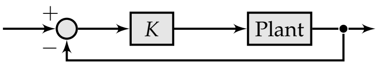
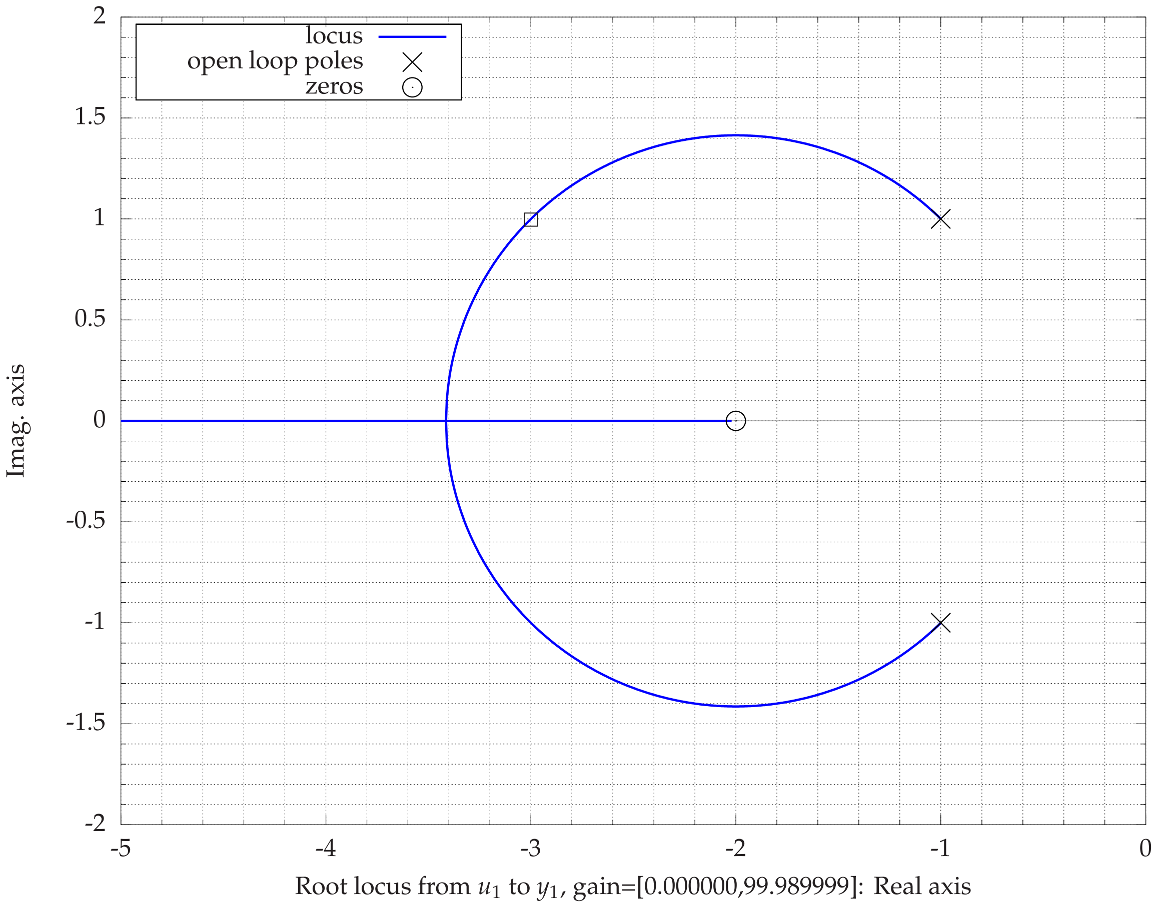
Final Exam, EE 250 (Control Systems Analysis), Spring 2013*

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

1) Write your name, roll number, and section on every page.
2) If you find the space provided beneath each problem to be insufficient, please continue only on the reverse of the page on which the problem is printed. Do not write on/behind any other printed page.
3) Pencil is OK for trial and error, but your final answer must use a pen.
4) Show all your assumptions.
5) You will need the following items: pen, pencil, ruler, eraser, calculator. Borrowing is prohibited.
6) Unclear answers will lead to arbitrary deduction of points. It is your responsibility to write legibly.
7) Giving or receiving help is prohibited.

1. A unity feedback control system 

has the following root locus for $K \geq 0$.



1.1. **[1 points]** Write the plant transfer function and the characteristic equation corresponding to this root locus.

1.2. **[1 points]** Place arrows on the root locus to show its direction of evolution as K goes from 0 to ∞.

1.3. **[2 points]** Calculate graphically the value of K at the point marked $\square$.

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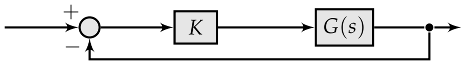
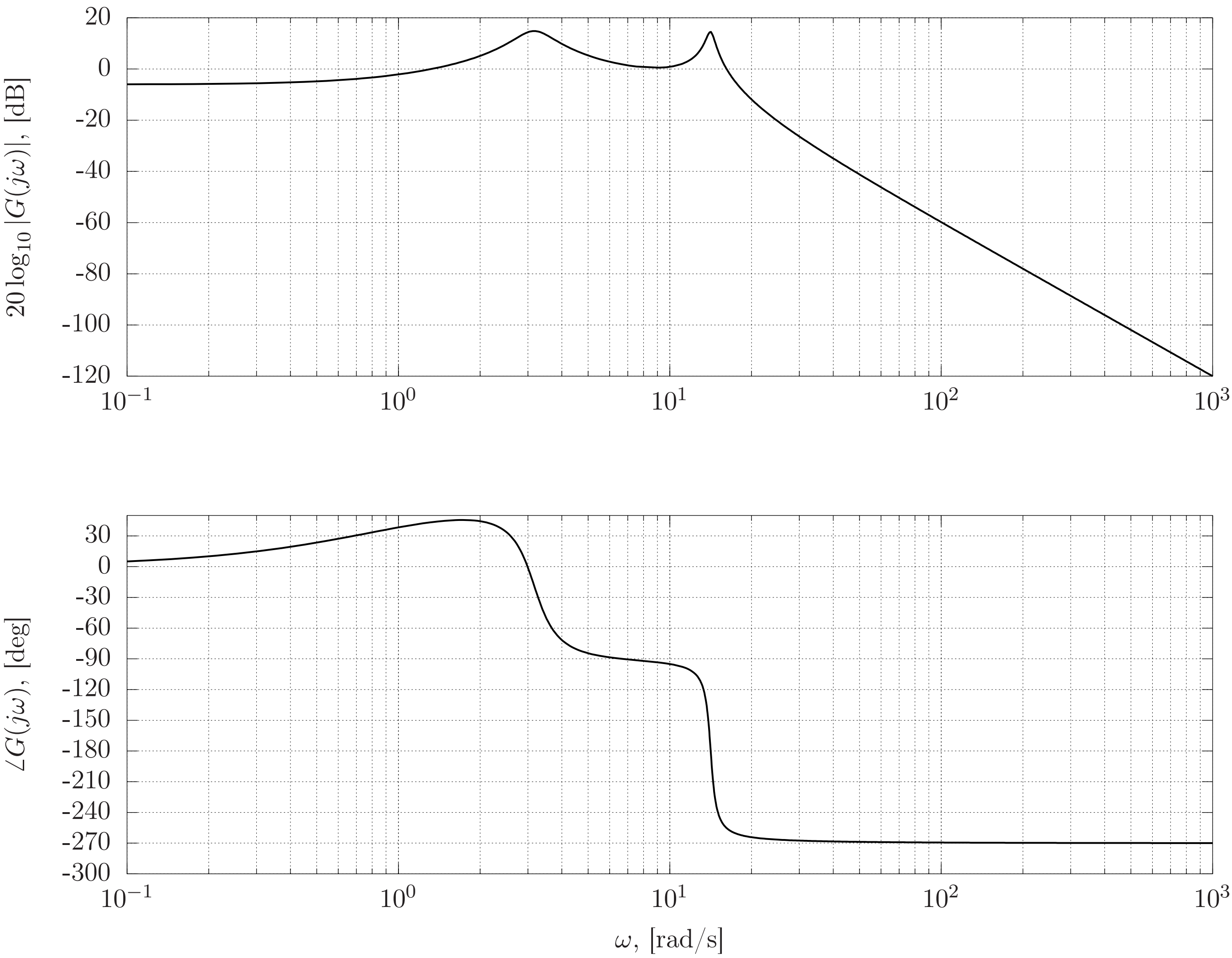
1.4. **[2 points]** Calculate analytically the angle of departure (or, is it arrival?) at the open loop poles.

1.5. **[2 points]** Calculate analytically the roots of multiplicity greater than 1.

1.6. **[4 points]** For the K found in Problem 1.3, determine the unit step response of the closed-loop system.

2. **[2 points]** The control system 

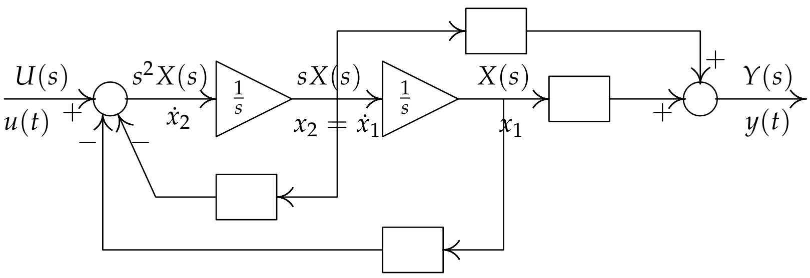
has the following Bode plot



Determine the largest value of $K > 0$ for which the control system is stable.

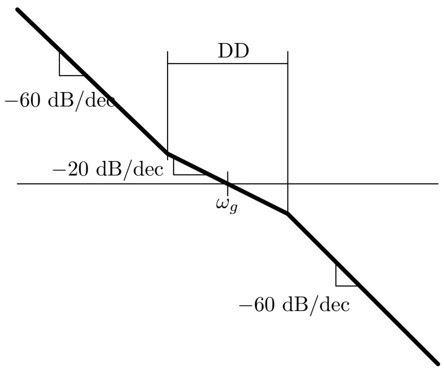
3. **[2 points]** Given $G_{\mathrm{CL}}(s) = \frac{G_{\mathrm{OL}}(s)}{1+G_{\mathrm{OL}}(s)}$. Evaluate $\left|G_{\mathrm{CL}}(j\omega_g)\right|$ for PM $= 45^\circ$; ω_g is the gain crossover frequency.

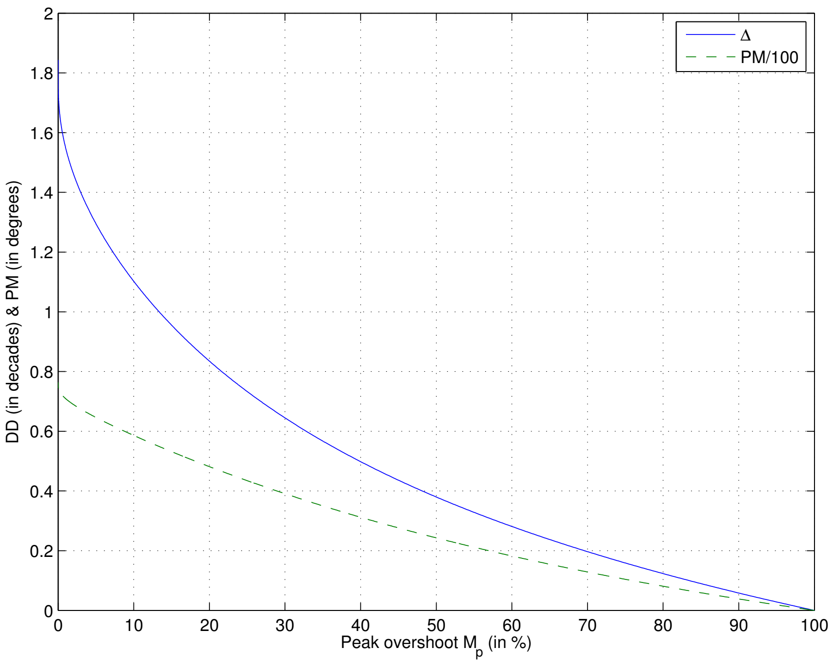
4. **[2 points]** In the process of numerically integrating the equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4y = 2\frac{du}{dt} + 7u$, you have arrived at the following diagram.



Fill the appropriate numbers into the appropriate blank boxes of this diagram.

5. The loop-shaping method that I taught you works only approximately in the region around ω_g. In this problem, we will see how approximately. Consider the following minimum-phase ABMP with DD = 1 decade.



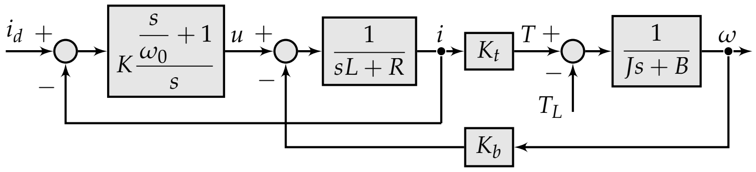


5.1. **[1 points]** Determine the phase margin that our loop-shaping technique attributes to this Bode plot.

5.2. **[1 points]** Given that the DD-to-$\Phi_{\max}$ relationship for a lead compensator is $\mathrm{DD} = \log_{10}\frac{1+\sin\Phi_{\max}}{1-\sin\Phi_{\max}}$, determine the $\Phi_{\max}$ of the lead compensator for DD = 1 decade.

5.3. **[3 points]** Now determine the true phase margin of the above Bode plot.

6. Consider the following proportional-integral control system for the armature current of a permanent magnet dc motor.



6.1. **[1 points]** Sketch the ABMP of the controller.

6.2. **[1 points]** We wish i to track step i_d with a small settling time. Assuming $K =$ fixed, select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

6.2.1. Large ω_0.

6.2.2. Small ω_0.

6.3. **[1 points]** We wish i to track step i_d with a small overshoot. Assuming $K =$ fixed, select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

6.3.1. Large ω_0.

6.3.2. Small ω_0.

6.4. **[1 points]** We wish i to track step i_d with a small settling time. Assuming $\omega_0 =$ fixed, select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

6.4.1. Large K.

6.4.2. Small K.

6.5. **[1 points]** We wish i to track step i_d with a small steady-state error. Select the one option that is most likely to result in this desired behavior, and justify your choice briefly.

6.5.1. Large K.

6.5.2. Small K.

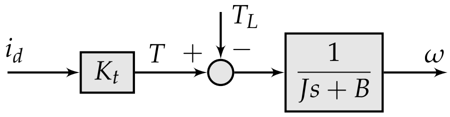
6.5.3. Large ω_0.

6.5.4. Small ω_0.

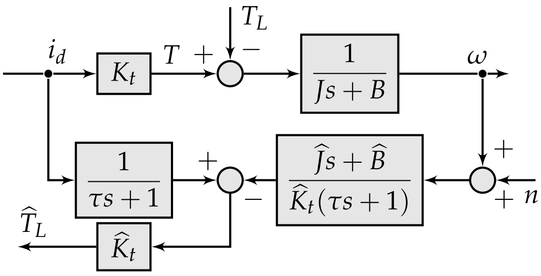
6.5.5. K and ω_0 do not affect the steady-state error.

7. The solution to the previous problem results in a well-regulated i.

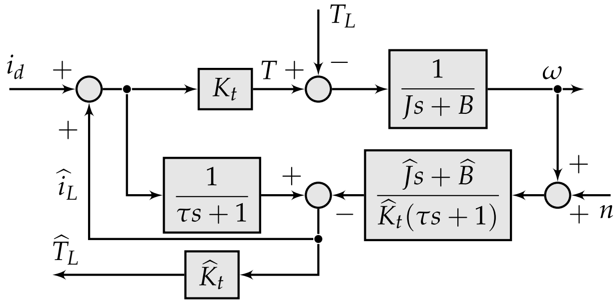
The motor with well-regulated i can be represented as



Here is an open-loop (OL) disturbance observer (DOB) scheme to generate an estimate $\widehat{T}_L$ of the load torque T_L.



On the other hand, the following scheme is a closed-loop (CL) DOB scheme.



Here, the quantities with the hats are estimates of the quantities without hats.

7.1. **[1 points]** Write the transfer function (TF) from T_L to $\widehat{T}_L$ for the OL DOB scheme.

7.2. **[2 points]** Determine the TF from T_L to $\widehat{T}_L$ for the CL DOB scheme.

7.3. **[2 points]** Assuming $\widehat{K}_t \approx K_t$, and that $\widehat{J}$ and $\widehat{B}$ are significantly different from J and B, for a unit step T_L, determine in which scheme $\widehat{T}_L$ is closer to T_L in steady state.
Hint: Use the final value theorem $y(t = \infty) = \lim_{s \to 0} sY(s)$.

7.4. **[2 points]** Assume that $i_d = 1$ A results in $\omega = 100$ rad/s while $T_L = 0$. If a non-zero T_L appears, determine which scheme rejects this disturbance better. That is, for a fixed i_d, in which scheme is ω less affected in steady state by the appearance of a step T_L?