INTRODUCTION TO GAME THEORY

Source: An Introduction to Game Theory by Martin J. Osborne

THEORY OF RATIONAL CHOICE

- A decision maker chooses the best action according to their preferences, among all the actions available to them.
- Every player/decision maker is aware of their **preferences**. The assumption here is that, every player when presented with any pair of actions, knows which of the pair they prefers, or regards both actions as equally preferable.
- **Rationality** lies within the consistency of the decisions or preferences made by the decision maker when faced with different sets of available actions.
- **Actions** is a set consisting of all the available choices the player can choose from. Generally, a subset of actions are available to the player depending on the restrictions of the game.

PREFERENCE & PAYOFF FUNCTIONS

- Preferences are represented by **payoff functions**.
- Payoff function associates a number with each action of the player in such a way that actions with higher numbers are preferred to the ones with lower numbers.
- The payoff function U represents a player's preferences for any action a in A and b in B

$$U(a) > U(b) \text{ if and only if the player prefer } a \text{ to } b$$

- The preferences convey **ordinal** information, that is the order of preferences. Hence so does the payoff functions too. It does not tell us by how much one action is preferred to the other.
- **Cardinal** payoff function provides information on the degree of preferences.

Example of payoff functions

- Person 1 cares about both her income and about person 2's income. The value she attaches to each unit of her own income is the same as the value she attaches to two units of the person 2's income. Order the outcomes according to her preferences given the first component is person 1's income and the second component is person 2's income.

$$(1,4)\ (2,1)\ (3,0)$$

THEORY OF RATIONAL CHOICE

- The action chosen by a player/decision maker (based on their preferences) is **at least as good as** every other available actions.
- In the consumption problem, where the consumer/decision makers choose what to consume, the set of available actions is the set of all bundles of goods that the consumer can afford.
- In the production problem, where the producer/decision makers choose the amount to produce, the set of available actions is the set of all input-output combinations.

GAME

- A **strategic game** (with ordinal preferences) is a model of interacting decision makers and consists of the following:
  1. **Players** or decision makers
  2. Each player has a possible set of **actions**
  3. Each player have **strategies**, which is an action profile chosen by the player given the actions of the other players.
  4. Each player has **preferences** about the action profile.

SIMULTANEOUS GAME

- Time element is absent in such games.
- Each player chooses their actions simultaneously. That is, no player while choosing their actions are informed of the actions chosen by the other players.
- Example : **PRISONER'S DILEMMA**

		Suspect 2	
		Quiet	*Fink*
Suspect 1	*Quiet*	2, 2	0, 3
	Fink	3, 0	1, 1

- Prisoner's dilemma models a situation in which there aer gains from cooperation but each player has an incentive to *not cooperate*, whatever action the other player does.

Contd...

- A lot of strategic interactions between individuals can be modeled as prisoner's dilemma.
- Working on a joint project

	Work hard	*Goof off*
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

Duoploy

	High	*Low*
High	1000, 1000	−200, 1200
Low	1200, −200	600, 600

NASH EQUILIBRIUM

- In a game, the best action for any given player depends, in general on the other players' actions.
- So players must have *beliefs* about the other players' actions.
- The action profile $a*$ in a strategic game with ordinal preferences is a **Nash Equilibrium** if, for every player i and every action a_i of player i, $a*$ is at least as good according to player i's preferences as the action profile (a_i, a^*_{-i}) in which each player i chooses a_i while every other player j chooses a^*_j. That is, for every player i,

$$u_i(a*) \geq u_i(a_i, a^*_{-i}) \text{ for every action } a_i \text{ of player } i$$

where u_i is a payoff function that represents player i's preferences.

- A **Nash Equilibrium** is an action profile $a*$ with the property than no player i can do better by choosing an action different from a_i^*, given that every other player j adheres to a_j^*

EXAMPLES OF NASH EQUILIBRIUM

- **Bach or Stravinsky**

	Bach	*Stravinsky*
Bach	2, 1	0, 0
Stravinsky	0, 0	1, 2

- **Matching Pennies**

	Head	*Tail*
Head	1, −1	−1, 1
Tail	−1, 1	1, −1

- **Stag Hunt**

	Stag	*Hare*
Stag	2, 2	0, 1
Hare	1, 0	1, 1

BEST RESPONSE FUNCTION

- **Best Response** is the action that will give the player the highest payoff given the actions of other players.
- **Best Response Function**:

$$B_i(a_{-i}) = \{a_i \ in \ A_i \ : \ u_i(a_i, a_{-i}) \geq u_i(a_i^{'}, a_{-i}) \text{ for all } a_i^{'} \in A_i\}$$

- Nash Equilibrium can be redefined using Best Response Functions

The action profile $a*$ is a NE of a strategic game with ordinal preferences if and only if every players' action is a best response to the other players' actions:

$$a_i * \in B_i(a_{-i}) \forall \text{ player} i$$

STRICT AND NON-STRICT NE

- An action profile $a*$ is a strict NE if for every player i we have $u_i(a*) > u_i(a_i, a*_{-i})$ for every action $a_i \neq a*_i$ of player i

	L	M	R
T	1,1	1,0	0,1
B	1,0	0,1	1,0

DOMINATED ACTIONS

- In a strategic game with ordinal preferences, player i's action $a_i^{"}$ strictly dominates her action $a_i^{'}$ if

  $u_i(a_i^{"}, a_{-i}) > u_i(a_i{'}, a_{-i}) \forall$ list a_{-i} of the other players' actions

- A strictly dominated action is not used in NE.