

Notational Convention

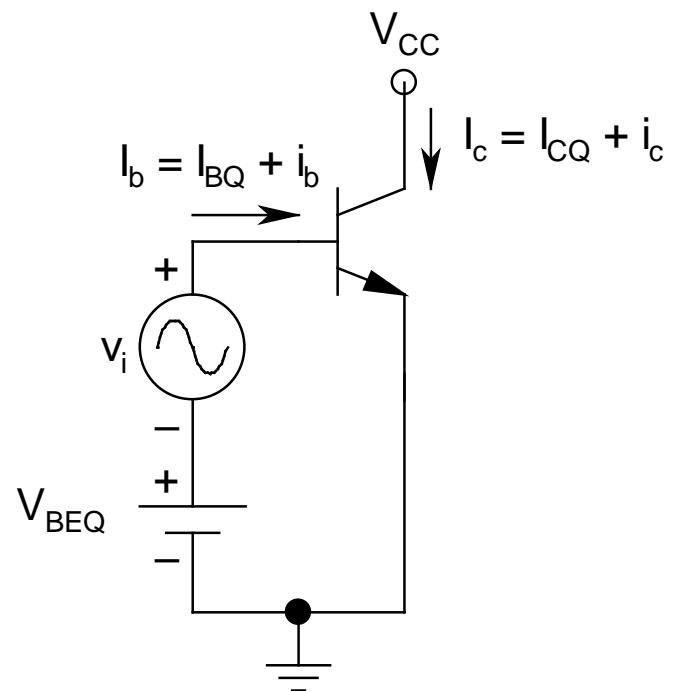
- *Pure DC quantities:*
 - *Capital letter with capital subscript* (e.g., V_{BE})
- *Pure ac quantities:*
 - *Small-case letter with small-case subscript*
(e.g., v_{be})
- *Instantaneous (DC + ac) quantities:*
 - Either *capital letter with small-case subscript*
(e.g., V_{be}) or *small-case letter with capital subscript* (e.g., v_{BE})

Small-Signal Model

- The *electrical equivalent* of the BJT at the *DC bias point*
- Basically an *electrical network*, having *passive and active elements*
- To obtain this model, *DC analysis* is needed, since the *information* regarding the *Q-point* (I_C , V_{CE}) is necessary
- *This model for npn and pnp BJT is same*

Validity of the Small-Signal Model

- Basically *linearization* of the *operating region* around the *Q-point*
- *This linearization should not contain any higher-order terms*



- To start with, assume $V_A \rightarrow \infty$

➤ $I_{CQ} = I_S \exp(V_{BEQ}/V_T)$

- Thus:

$$\begin{aligned} I_c &= I_S \exp\left(\frac{V_{be}}{V_T}\right) = I_S \exp\left(\frac{V_{BEQ} + V_i}{V_T}\right) \\ &= I_S \exp\left(\frac{V_{BEQ}}{V_T}\right) \exp\left(\frac{V_i}{V_T}\right) = I_{CQ} \exp\left(\frac{V_i}{V_T}\right) \end{aligned}$$

- Expand the ***exponential term*** in series:

$$\Rightarrow I_c = I_{CQ} \left[1 + \frac{V_i}{V_T} + \frac{1}{2!} \left(\frac{V_i}{V_T} \right)^2 + \frac{1}{3!} \left(\frac{V_i}{V_T} \right)^3 + \dots \right]$$

- Thus:

$$\Rightarrow i_c = I_c - I_{CQ} = I_{CQ} \left[\frac{V_i}{V_T} + \frac{1}{2!} \left(\frac{V_i}{V_T} \right)^2 + \frac{1}{3!} \left(\frac{V_i}{V_T} \right)^3 + \dots \right]$$

- ***True linearization*** of i_c - v_i relation ***can be achieved*** only if ***all higher-order terms*** can be ***neglected*** $\Rightarrow v_i$ should be $\ll V_T$

Small-Signal Model Parameters

- *Incremental Emitter Resistance* (r_E):

$$r_E = \left(\frac{\dot{i}_e}{v_i} \right)^{-1} = \left(\frac{\Delta I_E}{\Delta V_{BE}} \right)^{-1} \equiv \left(\frac{dI_E}{dV_{BE}} \right)^{-1} \Bigg|_{V_{CE} \text{ constant}} = \frac{V_T}{I_E}$$

- *Transconductance* (g_m):

$$g_m = \frac{\dot{i}_c}{v_i} = \frac{\Delta I_C}{\Delta V_{BE}} \equiv \frac{dI_C}{dV_{BE}} \Bigg|_{V_{CE} \text{ constant}} = \frac{I_C}{V_T}$$

- Thus, $g_m r_E = I_C/I_E = \alpha \approx 1$
- *A frequently used approximation:*
 - $g_m = 1/r_E$
- For $I_C = 1$ mA:
 - $r_E = 26 \Omega$ and $g_m = 1/26 \text{ A/V}$
- As $I_C \uparrow$:
 - $g_m \uparrow$ and $r_E \downarrow$
 - Also $P_D \uparrow$
- Gain = $f(g_m)$
 - ⇒ For *higher gain*, the circuit has to be fed *more power*

- ***Base-Emitter Resistance*** (r_π):

$$r_\pi = \frac{v_i}{i_b} = \frac{\Delta V_{BE}}{\Delta I_B} \equiv \left. \frac{dV_{BE}}{dI_C} \frac{dI_C}{dI_B} \right|_{V_{CE} \text{ constant}} = \frac{\beta}{g_m} \simeq \beta r_E$$

➤ For $I_C = 1 \text{ mA}$ and $\beta = 100$: $r_\pi = 2.6 \text{ k}\Omega$

- ***Output Resistance*** (r_0):

$$r_0 = \frac{v_{ce}}{i_c} = \left[\frac{dI_C}{dV_{CE}} \right]^{-1} \left|_{V_{BE} \text{ constant}} \right. = \frac{V_A}{I_C} = \frac{V_A}{V_T} \frac{V_T}{I_C} = \frac{1}{\eta g_m}$$

- For $I_C = 1 \text{ mA}$, $V_{AN} = 130 \text{ V}$, and $V_{AP} = 52 \text{ V}$:
 $r_0(\text{npn}) = 130 \text{ k}\Omega$ and $r_0(\text{pnp}) = 52 \text{ k}\Omega$
- $\eta (= V_T/V_A)$: 2×10^{-4} (npn) and 5×10^{-4} (pnp)
- $g_m r_o = \eta^{-1}$

- ***Collector-Base Resistance* (r_μ):**

$$r_\mu = \frac{V_{ce}}{i_b} = \left. \frac{\Delta V_{CE}}{\Delta I_B} \right|_{V_{BE} \text{ constant}} = \frac{dV_{CE}}{dI_C} \frac{dI_C}{dI_B} = \beta r_0$$

- *Oversimplification – actual value much higher* ($\sim 5\text{-}10\beta r_0$) $> 100\text{s of M}\Omega$

- **Emitter-Base Capacitance** (C_π):

$$C_\pi = C_{je} + C_b$$

➤ C_{je} : **Emitter-base depletion capacitance**

$$\approx 2C_{je0}$$

- C_{je0} : **Emitter-base depletion capacitance at zero bias**

➤ C_b : **Emitter-base diffusion capacitance**

(known as **base charging capacitance**)

$$= \tau_F g_m \quad (>> C_{je})$$

- τ_F : **Base transit time**

➤ $C_\pi \uparrow$ as $g_m \uparrow$ (**Problem!**)