Assignment 14

MSO-201: Probability and Statistics

1. Suppose $X_1, \ldots, X_n$ is a random sample from a Poisson distribution with parameter λ, i.e. it has the PMF as

$$P(X = i) = \frac{e^{-\lambda}\lambda^i}{i!}; \quad i = 0, 1, 2, \ldots.$$

Find the MLE of λ.

2. In the above problem if it is known that λ can take only positive integer values, find the MLE of λ.

3. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, 1)$ and $Y_1, \ldots, Y_m$ be a random sample from $N(2\theta, 1)$. Based on $X_1, \ldots, X_n, Y_1, \ldots, Y_m$ find the MLE of θ.

4. Let $X_1, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Find the MLE of σ^2. Find the MLE of σ.

5. Find a 95% confidence interval of σ^2.

6. If it is known that $1 \le \sigma^2 \le 2$, find the MLE of σ^2.

7. Suppose $X_1, \ldots, X_n$ is a random sample from the following PDF:

$$f(x|\mu) = e^{-(x-\mu)}; \qquad x > \mu,$$

and zero, otherwise. Find a 99% confidence interval of μ based on the MLE of μ.

8. Suppose $\{X_1, \ldots, X_n\}$ is a random from $\mathrm{Exp}(\theta_1)$, i.e. it has the PDF $f_X(x) = \frac{1}{\theta_1}e^{-x/\theta_1}$, for $x > 0$, and $\{Y_1, \ldots, Y_m\}$ is a random from $\mathrm{Exp}(\theta_2)$, i.e. it has the PDF $f_Y(y) = \frac{1}{\theta_2}e^{-x/\theta_2}$, for $y > 0$. If $\mu = P(X_1 < Y_1)$, find the MLE of μ.

9. Suppose $\{X_1, \ldots, X_n\}$ is a random from $\mathrm{Exp}(\theta)$, i.e. it has the PDF $f_X(x) = \frac{1}{\theta}e^{-x/\theta}$. Suppose for $i = 1, \ldots, n$,

$$Y_i = \begin{cases} 0 & \text{if} \quad X_i < 10 \\ 1 & \text{if} \quad X_i \ge 10 \end{cases}$$

Find the MLE of θ, based on $\{Y_1, \ldots, Y_n\}$.

10. Suppose $\{X_1, \ldots, X_n\}$ is a random from $\mathrm{Exp}(\theta)$, i.e. it has the PDF $f_X(x) = \frac{1}{\theta}e^{-x/\theta}$, for $x > 0$. Find a 95% confidence interval $1/\theta$.