Lecture-18

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

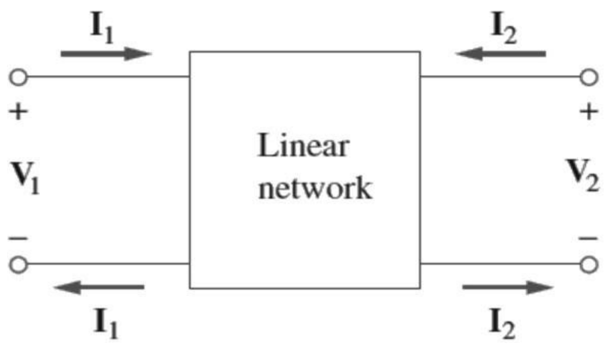
- Impedance Parameters (Cont…)
- Admittance Parameters.

Impedance Parameters (Cont…)

- The relation between the terminal voltage and the terminal current can be established using the following equations:

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$



- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{z}_{11} & \mathbf{z}_{12}\\ \mathbf{z}_{21} & \mathbf{z}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = [\mathbf{z}]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

- Here the **z** terms are known as the impedance parameters or the z parameters.

- Impedance parameters are expressed in ohms.

Impedance Parameters (Cont…)

- The values of the parameters can be evaluated by setting $\mathbf{I_1 = 0}$ (input port open-circuited) or $\mathbf{I_2 = 0}$ (output port open-circuited).
- Since the *z* parameters are obtained by open-circuiting the input or output port, they are also called the open-circuit impedance parameters.
- The individual parameters are evaluated as follows:

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

  - If $\mathbf{I}_2 = 0$, then

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$$

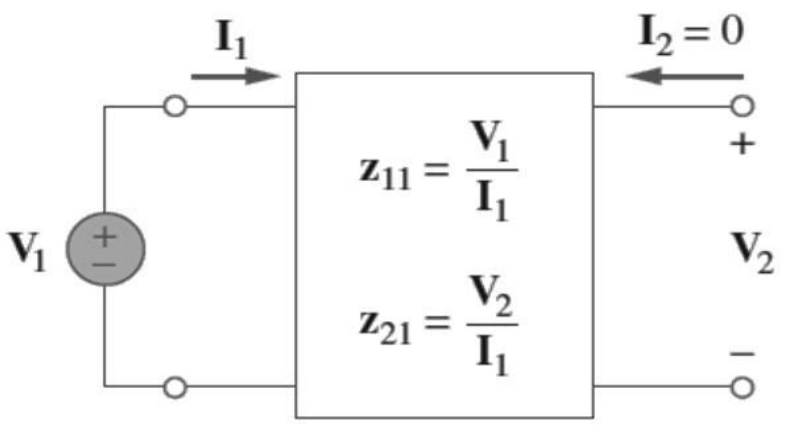
  - If $\mathbf{I}_1 = 0$, then

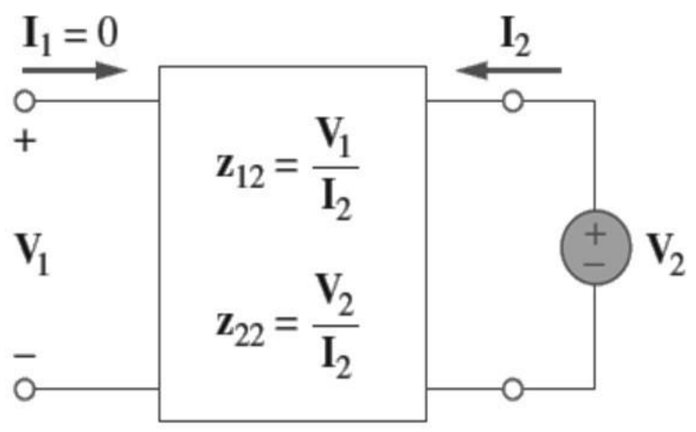
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

Impedance Parameters (Cont…)

- Specifically,
  - $\mathbf{z}_{11}$ = Open-circuit input impedance
  - $\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2
  - $\mathbf{z}_{21}$ = Open-circuit transfer impedance from port 2 to port 1
  - $\mathbf{z}_{22}$ = Open-circuit output impedance
  - The values of $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ can be obtained by connecting a voltage $\mathbf{V}_1$ (or current $\mathbf{I}_1$) to port 1 with port 2 open-circuited ($\mathbf{I}_2 = 0$).
  - $\mathbf{I}_1, \mathbf{V}_1$ and $\mathbf{V}_2$ are then obtained which can be used to evaluate $\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$ and $\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}$
  - Similarly, the values of $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ can be obtained by connecting a voltage $\mathbf{V}_2$ (or current $\mathbf{I}_2$) to port 2 with port 1 open-circuited ($\mathbf{I}_1 = 0$).
  - $\mathbf{I}_2, \mathbf{V}_2$ and $\mathbf{V}_1$ are then obtained which can be used to evaluate $\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}$ and $\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$

- The above procedure can be easily understood using the following Figure.





Impedance Parameters (Cont…)

- Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called driving-point impedances, while $\mathbf{z}_{12}$ and $\mathbf{z}_{21}$ are called transfer impedances.

- A driving-point impedance is the input impedance of a two-terminal (one-port) device.

- Thus, $\mathbf{z}_{11}$ is the input driving-point impedance with the output port open-circuited, while $\mathbf{z}_{22}$ is the output driving-point impedance with the input port open-circuited.

- When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be symmetrical.

- This implies that the network has mirror-like symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

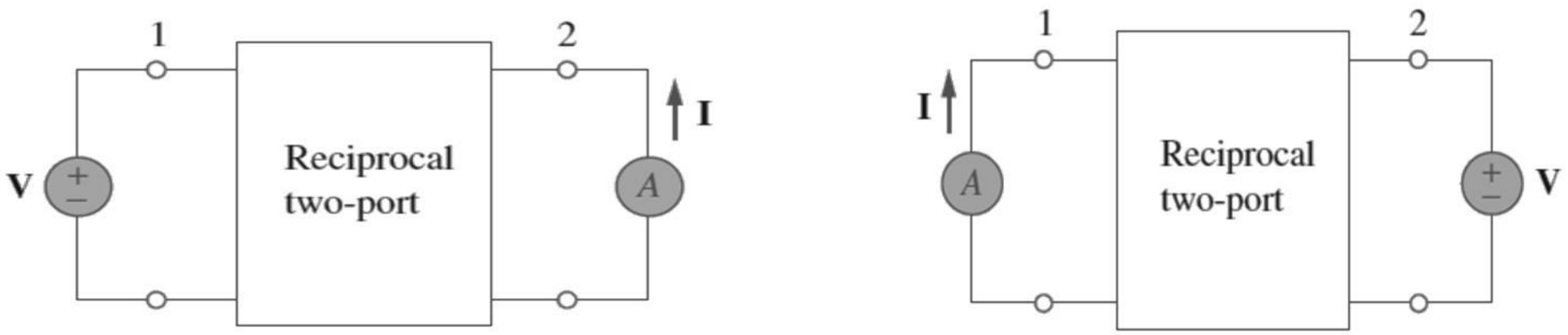
- In other words, a two port network is said to be symmetrical if the input and output ports can be interchanged without altering the port voltages and currents.

Impedance Parameters (Cont…)

- When the two-port network is linear and has no dependent sources, and the transfer impedances are equal ($\mathbf{z}_{12} = \mathbf{z}_{21}$), then the two-port is said to be reciprocal.

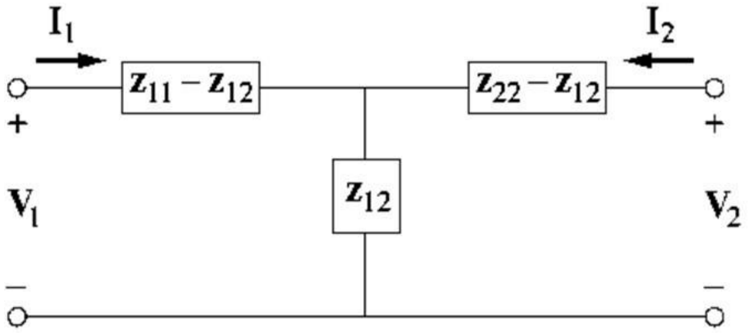
- This means that if the points of excitation and response are interchanged, the transfer impedances remain the same.

- As shown in the figure below, a two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

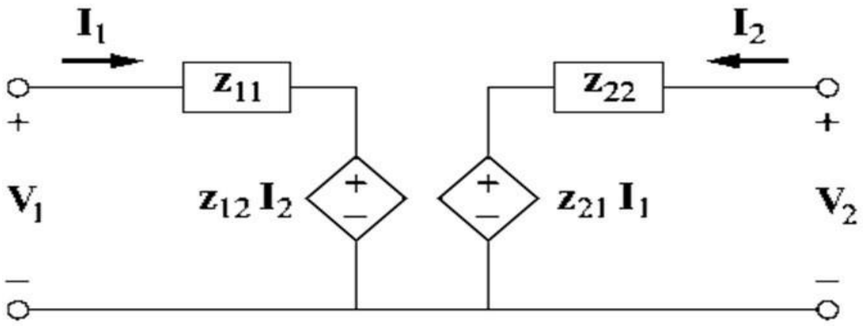


Impedance Parameters (Cont…)

- Any two-port that is made entirely of resistors, capacitors, and inductors must be reciprocal.
- A reciprocal network can be replaced by the T-equivalent circuit as shown in the below figure.



- If the network is not reciprocal, a more general equivalent network as shown below can be used.

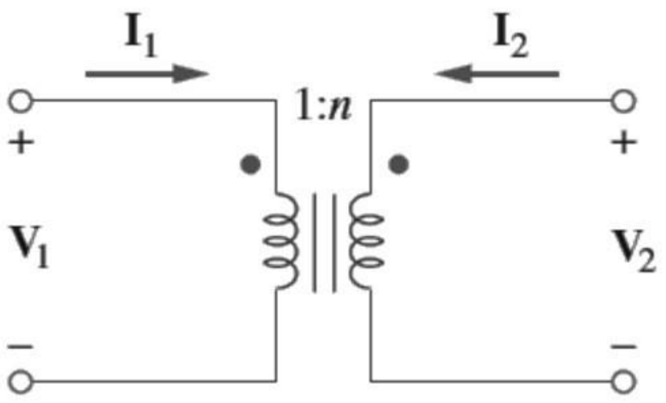


Impedance Parameters (Cont…)

- It should be mentioned that for some two-port networks, the **z**-parameters do not exist because they cannot be described by the **z**-parameter equations.
- As an example, consider the ideal transformer shown in the figure.
- The defining equations for the two-port network are:

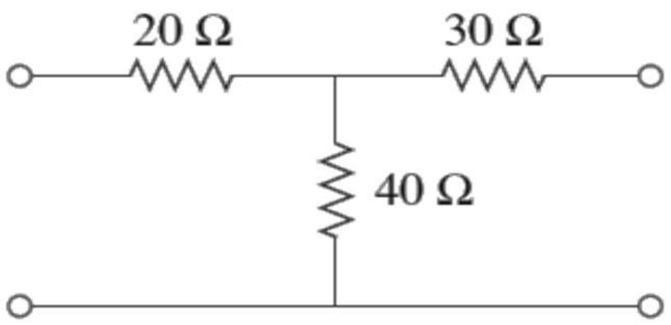
$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \mathbf{I}_1 = -n\mathbf{I}_2$$

- Observe that it is impossible to express the voltages in terms of the currents, and vice versa.
- Thus, the ideal transformer has no **z**-parameters.
- However, it does have hybrid parameter as will be discussed in the upcoming lectures.



❑ Example:

❖ Determine the **z**-parameters for the below circuit?



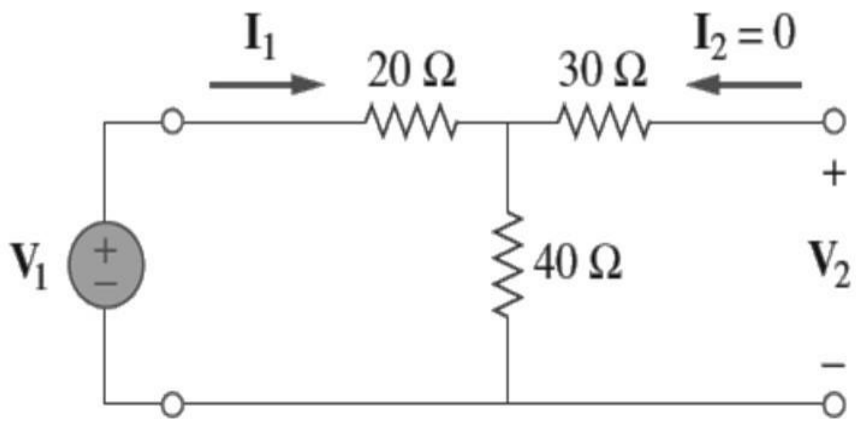
❑Solution:

The parameters are determined using the equations discussed earlier in the lecture

Impedance Parameters (Cont…)

❑Solution:

To determine $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we apply a voltage source $\mathbf{V}_1$ to the input port and leave the output port open as in the following figure.

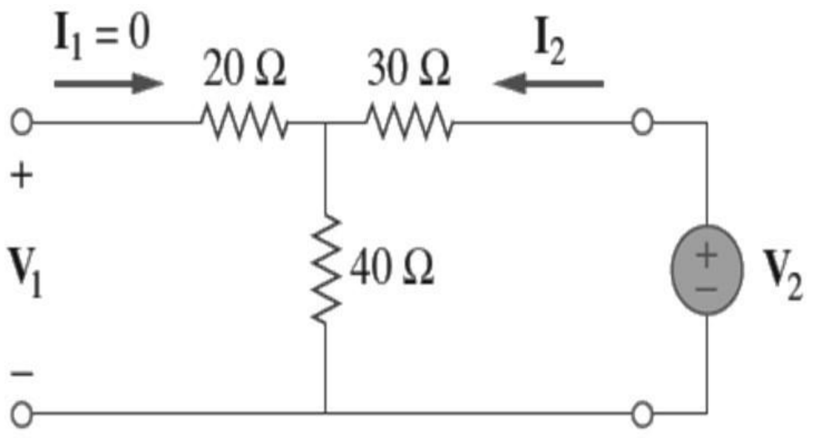


$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20+40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$

❑ Solution:

To determine $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we apply a voltage source $\mathbf{V}_2$ to the output port and leave the input port open as in the following figure.



$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega$$

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(40 + 30)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$

❑ Solution:

- This can alternatively be expressed in matrix form as,

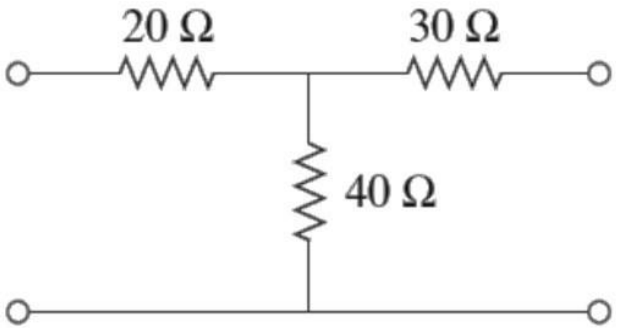
$$[\mathbf{z}] = \begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$$

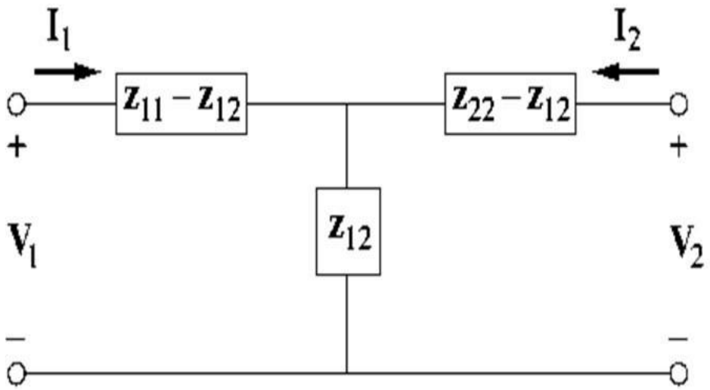
- Alternatively, since there is no dependent source in the circuit, $\mathbf{z}_{12} = \mathbf{z}_{21}$, we can use the T-equivalent circuit. Comparing the circuit in the question to the equivalent circuit,

$$\mathbf{z}_{12} = 40\ \Omega = \mathbf{z}_{21}$$

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 20 \Rightarrow \mathbf{z}_{11} = 20 + \mathbf{z}_{12} = 60\ \Omega$$

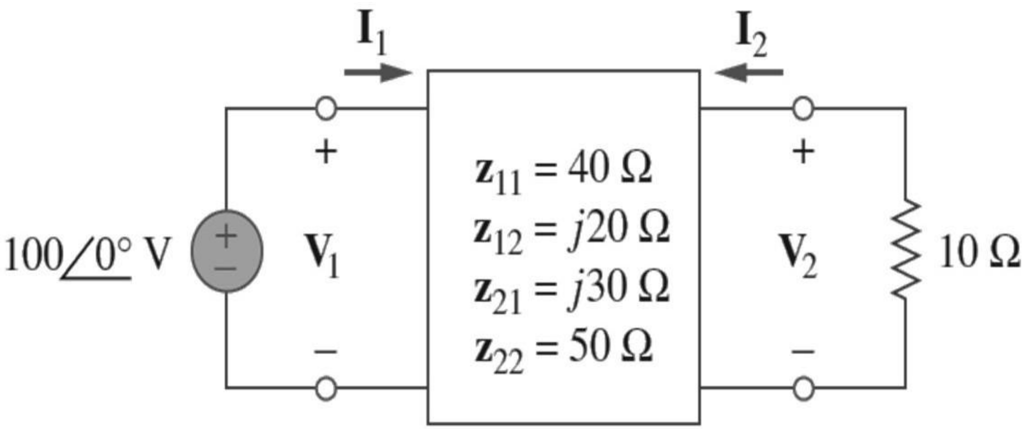
$$\mathbf{z}_{22} - \mathbf{z}_{21} = 30 \Rightarrow \mathbf{z}_{22} = 30 + \mathbf{z}_{12} = 70\ \Omega$$





❑Example:

- Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit given below?



❑Solution:

This is not a reciprocal circuit. The equivalent circuit discussed previously can be used to solve the problem. However, the **z**-parameter equations can be used directly in this scenario.

Impedance Parameters (Cont…)

❑ Solution:

- The relation between the terminal voltage and the terminal current can established using the following equations, as discussed previously:

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$
$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2$$

- But $\mathbf{V}_1 = 100\angle 0^\circ$ and $\mathbf{V}_2 = -10\mathbf{I}_2$

  - Therefore,

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \qquad (1)$$

$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \Rightarrow \mathbf{I}_1 = j2\,\mathbf{I}_2 \qquad (2)$$

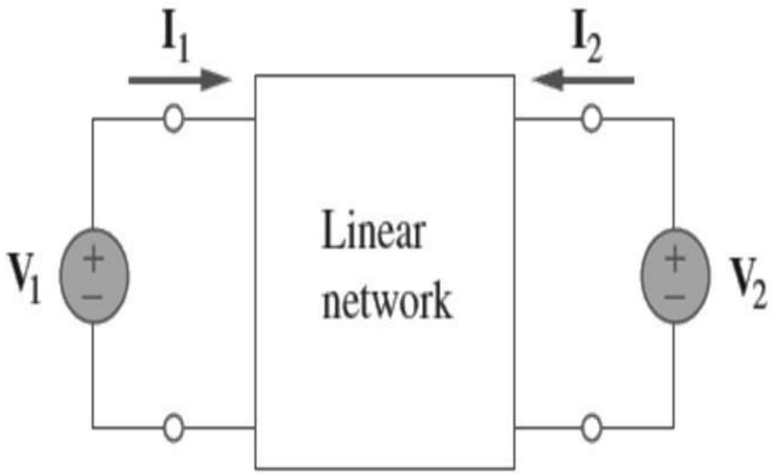
- Substituting the (2) in (1)

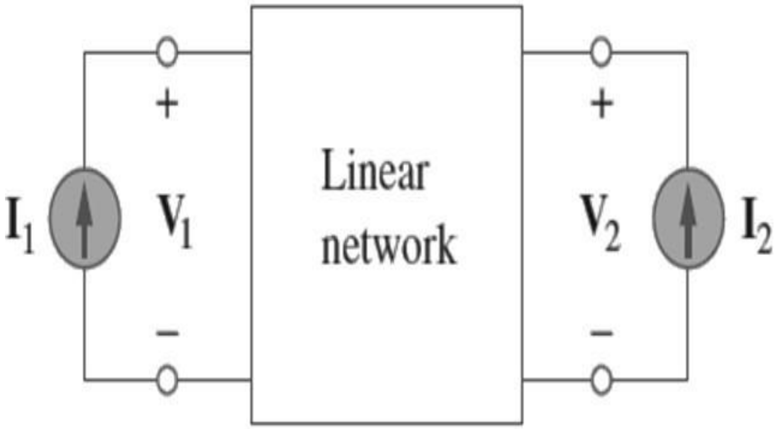
  - As $\mathbf{I}_1 = j2\,\mathbf{I}_2 = j2(-j) = 2$. Thus,

$$\mathbf{I}_1 = 2\angle 0^\circ \text{A} \qquad \text{and} \qquad \mathbf{I}_2 = 1\angle -90^\circ \text{A}$$

Admittance Parameters

- A two-port network may be voltage driven or current driven as illustrated in the figure below.
- To determine the admittance parameters the terminal currents are expressed in terms of the terminal voltages.





Admittance Parameters (Cont..)

- The relation between the terminal voltage and the terminal current can be established using following equations:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = [\mathbf{y}]\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

- Here the **y** terms are known as the admittance parameters or the y parameters.
- Admittance parameters are expressed in siemens.

Admittance Parameters (Cont..)

- The values of the parameters can be evaluated by setting $\mathbf{V_1 = 0}$ (input port short-circuited) or (output port short-circuited). $\mathbf{V_2 = 0}$

- Since the *y* parameters are obtained by short-circuiting the input or output port, they are also called the short-circuit admittance parameters.
- The individual parameters are evaluated as follows:

  - If $\mathbf{V}_2 = 0$, then

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$$

  - If $\mathbf{V}_1 = 0$, then

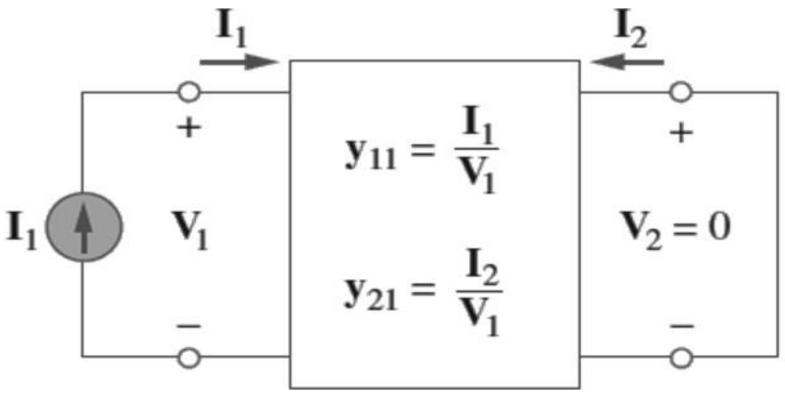
$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}, \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$$

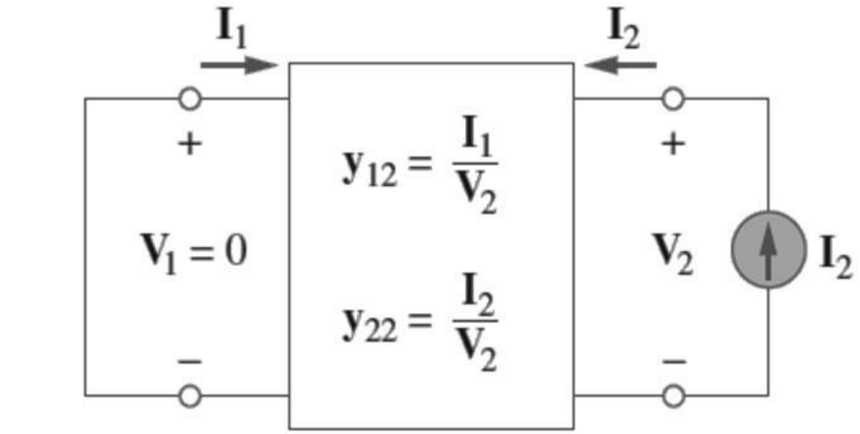
Admittance Parameters (Cont..)

- Specifically,
  - $\mathbf{y}_{11}$ = Short-circuit input admittance
  - $\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 1 to port 2
  - $\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 2 to port 1
  - $\mathbf{y}_{22}$ = Short-circuit output admittance
  - The values of $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ can be obtained by connecting a current $\mathbf{I}_1$ to port 1 with port 2 short-circuited.
  - $\mathbf{V}_1, \mathbf{I}_1$, and $\mathbf{I}_2$ are then obtained which can be used to evaluate $\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}$ and $\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$.
  - Similarly, the values of $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ can be obtained by connecting a current $\mathbf{I}_2$ to port 2 with port 1 short-circuited.
  - $\mathbf{V}_2, \mathbf{I}_2$, and $\mathbf{I}_1$ are then obtained which can be used to evaluate $\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}$ and $\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$.

Admittance Parameters (Cont..)

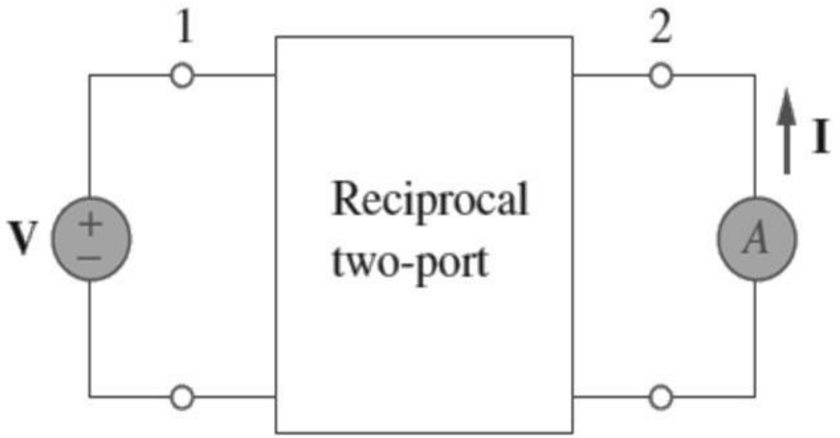
- The above procedure can be easily understood using the following Figures.

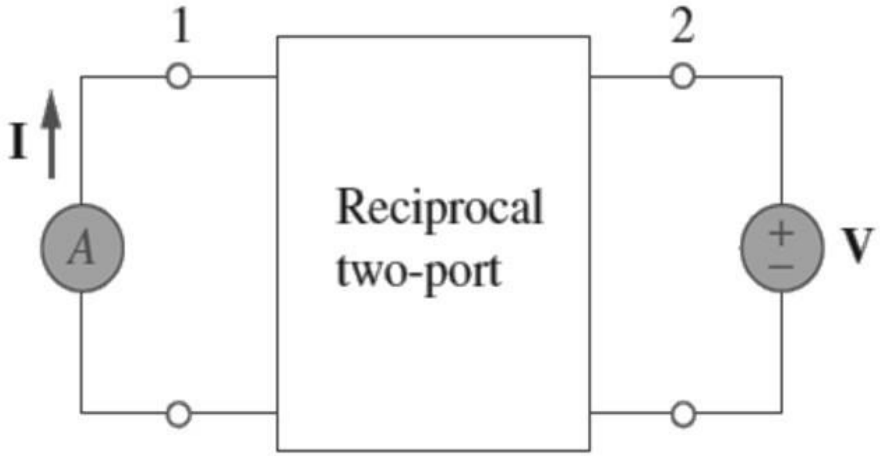




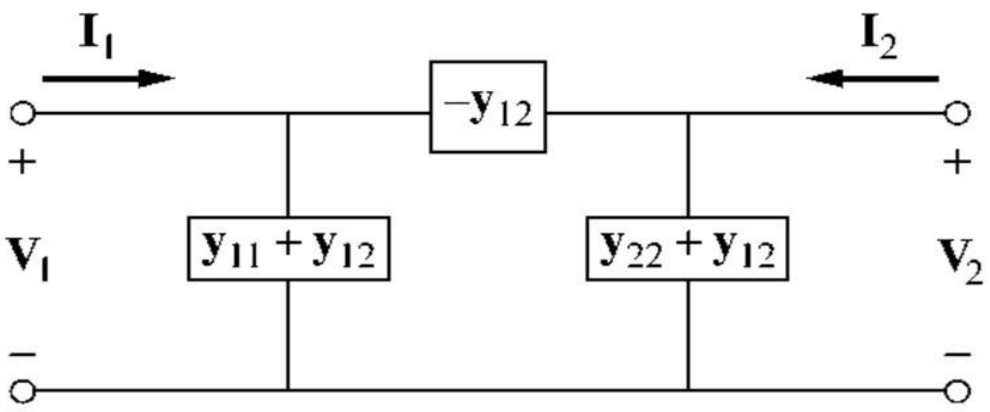
Admittance Parameters (Cont..)

- When the two-port network is linear and has no dependent sources, the transfer admittances are equal $(\mathbf{y}_{12} = \mathbf{y}_{21})$, and the two-port is said to be reciprocal.
- This can be proven the same way as in case of the **z**-parameters.
- As shown in the figure below, a two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

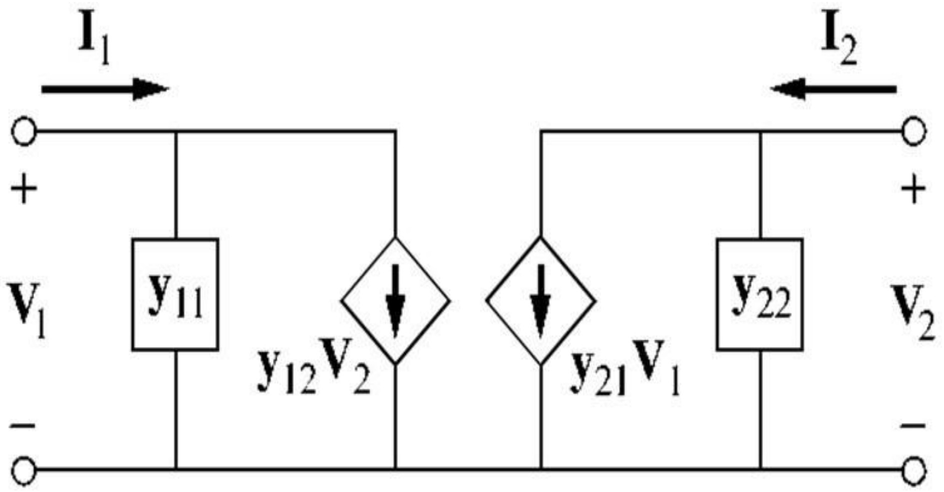




- In this case, the reciprocal network can be replaced by the π-equivalent circuit as shown in the below figure.

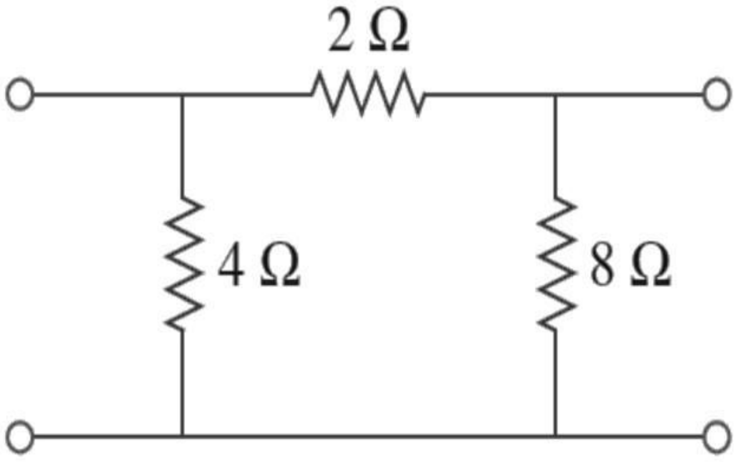


- If the network is not reciprocal, a more general equivalent network as shown below can be used.



Admittance Parameters (Cont..)

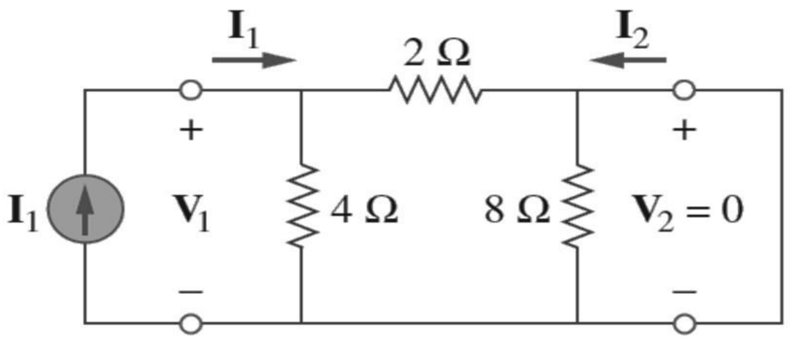
Example: Determine the **y**-parameters for the below circuit?



Solution: The parameters are determined using the equations discussed earlier in the lecture.

Admittance Parameters (Cont..)

- To determine $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$,we connect a current source $\mathbf{I}_1$ to the input port and short the output port as in the following figure.
- Since the 8 ohm resistor is shorted, 2 and 4 ohm resistors are in parallel.



$$\mathbf{V}_1 = \mathbf{I}_1(4||2) = \frac{4}{3}\mathbf{I}_1$$

- $$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = 0.75\ S$$

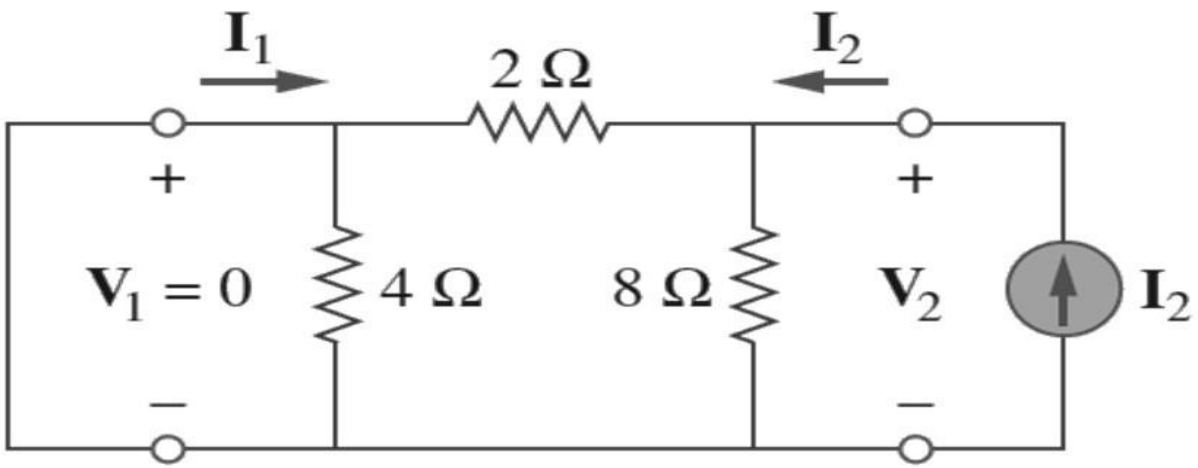
Admittance Parameters (Cont..)

- By current division,

$$-\mathbf{I}_2 = \frac{4}{4+2}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-\frac{2}{3}\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = -0.5\ S$$

- To determine $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$,we connect a current source $\mathbf{I}_2$ to the output port and short circuit the input port as in the following figure.



Admittance Parameters (Cont..)

- The 4 ohm resistor is shorted so that the 2 and 8 ohm resistors are in parallel.

$$\mathbf{V}_2 = \mathbf{I}_2(8\|2) = \frac{8}{5}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = 0.625\ S$$

- By current division,

$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5\ S$$

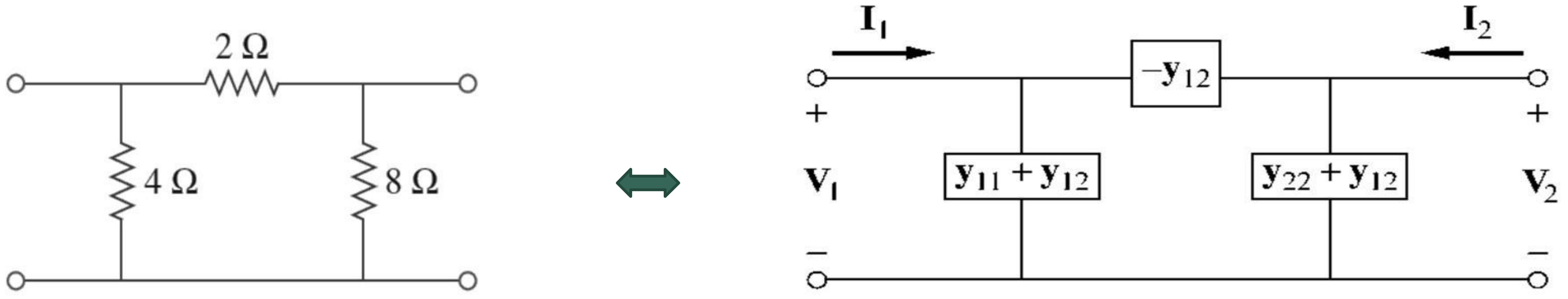
Admittance Parameters (Cont..)

- This can alternatively be expressed in matrix form as,

$$[y] = \begin{bmatrix} 0.75\ S & -0.5\ S \\ -0.5\ S & 0.625\ S \end{bmatrix}$$

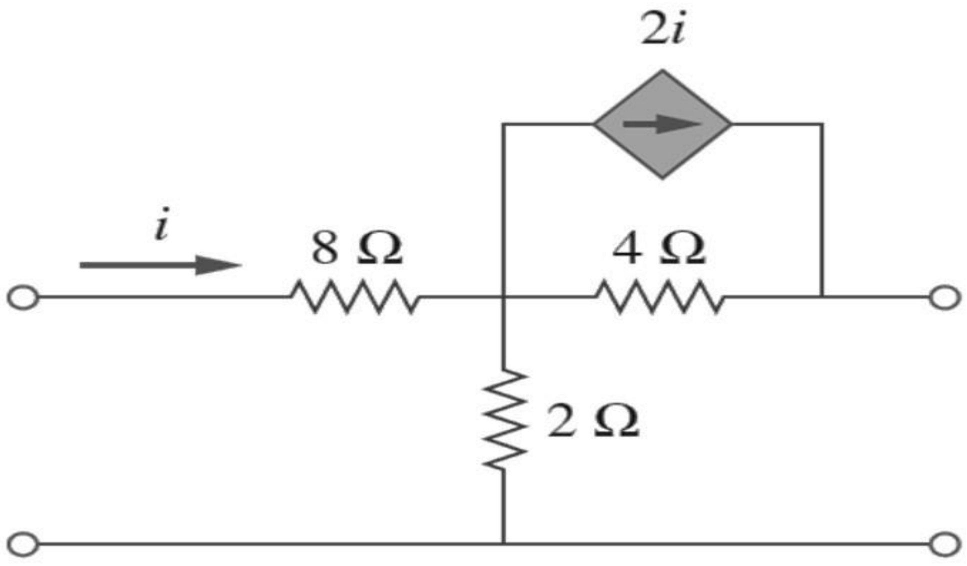
- Alternatively, since there is no dependent source in the circuit, and $\mathbf{y}_{12} = \mathbf{y}_{21}$, we can use the following π equivalent circuit to find parameters. Comparing the circuit in the question to the π equivalent circuit,

$$\mathbf{y}_{12} = -0.5\ S = \mathbf{y}_{21}$$
$$\mathbf{y}_{11} + \mathbf{y}_{12} = 0.25 \Rightarrow \mathbf{y}_{11} = 0.25 - \mathbf{y}_{12} = 0.75\ S$$
$$\mathbf{y}_{22} + \mathbf{y}_{21} = 0.125 \Rightarrow \mathbf{y}_{22} = 0.125 - \mathbf{y}_{21} = 0.625\ S$$



Admittance Parameters (Cont..)

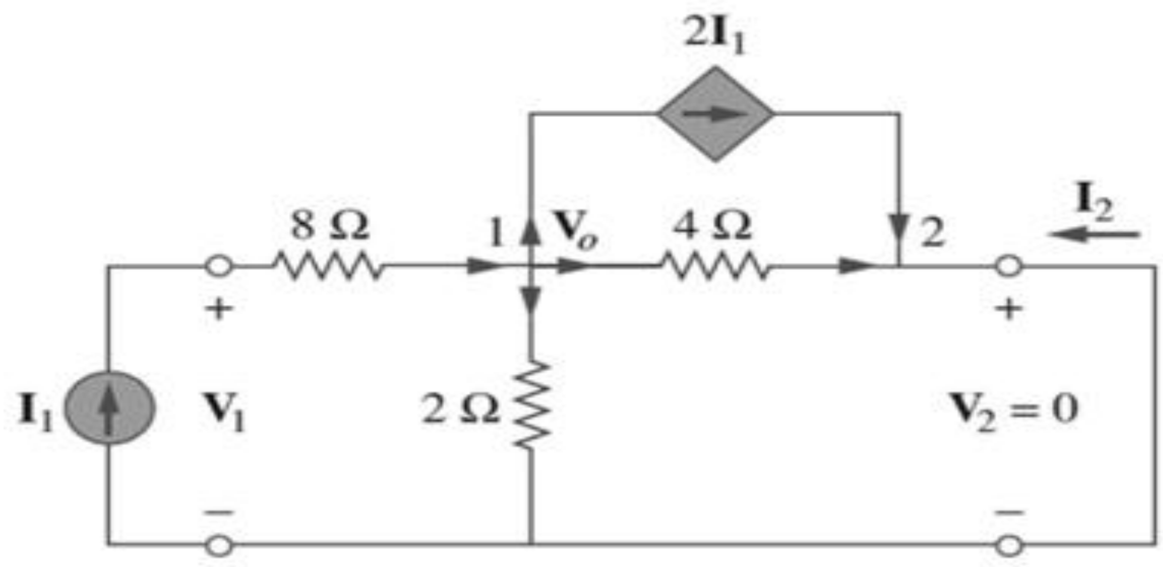
Example: Determine y-parameters for the following circuit?



Solution: The above circuit can be solved using the same procedure discussed in the previous example.

Admittance Parameters (Cont..)

- To determine $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, we connect a current source $\mathbf{I}_1$ to the input port and short the output port as in the following figure.



- At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_0}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - 0}{4}$$

- But $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_0)/8$, therefore

$$0 = \frac{\mathbf{V}_1 - \mathbf{V}_0}{8} + \frac{3\mathbf{V}_0}{4}$$

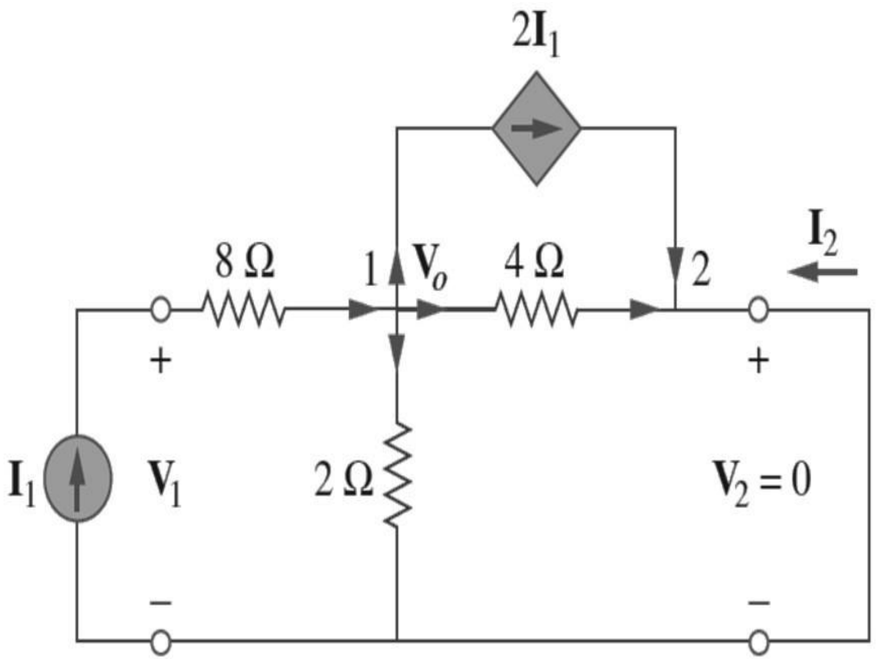
$$0 = \mathbf{V}_1 - \mathbf{V}_0 + 6\mathbf{V}_0 \Rightarrow \mathbf{V}_1 = -5\mathbf{V}_0$$

- Hence,

$$\mathbf{I}_1 = \frac{-5\mathbf{V}_0 - \mathbf{V}_0}{8} = -0.75\mathbf{V}_0$$

and,

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_0}{-5\mathbf{V}_0} = 0.15\ S$$



Admittance Parameters (Cont..)

- At node 2,

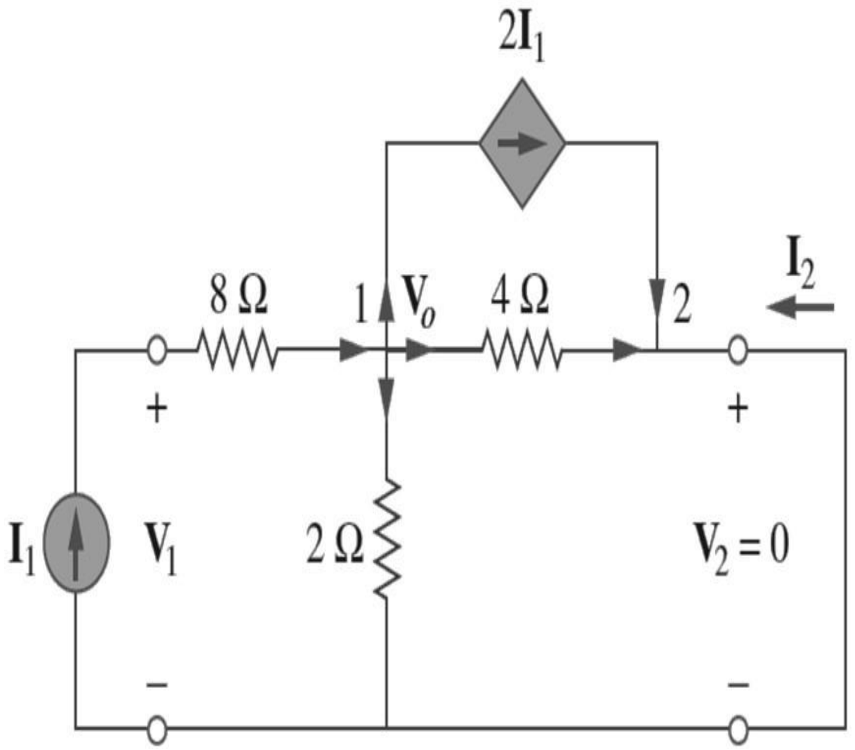
$$0 = \frac{\mathbf{V}_0 - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_0 - 1.5\mathbf{V}_0 = -1.25\mathbf{V}_0$$

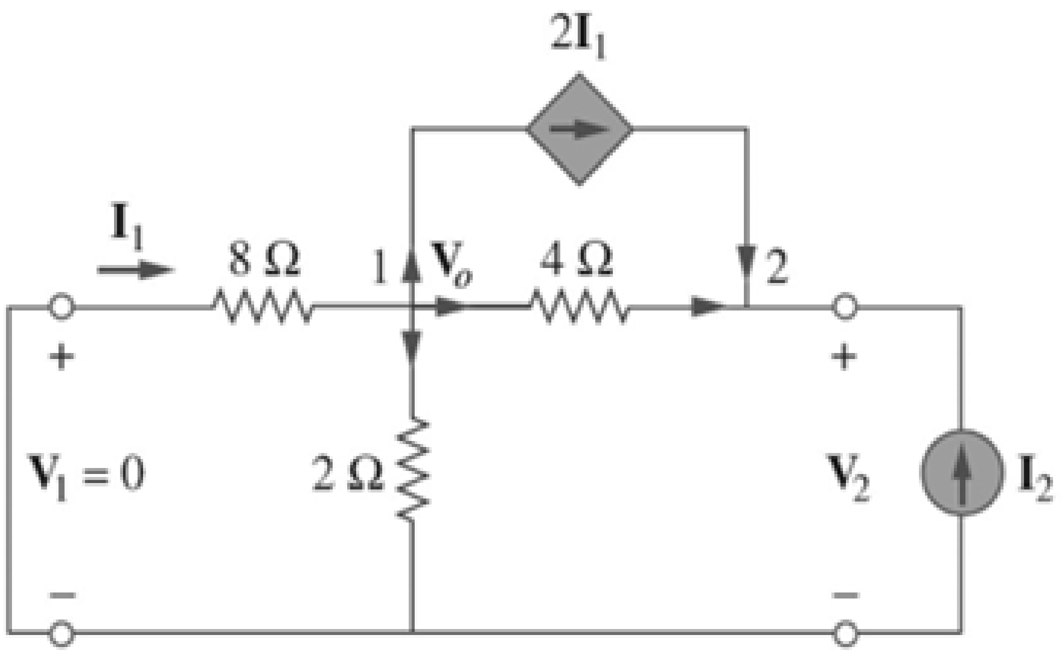
- Hence,

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1.25\mathbf{V}_0}{-5\mathbf{V}_0} = -0.25\ \mathrm{S}$$



Admittance Parameters (Cont..)

- To determine $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, we connect a current source $\mathbf{I}_2$ to the output port and short the input port as in the following figure.



- At node 1,

$$\frac{0-\mathbf{V}_0}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - \mathbf{V}_2}{4}$$

Admittance Parameters (Cont..)

- But $\mathbf{I}_1 = (0 - \mathbf{V}_0)/8$, therefore

$$0 = \frac{0 - \mathbf{V}_0}{8} + \frac{\mathbf{V}_0}{2} + \frac{\mathbf{V}_0 - \mathbf{V}_2}{4}$$
$$0 = -\mathbf{V}_0 + 4\mathbf{V}_0 + 2\mathbf{V}_0 - 2\mathbf{V}_2 \Rightarrow \mathbf{V}_2 = 2.5\mathbf{V}_0$$

- Hence,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_0/8}{2.5\mathbf{V}_0} = -0.05 \text{ S}$$

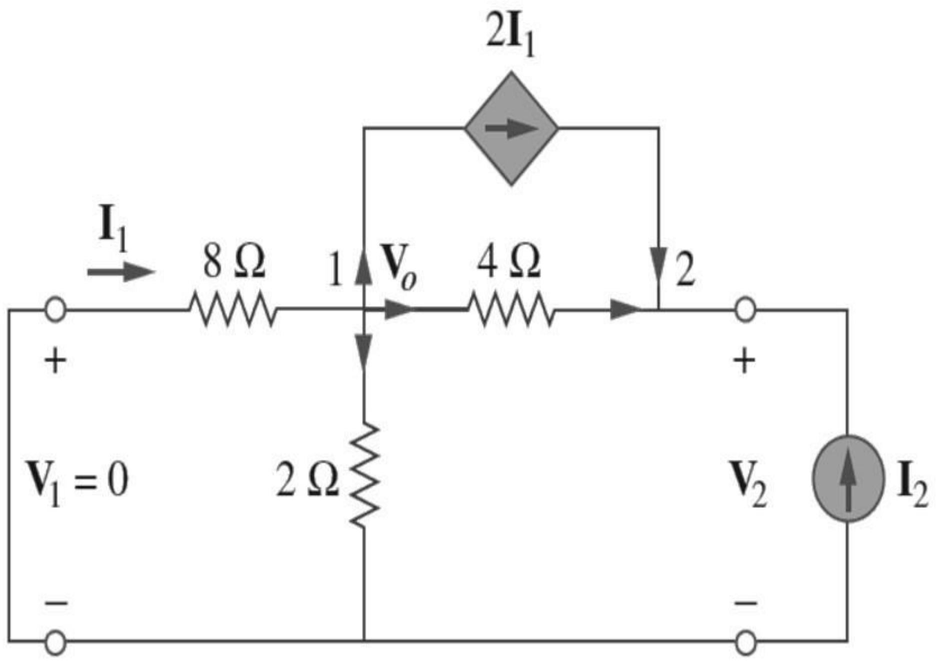
Admittance Parameters (Cont..)

- At node 2,

$$0 = \frac{\mathbf{V}_0 - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_0 - \frac{1}{4}(2.5\mathbf{V}_0) - \frac{2\mathbf{V}_0}{8}$$
$$= -0.625\mathbf{V}_0$$



- Hence,

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_0}{2.5\mathbf{V}_0} = 0.25\ S$$

- Here, $\mathbf{y}_{12} \neq \mathbf{y}_{21}$, hence the network is not reciprocal.


THANKS
FOR YOUR
ATTENTION