

Low-Frequency Response

- *The Infinite-Value Time Constant (IVTC) Technique:*
 - Used for obtaining the *lower cutoff frequency* (f_L)
 - If a circuit has n number of *capacitors*, then it would have n number of *time constants*
 - *This technique derives the information regarding f_L from these time constants*

- *The Algorithm:*
 - Null all independent sources to the circuit
 - Short all independent voltage sources
 - Open all independent current sources
 - DO NOT TOUCH DEPENDENT SOURCES
 - Name the capacitors C_i ($i = 1$ to n)
 - Consider C_1 and assign infinite values to all other capacitors (thus the name!)
 - Thus, except C_1 , all other capacitors will short out
 - Determine the Thevenin Resistance (R_1^∞)
across the two terminals of C_1

- *Find the time constant τ_1 associated with C_1*

$$(\tau_1 = R_1^\infty C_1)$$
- *Calculate the corresponding frequency $f_1 = 1/(2\pi\tau_1)$*
- *Repeat for all other capacitors, taking one at a time, and find all the rest of the frequencies (f_2, f_3, \dots, f_n)*
- Then the *Lower Cutoff Frequency* f_L can be expressed as:

$$f_L = \left[\sum_{i=1}^n f_i^2 \right]^{1/2}$$

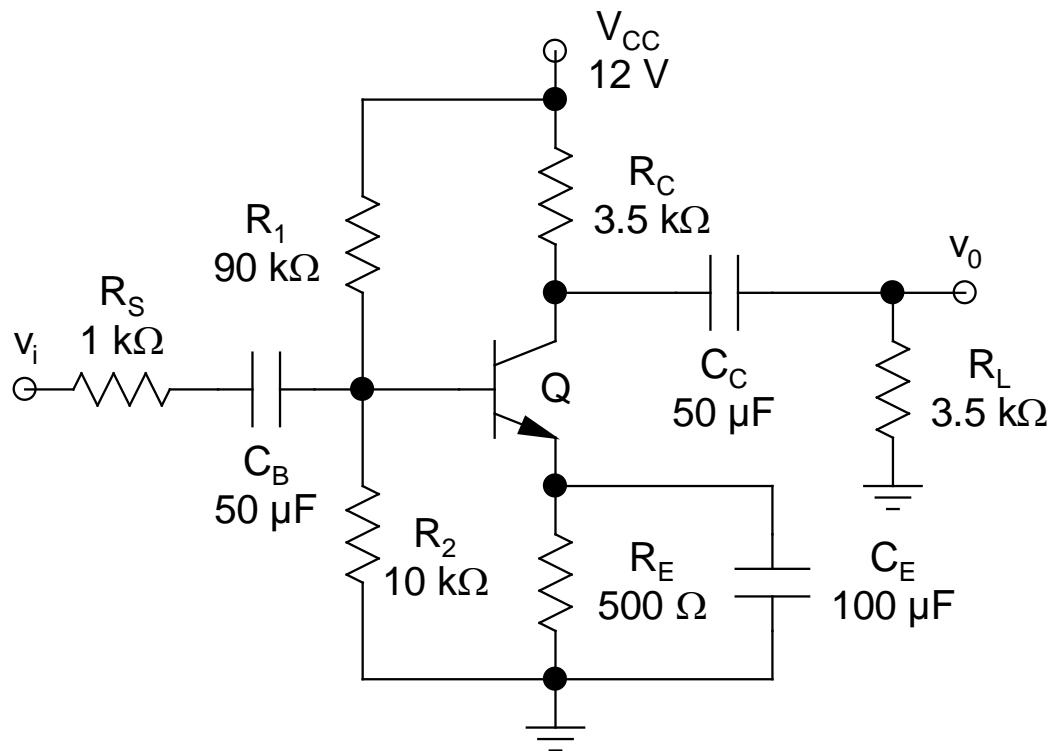
- In *discrete circuits*, a *major component of total cost* is due to the *cost of the capacitors* (*directly proportional to the value*)
- Hence, an attempt is made to *minimize* the *total capacitor requirement of the circuit*
- For this, the *Dominant Pole* (DP) technique is used
 - *One of the frequencies among f_1-f_n is made dominant*
 - *Others are made to lie at least 10 times away from it*

- For example, if f_d is chosen to be the DP, then all other poles are assumed to be at $f_d/10$

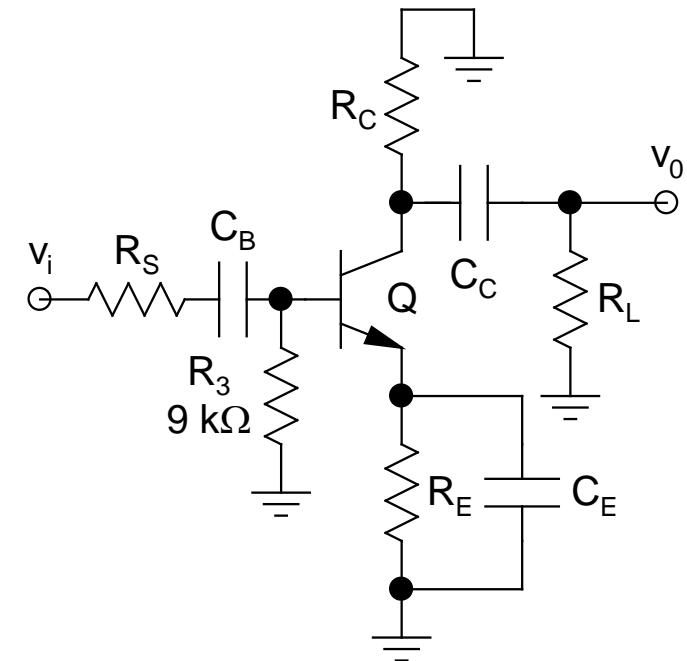
$$\Rightarrow f_L = \left[f_d^2 + \sum_{n=1} \left(\frac{f_d}{10} \right)^2 \right]^{1/2}$$

- C_d , which contributes f_d , is chosen to be that capacitor that sees the least Thevenin resistance across its terminals
- Reason is obvious:
 - If any other capacitor were chosen to contribute f_d , then C_d would have been ten times higher
- This choice is based on heuristics

- *Low-Frequency Response of RC-Coupled Amplifier:*



Complete Circuit



ac Schematic

- ***DC analysis*** gives $I_C = 1 \text{ mA}$ and $V_{CE} = 8 \text{ V}$
 $\Rightarrow r_E = 26 \Omega$ and $r_\pi = 2.6 \text{ k}\Omega$ (assuming $\beta = 100$)
- ***Neglect Early effect***
 $\Rightarrow r_0 \rightarrow \infty$
- 3 ***capacitors*** (C_B , C_E , C_C) with ***time constants*** τ_1 , τ_2 , τ_3 , and corresponding ***cutoff frequencies*** f_1 , f_2 , f_3
- To apply the ***IVTC technique***, we have to take
one capacitor at a time and ***treat other capacitors as short circuits***
- ***The analysis can be done by inspection!***

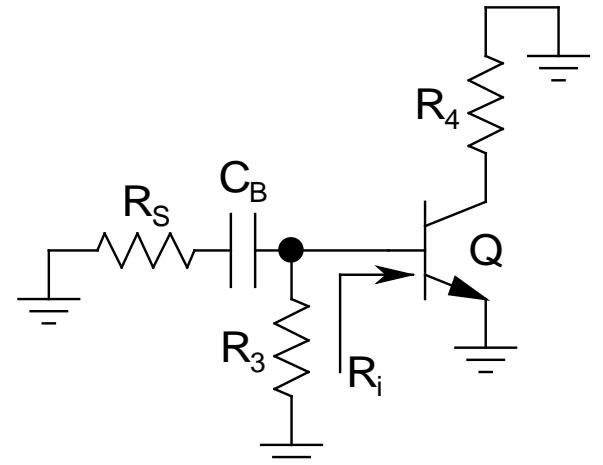
➤ **C_B :**

- *Short C_C and C_E*
- $R_3 = R_1 \parallel R_2 = 9 \text{ k}\Omega$
- $R_4 = R_C \parallel R_L = 1.75 \text{ k}\Omega$
- $R_i = r_\pi = 2.6 \text{ k}\Omega$
- By inspection, the *Thevenin resistance* seen by C_B :

$$R_B^\infty = R_S + (R_3 \parallel R_i) = 3 \text{ k}\Omega$$

$$\Rightarrow \tau_1 = R_B^\infty C_B = 150 \text{ ms}$$

$$\Rightarrow f_1 = 1/(2\pi\tau_1) = 1.06 \text{ Hz}$$



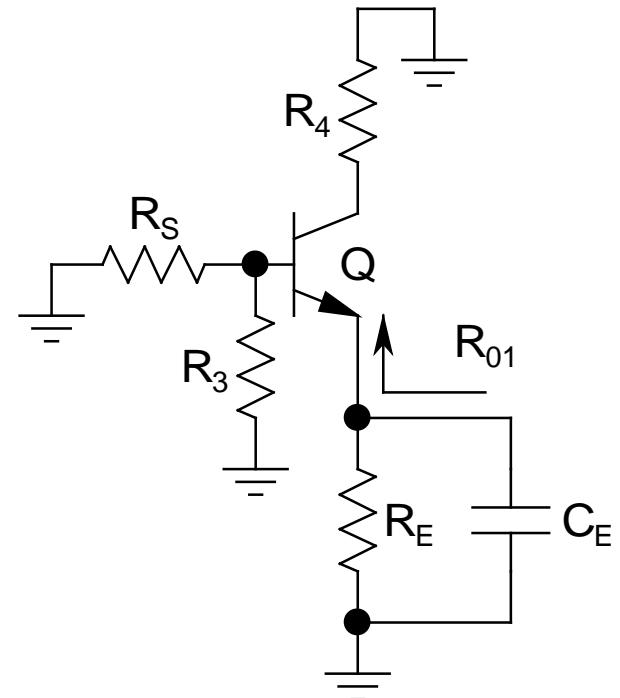
➤ **C_E :**

- *Short C_C and C_B*
- $R_{01} = r_E + (R_S \parallel R_3) / (\beta + 1)$
 $= 34.9 \Omega$
- By inspection, the *Thevenin resistance* seen by C_E :

$$R_E^\infty = R_E \parallel R_{01} = 32.6 \Omega$$

$$\Rightarrow \tau_2 = R_E^\infty C_E = 3.26 \text{ ms}$$

$$\Rightarrow f_2 = 1/(2\pi\tau_2) = 48.8 \text{ Hz}$$



➤ C_C :

- *Short C_E and C_B*
- By inspection, the *Thevenin resistance* seen by C_C :

$$R_C^\infty = R_C + R_L = 7 \text{ k}\Omega$$

$$\Rightarrow \tau_3 = R_C^\infty C_C = 350 \text{ ms}$$

$$\Rightarrow f_3 = 1/(2\pi\tau_3) = 0.45 \text{ Hz}$$

- Thus, the *lower cutoff frequency* of the circuit:

$$f_L = \left[f_1^2 + f_2^2 + f_3^2 \right]^{1/2} = 48.8 \text{ Hz}$$

- Note that *f_L is equal to f_2 (contributed by C_E)*
- Now let's attempt to *minimize* the *total capacitance requirement* of the circuit

➤ ***Minimization of the Total Capacitance:***

- From the previous analysis, we note that ***C_E sees the least Thevenin resistance across its two terminals***

⇒ ***Let's choose C_E to contribute the DP f_d, and let C_C and C_B each contribute poles at f_d/10***

$$\Rightarrow 48.8 = \sqrt{f_d^2 + 2(f_d/10)^2}$$

$$\Rightarrow f_d = 48.3 \text{ Hz and } f_d/10 = 4.83 \text{ Hz}$$

- Thus:

$$C_E = 1/(2\pi f_d R_E^\infty) = 101.1 \mu F$$

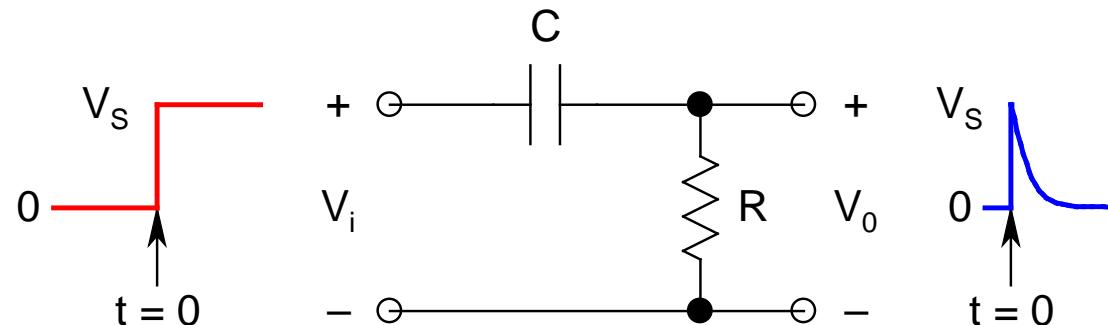
$$C_B = 1/[2\pi(f_d/10)R_B^\infty] = 11 \mu F$$

$$C_C = 1/[2\pi(f_d/10)R_C^\infty] = 4.7 \mu F$$

- Thus, the ***total capacitance*** requirement comes out to be ***116.8 μF*** , for the ***same f_L of 48.8 Hz***
- The ***original circuit*** had a ***total capacitance*** of ***200 μF***
- Thus, this approach gave a ***cost saving*** of almost ***42%*** in terms of the ***capacitors***
- As an ***exercise***, you can pick ***either C_C or C_B*** to ***contribute f_d*** , and find the ***total capacitance*** requirement for each case
- Finally, after all, this is a ***heuristic***
- To get the ***absolute minimum value*** of the ***total capacitance***, we need to ***formulate the problem***, and ***find the minima of the function mathematically***

- **Tilt/Sag:**

- For *pulse/square wave excitation*, f_L dictates the amount of *tilt/sag* present in the *output*
- Due to f_L , the circuit effectively behaves like a **HPF**, represented by a simple ***RC circuit***
- Under *step input*, the *output* would be a *spike*



➤ Thus:

$$V_0 = V_s \exp(-t/\tau_L)$$

$$\tau_L = RC = 1/\omega_L \quad (\omega_L = 2\pi f_L)$$

➤ For $t \ll \tau_L$:

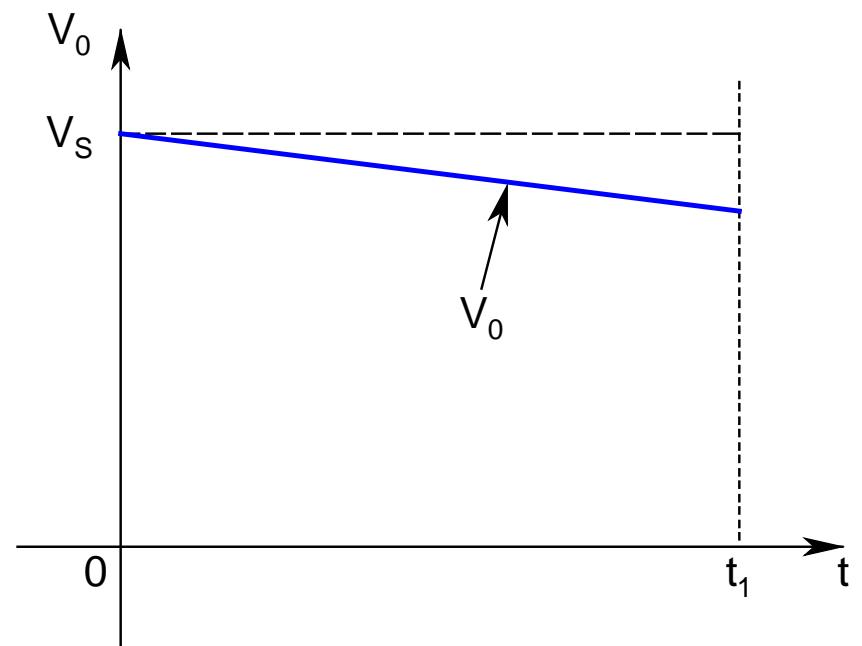
$$V_0 \approx V_s(1 - t/\tau_L)$$

$$= V_s(1 - \omega_L t)$$

$$= V_s(1 - 2\pi f_L t)$$

➤ Thus, ***V₀ drops linearly with time***

➤ ***Quantified by percent tilt/sag (P)***



$$\begin{aligned}\triangleright P &= [(V_S - V_0)/V_S] \times 100\% \\ &= (t_1/\tau_L) \times 100\%\end{aligned}$$

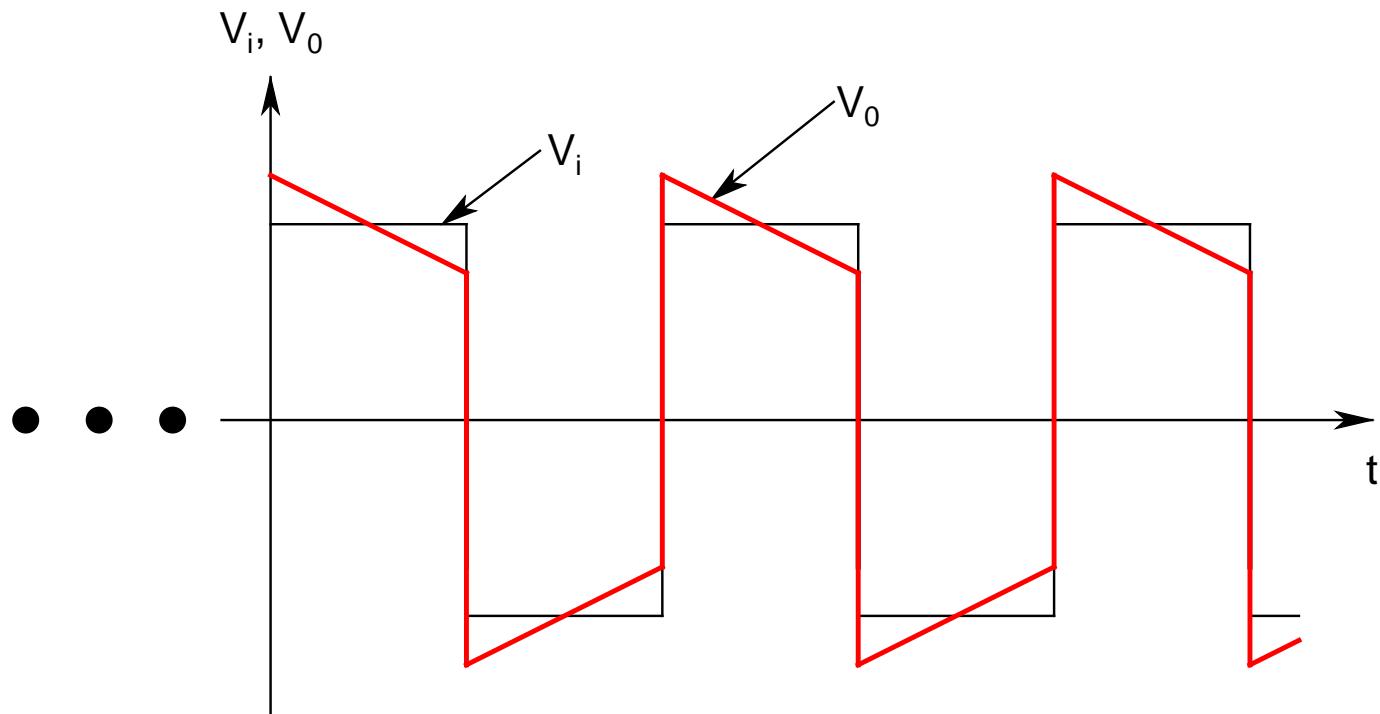
t_1 = ***Time at which the tilt is measured***

\triangleright For ***square wave input***, $t_1 = T/2$ ($T = \text{period} = 1/f$, $f = \text{cycle frequency}$)

$$\begin{aligned}\Rightarrow P &= [T/(2\tau_L)] \times 100\% = [\omega_L/(2f)] \times 100\% \\ &= (\pi f_L/f) \times 100\%\end{aligned}$$

\triangleright ***Note:*** P is directly proportional to f_L and inversely proportional to f

\Rightarrow ***Circuits having low f_L , will show significant amount of tilt/sag at low frequencies***



Tilt/Sag