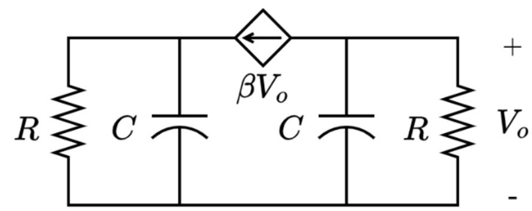
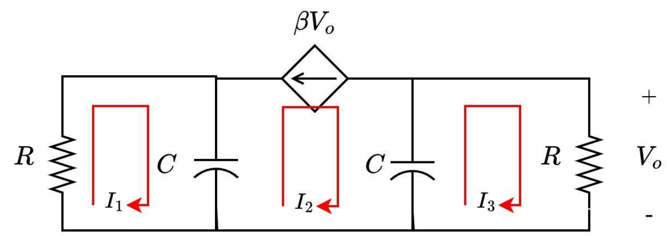
ESO203 Tutorial 5

Question 1:

For what value of β is the circuit in given figure stable?



Solution:



$$V_o = I_3 R$$

In Laplace transformation,

Applying KVL in mesh 1:

$$I_1\left(R+\frac{1}{sC}\right)-I_2\left(\frac{1}{sC}\right)=0$$

Applying KVL in mesh 2:

$$I_2=-\beta V_o=-\beta I_3 R$$

Applying KVL in mesh 3:

$$-I_2\left(\frac{1}{sC}\right)+I_3\left(R+\frac{1}{sC}\right)=0$$

Converting to matrix form:

$$\begin{bmatrix}\left(R+\frac{1}{sC}\right) & \left(-\frac{1}{sC}\right) & 0\\ 0 & 1 & \beta R\\ 0 & \left(-\frac{1}{sC}\right) & \left(R+\frac{1}{sC}\right)\end{bmatrix}\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

The determinant of the matrix is given by

$$\Delta=\left(R+\frac{1}{sC}\right)\left[\left(R+\frac{1}{sC}\right)+\frac{\beta R}{sC}\right]$$

Equating the determinant equal to 0, we get

$$\Delta=\left(R+\frac{1}{sC}\right)\left[\left(R+\frac{1}{sC}\right)+\frac{\beta R}{sC}\right]=0$$

$$\frac{s^2R^2C^2+s(2RC+\beta R^2C)+(\beta R+1)}{s^2C^2}=0$$

$$s^2R^2C^2 + s(2RC + \beta R^2C) + (\beta R + 1) = 0$$

For system to be stable, all the poles should be left hand side of y-axis of the s-plane.

By solving equation, we get

$$s_1 = -\frac{1}{RC} \quad , \quad s_2 = \frac{-2RC - 2\beta R^2C}{2R^2C^2}$$

For poles to be left hand side of s-plane,

$$s_2 = \frac{-2RC - 2\beta R^2C}{2R^2C^2} < 0$$

$$(\beta R + 1) < 0$$

$$\beta > -\frac{1}{R}$$

Question 2:

Synthesize the network that has an impedance function as given. Assume the resistance value is 1 ohm.

$$Z_1(s) = \frac{s^2 + 7s + 70}{s(s + 10)}$$

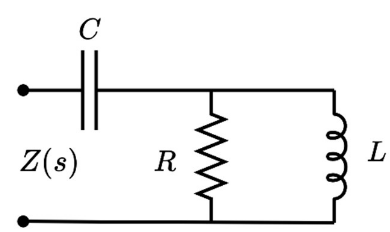
Solution:

For the given transfer function, do the partial fraction expansion, we get,

$$Z_1(s) = \frac{s^2 + 7S + 70}{S(S + 10)} = \frac{7}{s} + \frac{s}{s + 10}$$

The function is realized by a capacitor with a value of $\frac{1}{7}F$ and the second function can be realized by a parallel combination of a resistor and an inductor.

The possible synthesized circuit is given in the figure.



The impedance function can be derived from the circuit, as given below.

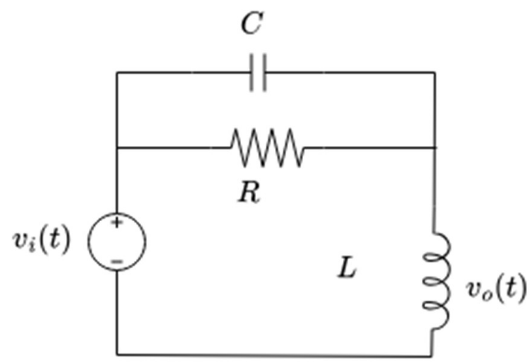
$$Z_2(s) = \frac{1}{sC} + \frac{sRL}{R + sL}$$

Comparing the $Z_1(s)$ and $Z_2(s)$, the obtained parameters with $R = 1\Omega$, are

$$C = \frac{1}{7}F \text{ and } L = \frac{1}{10}H$$

Question 3:

Given the transfer function $H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s^2+50s}{s^2+50s+}$. Realize the function using the circuit shown below. Find the value of L, for the given $R = 10\Omega$.



Solution:

Let Z_1 is equivalent to parallel combination of R and C.

$$Z_1(s) = \frac{R}{1 + sRC}$$

And similarly,

$$Z_2(s) = sL$$

The transfer function $H(s)$ can be obtained by voltage divide rule.

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$H(s) = \frac{sL}{\frac{R}{1 + sRC} + sL}$$

$$H(s) = \frac{sL(1 + sRC)}{\mathrm{R} + sL(1 + sRC)}$$

$$H(s) = \frac{s^2 + \frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Equating with given transfer function, we get

$$\frac{1}{RC} = 50$$

$$C = 2mF$$

$$\frac{1}{LC} = 20$$

$$L = 25H$$

Question 4:

A three-phase, four-wire system operating with a 208-V line voltage is shown in below figure. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method.

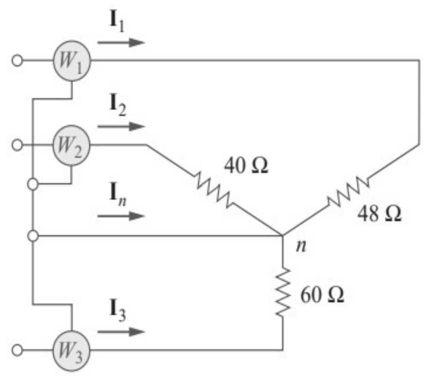
Calculate:

(a) The line to neutral voltage.

(b) The currents I_1, I_2, I_3 and I_n.

(c) The readings of the wattmeters.

(d) The total power absorbed by the load.



Solution:

Assume V_{AN}, V_{BN} and V_{CN} are the phase voltage and Z_{AN}, Z_{BN} and Z_{CN} are the per phase impedance of phase a, b and c respectively

(a) The Phase voltage

$$V_{Ph} = \frac{V_{LL}}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120.08\, V$$

(b) The currents $I_1,\ I_2,\ I_3$ and I_n

$$I_1 = \frac{V_{AN}}{Z_{AN}} = \frac{120.08\angle 0^\circ}{48} = 2.50\angle 0^\circ\ A$$

$$I_2 = \frac{V_{BN}}{Z_{BN}} = \frac{120.08\angle 120^\circ}{40} = 3.002\angle 120^\circ\ A$$

$$I_3 = \frac{V_{CN}}{Z_{CN}} = \frac{120.08\angle -120^\circ}{60} = 2\angle -120^\circ\ A$$

$$I_n = -(I_1 + I_2 + I_3)$$

$$I_n = -(2.50\angle 0^\circ + 3.002\angle 120^\circ + 2\angle -120^\circ)$$

$$I_n = 0.868\angle -90^\circ$$

(c) The readings of the wattmeter:

$$P_1 = V_{AN} I_1 \cos\left(\theta_{v_{AN}} - \theta_{i_1}\right) = 120.08 \times 2.50 \times \cos(0 - 0) = 300.2\ W$$

$$P_2 = V_{BN} I_2 \cos\left(\theta_{v_{BN}} - \theta_{i_2}\right) = 120.08 \times 3.002 \times \cos(120 - 120) = 360.48\ W$$

$$P_3 = V_{CN} I_3 \cos\left(\theta_{v_{CN}} - \theta_{i_3}\right) = 120.08 \times 2 \times \cos(-120 + 120) = 240.16\ W$$

(d) <u>The total power absorbed by the load:</u>

$$P_T = P_1 + P_2 + P_3$$

$$P_T = 300.2 + 360.48 + 240.16 = 900.84\ W$$

<u>Question 5:</u>

The primary current to an ideal transformer rated $\frac{2200}{110}V$ is $5A$ .Calculate

(a) the turns ratio,

(b) the KVA rating,

(c) the secondary current.

<u>Solution:</u>

This is a step-down transformer since $V_1 = 2200V > V_2 = 110V$

a.

$$n = \frac{V_2}{V_1}$$

$$n = \frac{110}{2200} = \frac{1}{20}$$

b.

$$S = V_1 * I_1 = V_2 * I_2$$

$$S = 2200V * 5A = 11kVA$$

c.

$$n = \frac{I_1}{I_2}$$

$$\frac{1}{20} = \frac{5}{I_2}$$

$$I_2 = 100A$$

Question 6:

For the Y-△ network shown in the figure, find the following.

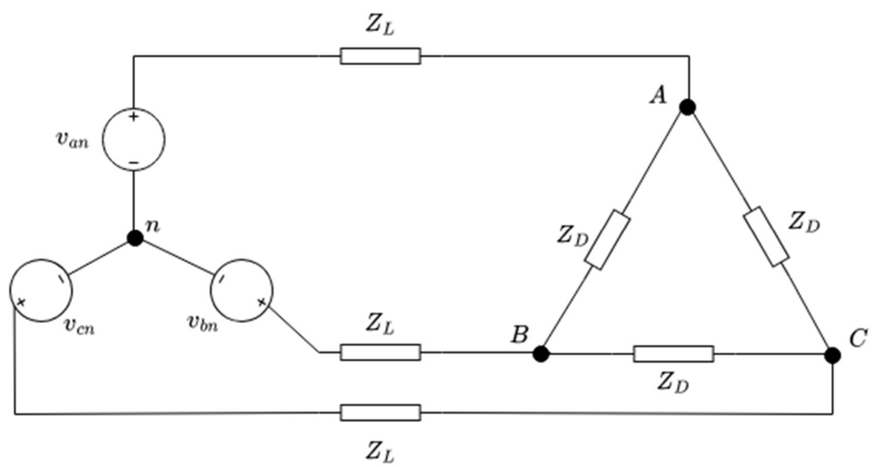
a. I_{AB} , $I_{BC,}$ and I_{CA}

b. I_a and I_β of $source$ v_{an} and v_{bn}

Given the following details of the network:

$$v_{an} = 100\angle 0^o\ V, v_{bn} = 100\angle -120^o\ V, and\ v_{cn} = 100\angle 120^o\ V$$

$$Z_L = 8 + j4\ \Omega, and\ Z_D = 12 + j6\ \Omega$$



Solution:

Step 1: Convert Δ Load to Y Load

The Y-equivalent impedance of a Δ load is given by:

$$Z_Y = \frac{Z_D}{3}$$

Substituting $Z_D = 12 + j6$

$$Z_Y = \frac{12 + j6}{3} = 4 + j2$$

Step 2: Total Y Load Impedance

The total impedance in each Y branch is

$$Z_{total} = Z_L + Z_Y$$

$$Z_{total} = (8 + j4) + (4 + j2) = 12 + j6$$

Step 3: Calculate Phase Currents

Using Ohm's law, the phase currents are:

$$I_\alpha = \frac{V_{an}}{Z_{total}}, \quad I_\beta = \frac{V_{bn}}{Z_{total}}, \quad I_\gamma = \frac{V_{cn}}{Z_{total}}$$

Magnitude of Z_{total}:

$$|Z_{total}| = \sqrt{(12^2 + 6^2)} = 13.42$$

Phase angle:

$$\theta_{Z_{total}} = \tan^{-1}(\frac{6}{12}) = 26.57°$$

Now, computing each current:

$$I_\alpha = \frac{100}{13.42}\angle(-26.57°) = 7.45\angle(-26.57°)\ A$$

$$I_\beta = 7.45\angle(-120° - 26.57°) = 7.45\angle(-146.57°)\ A$$

$$I_\gamma = 7.45\angle(120° - 26.57°) = 7.45\angle 93.43°\ A$$

Step 4: Convert Back to Δ Load Currents

To find the delta currents:

$$I_{AB} = I_\alpha - I_\beta$$

$$I_{BC} = I_\beta - I_\gamma$$

$$I_{CA} = I_\gamma - I_\alpha$$

Substituting values:

$$I_{AB} = 7.45\angle(-26.57°) - 7.45\angle(-146.57°)$$

$$I_{BC} = 7.45\angle(-146.57°) - 7.45\angle(93.43°)$$

$$I_{CA} = 7.45\angle(93.43°) - 7.45\angle(-26.57°)$$

These can be computed numerically using phasor subtraction.

Final Answer:

$$I_\alpha = 7.45\angle(-26.57°)\ A, \ I_\beta = 7.45\angle(-146.57°)\ A, \ I_\gamma = 7.45\angle(93.43°)\ A$$

$$I_{AB} = 12.9\angle(-3.43°)A, \ I_{BC} = 12.9\angle(-116.57°)A, \ I_{CA} = 12.9\angle(123.43°)A$$

<u>Question 7:</u>

A balanced three-phase Y-connected load is supplied by a 415V, 50Hz three-phase supply. The total power consumed by the load is 10 kW, and the power factor is 0.5 lagging. Using the two-wattmeter method, determine:

1. The readings of the two wattmeters (W_1 and W_2).

2. The phase current $\left(I_{ph}\right)$.

3. What happens to W_1 and W_2 if the power factor changes to 0.5 leading instead of lagging?

<u>Solution:</u>

<u>Step 1</u>: Given Data

• Line Voltage: $V_L = 415V$

• Power Factor: $cos\phi = 0.5$ (Lagging) ⇒ $\phi = 60°$

• Total Power: P = 10 kW

<u>Step 2:</u> Find the Line Current I_L

The total power in a three-phase system is given by:

$$P = \sqrt{3} \times V_L \times I_L \times cos\phi$$

Substituting the given values:

$$10{,}000 = \sqrt{3} \times 415 \times I_L \times 0.5$$

$$I_L = \frac{10000}{\sqrt{3} \times 415 \times 0.5} = 27.8\,A$$

Since the load is Y-connected, the phase current is:

$$I_{ph} = I_L = 27.8A$$

<u>Step 3:</u> Find the Wattmeter Readings W1 and W2

The wattmeter readings are given by:

$$W_1 = V_L \times I_L \times cos(30° + \phi)$$

$$W_2 = V_L \times I_L \times cos(30° - \phi)$$

Substituting $\phi = 60°$:

$$W_1 = 415 \times 27.8 \times cos(90°) = 0$$

$$W_2 = 415 \times 27.8 \times cos(-30°) = 10{,}000W$$

Thus, $W1 = 0\,W, \quad W2 = 10{,}000\,W$

This confirms that when cos ϕ = 0.5 lagging, one wattmeter reads zero and the other reads the full power.

Step 4: If Power Factor Becomes 0.5 Leading

For $cos\phi$ = 0.5 leading, the only change is the sign of ϕ (i.e., ϕ = -60° instead of +60°).

New wattmeter readings:

$$W_1 = V_L \times I_L \times cos(30° - (-60°)) = 0W$$

$$W_2 = V_L \times I_L \times cos(30° + (-60°)) = 10{,}000W$$

This means the wattmeter readings remain the same, as the power is unchanged.

Final Answer

1. For $cos\phi$ = 0.5 (lagging or leading):

$$W_1 = 0W, \quad W_2 = 10{,}000W$$

2. Phase Current: $I_{ph} = 27.8A$

3. If the power factor is leading instead of lagging, the wattmeter readings remain the same.

Key Learning

• At ϕ = 60° lagging, one wattmeter reads zero.

• If the power factor is leading instead of lagging, the wattmeter readings remain unchanged.

• For power factor angles closer to 90°, one wattmeter reading becomes negative.