

Transmission Lines - III

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Attenuation Constant from Power Relations

 The power flow along a lossy transmission line, in the absence of the reflection, may be written using the following expression:

$$P(z) = P_0 e^{-2\alpha z}$$

$$\alpha = \frac{\mathcal{P}_l(z)}{2P(z)} = \frac{\mathcal{P}_l(z=0)}{2P_0} \quad (\text{Np/m})$$

(Neper)*

*John Napier, who first proposed the use of algorithm.

$$V(z) = V_0 e^{-\alpha z}$$

$$\text{Voltage attenuation} \equiv \left| \frac{V_0}{V(z)} \right| = e^{\alpha z}$$

$$\alpha z = \ln \left| \frac{V_0}{V(z)} \right| \quad (\text{Np})$$

$$\text{Power attenuation} \equiv \left| \frac{P_0}{P(z)} \right| = e^{2\alpha z}$$

$$\equiv \frac{1}{2} \ln \left| \frac{P_0}{P(z)} \right| \quad (\text{Np})$$

☞ Logarithmic ratio of two signal amplitudes

$$\text{Voltage Gain dB} = 20 \log_{10} \left(\frac{v_{out}}{v_{in}} \right)$$

$$\text{Power Gain dB} = 10 \log_{10} \left(\frac{P_{out}}{P_{in}} \right)$$

➤ Decibel (dB) is primarily used to compress data

$$10^8 \equiv 20 \log 10^8 \equiv 160 \text{ dB}$$

☞ Ratio of two quantities



Attenuation Constant from Power Relations

$$P(z) = P_0 e^{-2\alpha z}$$

$$\text{Power Loss} = \ln \left[\frac{P_0}{P(z)} \right] \equiv 2\alpha z$$

$$\alpha z \equiv \frac{1}{2} \ln \left[\frac{P_0}{P(z)} \right] \quad (\text{Np})$$

$$\text{Power Loss dB} = 10 \log_{10} \left[\frac{P_0}{P(z)} \right] \equiv 10 \log_e \left[\frac{P_0}{P(z)} \right] \times \log_{10} [e]$$

$$\text{Power Loss dB} = 2 \times 10 \left\{ \frac{1}{2} \log_e \left[\frac{P_0}{P(z)} \right] \right\} \times \log_{10} [e]$$

$$\text{Loss dB} = 20 \log_{10} [e] \times \text{Loss Np}$$

$$dB \equiv 20 \log_{10} [e] \times Np$$

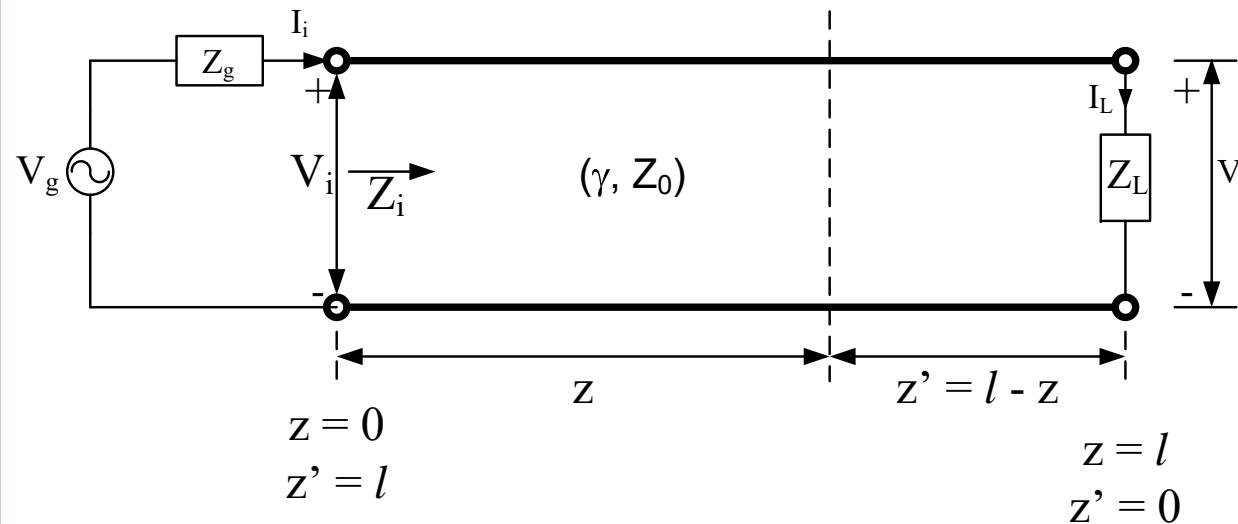
$$dB/m \equiv 8.686 \times Np/m$$

$$\text{dBm} \equiv \text{dB mW} \equiv 10 \log_{10} \left(\frac{\text{Watts}}{1\text{mW}} \right)$$

$$1 \text{ mW} \equiv 10 \log_{10} \left(\frac{1 \text{ mW}}{1 \text{ mW}} \right) = 0 \text{ dBm}$$



Wave Characteristic on a Finite Transmission Line



$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

$$\left(\frac{V}{I}\right)_{z=l} = \frac{V_L}{I_L} = Z_L$$

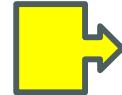


Reflected waves would be there for the unmatched case.

□ The line is said to be **matched**, when $Z_L = Z_0$

$$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$$

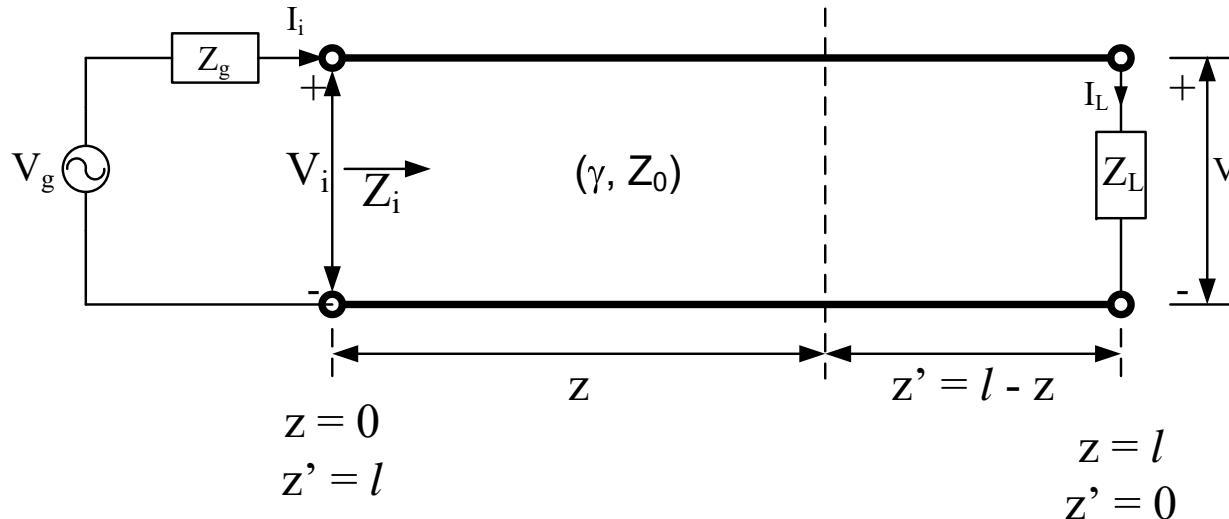
$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l}$$



$$V_0^+ = \frac{1}{2} (V_L + I_L Z_0) e^{\gamma l}$$

$$V_0^- = \frac{1}{2} (V_L - I_L Z_0) e^{-\gamma l}$$

Wave Characteristic on a Finite Transmission Line



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\frac{V_L}{I_L} = Z_L$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

$$V_0^+ = \frac{1}{2} (v_L + I_L Z_0) e^{\gamma l}$$

$$V_0^- = \frac{1}{2} (v_L - I_L Z_0) e^{-\gamma l}$$

$$I_0^+ = \frac{1}{2 Z_0} (v_L + I_L Z_0) e^{\gamma l}$$

$$I_0^- = -\frac{1}{2 Z_0} (v_L - I_L Z_0) e^{-\gamma l}$$

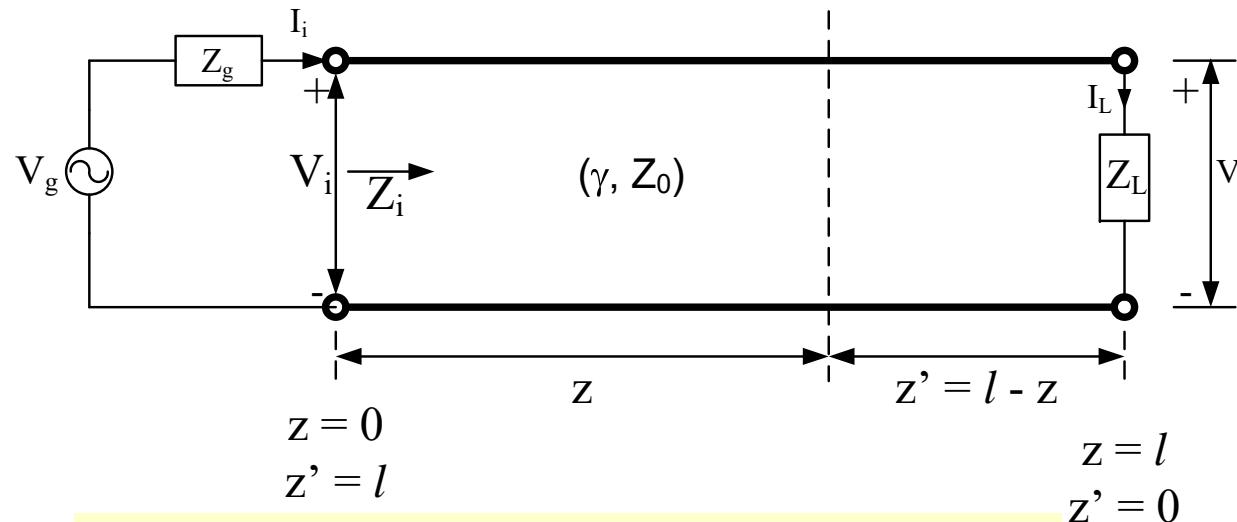
$$V(z) = \frac{1}{2} [(v_L + I_L Z_0) e^{\gamma(l-z)} + (v_L - I_L Z_0) e^{-\gamma(l-z)}]$$

$$I(z) = \frac{1}{2 Z_0} [(v_L + I_L Z_0) e^{\gamma(l-z)} - (v_L - I_L Z_0) e^{-\gamma(l-z)}]$$

$$V(z) = \frac{1}{2} [(I_L Z_L + I_L Z_0) e^{\gamma(l-z)} + (I_L Z_L - I_L Z_0) e^{-\gamma(l-z)}]$$

$$I(z) = \frac{1}{2 Z_0} [(I_L Z_L + I_L Z_0) e^{\gamma(l-z)} - (I_L Z_L - I_L Z_0) e^{-\gamma(l-z)}]$$

Wave Characteristic on a Finite Transmission Line



$$\frac{V_L}{I_L} = Z_L$$

$$z' = l - z$$

Distance from the Load

$$V(z) = \frac{I_L}{2} [(z_L + z_0)e^{\gamma(l-z)} + (z_L - z_0)e^{-\gamma(l-z)}]$$

$$I(z) = \frac{I_L}{2Z_0} [(z_L + z_0)e^{\gamma(l-z)} - (z_L - z_0)e^{-\gamma(l-z)}]$$

$$V(z') = \frac{I_L}{2} [(z_L + z_0)e^{\gamma z'} + (z_L - z_0)e^{-\gamma z'}]$$

$$I(z') = \frac{I_L}{2Z_0} [(z_L + z_0)e^{\gamma z'} - (z_L - z_0)e^{-\gamma z'}]$$

$$Z(z') \equiv \frac{V(z')}{I(z')} = Z_0 \frac{[z_L(e^{\gamma z'} + e^{-\gamma z'}) + z_0(e^{\gamma z'} - e^{-\gamma z'})]}{[z_0(e^{\gamma z'} + e^{-\gamma z'}) + z_L(e^{\gamma z'} - e^{-\gamma z'})]}$$

$$= Z_0 \frac{[z_L \cosh \gamma z' + z_0 \sinh \gamma z']}{[z_0 \cosh \gamma z' + z_L \sinh \gamma z']}$$

$$Z(z') \equiv \frac{V(z')}{I(z')} = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'}$$

At source end, $z' = l$, the line can be modeled as the input impedance.

Input impedance

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

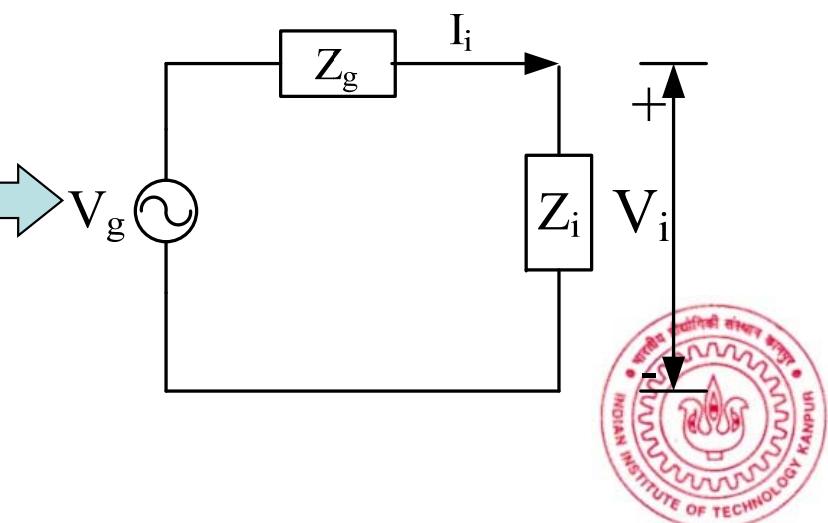
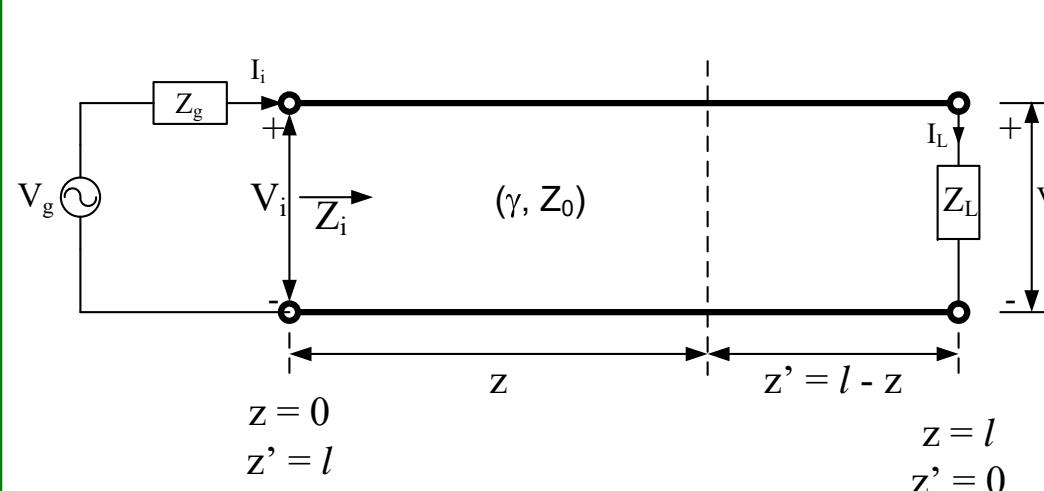
$$V_i = V_g \frac{Z_i}{Z_i + Z_g}$$

$$I_i = \frac{V_g}{Z_i + Z_g}$$

$$P_{avg,i} = \frac{1}{2} \Re e [V_i I_i^*]_{z=0, z'=l}$$

$$P_{avg,L} = \frac{1}{2} \Re e [V_L I_L^*]_{z=l, z'=0}$$

$$\equiv \frac{1}{2} \left| \frac{V_L}{Z_L} \right|^2 R_L$$



Input impedance

$$Z(z') = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'}$$

If $Z_L = Z_0$

$$z' = l \quad Z_{in} = Z_0$$

$$Z(z') = Z_0$$

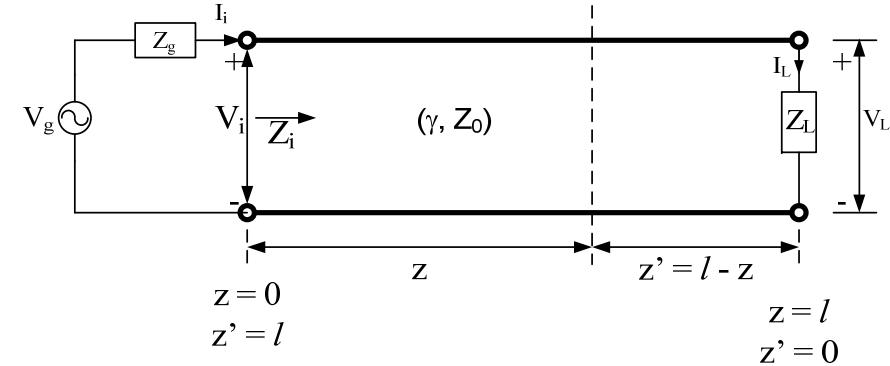
$$V(z) = I_L Z_0 e^{\gamma(l-z)}$$

$$I(z) = I_L e^{\gamma(l-z)}$$



$$V(z) = V_i e^{-\gamma z}$$

$$I(z) = I_i e^{-\gamma z}$$



$$V(z) = \frac{I_L}{2} [(z_L + z_0)e^{\gamma(l-z)} + (z_L - z_0)e^{-\gamma(l-z)}]$$

$$I(z) = \frac{I_L}{2Z_0} [(z_L + z_0)e^{\gamma(l-z)} - (z_L - z_0)e^{-\gamma(l-z)}]$$

- When a finite transmission line is **terminated with its own characteristic impedance** (the finite transmission line is **matched**), the voltage and current distribution on the line are the same as though the line has been extended to **infinity**.

Transmission Lines as Circuit Elements

Frequency	Wavelength
300 MHz	1 m
3 GHz	0.1 m
30 GHz	0.01 m

⌚ It is quite difficult to make Lumped circuit elements at higher frequencies.



Input impedance

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

⌚ For a lossless transmission line

$$\gamma = j\beta \quad Z_0 = R_0$$

$$\begin{aligned} \tanh \gamma l &= \tanh(j\beta l) \\ &= j \tan \beta l \end{aligned}$$

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$



Transmission Lines as Circuit Elements

Case-1: Open Circuit Termination

$$Z_L \rightarrow \infty$$

$$Z_{in} = R_0 \frac{1 + \frac{jR_0}{Z_L} \tan \beta l}{\frac{R_0}{Z_L} + j \tan \beta l}$$

$$Z_{io} = -jR_0 \cot \beta l \quad \text{Reactive}$$

$$Z_{io} = jX_{io} = -jR_0 \cot \beta l$$

$$\beta l = \frac{2\pi}{\lambda} l$$

$\beta l \ll 1$

Electrically short line

$$Z_{io} = jX_{io} = -j \frac{R_0}{\tan \beta l}$$

$$\approx -j \frac{R_0}{\beta l}$$

$$\equiv -j \frac{1}{\omega C l}$$

Impedance of a capacitor of $C l$ Farads.

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

