Lecture-21

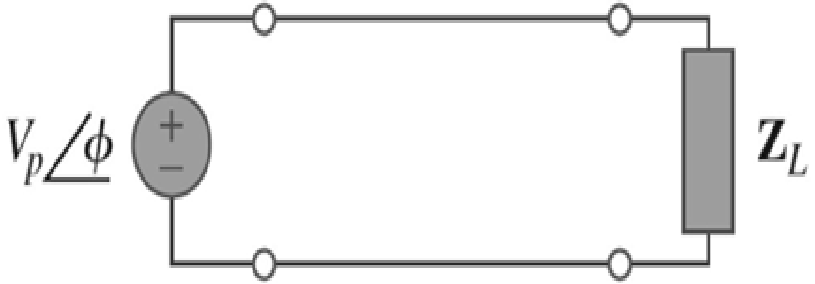
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Three phase system.
- Y-Y system.
- Y-Δ system.

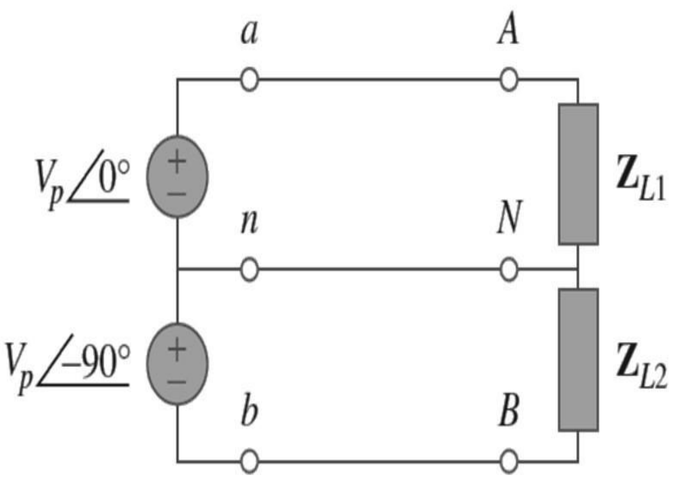
Three Phase Circuits

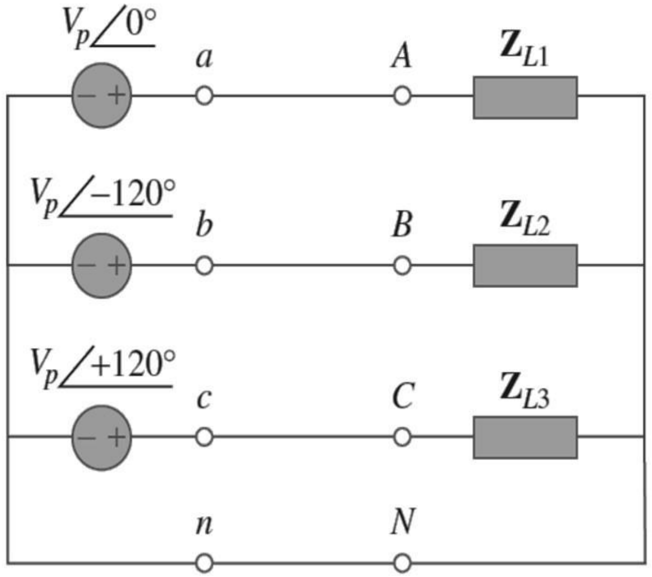
- So far in this course, we have been dealing with single-phase circuits.
- A single-phase AC power system consists of a generator connected through a pair of wires (a transmission line) to a load.
- This is illustrated in the below figure, where a single-phase two wire system is used.
- Here V_p is the rms magnitude of the source voltage and $\varnothing$ is the phase.



Three Phase Circuits (Cont…)

- Circuits or systems in which the ac sources operate at the same frequency but different phases are known as polyphase.
- The figure on the left below shows a two-phase three-wire system, and the figure on the right shows a three-phase four wire system.



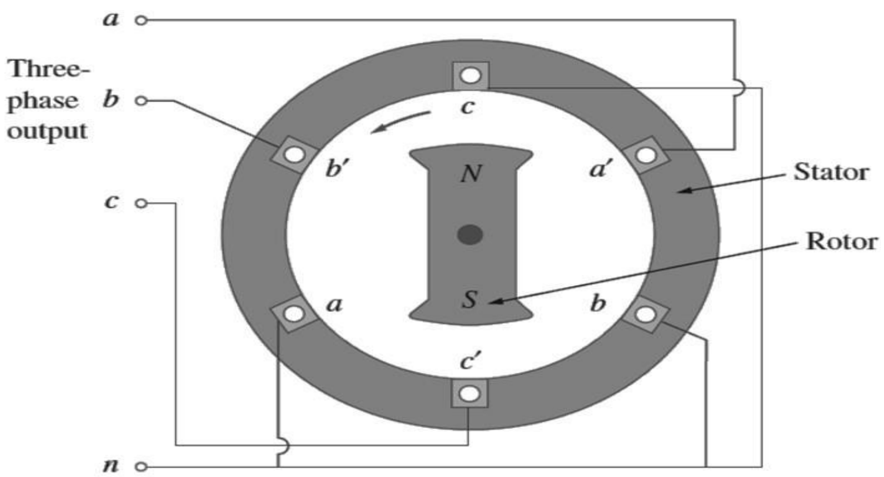


Three Phase Circuits (Cont…)

- As distinct from a single-phase system, a two-phase system is produced by a generator consisting of two coils placed perpendicular to each other so that the voltage generated by one lags the other by 90°.
- In the same way, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.
- Since the three-phase system is by far the most prevalent and most economical polyphase system, our discussion will mainly be on three-phase systems.
- Three-phase systems are important for at least three reasons.
  1. Nearly all electric power is generated and distributed in three-phase, at the operating frequency of 50 or 60 Hz.
  2. The instantaneous power in a three-phase system can be constant (not pulsating), as will be discussed later.
  3. The three-phase system is more economical than single phase system as the amount of wire required for a three-phase system is lesser than the amount of wire needed for an equivalent single-phase system.

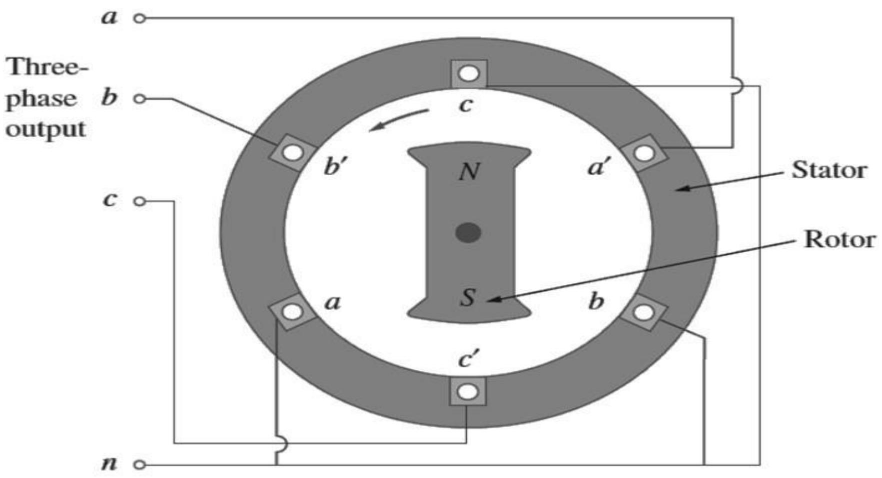
Balanced Three-Phase Voltages

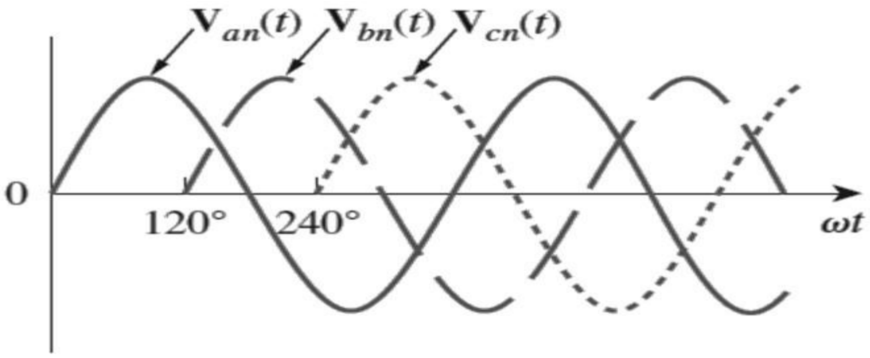
- Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross- sectional view is shown below.
- The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*).



Balanced Three-Phase Voltages (Cont..)

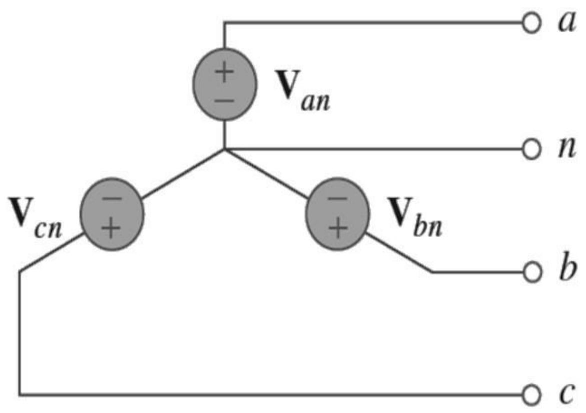
- Three separate windings or coils with terminals $a - a'$, $b - b'$, and $c - c'$ and are physically placed $120°$ apart around the stator.
- Terminals a and a' for example, stand for one of the ends of coils going into and the other end coming out of the page.
- As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils.
- Because the coils are placed $120°$ apart, the induced voltages in the coils are equal in magnitude but out of phase by $120°$ as shown below.

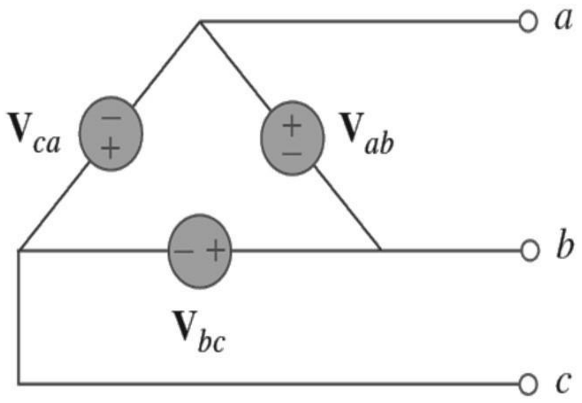




Balanced Three-Phase Voltages (Cont..)

- Since each coil can be regarded as a single-phase generator by itself, the three-phase generator can supply power to both single-phase and three-phase loads.
- A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines)
- A three-phase system is equivalent to three single-phase circuits.
- The voltage sources can be either wye (Y)-connected or delta (Δ)-connected as shown below.





Balanced Three-Phase Voltages (Cont..)

- Let us consider the wye-connected voltages for now.

- The voltages V_{an}, V_{bn}, and V_{cn} are respectively between lines *a*, *b*, and *c*, and the neutral line *n*.

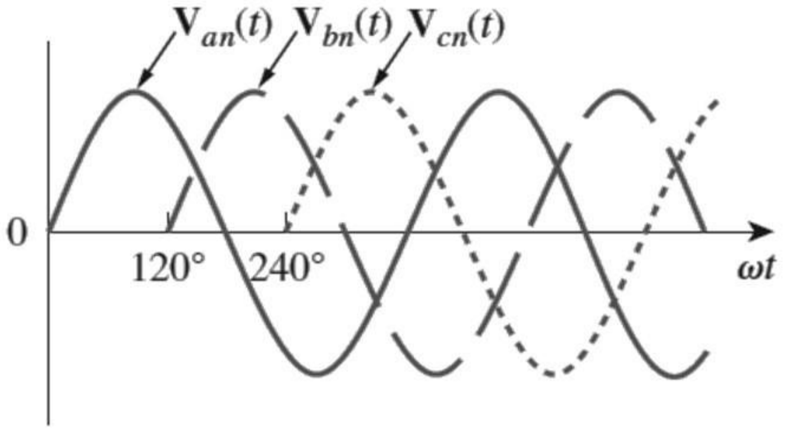
- These voltages are called *phase voltages*.

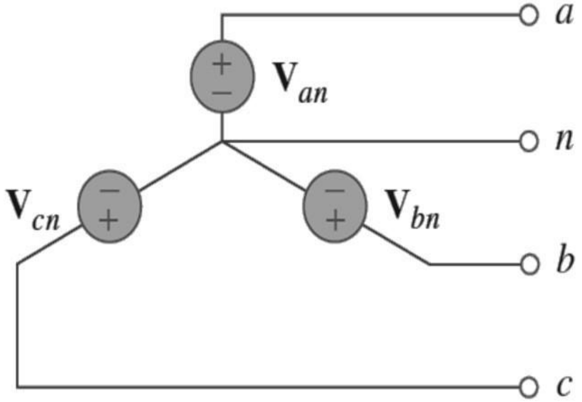
- If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120° the voltages are said to be *balanced*.

- This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

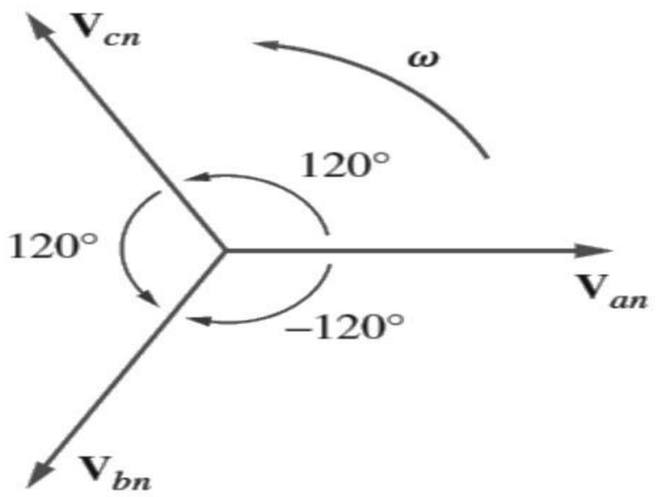




- Thus, balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

Balanced Three-Phase Voltages (Cont..)

- Since the three-phase voltages are 120° out of phase with each other, there are two possible combinations.



- One possibility is as shown above and expressed mathematically as.

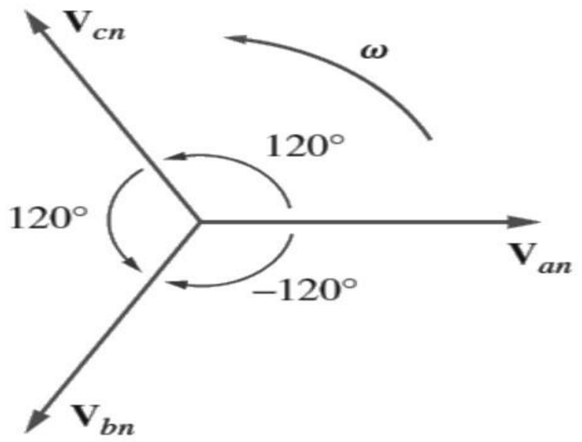
$$\mathbf{V}_{\mathrm{an}} = V_{\mathrm{p}}\angle 0°$$

$$\mathbf{V}_{\mathrm{bn}} = V_{\mathrm{p}}\angle -120°$$

$$\mathbf{V}_{\mathrm{cn}} = V_{\mathrm{p}}\angle -240° = V_{\mathrm{p}}\angle 120°$$

Balanced Three-Phase Voltages (Cont..)

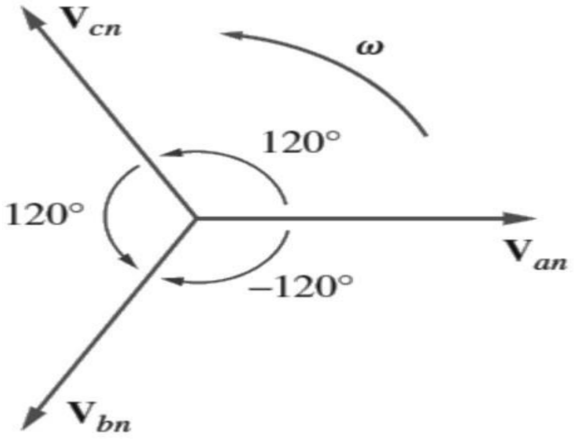
- Here V_p is the effective or RMS value of the phase voltage.
- Please note, as a common tradition in power systems, in this module voltage and current are considered in rms values unless otherwise stated.



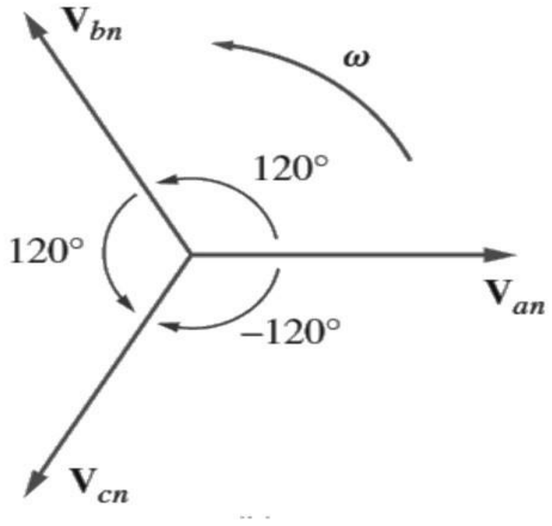
- This is known as the *abc sequence* or *positive sequence*.
- In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ which in turn leads $\mathbf{V}_{cn}$.
- This sequence is produced when the rotor in the figure, discussed earlier in this lecture, rotates counterclockwise.
- The phase sequence may also be regarded as the order in which the phase voltages reach their peak (or maximum) values with respect to time.

Balanced Three-Phase Voltages (Cont..)

- The second possible phase combination is as shown in the figure below.



(i) abc sequence or positive sequence.



(ii) acb sequence or negative sequence.

- This is expressed mathematically as.

$$\mathbf{V}_{\mathrm{an}} = V_{\mathrm{p}}\angle 0°$$

$$\mathbf{V}_{\mathrm{cn}} = V_{\mathrm{p}}\angle -120°$$

$$\mathbf{V}_{\mathrm{bn}} = V_{\mathrm{p}}\angle -240° = V_{\mathrm{p}}\angle 120°$$

Balanced Three-Phase Voltages (Cont..)

- This is known as the *acb sequence* or *negative sequence*.
- In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ which in turn leads $\mathbf{V}_{bn}$.
- This sequence is produced when the rotor in previous figure rotates clockwise.

$$\mathbf{V}_{\text{an}} + \mathbf{V}_{\text{bn}} + \mathbf{V}_{\text{cn}} = V_{\text{p}}\angle 0° + V_{\text{p}}\angle -120° + V_{\text{p}}\angle 120°$$

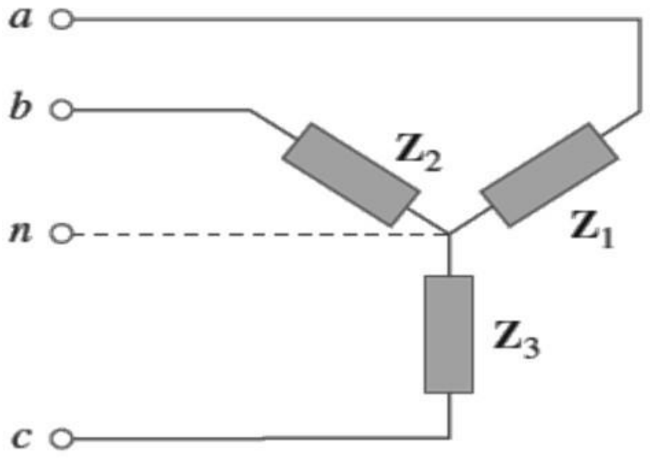
$$= V_{\text{p}}\,(1 - 0.5 - j0.866 - 0.5 + j0.866)$$

$$= 0$$

- The phase sequence is determined by the order in which the phasors pass through a fixed point in the phase diagram.

Balanced Three-Phase Voltages (Cont..)

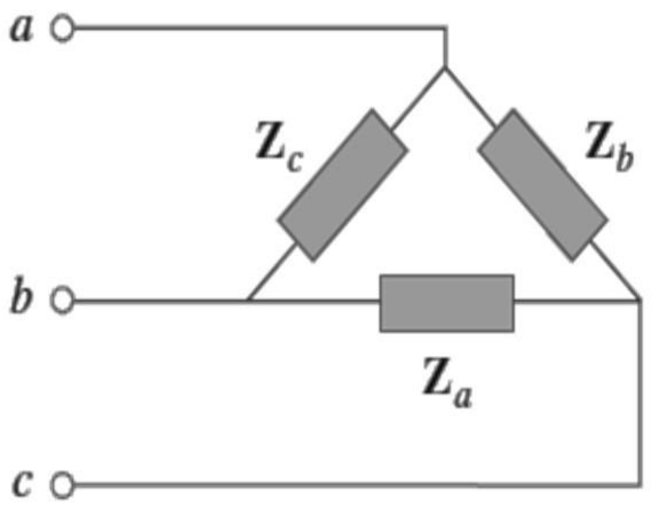
- Like the generator connections, a three-phase load can be either wye-connected or delta-connected, depending on the end application.
- Figure below shows a wye (Y)-connected load.



- The neutral line in the above figure may or may not be there, depending on whether the system is four- or three-wire.

Balanced Three-Phase Voltages (Cont..)

- Figure below shows a delta (Δ)-connected load.



- A neutral line is topologically impossible for a delta connection.

Balanced Three-Phase Voltages (Cont..)

- A wye- or delta-connected load is said to be *unbalanced* if the phase impedances are not equal in magnitude or out of phase.
- A balanced load is one in which the phase impedances are equal in magnitude and in phase.
- For a balanced wye (Y)-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

- For a balanced delta (Δ)-connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

- We recall from our previous discussions that,

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \text{ or } \mathbf{Z}_Y = \mathbf{Z}_\Delta/3$$

- So we can say that a wye (Y)-connected load can be transformed into a delta connected load, or vice versa using Y-Δ transformation discussed earlier.

Balanced Three-Phase Voltages (Cont..)

- Since both the three-phase source and the three-phase load can be either wye (Y)- or delta (Δ)-connected, we have four possible connections:
  1. Y-Y connection (i.e.,Y-connected source with a Y-connected load)
  2. Y- Δ connection
  3. Δ- Δ connection
  4. Δ -Y connection.
- It is important to note that a balanced delta-connected load is more common than a balanced wye- connected load.
- This is due to the ease with which loads may be added or removed from each phase of a delta- connected load.
- It is very difficult with a wye-connected load because the neutral may not be accessible.
- On the other hand, delta-connected sources are not common in practice because of the circulating current that will result in the delta-mesh if the three-phase voltages are slightly unbalanced.

Example:

- Determine the phase sequence of the set of voltages $v_{\mathrm{an}} = 200\cos(\omega t + 10)$, $v_{\mathrm{bn}} = 200\cos(\omega t - 230)$, $v_{\mathrm{cn}} = 200\cos(\omega t - 110)$?

Solution: We first convert the voltages to phasors as,

$$\mathbf{V}_{\mathrm{an}} = 200\angle 10°, \mathbf{V}_{\mathrm{bn}} = 200\angle -230°, \mathbf{V}_{\mathrm{cn}} = 200\angle -110°$$

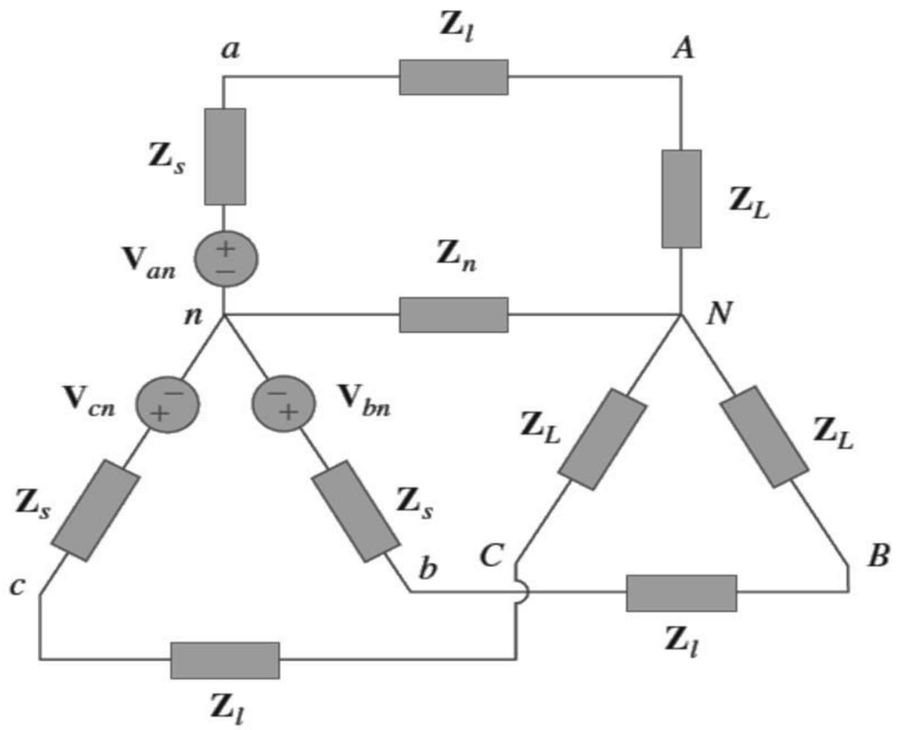
We notice that $\mathbf{V}_{\mathrm{an}}$ leads $\mathbf{V}_{\mathrm{cn}}$ by $120°$ and $\mathbf{V}_{\mathrm{cn}}$ in turn leads $\mathbf{V}_{\mathrm{bn}}$ by $120°$. Hence, we have an *acb* sequence.

Balanced Wye Wye Connection

- We begin with the Y-Y system, because any balanced three-phase system can be reduced to an equivalent Y-Y system.
- Therefore, analysis of this system is considered as the key to solve all balanced three-phase systems.
- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y- connected load.
- Consider a balanced four-wire Y-Y system, where a Y-connected load is connected to a Y-connected source.

Balanced Wye Wye Connection (cont…)

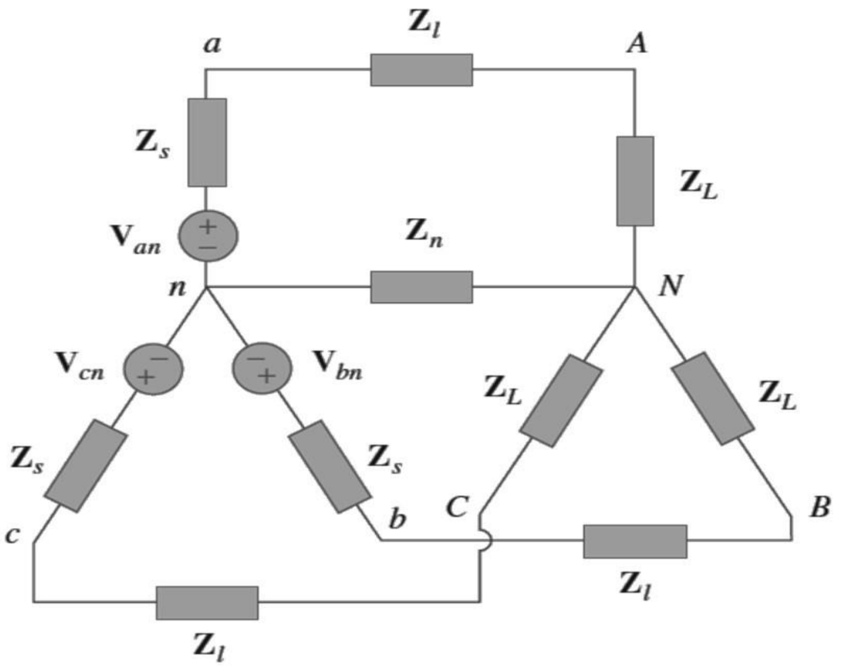
- We assume a balanced load so that load impedances are equal.
- Although the impedance $\mathbf{Z}_Y$ is the total load impedance per phase, it may also be regarded as the sum of the source impedance $\mathbf{Z}_s$, line impedance $\mathbf{Z}_l$, and load impedance $\mathbf{Z}_L$ for each phase, since these impedances are in series.



Balanced Wye Wye Connection (cont…)

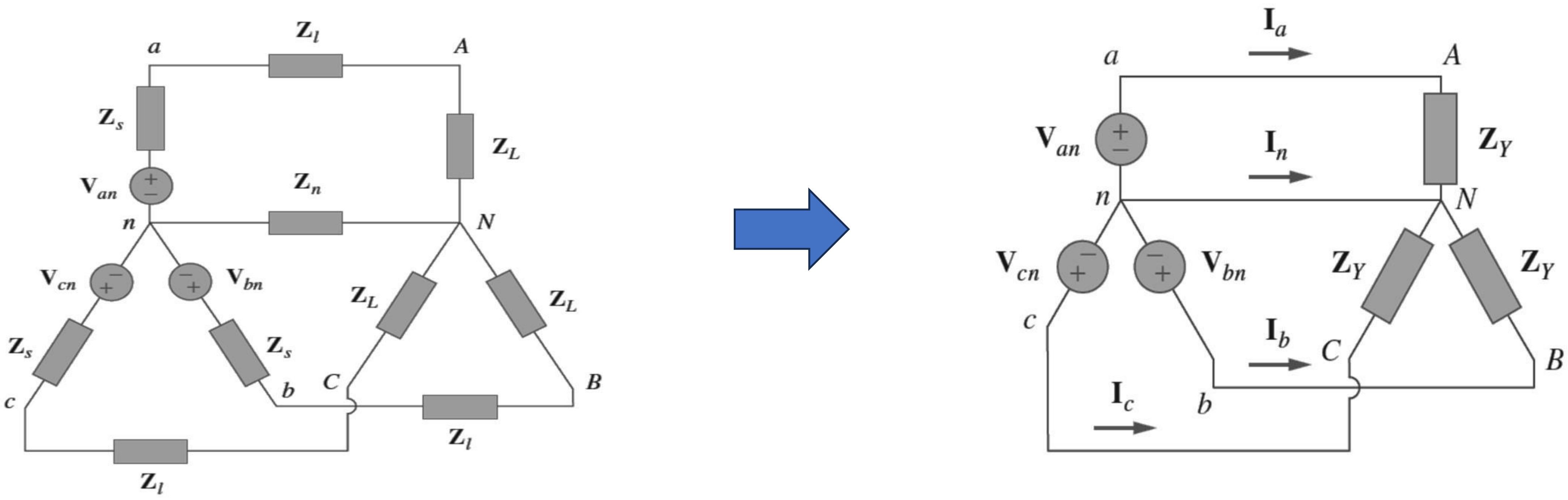
- As illustrated in the previous figure,
  - $\mathbf{Z}_s$ denotes the internal impedance of the phase winding of the generator;
  - $\mathbf{Z}_l$ is the impedance of the line joining a-phase of the source with a-phase of the load;
  - $\mathbf{Z}_L$ is the impedance of each phase of the load;
  - $\mathbf{Z}_n$ is the impedance of the neutral line.
- Thus, in general,

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_l + \mathbf{Z}_L$$



- $\mathbf{Z}_s$ and $\mathbf{Z}_l$ are often very small compared with $\mathbf{Z}_L$, so we can assume $\mathbf{Z}_Y = \mathbf{Z}_L$, if no source or line impedance is given.
- In any event, by lumping the impedances together, the Y-Y system can be simplified to that shown in the next slide.

Balanced Wye Wye Connection (cont…)



- Assuming the positive sequence, the phase voltage or line to neutral voltage are,

$$\mathbf{V}_{\mathrm{an}} = V_{\mathrm{p}}\angle 0°$$

$$\mathbf{V}_{\mathrm{bn}} = V_{\mathrm{p}}\angle -120°$$

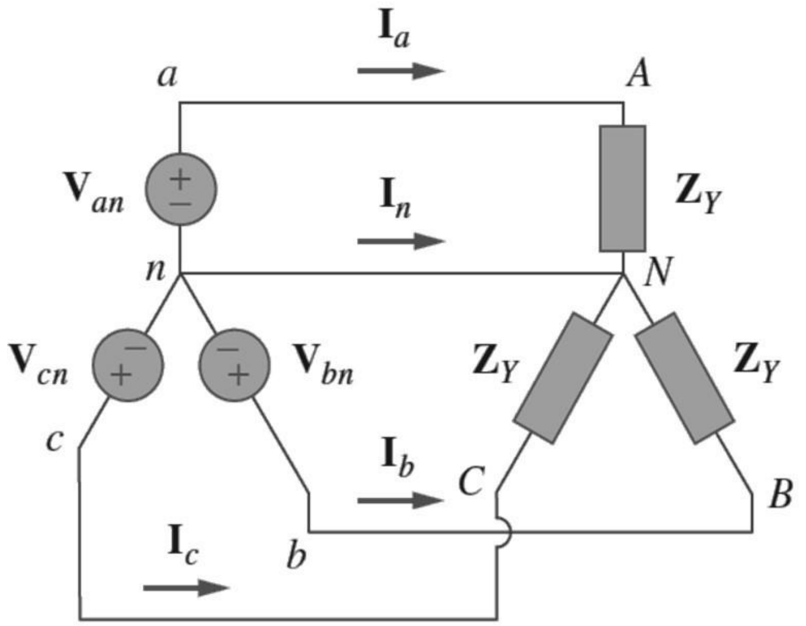
$$\mathbf{V}_{\mathrm{cn}} = V_{\mathrm{p}}\angle 120°$$

Balanced Wye Wye Connection (cont…)

- The line to line voltages or simply the line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ are related to the phase voltages as,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$

$$= V_p\left(1 + \frac{1}{2} + \frac{j\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ$$



- Similarly,

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} + \mathbf{V}_{nc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ$$

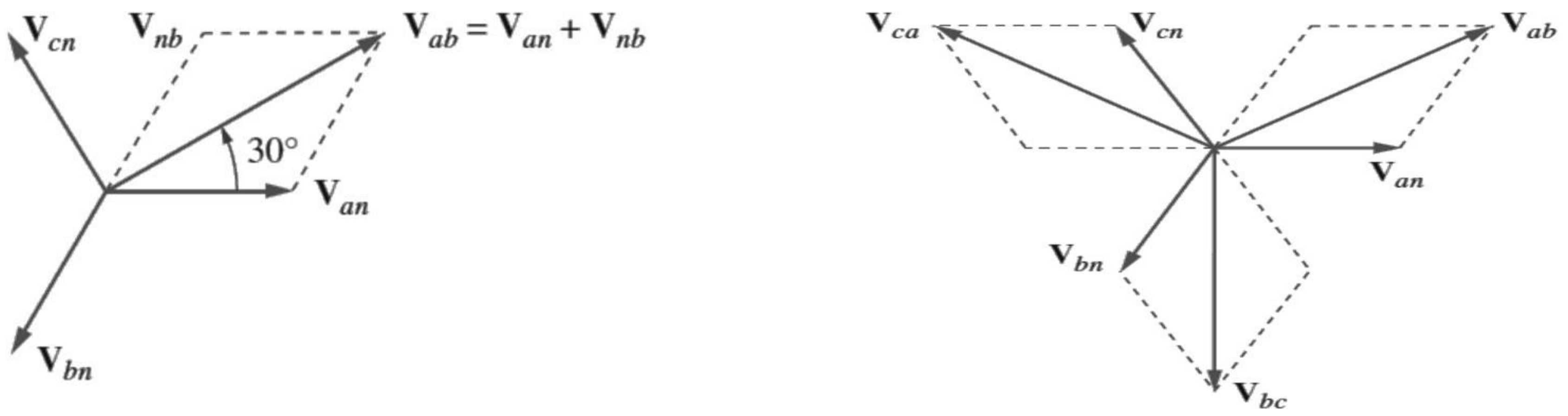
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} + \mathbf{V}_{na} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ$$

- Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_p or

$$V_L = \sqrt{3}V_p$$

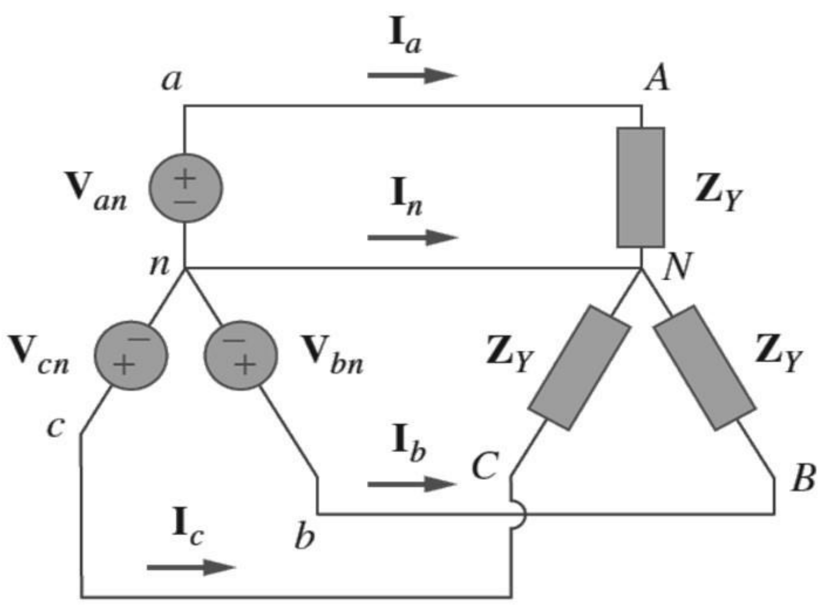
Balanced Wye Wye Connection (cont…)

- Also the line voltages lead their corresponding phase voltages by 30° as illustrated in the figure below.
- The figure also shows how to determine $\mathbf{V}_{ab}$ from the phase voltages.
- Notice that $\mathbf{V}_{ab}$ leads $\mathbf{V}_{bc}$ by 120° and $\mathbf{V}_{bc}$ leads $\mathbf{V}_{ca}$ by 120° so that the line voltages sum up to zero as do the phase voltages.



Balanced Wye Wye Connection (cont…)

- Applying KVL to each phase in the following figure,



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle-120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle-120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle-240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle-240^\circ$$

Balanced Wye Wye Connection (cont…)

- It can be readily inferred that,

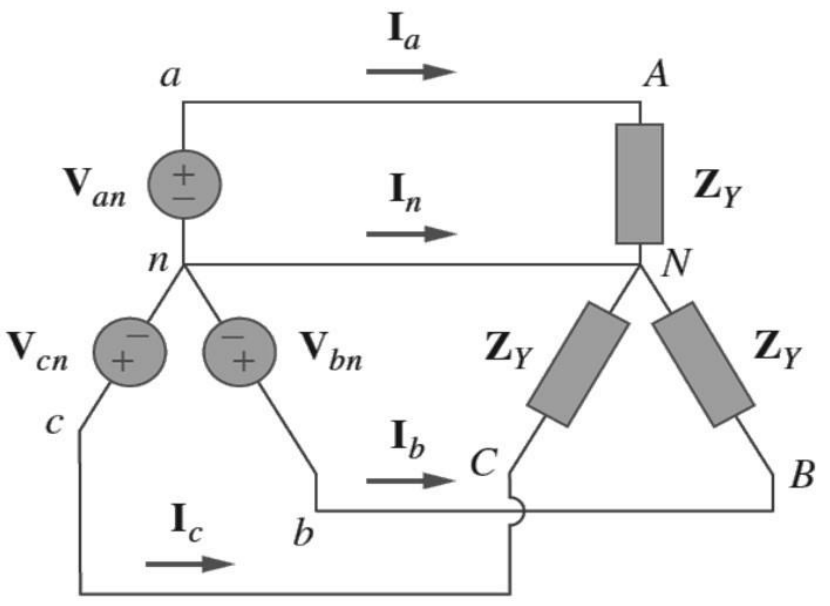
$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

- So that,

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

or

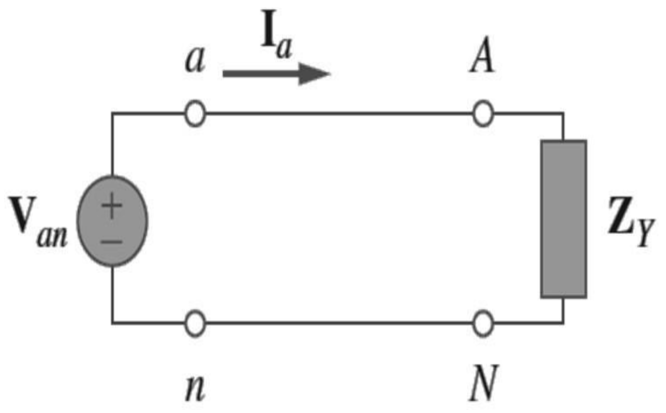
$$\mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0$$



- Therefore, the voltage across the neutral wire is zero.
- The neutral line can thus be removed without affecting the system.
- In long distance power transmission, conductors are used in multiples of three with the earth itself acting as the neutral conductor.
- Power systems designed in this way are well grounded at all critical points to ensure safety.

Balanced Wye Wye Connection (cont…)

- While the *line* current is the current in each line, the *phase* current is the current in each phase of the source or load.
- In the Y-Y system, the line current is the same as the phase current.
- We will use single subscripts for line currents because it is natural and conventional to assume that line currents flow from the source to the load.
- An alternative way of analyzing a balanced Y-Y system is to do so on a "per phase" basis.
- We look at one phase, say phase *a*, and analyze the single-phase equivalent circuit in the below figure.



Balanced Wye Wye Connection (cont…)

- The single-phase analysis yields the line current $\mathbf{I}_a$ as

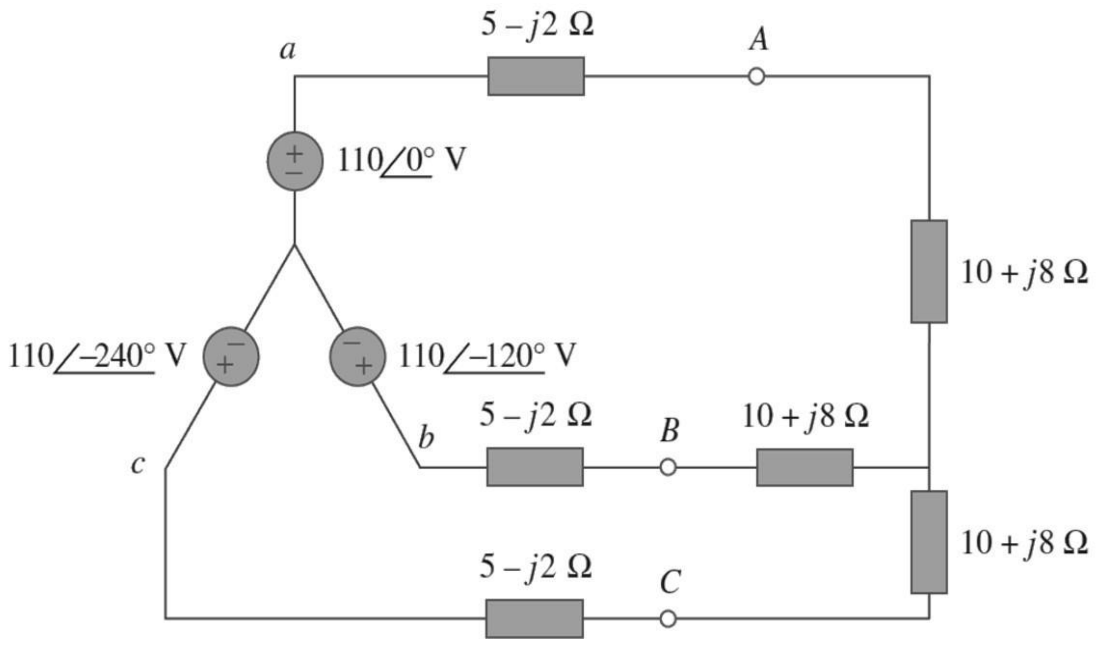
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

- We use the phase sequence to obtain other line currents.
- So, as long as the system is balanced, we need only analyze one phase.
- We may do this even if the neutral line is absent, as in the three-wire system.

Balanced Wye Wye Connection (cont…)

Example:

- Calculate the line currents in the three-wire Y-Y system given in the following figure?



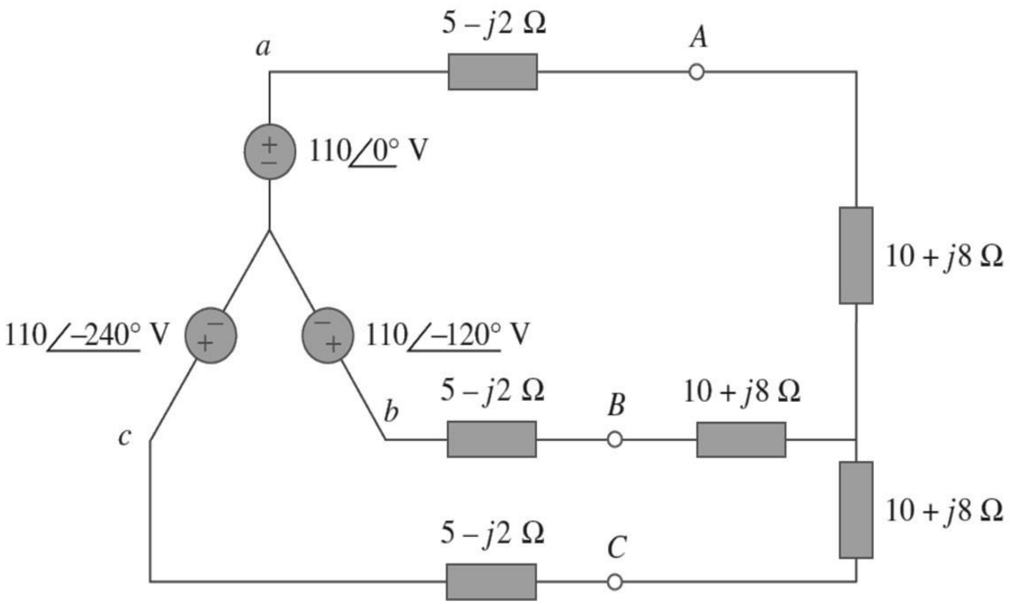
Solution: The three-phase circuit is balanced; we may replace it with its single-phase equivalent circuit as discussed previously.

Balanced Wye Wye Connection (cont…)

- The single-phase analysis yields the line current $\mathbf{I}_a$ as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

- Here,



$$\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8°.$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{110\angle 0°}{16.155\angle 21.8°} = 6.81\angle - 21.8° \mathrm{A}$$

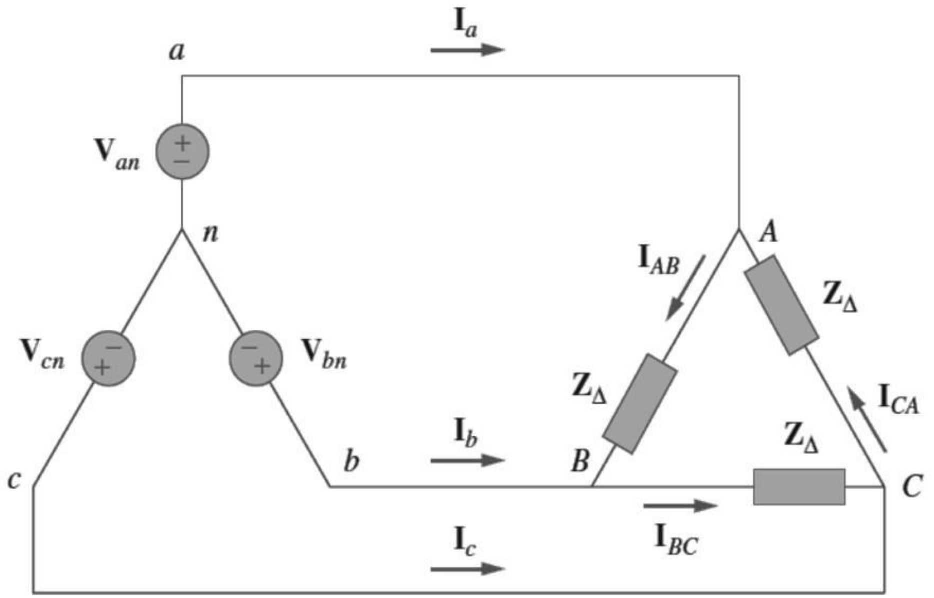
- Since the source voltages are in positive sequence, the line currents are also in positive sequence:

$$\mathbf{I}_b = \mathbf{I}_a\angle - 120° = 6.81\angle - 141.8° \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle - 240° = 6.81\angle - 261.8° = 6.81\angle 98.2° \text{ A}$$

Balanced Wye-Delta Connection

- A balanced Y-Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load.
- The balanced Y-delta system is shown in the below figure, where the source is Y-connected and the load is Δ-connected.
- There is, of course, no neutral connection from source to load for this case.



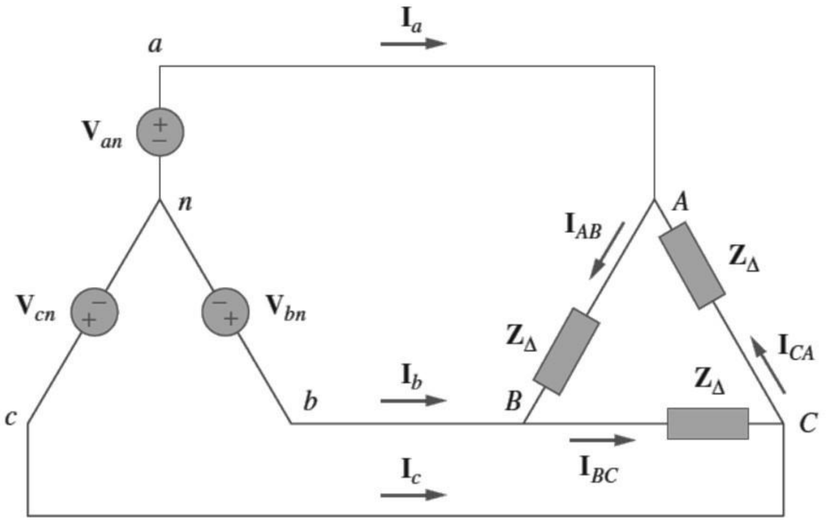
Balanced Wye-Delta Connection (Cont…)

- Assuming the positive sequence, the phase voltage or line to neutral voltage are,

$$\mathbf{V}_{\mathrm{an}} = V_{\mathrm{p}}\angle 0°$$

$$\mathbf{V}_{\mathrm{bn}} = V_{\mathrm{p}}\angle -120°$$

$$\mathbf{V}_{\mathrm{cn}} = V_{\mathrm{p}}\angle 120°$$



- As discussed previously the line voltages are,

$$\mathbf{V}_{\mathrm{ab}} = \sqrt{3}\, V_{\mathrm{p}}\angle 30° = \mathbf{V}_{\mathrm{AB}}$$

$$\mathbf{V}_{\mathrm{bc}} = \sqrt{3}\, V_{\mathrm{p}}\angle -90° = \mathbf{V}_{\mathrm{BC}}$$

$$\mathbf{V}_{\mathrm{ca}} = \sqrt{3}\, V_{\mathrm{p}}\angle -210° = \mathbf{V}_{\mathrm{CA}}$$

- This shows that the line voltages are equal to the voltages across the load impedances for this system configuration.

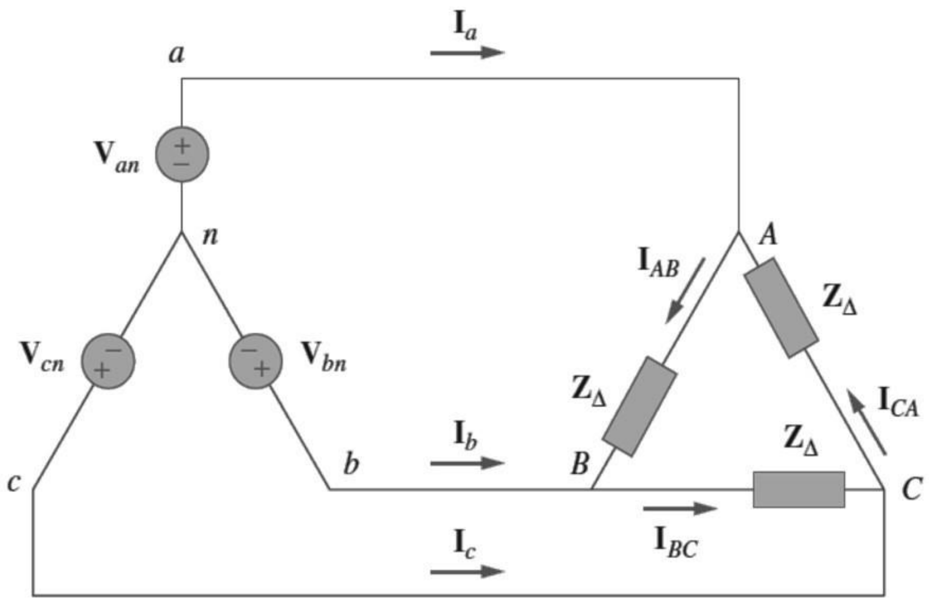
Balanced Wye-Delta Connection (Cont…)

- From these voltages, we can obtain the phase currents as,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}}$$



- These currents have the same magnitude but are out of phase with each other by 120°.
- Another way to obtain these phase currents is by applying KVL to the balance wye-delta connection.

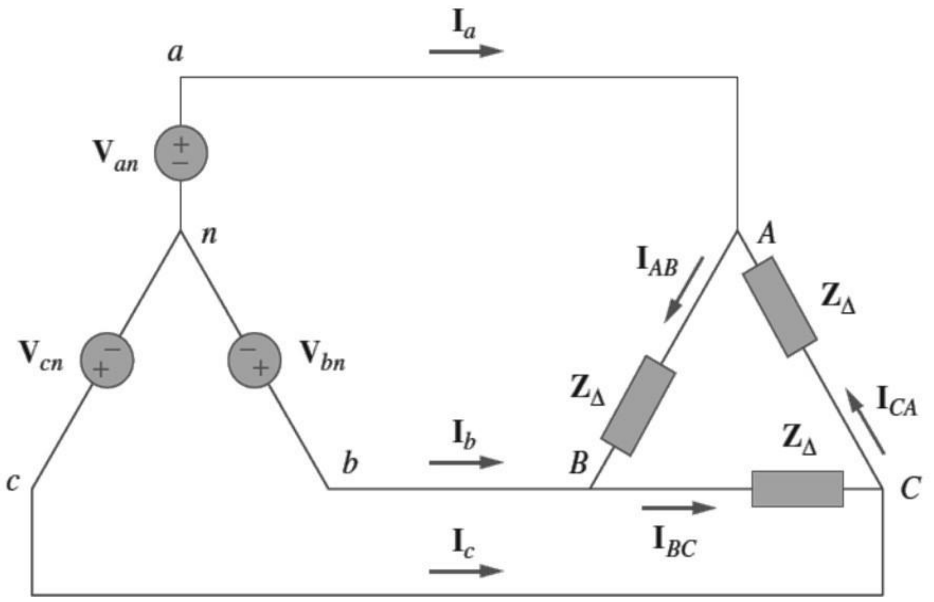
Balanced Wye-Delta Connection (Cont…)

- Applying KVL around the loop *aABbna* gives,

$$-Va_n + Z_{\Delta} I_{AB} + Vbn = 0$$

or

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}}$$



- The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C.

Balanced Wye-Delta Connection (Cont…)

- Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

- Since,

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240°$$

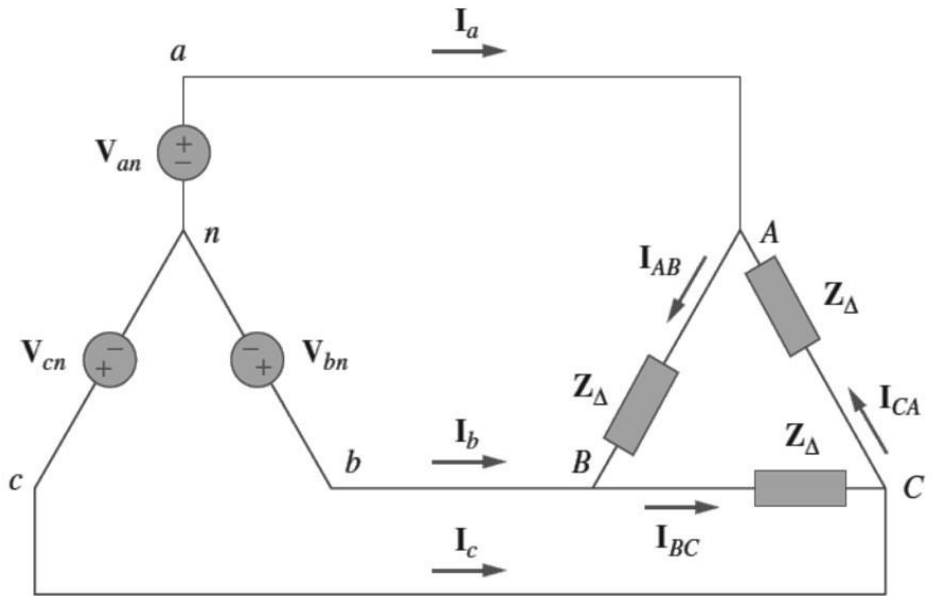
$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240°)$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

- Similarly,

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC}\sqrt{3}\angle -30°$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_{CA}\sqrt{3}\angle -30°$$



- This shows that the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, i.e.,

$$\mathbf{I}_\mathrm{L} = \sqrt{3}\,\mathbf{I}_\mathrm{p}$$

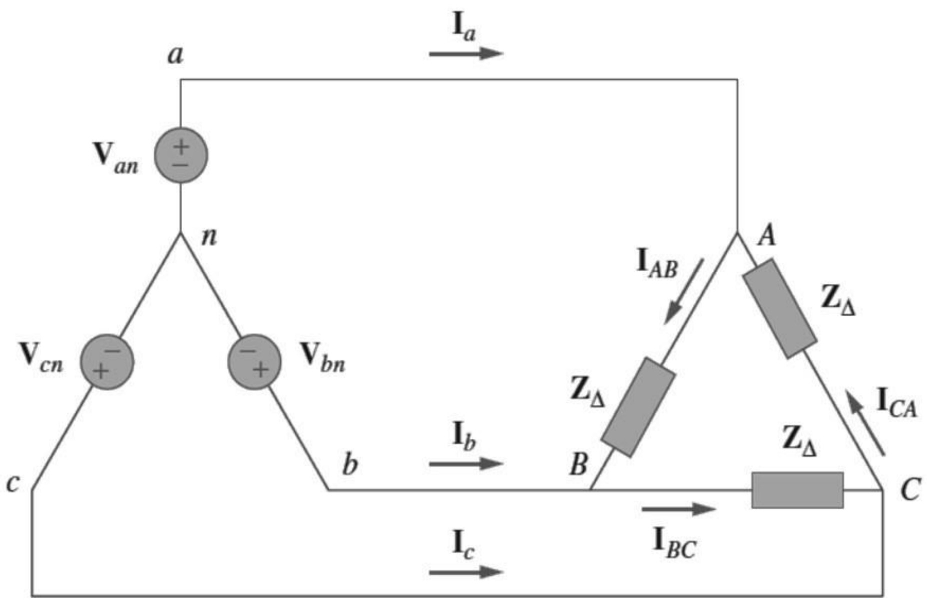
Balanced Wye-Delta Connection (Cont…)

- Here,

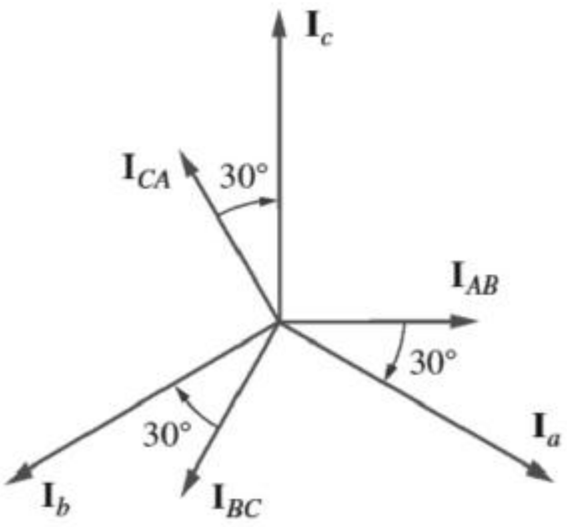
$$\text{line current}, \mathbf{I}_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

and

$$\text{phase current}, \mathbf{I}_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$



- Also, the line currents lag the corresponding phase currents by 30° assuming the positive sequence.
- This is illustrated in the phasor diagram shown below.



Balanced Wye-Delta Connection (Cont…)

- An alternative way of analyzing the circuit is to transform the Y-Δ connected load to an equivalent Y-Y connected load.
- Using the transformation formula

$$\mathbf{z}_Y = \frac{\mathbf{z}_\Delta}{3}$$

- After this transformation, we now have a Y-Y system.
- The three-phase system can then be replaced by the single phase equivalent circuit as discussed in the balanced Y-Y case.
- This allows us to calculate only the line currents.
- The phase currents are obtained using $\mathbf{I}_L = \sqrt{3}\,\mathbf{I}_p$ and utilizing the fact that each of the phase currents leads the corresponding line current by 30°.

Balanced Wye-Delta Connection (Cont…)

Example:

- A balanced abc sequence Y-connected source with $\mathbf{V}_{\text{an}} = 100\angle 10°$ V is connected to a delta connected balanced load $(8 + j4)\Omega$ per phase. Calculate the phase and line currents?

Solution: The load impedance is,

$$\mathbf{Z}_{\Delta} = (8 + j4)\ \Omega = 8.944\angle 26.57°\Omega$$

If the phase voltage $\mathbf{V}_{\text{an}} = 100\angle 10°$ V, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30° = 100\sqrt{3}\angle(10° + 30°)\mathbf{V} = 173.2\angle 40°\mathbf{V} = \mathbf{V}_{AB}$$

Balanced Wye-Delta Connection (Cont…)

- The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \ \mathbf{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^\circ = 19.36\angle 133.43^\circ \ \mathbf{A}$$

- The line currents are

$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle(13.43^\circ - 30^\circ) = 33.53\angle -16.57^\circ\text{A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120^\circ = 33.53\angle -136.57^\circ\text{A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120^\circ = 33.53\angle 103.43^\circ\text{A}$$


THANKS
FOR YOUR
ATTENTION