Lecture-20

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

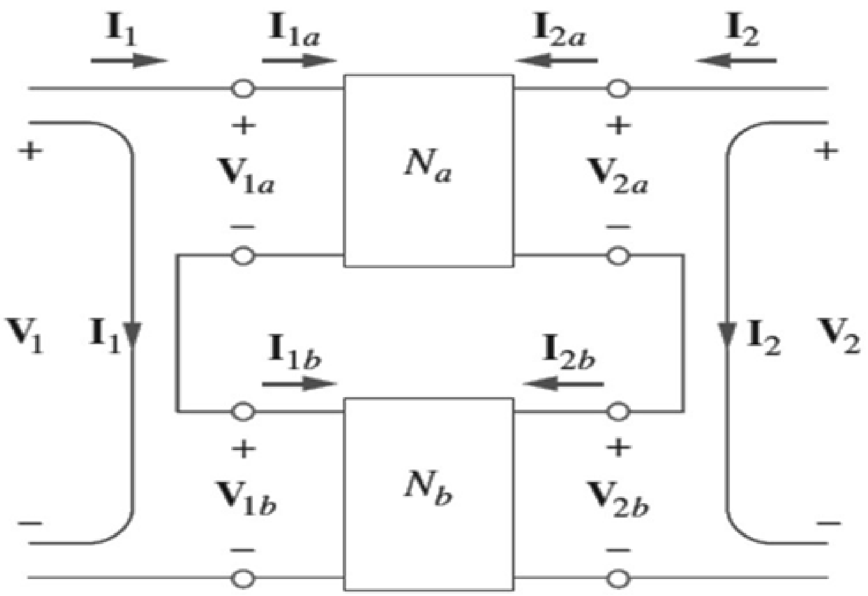
- Interconnection of Networks.
- Network Theorems in AC circuits.

Interconnection of Networks

- A large, complex network may be divided into sub-networks for the purposes of analysis and design.
- The sub-networks can be modeled as two-port networks, interconnected to form the original network.
- The two-port networks may therefore be considered as building blocks that can be interconnected to form a complex network.
- The interconnection can be in series, in parallel, or in cascade.
- Although the interconnected network can be described by any of the six parameter sets, a certain set of parameters may have a definite advantage.
- For example, when the networks are in series, their individual *z* parameters add up to give the *z* parameters of the larger network.

Interconnection of Networks (Cont…)

- When they are in parallel, their individual *y* parameters add up to give the *y* parameters of the larger network.
- When they are cascaded, their individual transmission parameters can be multiplied together to get the transmission parameters of the larger network.
- Consider the series connected network shown in the figure below.



Interconnection of Networks (Cont…)

- The networks are regarded to be in series as their input currents are the same and their voltages add.

  - For network N_a,

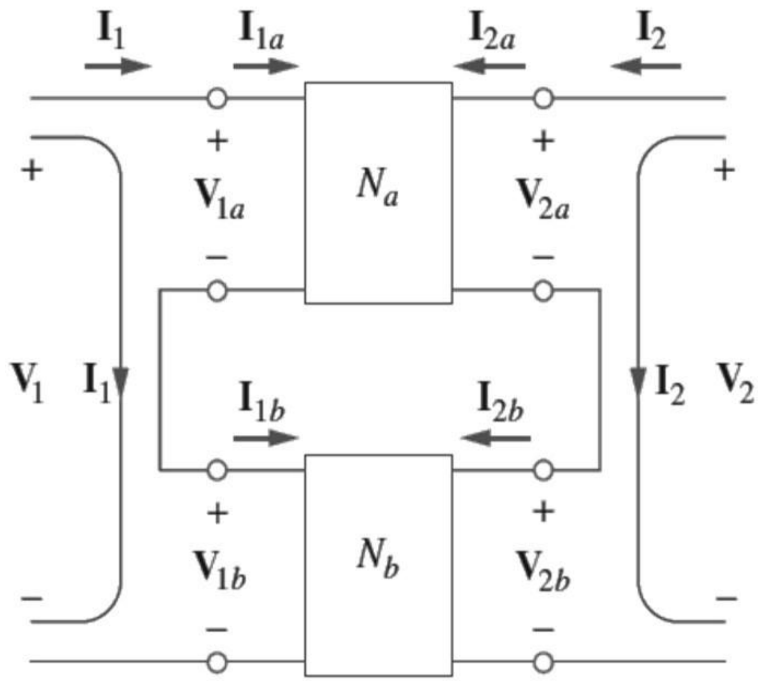
$$\mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a}$$

$$\mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a}$$

  - and for network N_b,

$$\mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b}$$

$$\mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b}$$



Interconnection of Networks (Cont…)

- From the figure it can be observed that, when circuits are in series,

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b},\ \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$$

- and that

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a}+\mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a}+\mathbf{z}_{12b})\mathbf{I}_2$$

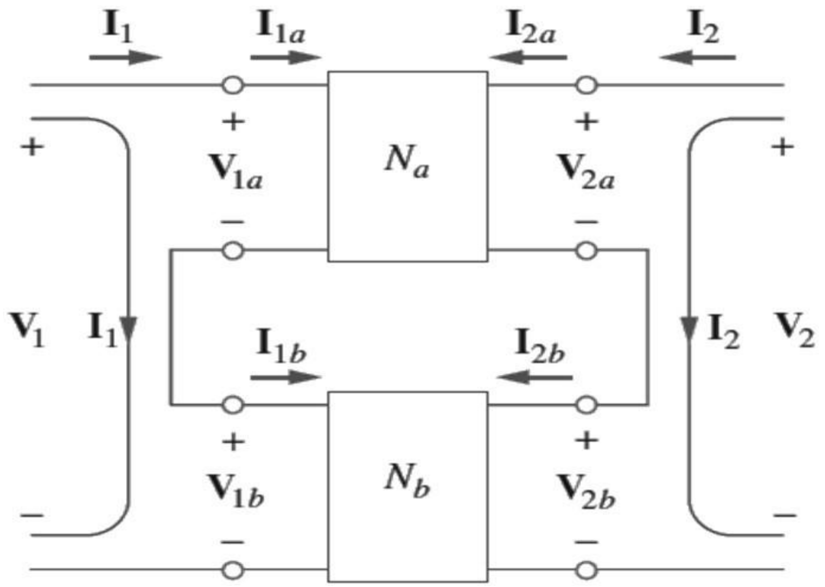
$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a}+\mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a}+\mathbf{z}_{22b})\mathbf{I}_2$$

- The z parameters of the overall network are,

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix}$$

or

$$[\mathrm{z}] = [\mathrm{z}_a] + [\mathrm{z}_b]$$

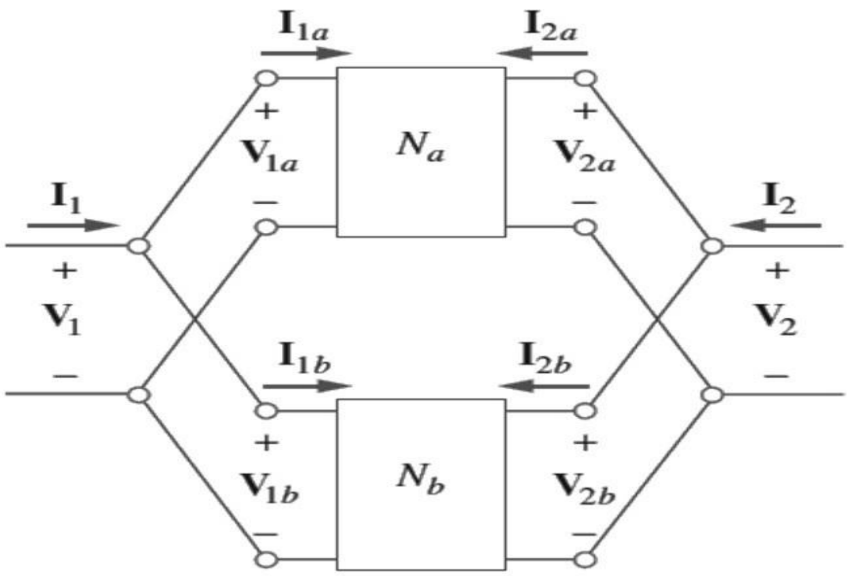


Interconnection of Networks (Cont…)

- Thus, the above derivation proves that the z parameters for the overall network are the sum of the z parameters for the individual networks.
- This can be extended to n networks in series.
- If two networks represented using the **[h]** parameters are connected in series, its equivalent can be computed by first converting the **[h]** parameters to **[z]** by comparing the equivalent circuit of both the models.
- Once the overall parameters of the circuit for the series network has been evaluated the **[z]** parameter of the equivalent circuit can then be converted back to the **[h]** parameters.
- This is applicable for all interconnections.

Interconnection of Networks (Cont…)

- Consider the parallel connected network shown in the figure below.



- Two two-port networks are in parallel when their port voltages are equal and the port currents of the larger network are the sums of the individual port currents.
- Here, each circuit must have a common reference and when the networks are connected together, they must all have their common references tied together.

Interconnection of Networks (Cont…)

- For network N_a,

$$\mathbf{I}_{1a} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{2a} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a}$$

- and for network N_b,

$$\mathbf{I}_{1b} = \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b}$$

$$\mathbf{I}_{2b} = \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b}$$

- From the figure it can be observed that,

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}, \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b}$$

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

Interconnection of Networks (Cont…)

- Using the above relations we get,

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} = (\mathbf{y}_{11a}+\mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a}+\mathbf{y}_{12b})\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} = (\mathbf{y}_{21a}+\mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a}+\mathbf{y}_{22b})\mathbf{V}_2$$

- The *y* parameters of the overall network are,

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix}$$

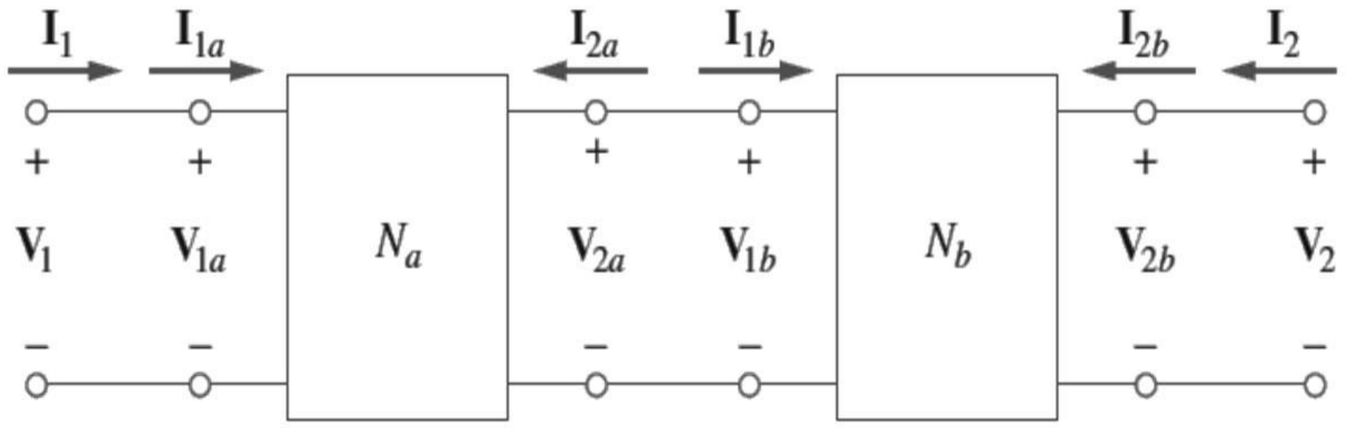
or

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]$$

- Thus, *y* parameters of the overall network are the sum of the *y* parameters of the individual networks.
- The result can be extended to *n* two-port networks in parallel.

Interconnection of Networks (Cont…)

- Consider the cascaded network shown in the figure below.

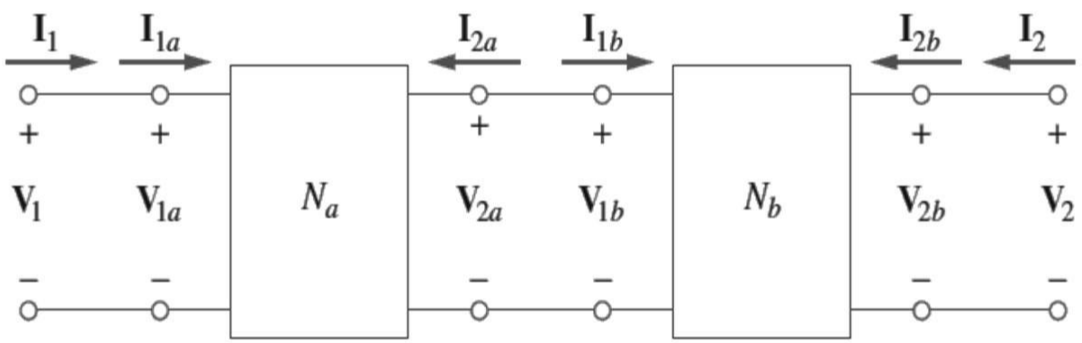


- Two networks are said to be *cascaded* when the output of one is the input of the other.

Interconnection of Networks (Cont…)

- For network N_a,

$$\begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix}$$



- and for network N_b,

$$\begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2b} \end{bmatrix}$$

- From the figure it can be observed that,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2b} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Introduction

- The concept of phasors, we discussed in the first module, finds application in AC circuit analysis.
- From the previous discussions we know that Ohm's law and Kirchhoff's laws can be applied to AC circuits as well.
- In this module, we will discuss how the theorems, discussed for DC circuits, can be used to analyze AC circuits.
- The techniques discussed previously are equally valid for AC circuits as well.
- This will be illustrated through various examples in the upcoming lectures.

❑ Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.

2. Solve the problem using circuit theorems.

3. Transform the resulting phasor to the time domain.

- Step 1 is not necessary if the problem is already specified in frequency domain.

- Step 2 is performed in a similar manner to the DC circuits except that complex numbers are involved.

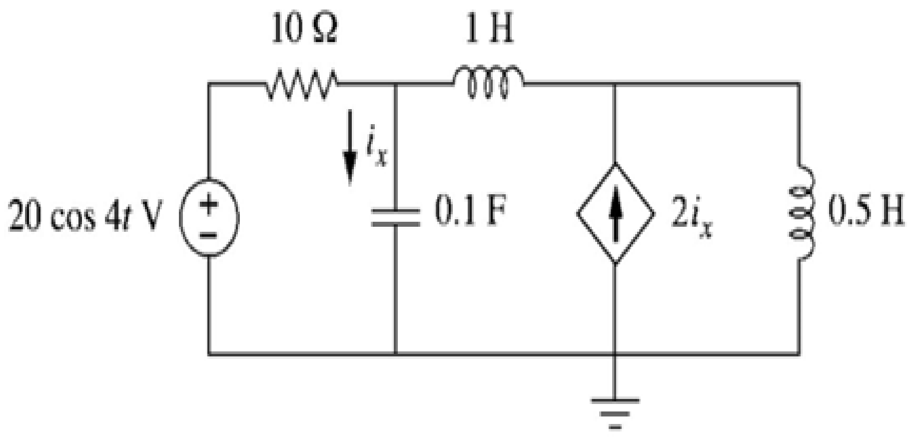
- Complex numbers can be easily handled using the techniques discussed previously.

❑ Nodal and Mesh Analysis:

- The basis of nodal analysis is Kirchhoff ’s current law (KCL) and the basis for mesh analysis is the Kirchhoff’s voltage law (KVL).
- Since both KCL and KVL are valid for complex numbers also, nodal and mesh analysis can be used to analyze AC circuits.
- The following examples illustrate circuit analysis using nodal and mesh analysis.

❑ Example:

- Find i_x in the circuit using nodal analysis?



Solution: We first convert the circuit to frequency domain,

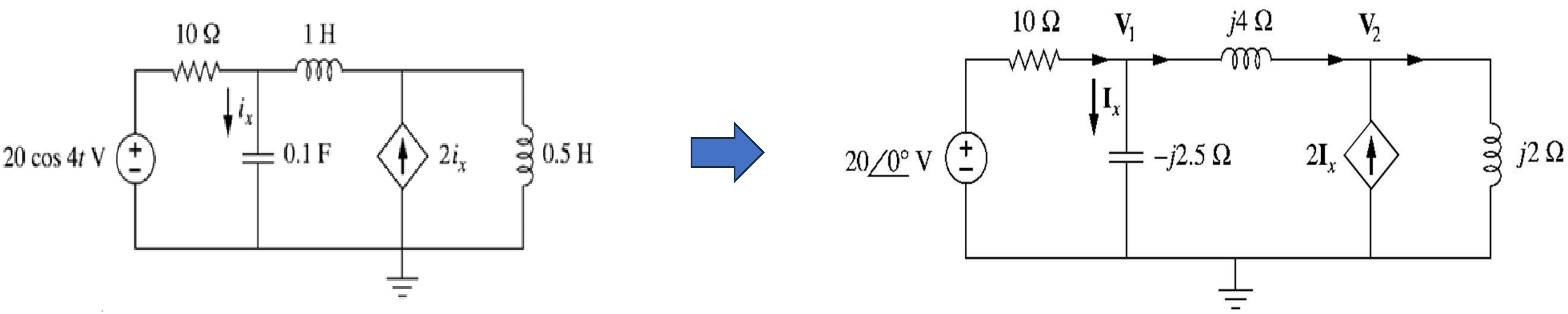
$$20 \cos 4t \Rightarrow 20\angle 0°, \ \ \omega = 4 \text{ rad/s}$$

$$1\text{H} \Rightarrow j\omega L = j4$$

$$0.5\text{H} \Rightarrow j\omega L = j2$$

$$0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j2.5$$

Thus, the frequency domain equivalent circuit is as shown below.



Applying KCL at node 1,

$$\frac{20-\mathbf{V}_1}{10}=\frac{\mathbf{V}_1}{-\mathrm{j}2.5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{\mathrm{j}4}$$

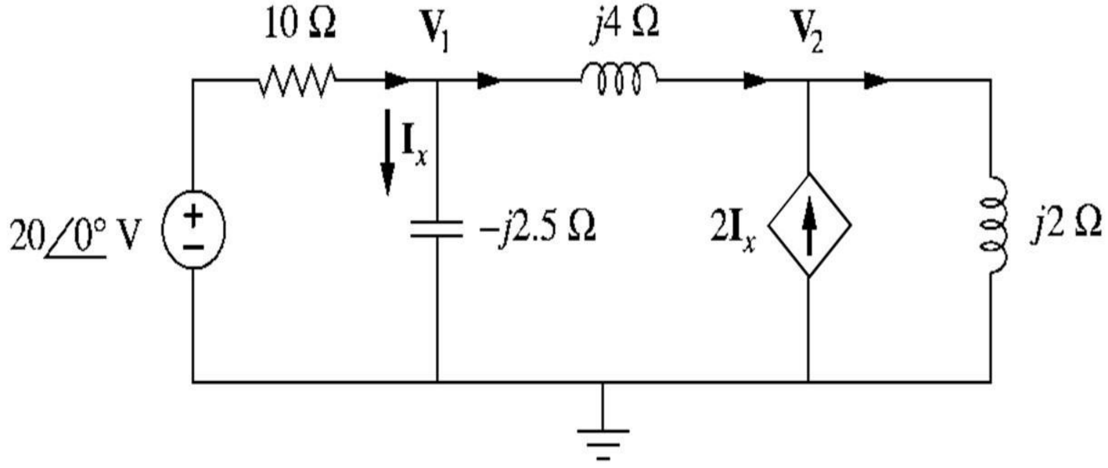
or

$$\mathbf{V}_1(1+j1.5)+j2.5\mathbf{V}_2=20$$

Applying KCL at node 2,

$$\frac{\mathbf{V}_2}{j2} = 2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{j}4}$$

But $\mathbf{I}_x = \mathbf{V}_1/{-j2.5}$. Substituting this gives,



$$\frac{\mathbf{V}_2}{j2} = \frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{j}4}$$

By simplifying, we get,

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

The nodal equations can be expressed in matrix form as,

$$\begin{bmatrix} 20 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

We obtain the determinants as,

$$\Delta = \begin{vmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5}$$
$$= 18.97\angle 18.43°\,\mathbf{V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5}$$
$$= 13.91\angle 198.3°\,\text{V}$$

The current $\mathbf{I}_x$ is given by,

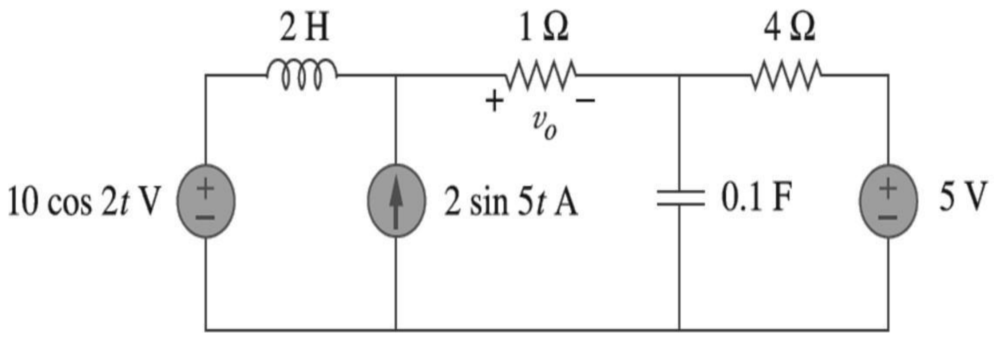
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43°}{2.5\angle -90°} = 7.59\angle 108.4° \Rightarrow i_x$$
$$= 7.59\cos(4t + 108.4°)\text{A}$$

Superposition Theorem:

- Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits.
- The problem becomes interesting if the circuit has sources operating at different frequencies.
- Since the impedances are dependent on frequency there will be separate circuits corresponding to each frequency domain.
- The total response is then obtained by adding the individual responses in time domain.
- Please note it is incorrect to add the responses in the phasor form or frequency domain

❑ Example:

- Find v_0 using superposition theorem?

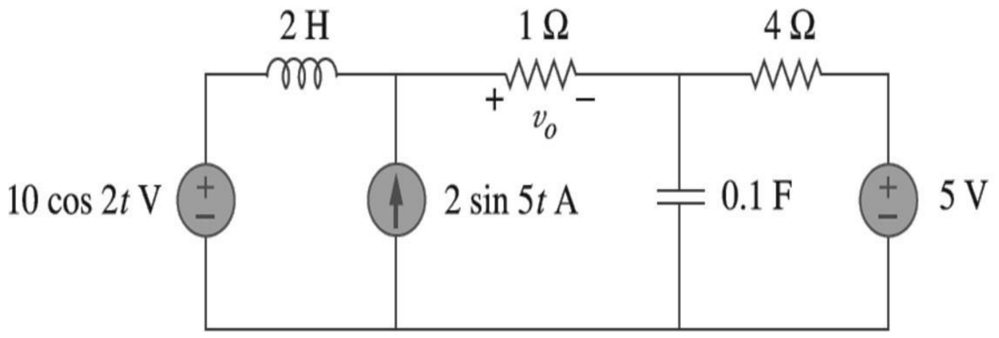


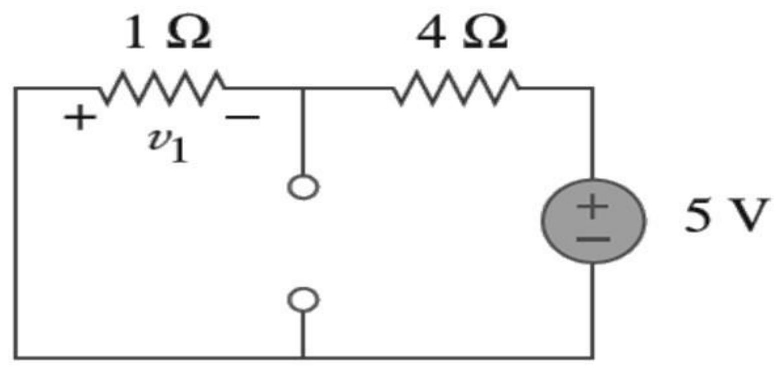
Solution: Since the circuit operates at 3 different frequencies, we can use superposition theorem to break the problem into 3 single frequency problems and express v_0 as,

$$v_0 = v_1 + v_2 + v_3$$

Here, v_1 is due to the 5 V dc source, v_2 is due to the $10\cos 2t$ voltage source, and v_3 is due to the $2\sin 5t$ current source.

- To find v_1, we set all sources except the 5 V DC source to zero.
- Please recall that for DC sources a capacitor acts as an open circuit and an inductor acts as a short circuit.
- The circuit can therefore, be represented as,





  - Hence, by voltage division,

$$-v_1 = \frac{1}{1+4}(5) = 1\text{V}$$

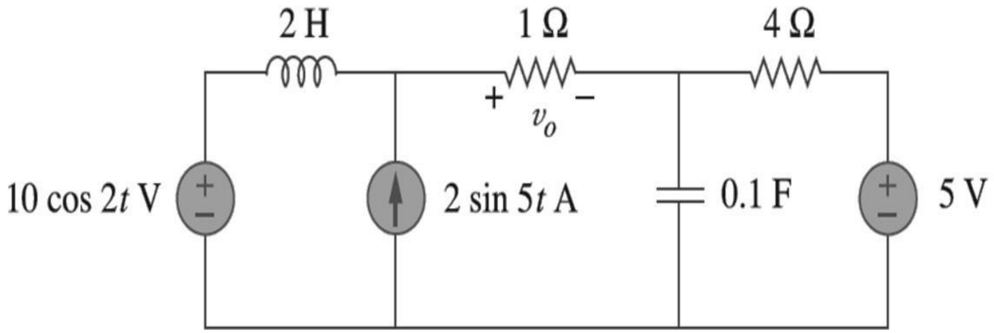
- To find v_2, we set both the 5 V DC source and $2 \sin 5t$ current source to zero.
- The circuit is then transformed to frequency domain, as,

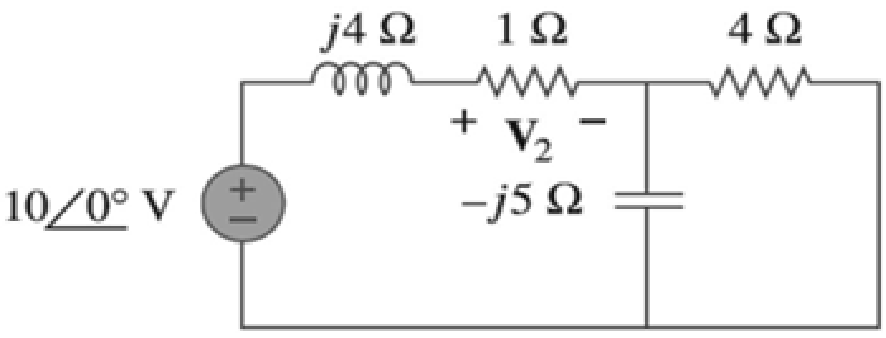
$$10 \cos 2t \Rightarrow 10\angle 0°,\ \omega = 2 \text{ rad/s}$$

$$2\text{H} \Rightarrow j\omega L = j4$$

$$0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j5$$

- The equivalent circuit is as shown in the figure below.





- The effective impedance is

$$\mathbf{Z} = -j5 || 4 = 2.439 - j1.951$$

- Hence, by voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\angle 0)$$
$$= 2.498\angle - 30.79\mathbf{V}$$

- The voltage in time domain is represented as,

$$v_2 = 2.498 \cos(2t - 30.79°)\ \mathrm{V}$$

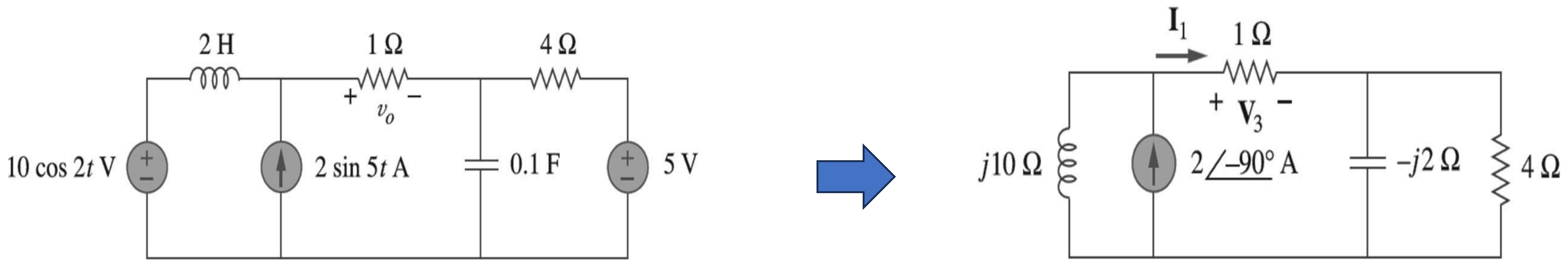
- To find v_3, we set both the $5\,\text{V}$ DC source and $10\cos 2t$ voltage source to zero.
- The circuit is then transformed to frequency domain, as,

$$2\sin 5t \Rightarrow 2\angle -90^\circ,\ \ \omega = 5\ \text{rad/s}$$

$$2\text{H} \Rightarrow j\omega L = j10$$

$$0.1\text{F} \Rightarrow \frac{1}{j\omega C} = -j2$$

- The equivalent circuit is as shown in the adjacent figure.



- The effective impedance is

$$\mathbf{Z}_1 = -j2||4 = 0.8 - j1.6$$

- Hence, by current division,

$$\mathbf{I}_1 = \frac{j10}{1 + j10 + \mathbf{Z}_1}(2\angle - 90)$$

$$\mathbf{V}_3 = \mathbf{I}_1 * 1 = 2.328\angle - 80 \text{ V}$$

- The voltage in time domain is represented as,

$$v_3 = 2.328\cos(5t - 80°) \text{ V} = 2.328\sin(5t + 10°) \text{ V}$$

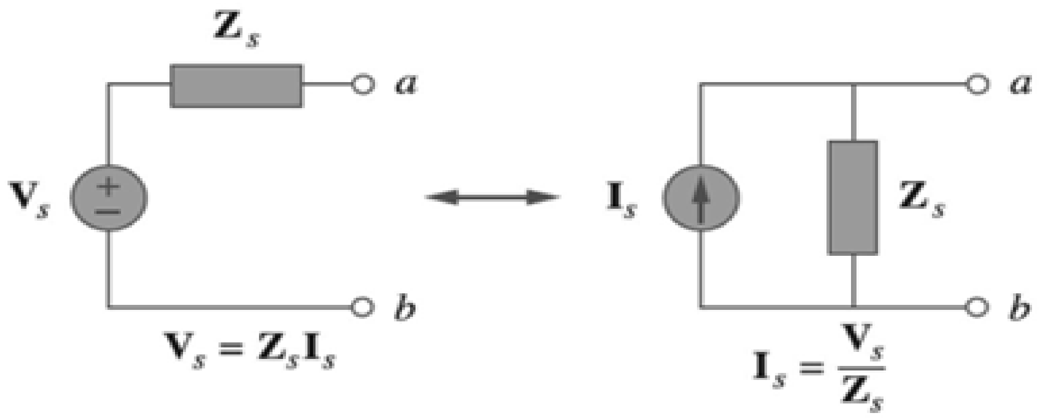
- Therefore,

$$v_0 = -1 + 2.498\cos(2t - 30.79°) + 2.328\sin(5t + 10°)\text{V}$$

❑ Source Transformation:

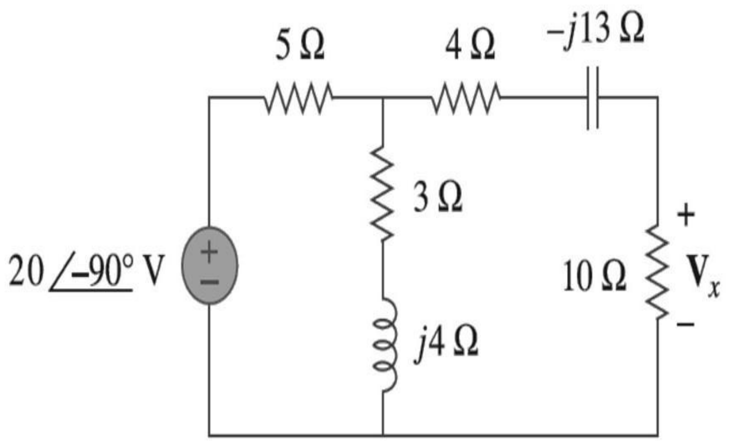
- Source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa.
- This is expressed as,

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_s \Leftrightarrow \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$



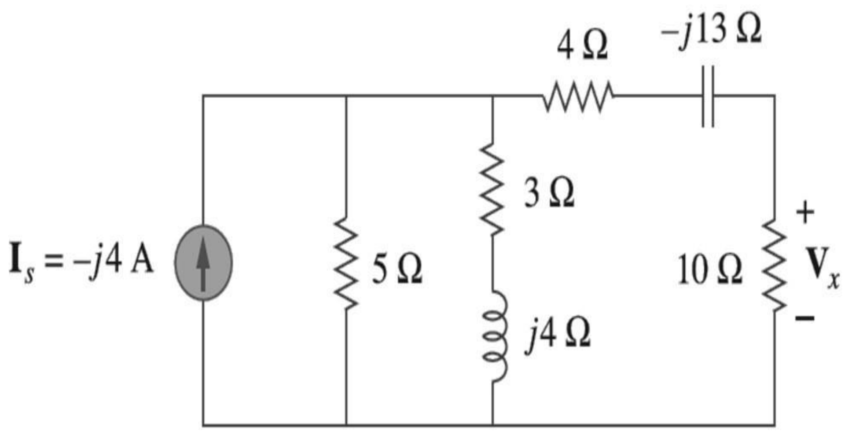
❑ Example:

- Find $\mathbf{V}_x$ using source transformation?



Solution: We transform the voltage source to a current source to obtain the circuit shown below. Here,

$$\mathbf{I}_s = \frac{20\angle-90^\circ}{5} = 4\angle-90^\circ$$



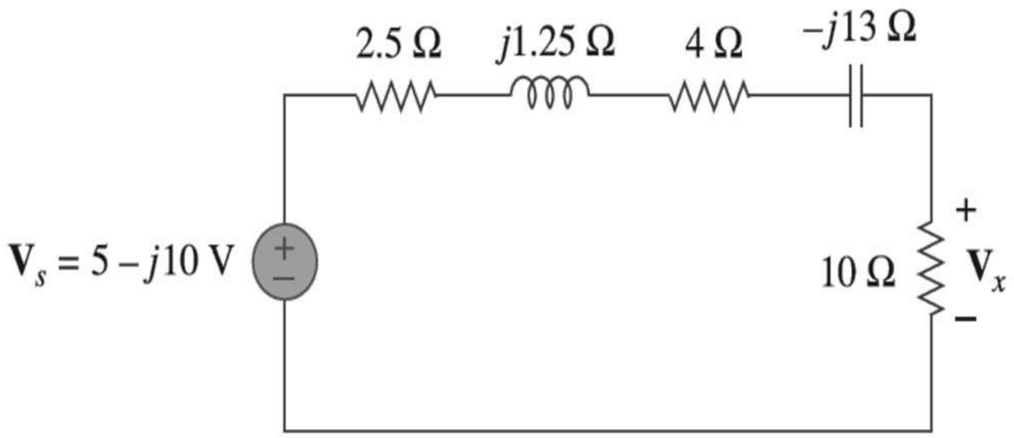
- The parallel combination 5 and $(3 + j4)$ impedances give

$$\mathbf{Z}_1 = 5||(3 + j4) = 2.5 + j1.25$$

- Converting the current source to voltage source, we get the following circuit.

- Here,

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_s = -j4(2.5 + j1.25)$$
$$= 5 - j10\mathbf{V}$$



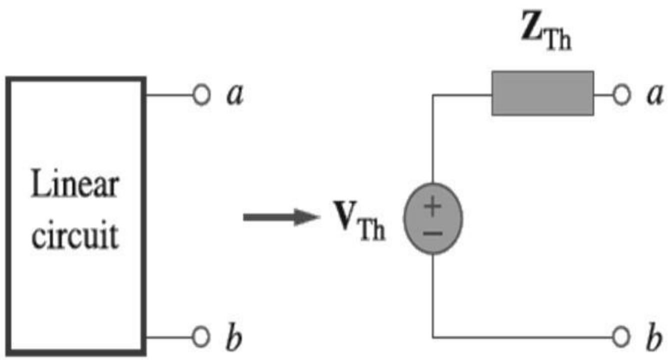
- By voltage division,

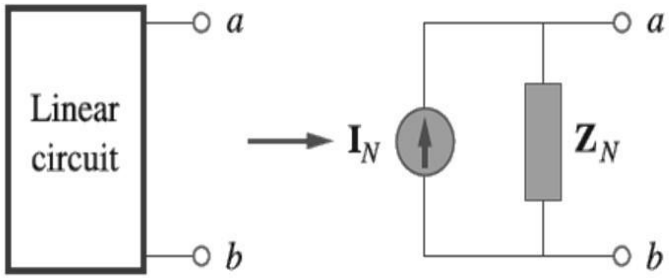
$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle - 28°\mathbf{V}$$

Thevenin and Norton Equivalent Circuit

- Thevenin's and Norton's theorems are applied to AC circuits in the same way as DC circuits.
- As in the case of superposition theorem, if there are sources operating at different frequencies there will be separate circuits corresponding to each frequency domain.
- The frequency domain equivalent circuits for Thevenin and Norton equivalent are are expressed as,

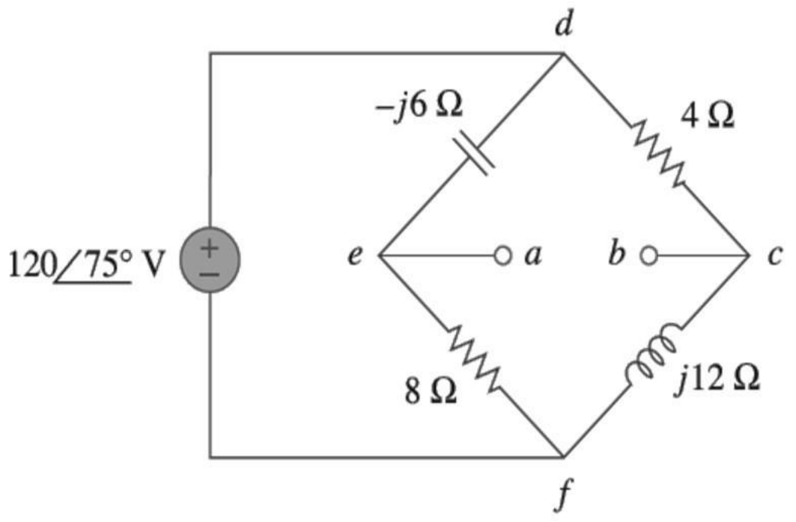
$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{N}}\mathbf{I}_{\mathrm{N}}, \qquad \mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{N}}$$





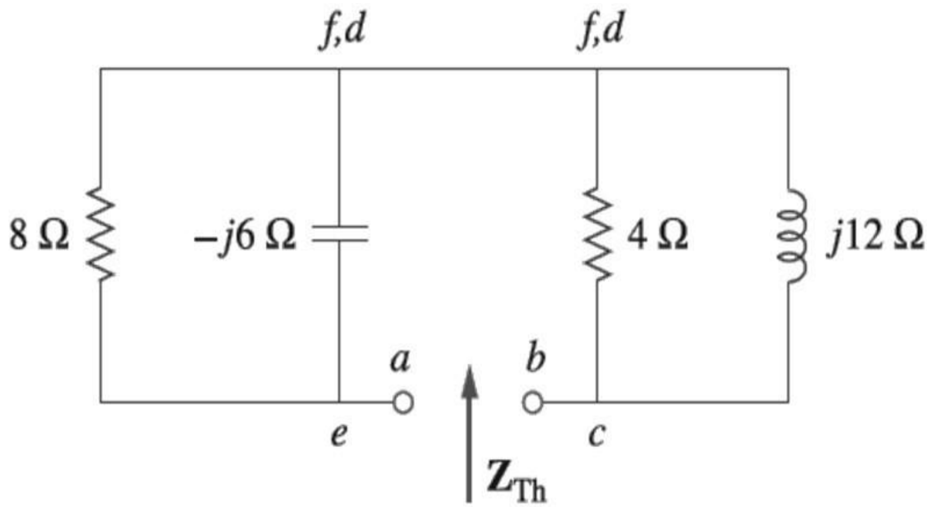
Example:

- Obtain Thevenin equivalent circuit across terminal a- b for the circuit given below?



Solution: $\mathbf{Z}_{\mathrm{Th}}$ is found by setting voltage source to 0.

The circuit then contains two parallel combinations of 8Ω and $-j6\Omega$ impedances in the first branch and 4Ω and $j12\Omega$ impedances in the second branch.



Then we have,

$$\mathbf{Z}_1 = -j6||8 = \frac{-j6(8)}{-j6+8} = 2.88 - j3.84\Omega\mathbf{Z}_2$$

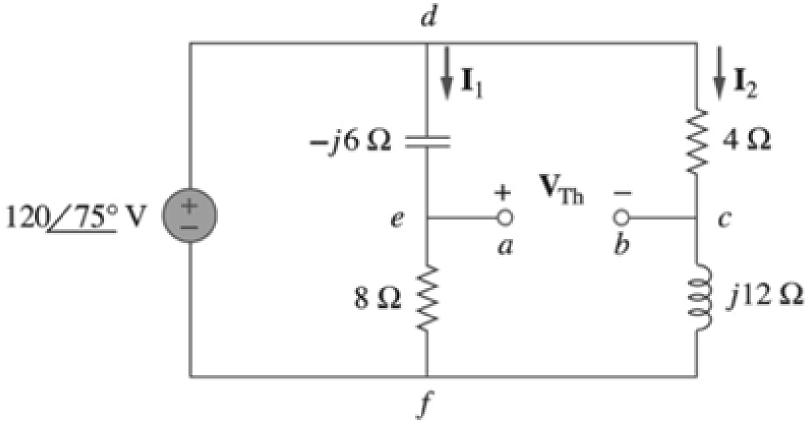
and

$$= j12||4 = \frac{j12(4)}{j12+4} = 3.6 + j1.2\Omega$$

Thevenin equivalent impedances is then given by,

$$\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\Omega$$

To find $\mathbf{V}_{\text{Th}}$ consider the below circuit.



Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as follows,

$$\mathbf{I}_1 = \frac{120\angle 75}{8 - j6}\,\mathbf{A}, \quad \mathbf{I}_2 = \frac{120\angle 75}{4 + j12}\,\mathbf{A}$$

Applying KVL around loop bcdeab,

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

Therefore,

$$\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1$$

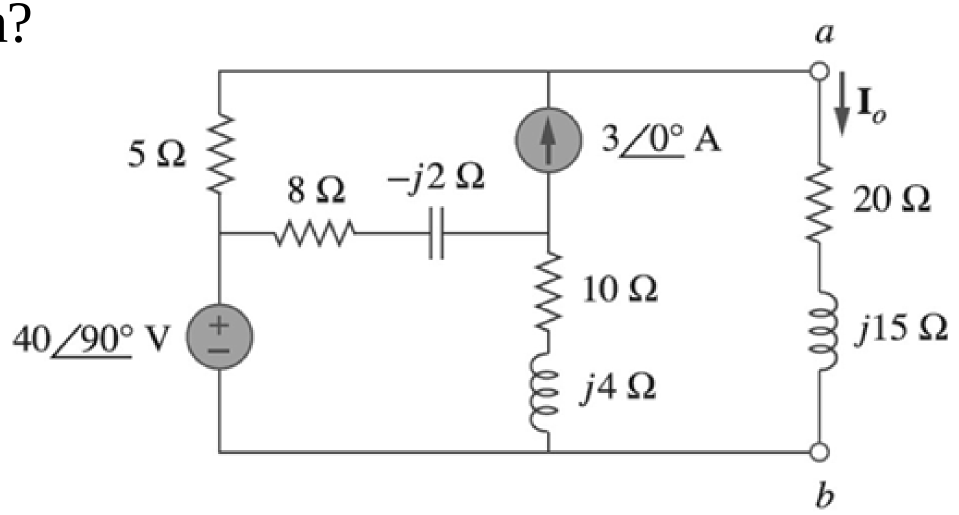
$$= \frac{480\angle 75}{4 + j12} + \frac{720\angle(75 + 90)}{8 - j6}$$

$$= 37.95\angle 3.43 + 72\angle 201.87$$

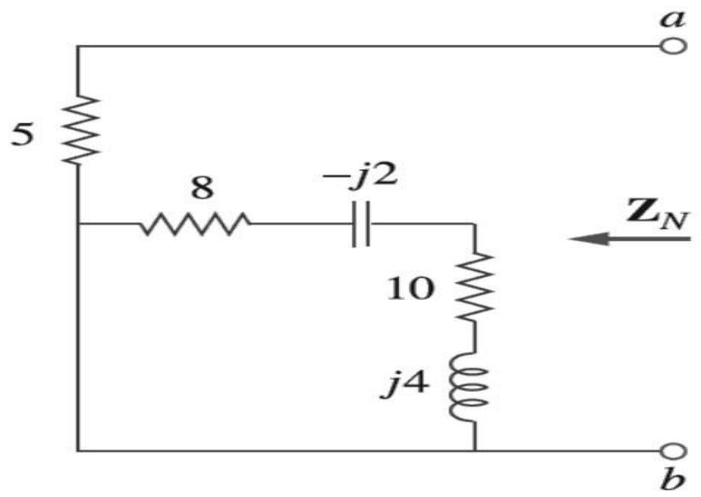
$$= 37.95\angle 220.31\text{V}$$

❑ Example:

- Find I_0 using Norton theorem?



Solution: To find Z_N we set the sources to 0 as shown in the below figure,

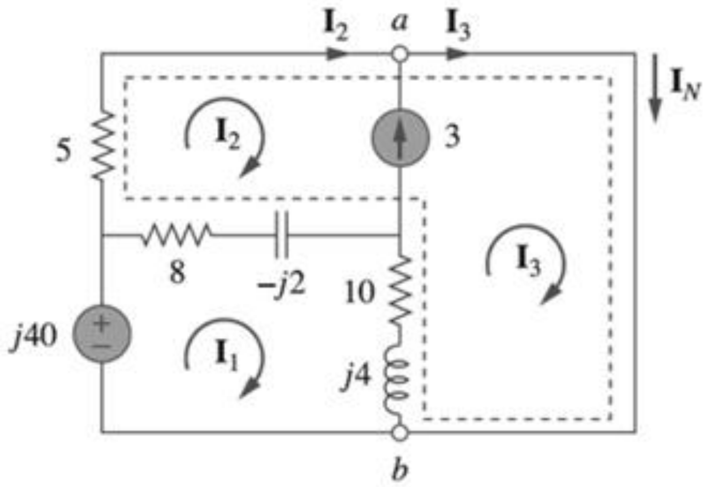


- As evident from the figure $(8 - j2)$ and $(10 + j4)$ impedances are short circuited.

- Therefore,

$$\mathbf{Z}_N = 5\ \Omega$$

- To get $\mathbf{I}_N$ we short-circuit terminals a-b as shown below and apply mesh analysis,



- It can be observed that meshes 2 and 3 form a supermesh because of the current source linking them.

- From mesh-1,

$$-j40 + \mathbf{I}_1(18 + j2) - \mathbf{I}_2(8 - j2) - \mathbf{I}_3(10 + j4) = 0$$

- From the supermsesh,

$$-\mathbf{I}_1(18 + j2) + \mathbf{I}_2(13 - j2) + \mathbf{I}_3(10 + j4) = 0$$

- At node a due to current source between meshes 2 and 3,

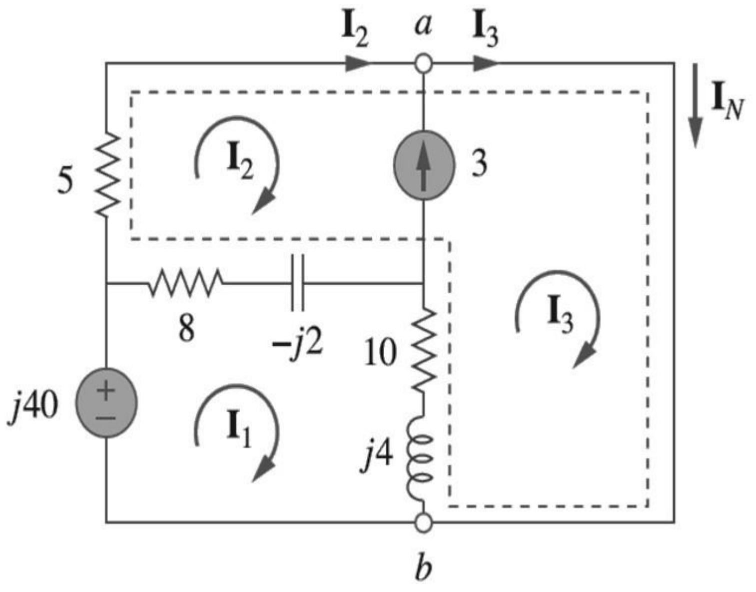
$$\mathbf{I}_3 = \mathbf{I}_2 + 3$$

- Adding the first two equations we get,

$$-j40 + 5\mathbf{I}_2 = 0$$

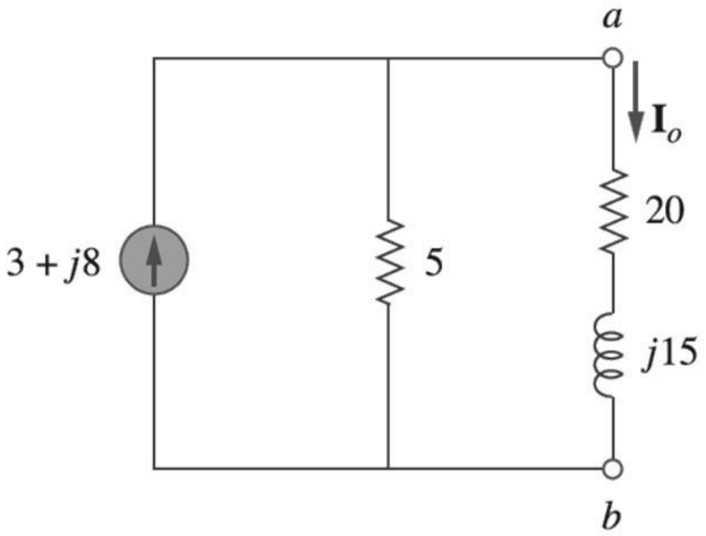
- Therefore,

$$\mathbf{I}_2 = j8$$



- But,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$



- The Norton current is ,

$$\mathbf{I}_N = \mathbf{I}_3 = 3 + j8$$

- The Norton equivalent circuit is as shown and by current division,

$$\mathbf{I}_0 = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48\mathbf{A}$$

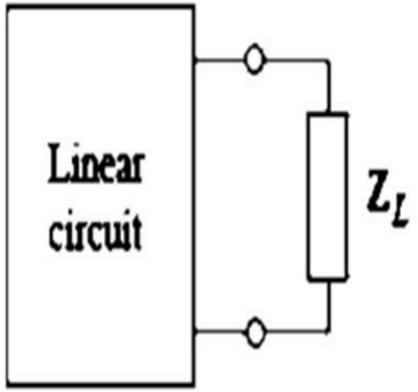
Maximum Power Transfer Theorem

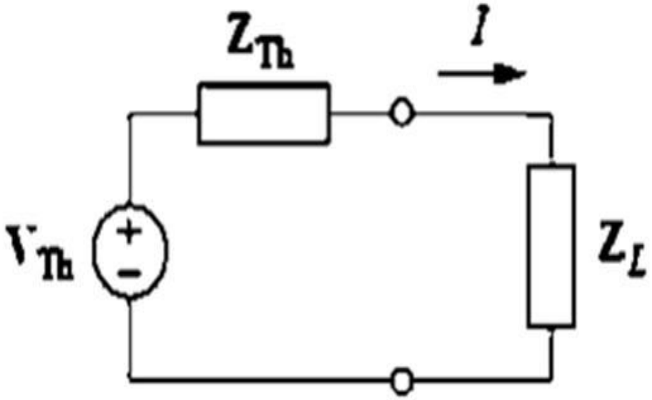
- Earlier, in DC circuit, we solved the problem of maximizing the power delivered by a power-supplying resistive network to a load R_L.

- Representing the circuit by it's Thevenin equivalent, we proved that the maximum power would be delivered to the load if the load resistance is equal to the Thevenin resistance, i.e. $R_L = R_{Th}$

- We now extend that result to AC circuits.

- Consider the circuit shown in figure below where an ac circuit is connected to a load Z_L and is represented by its Thevenin equivalent





- The load is usually represented by an impedance.
- In rectangular form, the Thevenin impedance Z_{Th} and the load impedance Z_L are –

$$Z_{Th} = R_{\mathrm{Th}} + jX_{\mathrm{Th}}$$

$$Z_L = R_{\mathrm{L}} + jX_{\mathrm{L}}$$

- The current through the load is –

$$I = \frac{V_{Th}}{Z_{Th} + ZL} = \frac{V_{Th}}{(R_{Th} + jXT_h) + (R_L + jXL)}$$

- The average power delivered to the load is given as –

$$P = \left(\frac{1}{2}\right) I^2 R_L = \frac{{V_{Th}}^2 \times \frac{R_L}{2}}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

- Our objective is to adjust the load parameters R_L and X_L so that P is maximum.
- To do this, we set $\partial P/\partial R_L$ and $\partial P/\partial X_L$ equal to zero.
- Using above Equation of P, we can obtain the values of $\partial P/\partial R_L$ and $\partial P/\partial X_L$

- Therefore,

$$\frac{\partial P}{\partial X_L} = \frac{{V_{Th}}^2 R_L (X_{Th} + X_L)}{[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{{V_{Th}}^2 [(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2 - 2R_L(R_{Th} + R_L)]}{2[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^2}$$

- Setting $\partial P/\partial X_{\mathrm{L}}$ equal to zero gives –

$$X_L = -X_{Th}$$

- Setting $\partial P/\partial R_{\mathrm{L}}$ equal to zero gives –

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} = R_{Th}$$

- Therefore, we can conclude that, for maximum average power transfer, Z_L must be selected so that $X_L = -X_{Th}$ and $R_L = R_{Th}$ i.e.,

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*$$

- So, for maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the Thevenin impedance Z_{Th}

- The maximum average power is given as –

$$P_{max} = \frac{V_{Th}^2}{8R_{Th}}$$

(Note: If V_{Th} is the RMS Value, then $P_{max} = \frac{V_{Th}^2}{4R_{Th}}$)

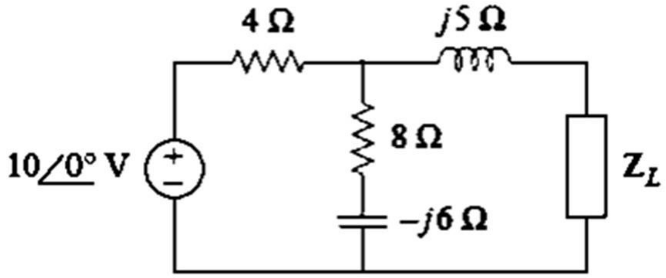
- If the load is purely real, the condition for maximum power transfer is obtained by setting $X_L = 0$; that is –

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2}$$

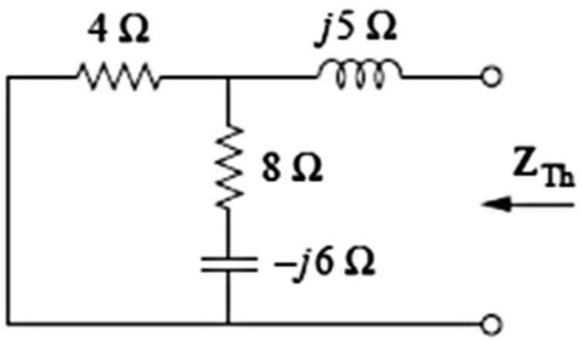
- This means that for maximum average power transfer to a purely resistive load, the load impedance (or resistance) is equal to the magnitude of the Thevenin impedance.

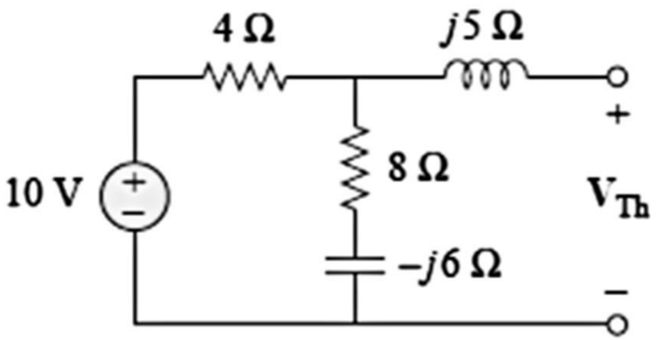
❑ Example:

- Determine the load impedance Z_L that maximizes the average power drawn from the circuit?



Solution: First we obtain the Thevenin equivalent at the load terminals,





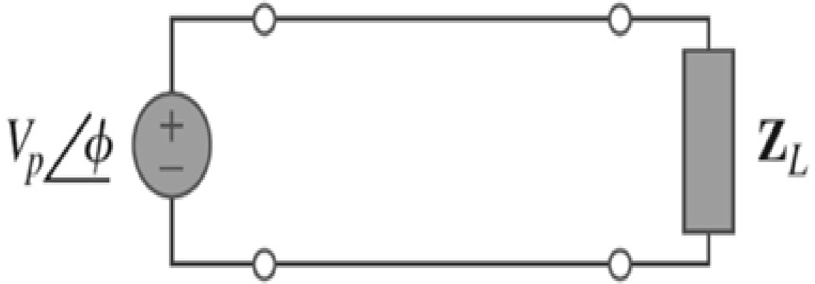
$$Z_{\mathrm{Th}} = 2.933 + j4.467\ \Omega \qquad V_{\mathrm{Th}} = 7.454\angle -10.3°\ V$$

So,

$$Z_L = Z_{Th}^{*} = 2.933 - j4.467\Omega$$

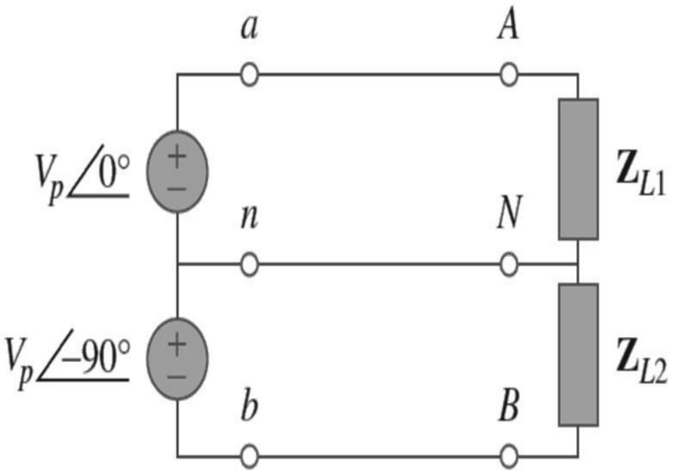
Three Phase Circuits

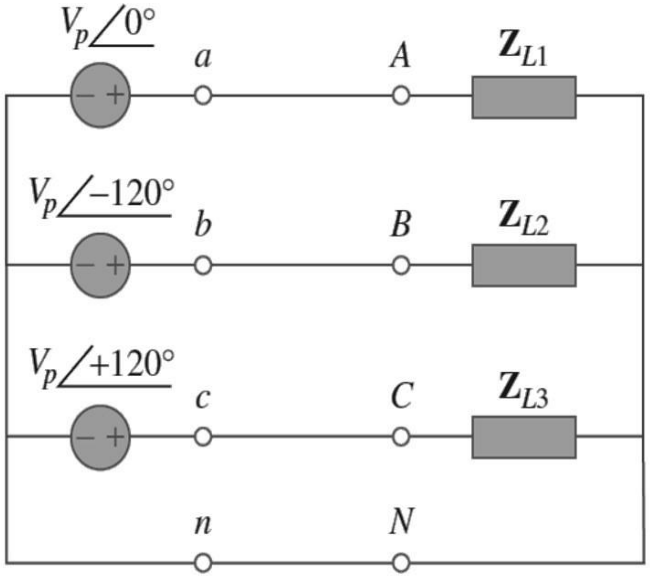
- So far in this course, we have been dealing with single-phase circuits.
- A single-phase AC power system consists of a generator connected through a pair of wires (a transmission line) to a load.
- This is illustrated in the below figure, where a single-phase two wire system is used.
- Here V_p is the rms magnitude of the source voltage and ϕ is the phase.



Three Phase Circuits (Cont…)

- Circuits or systems in which the ac sources operate at the same frequency but different phases are known as polyphase.
- The figure on the left below shows a two-phase three-wire system, and the figure on the right shows a three-phase four wire system.





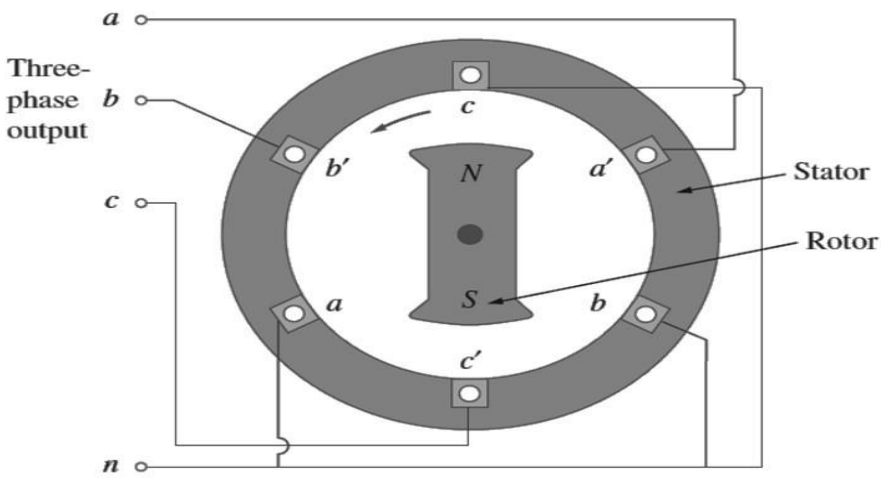
Three Phase Circuits (Cont…)

- As distinct from a single-phase system, a two-phase system is produced by a generator consisting of two coils placed perpendicular to each other so that the voltage generated by one lags the other by 90°.
- In the same way, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.
- Since the three-phase system is by far the most prevalent and most economical polyphase system, our discussion will mainly be on three-phase systems.
- Three-phase systems are important for at least three reasons.

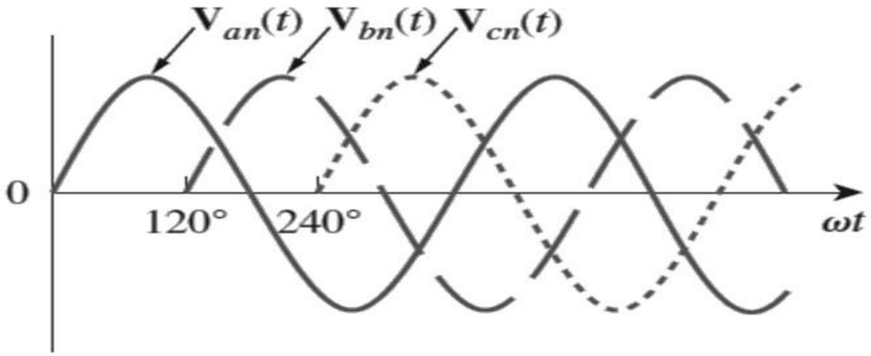
1. Nearly all electric power is generated and distributed in three-phase, at the operating frequency of 50 or 60 Hz.
2. The instantaneous power in a three-phase system can be constant (not pulsating), as will be discussed later.
3. The three-phase system is more economical than single phase system as the amount of wire required for a three-phase system is lesser than the amount of wire needed for an equivalent single-phase system.

Balanced Three-Phase Voltages

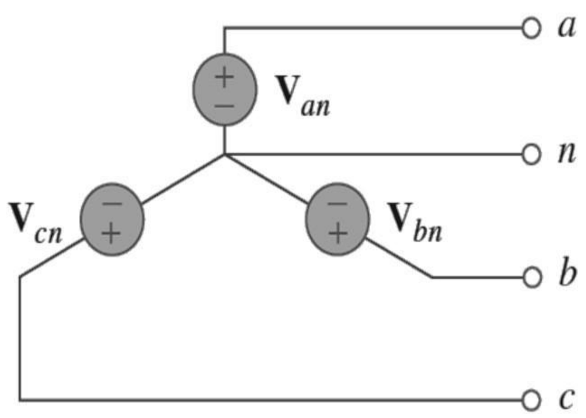
- Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross- sectional view is shown below.
- The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*).

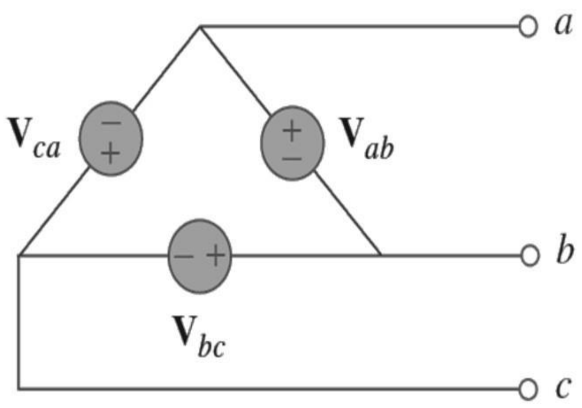


- Three separate windings or coils with terminals $a - a$', $b - b$', and $c - c$' and are physically placed $120°$ apart around the stator.
- Terminals a and a' for example, stand for one of the ends of coils going into and the other end coming out of the page.
- As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils.
- Because the coils are placed $120°$ apart, the induced voltages in the coils are equal in magnitude but out of phase by $120°$ as shown below.



- Since each coil can be regarded as a single-phase generator by itself, the three-phase generator can supply power to both single-phase and three-phase loads.
- A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines)
- A three-phase system is equivalent to three single-phase circuits.
- The voltage sources can be either wye (Y)-connected or delta (Δ)-connected as shown below.



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