**Lecture-8**

**On**

# INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Impedance and Admittance.
- Effective or RMS Voltage, Current and power.

# Average Power (Cont…)

- Next, we consider two special cases.
- When $\theta_v = \theta_i$, we know that the voltage and the current are in phase.
- This is a purely resistive circuit, and the average power is given by,

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$$

- This shows that a purely resistive load absorbs power at all times.
- When $\theta_v$ - $\theta_i$ = ±90°, the circuit is a purely reactive circuit, and the average power is.

$$P = \frac{1}{2} V_m I_m \cos 90 = 0$$

- Thus, a purely reactive circuit absorbs no average power.

## ❑ Example:

Find the instantaneous and average power absorbed by a passive linear circuit if it is excited by a source $v(t)$ = 120 cos(377$t$ + 45°) V and current in the circuit is $i(t)$ = 10 cos(377$t$ - 10°) A?

**Solution:** The instantaneous power is given by,

$$p(t) = vi = 1200 \cos(377t + 45°) \cos(377t — 10°)$$

$$=600 \cos[(754t + 35°) + \cos 55°]$$

The average power can be evaluated using:

$$\text{P} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} * 120*10*\cos(45-(-10)) = 344.2\text{W}$$

This is same as the constant part of $p(t)$ above.

# Impedance and Admittance

- The voltage-current relation for the circuit elements is given by,

$$\boldsymbol{V} = R\boldsymbol{I}, \qquad \boldsymbol{V} = j\omega L\boldsymbol{I}, \qquad \boldsymbol{V} = \frac{\boldsymbol{I}}{j\omega C}$$

- The above equations can be rewritten in the terms of the ratio of phasor voltage to the phasor current as,

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = R, \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L, \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{1}{j\omega C}$$

- From the above equations we obtain Ohm's law in phasor notation for any element as,

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = \boldsymbol{Z} \quad or \quad \boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}$$

- Here, $\boldsymbol{Z}$ is a frequency dependent quantity known as the impedance.

## Impedance and Admittance (cont…)

- The impedance $\boldsymbol{Z}$ of a circuit is the ratio of the phasor voltage $\boldsymbol{V}$ to the phasor current $\boldsymbol{I}$, measured in ohms (Ω).
- The impedance represents the opposition that the circuit exhibits to the flow of sinusoidal current.
- Although the impedance is the ratio of two phasors, it is not a phasor, because it does not correspond to a sinusoidally varying quantity.
- As a complex quantity, the impedance can be expressed in rectangular form as,

$$\boxed{\boldsymbol{Z} = R + jX}$$

- Here, $R$, the real part of $\boldsymbol{Z}$, is the resistance and $X$, the imaginary part of $\boldsymbol{Z}$, is the reactance.
- We say that the impedance is inductive when $X$ is positive and the impedance is capacitive when $X$ is negative.

# Impedance and Admittance (cont…)

- Thus, impedance $\boldsymbol{Z} = R + jX$ is said to be inductive or lagging since current lags the voltage.
- Similarly, the impedance $\boldsymbol{Z} = R - jX$ is said to be capacitive or leading since current leads the voltage.
- As a complex quantity, the impedance can be expressed in polar form as, $\boldsymbol{Z} = |Z| \angle\theta$.
- Therefore, impedance can be expressed as,

$$\boxed{\boldsymbol{Z} = R + jX = |\boldsymbol{Z}|\angle\theta}$$

$$|Z| = \sqrt{R^2 + X^2} \qquad R = |Z|\cos\theta$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) \qquad X = |Z|\sin\theta$$

# Impedance and Admittance (cont…)

- It is sometimes convenient to use reciprocal of the impedance is calculations.
- This quantity is known as the admittance and is measured in siemens (S).
- It is expressed mathematically as,

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = G + jB$$

- Here, $\boldsymbol{G}$, the real part of $\boldsymbol{Y}$, is the conductance and $\boldsymbol{B}$, the imaginary part of $\boldsymbol{Y}$, is called the susceptance.

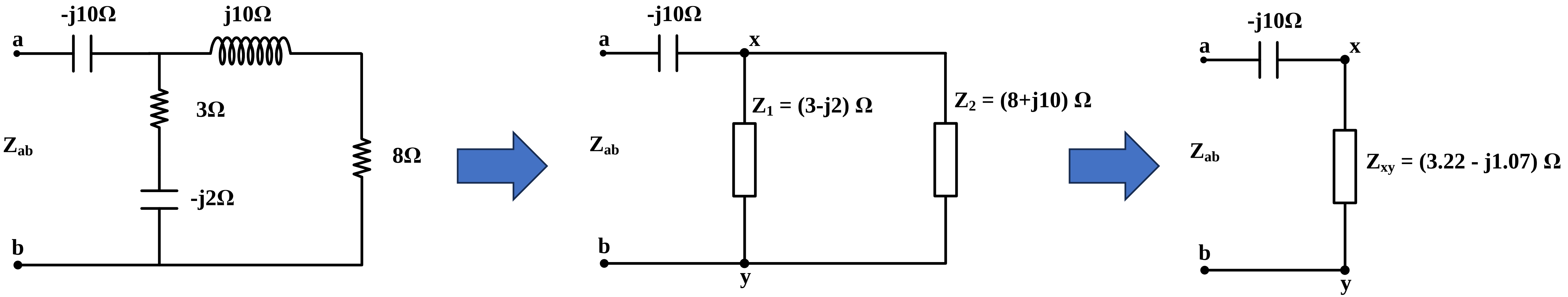
$$G + jB = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

- Therefore,

$$G = \frac{R}{R^2 + X^2}, B = -\frac{X}{R^2 + X^2}$$

# Impedance and Admittance (cont…)

- Find $Z_{ab}$?

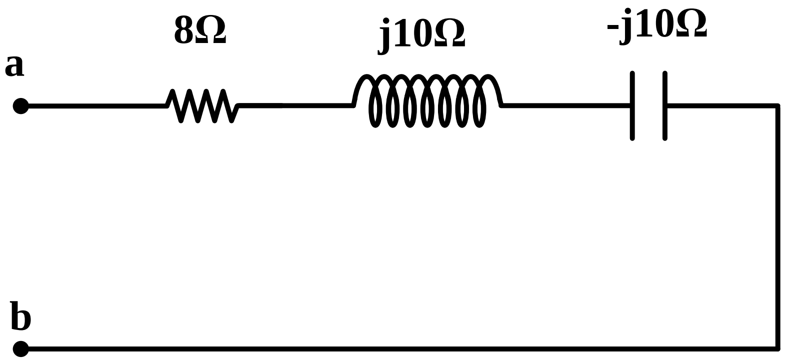


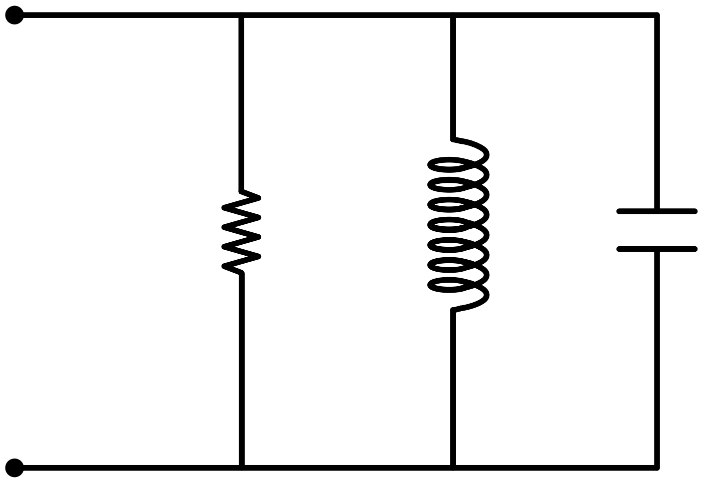

$$Z_{xy} = \frac{(3 - j2) \times (8 + j10)}{(3 - j2) + (8 + j10)} = 3.22 - j1.07$$

$$Z_{ab} = -j10 + 3.22 - j1.07 = 3.22 - j11.07$$

# Impedance and Admittance (cont…)

- Draw the phasor diagram for the circuits shown below:



# Effective or RMS Voltage and Current

- The idea of effective value arises from the need to measure the effectiveness of a sinusoidal voltage or current source in delivering power to a resistive load.
- The effective value of a periodic current is the equivalent DC current that delivers the same average power to the resistor as the periodic current does.
- The average power absorbed by a resistor in an AC circuit is given by,

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

$$\boxed{P = \frac{1}{T}\int_0^T i^2 R\,dt}$$

# Effective or RMS Voltage and Current (cont…)

- The average power absorbed by a resistor in DC circuit is given by,

$$P = I_{eff}^2 R$$

- From the above two equations we can observe that,

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$\therefore P = \frac{1}{T}\int_0^T i^2 R\, dt$$

- The effective value of voltage can be found out in a similar way as,

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

# Effective or RMS Voltage and Current (cont…)

- For the sinusoid $v(t) = V_m \cos \omega t$:

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T V_m^2 cos^2 \omega t dt} = \sqrt{\frac{V_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos(2\omega t))dt}$$

# Effective or RMS Voltage and Current (cont…)

- The effective value is the square root of the mean (or average) of the square of the periodic signal. Thus, the effective value is often known as the root-mean-square value, or rms value for short.

- Therefore, for the sinusoid $v(t)= V_m\cos\,\omega t$, the rms value is given by,

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 cos^2\omega t dt} = \sqrt{\frac{V_m^2}{T}\int_0^T \frac{1}{2}(1+\cos(2\omega t))dt} = \frac{V_m}{\sqrt{2}}$$

- For the sinusoid $i(t) = I_m\cos\,\omega t$,

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

- Please note that the above relationship holds only for **sinusoidal signals**.

# RMS Power

- We know that the average power absorbed by a circuit under sinusoidal excitation can be expressed as,

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

- This can be alternately written as,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

- Similarly, the average power absorbed by a resistor can be written as,

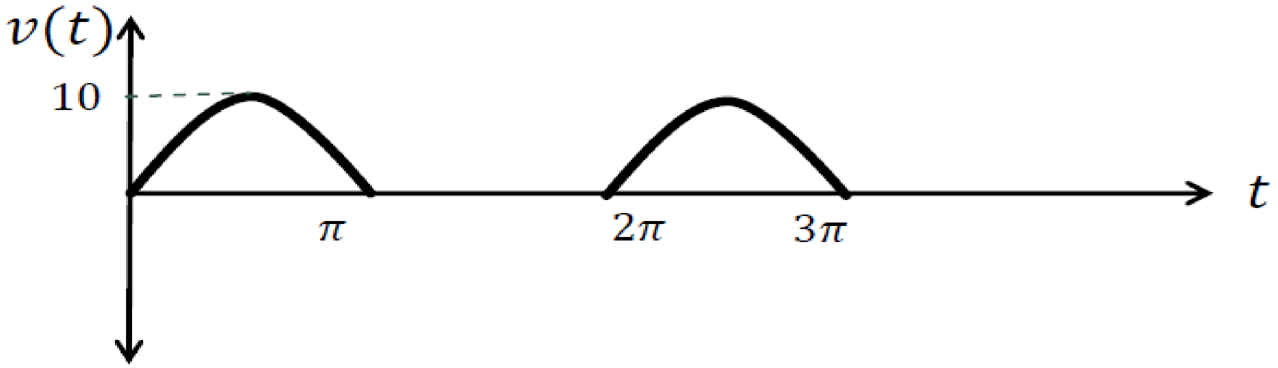
$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

# RMS Power (Cont…)

- When a sinusoidal voltage or current is specified, it is often in terms of its maximum (or peak) value or its rms value, since its average value is zero.
- The power industries specify phasor magnitudes in terms of their rms values rather than peak values.
- For instance, the 230 V available at every household is the rms value of the voltage from the power company.
- It is convenient in power analysis to express voltage and current in their rms values.
- Also, analog voltmeters and ammeters are designed to read directly the rms value of voltage and current, respectively.

❑ Example:

Find the rms value of the half wave rectified sine wave $v(t) = 10\sin t$ shown in the below figure? Also, find the average power dissipated in a 10 Ω resistor?




Solution: The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 sint, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value can be evaluated as:

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt = \frac{1}{2\pi}\left[\int_0^{\pi}(10 sint)^2 dt + \int_{\pi}^{2\pi} 0\ dt\right]$$

But $\sin^2(t) = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi} \frac{100}{2}(1-\cos 2t)dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^{\pi}$$

$$= \frac{50}{2\pi}\left(t - \frac{1}{2}\sin 2\pi - 0\right) = 25$$

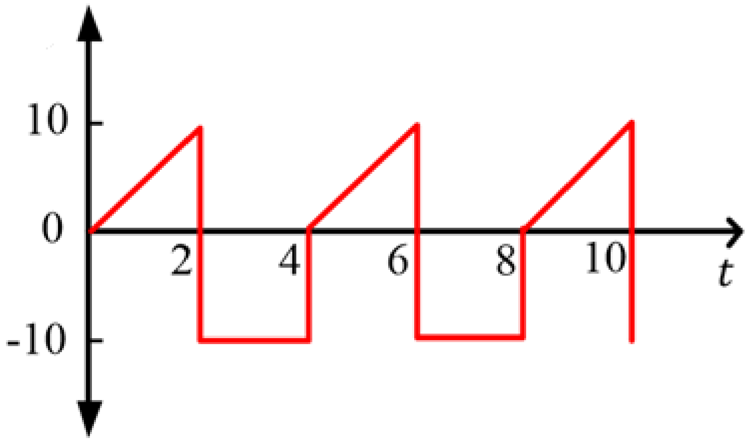
$$V_{rms} = 5\text{V}$$

The average power absorbed is given by,

$$P = \frac{V_{rms}^2}{R} = \frac{25}{10} = 2.5W$$

❑ Example:

Determine the rms value of the current waveform shown in figure below. If the current is passed through a $2\ \Omega$ resistor, find the average power absorbed by the resistor?




Solution: The period of the waveform is T=4. Over a period, we can write the current waveform as,

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value can be evaluated as:

$$\mathrm{I}_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)\,2dt + \int_2^4 (-10)2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\,\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165A$$

The power absorbed by a $2\ \Omega$ resistor is $= I_{rms}^2\, R = 133.3$ W

# Apparent Power and Power Factor

- Earlier, we derived that the average power absorbed by a circuit, excited by a sinusoidal signal, is expressed in terms of rms voltage and current as,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i) = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)$$

- The above equation can be rewritten as,

$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

- The quantity $S$ is known as the apparent power and is defined as the product of rms voltage and current.

## Apparent Power and Power Factor (Cont…)

- The apparent power is so called because it seems apparent that the power should be the voltage-current product, by analogy with DC resistive circuits.
- It is measured in volt-amperes or VA to distinguish it from the average or real power, which is measured in watts.
- From the previous expression, it can be observed that the apparent power needs to be multiplied by a factor to compute the real or average power.
- This factor is known as power factor and is expressed mathematically as,

$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$Pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

# Apparent Power and Power Factor (Cont…)

- The value of pf ranges between zero and unity.
- For a purely resistive load, the voltage and current are in phase, so that $\theta_v - \theta_i = 0$ and $pf = 1$.
- This implies that the apparent power is equal to the average power.
- For a purely reactive load, $\theta_v - \theta_i = 90°$ and $pf = 0$.
- In this case the average power is zero.
- In between these two extreme cases, pf is said to be leading or lagging.
- Leading power factor means that current leads voltage, which implies a capacitive load.
- Lagging power factor means that current lags voltage, implying an inductive load.

$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

## ❑ Example:

A series connected load draws a current $i(t)$=4 cos($100\pi t$ + 10°)A when the applied voltage is $v(t)$ = 120 cos($100\pi t$ - 20°)V. Find the apparent power and power factor of the load. Determine the element values that form the load.

Solution: The apparent power is given by,

$$S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} * \frac{4}{\sqrt{2}} = 240VA$$

The power factor can be evaluated using:

$$pf = \cos(\theta_v - \theta_i) = \cos(-20-10) = 0.866$$

The power factor is leading as the current leads the voltage.

$v(t) = 120\cos(100\pi t - 20°)$V $i(t)=4\cos(100\pi t + 10°)$A

The load impedance can be calculated as follows.

$$Z = \frac{V}{I} = \frac{120\angle -20°}{10\angle 10°} = 30\angle -30° = 25.98 - j15\Omega$$

The load impedance $\boldsymbol{Z}$ can therefore be modelled using a 25.98 Ω resistor in series with a capacitor whose impedance $\boldsymbol{X_c} = -\frac{1}{\omega C} = -15.$

Therefore,

$$C = \frac{1}{15\omega} = \frac{1}{15 * 100\pi} = 212.2\mu F.$$


THANKS
FOR YOUR
ATTENTION