2025-01-10 L03: Block Diagrams

LTI
* Linearity
* Time invariance
* Examples of linearity
* Examples of time invariance

LT
* Definitions of Laplace transform

BD
* Why only arrows, gain blocks, and summers in block diagrams
* Gain block can only be for nonloading elements.
* Why initial conditions dropped in forming block diagrams
* Problem: Application to BD of PMDC(M)

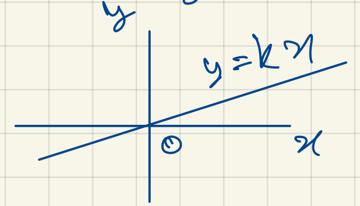
We already saw that the information content in the BD of the PMDC motor was much more than in the equations that gave that BD:

1) The BD shows what causes what causes what ...
2) BDs reveal natural feedback loops.
3) BDs easy to manipulate using BD algebra (to see in next lecture)

Linearity: Given $\xrightarrow{u} \boxed{f} \xrightarrow{y}$

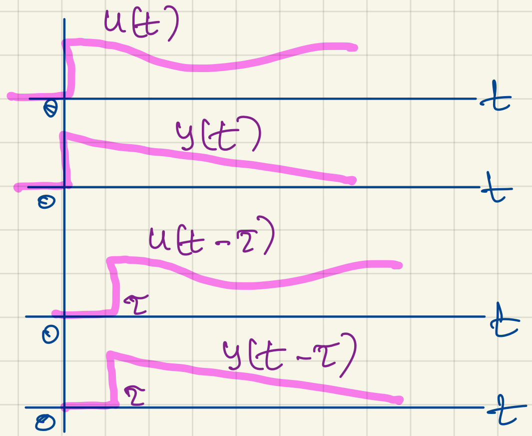
If $u_1 \longrightarrow y_1$ and $u_2 \longrightarrow y_2$, then if $\alpha_1 u_1 + \alpha_2 u_2 \longrightarrow \alpha_1 y_1 + \alpha_2 y_2$, then f is linear.

This concept of linearity is a generalization of the properties of a straight line through the origin.



Time invariance

For $\xrightarrow{u(t)} \boxed{f} \xrightarrow{y(t)}$, if $u(t-\tau) \longrightarrow y(t-\tau)$, then f is TI.



Examples for linearity: which of the foll. equations are linear?

$\dot{y}+y=u \qquad u_1 \to y_1 : \dot{y}_1+y_1=u_1$

$u_2 \to y_2 : \dot{y}_2+y_2=u_2$

$(\alpha_1\dot{y}_1+\alpha_2\dot{y}_2)+(\alpha_1 y_1+\alpha_2 y_2)=\alpha_1 u_1+\alpha_2 u_2$

$\Rightarrow \alpha_1 u_1+\alpha_2 u_2 \longrightarrow \alpha_1 y_1+\alpha_2 y_2$

$\therefore$ Yes, $\dot{y}+y=u$ is linear.

$\dot{y}+y^2=u \qquad u_1 \to y_1 : \dot{y}_1+y_1^2=u_1$

$u_2 \to y_2 : \dot{y}_2+y_2^2=u_2$

$(\alpha_1\dot{y}_1+\alpha_2\dot{y}_2)+(\alpha_1 y_1^2+\alpha_2 y_2^2)=\alpha_1 u_1+\alpha_2 u_2$

$\not\Leftarrow (\alpha_1\dot{y}_1+\alpha_2\dot{y}_2)+(\alpha_1 y_1+\alpha_2 y_2)^2=\alpha_1 u_1+\alpha_2 u_2$

$\therefore$ Not linear.

$\dot{y}+ty=u$ HW.

Examples for time invariance

$\dot{y}+y=u \qquad \dot{y}+y^2=u \qquad \dot{y}+ty=u$

Want a test that involves checking whether the definition $(u(t-\tau)\to y(t-\tau))$ is satisfied.

At this point, I have not come across such a test. If you have, pl. share with me. HW.

Instead, what is said is:

those of the above equations that have constant coefficients are TI.

Definition of Laplace Transform that is used in control theory.

$$F(s) := \mathcal{L}\{f(t)\} \triangleq \int_0^\infty f(t)\, e^{-st}\, dt$$

$$F_+(s) := \mathcal{L}_+\{f(t)\} \triangleq \int_{0+}^\infty f(t)\, e^{-st}\, dt$$

$$F_-(s) := \mathcal{L}_-\{f(t)\} \triangleq \int_{0-}^\infty f(t)\, e^{-st}\, dt$$

Normally, we use the first defn.

When we have action at the origin, then we use the third definition.

Almost never the second def.

Block diagrams

- The block diagrams that we see in this course are for transfer functions, which are in turn created for LTI algebro-differential equations:
  constant coefficient ODEs
  +
  Linear algebraic equations.

- The TFs that we see in this course have only 2 forms:

$$V(s) \to \boxed{G(s)} \xrightarrow{Y(s)} \qquad G(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

$$G(s) = e^{-t_d s}.$$

- The arrows, adders, and gain blocks arise naturally in working with const-coeff. algebro-diff. eq-ns.

Why ICs dropped in forming BDs

$$V(t) - E(t) = L\frac{di}{dt} + Ri.$$

Take LT of both sides:

$$\underbrace{V(s) - E(s)}_{\text{One driver}} = LsI(s) - \underbrace{Li(0)}_{\text{Second driver}} + RI(s)$$

of current.

$$LsI(s) + RI(s) = \underbrace{V(s) - E(s)}_{F(s)} + Li(0)$$

$$I(s) = \frac{F(s)}{sL + R} + \frac{L}{sL+R} i(0)$$

$$I(s) = \frac{1}{sL+R} F(s) + \frac{L}{sL+R} \mathcal{L}_{-}\{i(0)\delta(t)\}$$

Post-lecture Discussion

Q: ~~Does~~ linearity permit F & $\frac{1}{sL+R}$ to be functions of the same argument?

Q: I didn't bring out the aspect of linearity that it allows treating the effect of one stimulus at a time and then adding up these effects or a weighted combination of these effects.

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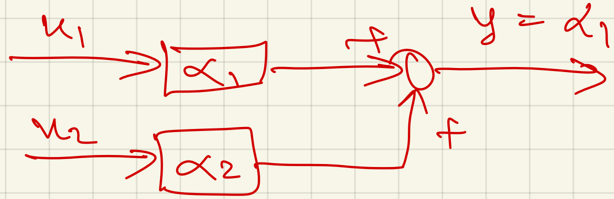
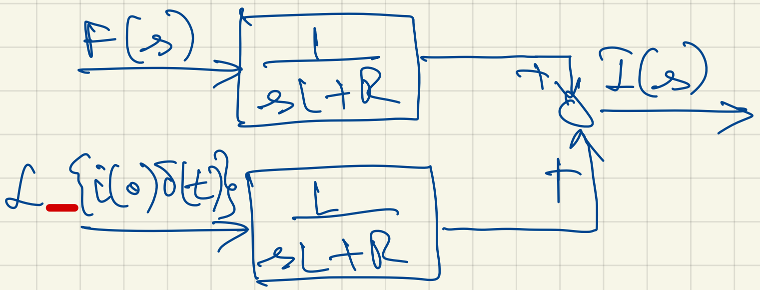
$$y = \alpha_1 u_1 + \alpha_2 u_2$$



$$I(s) = \frac{1}{sL+R}F(s) + \frac{L}{sL+R}\mathcal{L}_{-}\{i(0)\delta(t)\}$$



Krish Jain showed the BD of the generator.