Assignment 11
MSO-201: Probability and Statistics

1. Suppose X is a random variable show that $E(X) = E_Y(E_{X|Y}(X|Y))$ when all the expectations exist.

2. Suppose X is a random variable show that $V(X) = E_Y(V_{X|Y}(X|Y)) + V_Y(E_{X|Y}(X|Y))$, when all the variances exist.

3. Suppose X and Y are random variables with the following joint PDF

$$f_{X,Y}(x,y) = \frac{\lambda^\alpha}{\Gamma(\alpha)}(x-y)^{\alpha-1}e^{-\lambda(x-y)}e^{-y}; \quad x > y, y > 0.$$

   Find $E(X)$ and $V(X)$.

4. Suppose X, Y and Z are three independent exponential random variables with mean one. Let $U = \min\{X, Z\}$ and $V = \min\{V, Z\}$. Find the joint survival function of U and V. Find the marginal distributions of U and V. Are they independent?

5. Suppose X and Y are independent and identically distributed Gamma(2,1) random variables. Find the joint distribution of $U = X + Y$ and $V = \dfrac{X}{X+Y}$

6. In the above problem, find the PDF of V. It is known as Beta distribution.

7. Suppose X, Y and Z are independent and identically distributed Gamma(2,1) random variables. Find the joint distribution of $U = X + Y + Z$, $V = \dfrac{X}{X+Y+Z}$ and $W = \frac{Y}{X+Y+Z}$.

8. In the above problem, find the joint PDF of V and W. It is known as Dirichlet distribution.

9. Suppose X and Y are independent and identically distributed N(0,1) random variables. Suppose we have the following transformation $X = R\cos(\theta)$ and $Y = R\sin(\theta)$, where $R > 0$ and $0 < \theta < 2\pi$. Find the joint PDF of R and θ.

10. Suppose $X_1, \ldots, X_n$ are indendent and identically distributed N(0,1) random variables. Find the joint PDF of $Y = X_1 + \ldots + X_n$.