Assignment 1
MSO-201: Probability and Statistics

1. A box contains three marbles: one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box then replacing it in the box and drawing a second marble from the box. What is the sample space? If, at all times, each marble in the box is equally likely to be selected, what is the probability of each point in the sample space?

2. Repeat Exercise 1 when the second marble is drawn without replacing the first marble.

3. A coin is to be tossed until a head appears twice in a row. What is the sample space for this experiment? If the coin is fair, what is the probability that it will be tossed exactly four times?

4. Let E, F, G be three events. Find expressions for the events that of E, F, G

   (a) only F occurs,

   (b) both E and F but not G occur,

   (c) at least one event occurs,

   (d) at least two events occur,

   (e) all three events occur,

   (f) none occurs.

5. An individual uses the following gambling system at Las Vegas. He bets \$1 that the roulette wheel will come up red. If he wins, he quits. If he loses then he makes the same bet a second time only this time he bets with Dollar 2; and then regardless of the outcome, quits. Assuming that he has a probability of 1/2 of winning each bet, what is the probability that he goes home a winner?

6. Show that $E(F \cup G) = EF \cup EG$.

7. Show that $(E \cup F)' = E'F'$.

8. If $P(E) = 0.9$ and $P(F) = 0.8$, show that $P(EF) \geq 0.7$. In general, show that $P(EF) \geq P(E) + P(F) - 1$. This is known as Bonferroni's inequality.

9. We say that $E \subset F$ if every point in E is also in F. Show that if $E \subset F$, then $P(F) = P(E) + P(FE') \geq P(E)$.

10. Show that
$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i).$$
This is known as Boolefs inequality