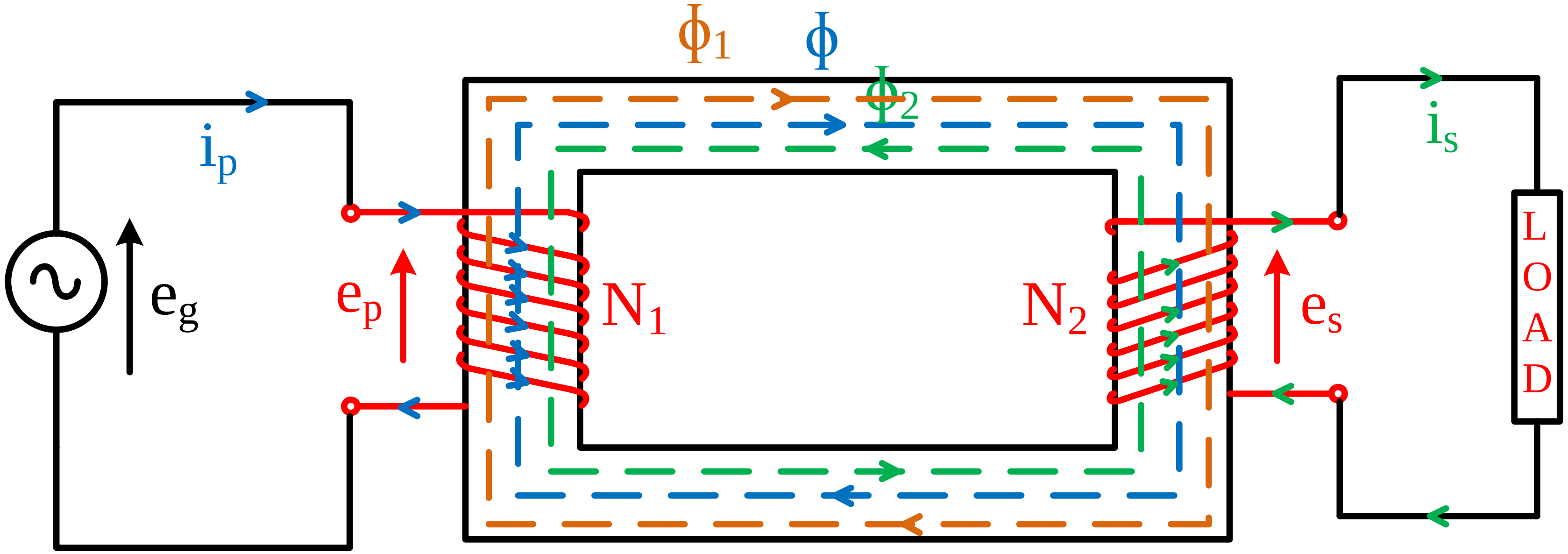
Lecture-24

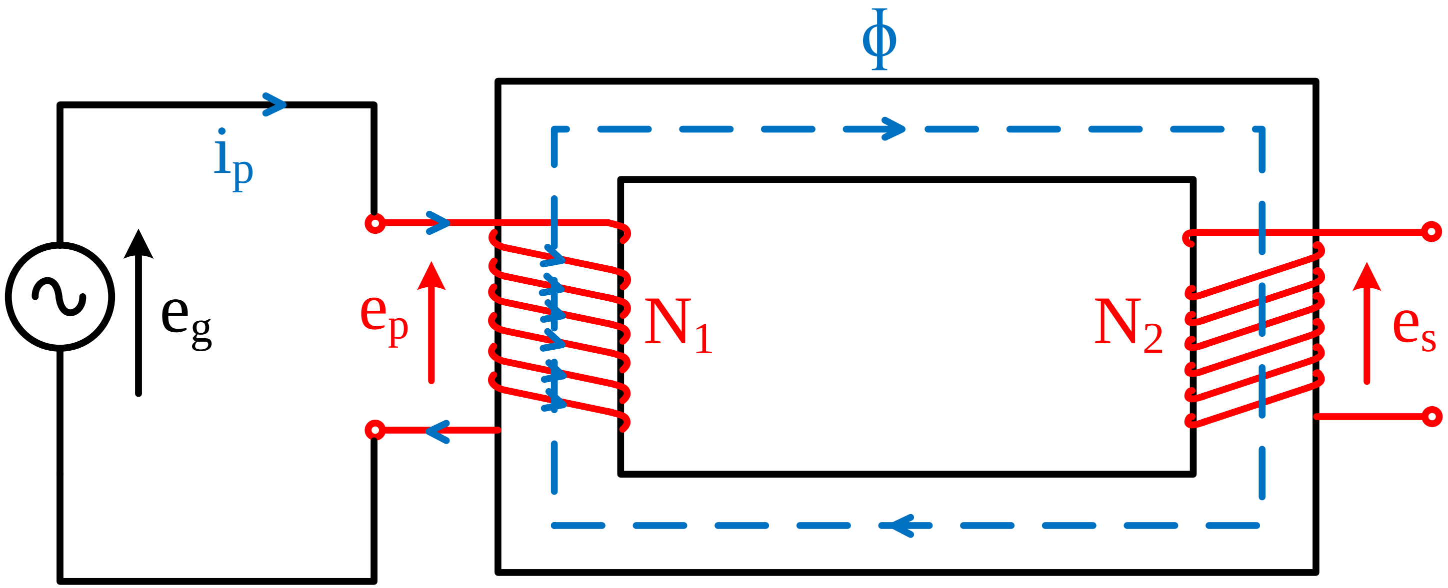
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Operating principle of transformer.
- Ideal Transformer.

Operating principle of transformer



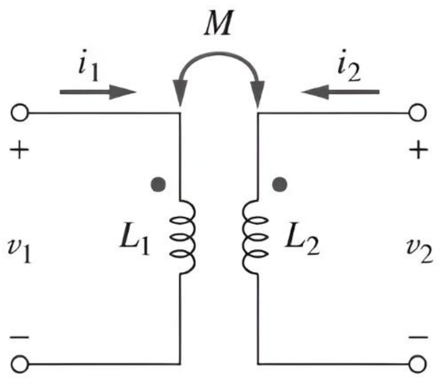


Ideal Transformer

- An ideal transformer is one with perfect coupling ($k = 1$).
- It consists of two (or more) coils with a large number of turns wound on a common core of high permeability.
- Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.
- Ideal transformer is the limiting case of two coupled inductors where the inductances approach infinity and the coupling is perfect

Ideal Transformer (cont…)

- Let us consider the circuit given below –



- In the frequency domain,

$$\boldsymbol{V}_1 = j\omega L_1 \boldsymbol{I}_1 + j\omega M \boldsymbol{I}_2 \Rightarrow \boldsymbol{I}_1 = \frac{\boldsymbol{V}_1 - j\omega M \boldsymbol{I}_2}{j\omega L_1}$$

$$\boldsymbol{V}_2 = j\omega M \boldsymbol{I}_1 + j\omega L_2 \boldsymbol{I}_2$$

- Combining both the equations,

$$\boldsymbol{V}_2 = j\omega L_2 \boldsymbol{I}_2 + \frac{M \boldsymbol{V}_1}{L_1} - \frac{j\omega M^2 \boldsymbol{I}_2}{L_1}$$

Ideal Transformer (cont…)

- But $M = \sqrt{L_1 L_2}$, as for perfect coupling k=1.
- Hence,

$$\boldsymbol{V}_2 = j\omega L_2 \boldsymbol{I}_2 + \frac{\sqrt{L_1 L_2}\boldsymbol{V}_1}{L_1} - \frac{j\omega L_1 L_2 \boldsymbol{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\boldsymbol{V}_1 = n\boldsymbol{V}_1$$

Where $n = \sqrt{\frac{L_2}{L_1}}$ is known as the turns ratio.

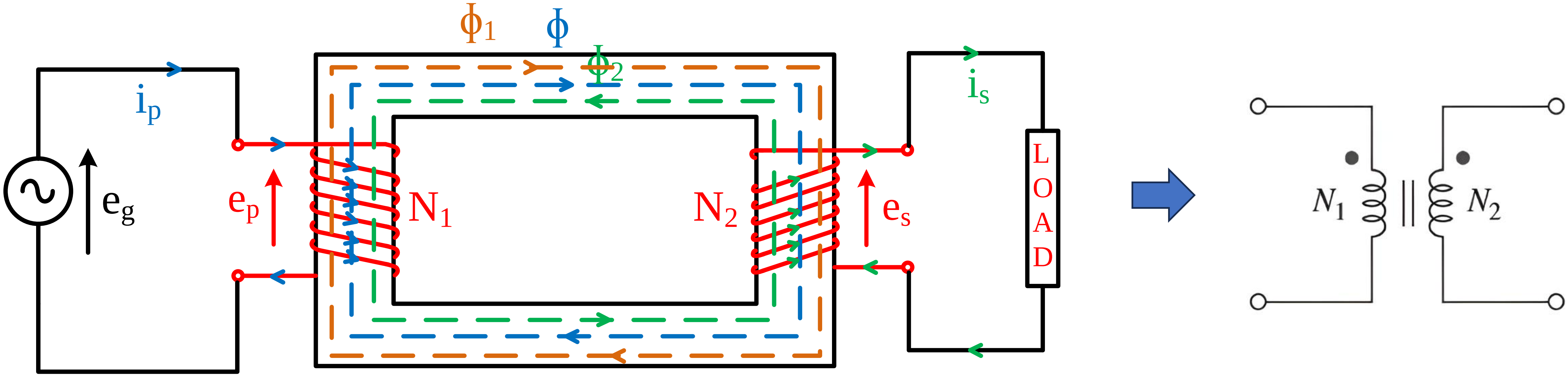
- As $L_1, L_2, M \to \infty$ such that n remains the same, the coupled coils become an ideal transformer.
- A transformer is said to be ideal if it has the following properties:
  1. Coils have very large inductances $L_1, L_2, M \to \infty$
  2. Coupling coefficient is equal to unity ($k = 1$)
  3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$)

Ideal Transformer (cont…)

- An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coilshave infinite self-inductances.

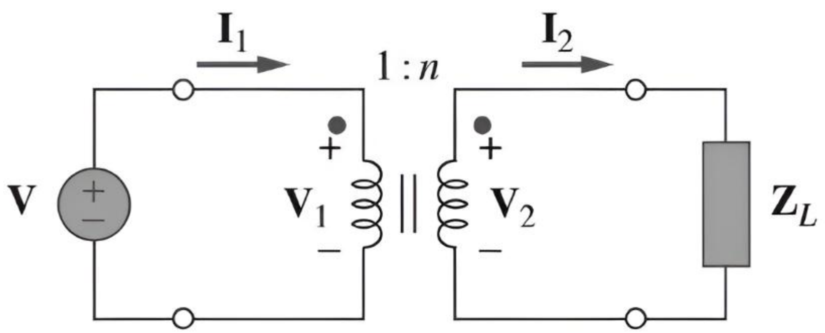
- The circuit symbol of a transformer is as shown in the figure below.

- The primary winding has N_1 turns and the secondary winding has N_2 turns.



Ideal Transformer (cont…)

- When a sinusoidal voltage is applied to primary winding as shown in the below figure, the magnetic flux ϕ goes through both the windings.



- According to Faraday's law, the voltage across the primary winding is

$$v_1 = N_1 \frac{d\phi}{dt}$$

- Similarly, the voltage across the secondary winding is

$$v_2 = N_2 \frac{d\phi}{dt}$$

Ideal Transformer (cont…)

$$v_1 = N_1 \frac{d\phi}{dt} \qquad v_2 = N_2 \frac{d\phi}{dt}$$

- Dividing the above two equations we get,

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

where n is the turns ratio or the transformation ratio.

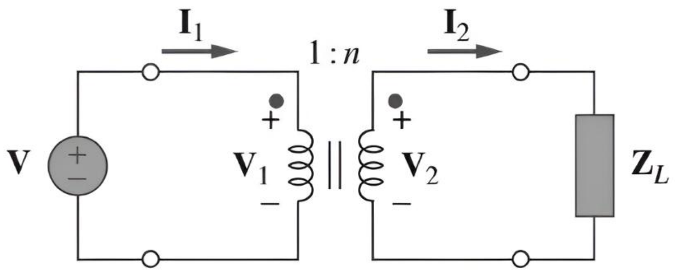
- Instead of the instantaneous voltages v_1 and v_2 we can use the phasor voltages $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$.

- Therefore,

$$\frac{\boldsymbol{V}_2}{\boldsymbol{V}_1} = \frac{N_2}{N_1} = n$$

Ideal Transformer (cont…)

- For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer.



- This implies that

$$v_1 i_1 = v_2 i_2$$

- In phasor form this is represented as,

$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

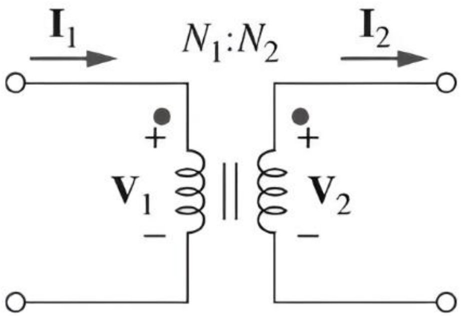
- When $n = 1$ the transformer is called an isolation transformer, if $n > 1$ it is called a step-up transformer, and if $n < 1$ it is called a step-down transformer.

- A step-down transformer is one whose secondary voltage is less than its primary voltage whereas a step-up transformer is one whose secondary voltage is greater than its primary voltage.

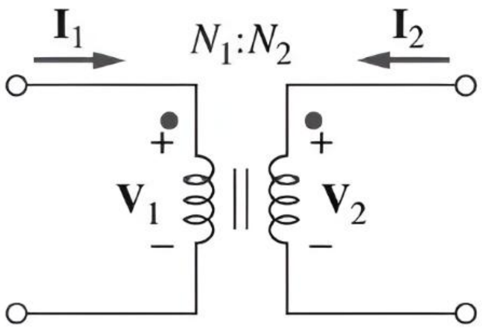
Ideal Transformer (cont…)

- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.
- If the polarity of $\boldsymbol{V}_1$ or $\boldsymbol{V}_2$ or the direction of $\boldsymbol{I}_1$ or $\boldsymbol{I}_2$ is changed, n may need to be replaced by $-n$.
- To get the polarity, the two simple rules to follow are:
  1. If $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ are both positive or both negative at the dotted terminals, use $+n$ in the transformer voltage equation. Otherwise, use $-n$.
  2. If $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ both enter into or both leave the dotted terminals, use $-n$ in the transformer current equation. Otherwise, use $+n$.
- These rules are demonstrated with the four circuits given in the next slide.

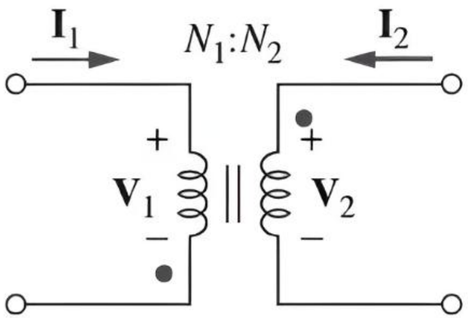
Ideal Transformer (cont…)



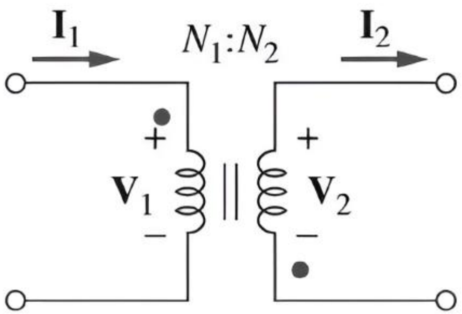
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

Ideal Transformer (cont…)

- Using the transformer voltage and current transformation equations we can represent $\boldsymbol{V}_1$ in terms of $\boldsymbol{V}_2$ and $\boldsymbol{I}_1$ in terms of $\boldsymbol{I}_2$, or vice versa as,

$$\boldsymbol{V}_1 = \frac{\boldsymbol{V}_2}{n} \ or \ \boldsymbol{V}_2 = n\boldsymbol{V}_1$$

$$\boldsymbol{I}_1 = n\boldsymbol{I}_2 \ \ or \ \boldsymbol{I}_2 = \frac{\boldsymbol{I}_1}{n}$$

- The complex power in the primary winding is

$$\boldsymbol{S}_1 = \boldsymbol{V}_1\boldsymbol{I}_1^* = \frac{\boldsymbol{V}_2}{n}(n\boldsymbol{I}_2)^* = \boldsymbol{V}_2\boldsymbol{I}_2^* = \boldsymbol{S}_2$$

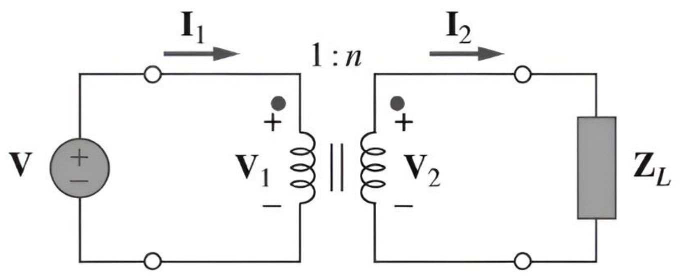
- This shows that the complex power supplied to the primary is delivered to the secondary without loss, as the transformer absorbs no power.

- It is expected since the ideal transformer is lossless.

Ideal Transformer (cont…)

- The input impedance as seen by the source is given by,

$$\boldsymbol{Z}_{in} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} = \frac{1}{n^2}\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2}$$



- But $\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2} = \boldsymbol{Z}_L$ so that,

$$\boldsymbol{Z}_{in} = \frac{1}{n^2}\boldsymbol{Z}_L$$

- The input impedance is also called the reflected impedance, since it appears as if the load impedance is reflected (referred) to the primary side.

- This ability of the transformer to transform a given impedance into another impedance provides us a means of impedance matching to ensure maximum power transfer.

❑ Example:

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution: (a) This is a step-down transformer as $\boldsymbol{V}_1$ = 2400 V > $\boldsymbol{V}_2$ = 120 V,

$$n = \frac{\boldsymbol{V}_2}{\boldsymbol{V}_1} = \frac{120}{2400} = 0.05$$

(b) To find the number of turns on the primary side, N_1, we know $n = N_2/N_1$ and $N_2 = 50$. Hence, N_1 = 50/0.05=1000 turns.

Ideal Transformer (cont…)

(c) KVA rating of the transformer is given by ,

$$S = V_1 I_1 = V_2 I_2 = 9.6kVA$$

- Therefore,

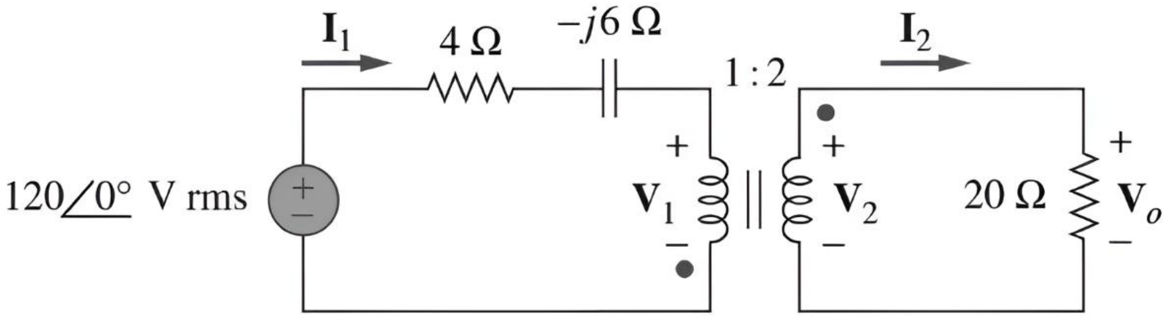
$$I_1 = \frac{9600}{V_1} = \frac{9600}{2400} = 4\ A$$

- The current I_2 given by,

$$I_2 = \frac{9600}{V_2} = \frac{9600}{120} = 80\ A$$

❑ Example:

For the ideal transformer circuit shown in the below figure, calculate: (a) the source current $\boldsymbol{I_1}$, (b) the output voltage $\boldsymbol{V_o}$, and (c) the complex power supplied by the source.



Solution: (a) The 20Ω impedance can be reflected to the primary side and we get,

$$\boldsymbol{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\,\Omega$$

- Thus,

$$\boldsymbol{Z}_{in} = 4 - j6 + \boldsymbol{Z}_R = 9 - j6 = 10.82\angle - 33.69\ \Omega$$

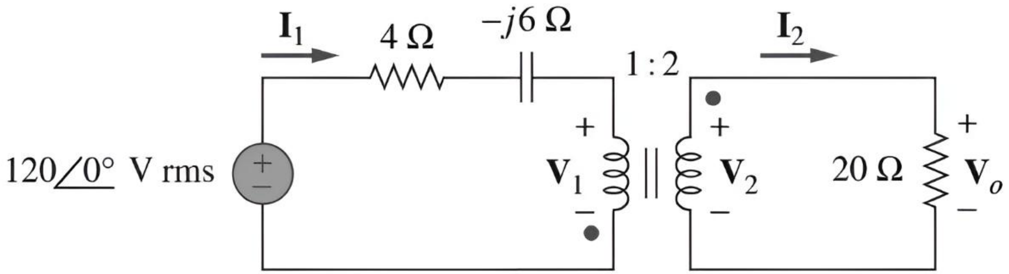
- Therefore,

$$\boldsymbol{I}_1 = \frac{120\angle 0}{\boldsymbol{Z}_{in}} = \frac{120\angle 0}{10.82\angle - 33.69} = 11.09\angle 33.69\ A$$

- (b) Since both $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ leave the dotted terminals,

$$\boldsymbol{I}_2 = -\frac{\boldsymbol{I}_1}{n} = -5.545\angle 33.69\ A$$

$$\boldsymbol{V}_o = 20\boldsymbol{I}_2 = 110.9\angle 213.60\ A$$



- The Complex power supplied is,

$$\boldsymbol{S} = \boldsymbol{V}_s \boldsymbol{I}_1^* = (120\angle 0)(11.09\angle - 33.69) = 1330.8\angle - 33.69\ VA$$

Transformer Working

❑ Transformer - Need

- Transformer is very important equipment in the power system network.
- It receives power at one voltage level and delivers it at another.
- This voltage conversion aids the efficient long-distance transmission of electrical power from generating stations to load end.
- Since power lines incur significant I^2R losses, it is important to minimize these losses by the use of high voltages.
- The same power can be delivered by high-voltage circuits at a fraction of the current required for low voltage circuits.
- At the generating end, voltage is stepped up by means of transformers for the transmission lines.
- At load end, transformers are used to step the voltage down to values suitable for motors, lights etc.

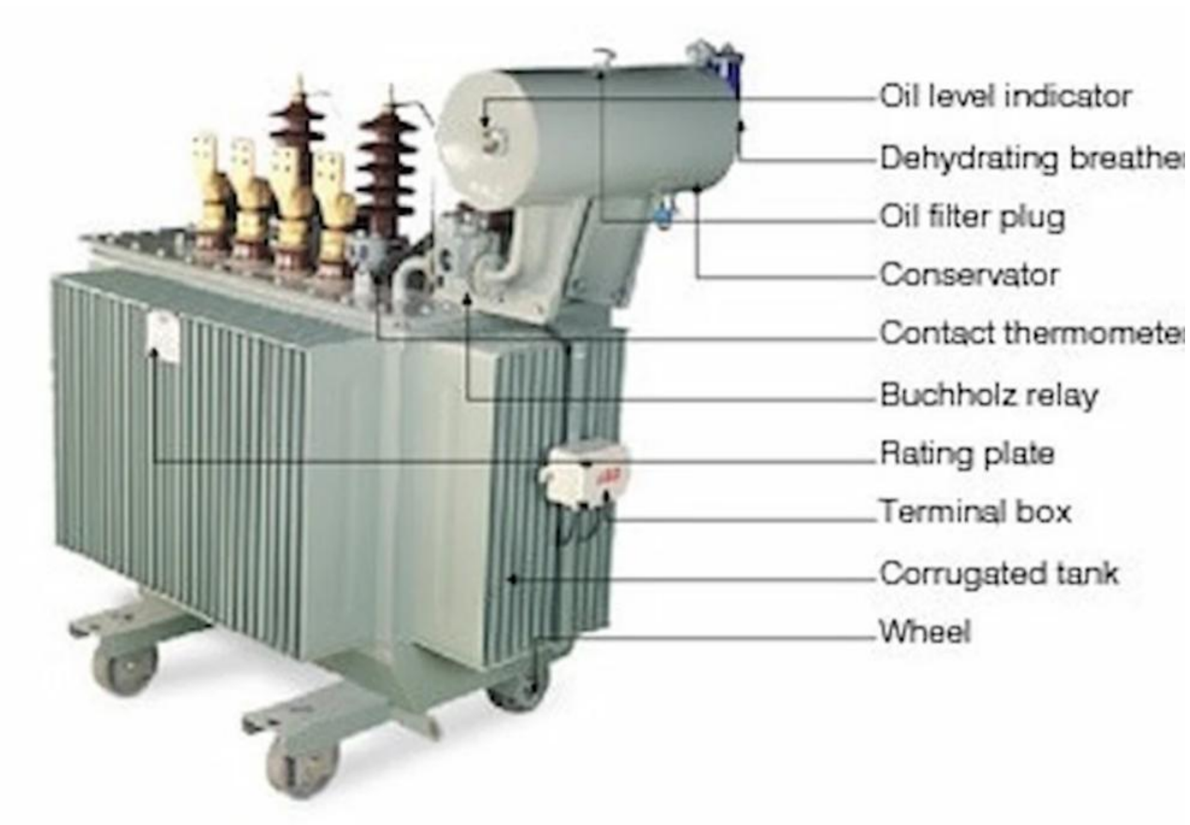
Transformer Working (cont…)

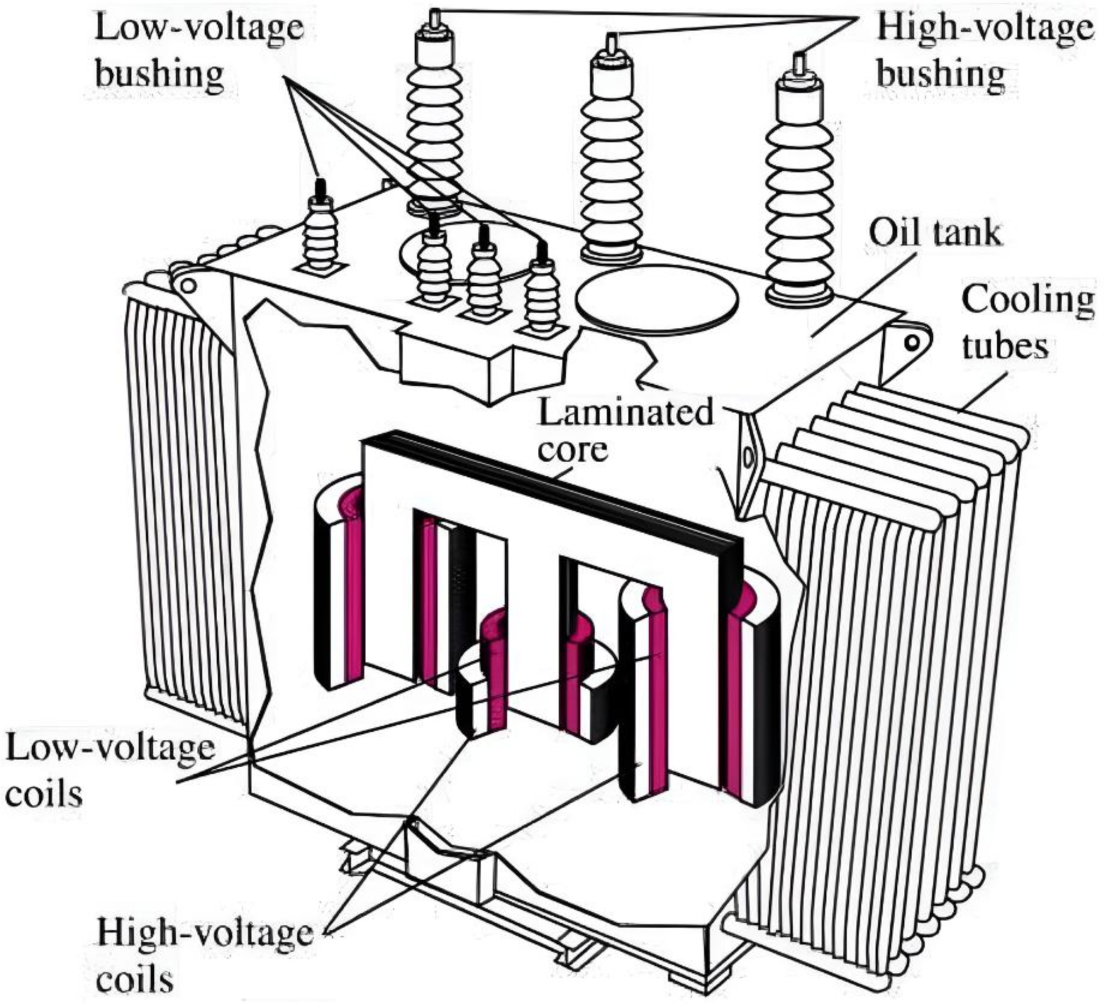
Transformer in a Substation

Transformer: Closer Look

Transformer Working (cont…)

Inside a Transformer

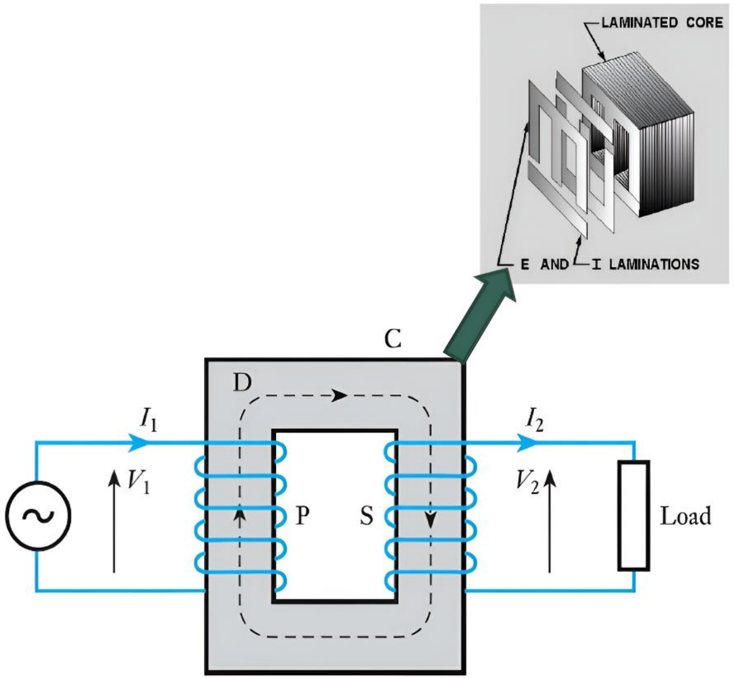




Transformer Working (cont…)

❑ Principle of Transformer Action

- A steel core C consists of laminated sheets, about 0.35–0.7 mm thick, insulated from one another to reduce the eddy-current loss.
- Vertical portions of the core are called as limbs, and the top and bottom portions are yokes.
- Coils P and S are wound on the limbs. Coil P is connected to the supply and is therefore termed the primary; coil S is connected to the load and is termed the secondary.
- An alternating voltage applied to P circulates a current through P and produces a flux in the steel core, the mean path of this flux being represented by the dotted line D.
- If the whole of the flux produced by P passes through S, the e.m.f. induced in each turn is the same for P and S.



Transformer Working (cont…)

- If, the primary winding (P) has N_1 turns and the secondary winding (S) has N_2 turns, then

$$\frac{\boldsymbol{V}_2}{\boldsymbol{V}_1} = \frac{N_2}{N_1} = n$$

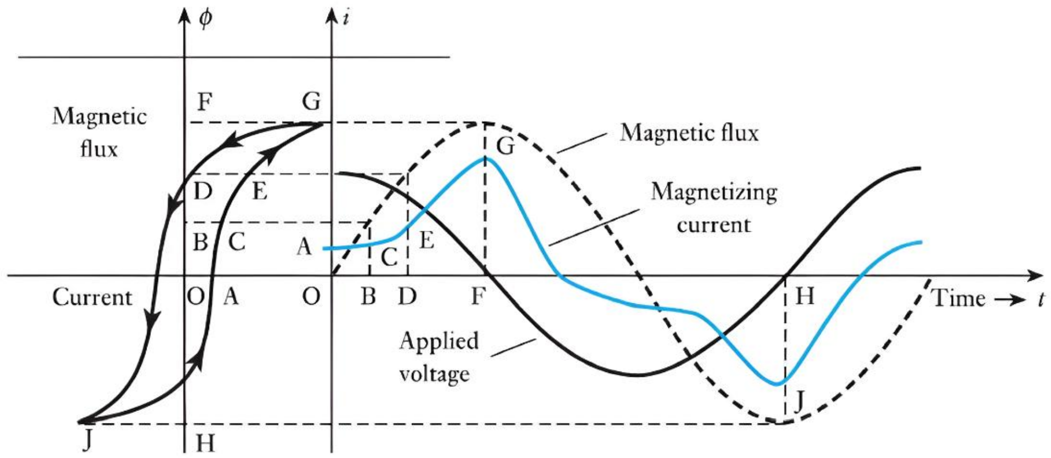
- If we assume the losses in the transformer are negligible, the energy supplied to the primary must be equal the energy absorbed by the secondary (Power conservation).
- This implies that

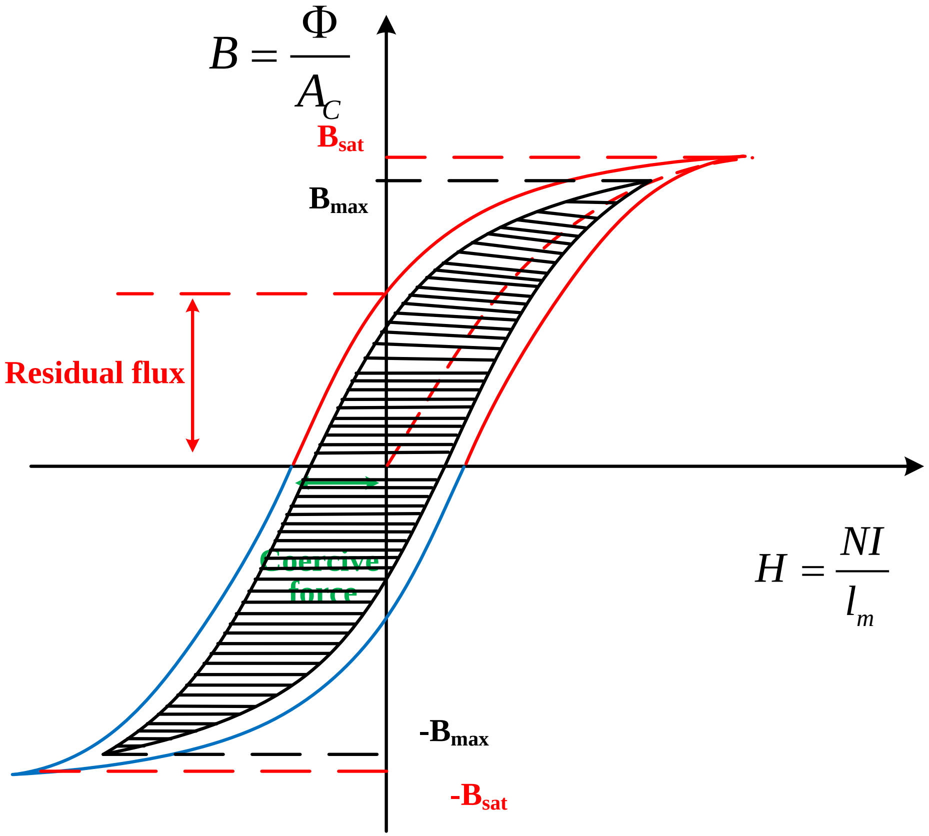
$$\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

- The magnetic flux forms the connecting link between the primary and secondary circuits.
- Any variation of the secondary current is accompanied by a small variation of the flux and therefore of the e.m.f. induced in the primary
- This enables the primary current to vary approximately in proportion to the secondary current.
- This balance of primary and secondary ampere-turns is an important relation for transformer action.

❑ Hysteresis Loss

- The relationship between the flux and the magnetizing current for the ferromagnetic core of transformer to be represented by the hysteresis loop.
- Let us assume that the waveform of the flux is sinusoidal as shown by the dotted curve in figure.
- When the flux is sinusoidal, the e.m.f. induced in the primary is also sinusoidal and lags the flux by a quarter of a cycle.
- The larger the loop, the greater the energy required to create the magnetic field.
- This energy has to be supplied during each cycle of magnetization. This requirement of supplying energy to magnetize the core is known as the hysteresis loss.





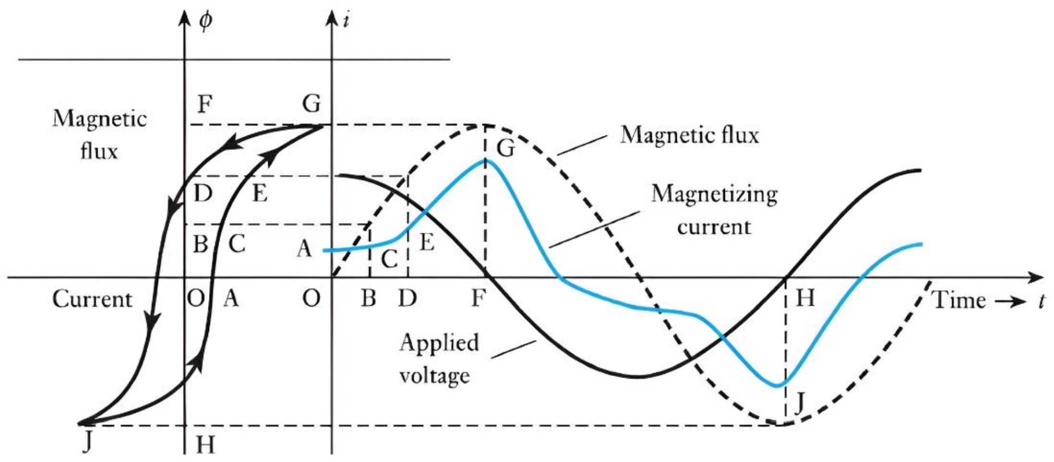
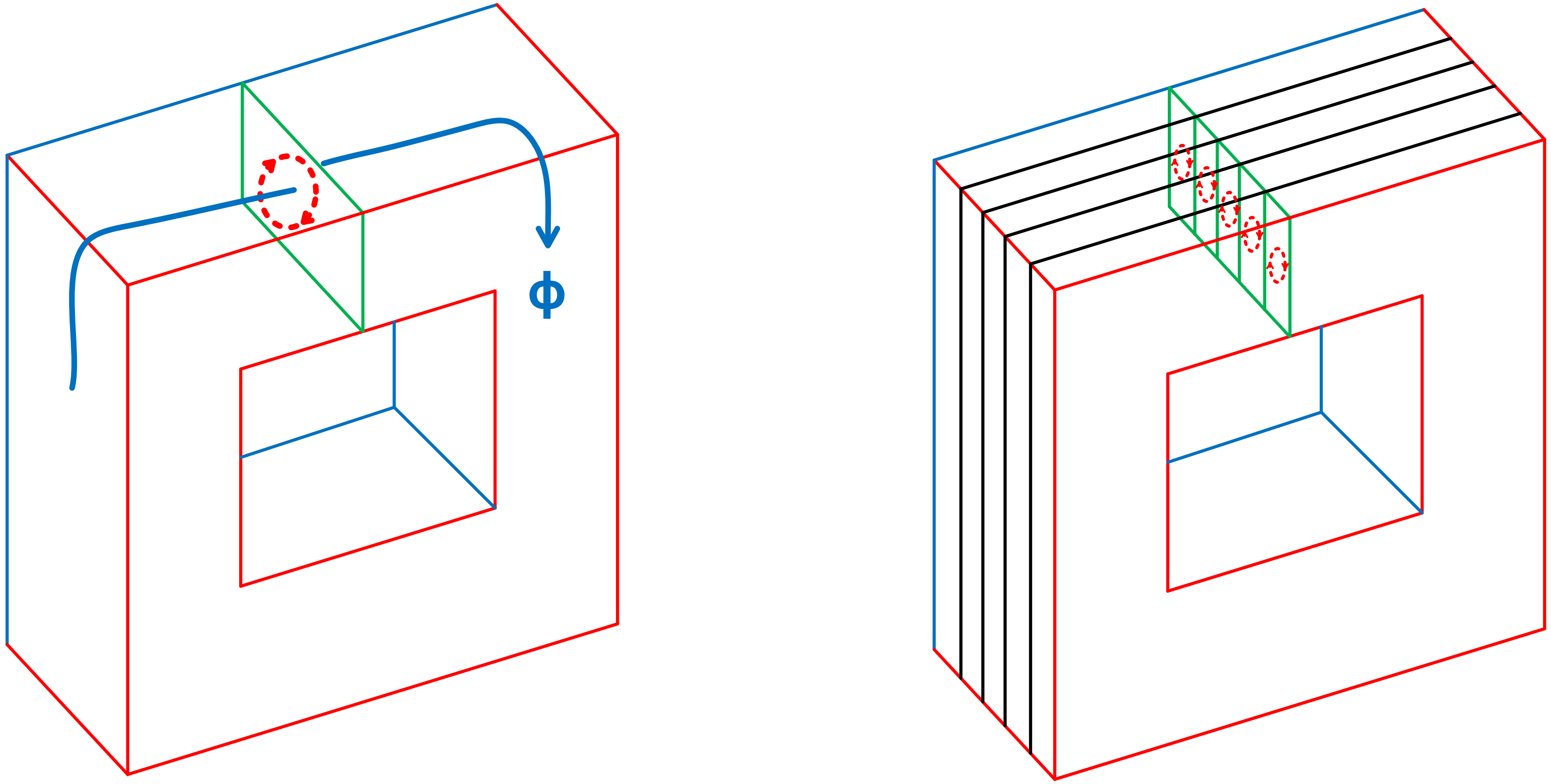
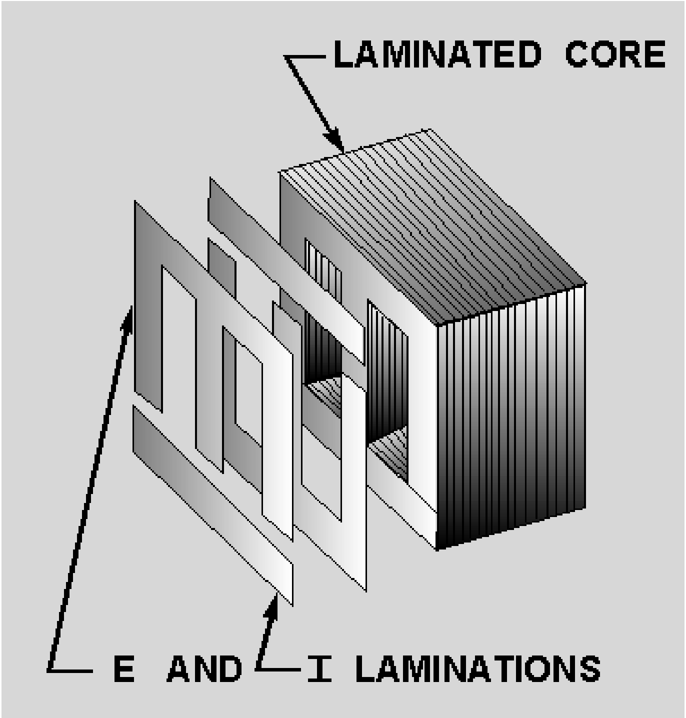


Fig. Hysteresis loss

Transformer Working (cont…)





0.35–0.7 mm thick

Fig. Eddy current loss


THANKS
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