Assignment 4

MSO-201: Probability and Statistics

1. Suppose cards numbered one through ten are placed in a hat, mixed up, and then one of the cards is drawn. If we are told that the number on the drawn card is at least five, then what is the conditional probability that it is ten?

2. A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy? Assume that the sample space $\Omega = (b,b),(b,g),(g,b),(g,g)$, and all outcomes are equally likely. Here, (b,g) means, for instance, that the older child is a boy and the younger child a girl.

3. Rahul can either take a course in computers or in chemistry. If Rahul takes the computer course, then he will receive an A grade with probability $1/2$; if he takes the chemistry course then he will receive an A grade with probability $1/3$. Rahul decides to base his decision on the flip of a fair coin. What is the probability that Rahul will get an A in chemistry?

4. For events $E_1, E_2, \ldots, E_n$, show that

$$P(E_1 \cap E_2 \ldots \cap E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2)\ldots P(E_n|E_1 \cap \ldots \cap E_n)$$

5. If A and B are independent events show that A and B' are also independent events.

6. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

7. If $P(A_1) > 0$ and $A_2, A_3, \ldots$ are mutually disjoint sets, show that

$$P(A_2 \cup A_3 \cup \ldots |A_1) = P(A_2|A_1) + P(A_3|A_1) + \ldots$$

8. Assume that $P(A_1 \cap A_2 \cap A_3 \cap A_4) > 0$, prove that

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)P(A_4|A_1 \cap A_2 \cap A_3)$$

9. Each of four persons res one shot at a target. Let C_k denote the event that the target is hit by person $k, k = 1,2,3,4$. If C_1, C_2, C_3, C_4 are independent and if $P(C_1) = P(C_2) = 0.7$, $P(C_3) = 0.9$, and $P(C_4) = 0.4$, compute the probability that (a) all of them hit the target; (b) exactly one hits the target; (c) no one hits the target; (d) at least one hits the target

10. Suppose $P(C_1) = 1/2$, $P(C_2) = 1/3$ and $P(C_3) = 1/4$ and they are independent events. Find $P(C_1 \cup C_2 \cup C_3)$.