Assignment 8

MSO-201: Probability and Statistics

1. Let X be a bounded random variables, i.e. there exists a K such that $|X| \leq K$, then the moment generating function of X exists for all $-\infty < t < \infty$.

2. Let X be a bounded random variables then for all $m > 0$, EX^m exists for all $m > 0$.

3. Let X be a random variable and there exists $K > 0$, such that $X > K$, then EX^m exists for all $m < 0$.

4. Let X have the following PDF

$$f_X(x) = \begin{cases} \frac{1}{2\sqrt{3}} & \text{if} \quad -\sqrt{3} < x < \sqrt{3} \\ 0 & \text{if} \quad |x| \geq \sqrt{3} \end{cases}$$

   Find the exact value of $P(|X| > 3/2)$. Find $P(|X| > 3/2)$ based on Chebyshev's inequality.

5. Let a random variable X has the following PMF

$$P(X = -1) = P(X = 1) = \frac{1}{8} \quad \text{and} \quad P(X = 0) = \frac{6}{8}.$$

   Find the exact value of $P(|X| \geq 1)$. Find $P(|X| \geq 1)$ based on Chebyshev's inequality.

6. Let X be a random variable such that $E(X) = 3$ and $E(X^2) = 13$, using Chebyshev's inequality find the lower bound for $P(-2 < X < 8)$.

7. Let X be a random variable such that $P(X \leq 0) = 0$ and let $E(X) = \mu$ exist. Show that $P(X \leq 2\mu) \geq \frac{1}{2}$

8. Let X be a random variable with MGF $M(t)$, for $-h < t < h$. Prove that for $0 < t < h$,

$$P(X \geq a) \leq e^{-at} M(t).$$

9. Suppose X is $N(0, 1)$ random variable. Show that $P(|X| > 4) \leq 0.1$.

10. Suppose $X \sim \text{Gamma}(10, 10)$, then show that $P(X > 2) \leq 0.1$.