Final Exam
EE 250 (Control Systems Analysis) Spring 2011 *

DEPARTMENT OF ELECTRICAL ENGINEERING, IIT KANPUR.

Instructions

1. You may use a pencil for trial and error, but your final drawing should use a pen.
2. Show all your assumptions.
3. You will need the following items: pen, pencil, ruler, eraser, calculator. Borrowing not permitted.
4. If we have difficulty in reading your answer book, we will deduct points arbitrarily. It is your responsibility to write legibly.

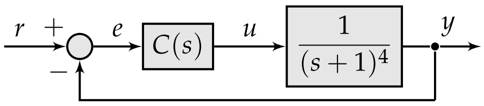
Useful information

Controller	k_p	T_i	T_D
P	$0.5k_{cr}$	∞	0
PI	$0.45k_{cr}$	P_{cr} / 1.2	0
PID	$0.6k_{cr}$	P_{cr} / 2	P_{cr} / 8

$$\frac{1}{\Delta}\sum_{i=1}^{N} P_i \Delta_i$$

Problems

1. We wish to use the ultimate gain method of Ziegler and Nichols to tune a PID controller for the following control system.



1.1. **[3 points]** Determine the critical period and critical gain of this system.

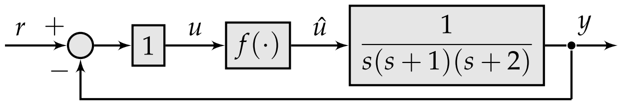
1.2. **[1 points]** Determine the closed-loop poles corresponding to the critical gain.

1.3. **[2 points]** Write the transfer function, including numerical values, of a practically implementable PID controller.

1.4. **[1 points]** In the step response of the closed-loop system, what would you expect the ratio of the first overshoot to the second overshoot to be? Explain.

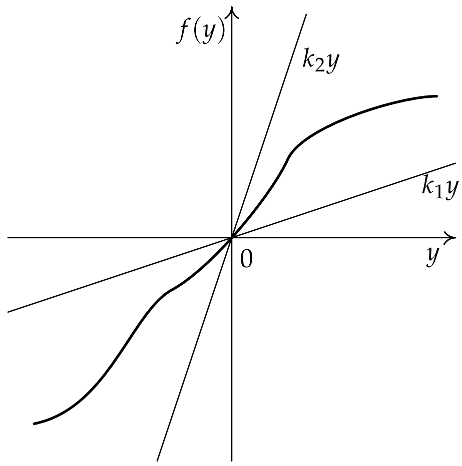
1.5. **[1 points]** If you used a PI controller, also tuned by the same method, instead of this PID controller, the closed-loop system corresponding to which controller would you expect to be more oscillatory? Explain.

2. **[4 points]** For the control system



determine the minimum value of $1/k_2$ for which the closed-loop system is guaranteed to be stable.

Here, $f(\cdot)$ is a sector-bounded nonlinearity characterized by nonnegative parameters k_1 and k_2 as shown below.



3. Two anti-wind-up schemes are proposed for a control system in which the controller contains an integrator and the plant contains a saturation element (limiter), which limits the magnitude of the input to the plant.

The first scheme is described by

$$\hat{u} = u + \left\{ k\varepsilon e - \left[\frac{\varepsilon s}{\varepsilon s + 1}\right] u \right\}$$

while the second scheme is described by

$$\hat{u} = u + \left\{ \frac{k\varepsilon}{(\varepsilon s + 1)} e - \left[\frac{\varepsilon s}{\varepsilon s + 1}\right] u \right\}$$

Here, e is the tracking error, $\hat{u}$ is the input to the limiter, u is the input to the plant, $k = 1$, and $\varepsilon = 0.1$ s.

Given that the scheme which involves less clipping by the limiter is the superior one,

3.1. **[2 points]** Which scheme is superior for the tracking of sinusoids of frequency greater than $2/\varepsilon$ rad/s? Why?

3.2. **[2 points]** which scheme is superior for the tracking of a step input? Why?

4. **[4 points]** In each of the following cases, set up coordinate axes, mark the locations of the poles and zeros, and write down adjacent to these roots the respective multiplicities, so that the respective root locus possesses the form mentioned

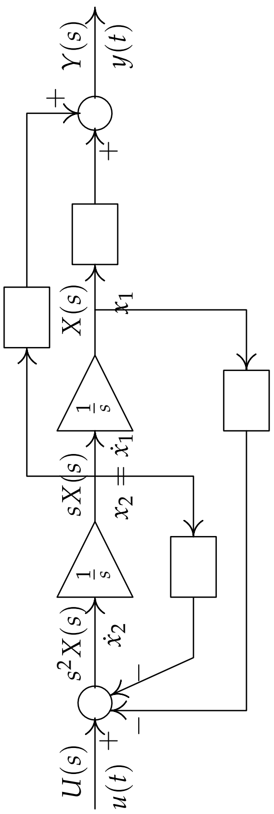
*Instructor: Ramprasad Potluri, E-mail: `potluri@iitk.ac.in`, Office: WL217A, Lab: WL217B, Phones: (0512) 259-8837, 259-7735.

4.1. Eight ("8").
4.2. Circle ("O") or ellipse.
4.3. "X".
4.4. Infinity ("∞") or eight turned 90°.

5. **[4 points]** In the process of numerically integrating the equation

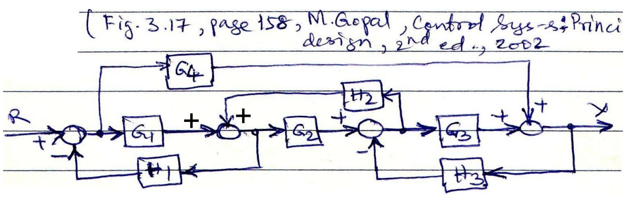
$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3 = 4\frac{du}{dt} + 5u,$$

you have arrived at the following diagram.



Fill the appropriate numbers into the appropriate blank boxes of this diagram.

6. **[4 points]** Determine the gain from R to Y in the following block diagram using Mason's rule.



7. The reaction curve of a certain plant gives the input-output transfer function

$$\frac{Y(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}$$

where L, T are fixed positive constants, and K is non-negative.

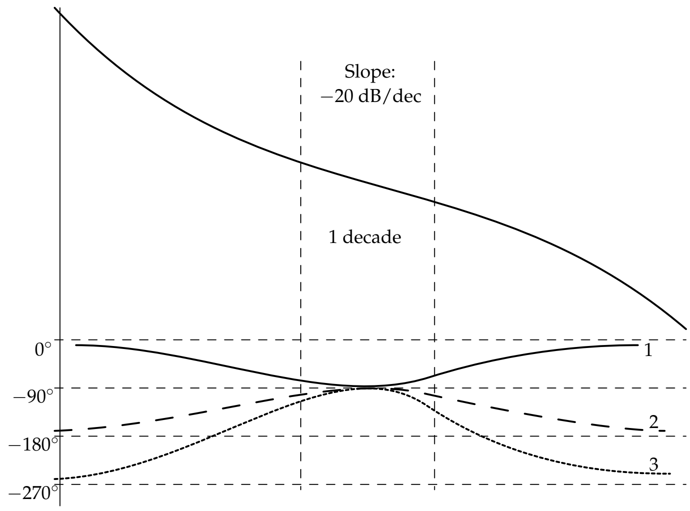
7.1. **[1 points]** For which values of K is this plant stable?
7.2. **[1 points]** Write down an arbitrary value of K for which closing a unity negative feedback loop around this plant will result in a stable system.
7.3. **[2 points]** Assuming for convenience that $T = 0$, determine the limiting value of K for which the previously-mentioned closed-loop system will be stable.

8. **[4 points]** Given

$$G_{CL}(s) = \frac{G_{OL}(s)}{1 + G_{OL}(s)}$$

Evaluate $|G_{CL}(j\omega_g)|$ for PM $= 30°$. Here, ω_g is the gain crossover frequency.

9. **[2 points]** Consider the BMP and BPPs shown below for a certain minimum-phase TF. Which of the three BPPs are valid candidates? Why?



10. **[2 points]** Determine the time at which the signal

$$y(t) = K\left(1 - e^{-\lambda t}\right)1(t)$$

settles to 80% of its final value.

11. Given that the unit step response of a certain system is

$$y(t) = \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\omega_n t + \theta)\right]\mathbb{1}(t)$$

determine

11.1. **[2 points]** the time instant at which the first peak overshoot occurs, and
11.2. **[2 points]** the value of this overshoot in percent.