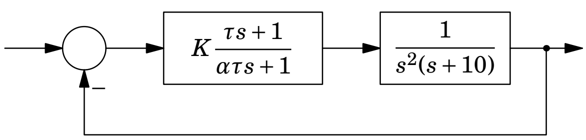
Final Exam EE 250 (Control System Analysis) Spring 2006 *

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

1. You may use pen, ruler, calculator. No pencils please!
2. Write your name on the graph paper provided and insert the graph paper into your answer booklet.
3. For all questions, show all the calculations you did to arrive at the final answer.
4. If you are forced to make any assumptions in answering any questions, you need to show all your assumptions.
5. **Please remain seated until we ask you to leave.**
6. Useful formula: $T_{PV} = M \cdot M_{PV}^{-1}$

1. **[10 points]** Determine, using asymptotic Bode plots, the values K, τ, α (do not forget to write units) such that the following control system has acceleration error constant $K_a \approx 1$ (rad/s)^2 while having $45° \leq \Phi\text{M} \leq 65°$. Use a ruler and **the semilog graph paper provided.**



2. **[10 points]** For the control system designed above, show the locus of the closed-loop poles for variation in K. From your root locus, determine the range of variation of K for the closed-loop system to be stable. **Use graph paper provided.**

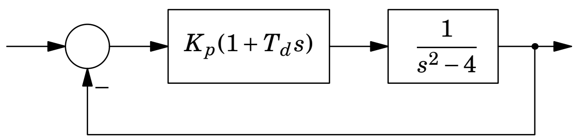
3. **[10 points]** Is the following state space model controllable?

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

If yes, then use the conversion-to-phase-variable-form technique of designing a feedback controller $u = -kx$ such that the desired closed-loop eigenvalues are at $\{-1, -2\}$. Verify your result using Ackerman's formula.

4. **[8 points]** Draw a Nyquist plot for the unity feedback control system with the open-loop transfer function $G(s) = K(1-s)/(1+s)$. **Show all steps involved in the construction.** Using Nyquist stability criterion, determine the range of values of $K > 0$ for which the closed-loop system is stable.

5. **[5 points]** For the following control system



use Routh-Hurwitz criterion to determine the range of values of K_p and T_d for the control system to be stable.

6. **[10 points]** Given the following equations that modeled the printer of your project, draw a block diagram that shows the signal flow from v_2 to y:

$$\begin{aligned}
T_1 &= k(r\theta - y) \\
T_2 &= k(y - r\theta) \\
T_1 - T_2 &= m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} \\
T_m &= \frac{K_m}{R} v_2 \\
T_m &= T + T_d \\
T &= J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + b\frac{\mathrm{d}\theta}{\mathrm{d}t} + r(T_1 - T_2)
\end{aligned}$$

Use Mason's gain rule to obtain a transfer function (TF) from u to y. Write the final form of the TF after plugging in the following numerical values:

$$\begin{aligned}
T_d &= 0 \\
m &= 0.2\,\text{kg} \\
k &= 20\,\text{N/m} \\
r &= 0.15\,\text{m} \\
b &= 0.25\,\text{N}\cdot\text{m}\cdot\text{s/rad} \\
R &= 2\,\Omega \\
K_m &= 2\,\text{N}\cdot\text{m/A} \\
J &= 0.01\,\text{kg}\cdot\text{m}^2
\end{aligned}$$

7. **[7 points]** Let us explore why "observer eigenvalues must be about 5 – 10 times as deep in left half plane as the regulator eigenvalues". Consider the scalar model:

$$\dot{x} = x + u, \quad y = x$$

After designing a regulator-observer pair for this model, the overall closed-loop model is:

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} &= \begin{bmatrix} A - BK & BK \\ 0 & A - K_o C \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix} \\
\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} &= \begin{bmatrix} 1 - 1\cdot K & 1 \cdot K \\ 0 & 1 - K_o 1 \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix}
\end{aligned}$$

Now, obtain the response $y(t)$ of the closed-loop system in each of the following two cases (**use only any of the techniques learned in EE250 for evaluating** e^{At}):
(a) $K = 5, K_o = 2$, (b) $K = 5, K_o = 20$
Which response is "better" and in what sense?

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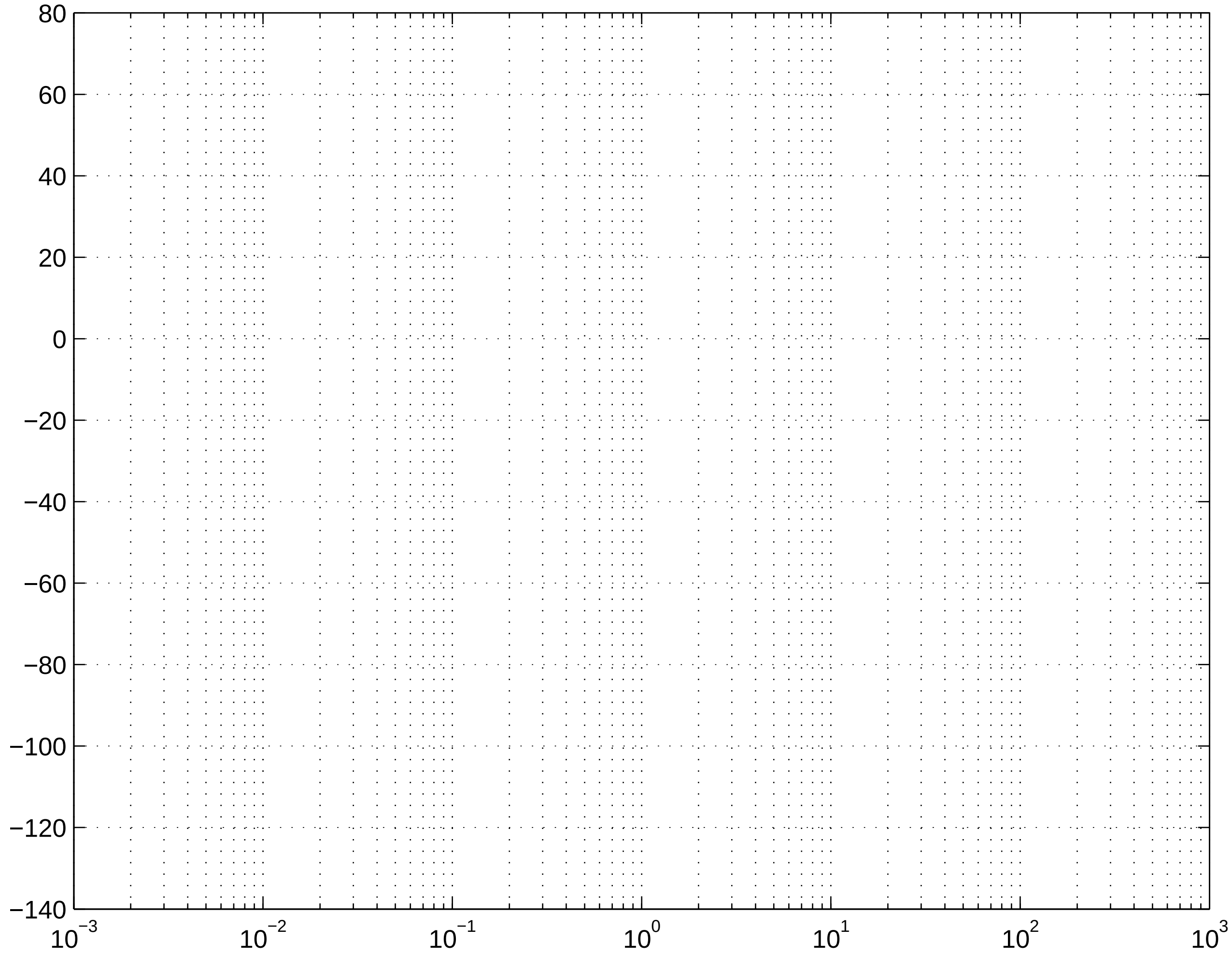
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