Lecture-7

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Phasor and power.

Evaluation Scheme:

S.N.	Lectures	Lab
1.	80-Marks	20-Marks
2.	Quizzes (surprise/announced) (25%) = 20	Each experiment 2.5 Marks $8 \times 2.5 = 20$
3.	Mid Semester Exam (30%) = 24	
4.	End Semester Exam (45%) = 36	

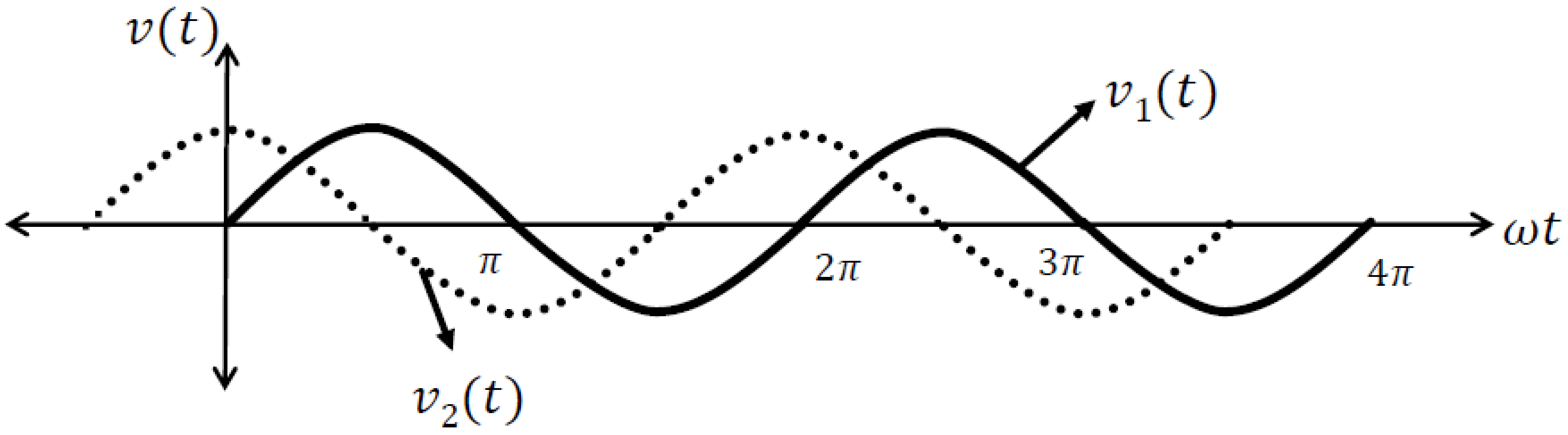
Sinusoids (Cont…)

- A more general expression for the sinusoid is given by,

$$v(t) = V_m \sin(\omega t + \emptyset)$$

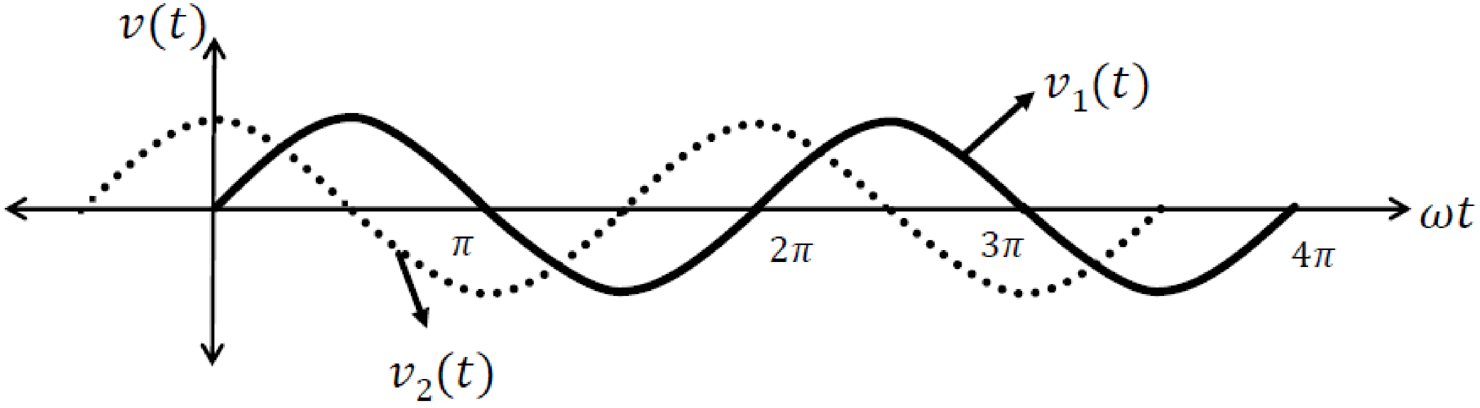
where, $\emptyset$ is the phase and is expressed in degrees or radians.

- To introduce the idea of phase, consider two sinusoids expressed as $\boldsymbol{v_1(t) = V_m sin\, \omega t}$ and $\boldsymbol{v_2(t) = V_m sin(\omega t + \emptyset)}$ as illustrated in the figure below.



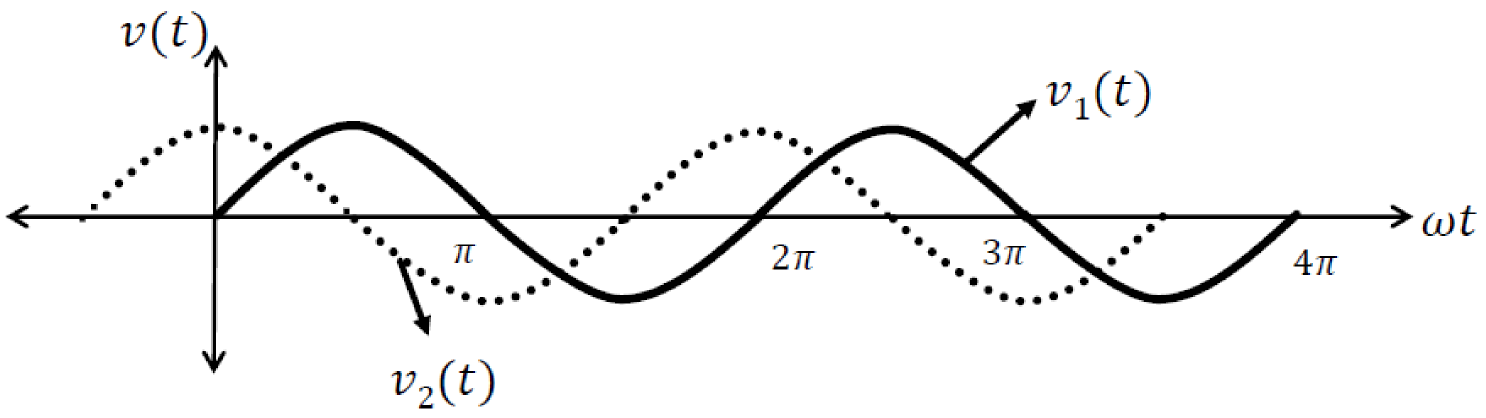
Sinusoids (Cont…)

- From the figure, it can be observed that the sinusoid v_2 occurs first in time.
- v_2 is, therefore, said to lead v_1 by $\emptyset$ or v_1 lags v_2 by $\emptyset$.
- If $\emptyset \neq 0$, then v_1 and v_2 are said to be out of phase.
- On the other hand, if $\emptyset = 0$, then v_1 and v_2 are said to be in phase, i.e. , they reach their maxima and minima at the same time.



Sinusoids (Cont…)

- A sinusoid can be represented in either sine or a cosine form.
- It can be transformed from one form to another using basic trigonometric identities.



Sinusoids (Cont…)

❑ Example:

- Find the amplitude, phase, period, and frequency of the sinusoid $v(t) = 12\cos(50t + 10°)$?

Solution:

The amplitude of the sinusoid is $V_m = 12\text{V}$.

The phase is $\emptyset = 10°$.

The angular frequency is $\omega = 50$ rad/s

The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257s$.

The frequency is given by, $f = \frac{1}{T} = 7.958\ Hz$.

❑ Example:

- Calculate the phase angle between v_1 = -10 cos(50t + 50) and v_2 = 12 sin(50t – 10)?

Solution: In order to compare the two sinusoid both of them needs to be expressed in the same form, i.e., in terms of either sine or cosine and with an amplitude of the same sign.

Here, we use cosine form with positive amplitudes to express the sinusoids as ,

$$v_1 = -10\cos(50t + 50) = 10\cos(50t + 50-180)$$

or, $$v_1 = -10\cos(50t + 50) = 10\cos(50t + 50+180)$$

and $$v_2 = 12\sin(50t - 10) = 12\sin(50t - 10-90)$$

Sinusoids (Cont…)

Using the above identities both v_1 and v_2 can be rewritten as,

$$v_1 = 10\cos(50t - 130)$$

or, $$v_1 = 10\cos(50t + 230)$$

and $$v_2 = 12\sin(50t - 100) = 12\cos(50t - 130 + 30)$$

or, $$v_2 = 12\sin(50t - 100) = 12\cos(50t + 260 - 360) = 12\cos(50t + 260)$$

Comparing the above equations, we can observe that v_2 leads v_1 by 30°. This can be verified for both the cases.

Note: The same sinusoids can be expressed in sine form as well to obtain the same answer.

Phasors

- Sinusoids are easily expressed in terms of phasors, which are more convenient to work with, than sine and cosine functions.
- A phasor is a complex number that represents the amplitude and phase of a sinusoid.
- A complex number z can be expressed in rectangular form as.

$$z = x + jy$$

- Here, x is the real part of z, y is the imaginary part of z, and $j = \sqrt{-1}$.
- The complex number, z can also be expressed in polar coordinates or exponential form as,

$$z = r\angle\emptyset = re^{j\phi} = r(\cos\emptyset + j\sin\emptyset) = x + jy$$

- where, r is the magnitude of z and $\emptyset$ is the phase of z.

Phasors (Cont…)

- From previous equation we can write $\cos\emptyset = Re(e^{j\emptyset})$ and $\sin\emptyset = \mathrm{Im}(e^{j\emptyset})$.
- Each representation is normally chosen according to its ease of use.
- For example, addition and subtraction of complex numbers are better performed in rectangular form; multiplication and division are better done in polar form.
- It is possible to convert the phasor from one form to another.

- Given $\boldsymbol{x}$ and $\boldsymbol{y}$ of the rectangular form, $\boldsymbol{r}$ and $\boldsymbol{\emptyset}$ of polar form can be obtained as,

$$r = \sqrt{x^2 + y^2},\ \emptyset = \tan^{-1} y/x$$

- Similarly, given $\boldsymbol{r}$ and $\boldsymbol{\emptyset}$, $\boldsymbol{x}$ and $\boldsymbol{y}$ can be evaluated as,

$$x = r\cos\emptyset,\ y = r\sin\emptyset$$

Phasors (Cont…)

- Sinusoid representation is phasor form -
  - Using $\cos\emptyset = Re(e^{j\emptyset})$ and $\sin\emptyset = \text{Im}(e^{j\emptyset})$, a given a sinusoid $v(t) = V_m\sin(\omega t + \emptyset)$ can be alternatively expressed as,

$$v(t) = V_m\cos(\omega t + \emptyset) = Re\left(V_m e^{j(\omega t + \emptyset)}\right) = Re\left(V_m e^{j\emptyset}e^{j\omega t}\right) = Re\left(\boldsymbol{V}e^{j\omega t}\right)$$

  - Here $\boldsymbol{V} = V_m e^{j\emptyset} = V_m\angle\emptyset$.
  - $\boldsymbol{V}$ is thus the phasor representation of the sinusoid $v(t)$.

- In other words, a phasor is a complex representation of the magnitude and phase of a sinusoid.
- Although the time factor, $\boldsymbol{e^{j\omega t}}$, is not explicitly mentioned in the phasor notation, it should be understood that the term is present, and the frequency of the phasor is $\boldsymbol{\omega}$.

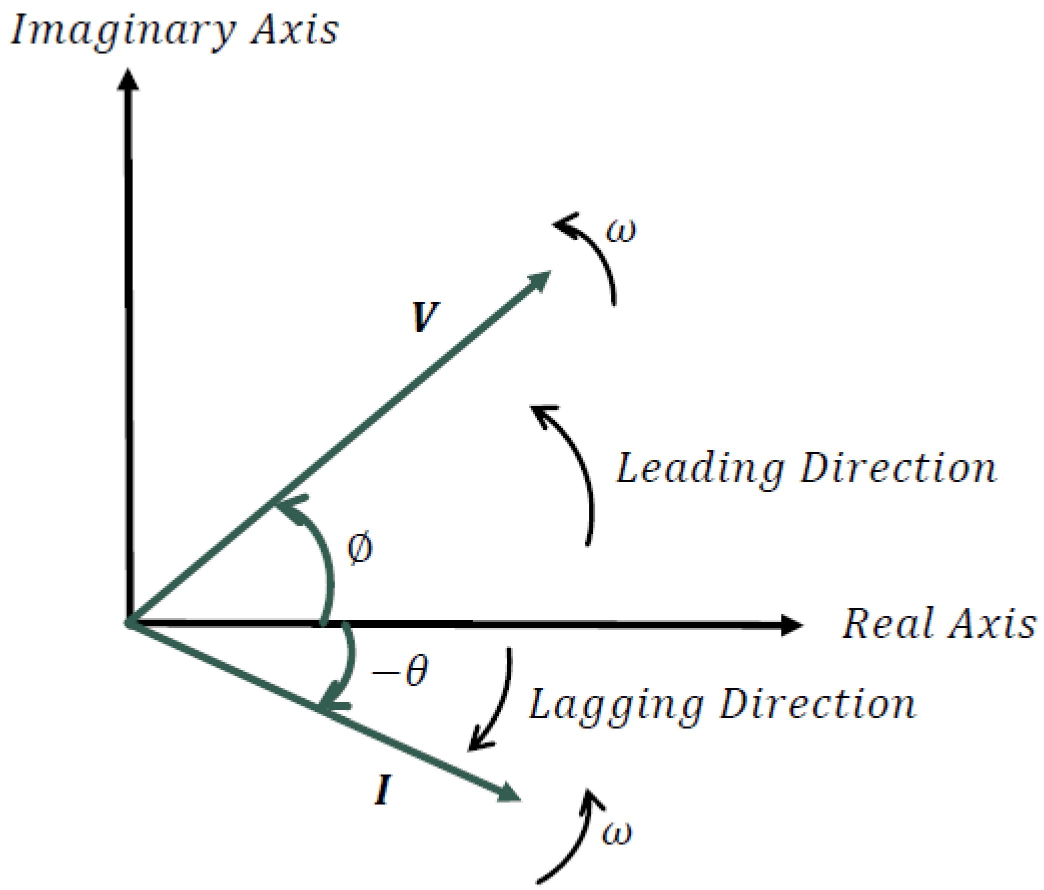
Phasors (Cont…)

- To get the phasor corresponding to a sinusoid, we first express the sinusoid in the cosine form.
- This ensures that the sinusoid can be written as the real part of a complex number.
- When the time factor, i.e. $e^{j\omega t}$, is removed, whatever is left is the phasor corresponding to the sinusoid.
- By suppressing the time factor, we transform the sinusoid from the time domain to the phasor domain.
- This transformation is summarized as:

$v(\mathrm{t}) = V_m \cos(\omega t + \emptyset)$	$\Leftrightarrow$	$\boldsymbol{V} = V_m \angle \emptyset$
(Time-domain representation)		(Phasor-domain representation)

Phasors (Cont…)

- A graphical representation of phasor is known as the phasor diagram.
- For example, the phasors $\boldsymbol{V} = V_m\angle\emptyset$ and $\boldsymbol{I} = I_m\angle\text{-}\theta$ are graphically represented as shown below:



Sinusoid – Phasor Transformation

Time Domain Representation	Phasor Domain Representation
$V_m \cos(\omega t + \emptyset)$	$V_m \angle \emptyset$
$V_m \sin(\omega t + \emptyset)$	$V_m \angle (\emptyset - 90°)$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle (\theta - 90°)$

standard form $v = V_m \cos(\omega t + \theta)$ and $i = I_m \cos(\omega t + \theta)$

❑ Example:

- Transform the sinusoid $i = 6\cos(50t - 10°)$ A to a phasor?

Solution: This sinusoid is in the standard form $i = I_m \cos(\omega t + \theta)$

For this form the corresponding phasor is represented as $I_m\angle\theta$.

Hence $i = 6\cos(50t - 10°)$ A can be represented in phasor form as

$$I = 6\angle{-10}.$$

❑ Example:

- Transform the sinusoid v = -4 sin(30t + 40°) V to a phasor?

Solution: This sinusoid is in the standard form $v = V_m \cos(\omega t + \theta)$

Using trigonometric identity - sin Ø = cos(Ø + 90°),

the sinusoid v = -4 sin(30t + 40°) = 4 cos(30t + 40° + 90°)

Thus, the sinusoid can be represented in phasor form as,

$$V = 4\angle 130.$$

❑ Example:

- Transform the phasor I = -3 + j4 A to a sinusoid?

Solution:

The above phasor is expressed in rectangular form and needs to be converted to polar form to transform it into a sinusoid.

To convert to polar form $I_m = \sqrt{3^2 + 4^2}$ =5, $\emptyset = \tan^{-1}\frac{-4}{3}$ = -53.13° or 126.87°

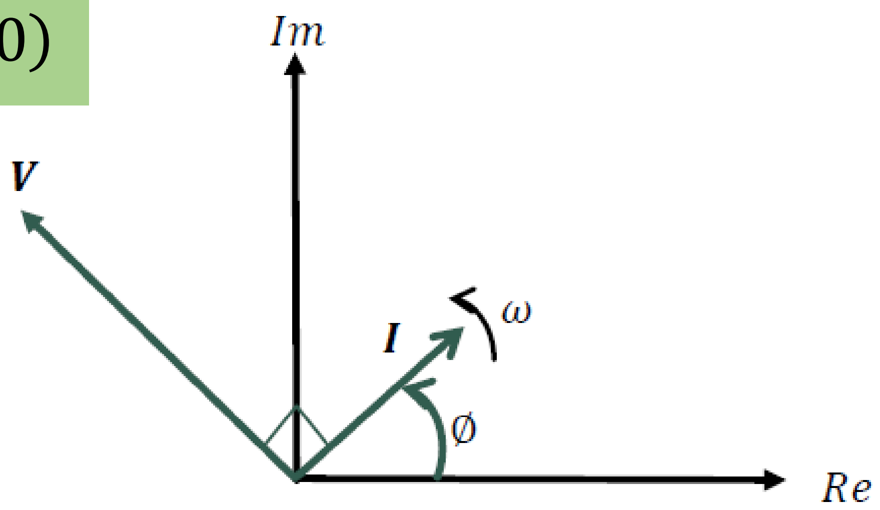
Therefore, I = 5∠126.87°

Transforming the above phasor into time domain, we get,

$$i = 5\cos(\omega t + 126.87°)\text{ A.}$$

- For **inductor**, let the current through an inductor L is $i = I_m \cos(\omega t + \emptyset)$.
- The voltage across it is given by,

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \emptyset) = \omega L I_m \cos(\omega t + \emptyset + 90)$$



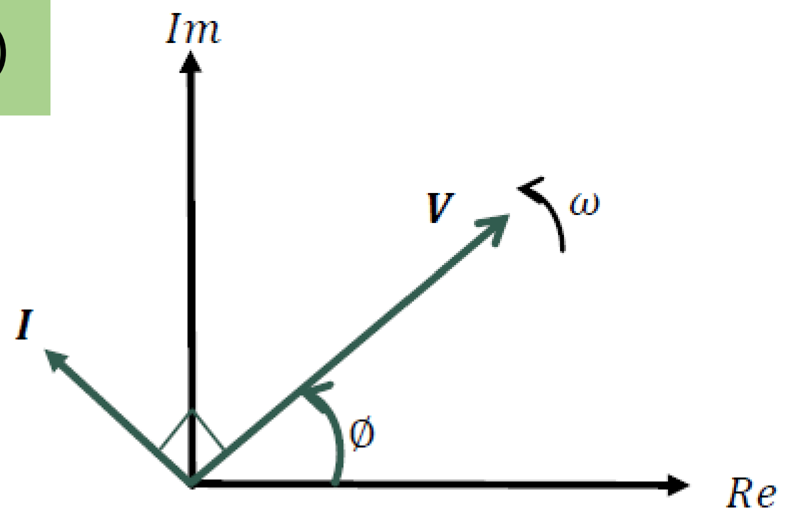
- This can be expressed in phasor form as,

$$\boldsymbol{V} = \omega L I_m e^{j(\emptyset+90)} = \omega L I_m e^{j\emptyset} e^{j90} = j\omega L I_m e^{j\emptyset} = j\omega L \boldsymbol{I}$$

- It can be observed from the above equation that voltage and current in case of inductor are 90° out of phase as shown in the phasor diagram.
- Specifically, the current lags the voltage by 90°.

- For **capacitor**, let the voltage across a capacitor ***C*** is $v = V_m \cos(\omega t + \emptyset)$.
- The current across it is given by,

$$i = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t + \emptyset) = \omega C V_m \cos(\omega t + \emptyset + 90)$$



- This can be expressed in phasor form as,

$$I = \omega C V_m\, e^{j(\emptyset+90)} = \omega C V_m\, e^{j\emptyset} e^{j90} = j\omega C V_m e^{j\emptyset} = j\omega C\, \boldsymbol{V}$$

- It can be observed from the above equation that voltage and current in case of capacitor are 90° out of phase as shown in the phasor diagram.
- Specifically, the current leads the voltage by 90°.

❑ Example:

A voltage $v(t)$= 12 cos(60t + 45°) is applied to a 0.1H inductor. Find the steady state current through the inductor?

Solution: For the inductor,

$$I = \frac{V}{j\omega L} = \frac{12\angle 45°}{j60 * 0.1} = \frac{12\angle 45°}{6\angle 90°} = 2\angle{-45°}$$

This can be rewritten in time domain as:

$$i(t) = 2\cos(60t - 45) \text{ A}$$

Instantaneous Power

- The instantaneous power, in watts, is the power at any instant of time.
- The instantaneous power $p(t)$ absorbed by an element is the product of the instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it.

$$\boxed{p(t) = v(t)\, i(t)}$$

- To derive a general case of instantaneous power absorbed by a combination of circuit elements consider

$$v(t)=V_{\mathrm{m}}\cos(\omega t + \theta_v) \text{ and } i(t)= I_m\cos(\omega t + \theta_i),$$

- Here, V_m and I_m are the amplitudes, and θ_v and θ_i are the phase angles of the voltage and current, respectively.

Instantaneous Power (Cont…)

- Therefore, the instantaneous power absorbed by the circuit is given by

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

- Using trigonometric identities, the above equation can be rewritten as,

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Instantaneous Power (Cont…)

- Therefore, the equation for power can be rewritten as,

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

- It can be observed that the instantaneous power has two parts, a constant time independent part and a sinusoidal part with twice the frequency of the voltage or current.
- The instantaneous power changes with time and is therefore difficult to measure.
- The average power is more convenient to measure.
- In fact, the wattmeter, the instrument used for measuring power, responds to average power.

Average Power

- The average power in watts, is the average of the instantaneous power over one period.
- The average power is expressed mathematically as,

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

- For the general signal described previously,

$$P = \frac{1}{T}\int_0^T \frac{1}{2}\,VmI_m\cos(\theta_v - \theta_i)\mathrm{dt} + \frac{1}{T}\int_0^T \frac{1}{2}\,VmI_m\cos(2\omega t + \theta_v + \theta_i)\,dt$$

Average Power (Cont…)

- In the above equation, the first integrand is constant, and the average of a constant is the same constant.
- The second integrand is a sinusoid.
- We know that the average of a sinusoid over its period is zero because the area under the sinusoid during a positive half-cycle is canceled by the area under it during the following negative half-cycle.
- Thus, the second term in the above equation vanishes and the average power is given by,

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Average Power (Cont…)

- Next, we consider two special cases.
- When $\theta_v = \theta_i$, we know that the voltage and the current are in phase.
- This is a purely resistive circuit, and the average power is given by,

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$$

- This shows that a purely resistive load absorbs power at all times.
- When θ_v - θ_i = ±90°, the circuit is a purely reactive circuit, and the average power is.

$$P = \frac{1}{2} V_m I_m \cos 90 = 0$$

- Thus, a purely reactive circuit absorbs no average power.


THANKS
FOR YOUR
ATTENTION