Final Exam, EE 250 (Control Systems Analysis), Spring 2012 *

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

Instructions

1. You may use a pencil for trial and error, but your final drawing should use a pen.
2. Show all your assumptions.
3. You will need the following items: pen, pencil, ruler, eraser, calculator. Borrowing not permitted.
4. If we have difficulty in reading your answer book, we will deduct points arbitrarily. It is your responsibility to write legibly.
5. Giving or receiving help may result in an F.
6. Write your name, etc on every page.

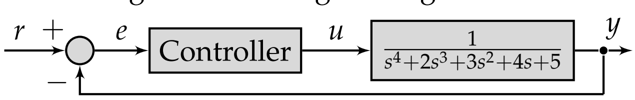
Useful information

Controller	k_p	T_i	T_D
P	$0.5k_{cr}$	∞	0
PI	$0.45k_{cr}$	$P_{cr}\,/\,1.2$	0
PID	$0.6k_{cr}$	$P_{cr}\,/\,2$	$P_{cr}\,/\,8$

$$\frac{1}{\Delta}\sum_{i=1}^{N} P_i \Delta_i$$

Problems

1. In using the ultimate gain Ziegler-Nichols tuning method for the control system



1.1. **[3 points]** Determine the critical period and critical gain.

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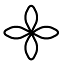
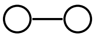
1.2. **[1 points]** Determine the closed-loop poles corresponding to the critical gain.

1.3. **[2 points]** Write the transfer function, including numerical values, of a practically implementable PID controller.

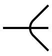
2. **[4 points]** In each of the following cases, set up coordinate axes, mark the locations of the poles and zeros, and write down adjacent to these roots the respective multiplicities, so that the respective root locus possesses the form mentioned. Here is an example.

Form:  or 

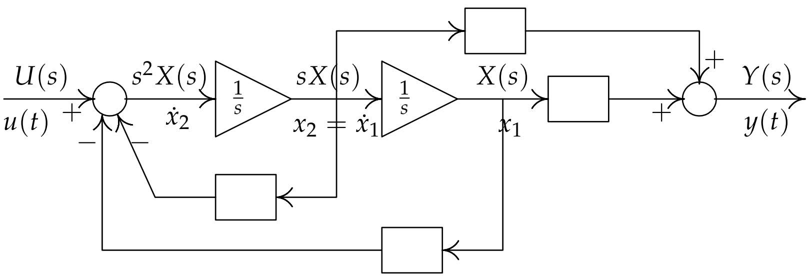
2.1. Form: 

2.2. Form: 

2.3. Form: 

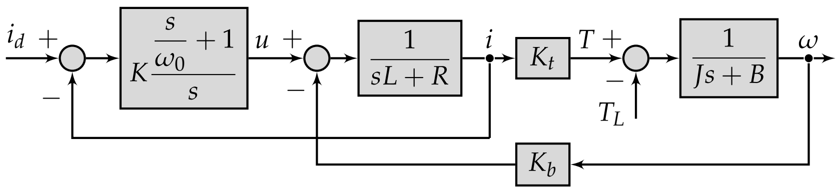
2.4. Form: 

3. **[4 points]** In the process of numerically integrating the equation $\frac{d^2y}{dt^2} + 1\frac{dy}{dt} + 2y = 3\frac{du}{dt} + 4u$, you have arrived at the following diagram.



Fill the appropriate numbers into the appropriate blank boxes of this diagram.

4. **[5 points]** Consider the following proportional-integral control system for the armature current of a permanent magnet dc motor.



4.1. We wish i to track step i_d with a small settling time. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

4.1.1. Large ω_0.

4.1.2. Small ω_0.

4.2. We wish i to track step i_d with a small overshoot. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

4.2.1. Large ω_0.

4.2.2. Small ω_0.

4.3. We wish i to track step i_d with a small settling time. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

4.3.1. Large K.

4.3.2. Small K.

4.4. We wish i to track step i_d with a small overshoot. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

4.4.1. Large K.

4.4.2. Small K.

4.5. We wish i to track step i_d with a small steady-state error. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

4.5.1. Large K.

4.5.2. Small K.

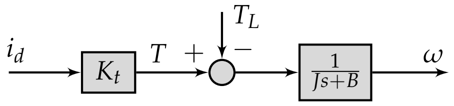
4.5.3. Large ω_0.

4.5.4. Small ω_0.

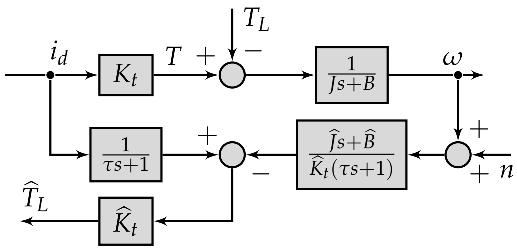
4.5.5. K and ω_0 do not affect the steady-state error.

5. The solution to the previous problem results in a well-regulated i.

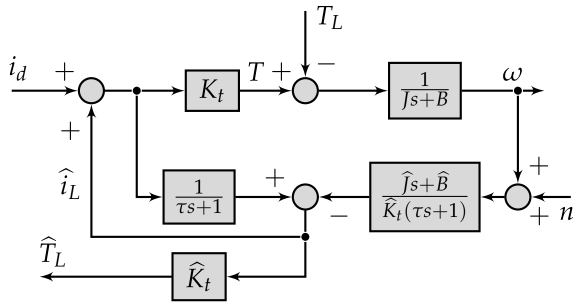
The motor with well-regulated i can be represented as



Here is an open-loop (OL) disturbance observer (DOB) scheme to generate an estimate $\widehat{T}_L$ of the load torque T_L.



On the other hand, the following scheme is a closed-loop (CL) DOB scheme.



Here, the quantities with the hats are estimates of the quantities without hats.

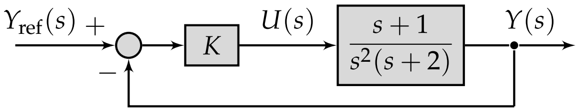
5.1. **[1 points]** Write the transfer function (TF) from T_L to $\widehat{T}_L$ for the OL DOB scheme.

5.2. **[2 points]** Determine the TF from T_L to $\widehat{T}_L$ for the CL DOB scheme.

5.3. **[2 points]** Assuming $\widehat{K}_t \approx K_t$, and that $\widehat{J}$ and $\widehat{B}$ are significantly different from J and B, for a unit step T_L, determine in which scheme $\widehat{T}_L$ is closer to T_L in steady state.
Hint: Use the final value theorem $y(t = \infty) = \lim_{s \to 0} sY(s)$.

5.4. **[2 points]** Assume that $i_d = 1$ A results in $\omega = 100$ rad/s while $T_L = 0$. If a non-zero T_L appears, determine which scheme rejects this disturbance better. That is, for a fixed i_d, in which scheme is ω less affected in steady state by the appearance of a step T_L?

6. The negative root locus (NRL) of the following system is the locus of its poles for $K \in [0, -\infty)$.

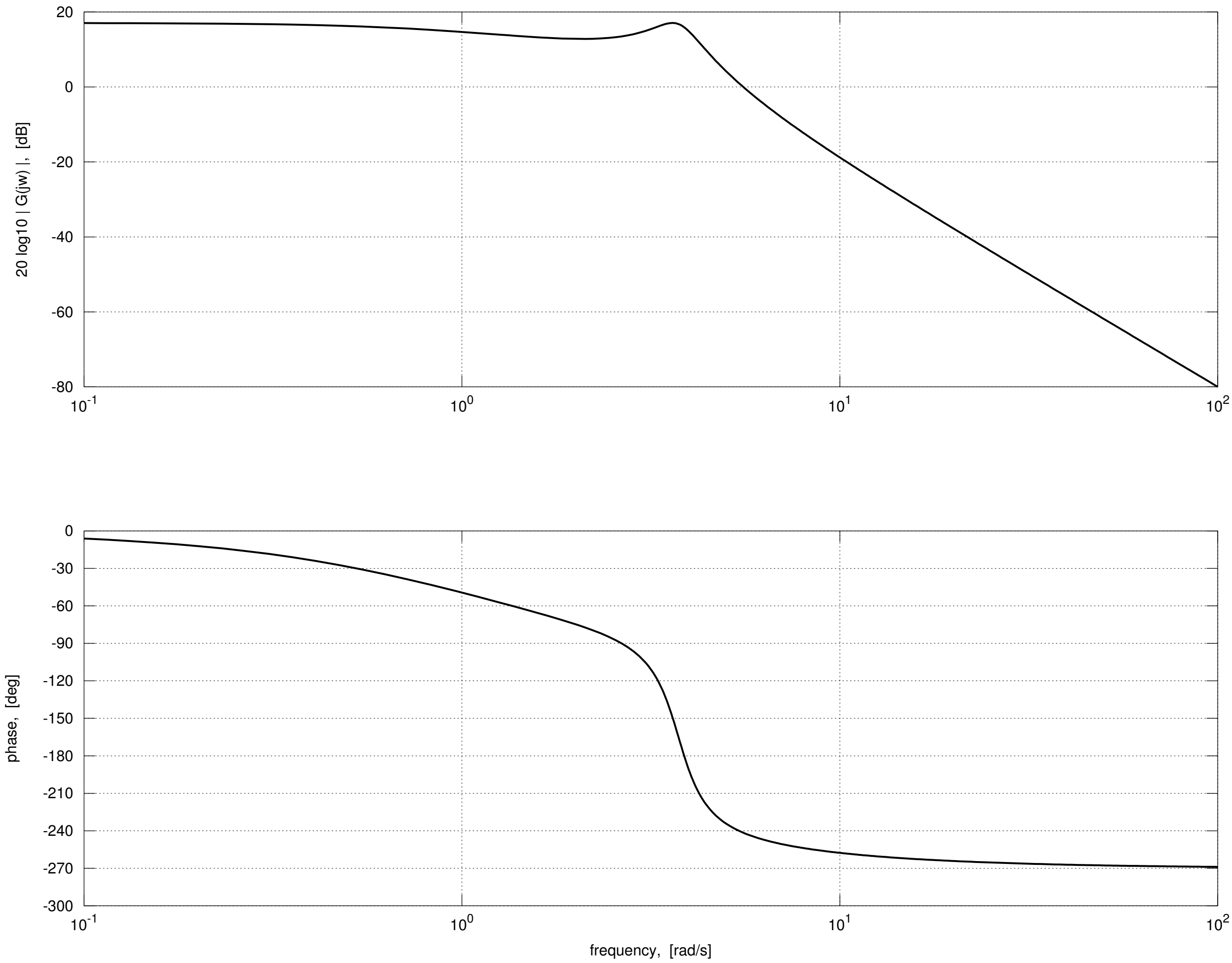


6.1. **[1 points]** Write the phase condition that the closed-loop poles of this system must satisfy.

6.2. **[2 points]** Mark the open loop poles and zeros on the s-plane and darken the sections of the real axis where the NRL lies.

6.3. **[2 + 2 points]** For this NRL, determine the centroid of the asymptotes and the angles at which they leave the centroid.

7. **[1+1 points]** Perform the steps shown for the following Bode plot.



7.1. Find the gain margin in dB and in absolute units.

7.2. Evaluate the phase margin in degrees.

8. **[2 points]** Given $G_{\mathrm{CL}}(s) = \frac{G_{\mathrm{OL}}(s)}{1+G_{\mathrm{OL}}(s)}$. Evaluate $\left|G_{\mathrm{CL}}(j\omega_g)\right|$ for PM $= 60^\circ$. Here, ω_g is the gain crossover frequency.

9. **[3 points]** Use Nyquist stability test to determine the values of $K \geq 0$ for which the following closed-loop system is stable. Here $0 < \zeta < 1$ and $\omega_n > 0$.