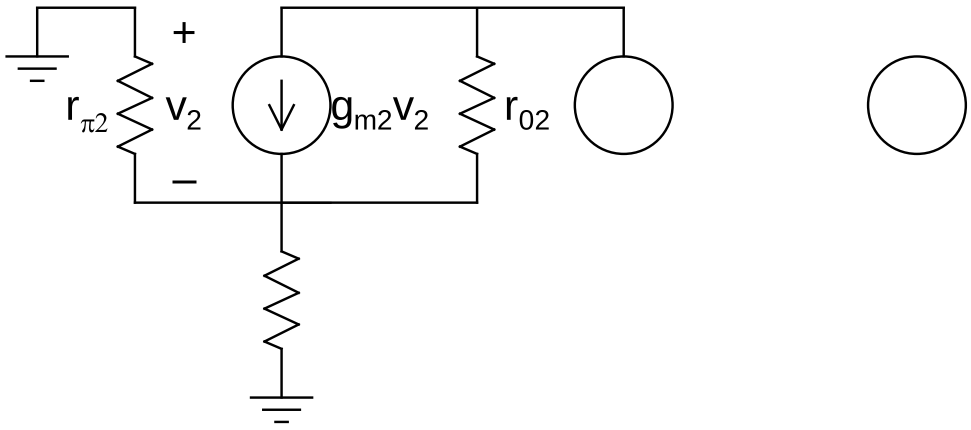
- ***Calculation of $R_0$*:**
  - ***Golden Rule can't be used since emitter of $Q_2$ is not grounded*** **(*$R_2$ present there*)**
  - ***Needs analysis***

    $\Rightarrow$ ***Leads to a module that is frequently encountered***
  - ***Base of $Q_1$-$Q_2$ at a fixed DC potential*** $\Rightarrow$ ***ac ground***



$$i_t = g_{m2} v_2 + \left(v_t + v_2\right)/r_{02}$$

$$= v_t/r_{02} + \left(g_{m2} + 1/r_{02}\right) v_2 \qquad v_t/r_{02} + g_{m2} v_2$$

$$v_2 = -i_t R_{eff}$$

$$\Rightarrow i_t = v_t/r_{02} - g_{m2} R_{eff} i_t$$

$$\Rightarrow R_0 = v_t/i_t = r_{02}(1 + g_{m2} R_{eff})$$

- This is a ***Golden Equation***, which would be ***used frequently***
  - ***Carefully note the topology that produces this result***
- ***Exercise***: ***Reverse $v_2$ and show that the expression for $R_0$ remains invariant***

- If $r_{\pi 2} >> R_2$, $R_0 = r_{02}(1 + g_{m2}R_2)$
- If $R_2 >> r_{\pi 2}$, $R_0 \approx \beta_2 r_{02}$ (since $\beta = g_m r_\pi >> 1$)
- ***Under the second condition, the circuit produces enormously large value of $R_0$ ~ 10s of MΩ or greater***
  - ***Almost approaches a constant current source***!
- ***It's good to check the relative values of $R_2$ and $r_{\pi 2}$ before using either of the equations***
- ***This circuit does not have any MOS counterpart for obvious reasons***!

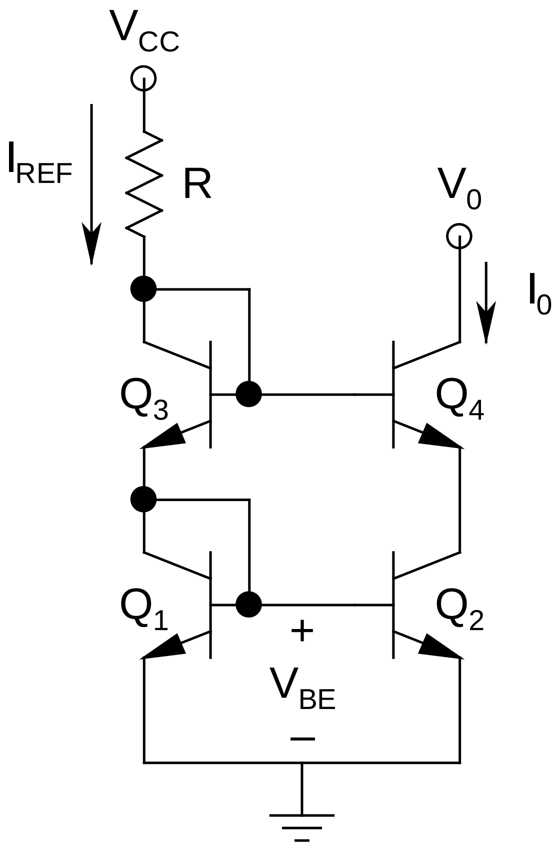
- ***Cascode Current Source***:
  - ***The best and most widely used***
  - ***Almost universal choice for biasing IC stages***
  - ***Produces extremely high $R_0$***
  - ***Original cascode needs higher values of $V_{0,min}$***
  - ***Modified cascode gets rid of this problem and pushes $V_{0,min}$ down***
  - ***The topology is basically two simple CMs stacked one upon the other***
  - ***Both npn and NMOS implementations exist***

- ***npn Cascode***:
  - ***All Qs are perfectly matched***
  - ***Neglecting $I_B$ and $V_A$***:

    $$I_0 = I_{REF} = (V_{CC} - 2V_{BE})/R$$

  - Show that if ***$I_B$ can't be neglected***, but ***all Qs have same β***:

    $$I_0 = \frac{I_{REF}}{1 + (4\beta + 2)/\beta^2}$$




  - Thus, ***β immunity is not that pronounced***

- ***All Qs operate with the same $V_{BE}$***

$$\Rightarrow V_{B1} = V_{B2} = V_{BE},\ V_{B3} = V_{B4} = 2V_{BE}$$

$$\Rightarrow V_{E4} = V_{C2} = V_{BE}$$

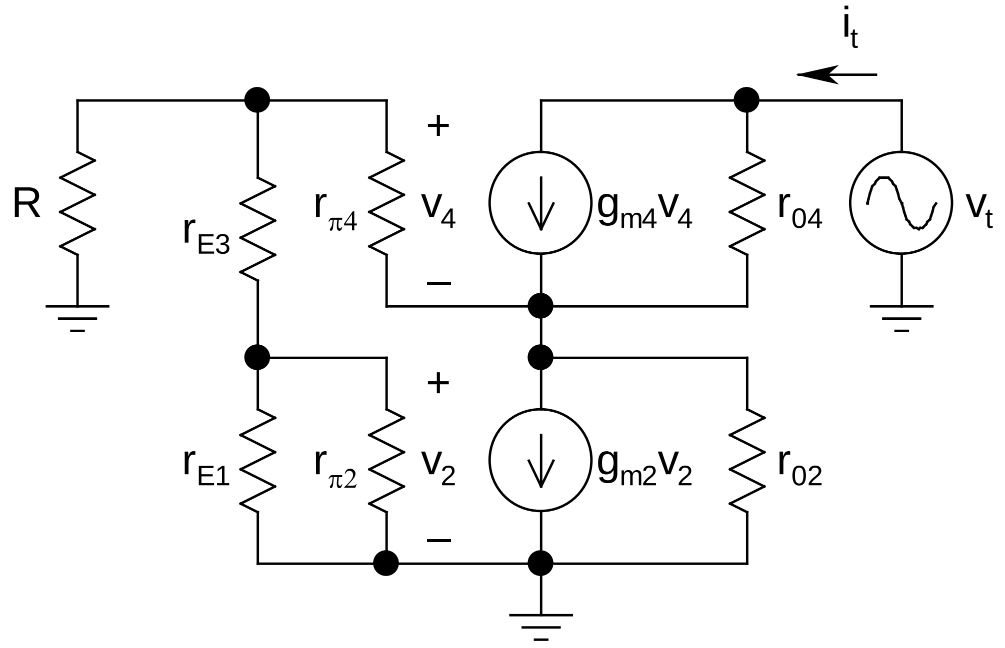
$$\Rightarrow V_{BE2} = V_{CE2}$$

$\Rightarrow$ ***$Q_2$ can never saturate, but $Q_4$ can***!

$$\Rightarrow V_{0,min} = V_{BE} + V_{CE4}(SS) = 0.7 + 0.2 = 0.9\ V$$

- ***The output voltage swing is sacrificed quite a bit***!
- ***However, the main advantage of this circuit is enormously large $R_0$***

## ➢ ***Calculation of $R_0$:***

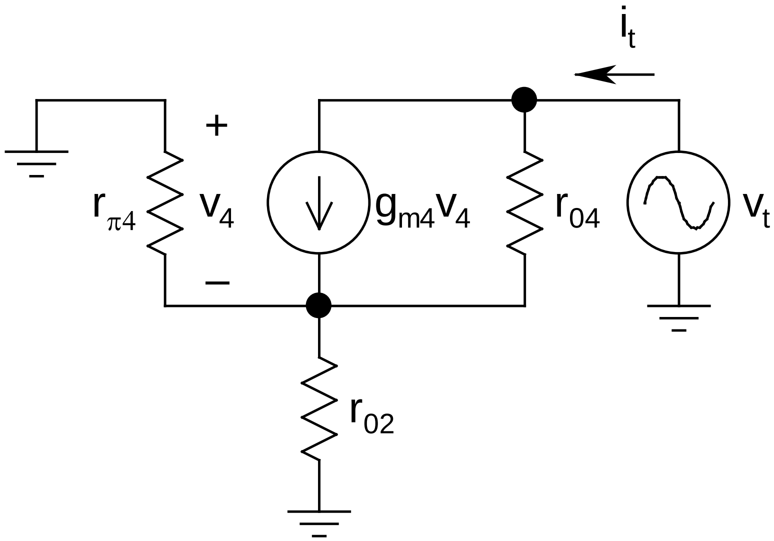



- ***$Q_1$ and $Q_3$ diode-connected*** $\Rightarrow$ $r_{E1}$ and $r_{E3}$

- Note that to a ***first-order estimate***, ***bases of $Q_1$-$Q_2$ and $Q_3$-$Q_4$*** can be considered to be at a ***fixed DC potential***, and thus, ***ac ground***

  $\Rightarrow v_2 = 0 \Rightarrow g_{m2}v_2 = 0$

  $\Rightarrow$ Leads to the ***simplified equivalent*** (looks familiar?)




- ***By inspection***:

  $R_0 \approx r_{o4}(1 + g_{m4}r_{\pi4})$

  $\approx \beta_4 r_{04}$

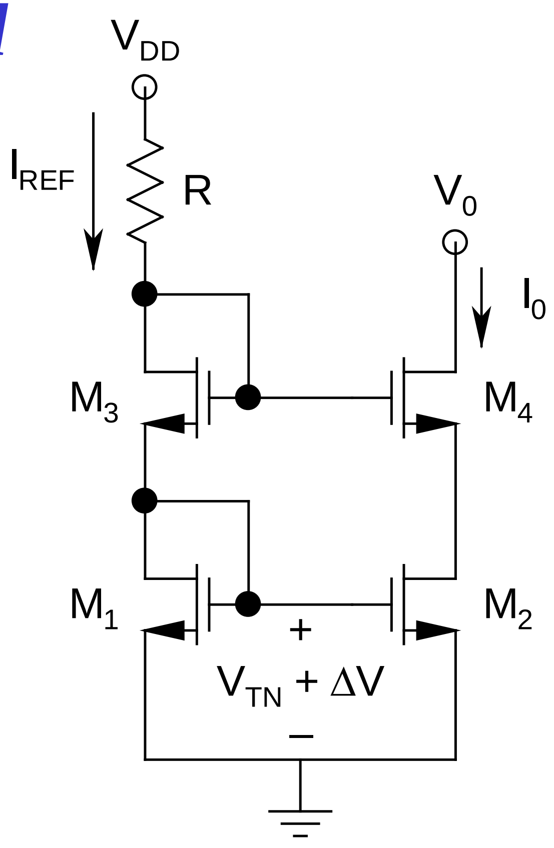
  (***assuming $r_{o2} >> r_{\pi4}$***)

- ***Actual analysis gives***:

  $R_0 = \beta_4 r_{04}/2$ (***large error***!)

- ***NMOS Cascode***:
  - *All Ms perfectly matched*
  - *All bodies connected to ground*
    - *$M_1$-$M_2$ does not have body effect*, *but $M_3$-$M_4$ does*!
    - *Makes hand analysis quite tedious*

      $\Rightarrow$ *Neglect body effect*
  - *All Ms operate with same $V_{GS}$*
  - Define $\Delta V = V_{GS} - V_{TN} = V_{GT}$
    - $\Delta V$ = *Gate Overdrive*



- ***The reference current***:

$$I_{REF} = \frac{V_{DD} - 2V_{GS}}{R} = \frac{k_N}{2} V_{GT}^2 \quad (\textit{\textbf{neglecting}}\ \lambda)$$

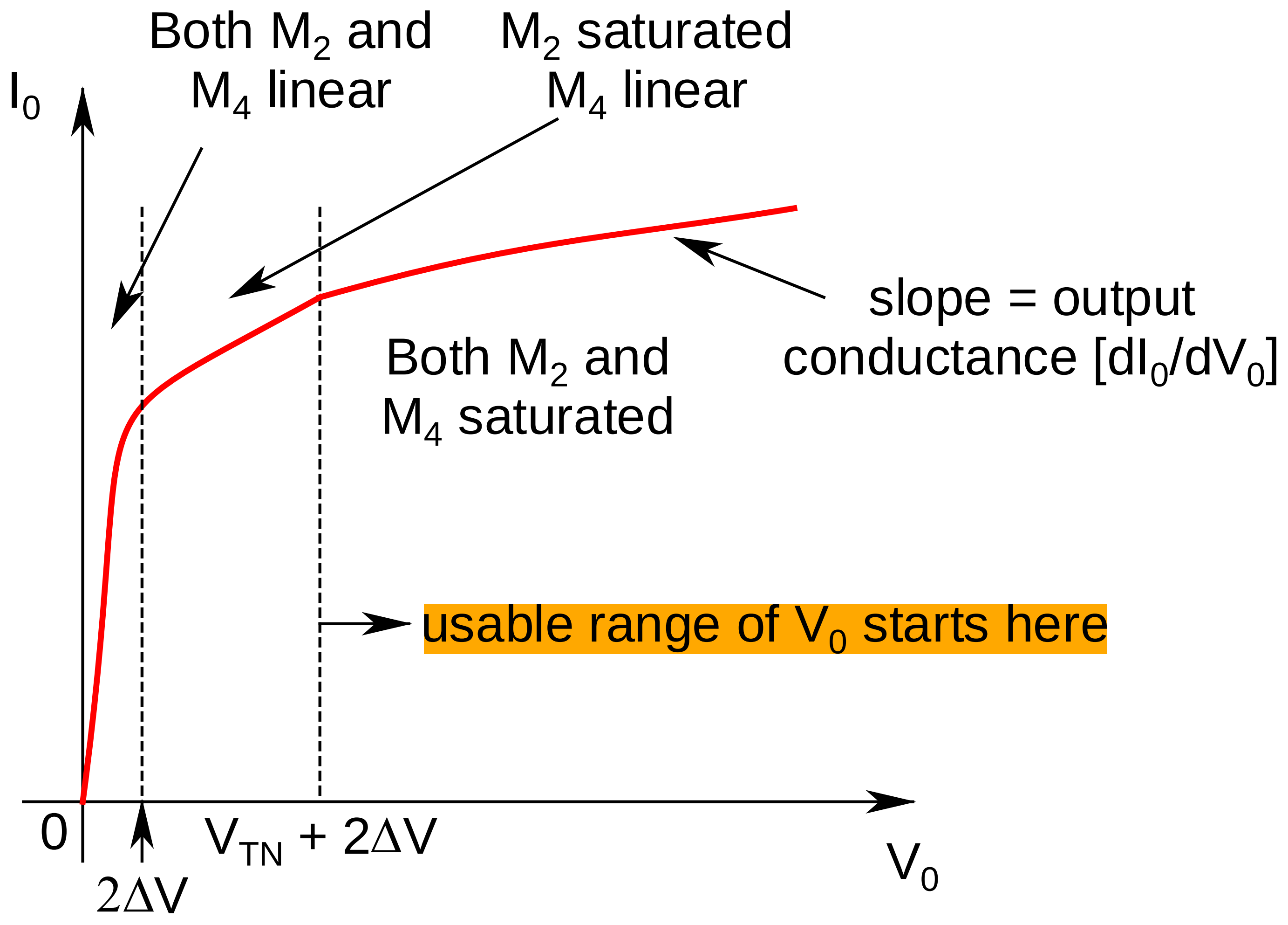
- ***$V_{GS}$ and $I_{REF}$ can be found*** $\Rightarrow I_0 = I_{REF}$
- $V_{G1} = V_{G2} = V_{GS} = V_{TN} + \Delta V$
- $V_{G3} = V_{G4} = 2V_{GS} = 2(V_{TN} + \Delta V)$
- $V_{S4} = V_{D2} = V_{TN} + \Delta V$

$\Rightarrow V_{GS2} = V_{DS2}$

$\Rightarrow$ ***$M_2$ can never enter linear region***

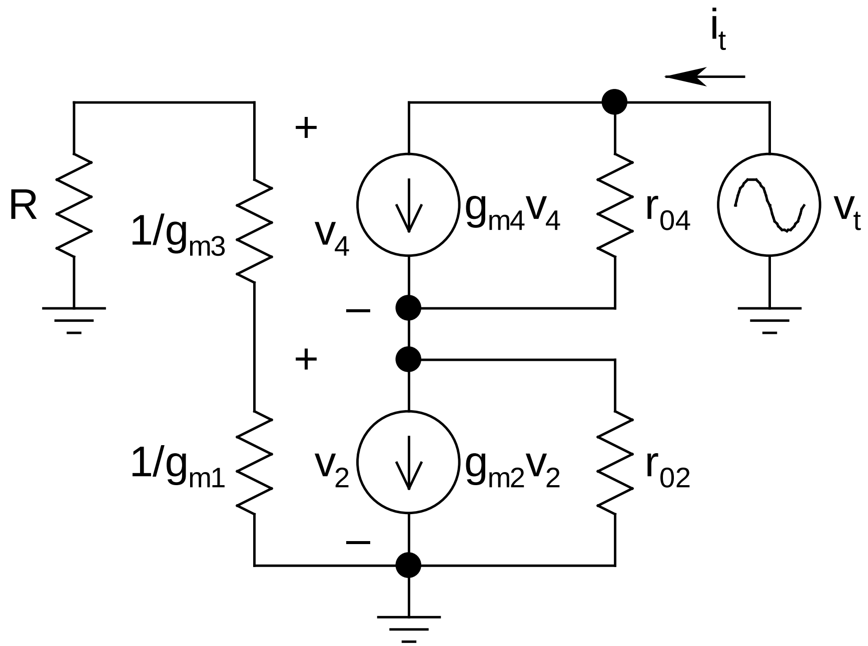
$\Rightarrow V_{0,min} = V_{DS2} + V_{DS4} = V_{TN} + 2\Delta V$

- ***This can be quite significant**, **since $V_{TN}$ is added to $\Delta V$***
  - ***Assuming $\Delta V \sim 0.1$ V and $V_{TN} \sim 0.7$ V, $V_{0,min} \sim 0.9$ V, which is very large***
  - ***This is one of the drawbacks of this simple cascode circuit** (**modified cascode doesn't have this problem**)*
- ***If $V_0$ drops below ($V_{TN} + 2\Delta V$), first $M_4$ enters linear region, and circuit performance starts to get affected***
- ***For further drop in $V_0$ , $M_2$ also enters linear region, and the current mirror collapses***!



Both $M_2$ and
$M_4$ linear
$M_2$ saturated
$M_4$ linear
$I_0$
slope = output
conductance $[dI_0/dV_0]$
Both $M_2$ and
$M_4$ saturated
usable range of $V_0$ starts here
0
$2\Delta V$
$V_{TN} + 2\Delta V$
$V_0$

## ➢ *Calculation of $R_0$*:



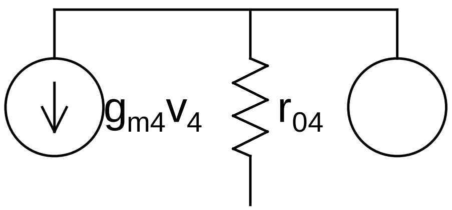

Exact Equivalent

- ***$M_1$ and $M_3$ diode-connected*** $\Rightarrow$ $1/g_{m1}$ and $1/g_{m3}$

- ***The left part of the circuit has no source***

  $\Rightarrow v_2 = 0 \Rightarrow g_{m2}v_2 = 0$

  $\Rightarrow$ ***Leads to the simplified equivalent*** (now should look very familiar!)




- ***By inspection***:

  $R_0 \approx r_{o4}(1 + g_{m4}r_{02})$

  $\approx g_{m4}r_{02}r_{04}$

- ***Can be huge***!