Lecture-5

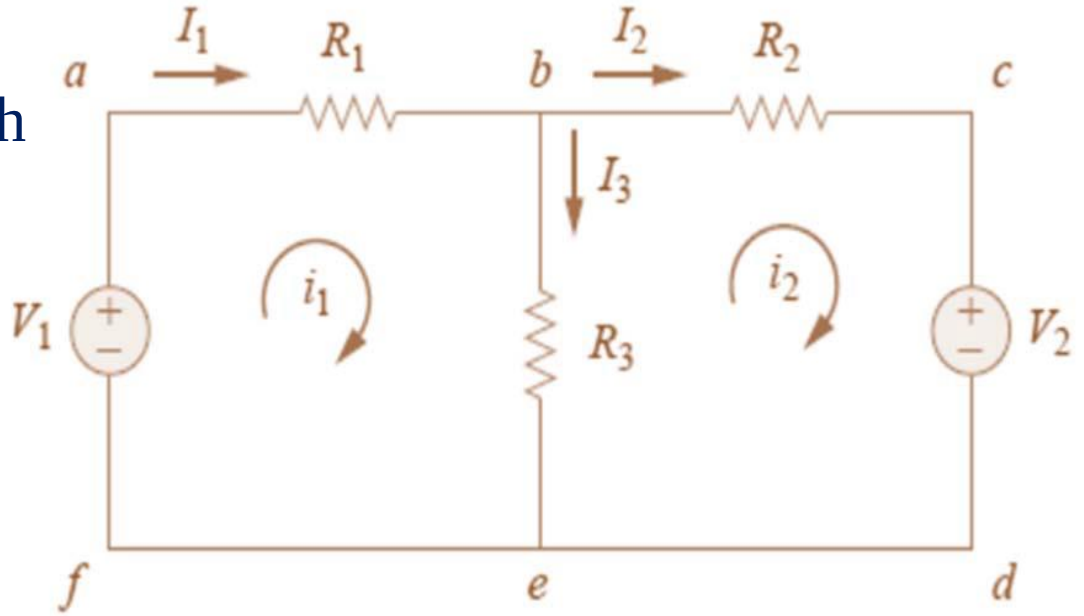
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Mesh analysis.
- Nodal analysis.
- Graph theory.

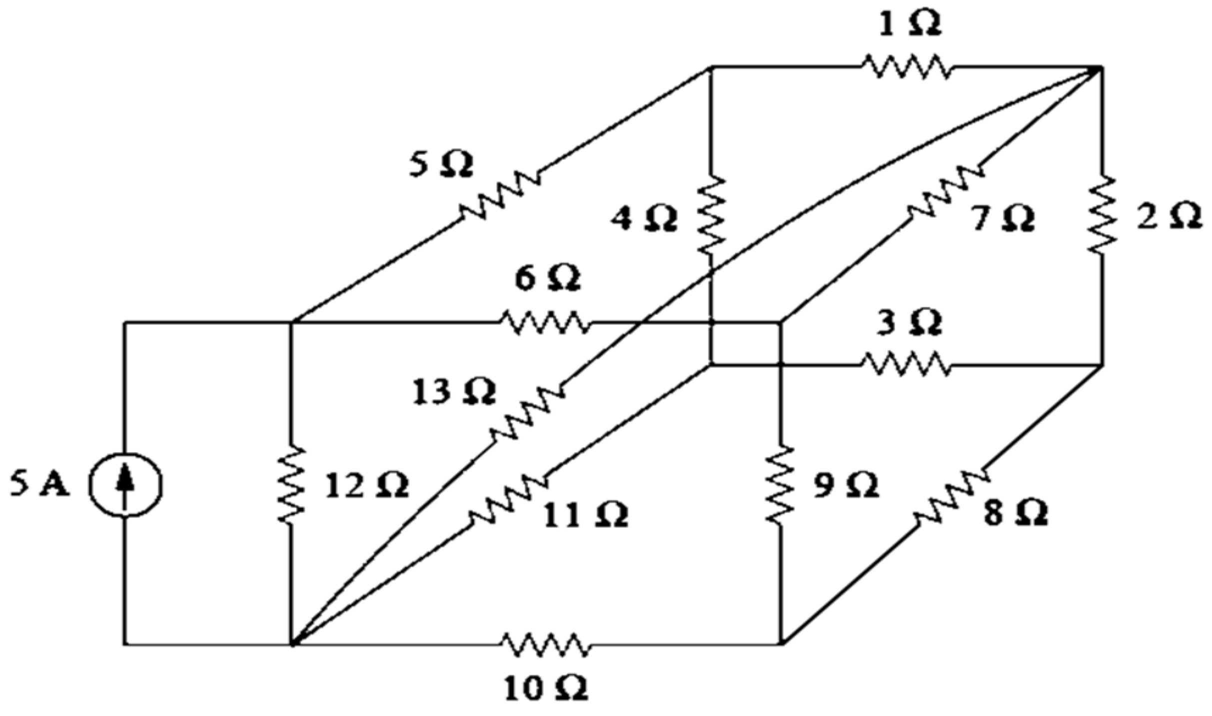
Mesh Analysis

- Mesh analysis provides another general procedure for analysing circuits, using mesh currents (or loop current) as the circuit variables
- Mesh analysis is also known as loop analysis or the mesh-current method
- Instead of element currents as circuit variables, using mesh currents is convenient and reduces the number of equations that must be solved simultaneously.
- Loop is a closed path with no node passed more than once while a mesh is a loop that does not contain any other loop within it.
- paths **abefa** and **bcdeb** are meshes, but path **abcdefa** is not a mesh.
- Mesh analysis applies KVL to find unknown currents.

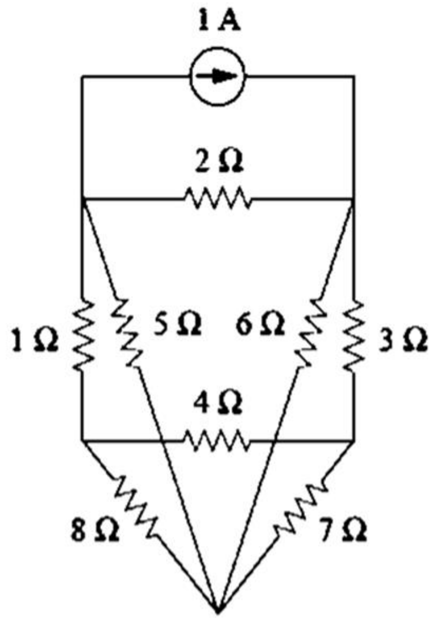


Mesh Analysis (Contd…)

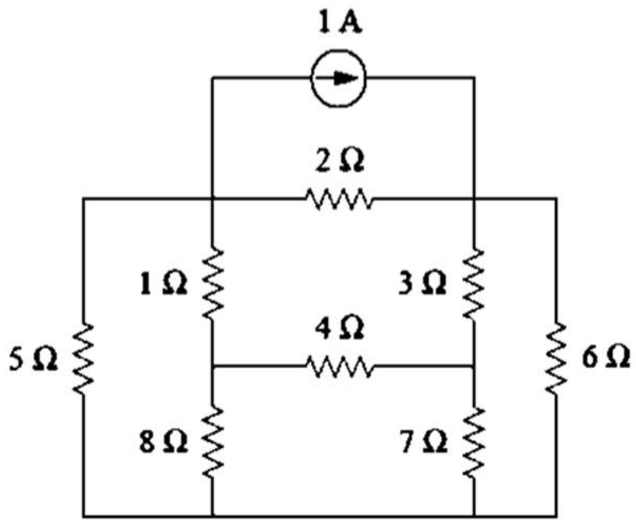
- Mesh analysis is only applicable to a circuit that is planar.
- A planar circuit is one that can be drawn in a plane with no branches crossing one another.
- A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches.



Non-planar circuit



(a)



Planar circuit

Mesh Analysis (Contd…)

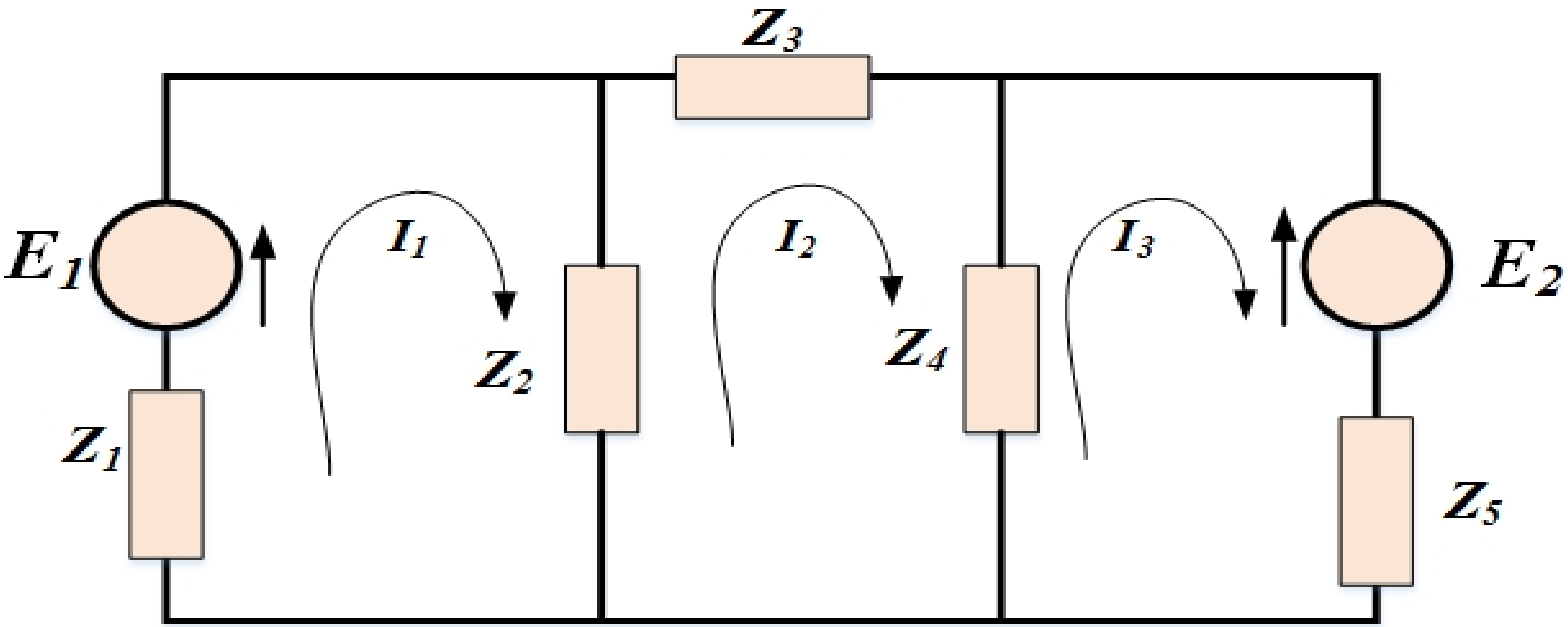
❑ Steps to Determine Mesh Currents:

1. Assign mesh currents i_1, i_2, i_3 ………… to the **n** meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

Mesh Analysis (Contd…)

- Mesh analysis is merely an extension of the use of Kirchhoff's laws.
- The figure given below shows a network whose circulating currents I_1, I_2 and I_3 have been assigned to closed loops in the circuit rather than to branches. Currents I_1, I_2 and I_3 are called mesh-currents.



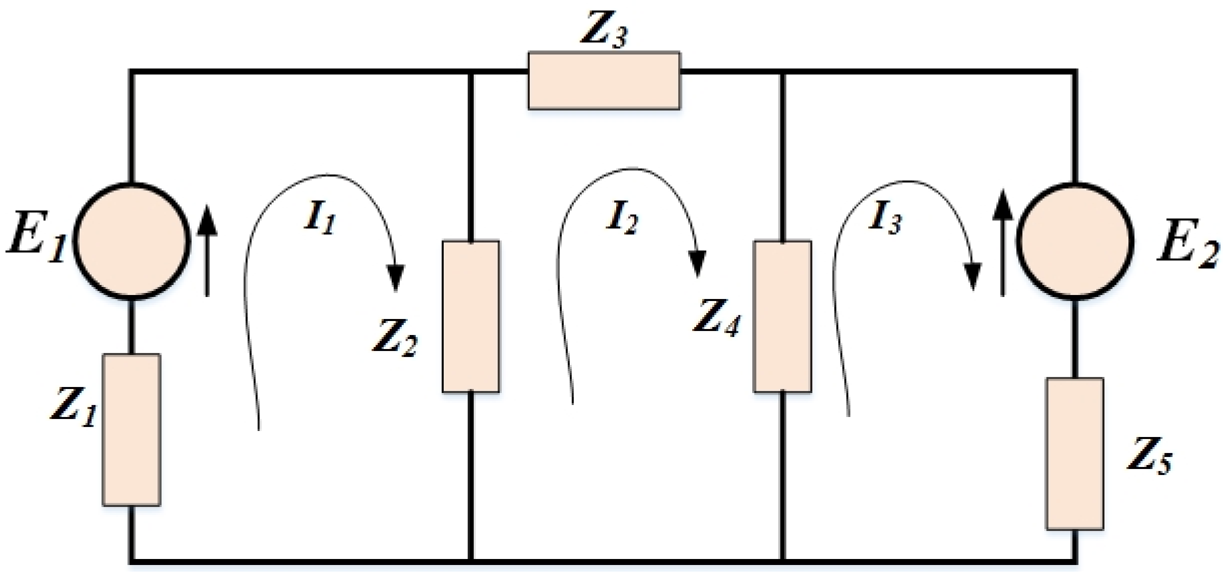
Mesh Analysis (Contd…)

- In the figure, mesh-currents are all arranged in clockwise direction to circulate in the same direction.
- Kirchhoff's second law (KVL) is applied to each of the mesh, which, in the circuit of figure below produces three equations with three unknowns which may be solved for I_1, I_2, and I_3.
- The three equations are :

$$I_1(Z_1+Z_2)-I_2Z_2=E_1$$

$$I_2(Z_2+Z_3+Z_4)-I_1Z_2-I_3Z_4=0$$

$$I_3(Z_4+Z_5)-I_2Z_4=-E_2$$



Mesh Analysis (Contd…)

- The branch currents are determined by taking the phasor sum of the mesh currents common to that branch.
- For example, the current flowing in impedance Z2, is given by $(I_1 - I_2)$ phasor.
- Notice that the branch currents are different from the mesh currents unless the mesh is isolated.
- **<u>Verification</u>**: Using theorem of network topology (i.e. m=b-n+1) in the previous figure –

$$b=7,\ n=5$$

so, $$m = 7-5+1 = 3$$

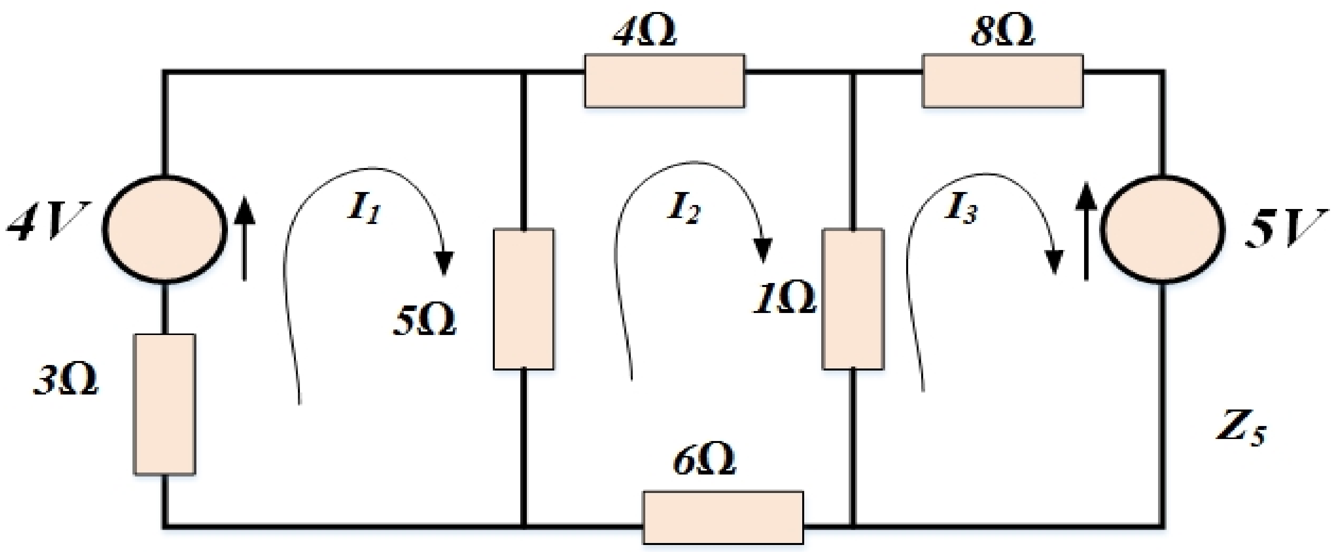
- So, 3 independent mesh, and therefore, three independent equations will be required to solve the circuit.

Problem

- **Problem** : For the DC circuit shown below, use mesh-current analysis to determine the current flowing in-

  (a) the 5Ω resistance

  (b) the 1 Ω resistance



Using Kirchhoff's voltage law:

For mesh 1, $(3 + 5)I_1 - 5I_2 = 4$

For mesh 2, $(4 + 1 + 6 + 5)I_2 - (5)I_1 - (1)I_3 = 0$

For mesh 3, $(1 + 8)I_3 - (1)I_2 = -5$

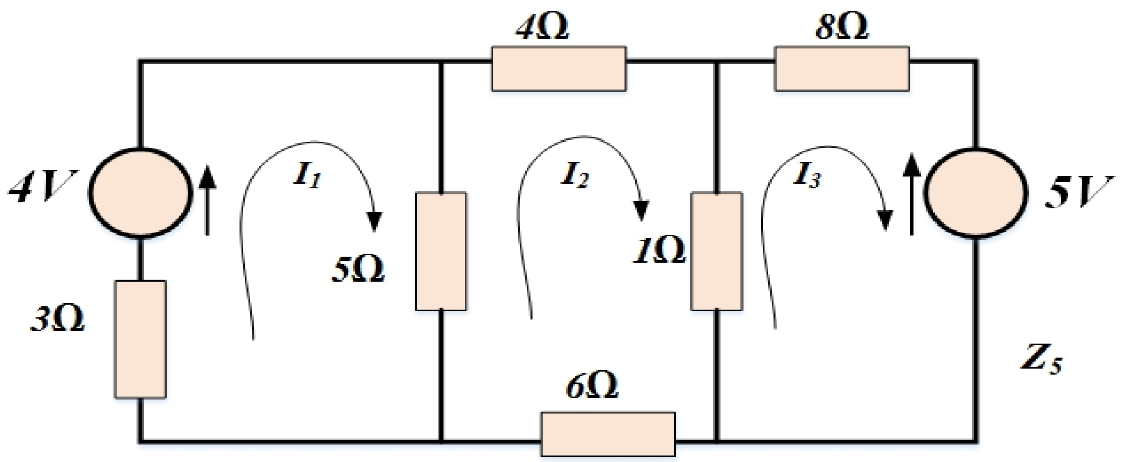
Problem Contd…

- Thus,

$$8I_1 - 5I_2 - 4 = 0 \qquad (1')$$

$$-5I_1 + 16I_2 - I_3 = 0 \qquad (2')$$

$$-I_2 + 9I_3 + 5 = 0 \qquad (3')$$



- Solving these equations, we get :

$$I_1 = 0.595\ A,$$

$$I_2 = 0.152\ A,$$

$$I_3 = -0.539\ A$$

(a) Current in the 5Ω resistance = $I_1 - I_2 = 0.595 - 0.152 = 0.44$A

(b) Current in the 1Ω resistance = $I_2 - I_3 = 0.152 - (-0.539) = 0.69$A

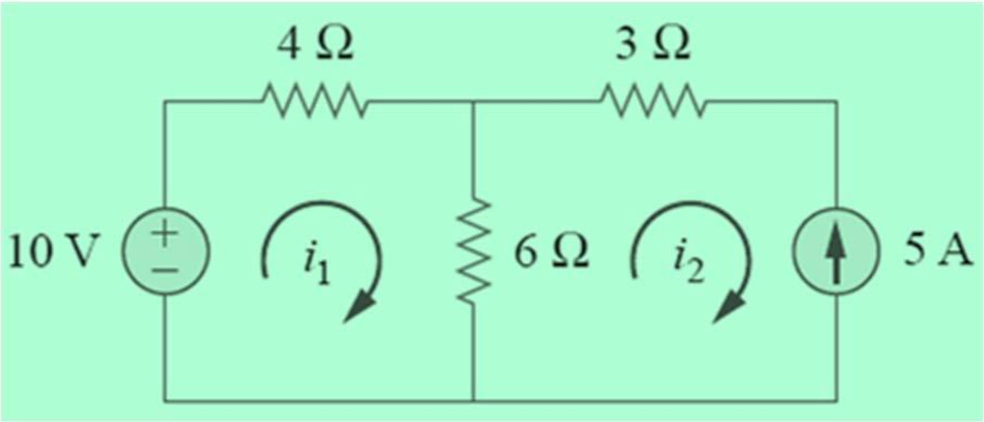
Mesh Analysis With Current Sources

- Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated
- But it is actually much easier than what we encountered in the previous section
- It is so because the presence of the current sources reduces the number of equations
- Lets consider the following two cases -

Case 1 - When a current source exists only in one mesh

- We set i_2 = -5 A and write a mesh equation for the other mesh in the usual way:

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

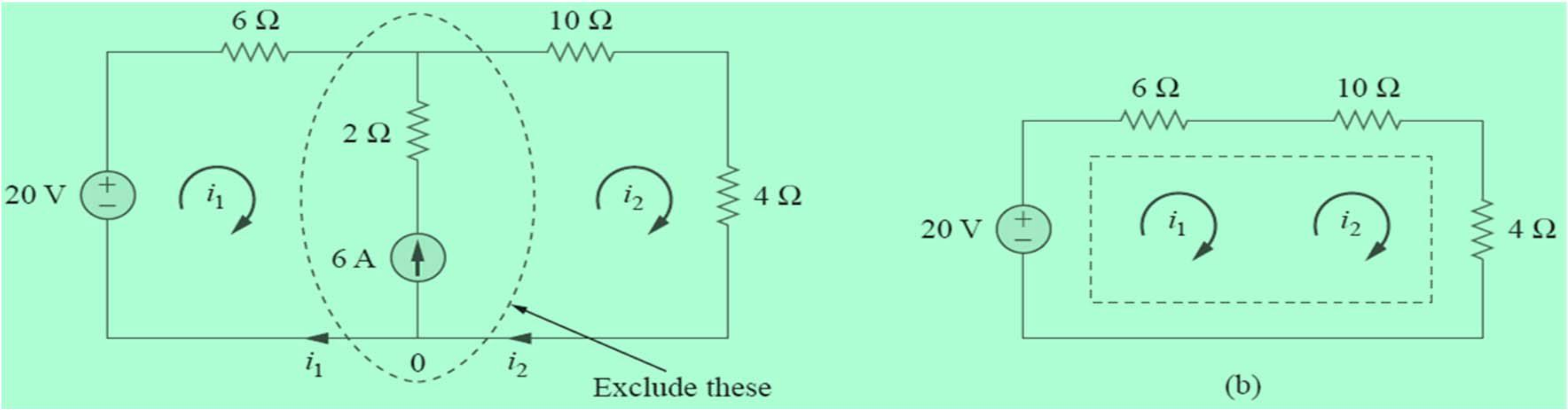


Putting the value of i_2 = -5 A, we get, i_1 = -2 A

Mesh Analysis With Current Sources (Contd…)

Case 2: When a current source exists between two meshes

- We create a supermesh by excluding the current source and any elements connected in series with it , as shown in Figure below.
- A supermesh results when two meshes have a (dependent or independent) current source in common.
- If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.



Mesh Analysis With Current Sources (Contd…)

- The supermesh is required to be created because mesh analysis applies KVL, which requires that we know the voltage across each branch. But we do not know voltage across a current source in advance.
- Supermesh must satisfy KVL like any other mesh.
- Applying KVL to the supermesh in the figure gives:

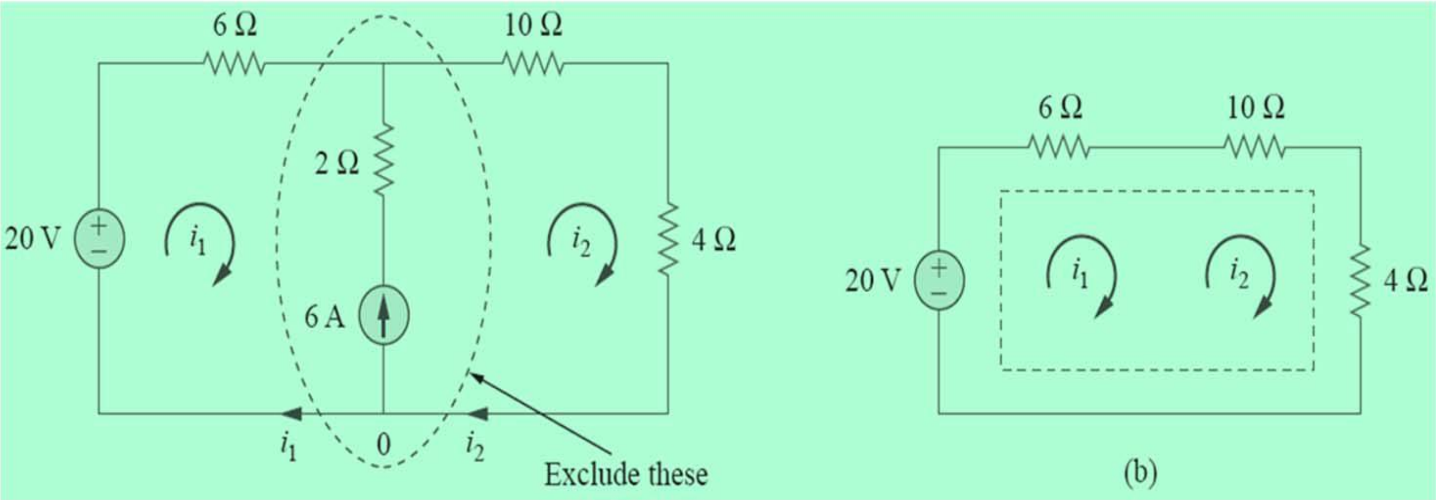
$$-20 + 6i_1 + 10i_2 + 4i_2 = 0 \;\rightarrow 6i_1 + 14i_2 = 20$$

- We apply KCL to a node in the branch where the two meshes intersect –

$$i_2 = i_1 + 6$$

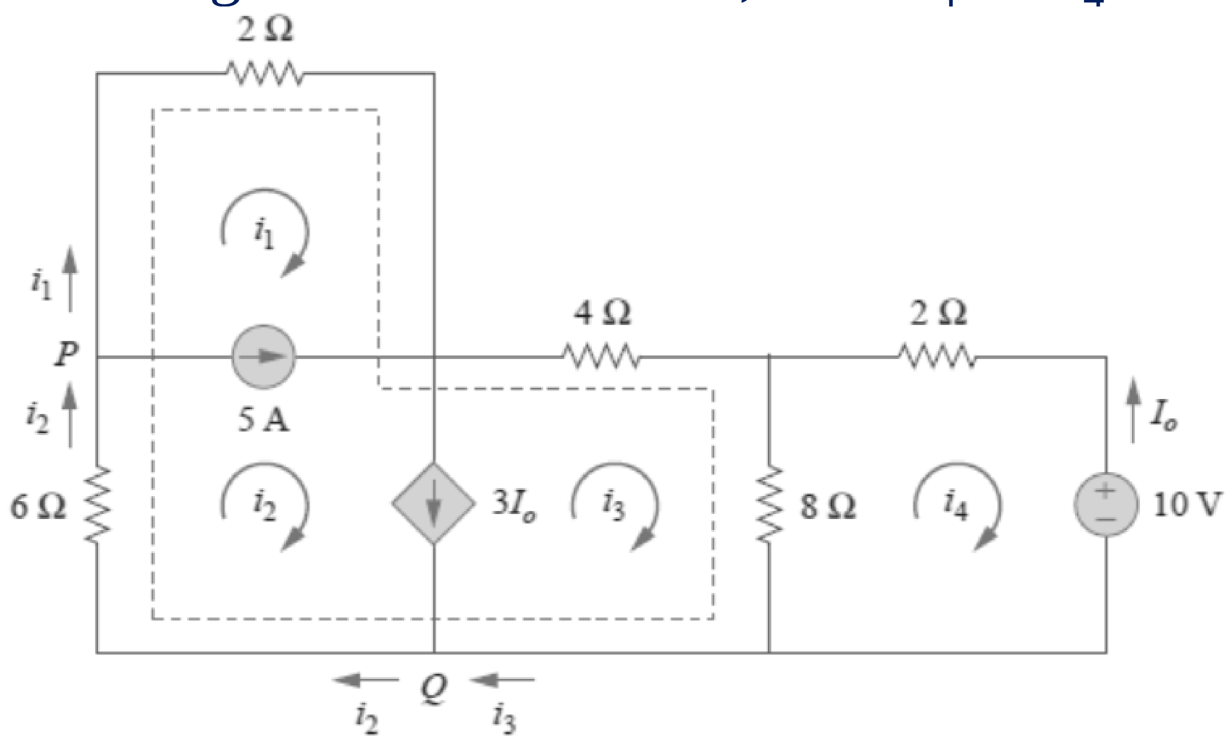
- From above two equations -

$$i_1 = -3.2\ A\ and\ i_2 = 2.8\ \mathrm{A}$$



Problem:

❑ **Problem**: For the circuit in Figure shown below, find i_1 to i_4 using mesh analysis.



- Meshes 1 and 2 form a supermesh since they have an independent current source in common.
- Meshes 2 and 3 form another supermesh since they have a dependent current source in common.
- The two supermeshes intersect and, therefore, form a larger supermesh.

Problem Contd…

- Applying KVL to larger supermesh:

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

- Applying KCL to node P:

$$i_2 = i_1 + 5$$

- Applying KCL to node Q:

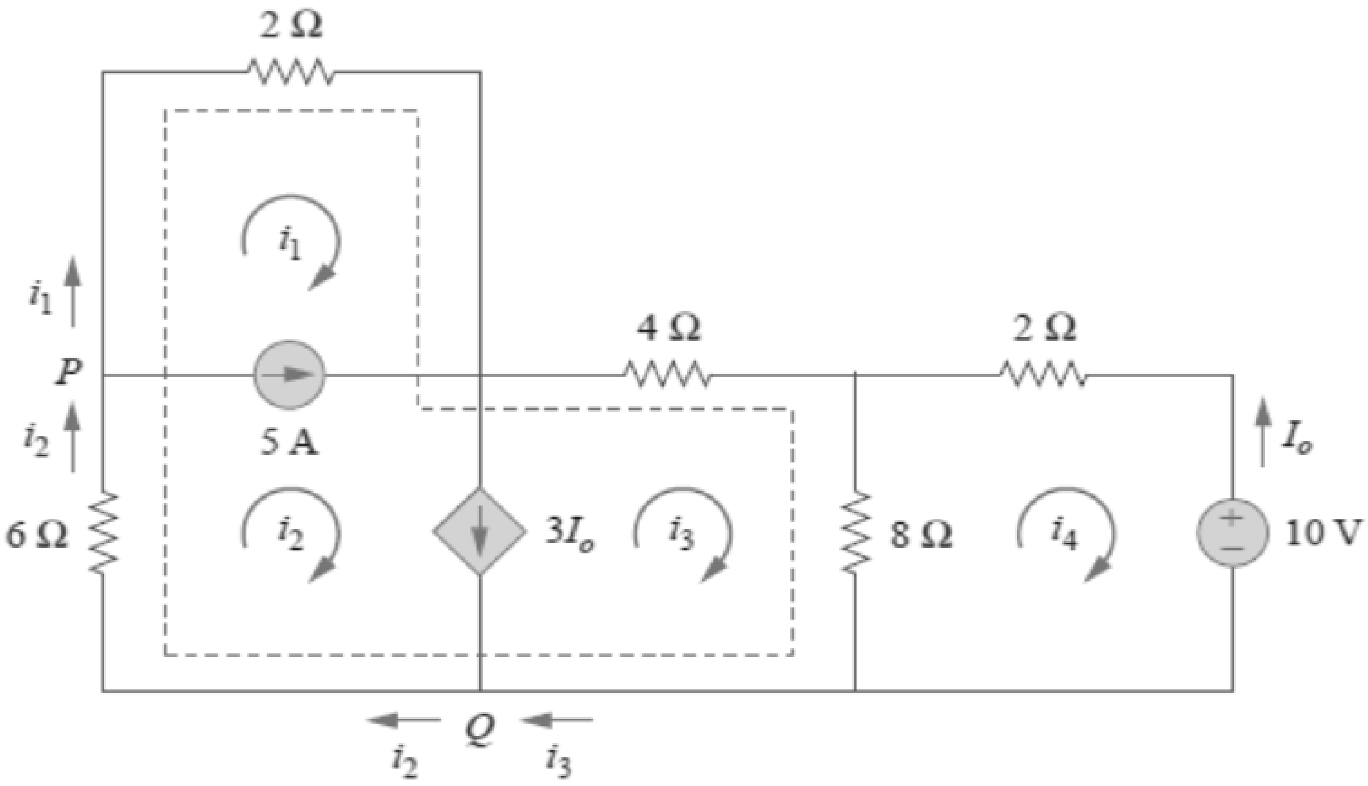
$$i_2 = i_3 + 3I_0$$

But,

$$i_4 = -I_0$$

So,

$$i_2 = i_3 - 3i_4$$



Problem Contd…

- Applying KVL in mesh 4:

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

- Four equations and 4-unknow:

$$2i_1 + 6i_2 + 12i_3 - 8i_4 = 0$$

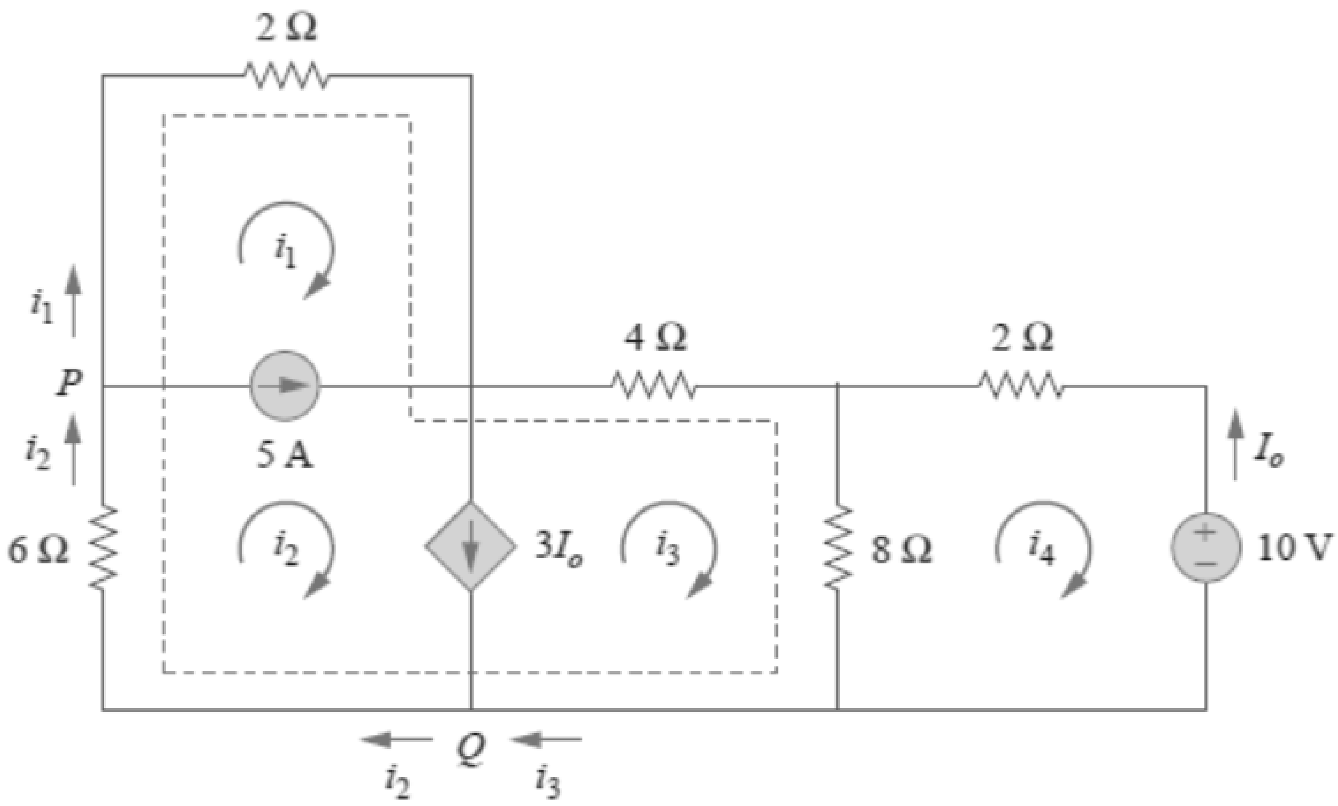
$$i_2 = i_1 + 5$$

$$i_2 = i_3 - 3i_4$$

$$10i_4 - 8i_3 = -10$$

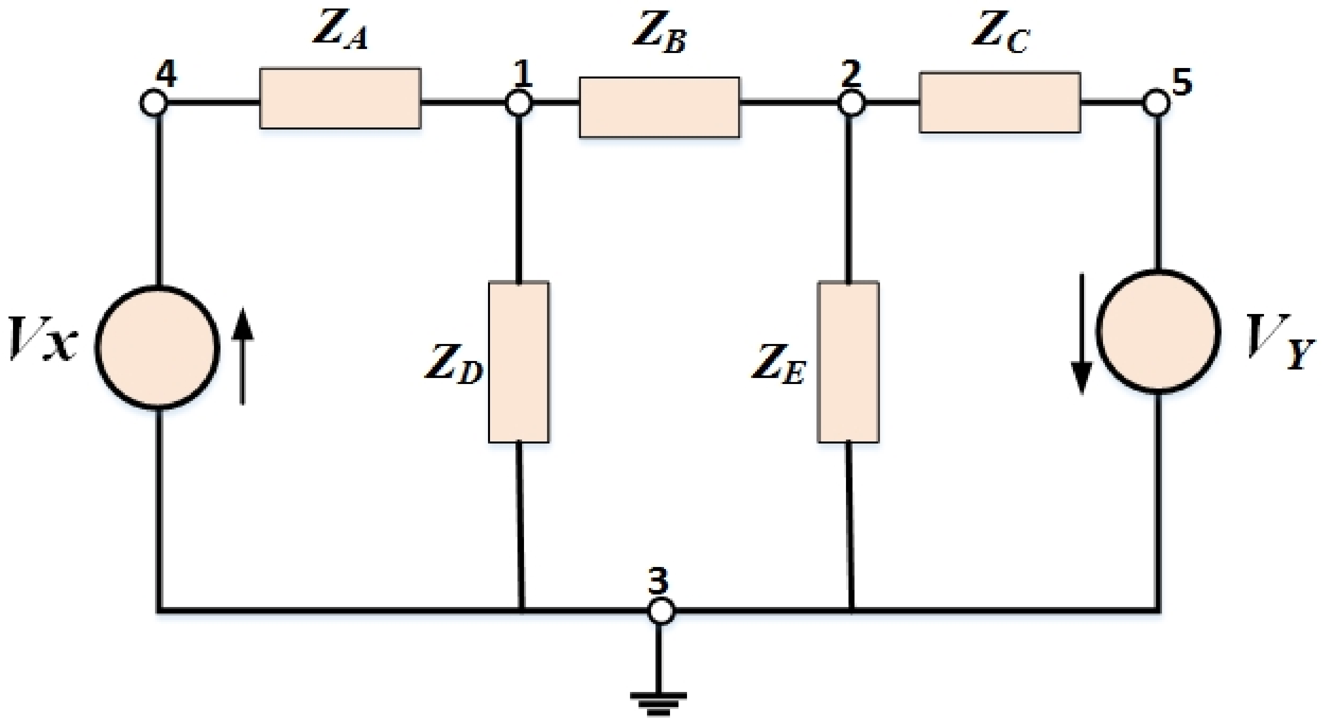
- Solving the equations, we get:

$$i_1 = -7.5\ A,$$
$$i_2 = -2.5\ A,$$
$$i_3 = 3.93\ A,$$
$$i_4 = 2.143\ A$$



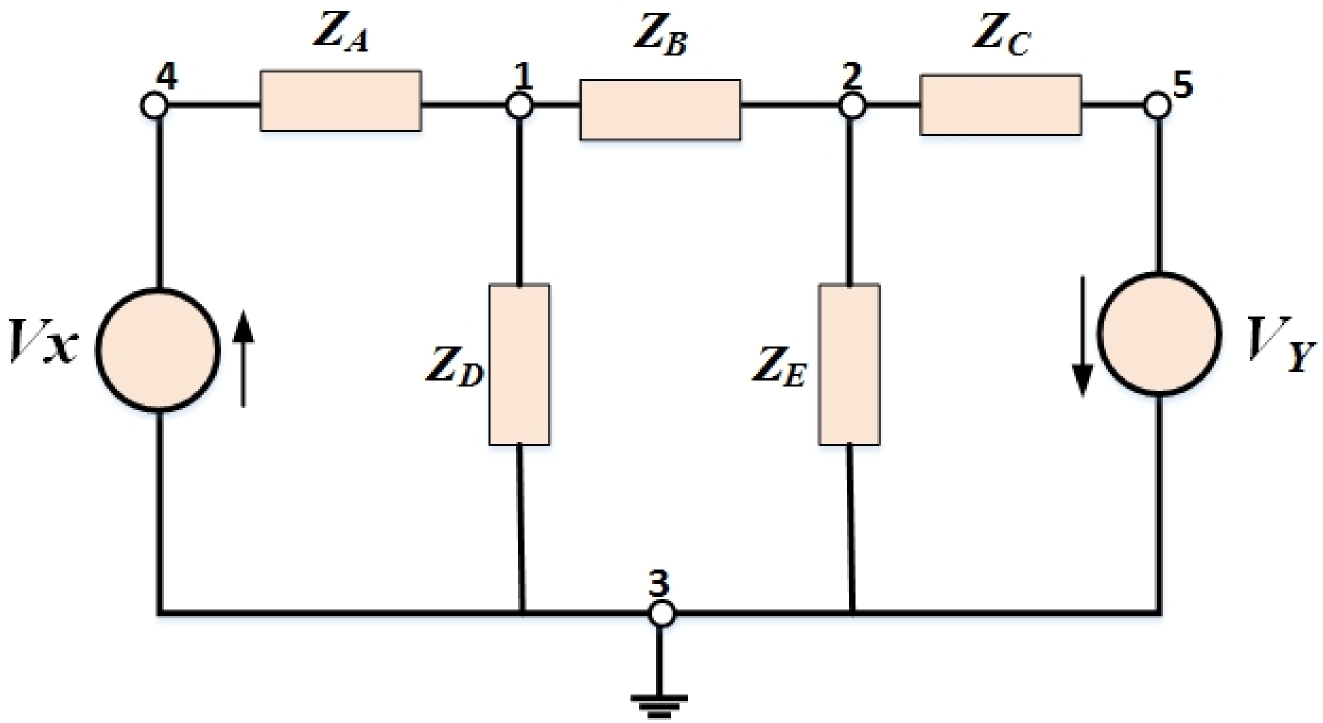
Nodal Analysis

- Nodal analysis provides a procedure for analysing circuits using node voltages as the circuit variables.
- A node of a network is defined as a point where two or more branches are joined.
- If three or more branches join at a node, then that node is called a **principal node**. Here, points 1, 2, 3, 4 and 5 are nodes, while points 1, 2 and 3 are principal nodes.



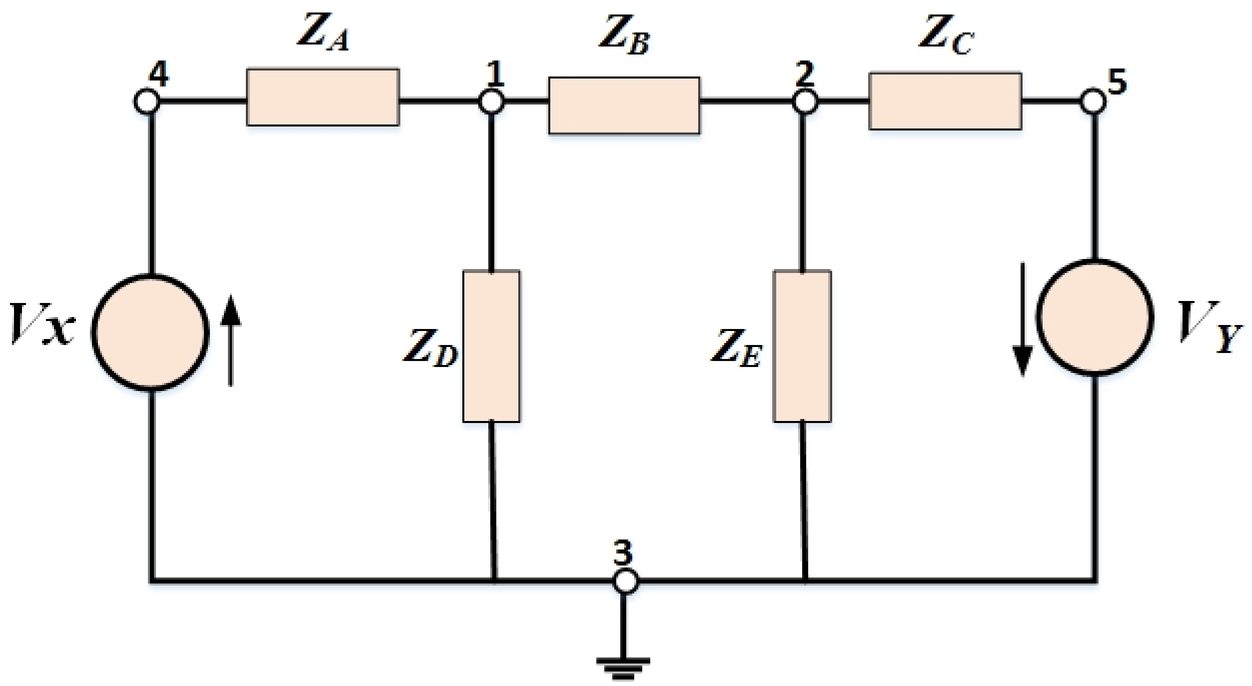
Nodal Analysis (Contd…)

- A node voltage is the voltage of a particular node with respect to another node called as **reference node.**
- In the figure shown below, node 3 is chosen as the reference node.
- Therefore, V_{13} is the voltage at node 1 with respect to node 3. Similarly, V_{23} is the voltage at node 2 with respect to node 3, and so on.



Nodal Analysis (Contd…)

- Since the node voltage is always determined with respect to a particular chosen reference node, the notation V_1 for V_{13} and V_2 for V_{23} can be used for simplicity.
- The objective of nodal analysis is to determine the values of voltages at all the principal nodes with respect to the reference node, e.g., to find voltages V_1 and V_2.
- Once voltages are determined, the currents flowing in each branch can be found.



Steps to Determine Node Voltages

- Select a node as the reference node. Assign voltages v_1, v_2, …. v_n to the remaining **(n-1)** nodes. The voltages are referenced with respect to the reference node.
- Apply KCL to each of the **(n-1)** non-reference nodes. Use Ohm’s law to express the branch currents in terms of node voltages.
- Solve the resulting simultaneous equations to obtain the unknown node voltages.

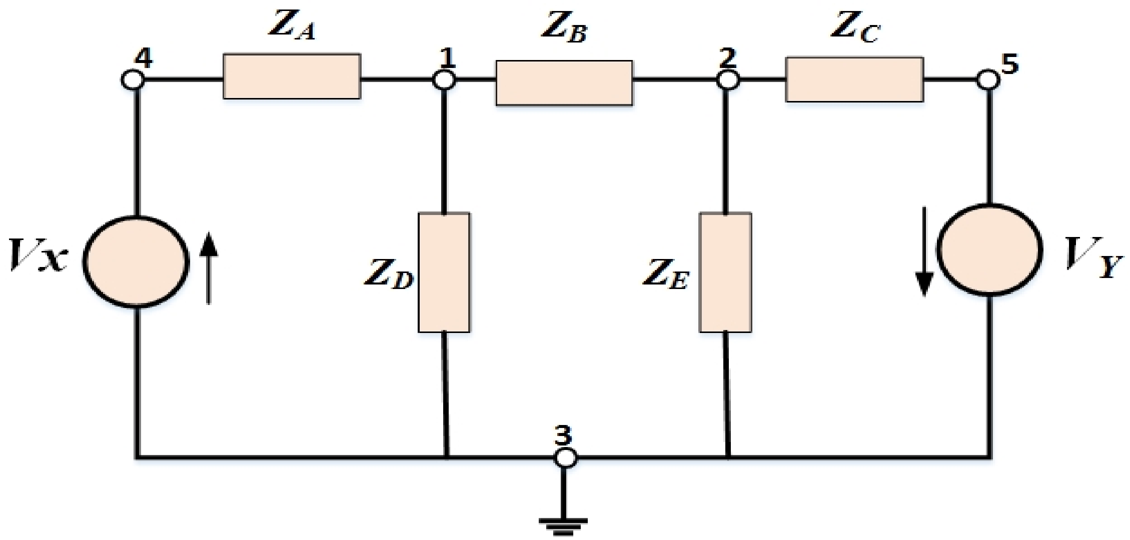
Node Analysis (Contd…)

- Let us assume that all branch currents are leaving the node, therefore, the sum of currents at a junction is zero,

$$\frac{V_1 - V_x}{Z_A} + \frac{V_1}{Z_D} + \frac{V_1 - V_2}{Z_B} = 0 \qquad (1)$$

- Similarly, for node 2, assuming all branch currents are leaving the node,

$$\frac{V_2 - V_1}{Z_B} + \frac{V_2}{Z_E} + \frac{V_2 + V_Y}{Z_C} = 0 \qquad (2)$$

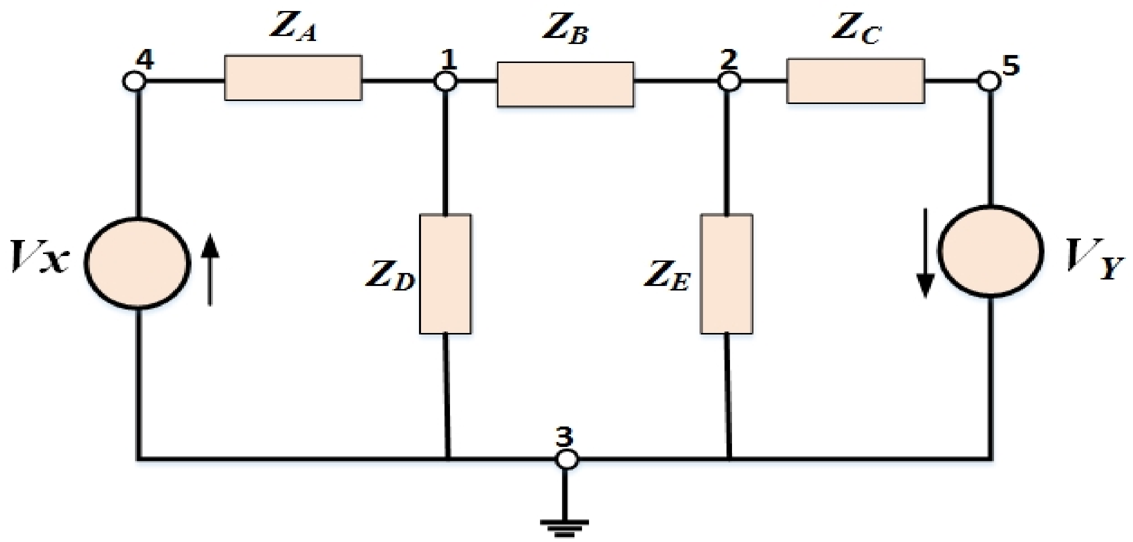


Node Analysis (Contd…)

- Rearranging equations (1) and (2) gives:

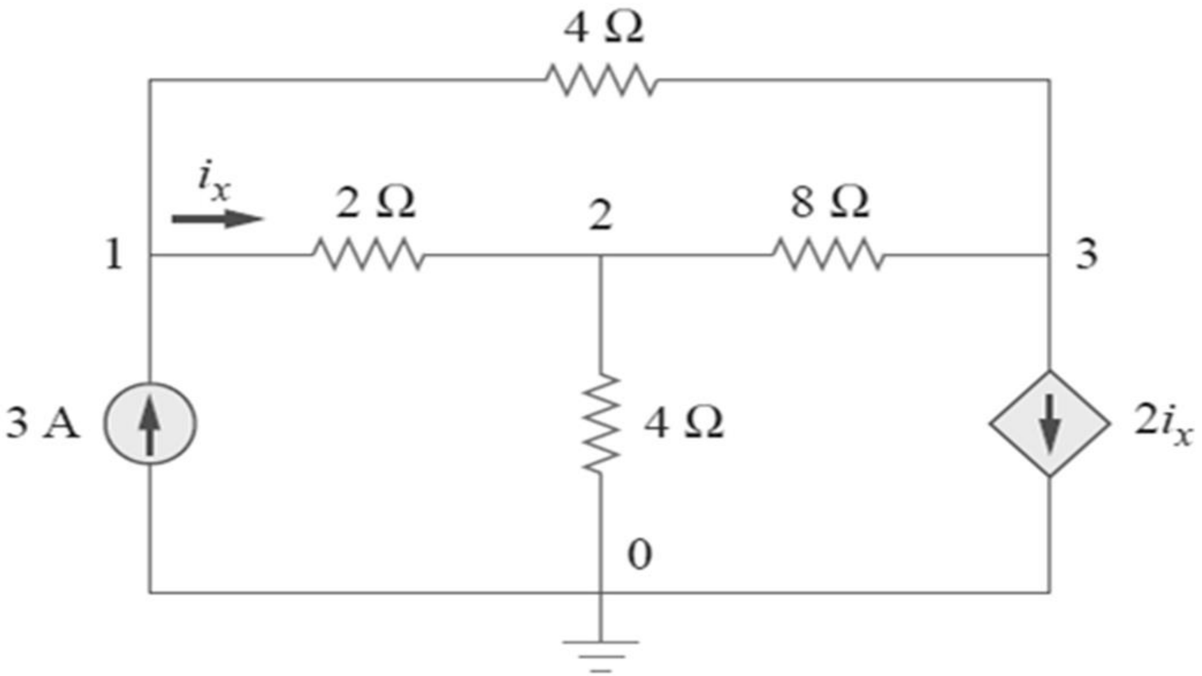
$$\left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_D}\right)V_1-\left(\frac{1}{Z_B}\right)V_2-\left(\frac{1}{Z_A}\right)V_x=0 \qquad (3)$$

$$-\left(\frac{1}{Z_B}\right)V_1-\left(\frac{1}{Z_B}+\frac{1}{Z_C}+\frac{1}{Z_E}\right)V_2+\left(\frac{1}{Z_C}\right)V_Y=0 \qquad (4)$$



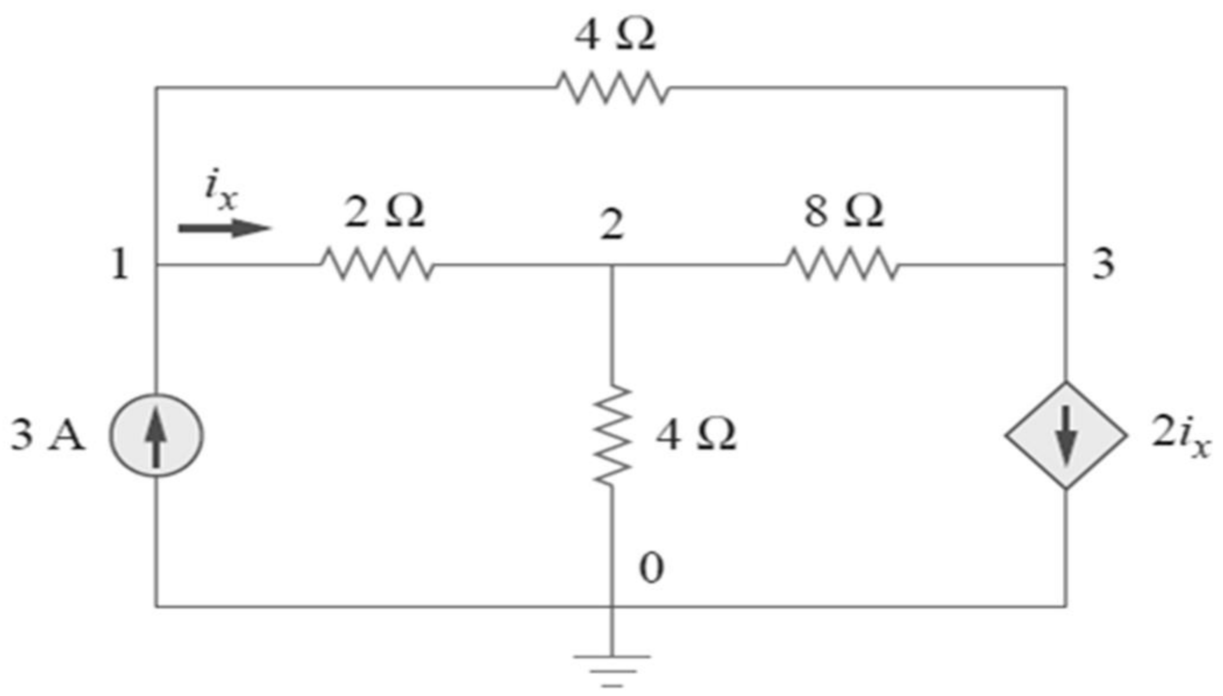
Problem:

❑ **Problem**: Determine the voltages at the nodes in Figure below.



Problem Contd…

The circuit in this example has three nonreference nodes. We assign voltages to the three nodes as shown in Figure below -



At node-1:

$$-3+\frac{v_1-v_2}{2}+\frac{v_1-v_3}{4}=0$$

$$2v_1-2v_2+v_1-v_3=12$$

$$3v_1-2v_2-v_3=12 \qquad (1)$$

At node-2:

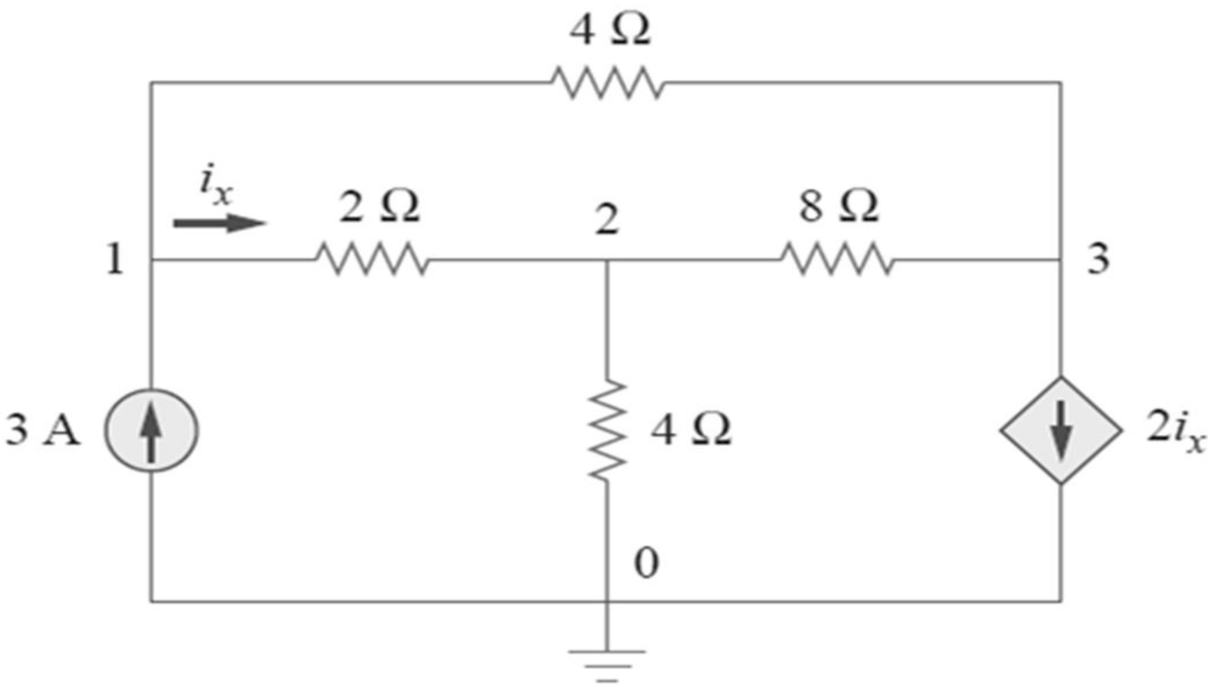
$$\frac{v_2-v_1}{2}+\frac{v_2-0}{4}+\frac{v_2-v_3}{8}=0$$

$$4v_2-4v_1+2v_2+v_2-v_3=0$$

$$-4v_1+7v_2-v_3=0 \qquad (2)$$

Problem Contd…

The circuit in this example has three nonreference nodes. We assign voltages to the three nodes as shown in Figure below -



$$i_x = \frac{v_1 - v_2}{2}$$

At node-3:

$$\frac{v_3 - v_2}{8} + 2i_x + \frac{v_3 - v_1}{4} = 0$$

$$\frac{v_3 - v_2}{8} + \frac{2(v_1 - v_2)}{2} + \frac{v_3 - v_1}{4} = 0$$

$$v_3 - v_2 + 8v_1 - 8v_2 + 2v_3 - 2v_1 = 0$$

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3)$$

Problem Contd…

The circuit in this example has three nonreference nodes. We assign voltages to the three nodes as shown in Figure below -

Three equations 3-unknows

$$3v_1 - 2v_2 - v_3 = 12 \qquad (1)$$

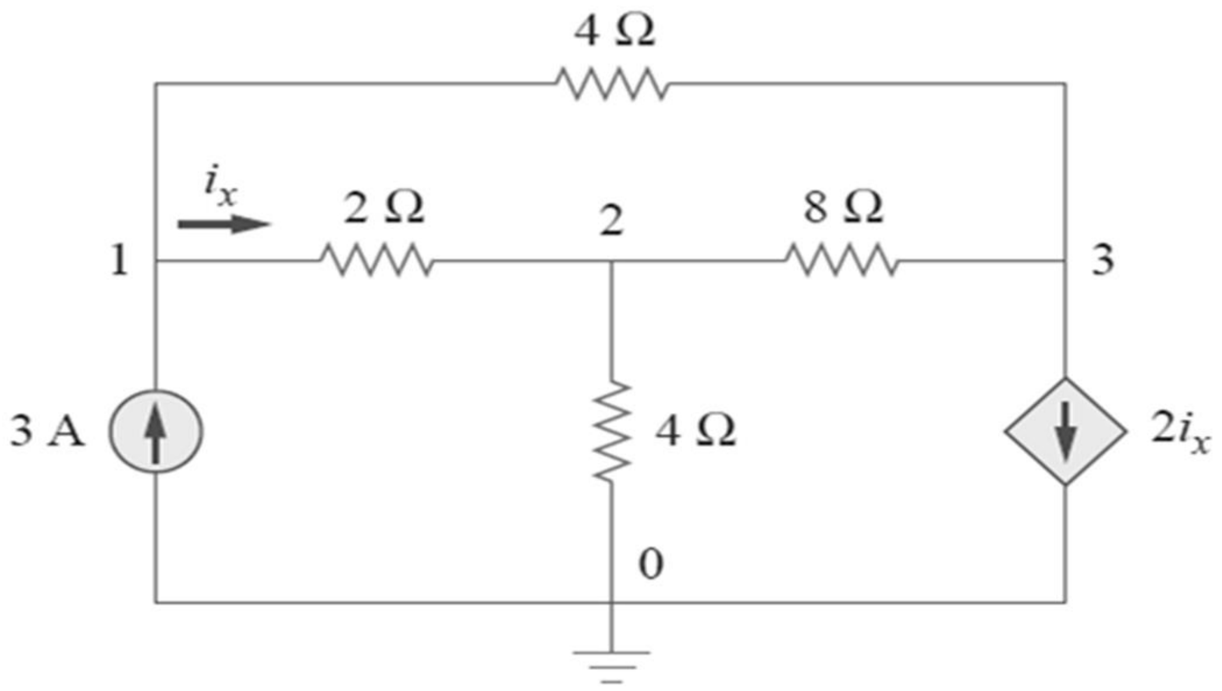
$$-4v_1 + 7v_2 - v_3 = 0 \qquad (2)$$

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3)$$

$$v_1 = 4.8$$

$$v_2 = 2.4$$

$$v_3 = -2.4$$

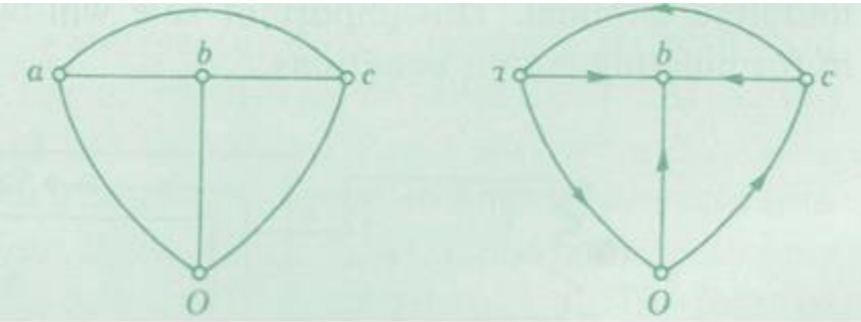


Network Vs. Circuit

- To differentiate between a circuit and a network, a network is regarded as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths.

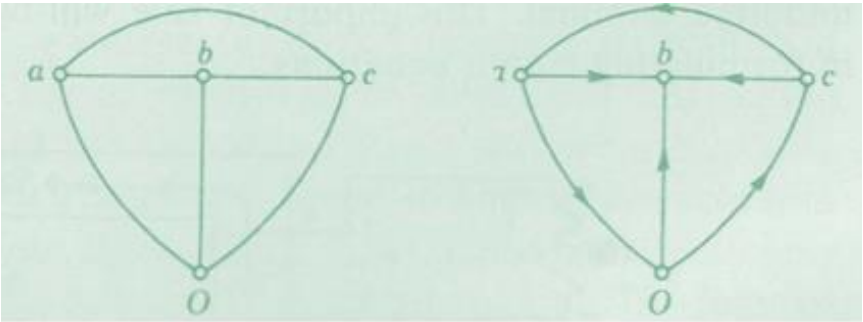
Topology:

- In network topology, properties relating to the placement of elements in the network and geometric configuration of the network are studied. Such elements include branches, nodes, and loops.
- We make use of graph in describing the topological properties of networks.
- To construct a graph corresponding to a given schematics of a network, we replace all elements of the network with lines. This shows the skeleton of the network.



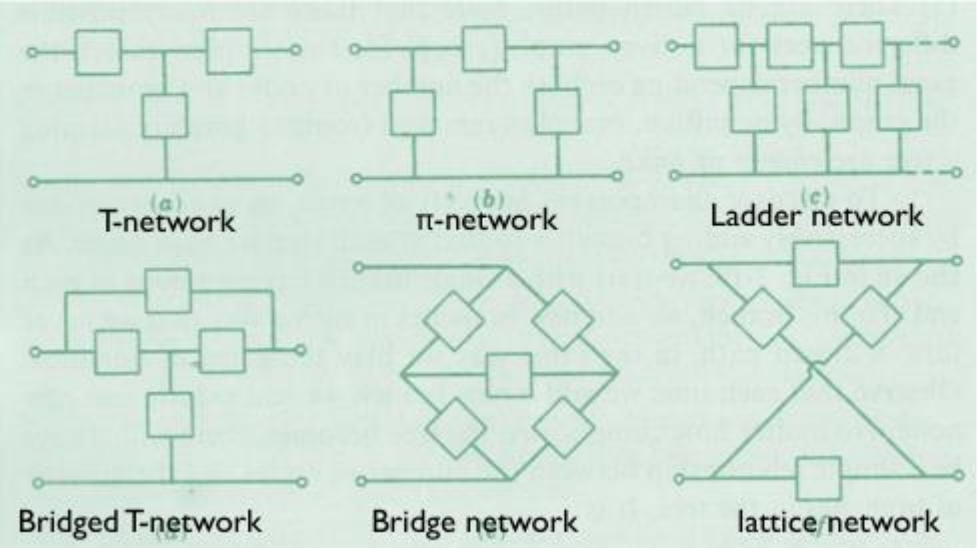
Topology:

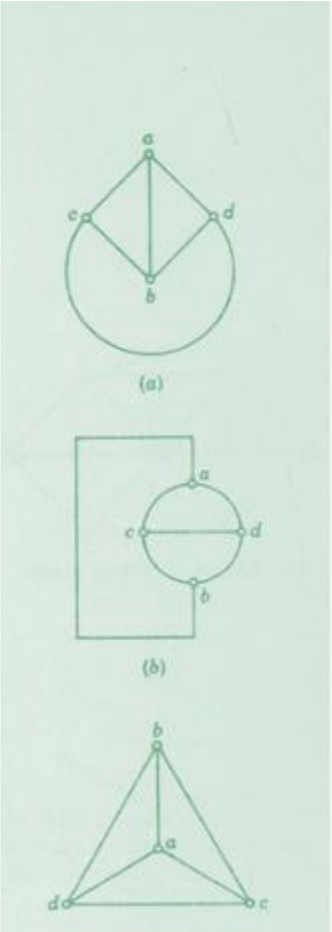
- If we also indicate a reference direction by an arrow for each of the lines of the graph, then it is known as oriented graph.
- Lines in the graph are called branches.
- Junction of two or more branches is called node.



Topology (Cont…)

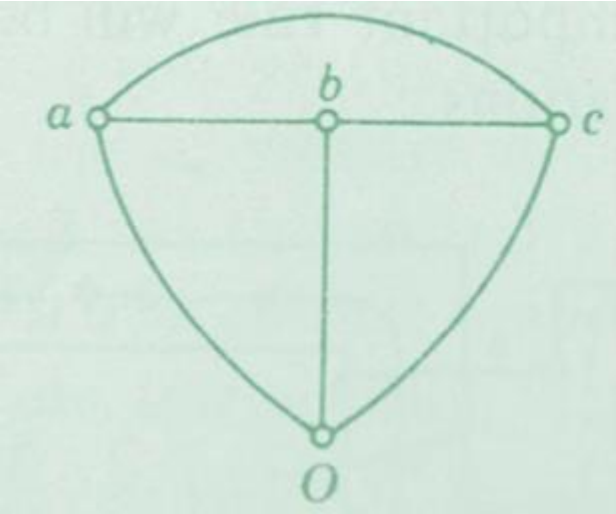
- Topology deals with properties of network which are unaffected when we stretch, twist, or distort the size and shape of the network
- The figure shows three graphs
- Although three graphs appear to be different, they are actually identical topologies
- The relationship between branches and nodes is identical
- Some topological structures occur so frequently in electrical engineering that they are given special names





Topology (Cont…)

- Subgraph of a given graph is formed by removing branches from the original graph.
- Tree is one kind of subgraph which is important for our circuit analysis.
- Tree is formed by removing a branch from the graph.
- A tree of **n** nodes has the following property:
    - It contains all nodes of the graph.
    - It contains n-1 branches.
    - There is no closed path.



- There are many possible different trees of a graph.
- Branches removed from the graph in forming a tree are called chords or links.
- Concept of tree is used for the proper choice of current variables for the analysis of a network.


THANKS
FOR YOUR
ATTENTION