Transmission Lines - V

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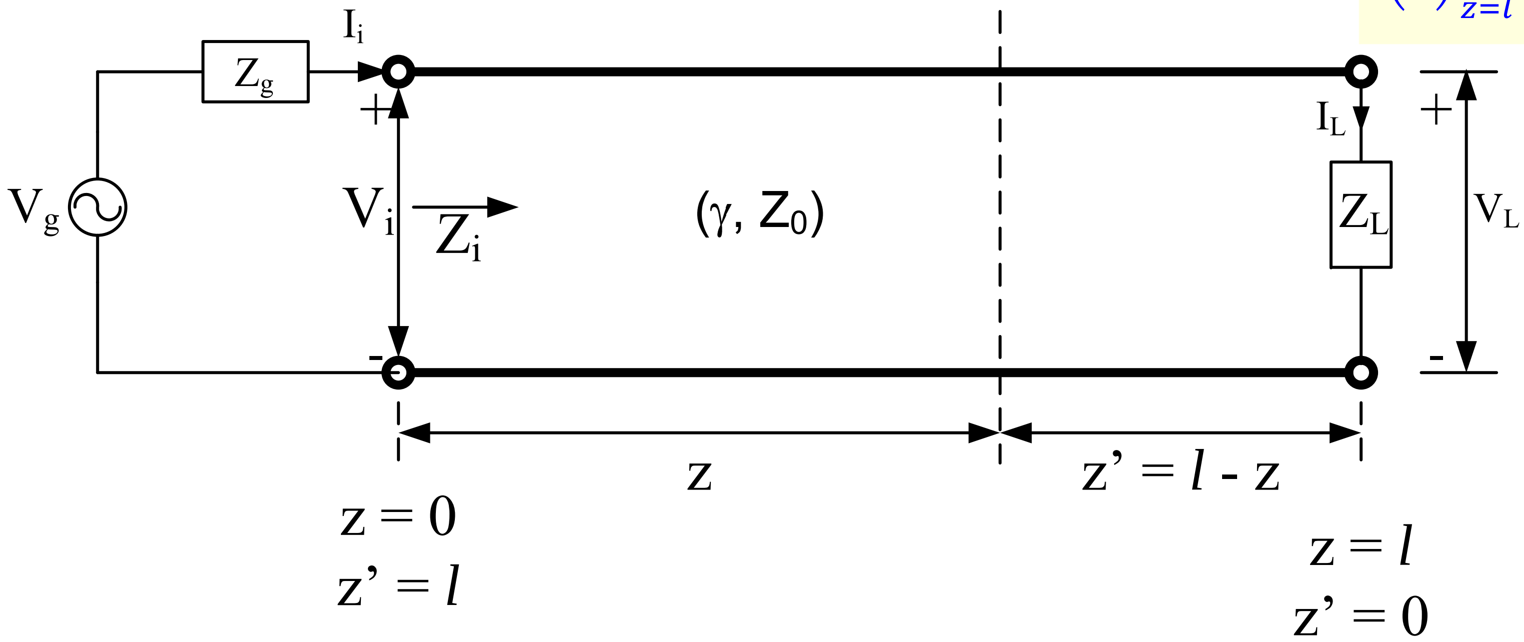
The Finite Transmission Line

$$\left(\frac{V}{I}\right)_{z=l} = \frac{V_L}{I_L} = Z_L$$

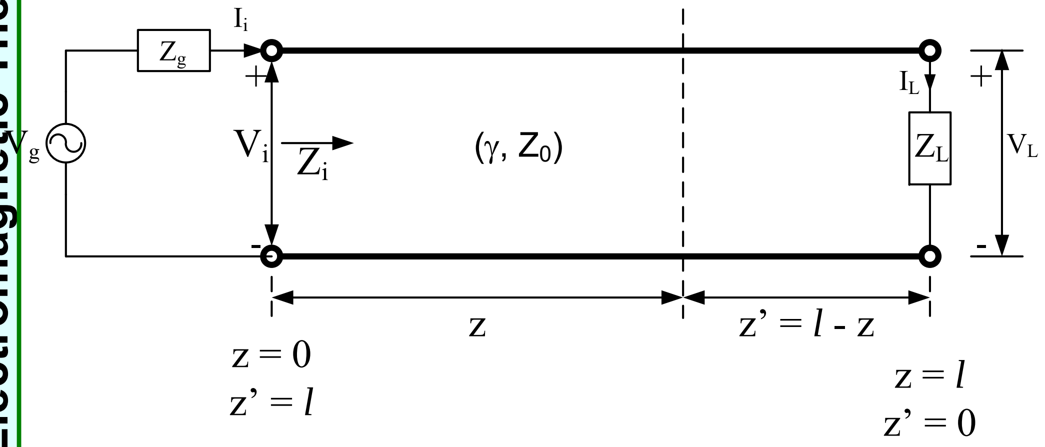


$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = Z_0$$

The line with the Resistive Termination

$Z_L \neq Z_0$



incident | reflected

$$V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$I(z') = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \frac{(Z_L - Z_0)}{(Z_L + Z_0)}e^{-2\gamma z'}\right]$$

$$I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma z'}\left[1 - \frac{(Z_L - Z_0)}{(Z_L + Z_0)}e^{-2\gamma z'}\right]$$

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \Gamma e^{-2\gamma z'}\right]$$

$$I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma z'}\left[1 - \Gamma e^{-2\gamma z'}\right]$$

$$\Gamma = \frac{(Z_L - Z_0)}{(Z_L + Z_0)} = |\Gamma| e^{j\theta_\Gamma}$$

Voltage reflection coefficient of the load

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}[1 + \Gamma e^{-2\gamma z'}]$$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = Z_0$$

$$\Gamma = \frac{(Z_L - Z_0)}{(Z_L + Z_0)} = |\Gamma|e^{j\theta_\Gamma}$$

Voltage reflection coefficient

☝ The ratio of the complex amplitudes of the reflected and incident voltage waves at load end (z' = 0).

Current reflection coefficient

❑ The ratio of the complex amplitudes of the reflected and incident current waves at the load end (z' = 0)

$$\Gamma_i = \frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma_V = -\Gamma$$

For a lossless transmission line

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + \Gamma e^{-j2\beta z'}\right]$$
$$I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{j\beta z'}\left[1 - \Gamma e^{-j2\beta z'}\right]$$

$$\gamma = j\beta \qquad \Gamma = |\Gamma|e^{j\theta_\Gamma}$$

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]$$
$$I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{j\beta z'}\left[1 - |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$Z_{in} \equiv \frac{V(z')}{I(z')} = Z_0\frac{\left[1 + |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]}{\left[1 - |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]}$$

$$Z_L \equiv \left[Z_{in}\right]_{z'=0} = Z_0\frac{\left[1 + |\Gamma|e^{j\theta_\Gamma}\right]}{\left[1 - |\Gamma|e^{j\theta_\Gamma}\right]}$$

- The voltage and current phasors on a lossless line can also be characterized using the following equation.

$$V(z') = I_L[Z_L \cosh\gamma z' + Z_0 \sinh\gamma z']$$
$$I(z') = \frac{I_L}{Z_0}[Z_L \sinh\gamma z' + Z_0 \cosh\gamma z']$$

$\gamma = j\beta$

$V_L = Z_L I_L$

$Z_L = R_L$ ⇐ Resistive termination

$$V(z') = V_L \cos\beta z' + jI_L R_0 \sin\beta z'$$
$$I(z') = I_L \cos\beta z' + j\frac{V_L}{R_0}\sin\beta z'$$

$$|V(z')| = V_L\sqrt{\cos^2\beta z' + \left(\frac{R_0}{R_L}\right)^2 \sin^2\beta z'}$$

$$|I(z')| = I_L\sqrt{\cos^2\beta z' + \left(\frac{R_L}{R_0}\right)^2 \sin^2\beta z'}$$

These plots as functions of z' are standing waves with the maxima and the minima at fixed locations from the load.

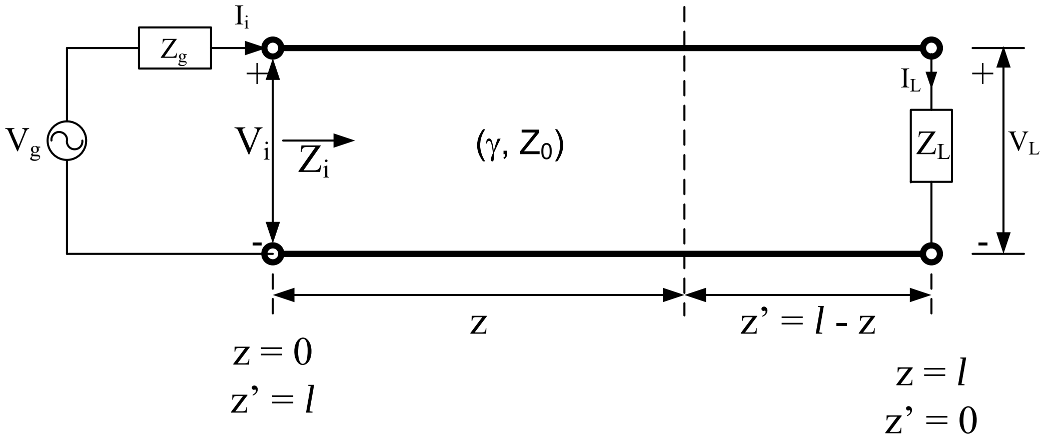
Standing Wave Ratio

Ratio of the maximum to minimum voltages along the finite terminated line

$$\Gamma = \frac{(Z_L - Z_0)}{(Z_L + Z_0)} = |\Gamma| e^{j\theta_\Gamma}$$

$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma| e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$S = \frac{|V_{\max}|}{|V_{min}|} \equiv \frac{1+|\Gamma|}{1-|\Gamma|}$$



$$|\Gamma| = \frac{S-1}{S+1}$$

$\Gamma = 0, \quad S = 1 \quad when \quad Z_L = Z_0 \quad (Matched\ Load)$

$\Gamma = -1, \quad S \to \infty \quad when \quad Z_L = 0 \quad (short\ circuit)$

$\Gamma = 1, \quad S \to \infty \quad when \quad Z_L \to \infty \quad (open\ circuit)$

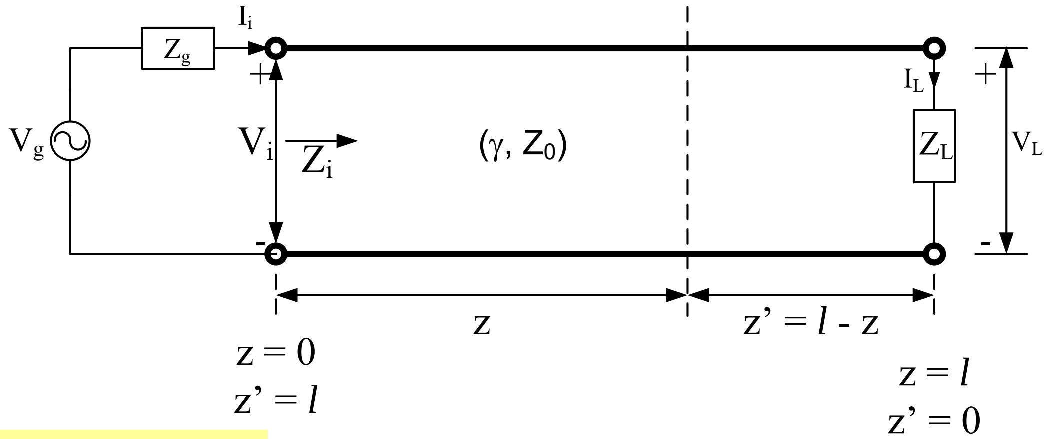
$$V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{j\beta z'}\left[1 - |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]$$

☜ **The voltage & current wave along a lossless line**

$|V_{\max}|, |I_{\min}|$ occur together when

$$\theta_\Gamma - 2\beta z'_M = -2n\pi, \qquad n = 0,1,2,\ldots..$$



$|V_{\min}|, |I_{\max}|$ occur together when

$$\theta_\Gamma - 2\beta z'_m = -(2n+1)\pi, \qquad n = 0,1,2,\ldots..$$

For resistive termination on a lossless transmission line

$$Z_L = R_L, \qquad Z_0 = R_0 \qquad V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma|e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$\Gamma = \frac{(R_L - R_0)}{(R_L + R_0)}$$

☞ Voltage reflection coefficient is purely real for the resistive load

1. $R_L > R_0$

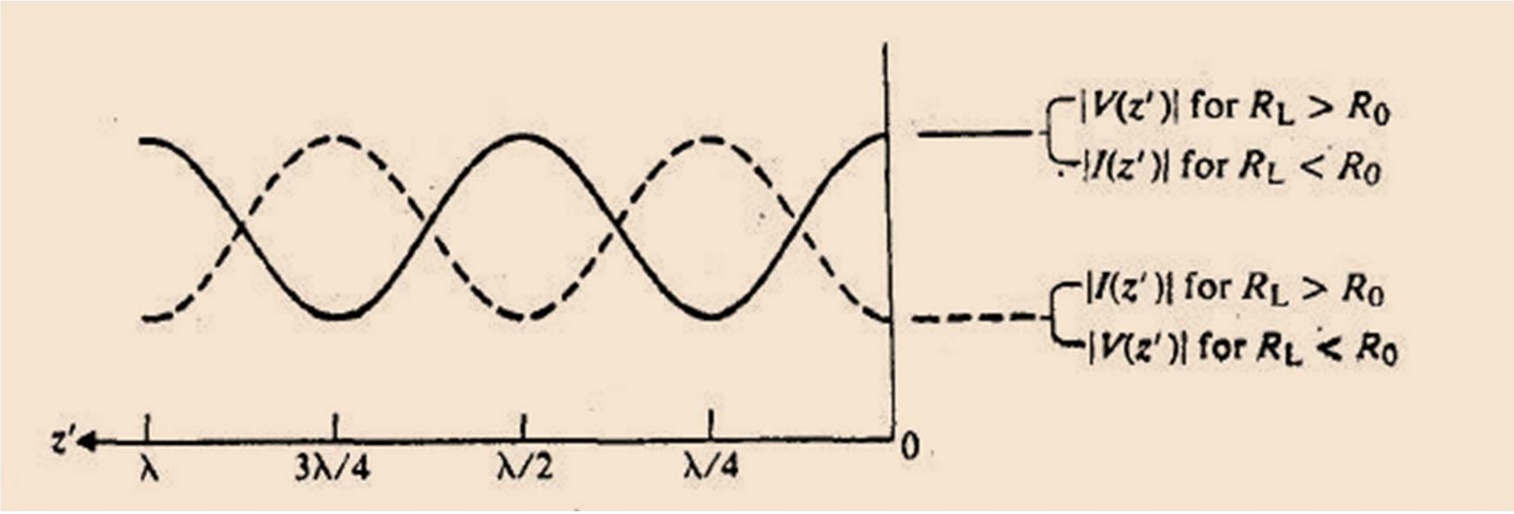
Γ is positive and real; $\theta_\Gamma = 0$

$$\theta_\Gamma - 2\beta z'_M = -2n\pi, \qquad n = 0,1,2,.....$$

Voltage maximum (Current minimum) will occur at termination (z'=0).

Other Voltage maximum (Current minimum) will be observed at

$$-2\beta z'_M = -2n\pi, \qquad z'_M = n\lambda/2$$
$$n = 1,2,.....$$



2. $R_L < R_0$

Γ is negative and real; $\theta_\Gamma = -\pi$

$$\theta_\Gamma - 2\beta z'_m = -(2n+1)\pi, \qquad n = 0,1,2,\ldots..$$

Current maximum (Voltage minimum) will occur at termination (z'=0).

Other Current maximum (Voltage minimum) will be observed at

$$-2\beta z'_m = -2n\pi, \qquad z'_m = n\lambda/2$$
$$n = 1,2,\ldots..$$

$$|V(z')| = V_L \sqrt{\cos^2 \beta z' + \left(\frac{R_0}{R_L}\right)^2 \sin^2 \beta z'}$$

$$|I(z')| = I_L \sqrt{\cos^2 \beta z' + \left(\frac{R_L}{R_0}\right)^2 \sin^2 \beta z'}$$

$R_L \to \infty$

$I_L = 0$, but V_L is finite

Open circuited line

$R_L > R_0$

$$|V(z')| = V_L|\cos \beta z'|$$

$$|I(z')| = \frac{V_L}{R_0}|\sin \beta z'|$$

Short circuited line

$R_L = 0$

$V_L = 0$, but I_L is finite

$$|V(z')| = I_L R_0|\sin \beta z'|$$

$$|I(z')| = I_L|\cos \beta z'|$$