Lecture-10

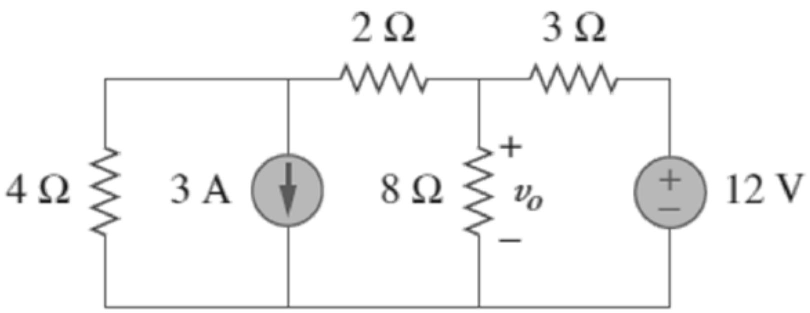
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Duality.
- Thevenin's Theorem.

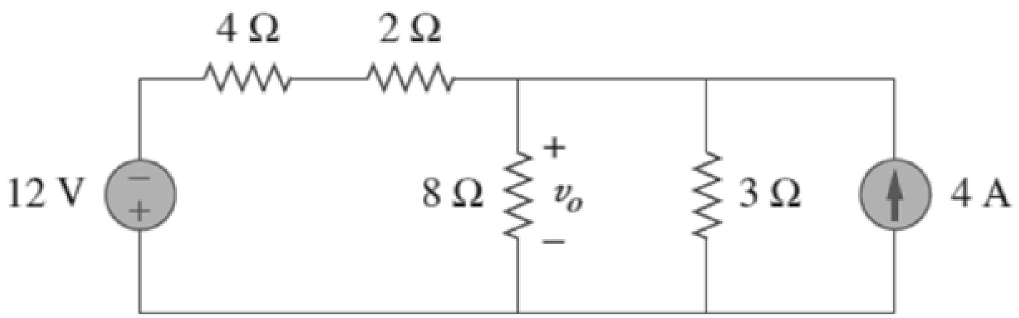
❑ Example:

For the adjacent circuit find v_0 using source transformation?



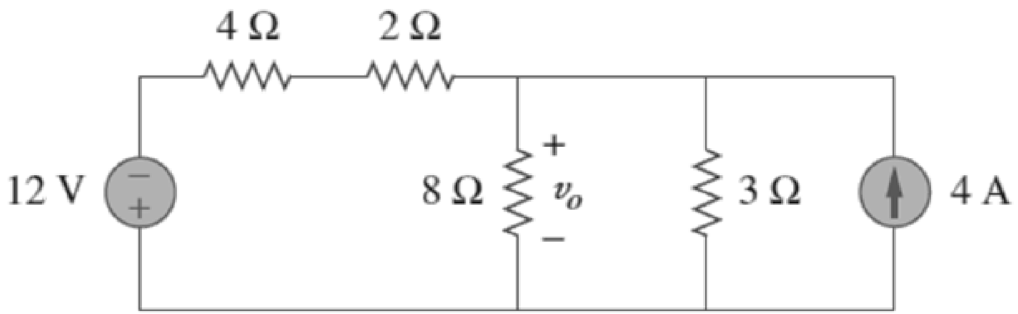
Solution: We first transform the current and voltage sources to obtain the circuit shown below.

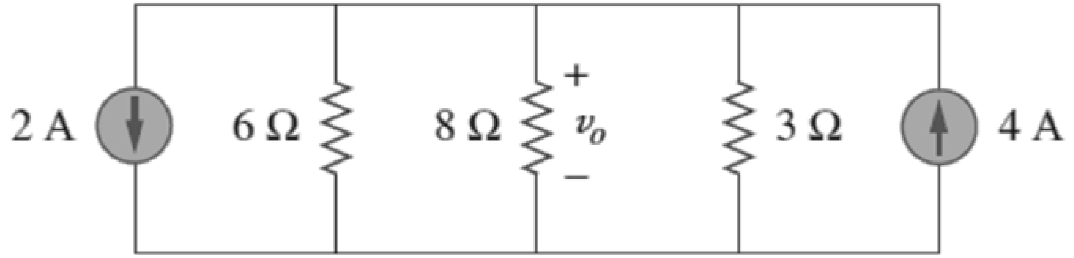
$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$



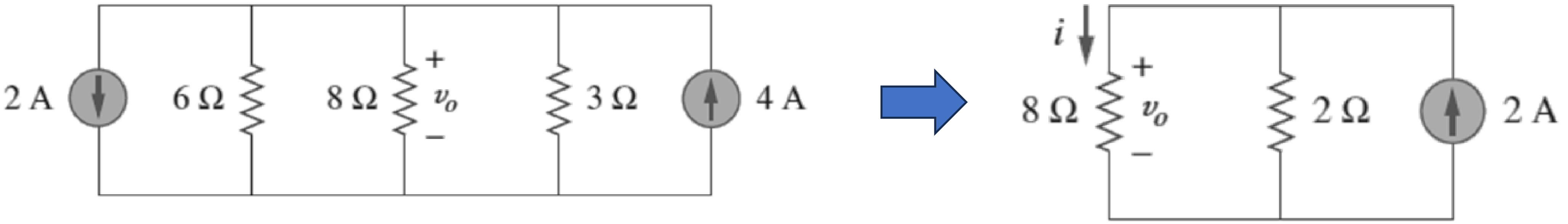
- Combining the 4Ω and 2Ω resistors in series and transforming the 12-V voltage source gives us the following circuit,

$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$





- We now combine the 6Ω and 3Ω resistors in parallel to get 2Ω.
- We also combine the 2-A and 4-A current sources to get a 2-A source.
- Thus, by repeatedly applying source transformations, we obtain the final circuit given as:



We use current division to get

$$i = \frac{2}{2+8} * 2 = 0.4A$$

and

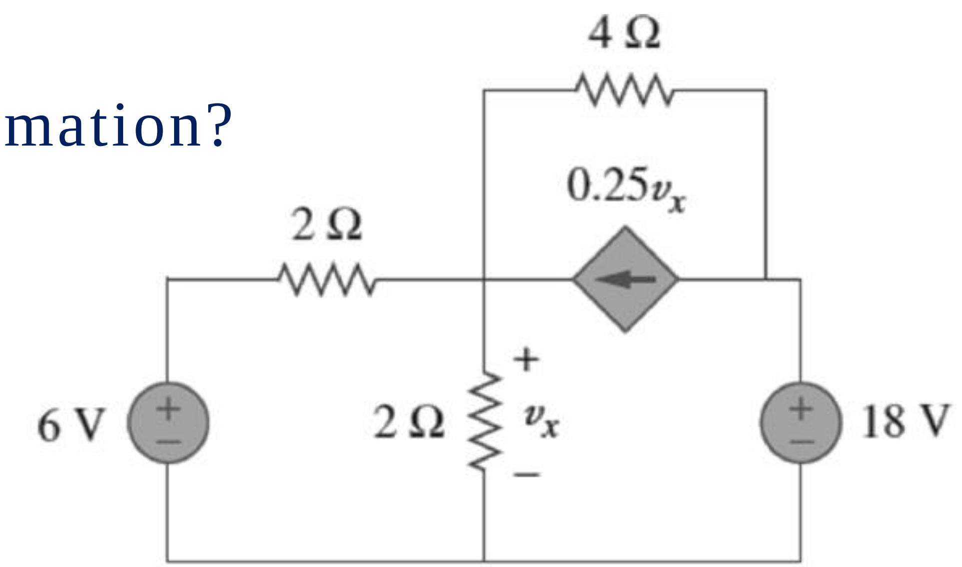
$$v_0 = 8i = 8 * 0.4 = 3.2V$$

Alternatively, since 8Ω and 2Ω are in parallel and they have the same voltage v_0 across. Hence,

$$v_0 = (8||2) * 2A = 3.2V$$

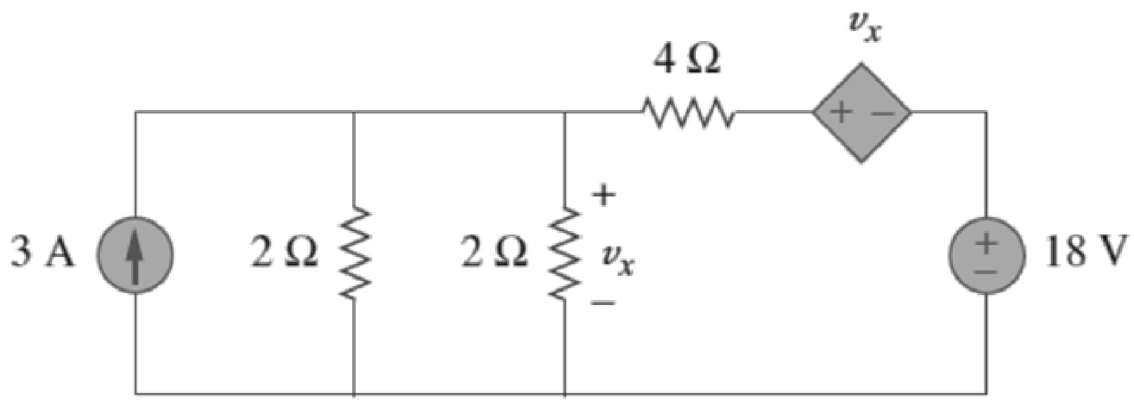
❑ Example:

For the adjacent circuit find v_x using source transformation?



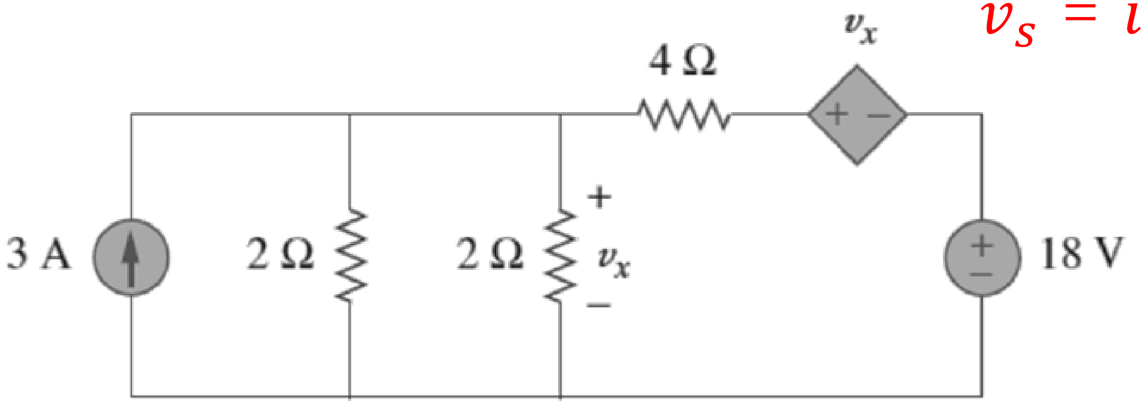
Solution: The circuit involves a voltage controlled dependent current source. Using source transformation for this dependent source as well as the 6V independent source, we get -

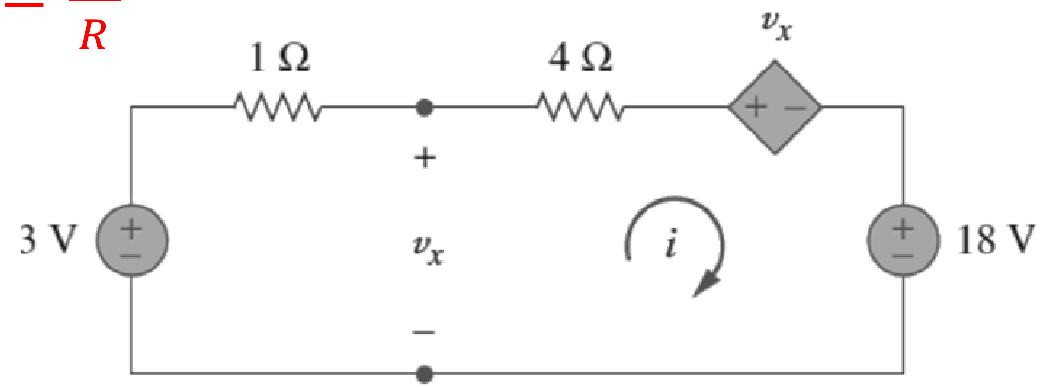
$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$



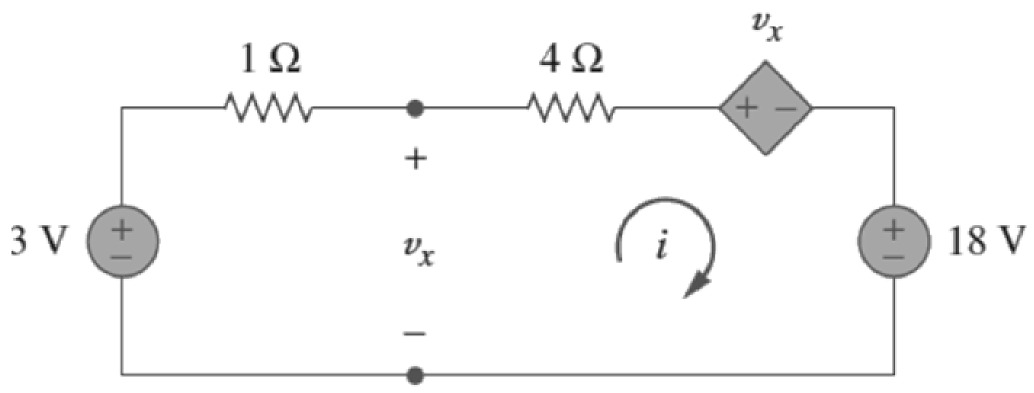
- The 18 V voltage source is not transformed because it is not connected in series with any resistor.
- The two 2Ω resistors combine parallel to give a 1Ω resistor, which is in parallel with a 3A current source.
- The current source is transformed to a voltage source as shown in the below figure.

$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$





- Notice that the terminals for v_x are still intact.



Applying KVL around the loop in the above figure gives,

$$-3 + 5i + v_x + 18 = 0$$

Applying KVL to the loop containing the 3V voltage source and the 1Ω resistor gives,

$$-3 + 1i + v_x = 0 \Rightarrow v_x = 3 - i$$

Substituting the above relation in the first equation, we get

$$15 + 5i + 3 - i = 0 \Rightarrow i = -4.5\text{A}$$

Thus, $v_x = 3 - i = 7.5\text{V}$.

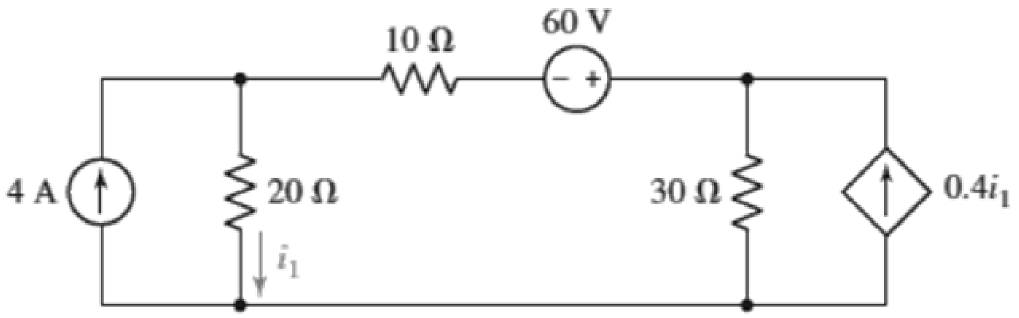
Some Key Points

- We conclude our discussion of practical sources and source transformations with a few observations.

   i. First, when we transform a voltage source, we must be sure that the source is in fact in series with the resistor under consideration.

   ii. For example, in the circuit below, it is perfectly valid to perform a source transformation on the voltage source using the 10Ω resistor, as they are in series.

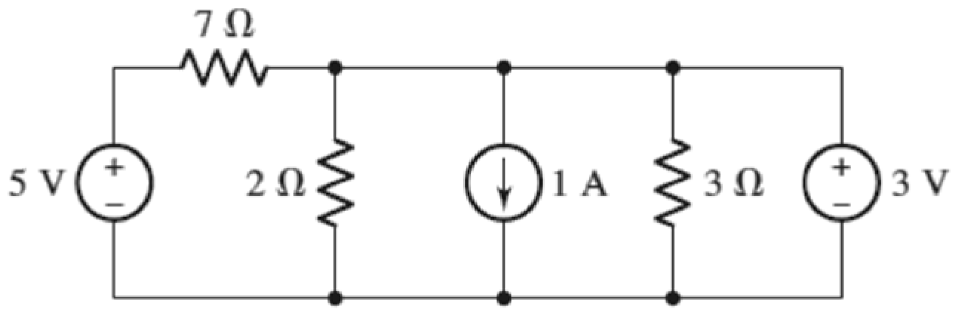
   iii. However, it would be incorrect to attempt a source transformation using the 60V source and the 30Ω resistor - a very common type of error.



Some Key Points (Cont…)

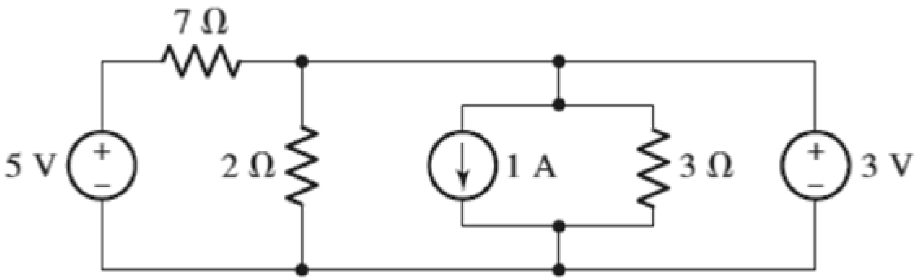
iv. In a similar fashion, when we transform a current source and resistor combination, we must be sure that they are in fact in parallel.

v. Consider the current source shown in the figure below.

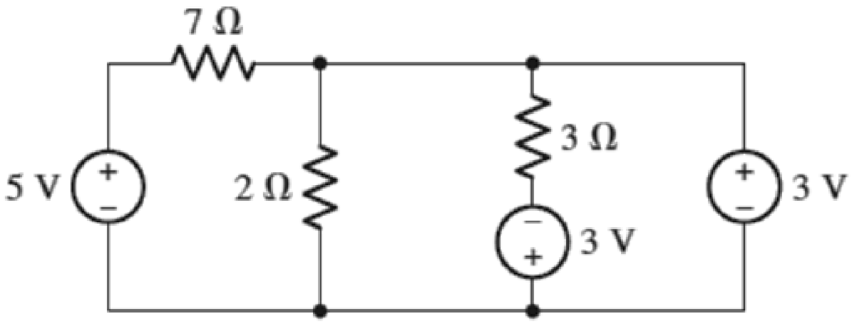


vi. We may perform a source transformation including the 3Ω resistor, as they are in parallel, but after the transformation there may be some ambiguity as to where to place the resistor.

vii. In such circumstances, it is helpful to first redraw the components to be transformed as shown the following figure.



viii. Then the transformation to a voltage source in series with a resistor may be drawn correctly as shown below; the resistor may in fact be drawn above or below the voltage source.

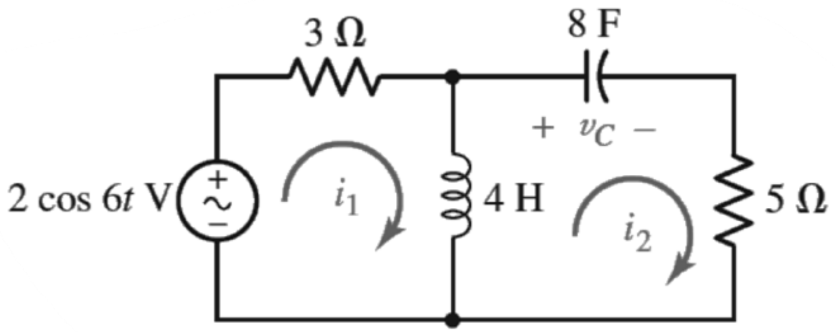


Duality

- The concept of duality applies to many engineering concepts.
- In this section, we shall define duality in terms of the circuit equations.
- Two circuits are “duals” if the mesh equations that characterize one of them have the same mathematical form as the nodal equations that characterize the other.
- They are said to be exact duals if each mesh equation of one circuit is numerically identical with the corresponding nodal equation of the other.
- Duality refers to any of the properties exhibited by dual circuits.

Duality (Cont…)

- Let us use the definition to construct an exact dual circuit by writing the two mesh equations for the circuit shown below.



$$3i_1 + \frac{4di_1}{dt} - \frac{4di_2}{dt} = 2cos6t \qquad (1)$$

$$-\frac{4di_1}{dt} + \frac{4di_2}{dt} + \frac{1}{8}\int_0^t i_2 dt + 5i_2 = -10 \qquad (2)$$

Duality (Cont…)

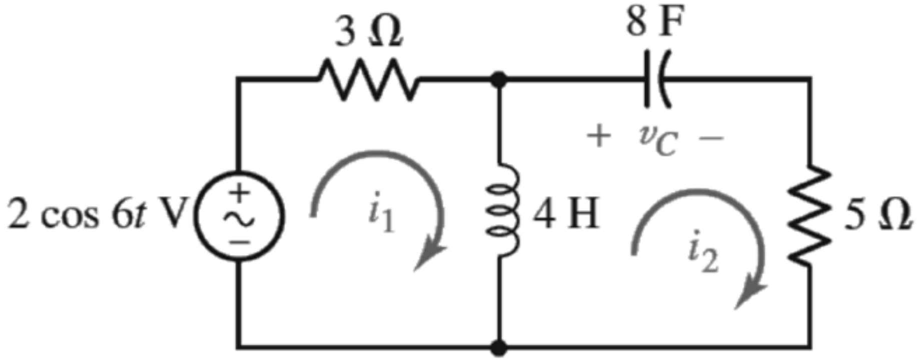
- We may now construct the two equations that describe the exact dual of our circuit.
- We wish these to be nodal equations and thus begin by replacing the mesh currents with nodal voltages.
- We obtain

$$3i_1 + \frac{4di_1}{dt} - \frac{4di_2}{dt} = 2cos6t \qquad (1)$$

$$-\frac{4di_1}{dt} + \frac{4di_2}{dt} + \frac{1}{8}\int_0^t i_2 dt + 5i_2 = -10 \qquad (2)$$

$$3v_1 + \frac{4dv_1}{dt} - \frac{4dv_2}{dt} = 2cos6t \qquad (3)$$

$$-\frac{4dv_1}{dt} + \frac{4dv_2}{dt} + \frac{1}{8}\int_0^t v_2 dt + 5v_2 = -10 \qquad (4)$$



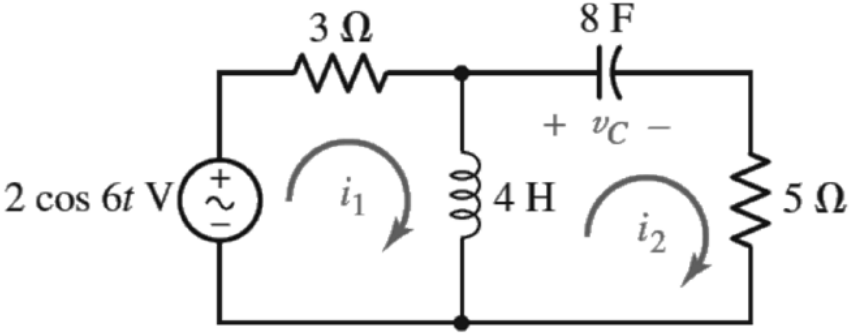
Duality (Cont…)

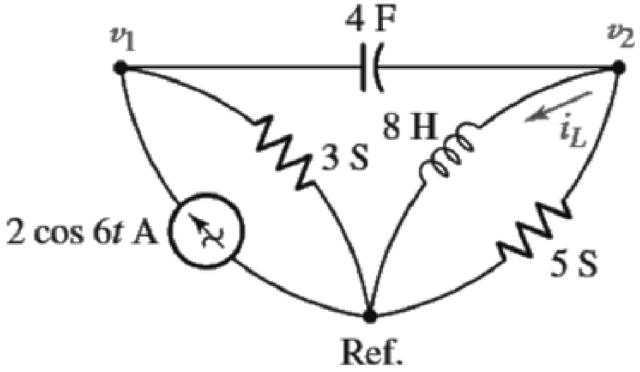
$$3v_1 + \frac{4dv_1}{dt} - \frac{4dv_2}{dt} = 2cos6t \qquad (3)$$

$$\frac{v_1 - 0}{3} + \frac{4dt(v_1 - v_2)}{dt} = 2cos6t$$

$$-\frac{4dv_1}{dt} + \frac{4dv_2}{dt} + \frac{1}{8}\int_0^t v_2 dt + 5v_2 = -10 \quad (4)$$

$$\frac{4dt(v_2 - v_1)}{dt} + \frac{1}{8}\int_0^t v_2 dt + \frac{v_2 - 0}{5} = -10$$

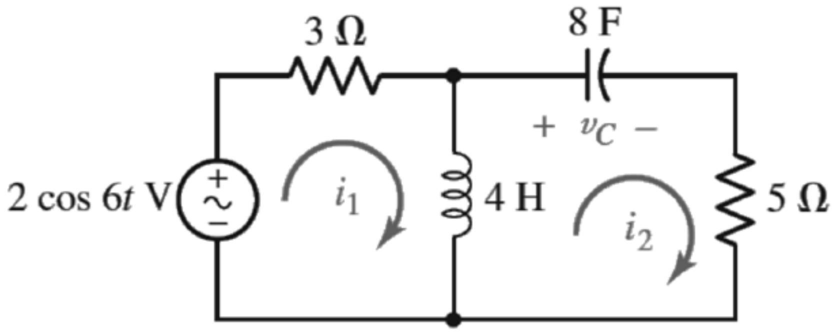


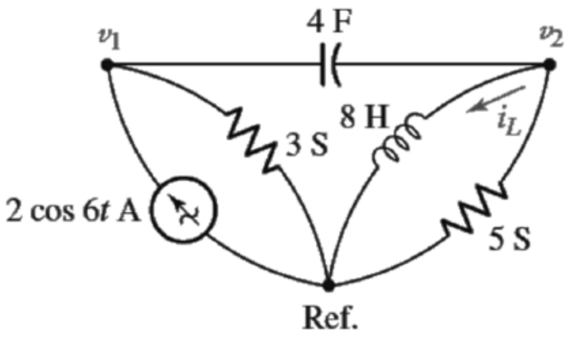


Duality (Cont…)

Next we need to determine the circuit represented by the above nodal equations –

- Let us first draw a line to represent the reference node, and then we may establish two nodes at which the positive references for v_1 and v_2 are located.
- Equation(3) indicates that a current source of $2\cos 6t$ A is connected between node 1 and the reference node, oriented to provide a current entering node 1.
- This equation also shows that a 3S conductance appears between node 1 and the reference node.
- From Eq.(4), we first consider the non mutual terms, i.e., those terms which do not appear in Eq.(3)
- This shows that an 8 H inductor and a 5S conductance are connected between node 2 and reference.

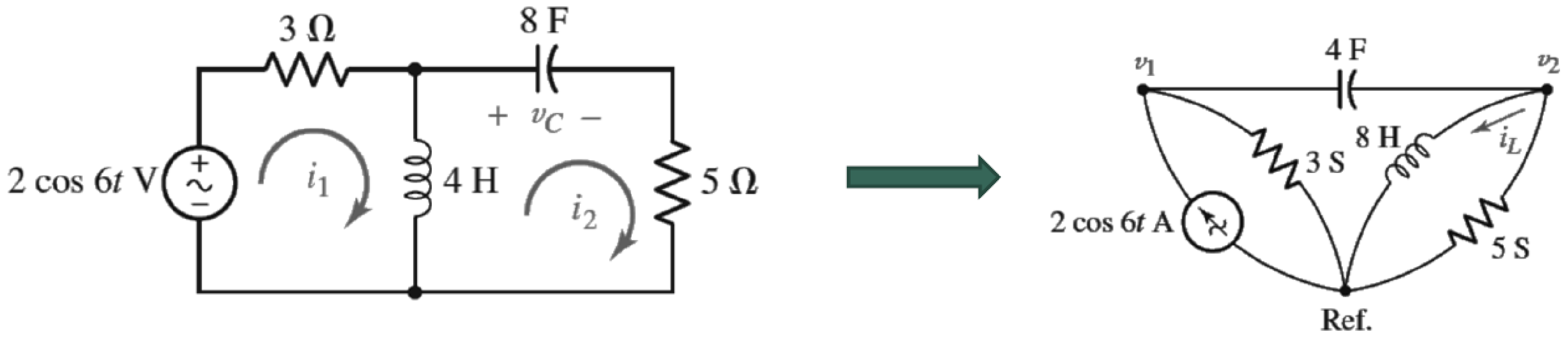




Duality (Cont…)

- The two similar terms, left in equations (3) and (4), represent a 4F capacitor present mutually at nodes1 and 2.
- The circuit is completed by connecting this capacitor between the two nodes.
- The constant term on the right side of equation (4) is the value of the inductor current at $t = 0$; in other words, $i_L(0) = 10$ A.
- The dual circuit is then as shown in the figure below.

$$-\frac{4dv_1}{dt} + \frac{4dv_2}{dt} + \frac{1}{8}\int_0^t v_2 dt + 5v_2 = -10 \qquad (4)$$

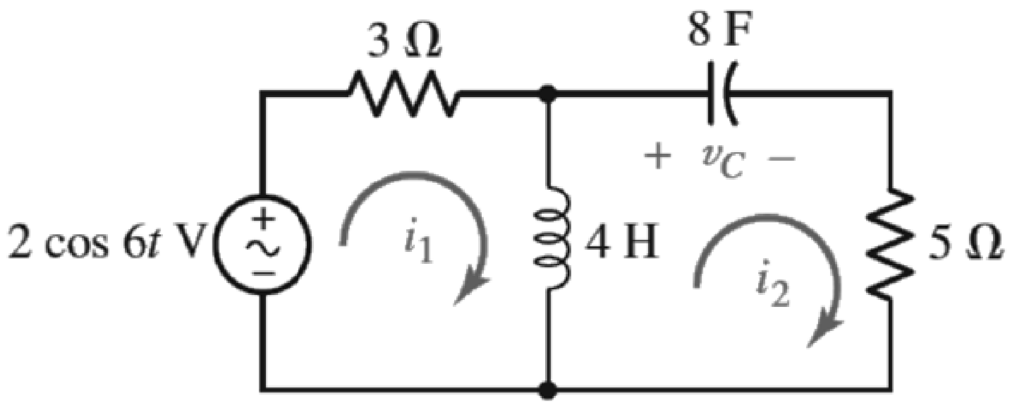


Duality (Cont…)

- Dual circuits may be obtained more readily than by this method, for that the equations also need not to be written.
- In order to construct the dual of a given circuit, we think of the circuit in terms of its mesh equations.
- With each mesh, we must associate a non reference node and, additionally, a reference node.
- On a diagram of the given circuit, we therefore place a node in the center of each mesh and supply the reference node circuit, near the diagram or a loop enclosing the diagram.

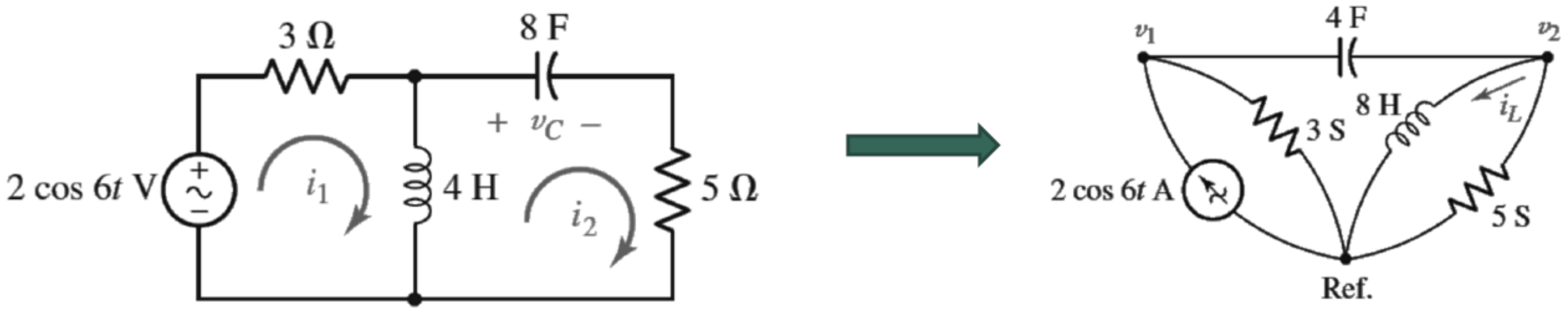
Duality (Cont…)

- Each element that appears jointly in two meshes is a mutual element. It must be replaced by an element that supplies the dual term in the two corresponding nodal equations. This dual element is connected directly between the two non reference nodes.



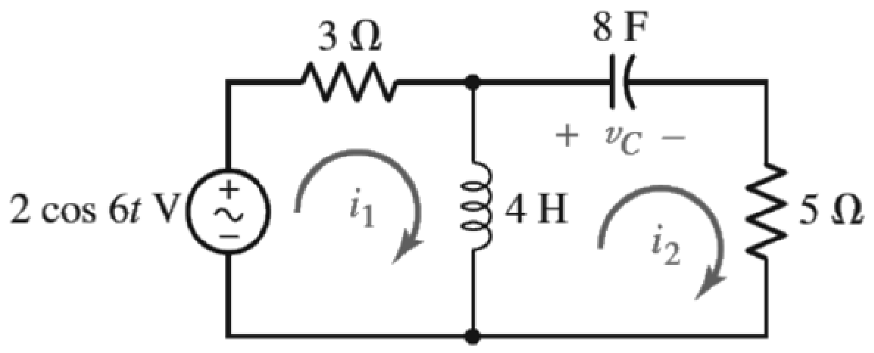
Duality (Cont…)

- The nature of the dual element itself is easily determined.
- The mathematical form of the equations will be the same only if inductance is replaced by capacitance, capacitance by inductance, conductance by resistance, and resistance by conductance.
- Thus, the 4H inductor which is common to meshes 1 and 2 in the circuit of the figure appears as a 4F capacitor connected directly between nodes 1 and 2 in the dual circuit.



Duality (Cont…)

- Elements that appear only in one mesh must have duals that appear between the corresponding node and the reference node.
- The voltage source $2\cos 6t$ V appears only in mesh 1, in figure, and its dual is a current source $2\cos 6t$ A. Therefore, it is connected only to node 1 and the reference node.
- As voltage source is clockwise-sensed, the current source must be into-the-non reference-node-sensed.
- Finally, provision must be made for the dual of the initial voltage present across the 8F capacitor in the given circuit.

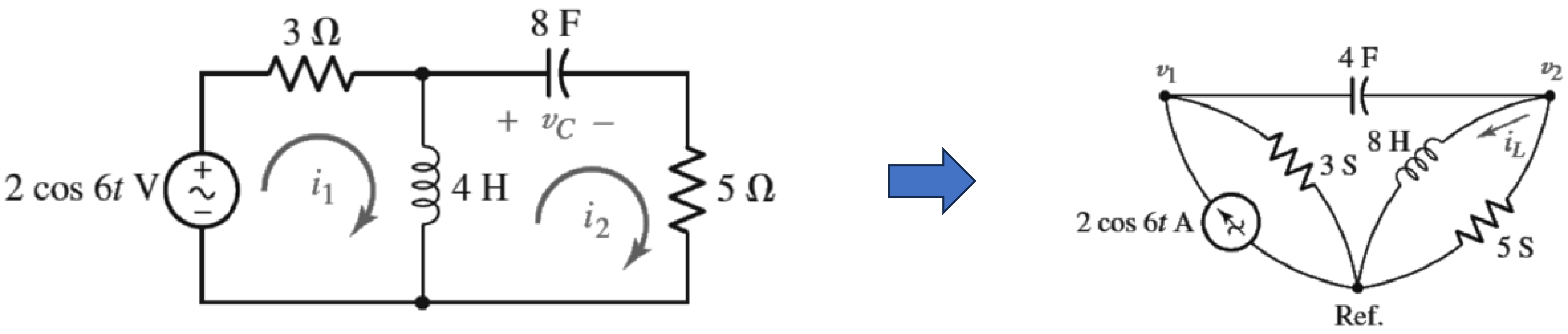


Duality (Cont…)

- From duality principle, the dual of initial voltage across capacitor is an initial current through the inductor in the dual circuit.
- The numerical values are same, and correct sign of the initial current may be determined most readily by considering the initial voltage in the given circuit and the initial current in the dual circuit as sources.
- Thus, if v_c is treated as a source, it would appear as $-v_c$ on the right side of the mesh equation.

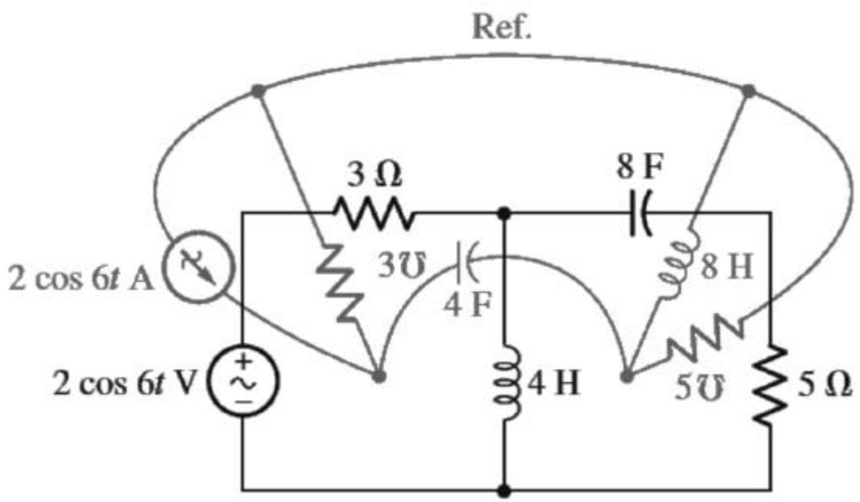
Duality (Cont…)

- In the dual circuit, treating the current i_L as a source will give $-i_L$ on the right side of the nodal equation.
- Since each has the same sign when treated as a source, then, if $v_c(0) = 10V$, $i_L(0)$ must be 10 A.



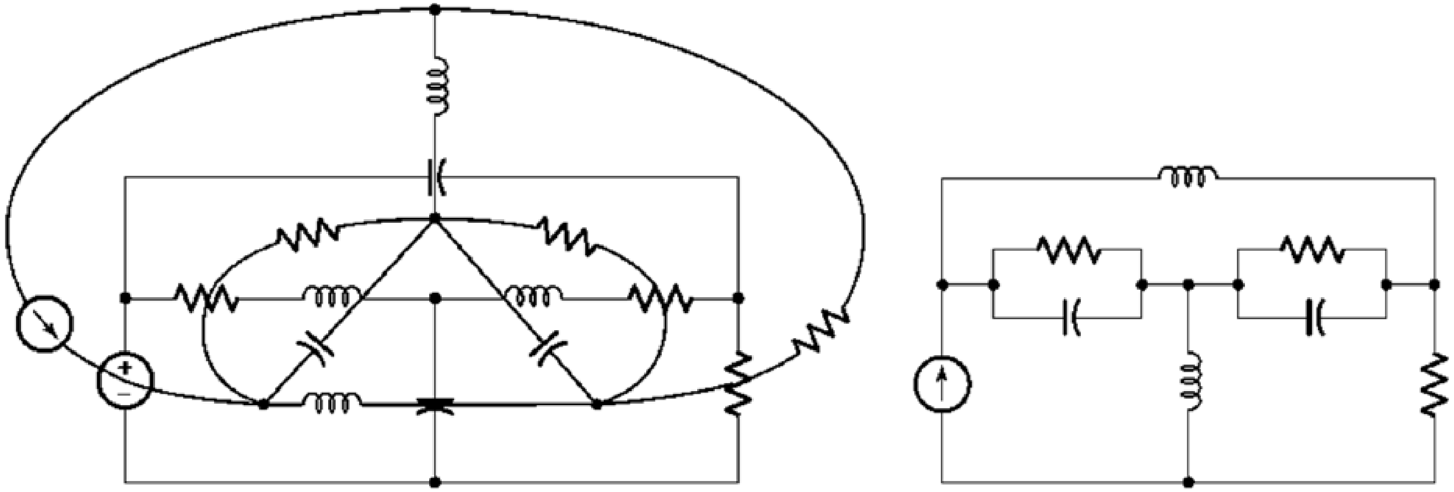
Duality (Cont…)

- Its exact dual is constructed on the circuit diagram itself by merely drawing the dual of each given element between the two nodes that are inside the two meshes and are common to the given element.
- A reference node that surrounds the given circuit may be helpful.



Duality (Cont…)

- An additional example of the construction of a dual circuit is shown in the figure below.
- Since no particular element values are specified, these two circuits are duals, but not necessarily exact duals.
- The original circuit may be recovered from the dual by placing a node in the center of each of the five meshes of the figure on the right and proceeding as before.



Duality (Cont…)

- A list of various dual pairs are tabulated in the below table.
- Power is not included in the table as it has no dual.

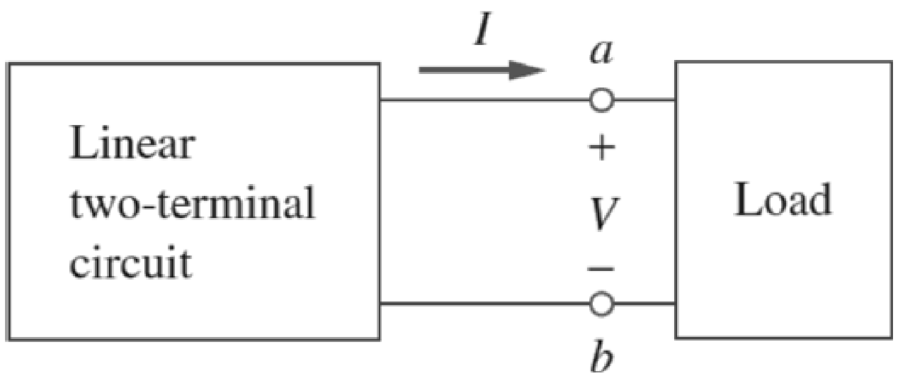
Resistance R	Conductance G
Inductance L	Capacitance C
Voltage V	Current I
Voltage Source	Current Source
Node	Mesh
Series Path	Parallel Path
Open circuit	Short Circuit
KVL	KCL

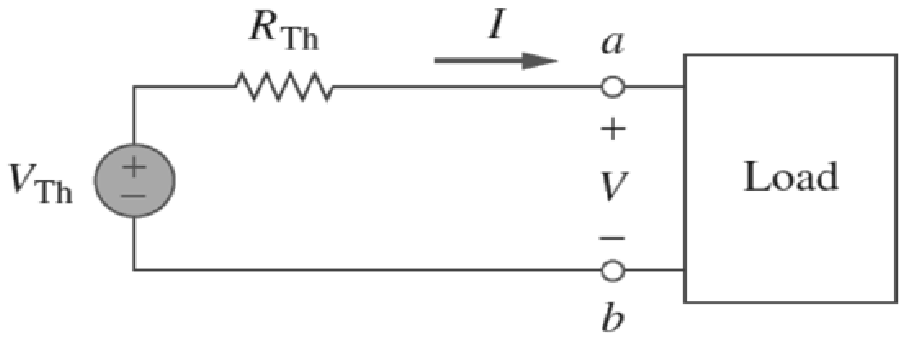
Thevenin’s Theorem

- It often occurs in practice that a particular element in a circuit is variable (usually called the load) while other elements are fixed.
- As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load.
- Each time the variable element is changed, the entire circuit has to be analyzed all over again.
- To avoid this problem, Thevenin’s theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

Thevenin's Theorem (Cont…)

- According to Thevenin's theorem, the linear circuit in the left of the figure below can be replaced by that shown in the right.
- The load in the figure may be a single resistor or another circuit.
- The circuit to the left of the terminals a-b in the figure is known as the Thevenin equivalent circuit.
- It was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.



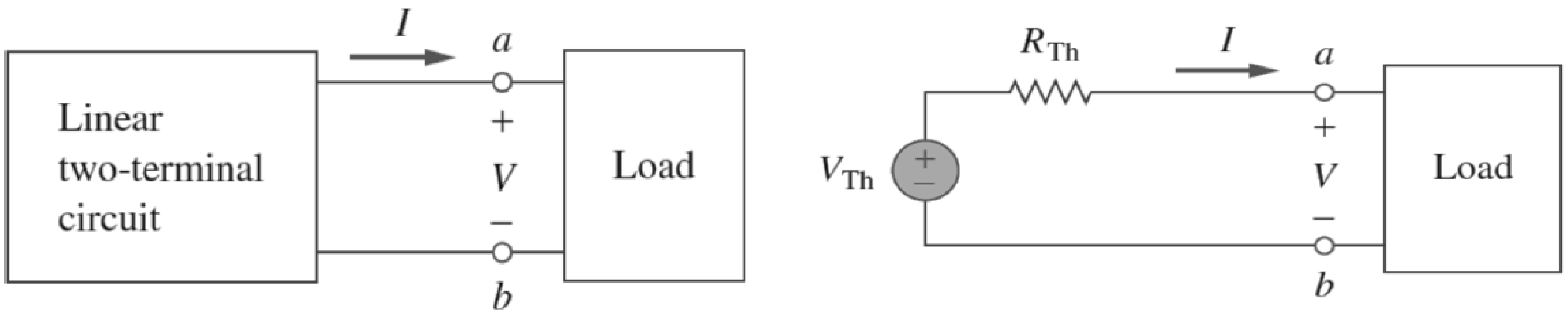


Thevenin's Theorem (Cont…)

- Thevenin's theorem: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{TH} in series with a resistor R_{TH}, where V_{TH} is the open-circuit voltage at the terminals and R_{TH} is the equivalent resistance at the terminals when the independent sources are turned off.
- Our major concern is to find out the values of V_{TH} and R_{TH}.
- To do so, assume the two circuits shown in the figure to be equivalent.
- They are said to be equivalent if they have the same voltage-current relationship at their terminals.

Thevenin's Theorem (Cont…)

- When terminals a - b are open circuited by removing the load, i.e. no current flows through the load, the open circuit voltage across the terminals a - b in the circuit on the left must be equal to the voltage source V_{TH} in the circuit on the right.



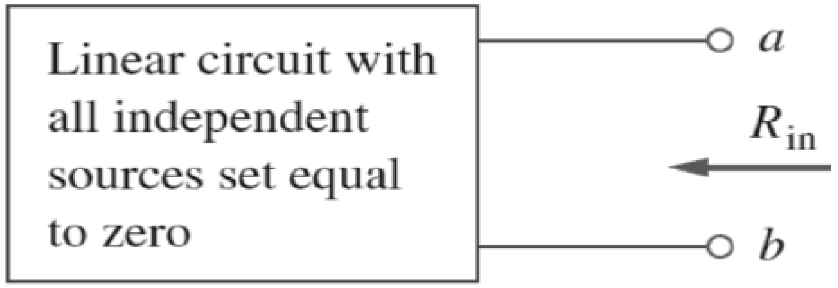
Thevenin's Theorem (Cont…)

- Thus, V_{TH} is the open circuit voltage across the terminals as shown in the circuit on the left in the figure below , that is,

$$V_{TH} = v_{oc}$$

- Again, with the load disconnected, terminals a-b open circuited, and all independent sources are turned off, the input resistance (or equivalent resistance) of the dead circuit at the terminals must be equal in both the circuits.
- Thus, R_{TH} is the input resistance at the terminal when the independent sources are turned off.

$V_{Th} = v_{oc}$



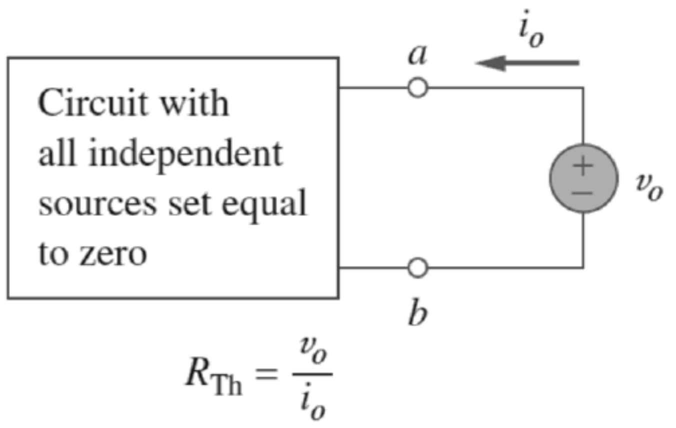
$R_{Th} = R_{in}$

Thevenin's Theorem (Cont…)

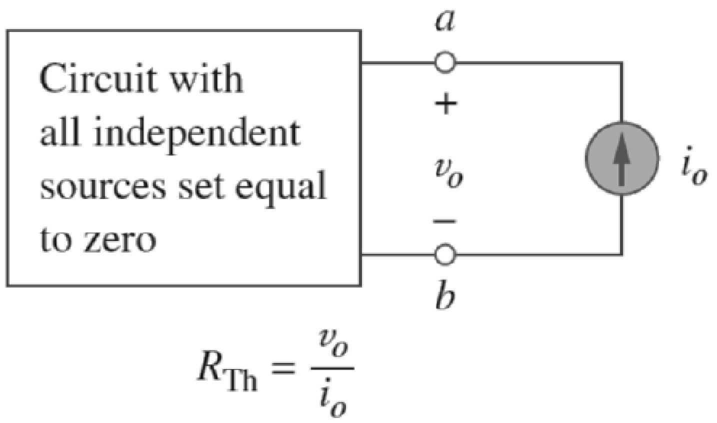
- To find out R_{TH} we need to consider two cases.
- **Case 1:**
  - ❖ If the network has no dependent sources, we turn off all the independent sources.
  - ❖ R_{TH} is the input resistance of the network looking between terminals a and b, as shown in the previous figure.
- **Case 2:**
  - ❖ If the network has dependent sources, we turn off only all independent sources.
  - ❖ The dependent sources cannot be turned off as they are controlled by circuit variables.
  - ❖ We apply a voltage v_0 at terminals a-b and determine the current i_0.

Thevenin's Theorem (Cont…)

- Then, R_{TH} is given by $\rightarrow R_{TH} = {}^{v_0}/_{i_0}$



- Alternatively, R_{TH} can be evaluated by inserting a current source as shown in the figure below.



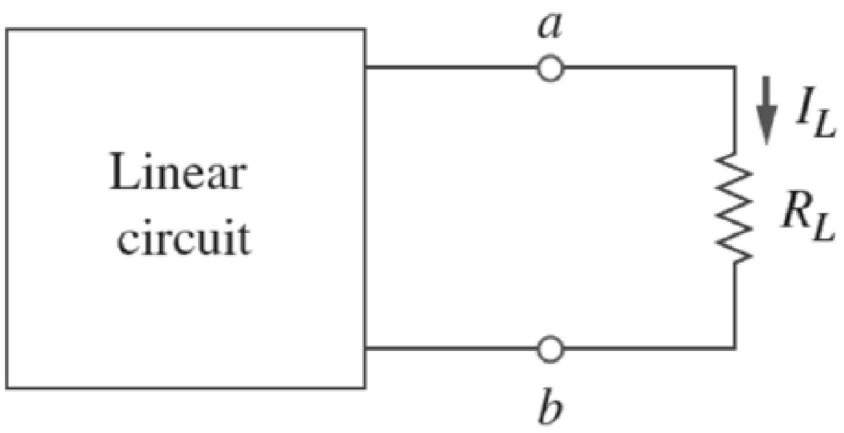
- Again, R_{TH} is ${}^{v_0}/_{i_0}$.

Thevenin’s Theorem (Cont…)

- Both approaches will give the same result.
- In either approach the value of v_0 and i_0 may be assumed to be equal to any value.
- It often occurs that R_{TH} takes a negative value. This implies that the circuit is delivering power.
- Negative value of R_{TH} is also possible in the circuit with dependent sources.
- Thevenin’s theorem is very important in the circuit analysis because helps in simplifying the circuit.
- A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

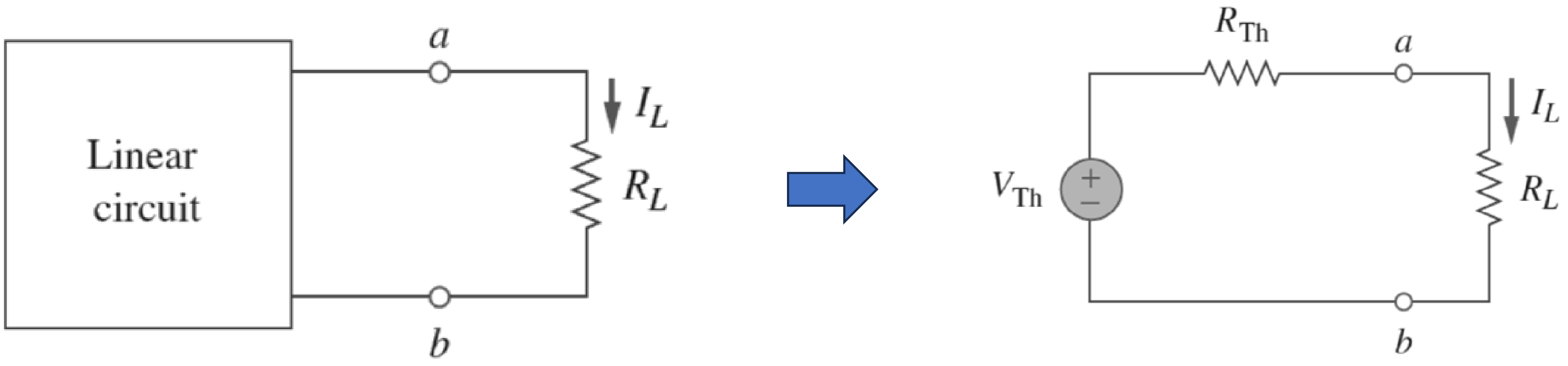
Thevenin's Theorem (Cont…)

- As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.
- The equivalent network behaves the same way externally as the original circuit.
- Let us consider a linear circuit terminated by a load R_L.



Thevenin's Theorem (Cont…)

- The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit are determined.
- The circuit can then be transformed as shown below.



- From the above figure we obtain,

$$I_L = {V_{Th}}/{(R_{Th} + R_L)}$$

$$V_L = I_L R_L = {V_{Th} R_L}/{(R_{Th} + R_L)}$$


THANKS
FOR YOUR
ATTENTION