If the variance of the random variable  $X$  exists, show that

$$E(X^2) \geq [E(X)]^2.$$

5. Let a random variable  $X$  of the continuous type have a PDF  $f_X(x)$  whose graph is symmetric with respect to  $x = c$ . If the mean value of  $X$  exists, show that  $E(X) = c$ .
6. Find the moment generating function of the random variable  $X$  having the pdf  $f_X(x) = 1/3$ ,  $-1 < x < 2$ , zero elsewhere.
7. Let  $X$  be a random variable such that  $E[(X - b)^2]$  exists for all real  $b$ . Show that  $E[(X - b)^2]$  is a minimum when  $b = E(X)$ .
8. Find the mean and the variance of the distribution that has the CDF

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{8} & \text{if } 0 \leq x < 2 \\ \frac{x^2}{16} & \text{if } 2 \leq x < 4 \\ 1 & \text{if } 4 \leq x. \end{cases}$$

9. Let  $X$  have the PDF  $f_X(x) = \frac{1}{2}e^{-|x|}$ , for  $-\infty < x < \infty$ . Find the MGF of  $X$ .
10. Let  $X$  be a random variable of the continuous type with PDF  $f_X(x)$ , which is positive provided  $0 < x < b < \infty$ , and is equal to zero elsewhere. Show that

$$E(X) = \int_0^\infty (1 - F_X(x))dx,$$

where  $F_X(x)$  is the CDF of  $X$ .