

Transmission Lines - IV

Prof. M. Jaleel Akhtar

mjakhtar@iitk.ac.in

<https://home.iitk.ac.in/~mjakhtar/>

**Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur – 208016 (U.P.)**



Transmission Lines as Circuit Elements

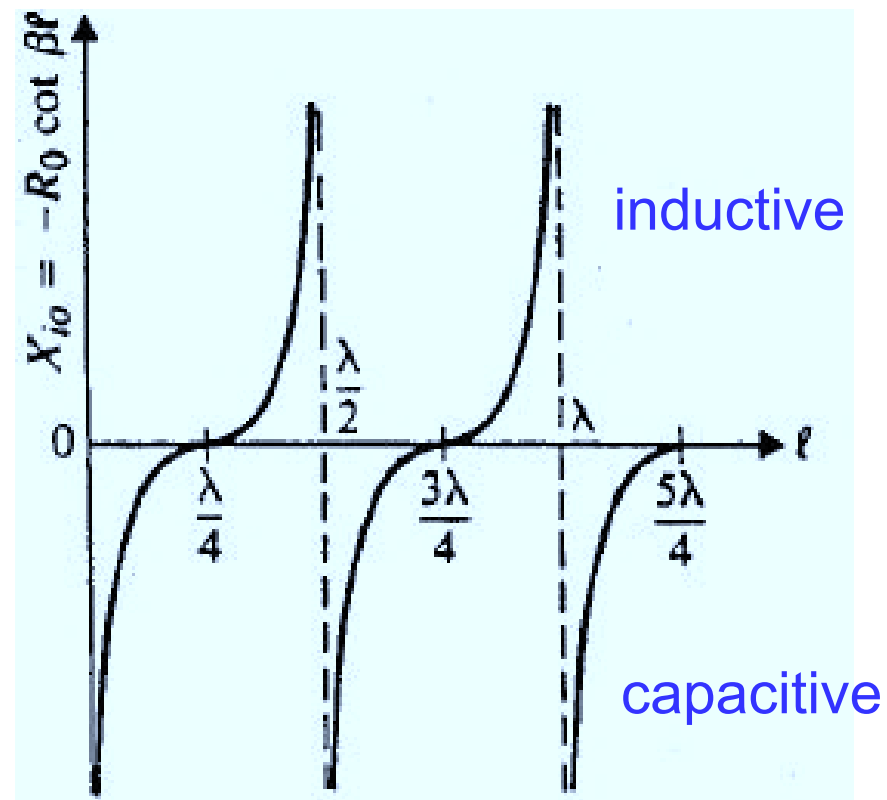
☞ Input impedance

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

Case-1: Open Circuit Termination

$$Z_{io} = jX_{io} \equiv -j \frac{1}{\omega C l}$$

☞ Impedance of a capacitor of Cl Farads.



Transmission Lines as Circuit Elements

Case-2: Short Circuit Termination

$$Z_L = 0$$

$$Z_{is} = jR_0 \tan \beta l$$

👉 Reactive

$$Z_{is} = jX_{is} = jR_0 \tan \beta l$$

$$\beta l = \frac{2\pi}{\lambda} l$$

$$\beta l \ll 1$$

Electrically short line

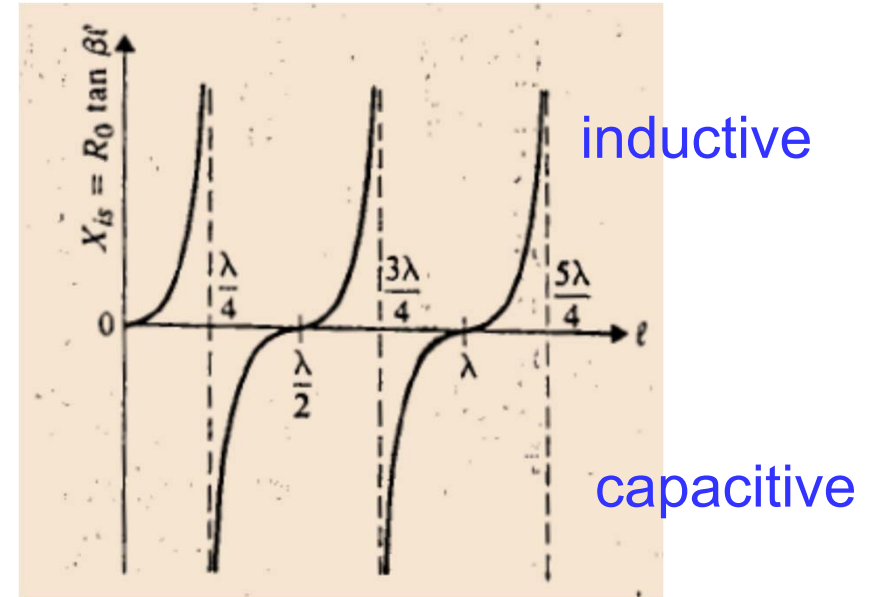
$$Z_{is} = jX_{is} = jR_0 \tan \beta l \cong jR_0 \beta l$$

$$\cong j\sqrt{\frac{L}{C}} \omega \sqrt{LC} l$$

$$\equiv j\omega L l$$

👉 Impedance of an inductor of Ll Henrys.

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$



Transmission Line Sections

Case-3: Quarter Wave Section

$$l = \frac{\lambda}{4}; \quad \beta l = \frac{\pi}{2} \quad \longleftrightarrow \quad l = (2n - 1) \frac{\lambda}{4}; \quad \beta l = (2n - 1) \frac{\pi}{2}$$

$$n = 1, 2, 3, \dots$$

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l} \quad \tan \beta l \rightarrow \pm \infty$$

$$Z_{in} = R_0 \frac{\frac{Z_L}{\tan \beta l} + jR_0}{\frac{R_0}{\tan \beta l} + jZ_L}$$



$$Z_i = \frac{R_0^2}{Z_L}$$

➤ An open-circuited, quarter wave line appears as a short circuit at the input terminals, and *vice-versa*.

□ The quarter wave line may be used as a **matching transformer**.



Transmission Line Sections

Case-4: Half Wave Section

$$l = n \frac{\lambda}{2}; \quad \beta l = n\pi$$

$$\tan \beta l = 0 \quad n = 1, 2, 3, \dots$$

$$Z_{in} = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$



$$Z_i = Z_L$$

- A Half wave lossless line transfers the load impedance to the input terminals without change.

Characterization of Transmission Lines

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

Short-circuited line:

$$Z_L = 0$$

$$Z_{is} = Z_0 \tanh \gamma l$$

Open-circuited line:

$$Z_L = \infty$$

$$Z_{io} = Z_0 \coth \gamma l$$

$$Z_0 = \sqrt{Z_{is} Z_{io}} \quad (\Omega)$$

$$\gamma = \frac{1}{l} \tanh^{-1} \sqrt{\frac{Z_{is}}{Z_{io}}} \quad (m^{-1})$$



Lossy Line - Short Circuit Termination

$$Z_L = 0$$

$$Z_{is} = Z_0 \tanh \gamma l$$

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

$$= Z_0 \frac{\sinh(\alpha + j\beta)l}{\cosh(\alpha + j\beta)l}$$

$$= Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$

$$l = n \frac{\lambda}{2}; \beta l = n\pi$$

$$\sin \beta l = 0$$

$$Z_{is} = Z_0 \tanh \alpha l \cong Z_0 \alpha l$$

☞ Half Wave Section

Small for a low loss line

☞ Series Resonant Circuit

$$l = (2n - 1) \frac{\lambda}{4}; \beta l = (2n - 1) \frac{\pi}{2}$$

$$\cos \beta l = 0$$

$$Z_{is} = \frac{Z_0}{\tanh \alpha l} \cong \frac{Z_0}{\alpha l}$$

☞ Quarter Wave Section

Large for a low loss line

☞ Parallel Resonant Circuit



Lossy Line - Short Circuit Termination

Frequency selective circuit

$$Z_L = 0$$

$$Z_{is} = Z_0 \tanh \gamma l$$

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

$$= Z_0 \frac{\sinh(\alpha + j\beta)l}{\cosh(\alpha + j\beta)l}$$

$$= Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$

$$l = n \frac{\lambda}{4}; \beta l = n \frac{\pi}{2}$$

$$f = f_0 + \delta f$$

$$\beta l = \frac{2\pi f}{v_p} l = \frac{2\pi(f_0 + \delta f)}{v_p} l = \frac{n\pi}{2} + \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right)$$

n is odd

$$\cos \beta l = \cos \left(\frac{n\pi}{2} + \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right) = -\sin \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right] \cong -\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right)$$

$$\sin \beta l = \sin \left(\frac{n\pi}{2} + \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right) = \cos \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right] \cong 1$$

$$Z_{is} = Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$



Lossy Line - Short Circuit Termination

$$Z_L = 0$$

Frequency selective circuit

$$Z_{is} = Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l}$$

$$\beta l = \frac{n\pi}{2} + \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right)$$

$$l = n \frac{\lambda}{4}; \beta l = n \frac{\pi}{2}$$

$$n \text{ is odd} \quad f = f_0 + \delta f$$

$$\cos \beta l \cong -\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \quad \sin \beta l \cong 1$$

Low loss line $\alpha l \ll 1$

$$\sinh \alpha l \cong \alpha l \quad \cosh \alpha l \cong 1$$

$$Z_{is} = Z_0 \frac{1 - j\alpha l \cos \beta l}{\alpha l - j \cos \beta l}$$

$$|\alpha l \cos \beta l| \ll 1$$

$$Z_{is} \cong Z_0 \frac{1}{\alpha l + j \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right]}$$

$$Z_{is} \cong \frac{Z_0 / \alpha l}{1 + j \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} \right]}$$

$$Z_{is, \max} = \frac{Z_0}{\alpha l}$$

Low loss line $\alpha l \ll 1$

$$Z_{is} \equiv \frac{Z_{is, \max}}{1 + j \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} \right]}$$



Lossy Line - Short Circuit Termination

Frequency selective circuit

$$Z_{is} \equiv \frac{Z_{is,max}}{1 + j \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} \right]}$$

$$\frac{|Z_{is}|^2}{|Z_{is,max}|^2} \equiv \frac{1}{1 + \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} \right]^2}$$

$$\delta f = \pm \frac{\Delta f}{2}$$

$$\frac{1}{1 + \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} \right]^2} = \frac{1}{2}$$

$$f_1 = f_0 - \delta f$$

$$f_2 = f_0 + \delta f$$

$$\Delta f = f_2 - f_1$$

👉 Half Power Bandwidth

👉 Half Power Frequencies

$$l = n \frac{\lambda}{4}; \beta l = n \frac{\pi}{2}$$

$$\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \frac{1}{\alpha l} = \frac{n\pi}{2l} \left(\frac{\Delta f}{2f_0} \right) \frac{1}{\alpha} = \frac{\beta}{2\alpha} \left(\frac{\Delta f}{f_0} \right) = 1$$

n is odd

$$Q \equiv \frac{f_0}{\Delta f} = \frac{\beta}{2\alpha}$$



Lossy Line - Short Circuit Termination

Frequency selective circuit

$$Q \equiv \frac{f_0}{\Delta f} = \frac{\beta}{2\alpha}$$

Low -Loss line ($R \ll \omega L$, $G \ll \omega C$)

Low loss line

$$\alpha l \ll 1$$

$$\alpha = \frac{1}{2} \left(R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right)$$

$$\beta = \omega \sqrt{LC}$$

$$Q \equiv \frac{\beta}{2\alpha} = \frac{\omega \sqrt{LC}}{\left(R \sqrt{\frac{C}{L}} + G \sqrt{\frac{L}{C}} \right)} = \frac{\omega \sqrt{LC} \sqrt{\frac{L}{C}}}{\left(R + G \sqrt{\frac{L}{C}} \sqrt{\frac{L}{C}} \right)} = \frac{\omega L}{\left(R + G \frac{L}{C} \right)}$$

At resonance:

$$Q \cong \frac{\omega_0 L}{\left(G \frac{L}{C} \right)}$$

Dielectric Losses

$$Q \cong \omega_0 RC$$

Parallel Resonant Circuit

Dielectric Losses

Quarter Wave Shorted Section

