- ***However, there may be situations when they may become unstable and break out into spontaneous oscillations***
- ***Potentially dangerous situation***, and the ***system should be protected against it***

- ***How does a negative feedback system become unstable***?
  - Write the ***loop gain*** expression in ***polar form***:

    $L(j\omega) = f(j\omega)A(j\omega) = |f(j\omega)A(j\omega)|\exp[j\phi(\omega)]$

    $\phi(\omega)$: ***Frequency dependent phase of the system***

- Consider a ***particular frequency*** ω_x, at which $\phi(\omega_x) = 180°$
- At ω_x, L would be a ***real number*** with ***negative sign***

  $\Rightarrow$ ***The feedback turns positive at this frequency***
- ***3 conditions may arise at ω_x***:
  - $|L| < 1$:
    - $A_f(j\omega_x) > A(j\omega_x)$, but the ***system will be stable***
  - $|L| = 1$:
    - $A_f(j\omega_x) \rightarrow \infty$, and ***output will appear without any input*** $\Rightarrow$ ***Oscillator***

- $|L| > 1$:
    - $A_f(j\omega_x) < A(j\omega_x)$, but the ***output will oscillate with gradually increasing amplitude***, and will ***eventually get limited by the nonlinearities present in the system***

- Thus, for a ***negative feedback system*** to turn into a ***positive feedback one***, the ***loop gain*** (L = fA) being ***equal to or less than –1*** is a ***sufficient and necessary condition***
- ***For this to happen***, the ***magnitude of the loop gain*** (L) ***should be equal to or greater than unity***, and the ***total phase around the loop should be 180°***

Transfer Function & Stability

- There is a ***strong correlation*** between the ***transfer function*** and ***stability*** of a system
- ***Single-Pole System***:
  - ***Transfer function*** with a ***negative real pole*** at ω_p:

$$A(j\omega) = \frac{A_0}{1 + j\omega/\omega_p}$$

A_0: ***Low-Frequency Gain***

- Now, assume that the ***system*** is ***connected*** in a ***feedback loop***, with the ***feedback network*** having ***feedback factor*** f

$\Rightarrow$ The ***closed-loop transfer function***:

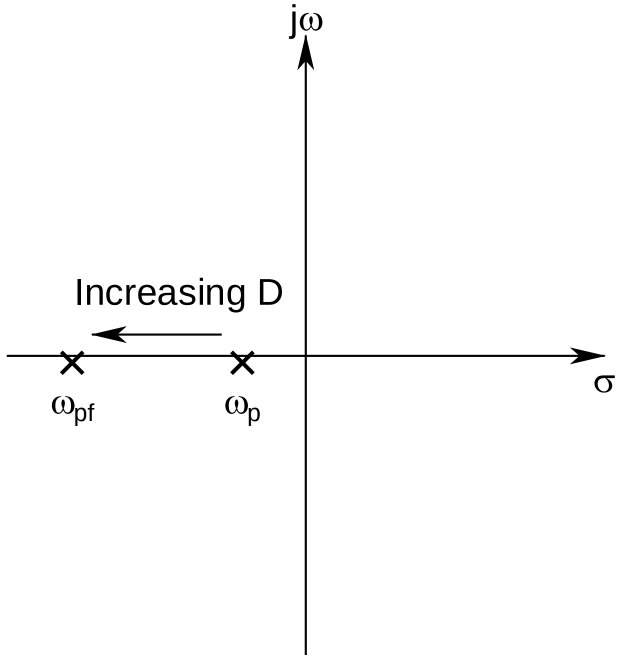
$$A_f = \frac{A_{f0}}{1 + j\omega/\omega_{pf}}$$

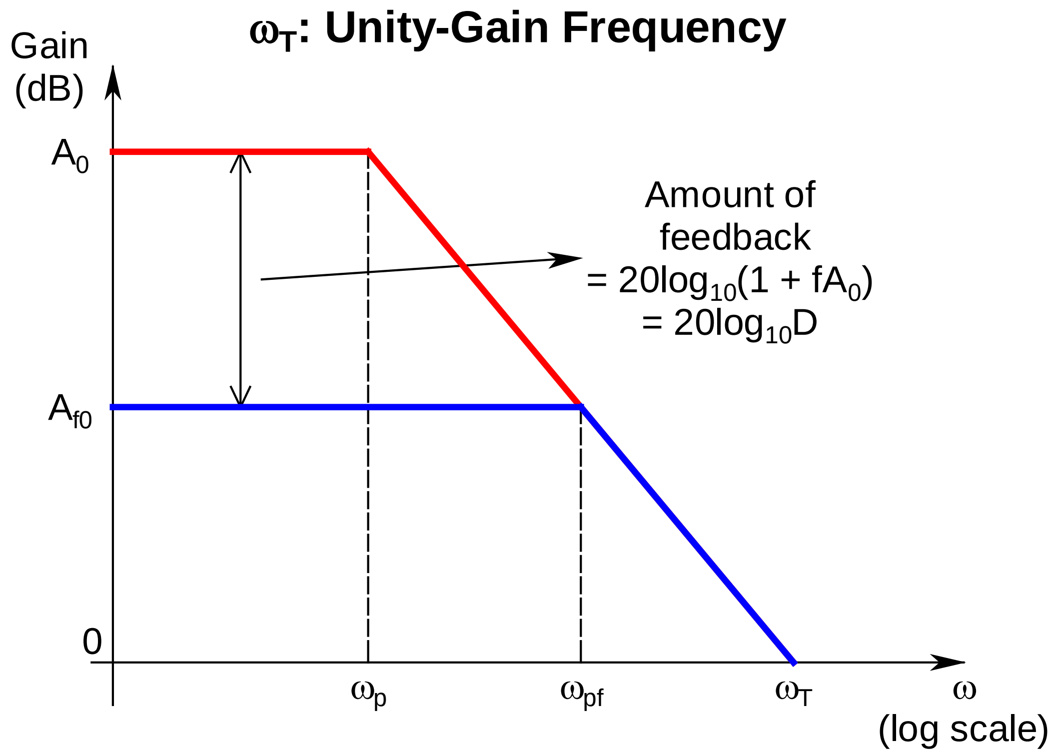
$$A_{f0} = A_0/(1 + fA_0) \text{ and } \omega_{pf} = \omega_p(1 + fA_0)$$

- The ***gain with feedback reduces by the same amount as the bandwidth gets increased***, ***keeping the GBP constant***

- Thus, the ***new pole frequency*** is D (the ***return difference***) times the ***old pole frequency***
  - $\Rightarrow$ It shifts ***left*** along the σ ***axis*** in the ***s-plane***, and ***remains on the LHP without any imaginary component***
  - $\Rightarrow$ ***The system remains stable even with feedback***
- Also, the ***phase*** of the system ***can never fall below*** $-90°$
- Here, of course we are assuming a ***passive feedback network***, i.e., ***f is a real number***

- Thus, ***f does not add any phase to the system***
- Hence, ***Barkhausen's criteria can never be satisfied for this case***
- Also, the ***pole can never enter the RHP***
- Thus, we ***conclude***:
    - A system with ***single-pole transfer function*** is ***Unconditionally Stable***, i.e., it will ***remain stable*** for ***values of f*** all the way ***up to unity*** (i.e., ***the entire output fed back to the input***)





Movement of the Pole for a Single-Pole System Under Negative Feedback and the Bode Plot of the Gain

- ***Two-Pole System***:
  - ***Transfer Function***:

$$A(s) = \frac{A_0}{\left(1 + j\omega/\omega_{p1}\right)\left(1 + j\omega/\omega_{p2}\right)}$$

A_0: ***Low-Frequency Gain***

ω_{p1}, ω_{p2}: ***Two negative real poles***, ***lying on the σ axis***, ***with $\omega_{p2} > \omega_{p1}$***

  - Now, with ***passive feedback*** with ***feedback factor*** f, the ***locations*** of the ***closed-loop poles*** can be found from: $1 + fA(s) = 0$

- Thus:

$$s^2 + (\omega_{p1} + \omega_{p2})s + (1 + fA_0)\omega_{p1}\omega_{p2} = 0$$

- ***Solution*** gives the ***locations*** of the ***two closed-loop poles***:

$$s_1,\ s_2 = -\frac{\omega_{p1} + \omega_{p2}}{2} \pm \frac{1}{2}\sqrt{\left(\omega_{p1} + \omega_{p2}\right)^2 - 4\left(1 + fA_0\right)\omega_{p1}\omega_{p2}}$$

- With ***increase in feedback***, the ***second term reduces***

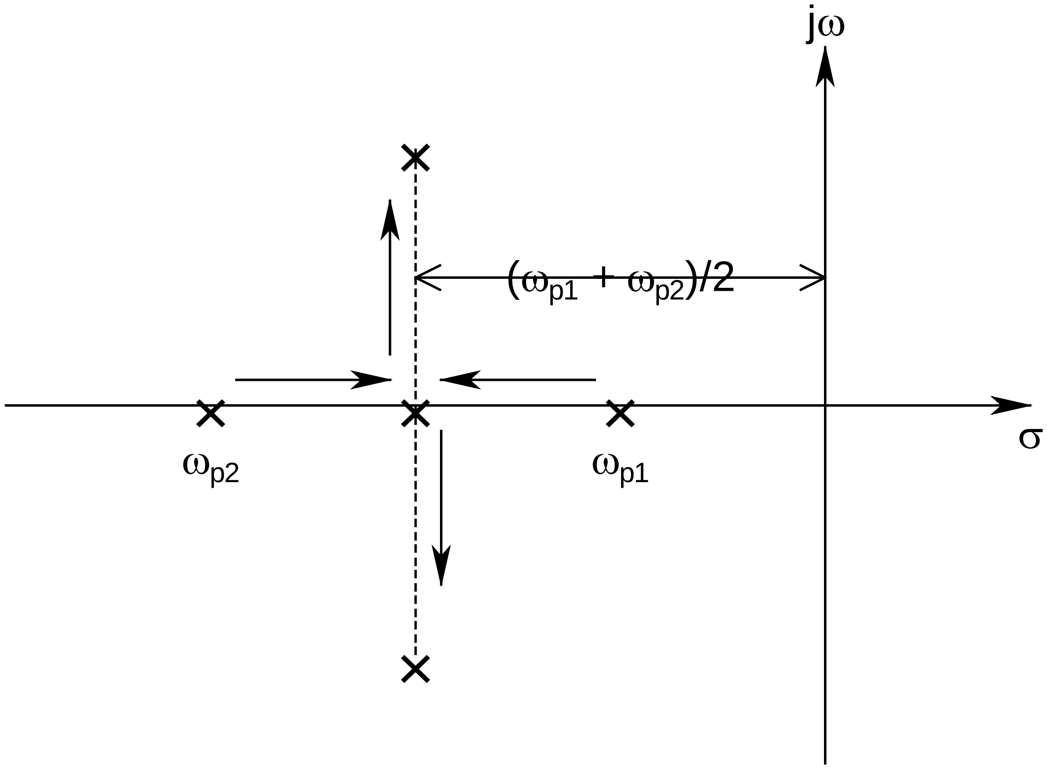
  $\Rightarrow$ ***s_1 and s_2 start to move towards each other along the σ axis***

- Eventually, at a ***particular feedback***, the ***second term would vanish***

- ***At this point***, the ***two poles will merge*** at $(\omega_{p1} + \omega_{p2})/2$
- With ***further increase in feedback***, the ***second term becomes imaginary***, while the ***first term remains constant***

  $\Rightarrow$ ***The poles remain complex conjugates***
- ***Even for f all the way up to unity***, ***when the entire output is fed back to the input***, the ***poles remain in the LHP*** and ***can never enter RHP***

  $\Rightarrow$ ***The system remains unconditionally stable***



Movement of the Poles for a Two-Pole System Under Negative Feedback With Increasing D

- Also, for a ***two-pole system***, the ***phase reaches*** $-180°$ ***only when the frequency becomes infinite*** (***mathematically***)
  - $\Rightarrow$ ***There is no physically achievable frequency when this can happen***
  - $\Rightarrow$ ***Unconditional Stability***

- ***System With Three (or More) Poles***:
  - ***Actual mathematical analysis quite tedious***
  - It can be shown that as the ***amount of feedback (D) is increased***:
    - ***The highest frequency pole (ω_{p3}) moves outward along the $-\sigma$-axis***

- ***The other two poles (ω_{p1} and ω_{p2}) move towards each other*** (***similar to a two-pole system***)
- As ***D is increased further***, these ***two poles eventually merge***, and then start having ***imaginary components***
- Their ***real part*** also ***keeps on changing with D***, ***keeping the nature of complex conjugacy intact***, and ***moves right in the s-plane***
- ***The path traced out by these poles is known as the root locus***
- ***For a particular value of D***, this ***root locus intersects the imaginary axis of the s-plane at two symmetric points***

- ***Under this condition***, ***sustained sinusoidal oscillation can be achieved***, ***since it now has a complex conjugate pair of poles without any real part*** (ω_{p3} ***will be so large that it will be inconsequential***)
- ***With further increase in D***, the ***root locus enters the RHP*** with the ***poles now having positive real part***

  $\Rightarrow$ ***Potentially dangerous situation in terms of stability***
- ***In terms of phase***, the ***total can be –270°***

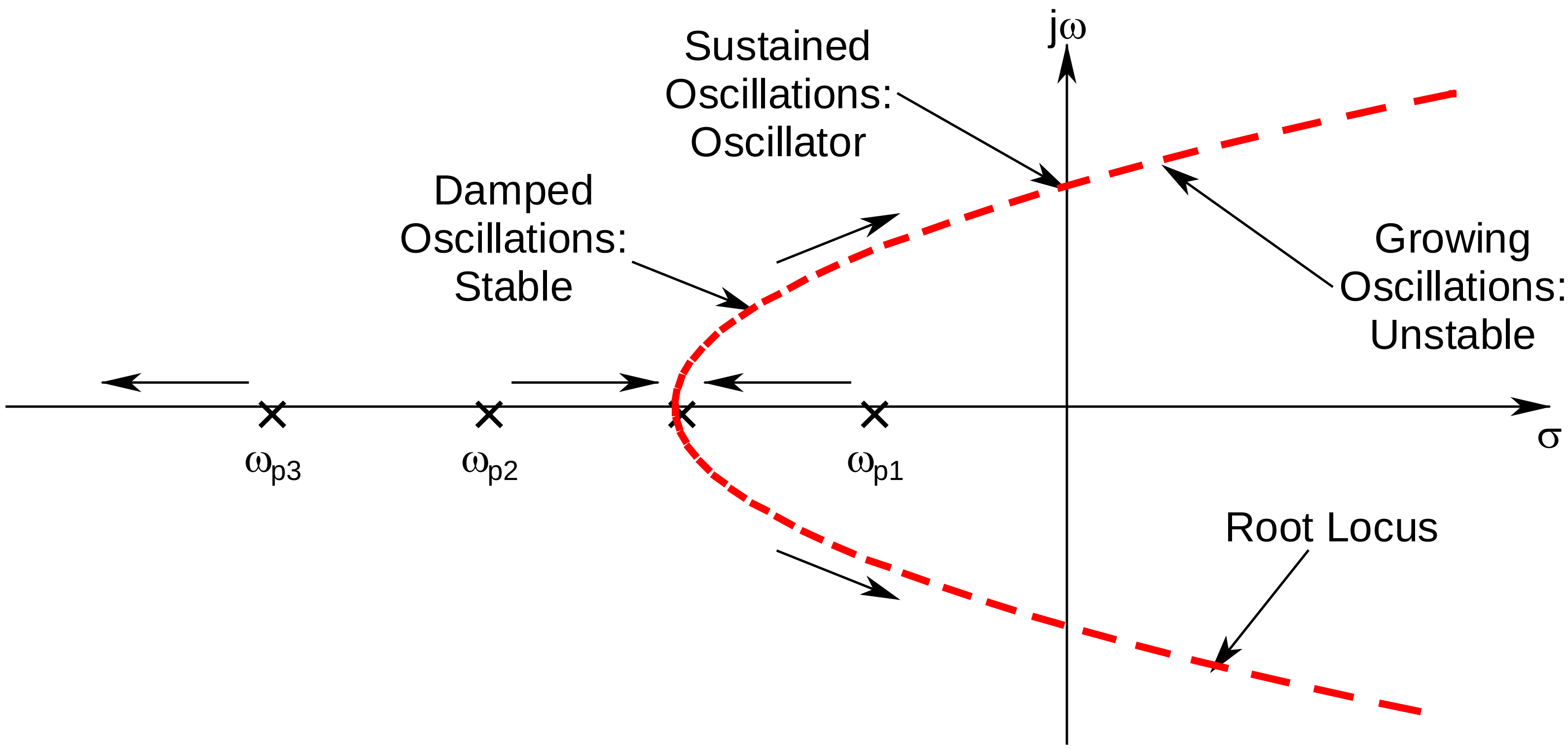
  $\Rightarrow$ ***There exists a particular value of f***, for which the ***phase will become –180°***

- ***Under this condition***, ***if the magnitude of the loop gain is exactly unity***, ***then the system will break out into spontaneous oscillation***, however, ***the amplitude will be controlled***

  $\Rightarrow$ ***Sustained sinusoidal oscillation***

- ***This particular value of f is known as the critical feedback factor (f_{crit}) for oscillation***
  - ❖ For $f < f_{crit}$, the ***system will be stable***
  - ❖ For $f > f_{crit}$, the ***system will be unstable***

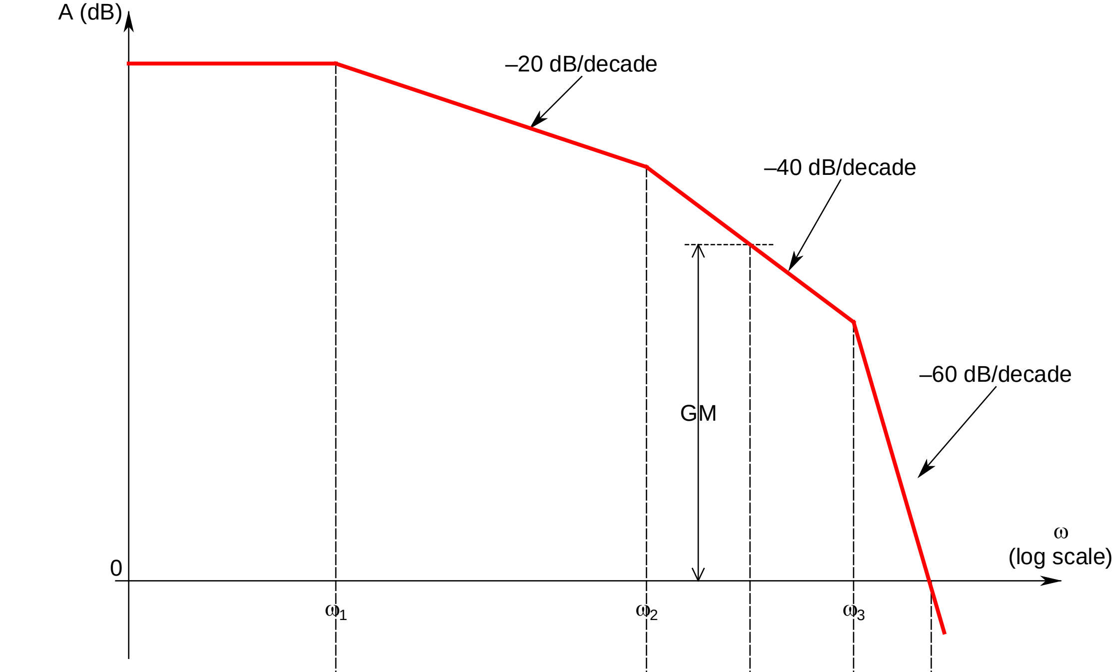
- Thus, the system is ***NOT Unconditionally Stable***, but ***stable only till a specific value of f***
  - ❖ Known as ***Conditionally Stable System***



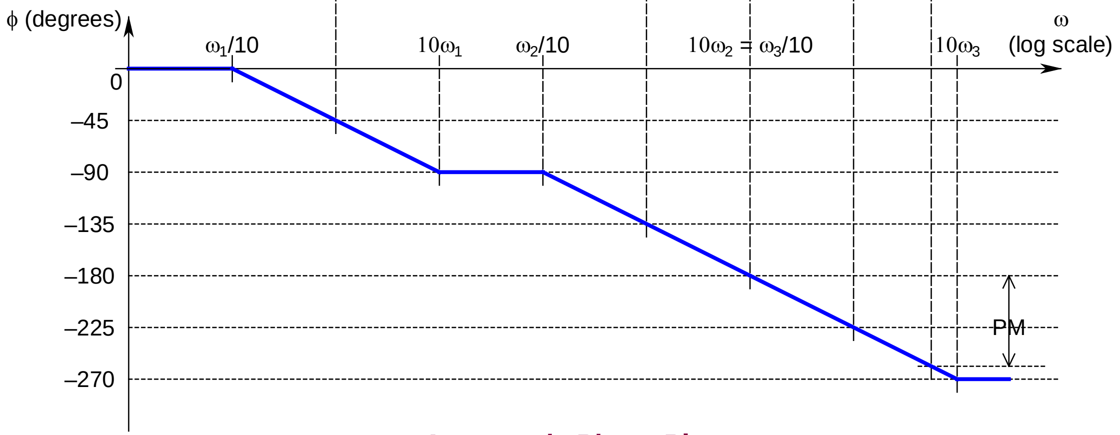
Root Locus of the Poles of a Three-Pole System as D is Increased

Stability Study Using Bode Plot

- The ***most convenient*** and the ***most useful***
- ***Recall***: ***Single- and Two-Pole Systems are unconditionally stable***
- Consider a ***Three-Pole System***, with the ***pole frequencies*** at ω_1, ω_2, and ω_3, with $\omega_3 = 100\omega_2$, and $\omega_2 > 100\omega_1$
- ***Note***: ***A = L if f = 1*** (***100% feedback***)
- Refer to the next slide (***Bode Plot***)



Asymptotic Gain Plot



Asymptotic Phase Plot

- ***Profile of A*:**
  - ***Remains constant at its low-frequency value for $\omega \leq \omega_1$***
  - ***Then drops @ 20 dB/decade till ω_2***
  - ***Followed by a drop @ 40 dB/decade till ω_3***
  - ***Then drops @ 60 dB/decade***
  - ***Finally crosses 0 dB at ω slightly less than $10\omega_3$***
- ***Profile of ϕ*:**
  - ***Remains zero till $\omega_1/10$***
  - ***Then drops @ 45°/decade***

- ***Reaches*** $-90°$ ***at*** $10\omega_1$
- ***Stays constant at*** $-90°$ ***till*** $\omega_2/10$
- ***Then starts to drop again @ 45°/decade till*** $10\omega_3$
- ***Reaches*** $-180°$ ***at*** $10\omega_2$ $(= \omega_3/10)$ ***and*** $-270°$ ***at*** $10\omega_3$

• ***Gain Margin*** (***GM***) and ***Phase Margin*** (***PM***):

- ***Extremely important terms with regard to stability of a system***
- ***From the sign and magnitude of these terms***, ***the stability of the system can be predicted***

- ***GM = A(dB)*** (***when*** $\phi = -180°$)
- $PM = 180° - |\phi|$ (***when A = 0 dB***)
- In our example, ***GM is positive*** (as shown in the figure)
- This is ***potentially a dangerous situation***, and characterizes a ***highly unstable system***
  - For ***positive GM***, ***with each pass around the loop***, the ***output amplitude will keep on growing***
- On the contrary, if ***GM were negative***, ***with each pass around the loop***, the ***output amplitude would have decreased***

- *The system would have come out of any unwanted oscillations*

- ***The GM dictates the maximum amount of feedback that can be allowed for the system to remain stable***
- ***For an unconditionally stable system, GM must be negative***

  $\Rightarrow$ ***A must be negative when*** $\phi = -180°$
- ***With regard to phase, when A crossed 0 dB,*** ϕ ***is close to*** $-270°$

  $\Rightarrow$ ***PM is negative, with a value of*** $\sim -90°$

- ***This also implies that when ϕ crossed $-180°$***, ***A of the system was greater than unity*** (***0 dB***)
  - ***A potentially dangerous situation in terms of stability***
- Therefore, ***for an unconditionally stable system***, ***PM must be positive***
- ***The two conditions with regard to GM and PM are actually correlated***
- ***Rule of Thumb***:
  - ***For a stable system***, ***GM ~ -10 dB and PM ~ $45°$ are generally good enough***