Lecture-6

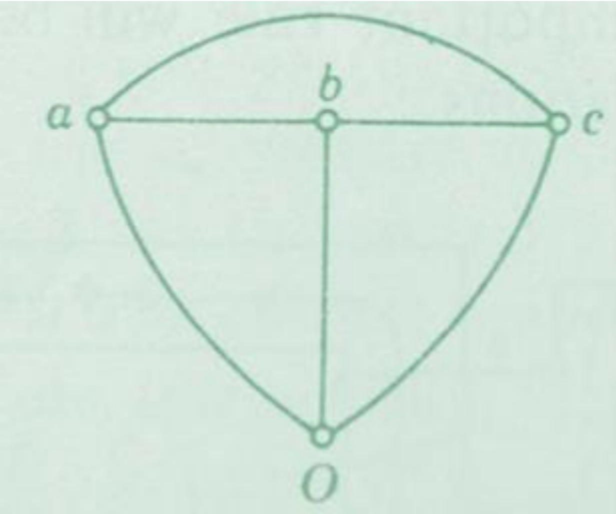
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Graph theory.
- Delta to star and star to delta conversion.
- AC Network.
- Phasor and power.

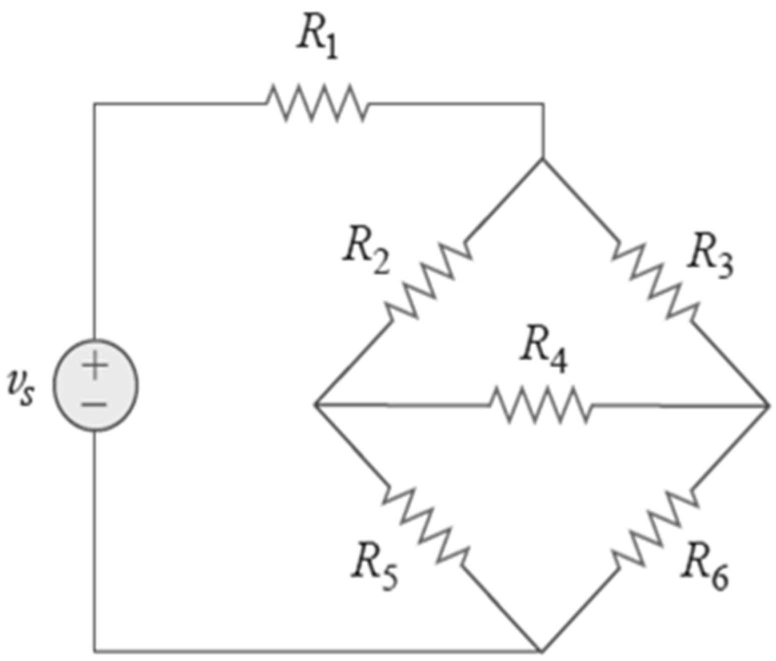
Topology (Cont…)

- Subgraph of a given graph is formed by removing branches from the original graph.
- Tree is one kind of subgraph which is important for our circuit analysis.
- Tree is formed by removing a branch from the graph.
- A tree of **n** nodes has the following property:
  - It contains all nodes of the graph.
  - It contains n-1 branches.
  - There is no closed path.

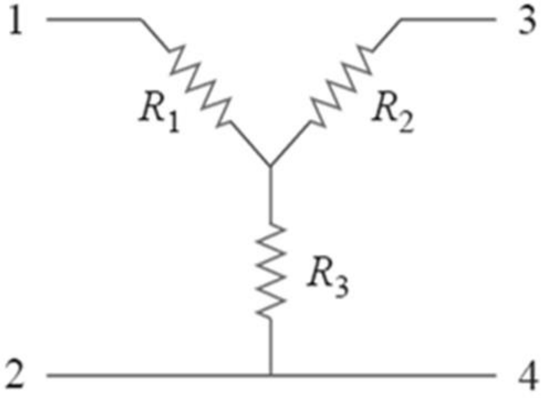


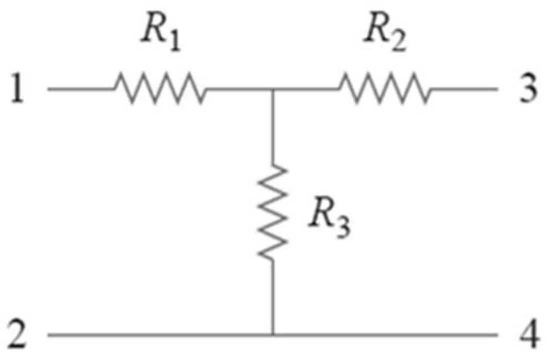
- There are many possible different trees of a graph.
- Branches removed from the graph in forming a tree are called chords or links.
- Concept of tree is used for the proper choice of current variables for the analysis of a network.

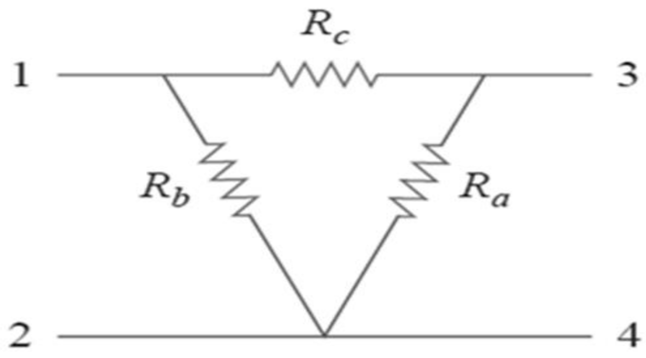
Star (Y) – Delta (Δ) Transformation

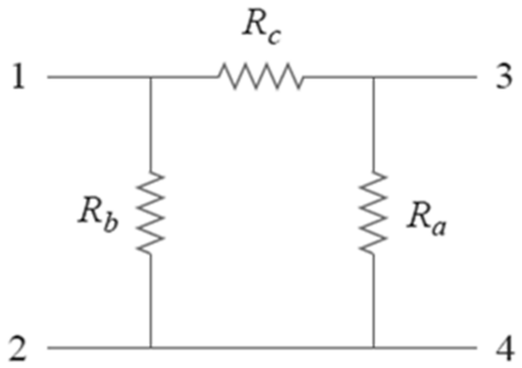


- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series, as shown in the figure
- Many circuits of the type shown in figure can be simplified by using three terminal equivalent networks
- These are wye (Y) or tee (T) network and the delta (Δ) or pi (Π) network
- These networks occur by themselves or as part of a larger network/circuit







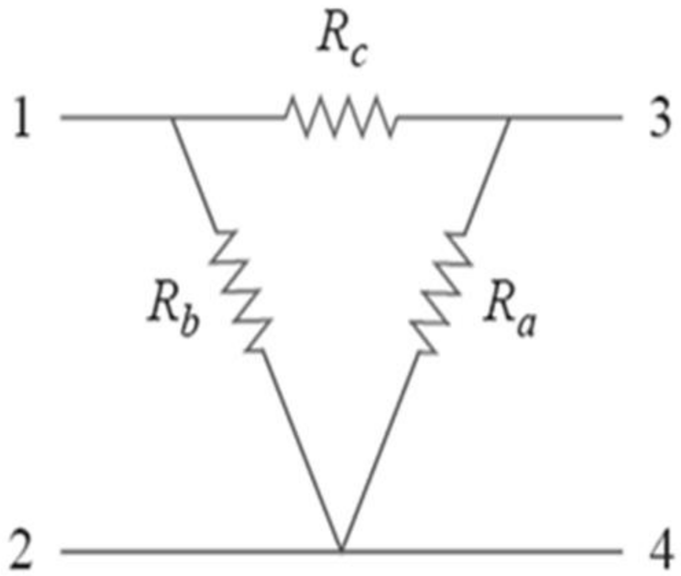


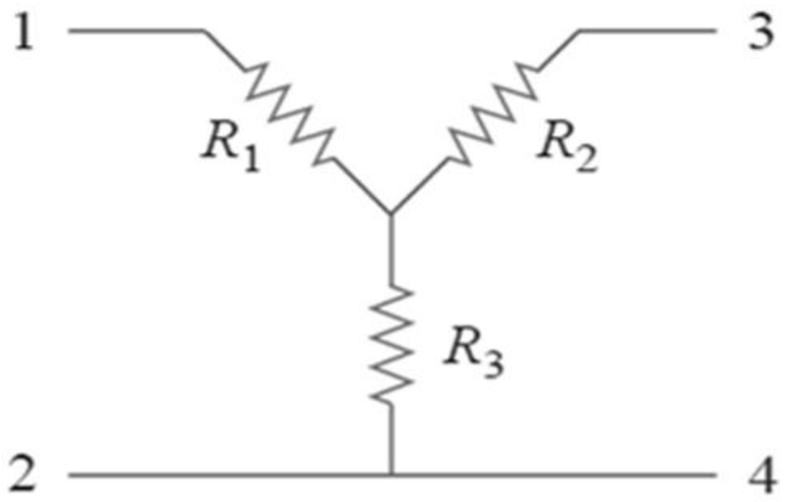
Delta to Star Conversion

- The conversion from delta to star is needed if it is more convenient to work with a star network in a place where the circuit contains a delta configuration.
- We superimpose a star network on the existing delta network and find the equivalent resistances in the star network.
- To obtain the equivalent resistances in the star network, make sure that the resistance between each pair of nodes in the delta network is the same as the resistance between the same pair of nodes in the star (or Y/T) network.

$R_{12}(Y) = R_1 + R_3$

$R_{12}(\Delta) = R_b || R_a + R_c$





Delta to Star Conversion (Cont…)

- From equations –

$$R_{12}(Y) = R_{12}(\Delta) \quad (1)$$

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} \quad (2)$$

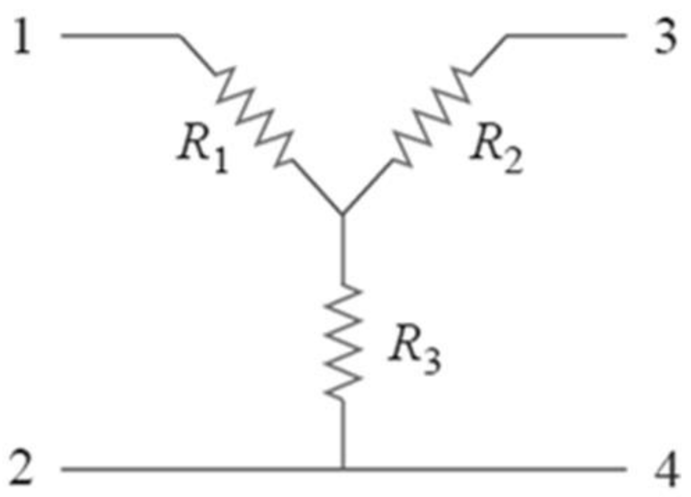
- Similarly,

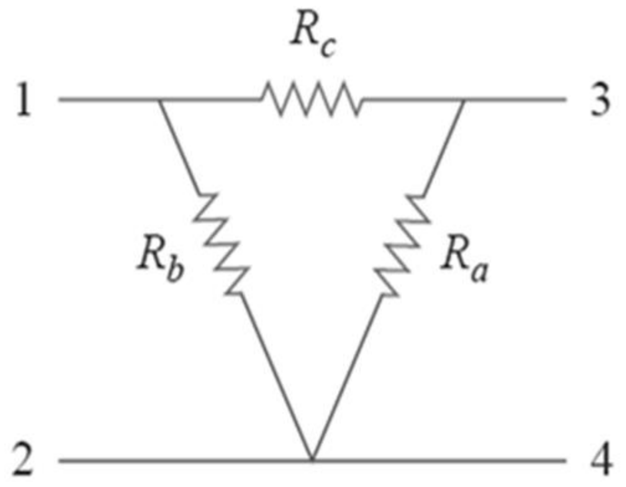
$$R_{13} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} \quad (3)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \quad (4)$$

- Subtracting Eq. 4 from Eq. 2 –

$$R_1 - R_2 = \frac{R_c(R_b-R_a)}{R_a+R_b+R_c}$$





Delta to Star Conversion (Cont…)

- Adding the following two equations, i.e. –

$$R_1+R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} \qquad R_1-R_2 = \frac{R_c(R_b-R_a)}{R_a+R_b+R_c}$$

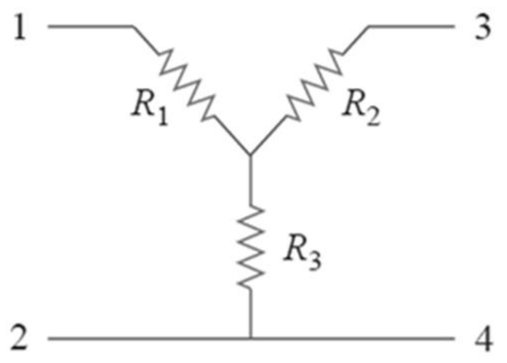
- We will get the values of R_1:

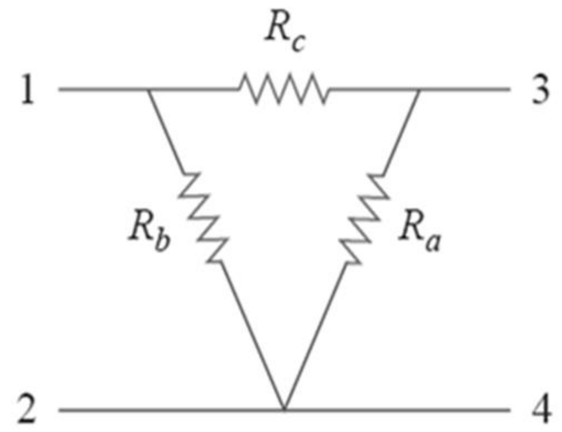
$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c}$$

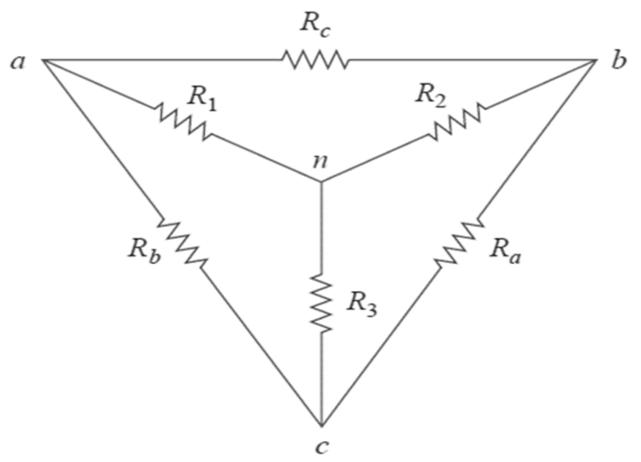
- Similarly

$$R_2 = \frac{R_cR_a}{R_a+R_b+R_c} \qquad R_3 = \frac{R_aR_b}{R_a+R_b+R_c}$$

- No need to memorize above equations, follow the following conversion rule
- **Each resistor in the star (Y) network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.**







Star to Delta Conversion

- To obtain the conversion formulas for transforming a star (Y) network to an equivalent delta network, we use the values of R1, R2, and R3, calculated recently.
- Using those values, we can calculate -

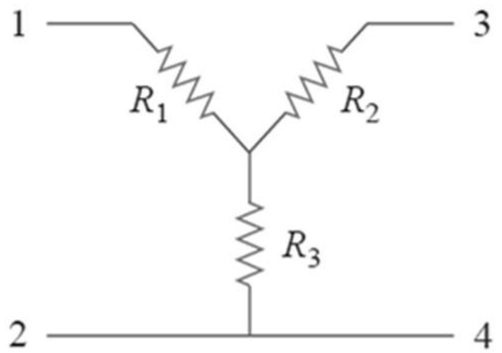
$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a+R_b+R_c)}{(R_a+R_b+R_c)^2} = \frac{R_aR_bR_c}{(R_a+R_b+R_c)}$$

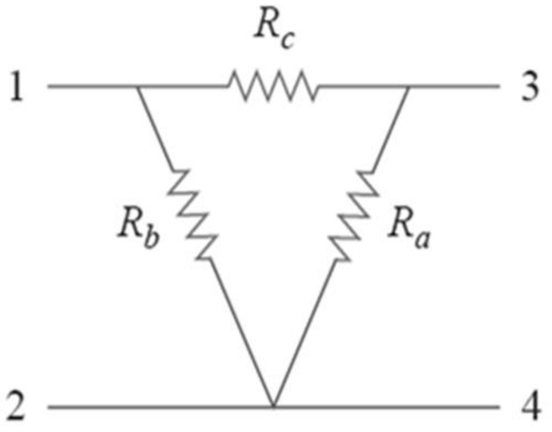
- Dividing above equation by the equation of R1, we get –

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

- Similarly,

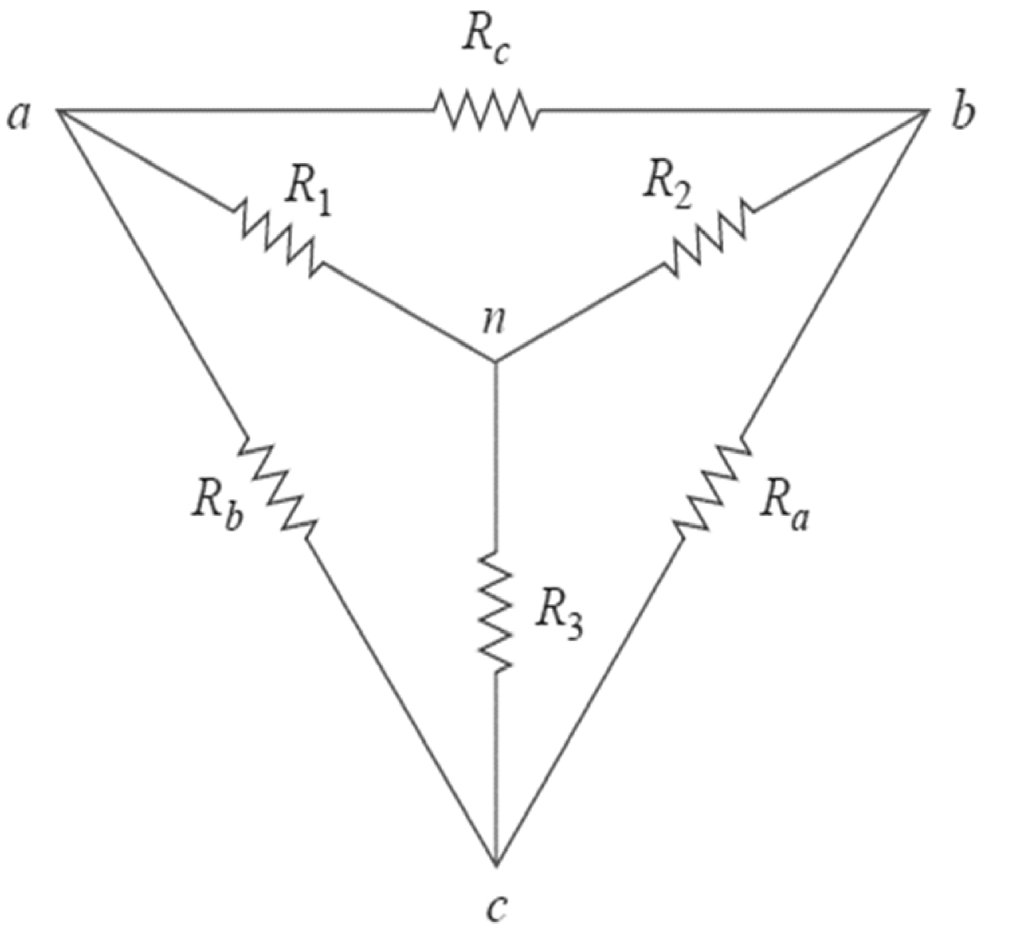
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$





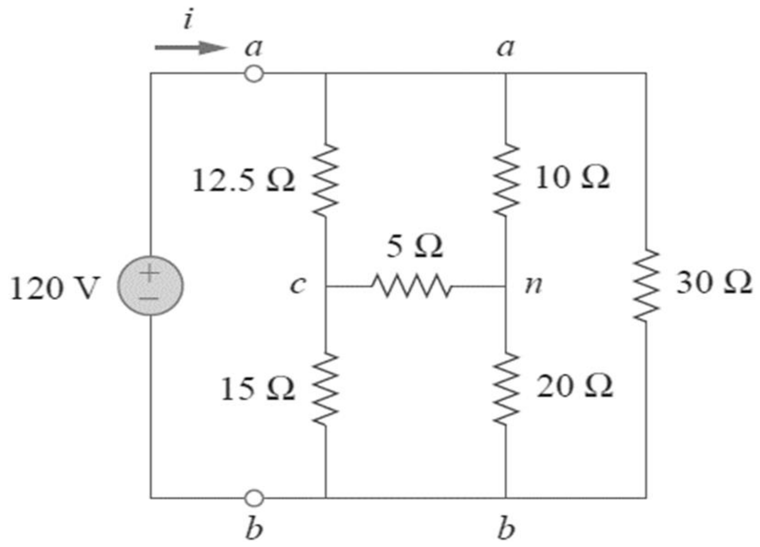
Star to Delta Conversion (Cont…)

- No need to memorize above equations, follow the following conversion rule
- Each resistor in the Delta (Δ) network is the sum of all possible products of star (Y) resistors taken two at a time, divided by the opposite star (Y) resistor.



Example

- Obtain the equivalent resistance R_{ab} for the circuit shown in the figure and find current i ?



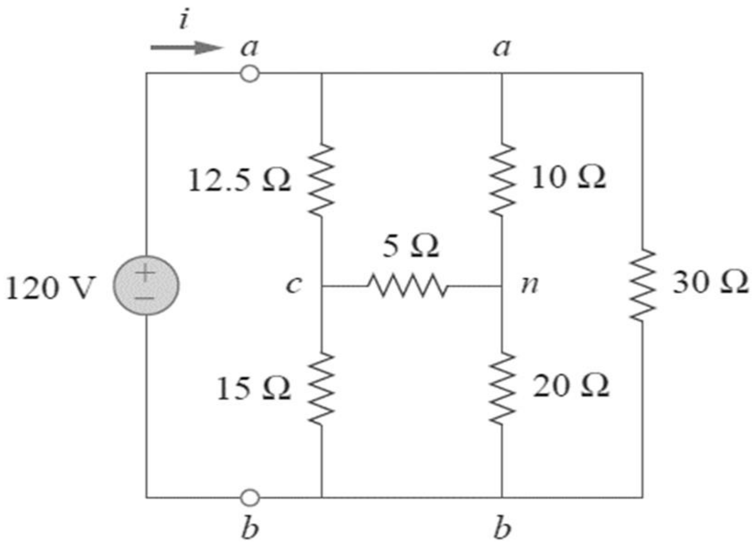
Solution:

- In this circuit, there are two star (Y) networks and three Delta networks. Transforming just one of these will simplify the circuit.
- If we convert the star network comprising the 5 Ω, 10 Ω, and 20 Ω resistors, we may select –R1 = 10 Ω, R2 = 20 Ω, R3 = 5 Ω.

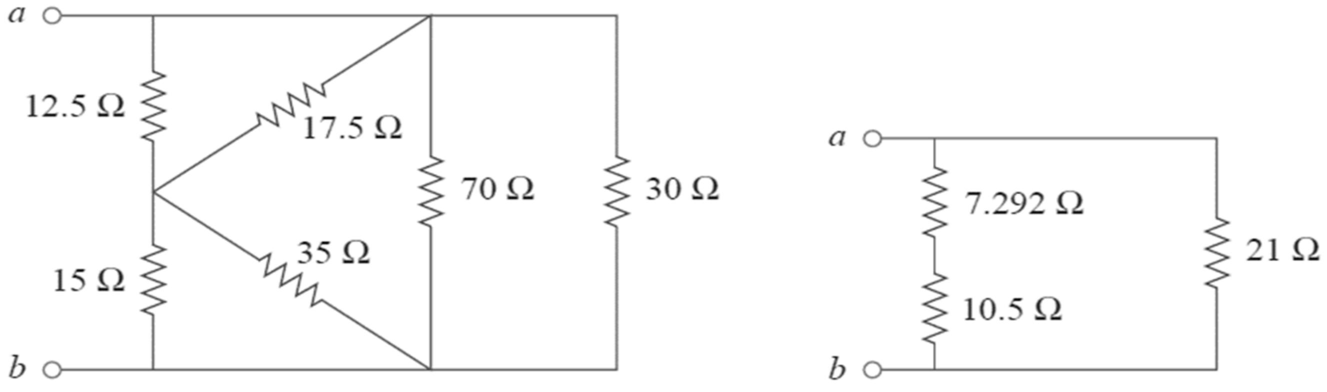
Example (Cont…)

- Using the following equations –

$$R_a = \frac{R_1R_2 + R_2R_3 \ + \ R_3R_1}{R_1}$$
$$R_b = \frac{R_1R_2 + R_2R_3 \ + \ R_3R_1}{R_2}$$
$$R_c = \frac{R_1R_2 + R_2R_3 \ + \ R_3R_1}{R_3}$$



- We get the values as –
- Ra = 35 Ω, Rb = 17.5 Ω, Rc = 70 Ω
- The equivalent circuit would be -

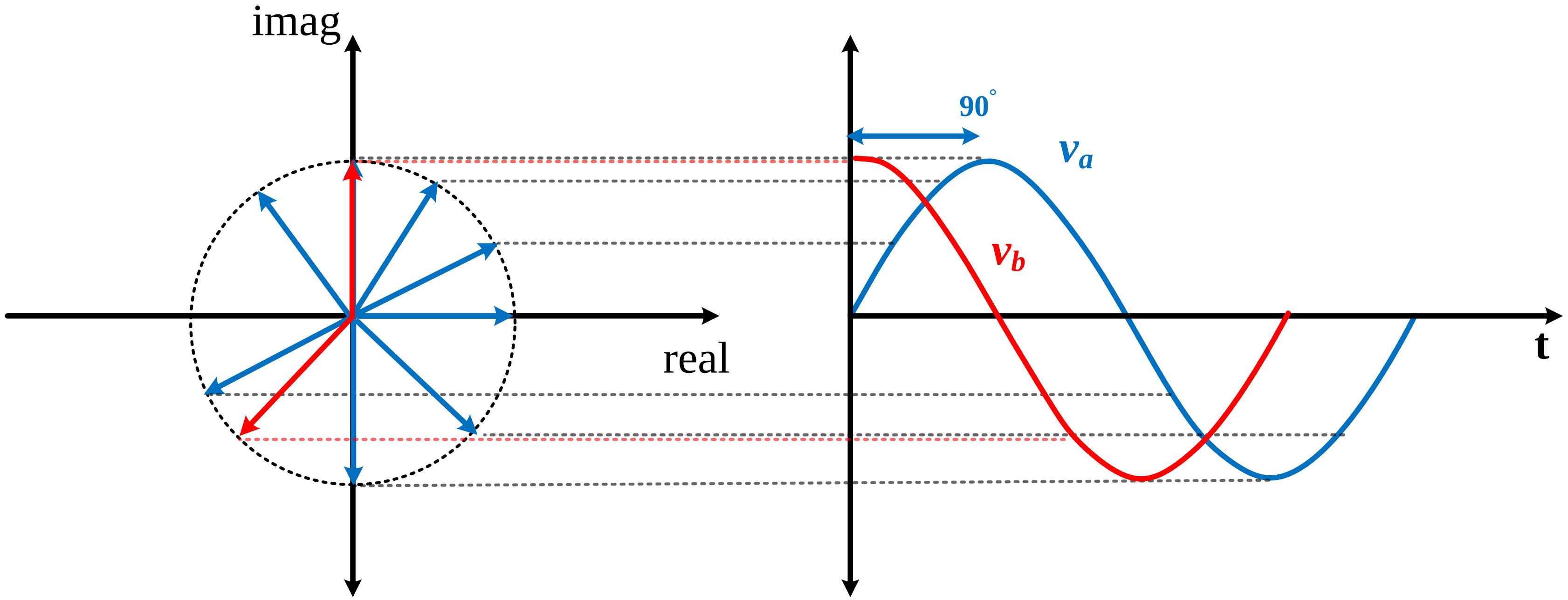


(a) (b)

- Therefore, Rab = 9.632 Ω
- and, I = V/Rab = 12.458 A

Sinusoids

Sinusoids (cont…)



- Space phasor, because it's in the complex domain.

$$v_a = V_m \sin \omega t$$

$$v_b = V_m \cos \omega t$$

$$v_b = v_a e^{\frac{j\pi}{2}} = v_a \left(\cos \frac{\pi}{2} + j \sin \frac{\pi}{2} \right)$$

$$v_b = j v_a$$

Sinusoids (cont…)

$$\boxed{v_b = jv_a = jV_m \sin \omega t}$$

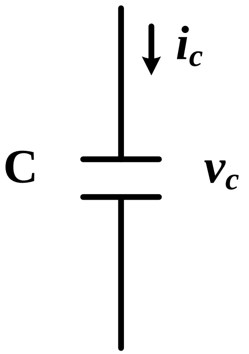
$$\frac{d(v_a)}{dt} = \frac{d(V_m \sin \omega t)}{dt} = V_m \omega \cos \omega t$$

$$= \omega V_m \cos \omega t = \omega jV_m \sin \omega t \quad \{\therefore v_b = jv_a\}$$

$$= j\omega v_a$$

$$\boxed{\frac{d}{dt} \Rightarrow j\omega}$$

Capacitor in AC-Network



$$c\frac{dv_c}{dt}=i_c$$

$$c\frac{d\left(V_m\sin\left(\omega t-\phi\right)\right)}{dt}=i_c$$

$$\omega cV_m\cos\left(\omega t-\phi\right)=i_c$$

$$\omega cjV_m\sin\left(\omega t-\phi\right)=i_c \qquad \left\{\therefore v_b = jv_a\right\}$$

$$j\omega c\times v_c=i_c \qquad (1)$$

$$\frac{volts}{ohms}=Amps$$

From eqn (1)

$$j\omega c\times v_c=i_c$$

$$\frac{v_c}{1/\,j\omega c}=i_c$$

$$\frac{1}{j\omega c}=\frac{-j}{-j^2\omega c}=-j\left(\frac{1}{\omega c}\right)$$

$$-jX_c$$

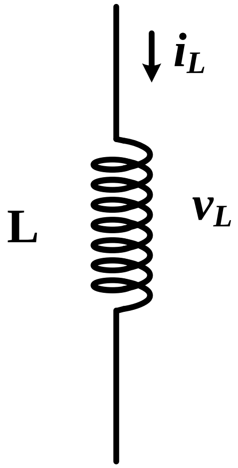
X_C capacitive reactance and its unit is ohms.

$$\boxed{X_c=\frac{1}{\omega c}}$$

'j' rotation parameter in complex domain.

'-ve' indicate leading phase.

Inductor in AC-Network



$$L\frac{di_L}{dt} = v_L$$

$$L\frac{d\left(I_m \sin(\omega t - \phi)\right)}{dt} = v_L$$

$$\omega L I_m cos(\omega t - \phi) = v_L$$

$$j\omega L I_m \sin(\omega t - \phi) = v_L \qquad \{\therefore i_b = j i_a\}$$

$$j\omega L i_L = v_L \qquad (1)$$

$$\frac{volts}{Amps} = ohms$$

From eqn (1)

$$j\omega L i_L = v_L$$

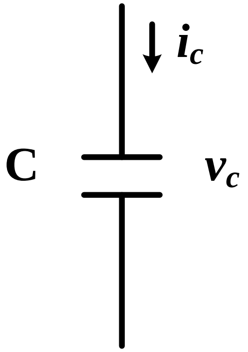
$$\frac{v_L}{i_L} = j\omega L$$

$$jX_L$$

X_L inductive reactance and its unit is ohms.

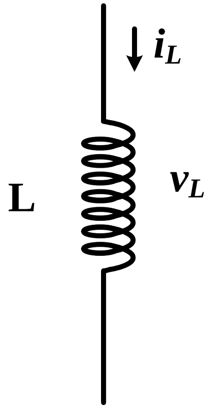
$$\boxed{X_L = \omega L}$$

Capacitor and Inductor in AC-Network



$$c\frac{dv_c}{dt} = i_c$$

$$\boxed{X_c = \frac{1}{\omega c}}$$



$$L\frac{di_L}{dt} = v_L$$

$$\boxed{X_L = \omega L}$$

Sinusoids (cont…)

- In this lecture we will discuss about the basic concepts needed to analyses circuits in which the source voltage or current is sinusoidal time varying.
- Sinusoidal time varying excitation means excitation given by a sinusoid.
- A sinusoid is a signal that has the form of a **sine** or a **cosine** function.
- A sinusoidal current is usually referred to as **alternating current (AC)**.
- Such a current reverses at regular time intervals and has alternately positive and negative values.
- Circuits driven by **sinusoidal current** or **voltage sources** are called AC circuits.

Sinusoids (cont…)

- We are interested in sinusoids because -

  - Its characteristic is sinusoidal.
  - A sinusoidal signal is easy to generate and transmit.
  - It is the form of voltage generated throughout the world and supplied to various types of loads.
  - Through Fourier analysis, any practical periodic signal can be represented by a sum of **sinusoids**.
  - A sinusoid is easy to handle mathematically.
  - The derivative and integral of a sinusoid are also sinusoids.

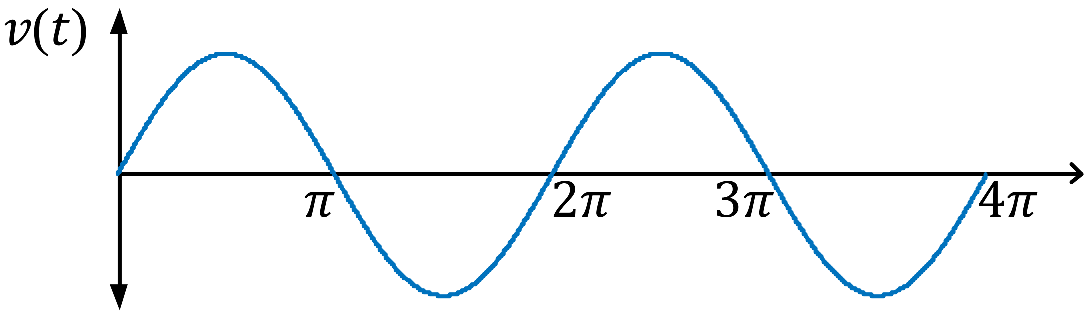
- Therefore, the sinusoid is an extremely important function in circuit analysis.

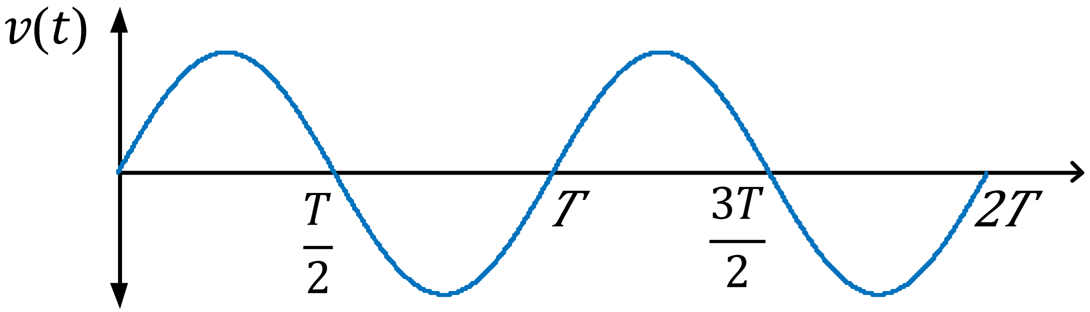
Sinusoids (Cont…)

- Consider the sinusoidal voltage expressed as follows:

$$v(t) = V_m \sin \omega t$$

- Here, $\boldsymbol{V_m}$ is the amplitude of the signal and $\boldsymbol{\omega}$ is the angular frequency in radians/sec.
- The figures given below represent the sinusoid as a function of $\boldsymbol{\omega t}$ and $\boldsymbol{t}$.
- It can be observed that the signal repeats itself after every $\boldsymbol{T}$ seconds.
- $\boldsymbol{T}$ is known as the period of the sinusoid.





Sinusoids (Cont…)

- From the previous figures it can be observed that $\omega T=2\pi$. Therefore, T can be expressed as

$$T = {}^{2\pi}/_{\omega}$$

- The fact that the signal repeats itself after T seconds can be demonstrated as follows:

$$v(t+T) = V_m \sin \omega(t+T) = V_m \sin \omega(t + {}^{2\pi}/_{\omega}) = V_m \sin(\omega t + 2\pi) = V_m \sin \omega t = v(t)$$

- Hence, v has the same value at $t+T$ as it does at t and $v(t)$ is said to be periodic.
- A periodic signal can, thus, be defined as a function that satisfies $f(t) = f(t + nT)$, for all t and all integers n.

Sinusoids (Cont…)

- As mentioned, the period $\boldsymbol{T}$ of the periodic function is the time of one complete cycle or the number of seconds per cycle.
- The reciprocal of this quantity is the number of cycles per second, known as the cyclic frequency $\boldsymbol{f}$ of the sinusoid.
- Thus,

$$f = {}^{1}/_{T}$$

- Using the above equation, we can conclude that,

$$\omega = 2\pi f$$

- Here, $\boldsymbol{\omega}$ is expressed in radians/second and $\boldsymbol{f}$ is in hertz (Hz).

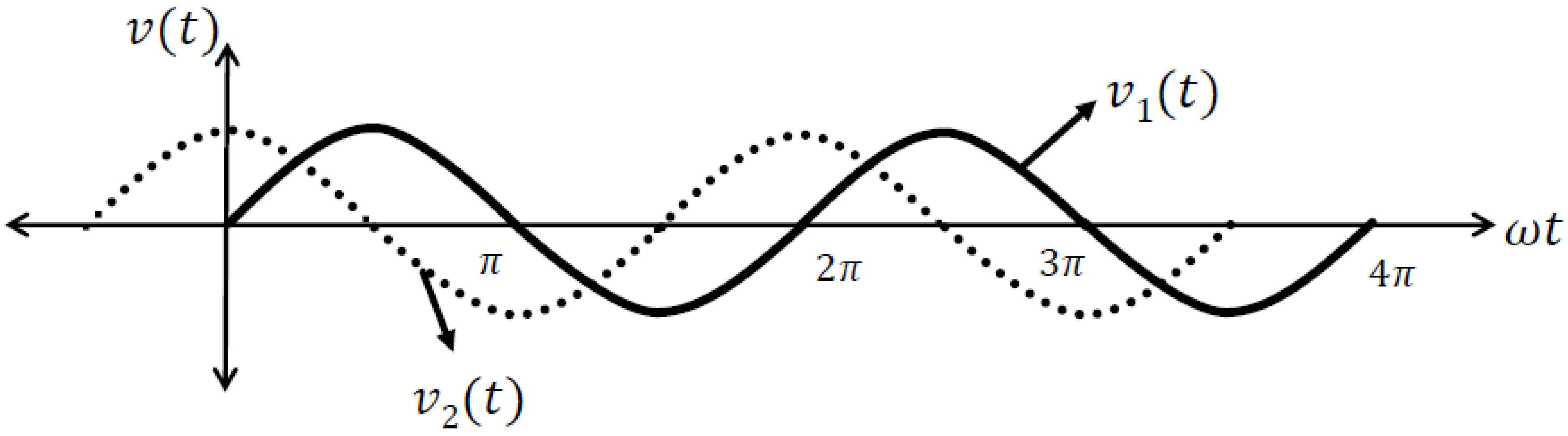
Sinusoids (Cont…)

- A more general expression for the sinusoid is given by,

$$v(t) = V_m \sin(\omega t + \emptyset)$$

where, $\emptyset$ is the phase and is expressed in degrees or radians.

- To introduce the idea of phase, consider two sinusoids expressed as $\boldsymbol{v1(t) = V_m sin\, \omega t}$ and $\mathbf{v_2}(\boldsymbol{t}) = \boldsymbol{V_m sin(\omega t + \emptyset)}$ as illustrated in the figure below.

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