Lecture-9

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Complex Power.
- Linearity Property.
- Superposition Theorem.
- Source Transformation Theorem.

Complex Power

- Given the phasors $\boldsymbol{V} = V_m\angle\theta_v$ and $\boldsymbol{I} = I_m\angle\theta_i$ of voltage $v(t)$ and current $i(t)$, the complex power $\boldsymbol{S}$ absorbed by the AC load is the product of the voltage and complex conjugate of the current, as follows:

$$S = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = V_{rms}I_{rms}^* = V_{rms}I_{rms}\angle\theta_v - \theta_i$$

- We can notice that the magnitude of the complex power, $\boldsymbol{S}$, is the apparent power S.

$$S = V_{rms}I_{rms}\cos(\theta_v - \theta_i) + jV_{rms}I_{rms}\sin(\theta_v - \theta_i)$$

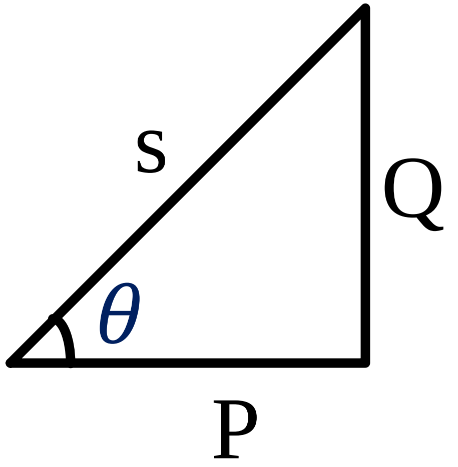
- The above equation has a real term and an imaginary term.
- As the magnitude of the complex power is the apparent power, complex power is also measured in voltamperes (VA).

Complex Power (Cont…)

- Complex power being a complex quantity can be resolved into real and imaginary terms.

$$S = V_{rms}I_{rms}\cos(\theta_v - \theta_i) + jV_{rms}I_{rms}\sin(\theta_v - \theta_i) = P + jQ$$

- The real term P represents the average power delivered to the load and is the only useful power.
- The imaginary term Q is known as the reactive power and represents the energy exchange between the source and the reactive part of the load (capacitance and inductance).
- Q is measured in volt-ampere reactive (VAR) to distinguish it from P measured in watts (W).
- It is a standard practice to represent S, P, and Q together with the power factor angle $\theta = \theta_v - \theta_i$ using a **power triangle** as shown in the adjacent figure.



Linearity Property

- Linearity is the property of an element describing a linear relationship between cause and effect.
- Although the property applies to many circuit elements, we shall limit its applicability to resistors in this lecture.
- The linearity property is a combination of both the homogeneity (scaling) property and the additivity property.
- The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant.

Linearity Property (Cont…)

- For a resistor, for example, Ohm's law relates the input i to the output v,

$$v = iR$$

- If the current is increased by a constant k, then the voltage increased correspondingly by k, that is

$$kiR = kv$$

- The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

- Using the voltage-current relationship of a resistor, if

$$v_1 = i_1 R$$

and

$$v_2 = i_2 R$$

then applying (v_1+v_2) gives

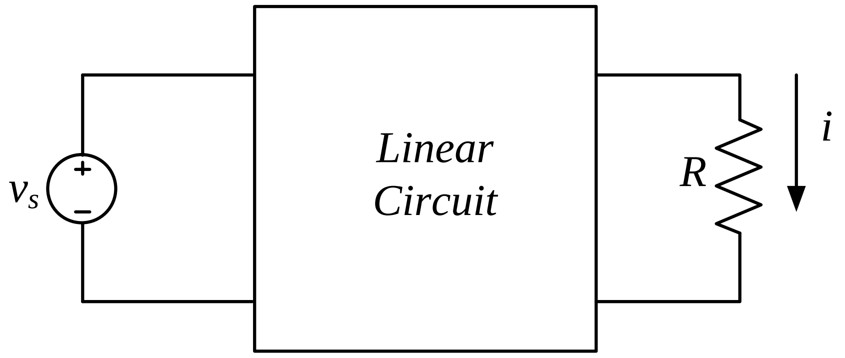
$$v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

Linearity Property (Cont…)

- We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.
- In general, a circuit is linear if it is both additive and homogeneous.
- A linear circuit consists of only linear elements, linear dependent sources, and independent sources.
- A linear circuit is the one whose output is linearly related (or directly proportional) to its input.
- Note that since $P = i^2R = \frac{v^2}{R}$ (making it a quadratic function rather than a linear one), the relationship between power and voltage (or current) is nonlinear.
- Therefore, the theorems covered in this module are not applicable to power.

Linearity Property (Cont…)

- To illustrate the linearity principle, consider the linear circuit shown in the adjacent figure.
- The linear circuit has no independent sources inside it.
- It is excited by a source voltage v_s, which serves as the input.
- The circuit is terminated by a resistance R.
- Let the current through the load R be the output.
- Suppose $v_s = 10V$ give $i = 2A$.
- According to linearity property $v_s = 1V$ give $i = 0.2A$.
- Similarly, $i = 1A$ must be due to an input $v_s = 5V$.



❑ Example:

For the circuit shown below find I_0 when $v_s = 12V$ and $v_s = 24V$?

Solution:

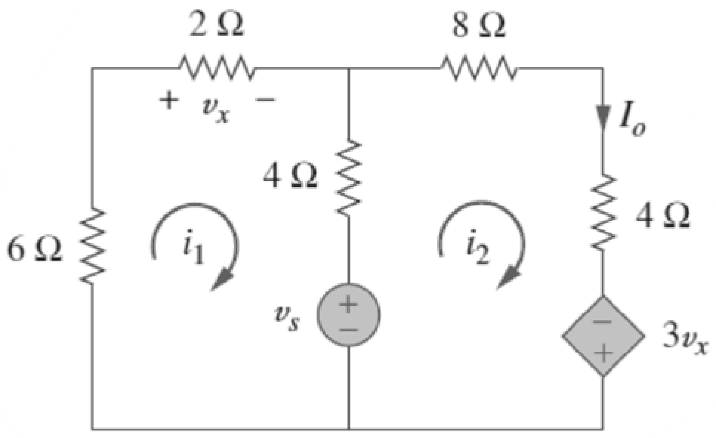
Applying KVL to the two loops

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0$$

As $v_x = 2\, i_1$, the above equation can be rewritten as,

$$-10i_1 + 16i_2 - v_s = 0$$



Solving the above equations we get,

$$i_2 = \frac{v_s}{76}$$

When v_s = 12V

$$I_0 = i_2 = \frac{12}{76} A = 0.158 \text{ A}$$

Similarly, when v_s = 24V

$$I_0 = i_2 = \frac{24}{76} A = 0.316\, A$$

showing that when the source value is doubled I_0 is doubled.

Superposition Theorem

- If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis.
- In another way, determine the contribution of each independent source to the variable separately and then add them up.
- The latter approach is known as the superposition.
- The idea of superposition rests on the linearity property.
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Superposition Theorem (Cont…)

- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.
- However, to apply the superposition principle, we must keep two things in mind:

    i. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0A (or an open circuit). In this way, we obtain a simpler and more manageable circuit.

    ii. Dependent sources are left intact because they are controlled by circuit variables.

Superposition Theorem (Cont…)

Steps to Apply Superposition Principle:

i. Turn off all independent sources except one source.
ii. Find the output (voltage or current) due to that active source using any network analysis technique.
iii. Repeat step 1 and 2 for each of the other independent sources.
iv. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Superposition Theorem (Cont…)

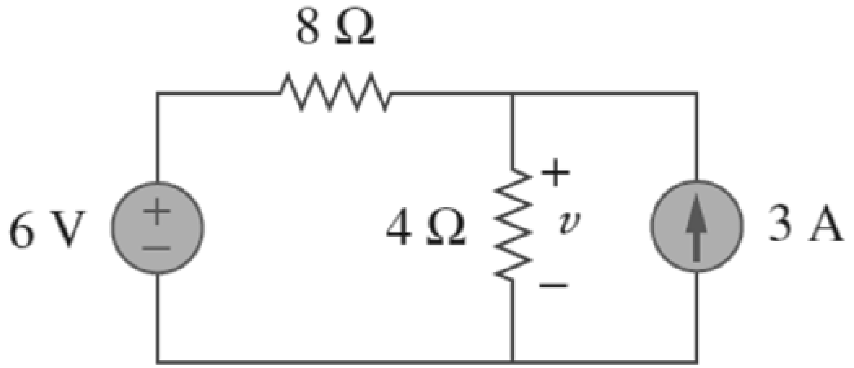
- Analyzing a circuit using superposition has one major disadvantage.
- It may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source.
- However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.
- Keep in mind that superposition is based on linearity.
- However, it is not applicable to the effect on power, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

❑ Example:

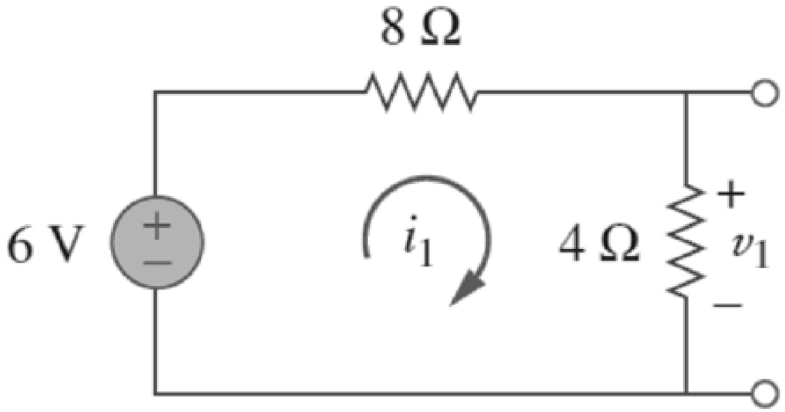
For the circuit shown use superposition theorem to find v?

Solution: Since there are two source let

$$v_1 + v_2 = v$$

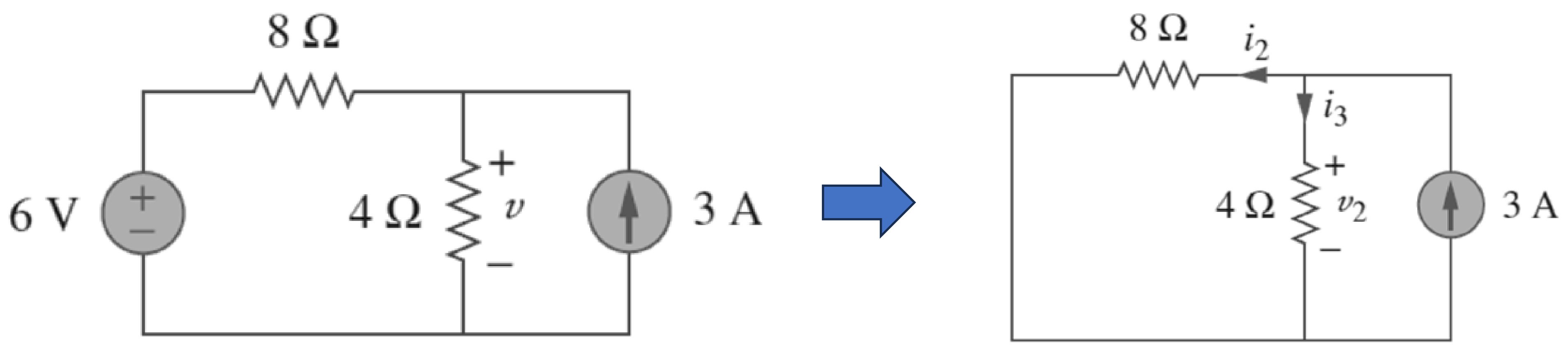


Here v_1 and v_2 are the contributions due to the 6V and 3A sources, respectively. To obtain v_1 the current source is set to 0A as shown in the following figure.



Applying KVL to the loop, $12i_1 - 6 = 0$

Therefore, $i_1 = 0.5$A and $v_1 = 4\ i_1 = 2$V.



To obtain v_2 the voltage source is set to 0 V as shown in the above figure.

Using current division, $i_3 = \frac{8}{4+8} * 3 = 2A$

Therefore, $v_2 = 4i_3 = 8V$.

Hence, $v = v_1 + v_2 = 2 + 8 = 10V$.

❑ Example:

For the circuit shown use superposition theorem to find i_0?

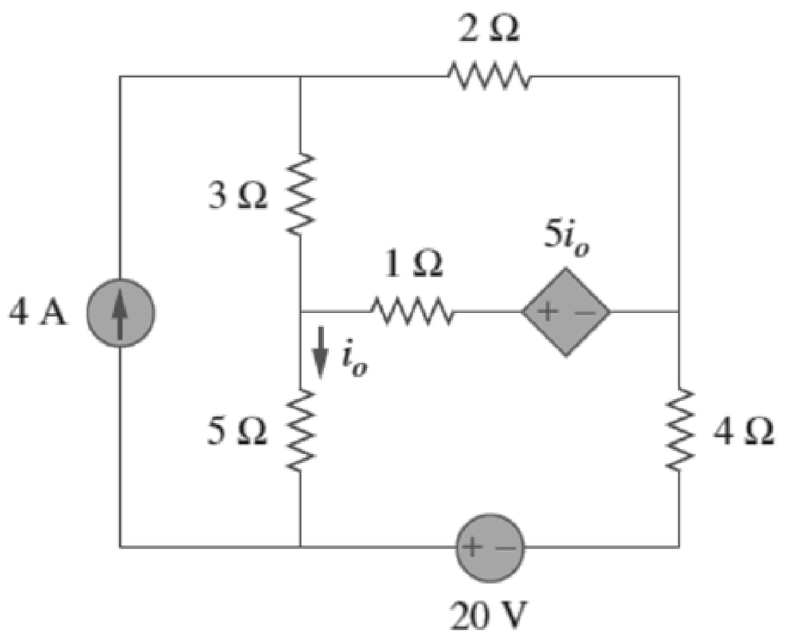
Solution: The circuit involves a dependent source which must be left intact.

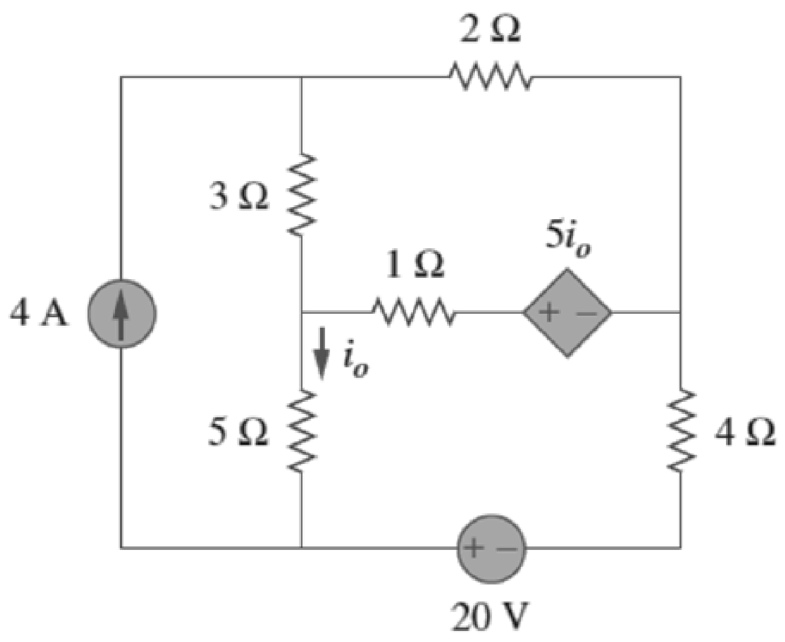
Let,

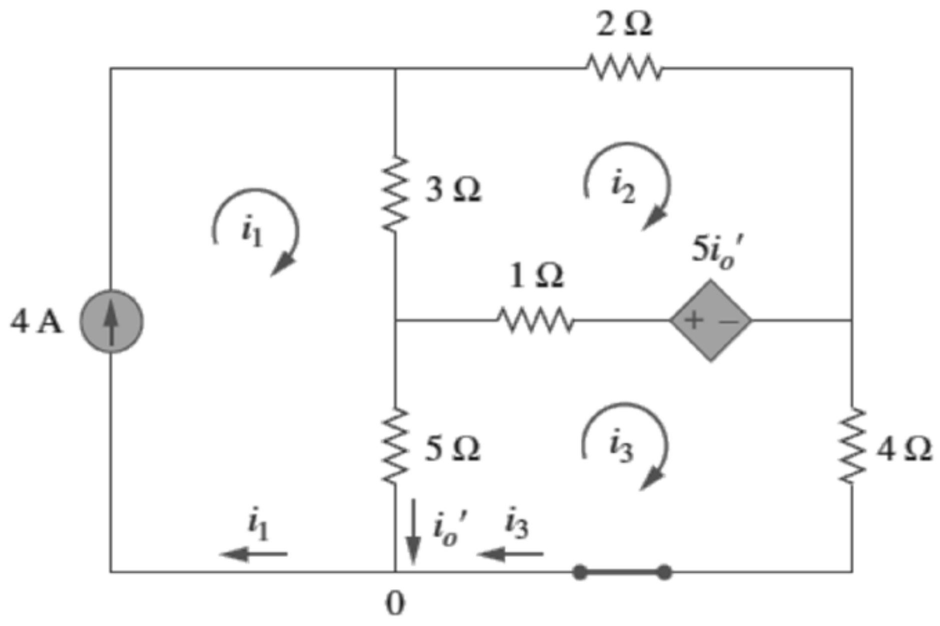
$$i_0 = i_0' + i_0''$$

Here i_0' and i_0'' are the contributions due to the 4A and 20V sources, respectively.

To obtain i_0' we turn off the 20V source to obtain the circuit shown in the next slide.







We apply mesh analysis to obtain i_0'.

For loop 1,

$$i_1 = 4$$

For loop 2,

$$-3i_1 + 6i_2 - i_3 - 5\, i_0' = 0$$

For loop 3,

$$-5i_1 - i_2 + 10i_3 + 5\,i_0' = 0$$

But at node ‘0’,

$$i_3 = i_1 - i_0' = 4 - i_0'$$

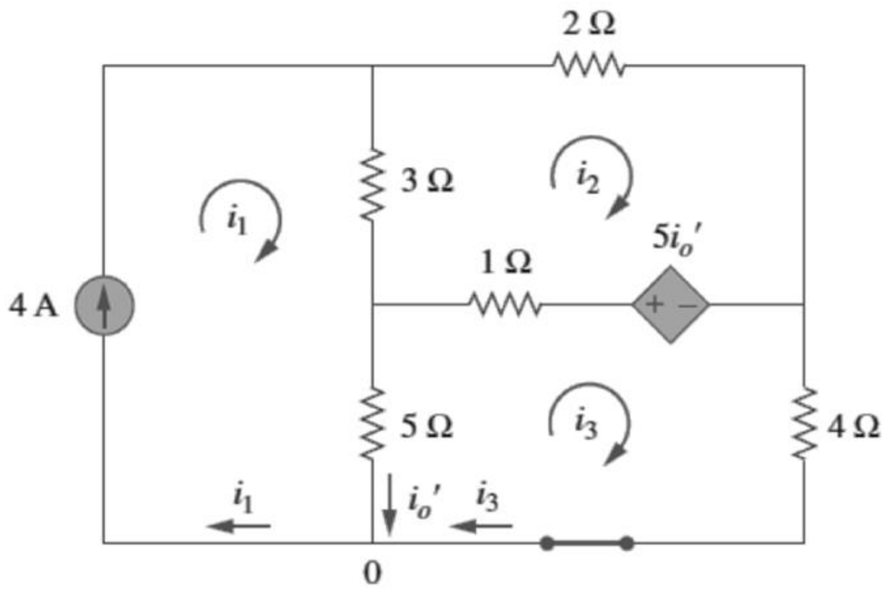
Using above equations we get,

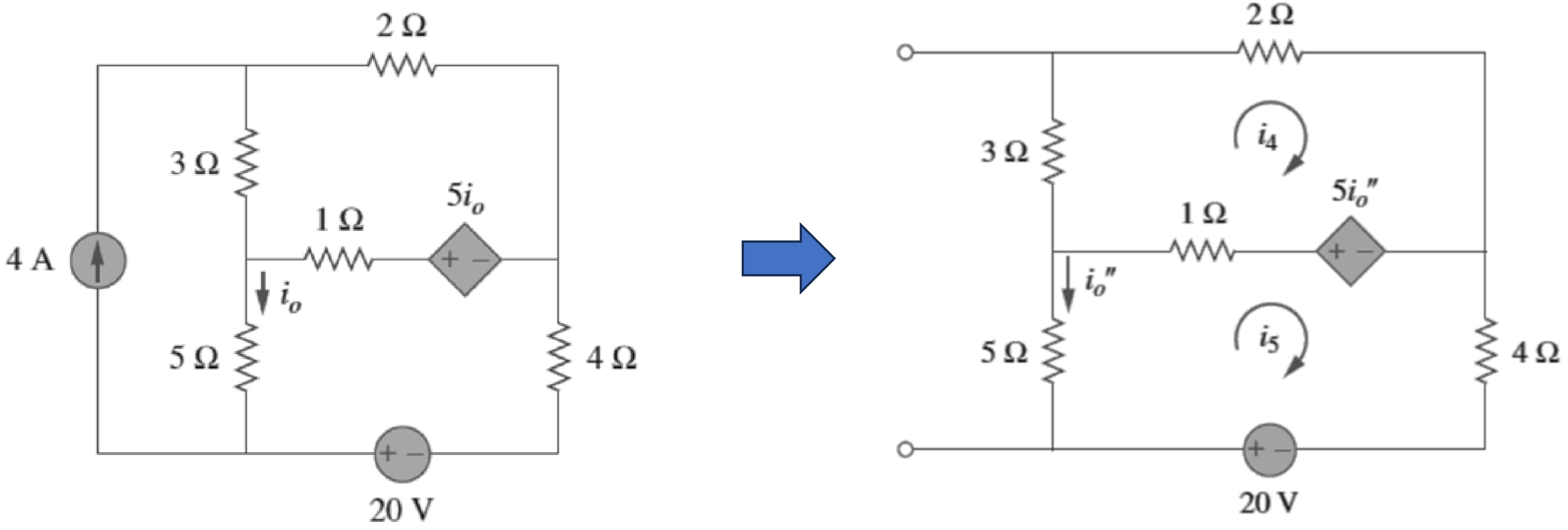
$$3i_2 - 2i_0' = 8$$

$$i_2 + 5i_0' = 20$$

This can be solved to get,

$$i_0' = \frac{52}{17}A$$





To obtain i_0'' we turn off the 4A source to obtain the circuit shown below.

For loop 4,

$$6i_4 - i_5 - i_0'' = 0$$

For loop 5,

$$-i_4 + 10i_5 - 20 + 5i_0'' = 0$$

But,

$$i_5 = -\,i_0''$$

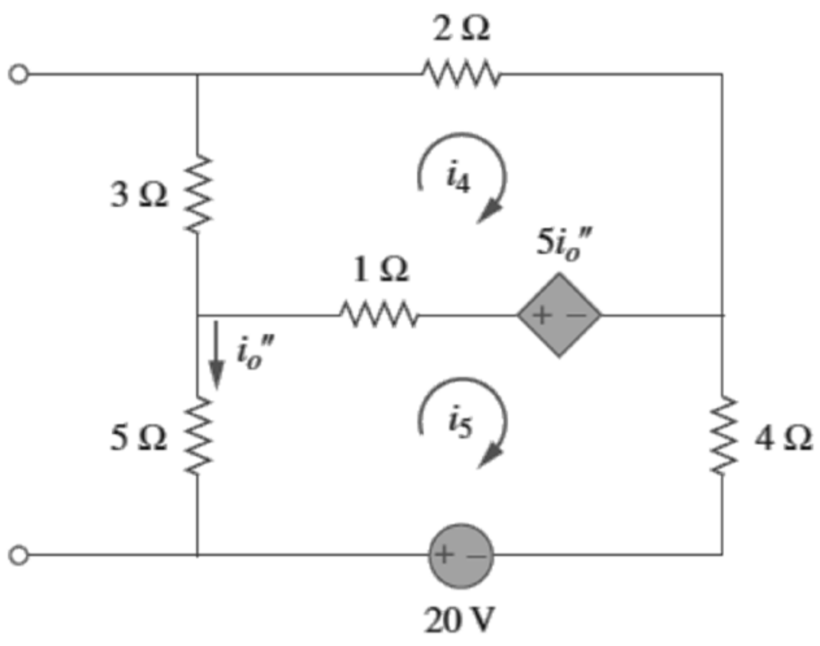
Using the above equations, we get,

$$6i_4 - 4i_0'' = 0$$

$$4i_4 + 5i_0'' = -20$$

This can be solved to get,

$$i_0'' = -\frac{60}{17}A$$



Now,

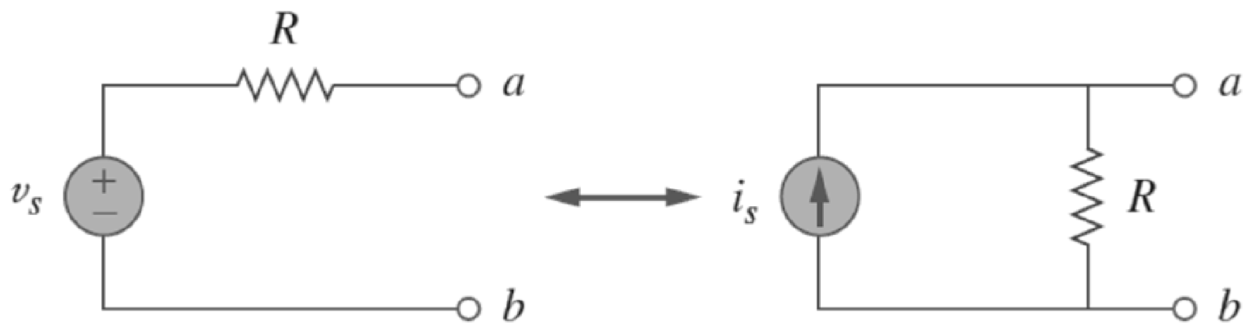
$$i_0 = i_0' + i_0'' = -\frac{8}{17} = -0.4076A$$

Source Transformation Theorem

- We have discussed that series-parallel combination and star-delta transformation help simplify circuits.
- Source transformation is another tool for simplifying circuits.
- Basic to these tools is the concept of equivalence.
- We recall that an equivalent circuit is one whose v - i characteristics are identical with the original circuit.
- A source transformation is the process of replacing a "voltage source in series with a resistor" by a "current source is in parallel with a resistor", or vice versa.

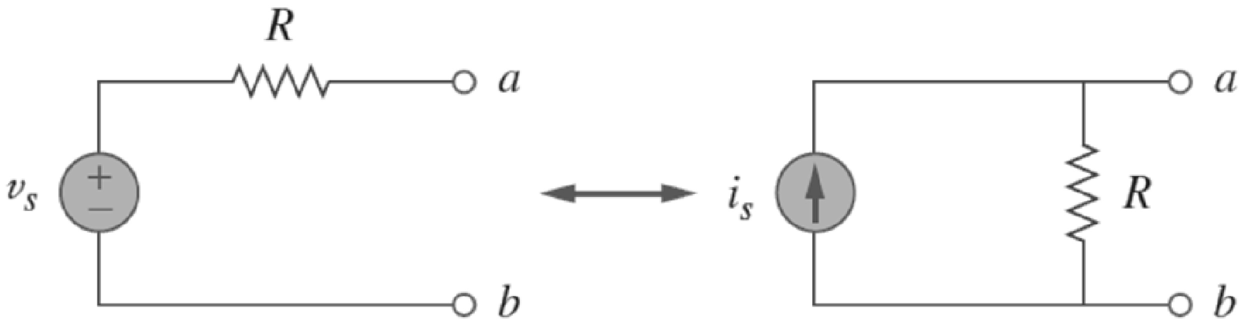
Source Transformation Theorem (Cont…)

- Earlier we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources.
- To expedite the circuit analysis, substitute a voltage source in series with a resistor for a current source in parallel with a resistor or vice versa.
- Each substitution is known as a source transformation.



Source Transformation Theorem (Cont…)

- The two circuits in the previous figure are equivalent, provided they have the same voltage-current relationship at terminals a - b.
- It is easy to show that they are equivalent.
- If the sources are turned off, the equivalent resistances across the terminals a-b in both circuits is R.
- Also, when terminals a-b are shorted, the short circuit current flowing from a to b is, $i_{sc} = \frac{v_s}{R}$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side.
- Thus, $\frac{v_s}{R} = i_s$ in order for the two circuits to be equivalent.

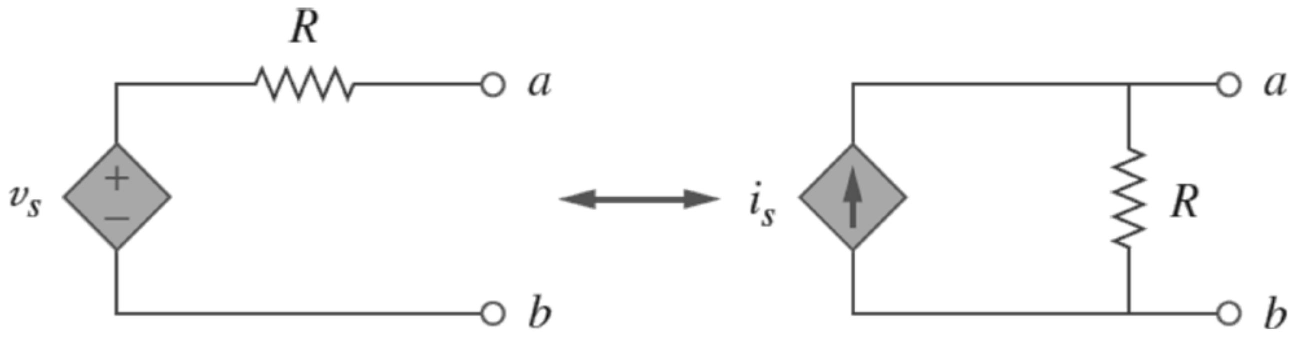


Source Transformation Theorem (Cont…)

- Hence, source transformation requires that.

$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$

- Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.
- As shown in the below figure a "dependent voltage source in series with a resistor" can be transformed to a "dependent current source in parallel with the resistor" or vice versa.



Source Transformation Theorem (Cont…)

- Like the star-delta transformation, a source transformation does not affect the remaining part of the circuit.
- When applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis.
- However, we should keep the following points in mind when dealing with source transformation:

  i. The arrow of the current source is directed toward the positive terminal of the voltage source.

  ii. Source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source.


THANKS
FOR YOUR
ATTENTION