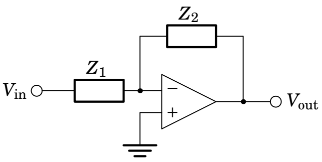
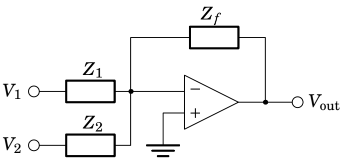
Final Exam, EE250 (Control System Analysis) Spring 2007 *

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

1. The exam is of 3 hours and 35 points.
2. Each question is for 1 point.
3. Please verify that your answer booklet contains 1 blank graph paper (containing 1 semilog and 1 linear) and 1 Bode plot.
4. Questions marked **NA** need answers that are within ±5% of the actual values. On such questions, demonstration of knowledge of concept is not enough.
5. Use a straight edge and a calculator where necessary.
6. Wherever you need to use Bode plot methods, construct **asymptotic** magnitude plots and **actual** phase plots.
7. Use a pen for writing and a pencil for drawing.
8. Show all the assumptions that you make.
9. There is **no** partial credit.
10. Useful op-amp circuits:



$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$$



$$V_{out} = -\frac{Z_f}{Z_1}V_1 - \frac{Z_f}{Z_2}V_2$$

11. Walter R. Evans' rules for sketching root locus (RL):

Rule 1 Number of branches of RL.

Rule 2 Origin & destination of RL.

Rule 3 Section of RL on real axis.

Rule 4 Form of RL about the real axis.

Rule 5 Centroid ($-\sigma$) of asymptotes of RL.

Rule 6 Angle of asymptotes of RL.

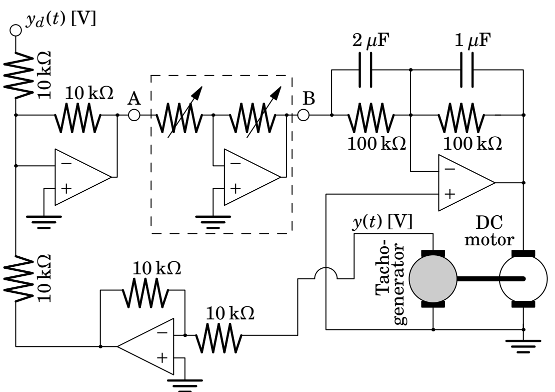
Rule 7 Break-away & break-in points on the RL.

Rule 8 Angles of departure from poles & arrival at zeros.

Rule 9 Intersection of RL with $j\omega$ axis.

Rule 10 Calculate a few test points that will satisfy the angle criterion with an error of $\le 5° - 7°$, and use these test points in order to make the sketch reasonably accurate.

DC motor control A DC motor is know to have the transfer function (TF) $\frac{1}{s(s+2)}$ $\frac{\text{rad/s}}{\text{V}}$. The motor's tachogenerator is known to provide 1 mV per rad/s. You built the following circuit (named "DCMC1"):

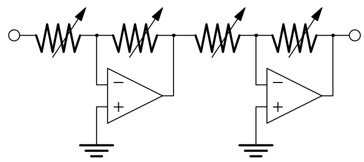


1.1. Draw a block diagram representation of this circuit that will help you write the open-loop (OL) and closed-loop (CL) TFs.

1.2. Write the OL and CL TFs.

1.3. Sketch the root locus (RL) of this circuit for variation in the gain between points A and B.

Next, you want to study the effect of replacing the portion of DCMC1 between points A and B with the following circuit:



The resulting circuit will be called "DCMC2".

2.1. Draw a block diagram representation of this circuit that will help you write the OL and CL TFs.

2.2. Write the OL and CL TFs.

2.3. Sketch the RL of this circuit for variation in the gain between points A and B.

2.4. Which of DCMC1 and DCMC2 is your final choice for your motor control circuit? Give a brief reason.

Lag Compensator A lag compensator is described by the equation: $D(s) = \frac{Ts+1}{\alpha Ts+1}$, $1 < \alpha$.

3.1. Sketch the Bode plot of $D(s)$. Label the axes and the corner frequencies.

3.2. Find the frequency $\omega_{\min}$ at which $D(s)$ has minimum phase for a given value of α.

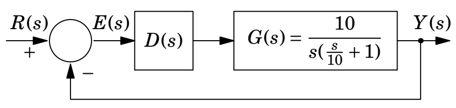
3.3. Find the phase $\phi_{\min}$ that $D(s)$ provides at $\omega_{\min}$.

3.4. Find the value of α for a given $\phi_{\min}$.

3.5. Find the width of $D(s)$ in decades (distance between the frequencies $1/T$ and $1/(\alpha T)$).

3.6. Suppose you wish $D(s)$ to provide you a minimum phase of $\phi_{\min} = -60°$. What should the distance (in decades) between the two corner frequencies of $D(s)$ be?

Loop-shaping Consider the following system:



We wish to design a compensator $D(s)$ that satisfies the following design specifications:

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i. $K_v = 100$.

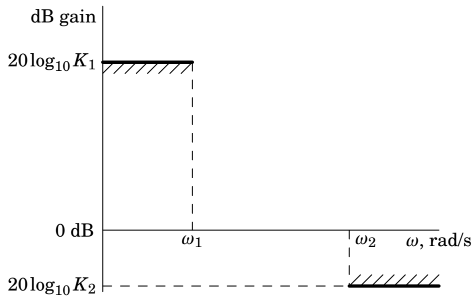
ii. $\Phi\text{M} = 45°$.

iii. Sinusoidal inputs of up to 1 rad/sec to be reproduced with $\leq 2\%$ error.

iv. Sinusoidal inputs with a frequency of greater than 100 rad/sec to be attenuated at the output to $\leq 5\%$ of their input value.

Do all construction on the blank semilog grid.

4.1. Determine K_1 and K_2 in the following figure.



4.2. Write the numerical values of ω_1 and ω_2 of the above figure.

4.3. Calculate the needed distance (in decades) between the corner frequencies of the −20 dB/dec section around ω_g for the desired Bode plot. **NA**.

4.4. Draw the asymptotic Bode magnitude plots of the desired $D(s)G(s)$ and of $G(s)$. Your figure must contain all the necessary labels. **NA**.

4.5. Show the Bode magnitude plot of the resulting $D(s)$. Write the TF of $D(s)$. **NA**.

4.6. For the resulting CL system, given that $\omega_B \in [\omega_{lo}, \omega_{hi}]$, where ω_B is the bandwidth, what are the numerical values of ω_{lo} and ω_{hi}? **NA**.

4.7. Calculate the magnitude of the CL TF at ω_g. **NA**.

4.8. Sketch the Nyquist plot of the $D(s)G(s)$. Label the points corresponding to $\omega = 0_+$, $\omega = +\infty$, ω_g, ω_ϕ.

Lead/lag compensator design The OL TF of a unity-feedback system is $G(s) = \frac{K}{s(s/5+1)(s/50+1)}$. You want to design a lag compensator $D(s)$ so that the CL system satisfies the following specifications:

i. Steady-state error to a unit ramp reference input (e_{ssr}) is less than 0.01.

ii. $\Phi\text{M} \geq 40°$.

The Bode plot of $G(s)|_{K=1}$ is provided on a separate page. **Do all required construction on that page.**

5.1. Determine a suitable value of K.

5.2. Draw the Bode plot of $\widehat{G}(s)$. $\widehat{G}(s)$ is $G(s)$ with the value of K calculated in question 5.1. **NA**.

5.3. Choose the width of $D(s)$ in decades (the distance between the corner frequencies). Give reason for choosing this width.

5.4. Where — relative to the Bode plot of $\widehat{G}(s)$ — would you position ω_{min} of $D(s)$? Give reason.

5.5. Draw the Bode plot of $D(s)$. Show a table of at least 5 points for the phase plot. **NA**.

5.6. Draw the Bode plot of the resulting $D(s)\widehat{G}(s)$. **NA**.

5.7. If this $D(s)\widehat{G}(s)$ has satisfied your specifications, then congratulations with a bonus point! Otherwise, explain briefly what you will do next to complete the design successfully.

5.8. Sketch the Nyquist plot of $D(s)\widehat{G}(s)$. Label the points corresponding to $\omega = 0_+$, $\omega = +\infty$, ω_g, ω_ϕ.

Root Locus sketching On the attached linear grid, sketch the PRL for the following equation: $1 + \frac{K}{s(s+2)(s^2+2s+5)} = 0$. Your calculations will contain the following items:

6.1. Rules 1, 2, 3, 4.

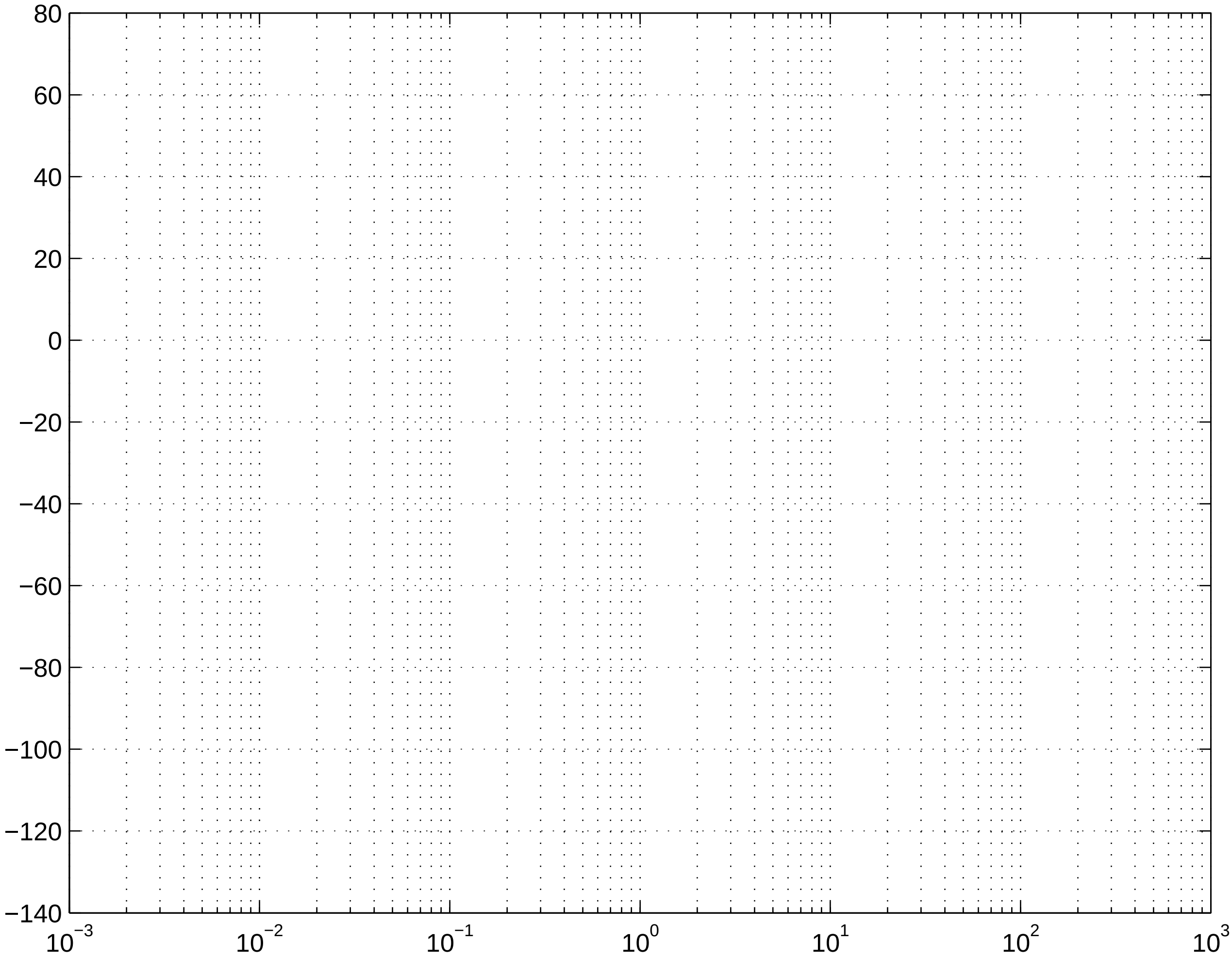
6.2. Rule 5, 6. **NA**.

6.3. Rule 7. **NA**.

6.4. Rule 8. **NA**.

6.5. Rule 9. **NA**.

6.6. Rule 10. Calculate 3 test points. **NA**.


80
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