Lecture-15

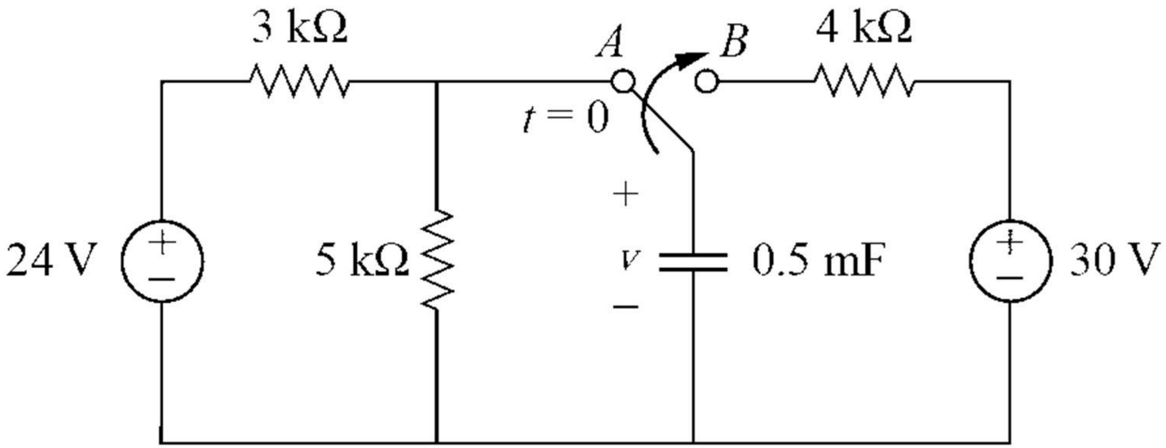
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Step Response of an RC Circuit.
- Step Response of an RL Circuit.

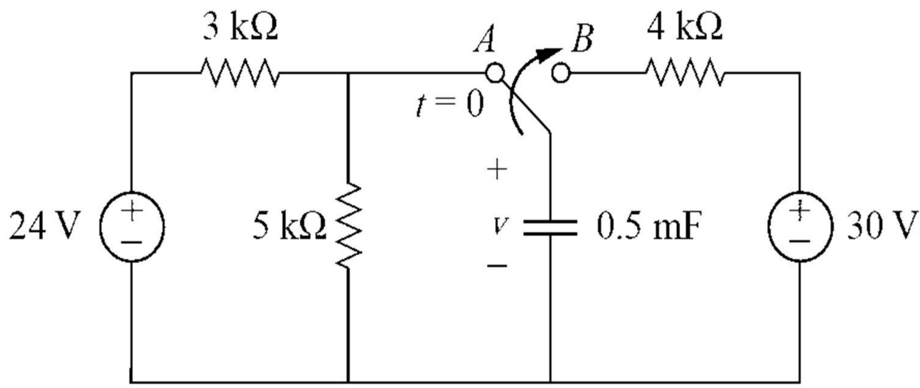
❑ EXAMPLE:

- The switch in Fig. below has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1s$ and $4s$.



Step Response of an RC Circuit (Cont..)

- For $t < 0$, the switch is at position A. Since v is the same as the voltage across the $5k\,\Omega$ resistor, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as



$$v = \frac{5}{5+3}(24) = 15\ V$$

- As the capacitor voltage cannot change instantaneously, so,

$$v(0^-) = v(0^+) = \mathrm{v}(0) = 15\ V$$

- For $t > 0$, the switch is in position B. The thevenin resistance connected to the capacitor is $R_{Th} = 4\ k\,\Omega$, and the time constant can be calculated as,

$$\boldsymbol{\tau} = Rt_hC = 4 \times 103 \times 5 \times 10^{-4} = 2s$$

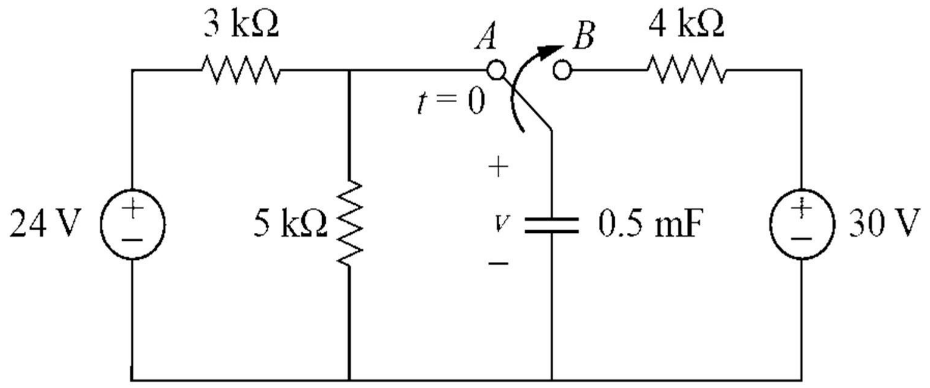
Step Response of an RC Circuit (Cont..)

- The capacitor acts like an open circuit to DC at steady state, therefore, $v(\infty) = 30V$.

So,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-\frac{t}{\tau}}$$

$$= 30 + (15 - 30)e^{-\frac{t}{\tau}}$$



At t=1,

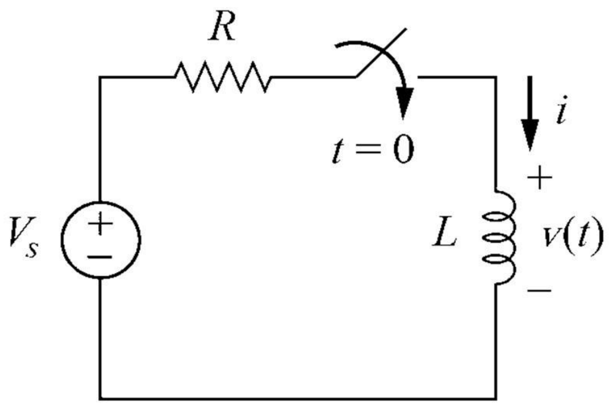
$$v(1) = 30 - 15\, e^{-0.5} = 20.902\ V$$

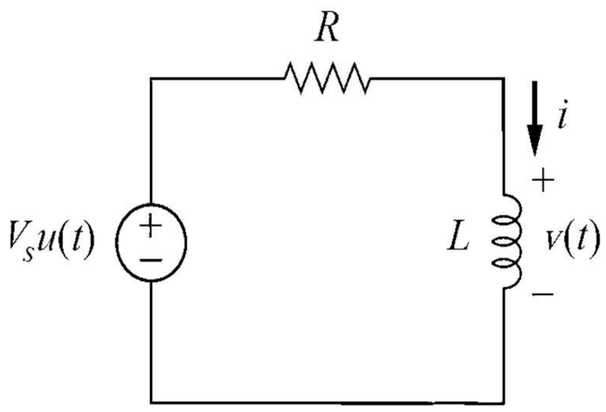
At t=4,

$$v(4) = 30 - 15\, e^{-2} = 27.97\ V$$

Step Response of an RL Circuit

- Consider the *R-L* circuit in left Figure below, which is replaced by the circuit in right Figure.
- We need to find the inductor current *i* as the circuit response.





Step Response of an RL Circuit (Cont..)

- There are two Methods to find the R-L response of the circuit:
  - Apply Kirchhoff's laws,
  - Apply alternate technique as explained earlier

Let the response be the sum of the natural current and the forced current,

$$i = i_f + i_n$$

As the natural response is always a decaying exponential, that is,

$$i_n = Ae^{-\frac{t}{\tau}}, \qquad \tau = L/R$$

where A is a constant to be determined.

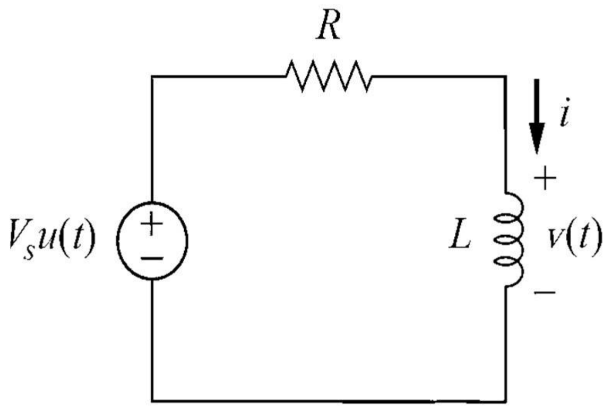
Step Response of an RL Circuit (Cont..)

- The forced response is the value of the current a long time after the switch is closed as shown in the Figure.
    - The natural response essentially dies out after five time constants.
    - At that time, the inductor becomes a short circuit, and the voltage across it will be zero.
    - The entire source voltage V_s appears across R.

Thus, the forced response is -

$$i_f = \frac{V_S}{R}$$

$$i = \frac{V_S}{R} + Ae^{-\frac{t}{\tau}}$$



Step Response of an RL Circuit (Cont..)

- **Next, determine the constant *A* from the initial value of *i*.**
- Let I_0 be the initial current through the inductor, which may come from a source other than Vs. Since the current through the inductor cannot change instantaneously,

$$i(0^-) = i(0^+) = I_0$$

Thus, at $t = 0$, Equation $i = \frac{V_S}{R} + Ae^{-\frac{t}{\tau}}$ becomes

$$I_0 = \frac{V_S}{R} + A$$

From this, we obtain A as

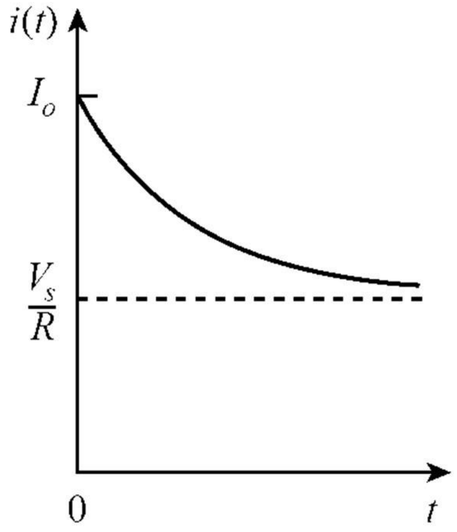
$$A = I_0 - \frac{V_S}{R}$$

Step Response of an RL Circuit (Cont..)

- Substituting for *A*, we get

$$\boxed{i = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right) e^{-\frac{t}{\tau}}}$$

- This is the complete response of the *R-L* circuit as shown in the Figure



- The response in above equation may be written as

$$\boldsymbol{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}}$$

where $i(0)$ and $i(\infty)$ are the initial and final values of i

Step Response of an RL Circuit (Cont..)

If $I_0 = 0$, then

$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{V_s}{R}\left(1 - e^{-\frac{t}{\tau}}\right), & t > 0 \end{cases}$$

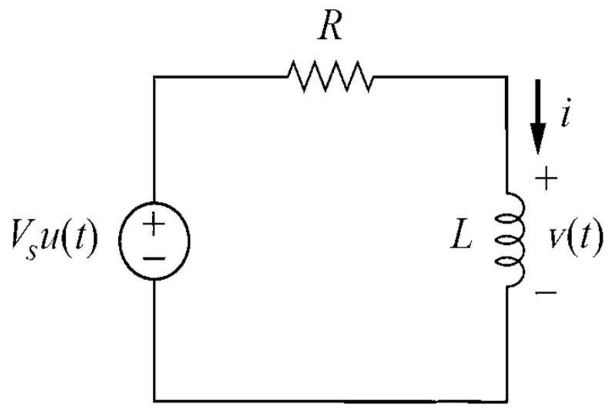
or

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{t}{\tau}}\right)u(t)$$

This equation is called the step response of the *R-L* circuit.

Step Response of an RL Circuit (Cont..)

The voltage across the inductor is, $v = L\frac{di}{dt}$

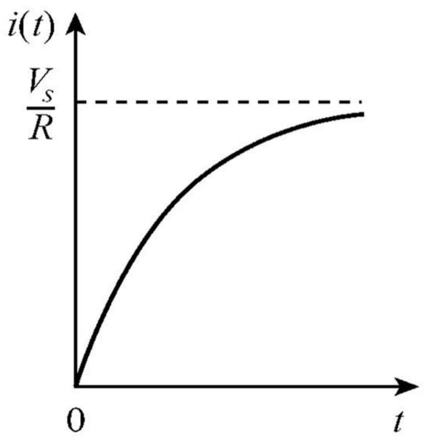


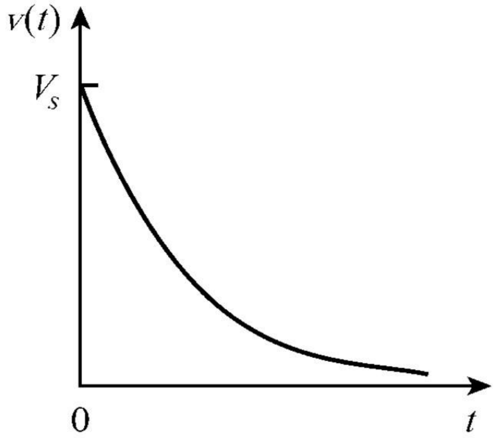
$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{t}{\tau}}\right)u(t)$$

$$v(t) = L\frac{di}{dt} = \frac{L}{\tau R}V_S e^{-\frac{t}{\tau}} \quad , \qquad t > 0 \qquad or$$

$$v(t) = V_S e^{-\frac{t}{\tau}}u(t)$$

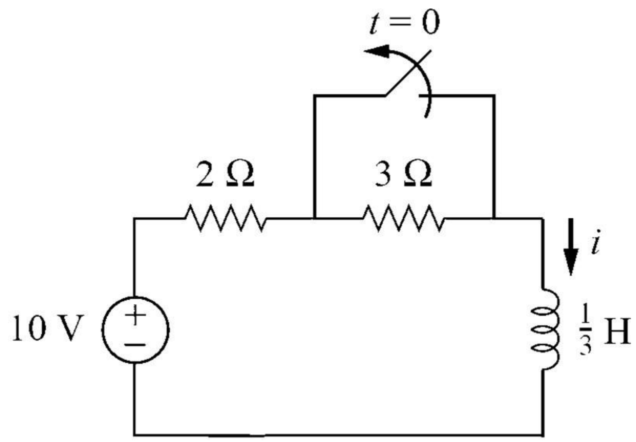
Figure, below, shows the step responses of inductor current and voltage





EXAMPLE:

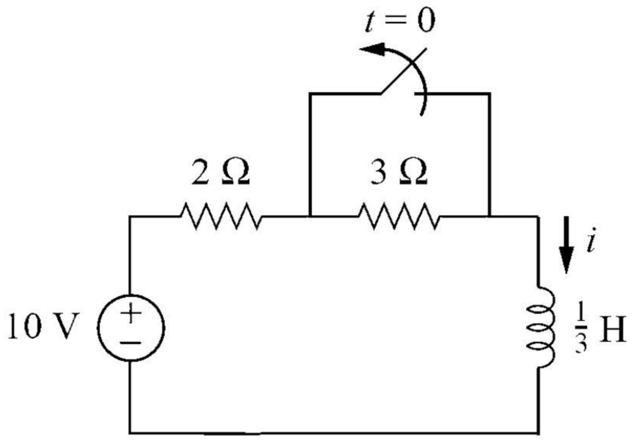
- Find $i(t)$ in the circuit in Fig. below for $t > 0$. Assume that the switch has been closed for a long time.



Step Response of an RL Circuit (Cont..)

Solution:

- When $t < 0$, the 3Ω resistor is short-circuited, and the inductor also acts like a short circuit. The current through the inductor at $t = 0^-$ is



$$i(0^-) = \frac{10}{2} = 5\ A$$

- Since the inductor current cannot change instantaneously, so,

$$i(0) = i(0^-) = i(0^+) = 5A$$

- When $t > 0$, the switch is open. Therefore, 2Ω and 3Ω resistors are now in series, so,

$$i(\infty) = \frac{10}{2+3} = 2\ A$$

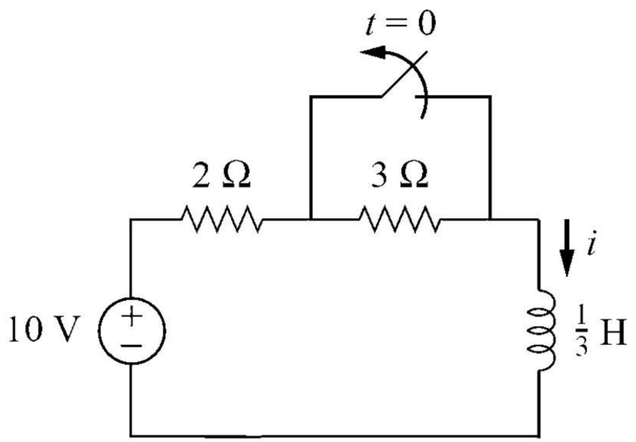
- The Thevenin resistance across the inductor terminals is

$$R_{Th} = 2 + 3 = 5\ \Omega$$

Step Response of an RL Circuit (Cont..)

The value of time constant,

$$\tau = \frac{L}{R_{Th}} = \frac{\frac{1}{3}}{5} = \frac{1}{15}s$$



Thus,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$$

$$= 2 + (5 - 2)e^{-15t}$$

$$= 2 + 3e^{-15t}\text{A}, \quad t > 0$$


THANKS
FOR YOUR
ATTENTION