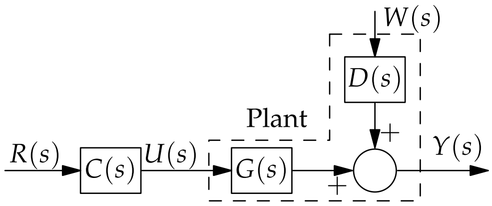
TA # 01, EE 250 (Control System Analysis) - Spring 2025*

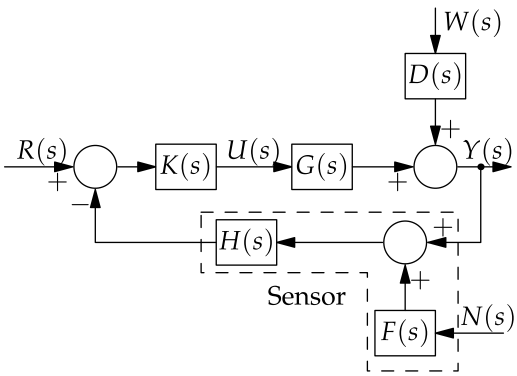
DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

1 Tutorial Problems

Consider the following open-loop system



and the following closed-loop version of this open-loop system:



What do these figures show?

Here, $C(s)$ is a suitable open-loop controller such that the $R(s)$ that is applied to the closed-loop system may also be applied to the open-loop system.

Our objective is that $y(t)$ should respond to the command input $r(t)$. So, for example, if $G(s)$ represents the TF of a PM DC motor, then $y(t)$ may be its shaft speed, while $u(t)$ may be the voltage applied to its armature (upto, say, 100 V), and $r(t)$ may be the small voltage (say 5 V) that will serve as the command input corresponding to $y_{SS} = 1000$ rpm.

So, as you see, we are achieving this objective in two ways — through an OL configuration, and through a CL configuration. In the first, we apply $r(t) = 5 \cdot \mathbb{1}(t)$ and let $y(t)$ go to 1000 rpm. In the second, we apply $r(t) = 5 \cdot \mathbb{1}(t)$ and take a more active approach — we guide $y(t)$ towards 1000 rpm using the negative feedback.

With this background, answer the following questions:

1. **Sensitivity to plant parameter variations:**
   (a) Assuming G changes to $G + \Delta G$, where $\Delta G \ll G$, and assuming that this change results in Y caused by R changing to $Y + \Delta Y$, determine ΔY in each of the two cases in terms of the control system parameters and R.
   (b) Now, assuming that you can suitably choose K, H, and C, ΔY in which case can be made smaller of the two? What is your choice of K, H, C?
   (c) From this analysis, what conclusion can we draw about the robustness and sensitivity of each of the two schemes to plant parameter variations?

   Note that the property of linearity of a system allows us to treat the response of this system to one signal at a time. So, in the above discussion, you can safely assume that W and N are zero.
2. **Sensitivity to disturbances:**
   (a) In each case, determine the gain from W to Y.
   (b) Now, assuming that you can suitably choose K, H, and C, in which case is $y(t)$ less affected by $w(t)$? What is your choice of K, H, C?
   (c) From this analysis, what conclusion can we draw about the robustness and sensitivity of each of the two schemes to disturbances?
3. **Sensitivity to sensor noise:**
   (a) Determine the gain from N (sensor noise) to Y in the closed-loop case.
   (b) Suppose we vary K in order to help the closed-loop system reject W. What effect does this have on the robustness to sensor noise?
   (c) Is there any way to robustify the closed-loop system to sensor noise?
4. **Presence of derivatives in the feedback path:**
   (a) Suppose F is the transfer function of a term that involves the time derivative ($\frac{d}{dt}$). Suppose $n(t) = A \sin 100t$ (with $N(s) = \mathcal{L}\{n(t)\}$). What effect will this have on Y?
   (b) What can you conclude from this about having derivative elements in the feedback path?
5. Overall, what is your assessment of the pros and cons of the negative feedback configuration?

2 Non-Tutorial Problems

These problems will not be discussed in the tutorials.

You should be able to determine the Laplace transform of each of the following functions based on only the lectures. If you find the lectures to be inadequate for any of these items, please inform me.

(1) $\delta(t)$ (2) $\cosh(at)$ (3) $\sinh(at)$ (4) $e^{at}\cos(bt)$ (5) $\mathrm{d}^k f(t)/\mathrm{d}t^k$ knowing only the expression for $\mathfrak{L}_{-}\{\mathrm{d}f/\mathrm{d}t\}$. (6) $\delta(t-c)$ (7) $ty(t)$, given $\mathcal{L}[y(t)] = Y(s)$ (8) $y(t-t_0)\mathbf{1}(t-t_0)$, given that $\mathcal{L}[y(t)] = Y(s)$, $t_0 > 0$

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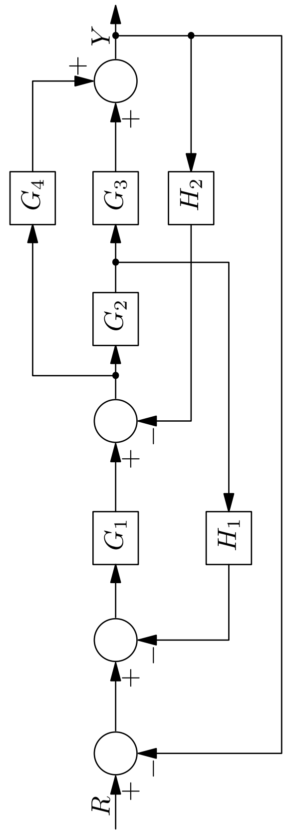
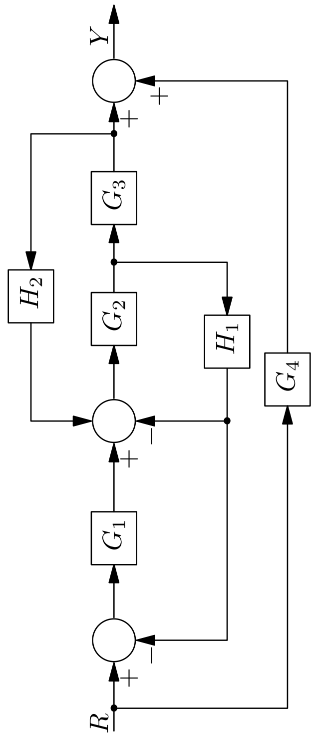
3 Tutorial Problems

1. In each of the following figures, determine the transfer function (TF) $Y(s)/R(s)$ through block diagram manipulation (the figures have been borrowed from [Gop93]):

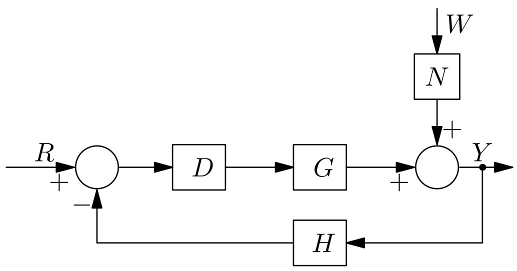




2. The property of linearity is stated as follows:

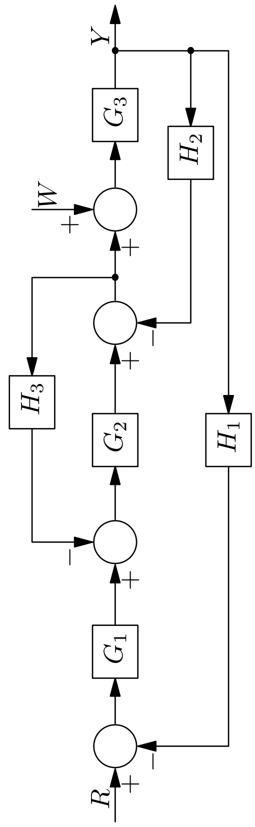
   Given $u \longrightarrow y$, if $u_1 \longrightarrow y_1$ and $u_2 \longrightarrow y_2$, then $\alpha u_1 + \beta u_2 \longrightarrow \alpha y_1 + \beta y_2$, for arbitrary scalars α and β. Here, "$\longrightarrow$" reads "causes".

   This statement seems to require that the point of application of u_1 and u_2 should be the same. In the following block diagram, the points of application of R and W are different, and yet we claim that this block diagram represents a linear system.

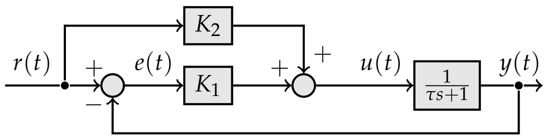


   Can you illustrate, through the example of this block diagram, the following statement from [Che59, page 2]: "We note in passing that the different excitations do not have to be applied on the same part of the system"?

3. The output of the following block diagram is $Y(s) = M(s)R(s) + M_w(s)W(s)$. Here, $M(s) = \left.\frac{Y(s)}{R(s)}\right|_{W(s)=0}$, and $M_w(s) = \left.\frac{Y(s)}{W(s)}\right|_{R(s)=0}$. Find M(s) and $M_w(s)$ through block diagram manipulation (the problem has been borrowed from [Gop93]).



4. **Command feedforward:** Determine the expressions for the unit step responses $y(t)$ and $u(t)$ in the following command feedfoward unity-feedback control system.



   For a given K_1, what should the value of K_2 be for zero steady-state error between $r(t)$ and $y(t)$?

4 Non-Tutorial Problems: GNU Octave

GNU Octave exercises are not discussed in tutorials, unless the tutor wishes. They can be discussed in the M-tutorials, or with me.

1. Copy and paste the code shown below into a file named `lin1.m` that you edit in `Notepad` if you work under MS Windows, and answer the following questions.
   (a) Format the lines until the the words match those written in this file.
   (b) What does this file do?

(c) Which of the words in the file are GNU Octave commands, and which are names of variables?
(d) What does each of the commands do?

Run this file in GNU Octave, and perform the following.

(a) If GNU Octave returns errors, then type `help` followed by the command name at GNU Octave's command prompt, hit the `enter` button of your keyboard, read about the command, and correct the file until GNU Octave does not return any errors when you try to execute this file.
(b) Modify this file minimally so that you now have the plant with the TF $\frac{s^2+2s+3}{s^2+3s+4}$. Run the file, and comment on the resulting plot.
(c) If, instead of a negative feedback, as has been applied in the file, you wish to apply positive feedback, what would be the minimal change you would make?

Check if this file will execute in MATLAB too.

```
% lin1.m: m-file to see the step response of a
% certain closed-loop system
%++++++++++++++++++++++++++++++++++++++++++++

close all
clear all

K = 1; % Controller gain.

nc = K*[1 2]; % Numerator of controller's TF.
dc = [1 10];  % Denominator of controller's TF.

np = 1;       % Numerator of plant's TF.
dp = [1 1 0]; % Denominator of plant's TF.

con = tf(nc,dc);
plant = tf(np,dp);

olsys = series(con,plant);
% olsys is the open-loop system

clsys = feedback(olsys,1,-1)
% clsys is the closed-loop system

step(clsys);
grid
```

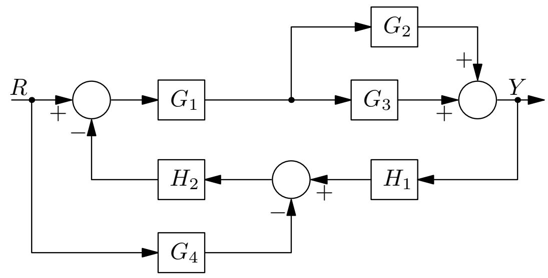
5 Tutorial Problems

1. Draw an SFG for the following linear system assuming u_1 and u_2 as the inputs:

$$x_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1$$
$$x_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2$$
$$x_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$

2. Find x_3/u_1 for the above SFG using Mason's gain formula.

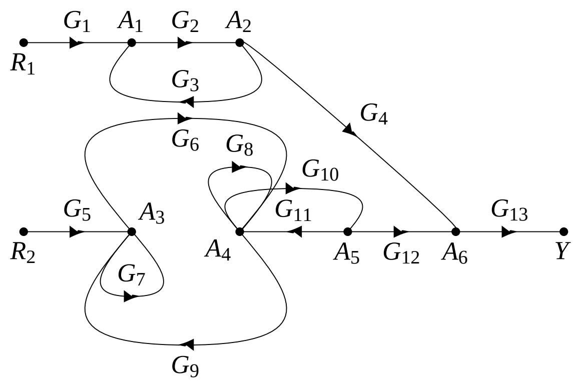
3. Convert the following block diagram to a signal flow graph, and therefrom obtain the input-output transfer function using Mason's gain formula.



What is the loop gain of the block diagram in SFG language?

What is the loop gain of the block diagram in block diagram language?

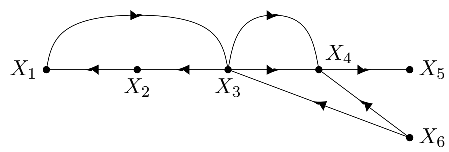
4. For the following signal flow graph, determine the gain Y/R_2 using Mason's rule.



6 Non-Tutorial Problems

1. Which of the following could be the input node(s) of the below graph (circle the most correct answer):

(a) X_1 (b) X_5 (c) X_6 (d) X_1, X_6
(e) X_1, X_2, X_3, X_4, X_6 (f) Any of the nodes

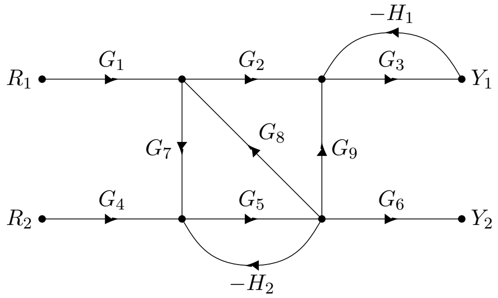


2. Which of the following could be the output node(s) of the above graph (circle the most correct answer):

(a) X_1 (b) X_5 (c) X_6 (d) X_1, X_6
(e) X_1, X_2, X_3, X_4, X_5 (f) Any of the nodes

3. Calculate the following quantities in obtaining the transmittance $Y(s)/R_1(s)$ using Mason's Gain rule for the following graph.

(a) Δ
(b) N
(c) $P_1, P_2, P_3, \ldots$ (whichever exist)
(d) $\Delta_1, \Delta_2, \Delta_3, \ldots$ (whichever exist)



References

[Che59] David K. Cheng. *Analysis of Linear Systems*. Addison-Wesley, 1959. Indian Student Edition, 11th Reprint, 1998, Narosa.

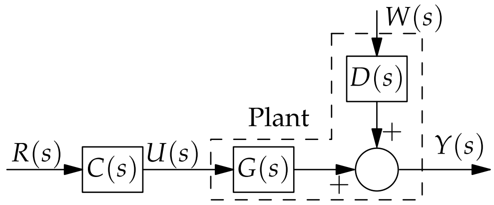
[Gop93] Madan Gopal. *Modern Control System Theory*. New Age International (P) Ltd., New Delhi, India, second edition, 1993. 2003 Reprint.

Solution to TA # 01, EE 250 Spring 2025*

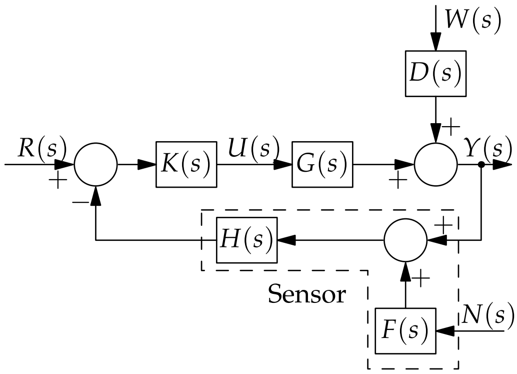
DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR

1 Tutorial Problems: Section 1

Consider the following open-loop system



and the following closed-loop version of this open-loop system:



What do these figures show?

Here, $C(s)$ is a suitable open-loop controller such that the $R(s)$ that is applied to the closed-loop system may also be applied to the open-loop system.

Our objective is that $y(t)$ should respond to the command input $r(t)$. So, for example, if $G(s)$ represents the TF of a PM DC motor, then $y(t)$ may be its shaft speed, while $u(t)$ may be the voltage applied to its armature (upto, say, 100 V), and $r(t)$ may be the small voltage (say 5 V) that will serve as the command input corresponding to $y_{SS} = 1000$ rpm.

So, as you see, we are achieving this objective in two ways — through an OL configuration, and through a CL configuration. In the first, we apply $r(t) = 5 \cdot \mathbb{1}(t)$ and let $y(t)$ go to 1000 rpm. In the second, we apply $r(t) = 5 \cdot \mathbb{1}(t)$ and take a more active approach — we guide $y(t)$ towards 1000 rpm using the negative feedback.

With this background, answer the following questions:

1. **Sensitivity to plant parameter variations:**
   (a) Assuming G changes to $G + \Delta G$, where $\Delta G \ll G$, and assuming that this change results in Y caused by R changing to $Y + \Delta Y$, determine ΔY in each of the two cases in terms of the control system parameters and R.

   In the OL case, $Y + \Delta Y = (G + \Delta G)CR$. $\therefore \Delta Y = \Delta GCR$. In the CL case, $Y + \Delta Y = \frac{K(G+\Delta G)}{1+HK(G+\Delta G)}R \approx \frac{K(G+\Delta G)}{1+HKG}R$. $\therefore \Delta Y = \frac{K\Delta G}{1+HKG}R$.

   (b) Now, assuming that you can suitably choose K, H, and C, ΔY in which case can be made smaller of the two? What is your choice of K, H, C?

   If we choose K and H such that $\mathrm{Re}\{HKG\} \gg 1$, then $\Delta Y = \frac{\Delta G}{1+HG}R$. On the other hand, we do not have much flexibility in the choice of C; its magnitude has to be at least equal to 1 in the example of the PM DC motor. So, we can conclude that ΔY in the CL case is the smaller of the two. Note that, in practice, even H is mostly fixed, and we are free to choose only K.

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(c) From this analysis, what conclusion can we draw about the robustness and sensitivity of each of the two schemes to plant parameter varations?

`The CL scheme is the more robust one, or the less sensitive one, of the two to plant parameter variations.`

Note that the property of linearity of a system allows us to treat the response of this system to one signal at a time. So, in the above discussion, you can safely assume that W and N are zero.

2. **Sensitivity to disturbances:**

(a) In each case, determine the gain from W to Y.

In the OL case, $Y = DW$. In the CL case, $Y = \frac{D}{1+KGH}W$.

(b) Now, assuming that you can suitably choose K, H, and C, in which case is $y(t)$ less affected by $w(t)$? What is your choice of K, H, C?

If we choose K and H such that $\mathrm{Re}\{HKG\} \gg 1$, then $Y = \frac{D}{KGH}W$. On the other hand, we do not have much flexibility in the choice of C, as mentioned in solution 1b.

(c) From this analysis, what conclusion can we draw about the robustness and sensitivity of each of the two schemes to disturbances?

`The CL scheme is more robust, or less sensitive, to disturbances.`

3. **Sensitivity to sensor noise:**

(a) Determine the gain from N (sensor noise) to Y in the closed-loop case.

$Y = \frac{-FHKG}{1+HKG}N$.

(b) Suppose we vary K in order to help the closed-loop system reject W. What effect does this have on the robustness to sensor noise?

If we choose K the way we did in solution 2b, then we have $Y \approx -FN$. This means that the sensor noise can potentially appear without mitigation in Y.

(c) Is there any way to robustify the closed-loop system to sensor noise?

`Choose a very good sensor that will not introduce noise, e.g., one that will be adequately unaffected by EMI. Use a low pass filter, as sensor noise is usually a high-frequency phenomenon. In the absence of either of these two measures, a closed-loop system is not robust to sensor noise, and this is its one drawback.`

4. **Presence of derivatives in the feedback path:**

(a) Suppose F is the transfer function of a term that involves the time derivative ($\frac{d}{dt}$). Suppose $n(t) = A\sin 100t$ (with $N(s) = \mathcal{L}\{n(t)\}$). What effect will this have on Y?

As we saw above, $Y \approx -FN$. So, if F is the LT of a derivative term, then $y(t) \propto \frac{d}{dt}\{n(t)\}$. In this specific example, $y(t) \propto 100A\cos 100t$. So, the derivative element has amplified a possibly small noise a 100 times.

(b) What can you conclude from this about having derivative elements in the feedback path?

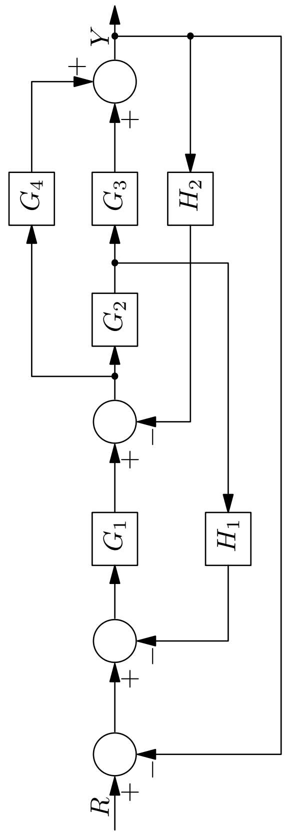
`This is a problem with having dervative elements anywhere in the control system. So, we need to be careful with derivative elements.`

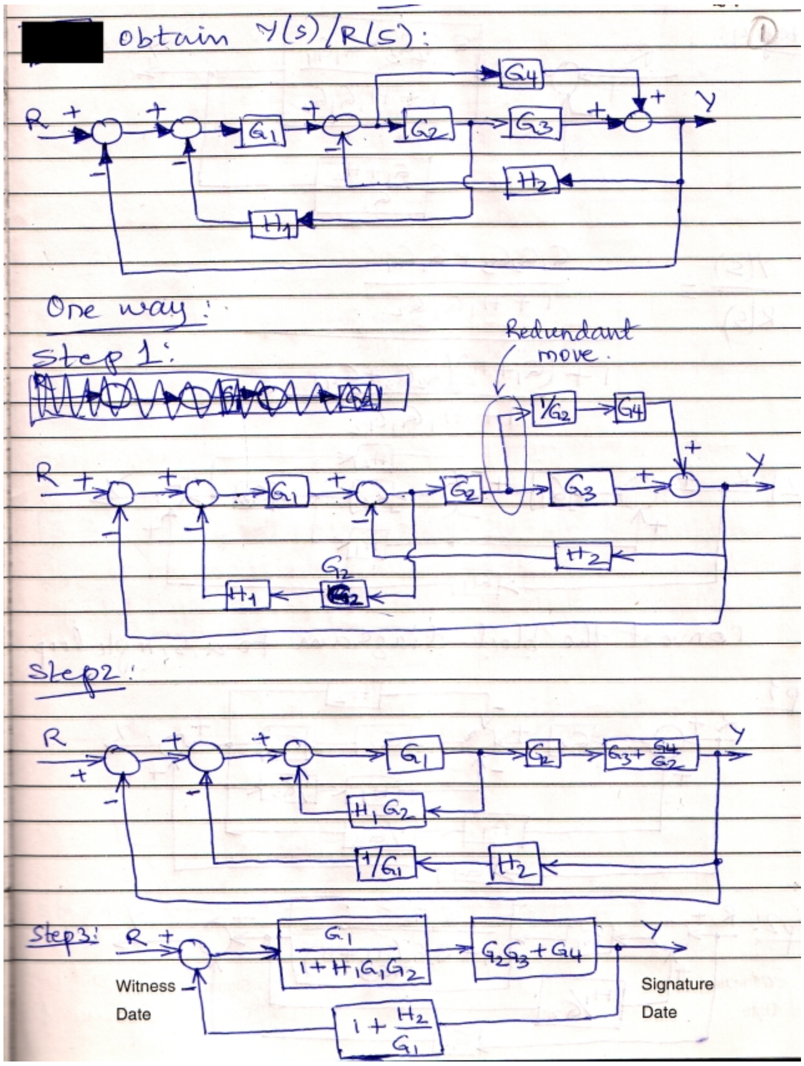
5. Overall, what is your assessment of the pros and cons of the negative feedback configuration?

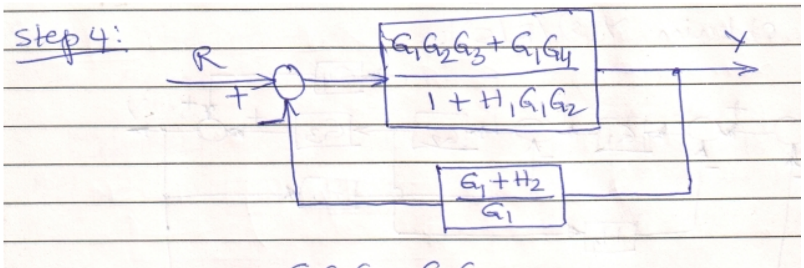
`The negative feedback configuration is useful for disturbance rejection, and for robustness to plant parameter variations, and, as we saw in Lecture 06, for improving the time constant of the system and for stabilizing unstable systems. However, it needs to be used carefully. Feedback can make the system behave poorly if we choose a poor sensor, or if we ignore some derivative elements.`

2 Tutorial Problems: Section 3

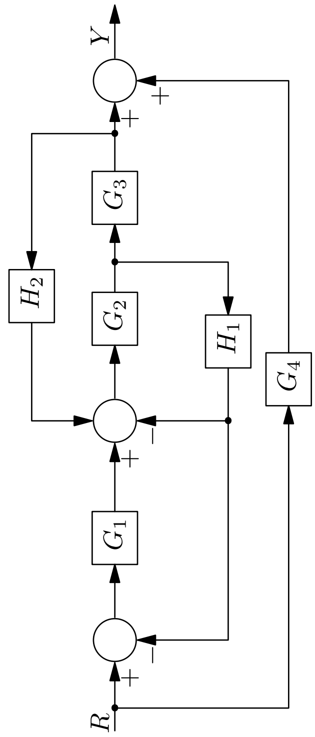
1. In each of the following figures, determine the transfer function (TF) $Y(s)/R(s)$ through block diagram manipulation (the figures have been borrowed from [Gop93]):

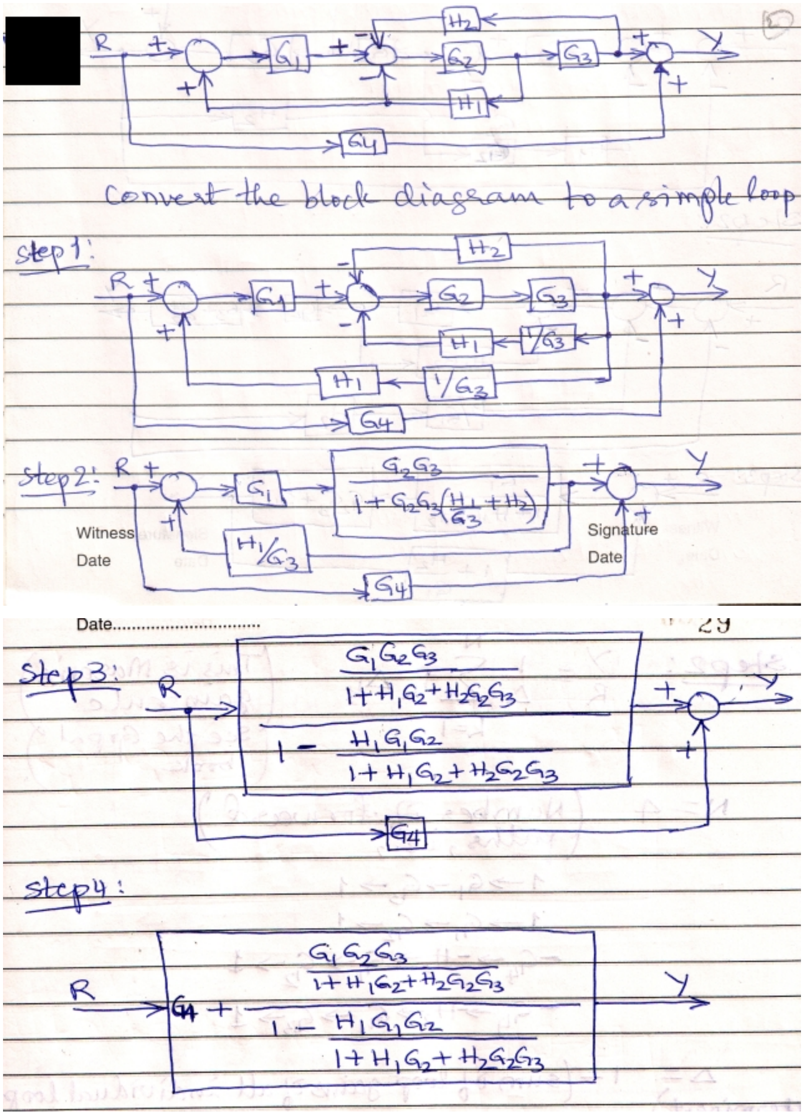






$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+H_1G_1G_2}}{1+\dfrac{(G_1+H_2)(G_2G_3+G_4)}{1+H_1G_1G_2}}$$



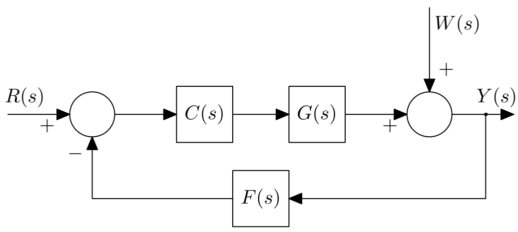


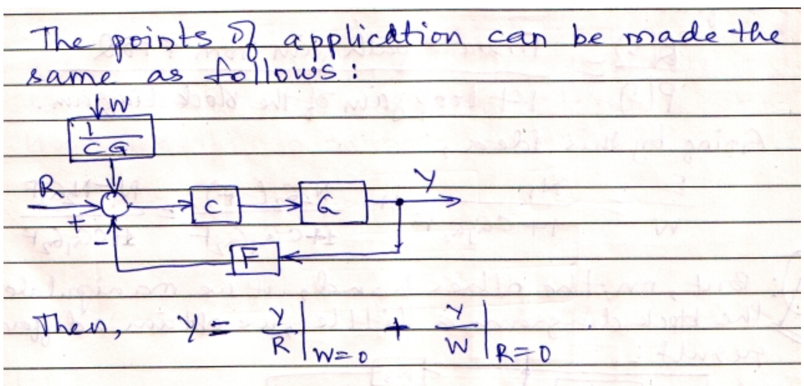
2. The property of linearity is stated as follows:

   Given $u \longrightarrow y$, if $u_1 \longrightarrow y_1$ and $u_2 \longrightarrow y_2$, then $\alpha u_1 + \beta u_2 \longrightarrow \alpha y_1 + \beta y_2$, for arbitrary scalars α and β. Here, "$\longrightarrow$" reads "causes".

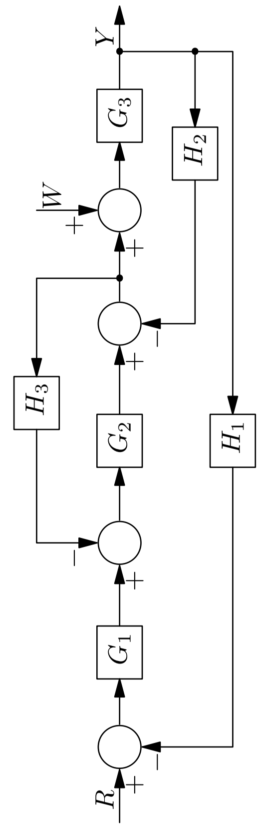
   This statement seems to require that the point of application of u_1 and u_2 should be the same. In the following block diagram, the points of application of R and W are different, and yet we claim that this block diagram represents a linear system.

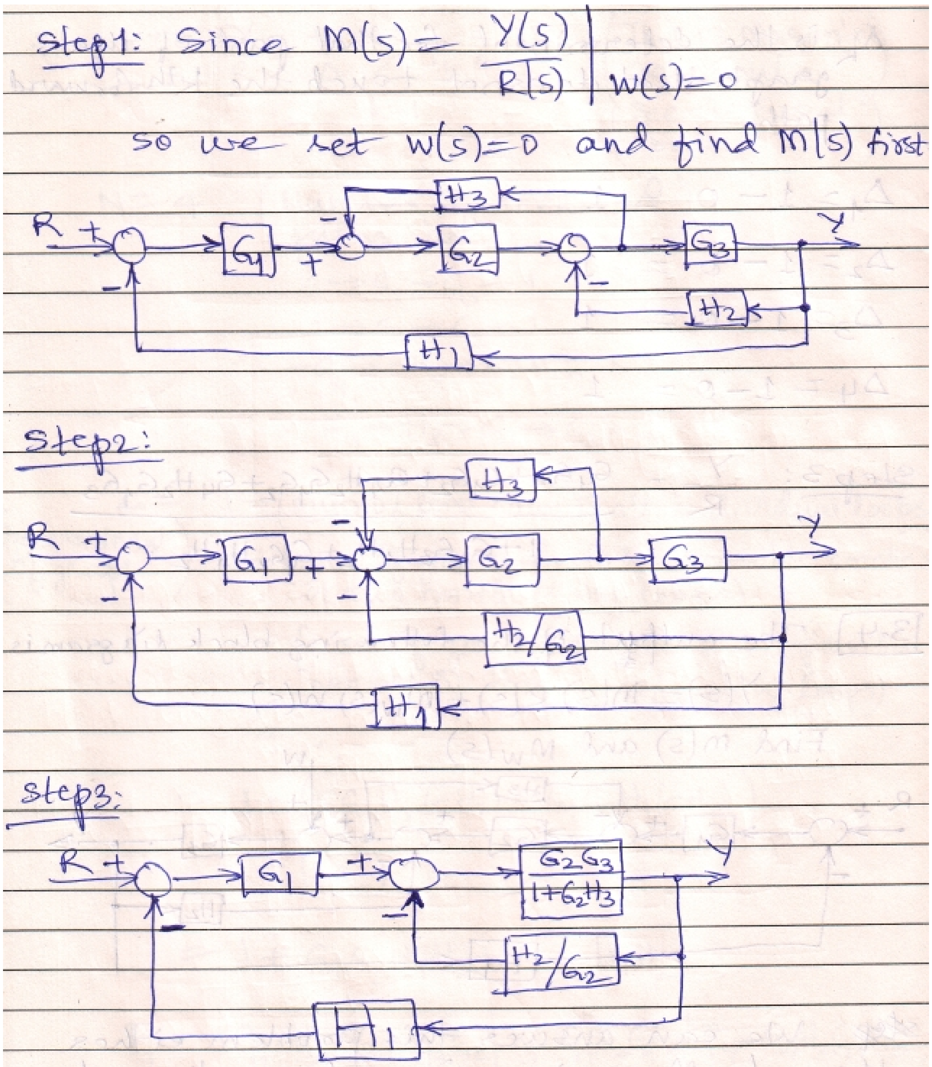
   Can you illustrate, through the example of this block diagram, the following statement "We note in passing that the different excitations do not have to be applied on the same part of the system" [Che59, page 2]?



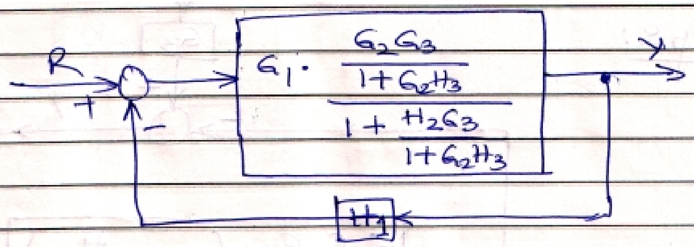


3. The output of the following block diagram is $Y(s) = M(s)R(s) + M_w(s)W(s)$. Here, $M(s) = \left.\frac{Y(s)}{R(s)}\right|_{W(s)=0}$, and $M_w(s) = \left.\frac{Y(s)}{W(s)}\right|_{R(s)=0}$. Find M(s) and $M_w(s)$ through block diagram manipulation (the problem has been borrowed from [Gop93]).





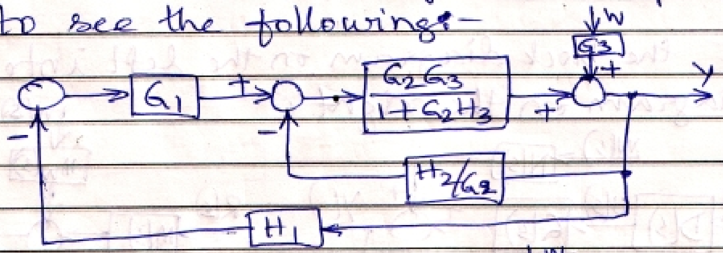
Step 4:



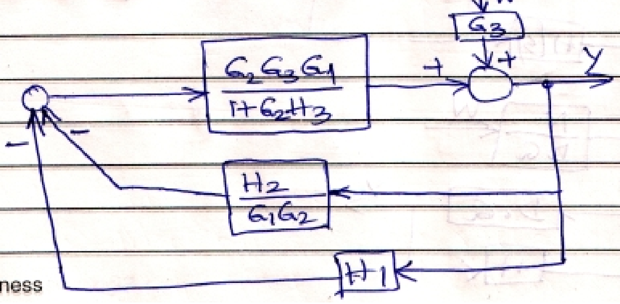
Step 5: $$\left.\frac{Y(s)}{R(s)}\right|_{W(s)=0} = M(s) = \frac{\dfrac{G_1G_2G_3}{1+G_2H_3+H_2G_3}}{1+\dfrac{G_1G_2G_3H_1}{1+G_2H_3+H_2G_3}}$$

Step 6: $$M_W(s) = \left.\frac{Y(s)}{W(s)}\right|_{R(s)=0}$$ This too needs some block diagram manipulations.

We can utilize step1 and step2 and step3 to see the following:-



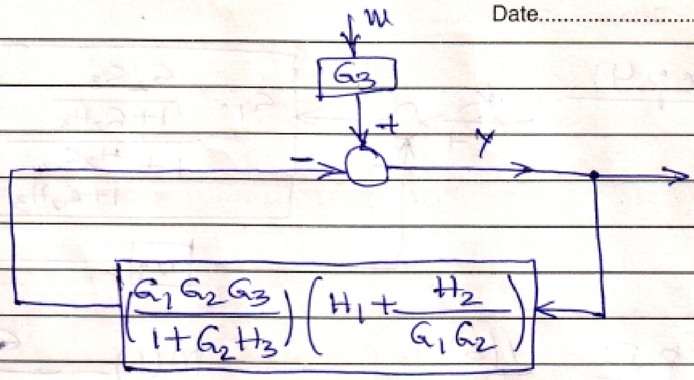
Step 7:



Witness Signature

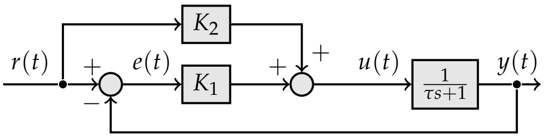
34 Date..............................

Step 8:



Step 9: $$\left.\frac{Y(s)}{W(s)}\right|_{R(s)=0} = M_W(s) = \frac{G_3}{1+\left(\dfrac{G_1G_2G_3}{1+G_2H_3}\right)\left(H_1+\dfrac{H_2}{G_1G_2}\right)}$$

4. **Command feedforward:** Determine the expressions for the unit step responses $y(t)$ and $u(t)$ in the following command feedfoward unity-feedback control system.



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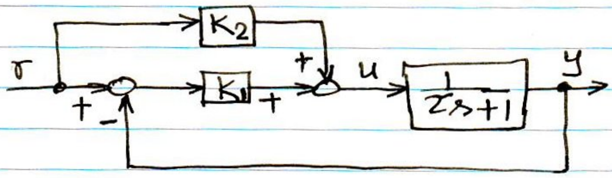


$$\frac{Y(s)}{R(s)} = \frac{(K_1+K_2)\cdot\frac{1}{\tau s+1}}{1+\frac{K_1}{\tau s+1}} = \frac{K_1+K_2}{\tau s+1+K_1}$$

$$= \frac{K_1+K_2}{\tau}\cdot\frac{1}{s+\frac{1+K_1}{\tau}}.$$

With $R(s)=\frac{1}{s}$, we have:

$$Y(s) = \frac{K_1+K_2}{\tau}\cdot\frac{1}{s+a}\cdot\frac{1}{s}, \text{ where } a=\frac{1+K_1}{\tau}.$$

$$= \frac{K_1+K_2}{a\tau}\left[\frac{-1}{s+a}+\frac{1}{s}\right] \qquad (*)$$

$$\Rightarrow y(t) = \frac{K_1+K_2}{a\tau}\left[1-e^{-at}\right]1(t)$$

$$= \frac{K_1+K_2}{K_1+1}\left[1-e^{-at}\right]1(t), \quad a=\frac{1+K_1}{\tau}.$$

For zero steady-state error, choose $K_2=1$.

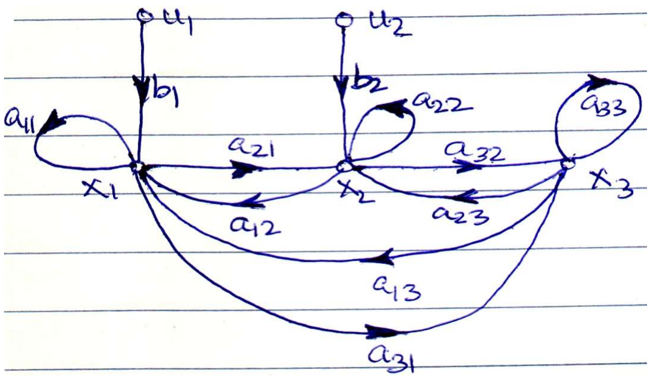
Note that the steady-state value of $y(t)$ could also have been determined from $(*)$ using final value thm.

$$y(\infty) = \lim_{s\to 0} sY(s) = \frac{K_1+K_2}{a\tau} = \frac{K_1+K_2}{K_1+1}.$$

3 Tutorial Problems: Section 5

1. Draw an SFG for the following linear system assuming u_1 and u_2 as the inputs:

$$
\begin{aligned}
x_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1 \\
x_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2 \\
x_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3
\end{aligned}
$$



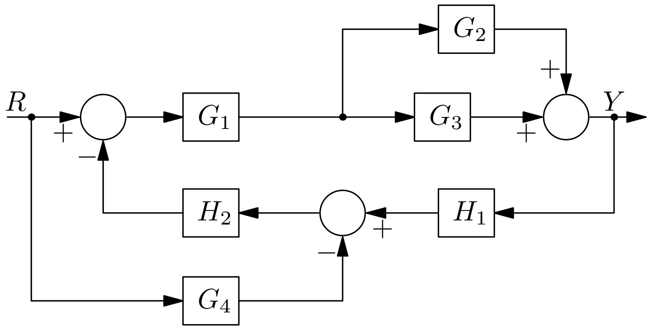
2. Find x_3/u_1 for the above SFG using Mason's gain formula.

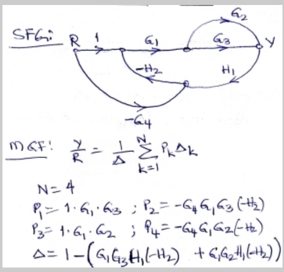
$$\frac{x_3}{u_1} = \frac{1}{\Delta}\sum_{k=1}^{N} P_k\Delta_k$$

where,

$$
\begin{aligned}
N &= 2 \\
P_1 &= b_1a_{21}a_{32} \\
P_2 &= b_1a_{31} \\
\Delta &= 1 - (a_{11} + a_{21}a_{12} + a_{22} + a_{32}a_{22} + a_{33} + \\
&\qquad a_{21}a_{32}a_{13} + a_{31}a_{23}a_{13} + a_{31}a_{13}) \\
&\quad + (a_{11}a_{22} + a_{11}a33 + a_{22}a_{33} + \\
&\qquad a_{11}a_{32}a_{23} + a_{21}a_{12}a_{33} + a_{22}a_{33}) \\
&\quad - (a_{11}a_{22}a_{33}) \\
\Delta_1 &= 1 - 0 = 1 \\
\Delta_2 &= 1 - a_{22}
\end{aligned}
$$

3. Convert the following block diagram to a signal flow graph, and therefrom obtain the input-output transfer function using Mason's gain formula.





MGF: $\frac{Y}{R} = \frac{1}{\Delta}\sum_{k=1}^{N} P_k \Delta_k$

$N = 4$

$P_1 = 1\cdot G_1 \cdot G_3$; $P_2 = -G_4 G_1 G_3 (-H_2)$

$P_3 = 1 \cdot G_1 \cdot G_2$; $P_4 = -G_4 G_1 G_2 (-H_2)$

$\Delta = 1 - \left(G_1 G_3 H_1 (-H_2) + G_1 G_2 H_1 (-H_2)\right)$

$\Delta_1 = 1, \quad \Delta_2 = 1, \quad \Delta_3 = 1, \quad \Delta_4 = 1.$

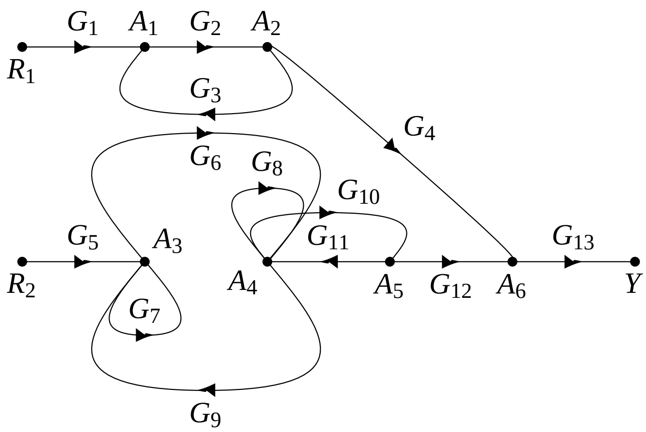
What is the loop gain of the block diagram in SFG language?

$$G(G_2 + G_3)H_1(-H_2)$$

What is the loop gain of the block diagram in block diagram language?

$$G(G_2 + G_3)H_1 H_2$$

4. For the following signal flow graph, determine the gain Y/R_2 using Mason's rule.



$$N = 1$$
$$P_1 = G_5 G_6 G_{10} G_{12} G_{13}$$
$$\Delta_1 = 1 - 0 = 1$$
$$\Delta = 1 - (G_7 + G_8 + G_{10}G_{11} + G_9 G_6) + (G_7 G_8 + G_7 G_{10} G_{11})$$
$$\frac{Y}{R_2} = \frac{P_1 \Delta_1}{\Delta}$$

References

[Che59] David K. Cheng. *Analysis of Linear Systems*. Addison-Wesley, 1959. Indian Student Edition, 11$^{\text{th}}$ Reprint, 1998, Narosa.

[Gop93] Madan Gopal. *Modern Control System Theory*. New Age International (P) Ltd., New Delhi, India, second edition, 1993. 2003 Reprint.