

Low-Frequency Response

- *The Infinite-Value Time Constant (IVTC) Technique:*
 - Used for obtaining the *lower cutoff frequency* (f_L)
 - If a circuit has n number of *capacitors*, then it would have n number of *time constants*
 - *This technique derives the information regarding f_L from these time constants*

- *The Algorithm:*

- *Null all independent sources to the circuit*

- *Short all independent voltage sources*

- *Open all independent current sources*

- *DO NOT TOUCH DEPENDENT SOURCES*

- *Name the capacitors C_i ($i = 1$ to n)*

- *Consider C_1 and assign infinite values to all other capacitors (thus the name!)*

- *Thus, except C_1 , all other capacitors will short out*

- *Determine the Thevenin Resistance (R_1^∞) across the two terminals of C_1*

- *Find the time constant τ_1 associated with C_1*

$$(\tau_1 = R_1^\infty C_1)$$
- *Calculate the corresponding frequency $f_1 = 1/(2\pi\tau_1)$*
- *Repeat for all other capacitors, taking one at a time, and find all the rest of the frequencies (f_2, f_3, \dots, f_n)*
- Then the *Lower Cutoff Frequency* f_L can be expressed as:

$$f_L = \left[\sum_{i=1}^n f_i^2 \right]^{1/2}$$

- In *discrete circuits*, a *major component of total cost* is due to the *cost of the capacitors* (*directly proportional to the value*)
- Hence, an attempt is made to *minimize* the *total capacitor requirement of the circuit*
- For this, the *Dominant Pole* (DP) technique is used
 - *One of the frequencies among f_1 - f_n is made dominant*
 - *Others are made to lie at least 10 times away from it*

- For example, if *f_d is chosen to be the DP*, then *all other poles are assumed to be at $f_d/10$*

$$\Rightarrow f_L = \left[f_d^2 + \sum_{n=1} \left(\frac{f_d}{10} \right)^2 \right]^{1/2}$$

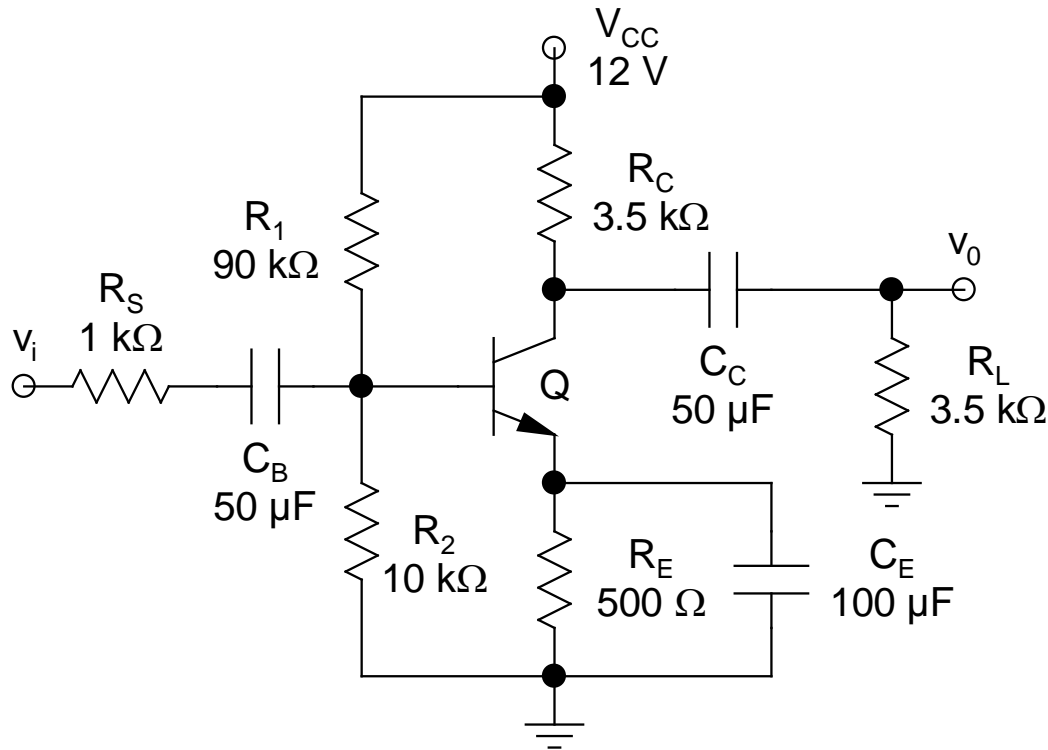
- C_d , which *contributes* f_d , is *chosen* to be that capacitor that *sees* the *least Thevenin resistance* across its terminals

- *Reason is obvious:*

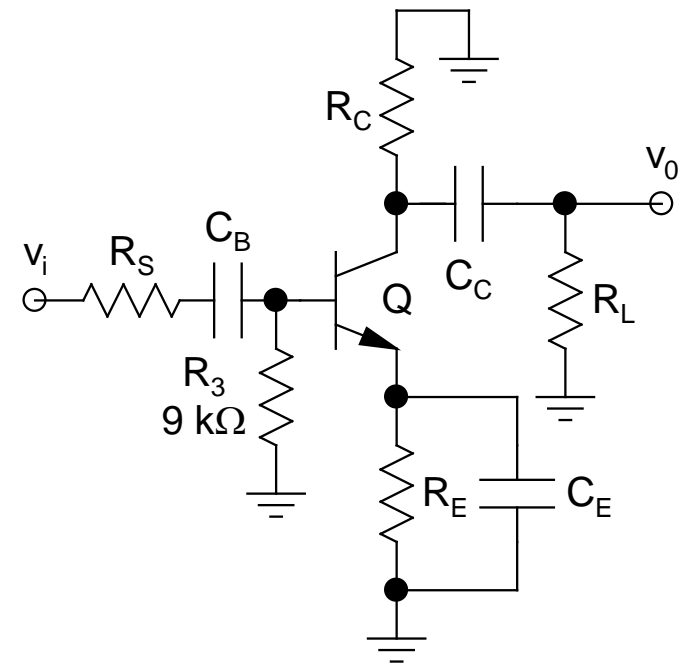
- If *any other capacitor* were *chosen* to *contribute* f_d , then C_d would have been *ten times higher*

- *This choice is based on heuristics*

- Low-Frequency Response of RC-Coupled Amplifier:***



Complete Circuit

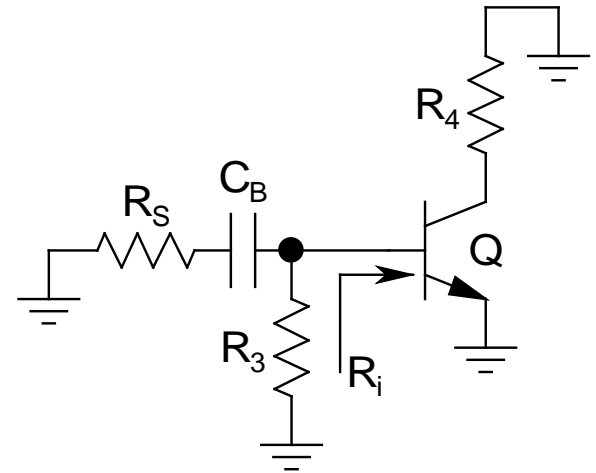


ac Schematic

- *DC analysis* gives $I_C = 1 \text{ mA}$ and $V_{CE} = 8 \text{ V}$
 $\Rightarrow r_E = 26 \Omega$ and $r_\pi = 2.6 \text{ k}\Omega$ (assuming $\beta = 100$)
- *Neglect Early effect*
 $\Rightarrow r_0 \rightarrow \infty$
- 3 *capacitors* (C_B, C_E, C_C) with *time constants* τ_1, τ_2, τ_3 , and corresponding *cutoff frequencies* f_1, f_2, f_3
- To apply the *IVTC technique*, we have to take *one capacitor at a time* and *treat other capacitors as short circuits*
- *The analysis can be done by inspection!*

➤ C_B :

- *Short C_C and C_E*
- $R_3 = R_1 \parallel R_2 = 9 \text{ k}\Omega$
- $R_4 = R_C \parallel R_L = 1.75 \text{ k}\Omega$
- $R_i = r_\pi = 2.6 \text{ k}\Omega$
- By inspection, the *Thevenin resistance* seen by C_B :
$$R_B^\infty = R_S + (R_3 \parallel R_i) = 3 \text{ k}\Omega$$
$$\Rightarrow \tau_1 = R_B^\infty C_B = 150 \text{ ms}$$
$$\Rightarrow f_1 = 1/(2\pi\tau_1) = 1.06 \text{ Hz}$$



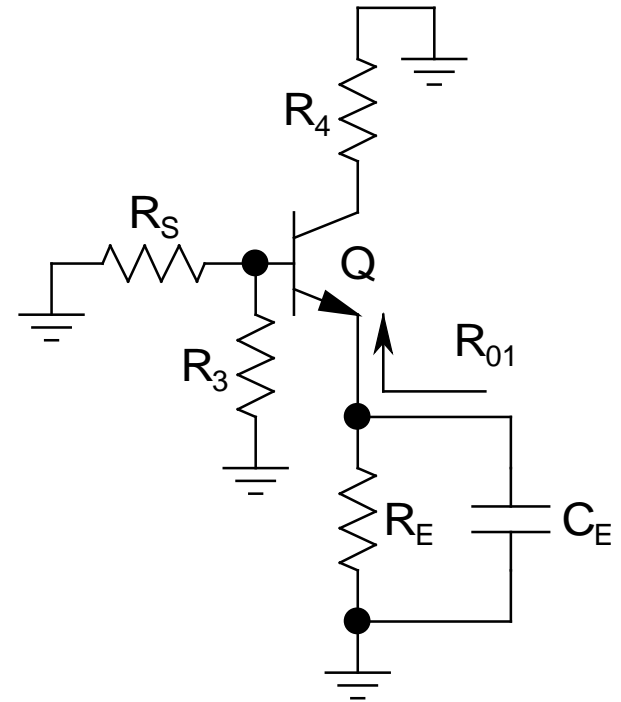
➤ C_E :

- *Short C_C and C_B*
- $R_{01} = r_E + (R_S \parallel R_3)/(\beta + 1)$
 $= 34.9 \Omega$
- By inspection, the *Thevenin resistance* seen by C_E :

$$R_E^\infty = R_E \parallel R_{01} = 32.6 \Omega$$

$$\Rightarrow \tau_2 = R_E^\infty C_E = 3.26 \text{ ms}$$

$$\Rightarrow f_2 = 1/(2\pi\tau_2) = 48.8 \text{ Hz}$$



➤ C_C :

- *Short C_E and C_B*

- By inspection, the *Thevenin resistance* seen by C_C :

$$R_C^\infty = R_C + R_L = 7 \text{ k}\Omega$$

$$\Rightarrow \tau_3 = R_C^\infty C_C = 350 \text{ ms}$$

$$\Rightarrow f_3 = 1/(2\pi\tau_3) = 0.45 \text{ Hz}$$

- Thus, the *lower cutoff frequency* of the circuit:

$$f_L = \left[f_1^2 + f_2^2 + f_3^2 \right]^{1/2} = 48.8 \text{ Hz}$$

➤ Note that f_L is equal to f_2 (contributed by C_E)

➤ Now let's attempt to *minimize* the *total capacitance requirement* of the circuit

➤ ***Minimization of the Total Capacitance:***

- From the previous analysis, we note that C_E *sees the least Thevenin resistance across its two terminals*

⇒ *Let's choose C_E to contribute the DP f_d , and let C_C and C_B each contribute poles at $f_d/10$*

$$\Rightarrow 48.8 = \sqrt{f_d^2 + 2(f_d/10)^2}$$

$$\Rightarrow f_d = 48.3 \text{ Hz and } f_d/10 = 4.83 \text{ Hz}$$

- Thus:

$$C_E = 1 / (2\pi f_d R_E^\infty) = 101.1 \text{ } \mu\text{F}$$

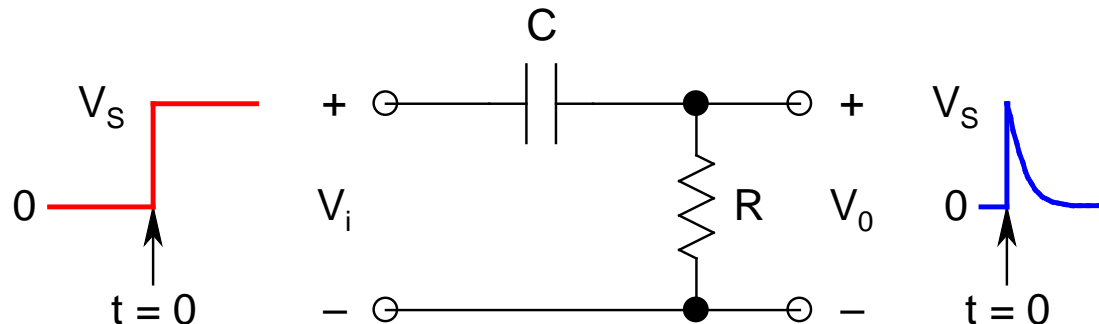
$$C_B = 1 / [2\pi (f_d/10) R_B^\infty] = 11 \text{ } \mu\text{F}$$

$$C_C = 1 / [2\pi (f_d/10) R_C^\infty] = 4.7 \text{ } \mu\text{F}$$

- Thus, the *total capacitance* requirement comes out to be *116.8 μF* , for the *same f_L of 48.8 Hz*
- The *original circuit* had a *total capacitance* of *200 μF*
- Thus, this approach gave a *cost saving* of almost *42%* in terms of the *capacitors*
- As an *exercise*, you can pick *either C_C or C_B* to *contribute f_d* , and find the *total capacitance* requirement for each case
- Finally, after all, this is a *heuristic*
- To get the *absolute minimum value* of the *total capacitance*, we need to *formulate the problem*, and *find the minima of the function mathematically*

- **Tilt/Sag:**

- For **pulse/square wave excitation**, f_L dictates the amount of **tilt/sag** present in the **output**
- **Due to f_L** , the circuit effectively behaves like a **HPF**, represented by a simple **RC circuit**
- Under **step input**, the **output** would be a **spike**



➤ Thus:

$$V_0 = V_S \exp(-t/\tau_L)$$

$$\tau_L = RC = 1/\omega_L \quad (\omega_L = 2\pi f_L)$$

➤ For $t \ll \tau_L$:

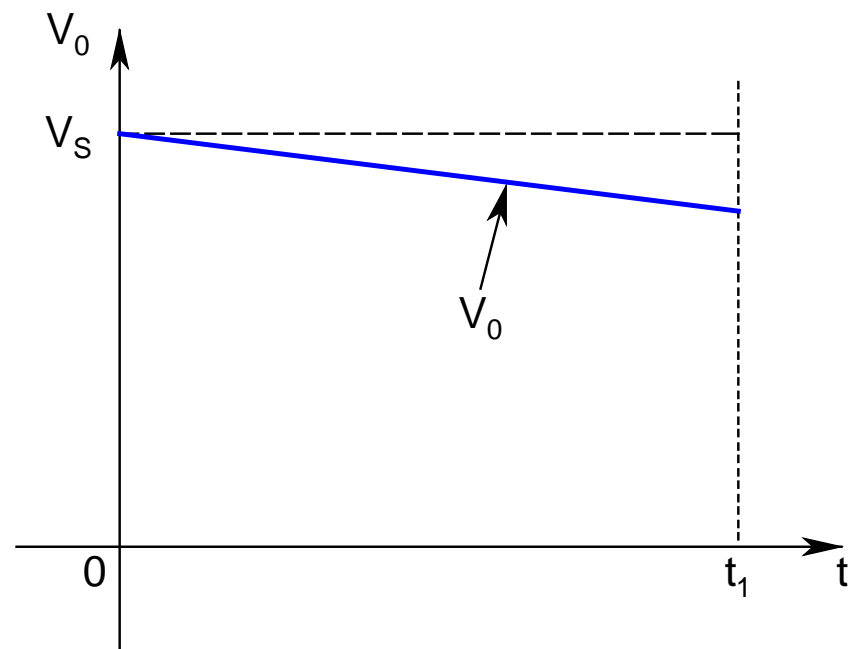
$$V_0 \approx V_S(1 - t/\tau_L)$$

$$= V_S(1 - \omega_L t)$$

$$= V_S(1 - 2\pi f_L t)$$

➤ Thus, *V_0 drops linearly with time*

➤ *Quantified by percent tilt/sag (P)*



$$\begin{aligned} \text{➤ } P &= [(V_S - V_0)/V_S] \times 100\% \\ &= (t_1/\tau_L) \times 100\% \end{aligned}$$

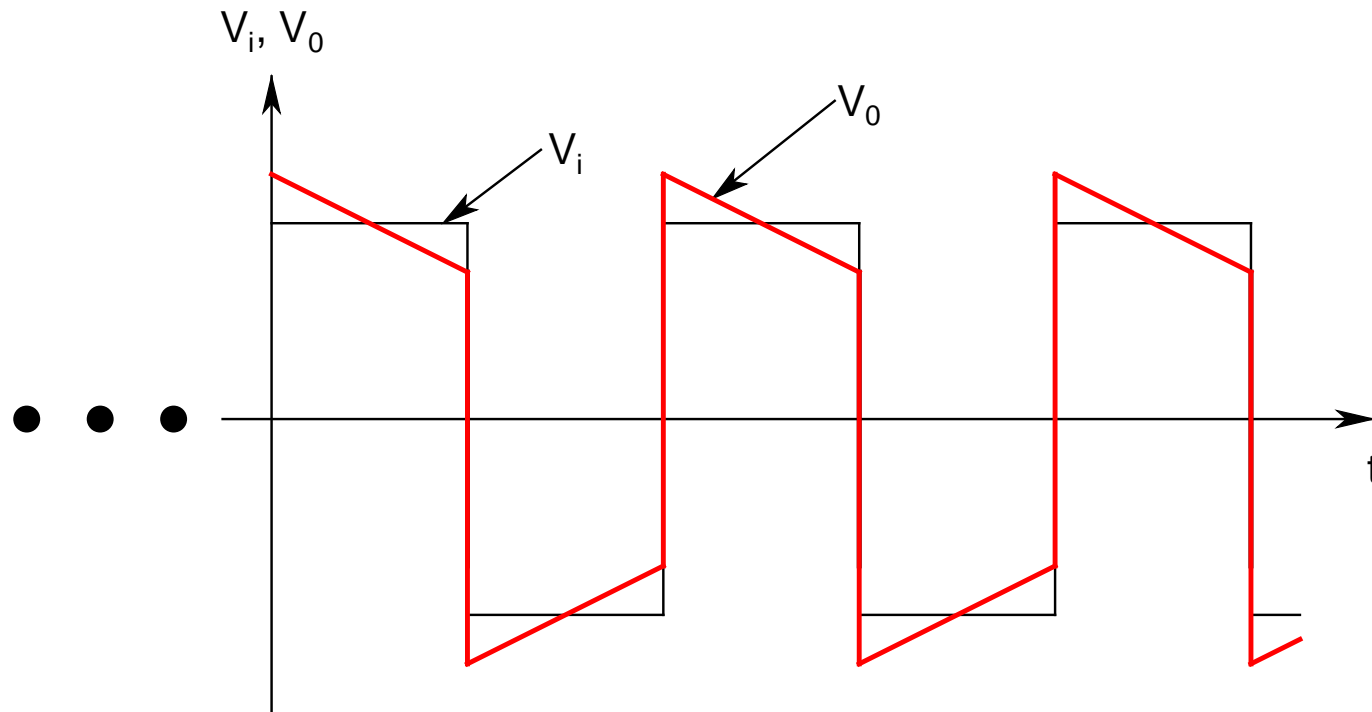
$t_1 = \text{Time at which the tilt is measured}$

➤ For *square wave input*, $t_1 = T/2$ ($T = \text{period} = 1/f$, $f = \text{cycle frequency}$)

$$\begin{aligned} \Rightarrow P &= [T/(2\tau_L)] \times 100\% = [\omega_L/(2f)] \times 100\% \\ &= (\pi f_L/f) \times 100\% \end{aligned}$$

➤ *Note: P is directly proportional to f_L and inversely proportional to f*

\Rightarrow *Circuits having low f_L , will show significant amount of tilt/sag at low frequencies*



Tilt/Sag