Tutorial 1 Solution

1) **Three light bulbs are connected to a 200 V battery in series. The bulbs are consuming 60 W, 80 W, and 100 W power as shown below. Find the current 'I' through it.**

   Solution:

   Let, the resistance of Bulb B_1, B_2, and B_3 are R_1, R_2, and R_3 respectively

   The bulbs are in series. So, the current I is the same for all the bulbs.

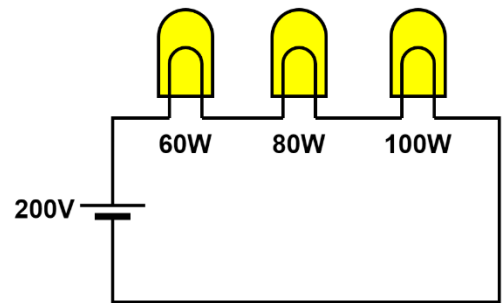 $$R_1 = \frac{P_1}{I^2} \qquad R_2 = \frac{P_2}{I^2} \qquad R_3 = \frac{P_3}{I^2}$$

   We know that,

   $$V = IR$$

   $$200 = I(R_1 + R_2 + R_3) = I\left(\frac{60}{I^2} + \frac{80}{I^2} + \frac{100}{I^2}\right)$$

   $$200 = I\left(\frac{240}{I^2}\right)$$

   $$I = 1.2\ A$$

2) **The current through a 0.2 Henry inductor is $i = 5te^{-2t}\ A$. Find the voltage across it and the energy stored in the inductor?**

   Solution:

   Given,

   $$L = 0.2\ H$$

   $$i(t) = 5te^{-2t}\ A$$

   We know that, voltage across the inductor is given by

   $$v(t) = L\frac{di}{dt} = 0.2\frac{d}{dt}(5te^{-2t})$$

   $$v(t) = 0.2\{5t(-2)e^{-2t} + e^{-2t}(5)\}$$

   $$v(t) = (1 - 2t)e^{-2t}$$

3) **A certain circuit element has the current $i = 10e^{-5000t}\ A$ and voltage $v = 50(1 - e^{-5000t})\ V$. Find the total energy transferred during $t \geq 0$.**

   Solution:

   The instantaneous power is

   $$p(t) = v(t) * i(t)$$

   $$p(t) = 500(e^{-5000t} - e^{-10000t})$$

   The total energy is given by

   $$E = \int_0^{\infty} p(t)dt$$

   $$E = \int_0^{\infty} 500(e^{-5000t} - e^{-10000t})dt = 500\left[\frac{e^{-5000t}}{-5000} - \frac{e^{-10000t}}{-10000}\right]_0^{\infty} = 500\left[\frac{1}{5000} - \frac{1}{10000}\right]$$

   $$E = 0.05\ Joules$$

4) Find the number of node, branch, mesh, and loop in the given figures 4(a) and 4(b).

Solution:

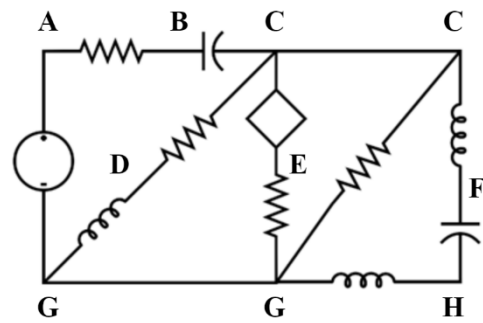**4(a):** The nodes are A, B, C, D, E, F, G, and H.

The number of nodes 'n'= 8

The number of branches 'b'= 11

The number of mesh 'm' is given by

$$m = b - n + 1$$

$$m = 4$$

The number of loops 'l' is 10.

4(b): The nodes are A, B, C, D, E, F, G, and H.

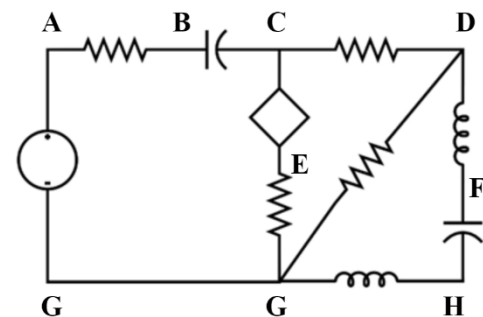The number of nodes 'n'= 8

The number of branches 'b'= 10

The number of mesh 'm' is given by

$$m = b - n + 1$$

$$m = 3$$

The number of loops 'l' is 6.

5) Calculate the power supplied or absorbed by each element in the figure and show $\sum P_{absorbed} = \sum P_{supplied}$

Solution:

The current goes out from the positive terminal of the battery. So, power is supplied by the voltage source.

$$P_1 = VI$$
$$P_1 = 25 \times 10 = 250W$$

The current goes in the positive terminal of the voltage of the resistor. So, the power is absorbed by the component 2 and 3.

The voltage appeared across the dependent source is positive with respect to current direction. So, power is supplied by the dependent source.

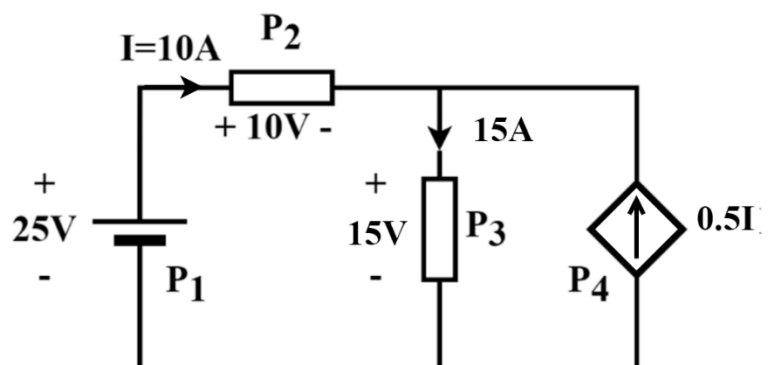$$P_2 = VI$$
$$P_2 = 10 \times 10 = 100W$$
$$P_3 = VI$$
$$P_3 = 15 \times 15 = 225W$$
$$P_4 = VI$$
$$P_4 = 15 \times 0.5 \times 10 = 75W$$
$$P_{absorbed} = P_2 + P_3 = 325W$$
$$P_{supplied} = P_1 + P_4 = 325W$$

6) **In the interval $0 < t < 5\pi\ ms$, a $20\ \mu F$ capacitor has a voltage $V = 100 \sin 200t$ (V). Find the charge, power, and energy.**

   Solution:

   Charge, $Q = CV = 20 \times 100 \sin 200t\ \mu C = 2 \sin 200t\ mC$

   Current, $i = C\frac{dV}{dt} = 20 \times 100 \times 200 \cos 200t\ \mu C = 400 \cos 200t\ mA$

   Power, $p = Vi = 400 \times 100 \sin 200t \times \cos 200t\ mW = 20 \sin 400t\ W$

   Energy, $E = \frac{1}{2}CV^2 = \frac{1}{2} \times 20\ \mu \times (100 \sin 200t)^2 = 100 \sin^2 200t\ mJ$

   For the interval $0 < t < 5\pi\ ms$,
   $$E(5\pi) - E(0) = 0$$

7) **The two electrical network N$_1$ and N$_2$ are connected through three resistors as shown in figure. Find the voltage across 3Ω resistor.**
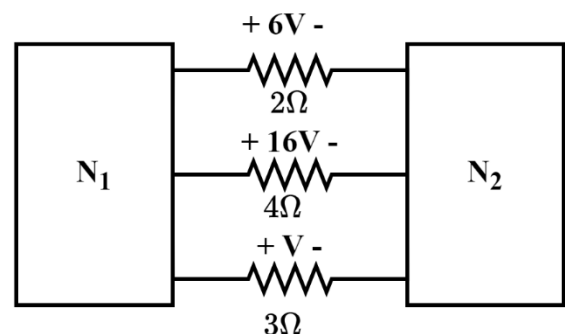
   Solution:

   Current flows in closed path.

   Current flows from N$_1$ to N$_2$ = Current flows from N$_1$ to N$_2$

   The net current flows from N$_1$ to N$_2$ is given by

   $$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V}{R_3} = 0$$

   $$\frac{6}{2} + \frac{16}{4} + \frac{V}{3} = 0$$

   $$V = -21\ V$$

8) **Find the current going out from the positive terminal of the 2V battery using mesh analysis.**

   Solution:

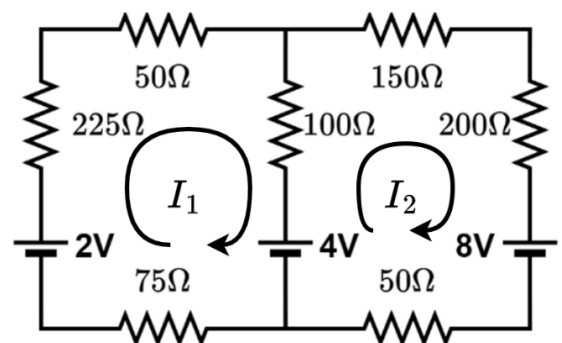   For mesh 1,

   $$-2 + 225I_1 + 50I_1 + 100(I_1 - I_2) + 4 + 75I_1 = 0$$

   $$450I_1 - 100I_2 = -2 \qquad \text{... ... ...} (i)$$

   For mesh 2,

   $$-4 + 100(I_2 - I_1) + 150I_2 + 200I_2 + 8 + 50I_2 = 0$$

   $$-100I_1 + 500I_2 = -4 \qquad \text{... ... ...} (ii)$$

   From equation (i) and (ii), The current going out from the positive terminal of 2V battery is
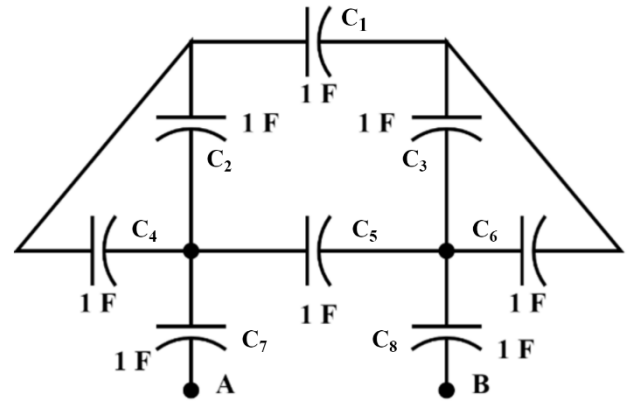 $$I_1 = -6.51\ mA$$

9) Find the equivalent capacitance between A and B.

Solution:

C_2 and C_4 are in parallel.

Similarly, C_3 and C_6 are in parallel.
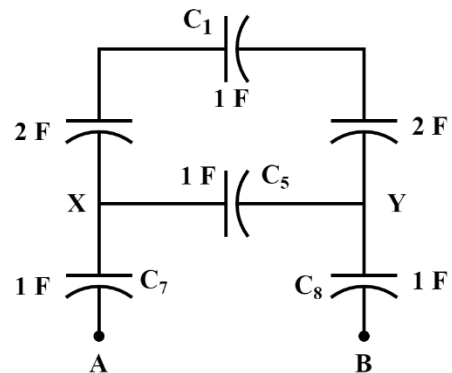
Equivalent is $C_2 + C_4 = 2\ F$

Now, equivalent circuit is

The equivalent capacitance between X and Y is given by

$$C_{XY} = \left(\frac{1}{\frac{1}{C_1} + \frac{1}{2} + \frac{1}{2}} \right) + C_5$$
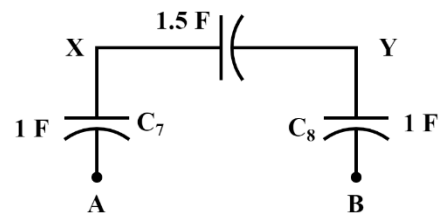
$$C_{XY} = 1.5\ F$$

The equivalent capacitance between A and B is given by

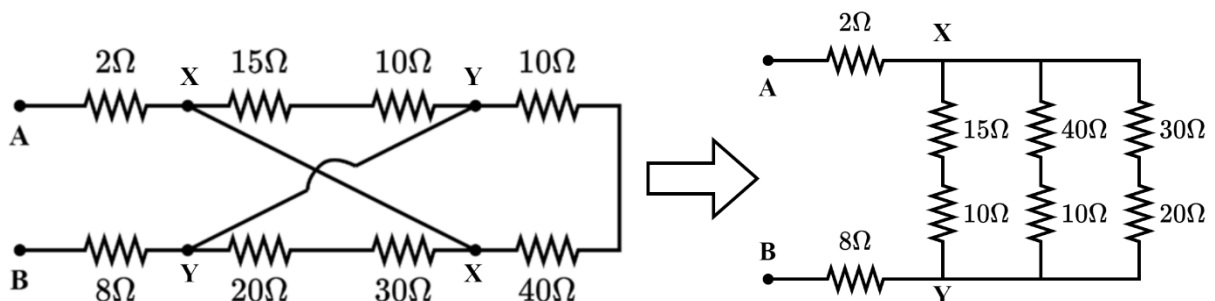$$C_{AB} = \left(\frac{1}{\frac{1}{C_7} + \frac{1}{C_8} + \frac{1}{1.5}} \right)$$

$$C_{AB} = 0.375\ F$$

10) Find the equivalent resistance between A and B.

Solution:

The equivalent resistance across terminal A and B is given by

$$R_{AB} = 2 + 8 + (50||50||25)$$

$$R_{AB} = 2 + 8 + (12.5)$$

$$R_{AB} = 22.5\ \Omega$$