

Lecture-2

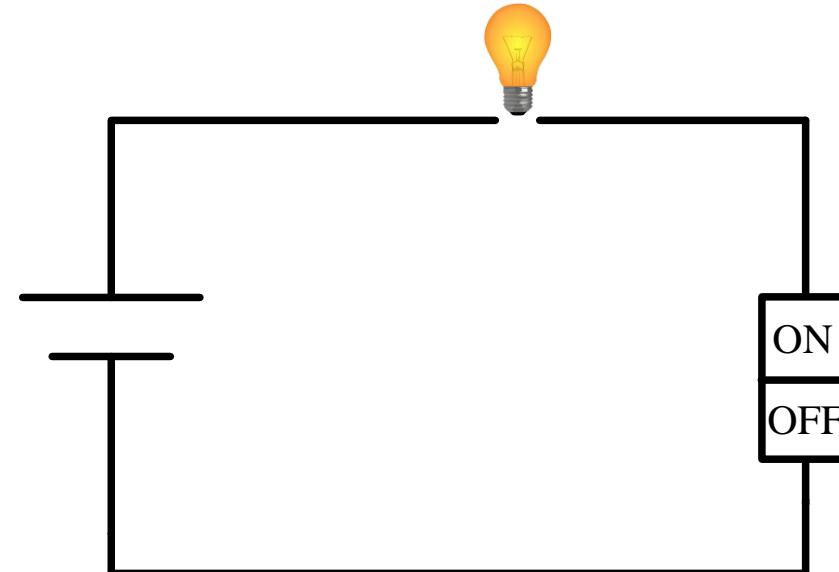
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Electric charge.
- Current.
- Voltage.
- Power and energy.
- Resistance and Conductance.

ELECTRIC CURRENT

- The battery acts as a source of electromotive force (emf).
- Connecting a conducting wire to a battery causes the electrons to move in one direction.
- This motion of charges create an electric current.
- Although, the current in a metallic conductor is due to the flow of electrons it is conventional to take the current as the net flow of positive charge in a direction opposite to the flow of electrons. (**Benjamin Franklin and Joseph Thompson**)



ELECTRIC CURRENT (Cont...)

- Hence, electric current can be defined as the time rate of change of charge.
- Mathematically, the relationship between charge, q and current, i can be expressed as,

$$i \triangleq \frac{dq}{dt}$$

- The unit of current is ampere (A), where $1 \text{ ampere} = 1 \text{ coulomb / second}$.
- The charge transferred between time t_0 and t is evaluated by integrating the above equation.

$$Q \triangleq \int_{t_0}^t i \, dt$$

ELECTRIC CURRENT (Cont...)

□ Example:

- The total charge entering a terminal is given by $q=5t\sin 4\pi t$ mC. Calculate the current at $t=0.5$ s?

□ Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) = 5 \sin 4\pi t + 20\pi t \cos 4\pi t$$

At $t=0.5$

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42\text{mA.}$$

ELECTRIC CURRENT (Cont...)

□ Example:

- Determine the total charge entering a terminal between $t=1\text{s}$ and $t=2\text{s}$ if the current passing the terminal is $i=(3t^2-t) \text{ A}$?

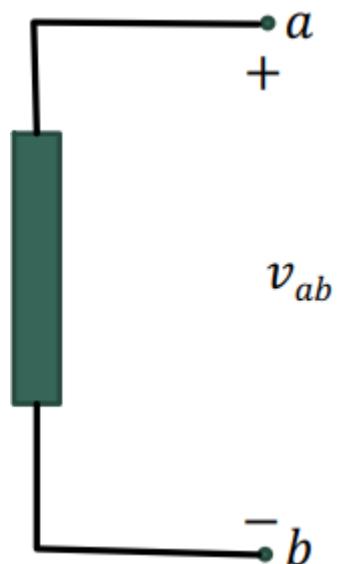
□ Solution:

$$\begin{aligned} Q &= \int_{t=1}^2 idt \\ &= \int_{t=1}^2 (3t^2 - t)dt \\ &= \left(t^3 - \frac{t^2}{2} \right) \Big|_1^2 \\ &= 5.5\text{C} \end{aligned}$$

VOLTAGE

- Energy transfer or work is needed to move electrons in a particular direction.
- This work is performed by an external emf generally provided by the battery.
- The emf is also known as voltage or potential difference.
- Voltage is defined as the energy required to move a unit charge through an element.
- It is expressed mathematically as,

$$v_{ab} \triangleq \frac{dw}{dq}$$



- Here, v_{ab} is the voltage between two points a and b , w is the energy in joules (J), and q is the charge in coulombs (C).

VOLTAGE (Cont..)

- Voltage is measured in volts (V).
- Here, 1 volt = 1 joule / coulomb = 1 newton-meter / coulomb.
- The +ve and -ve signs are used to represent the voltage polarity or the reference direction.
- It follows logically that $v_{ab} = -v_{ba}$.
- Current and voltage are considered as the two basic variables in the electric circuit.

POWER AND ENERGY

- Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves.
- For practical purposes, we need to know how much power an electric device can handle.
- When we pay our bills to the electric utility companies, we are paying for the electric energy consumed over a certain period of time.
- Thus, power and energy calculations are also important in circuit analysis.

POWER AND ENERGY (Cont...)

- To relate power and energy to voltage and current, we recall from physics the following:
- Power is the time rate of expending or absorbing energy and can be expressed as

$$p \triangleq \frac{dw}{dt}$$

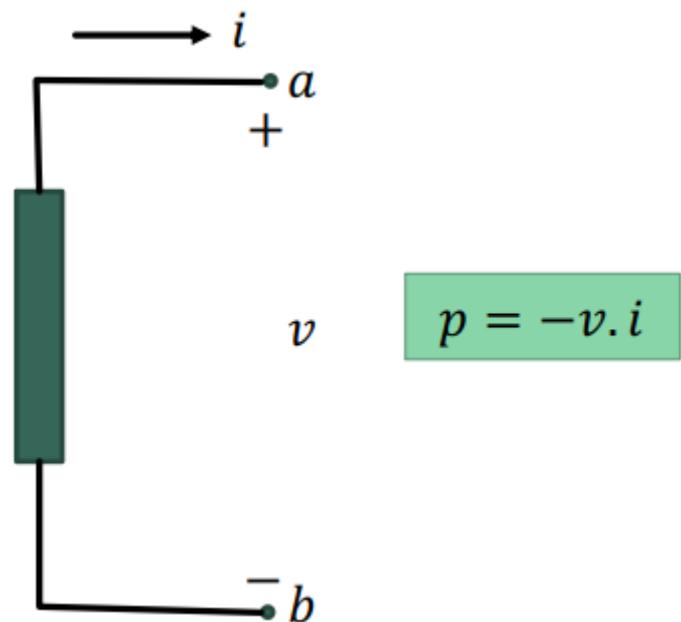
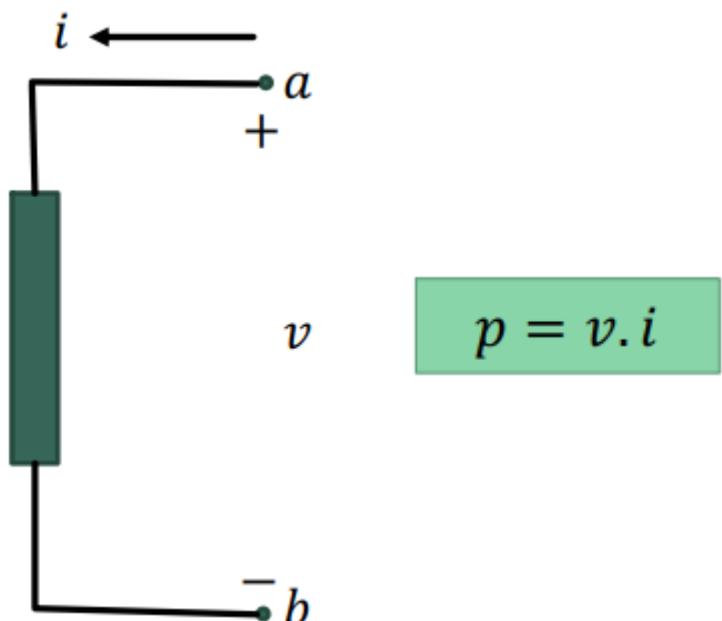
- Power, p , is measured in watts (W), w is the energy in joules (J), and t is the time in seconds (s).
- Using the previous equations:-

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$$

- Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it.
- The power p in the above equation is a time-varying quantity and is called the **instantaneous power**.

POWER AND ENERGY (Cont...)

- If the power has a +ve sign, power is being delivered to or absorbed by the element.
- If, on the other hand, the power has a -ve sign, power is being supplied by the element.
- Current direction and voltage polarity play a major role in determining the sign of power.



POWER AND ENERGY (Cont...)

- It is therefore important that we pay attention to the relationship between current and voltage.
- The voltage polarity and current direction must conform with those shown in the previous figure in order for the power to have a positive sign.
- This is known as the **passive sign** convention.
- Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = vi$.
- If the current enters through the negative terminal, $p = -vi$.

POWER AND ENERGY (Cont...)

- Law of conservation of energy must be obeyed in any electric circuit.
- For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0$$

- This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.
- Using energy power equation, the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt$$

- Energy is, thus, defined as the capacity to do work and is measured in joules (J).

POWER AND ENERGY (Cont...)

□ Example:

- An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

□ Solution:

The total charge is

$$\Delta q = i\Delta t = 2 * 10 = 20\text{C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3*10^3}{20} = 115\text{V}$$

POWER AND ENERGY (Cont...)

□ Example:

- Find the power delivered to an element at $t = 3\text{ms}$ if the current entering its positive terminal is $i = 5\cos 60\pi t \text{ A}$ and the voltage is $v = 3i$?

□ Solution: The voltage is

$$v = 3i = 15 \cos 60\pi t$$

Hence, power

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3\text{ms}$

$$\text{Power } p = 75 \cos^2(60\pi * 3 * 10^{-3}) = 53.48 \text{ W}$$

POWER AND ENERGY (Cont...)

□ Example:

- For the previous problem find the power at $t = 3\text{ms}$ if the voltage is $v = 3\frac{di}{dt}$?

□ Solution: The voltage is

$$v = 3 \frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t$$

Hence, power

$$p = vi = -4500 \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3\text{ms}$

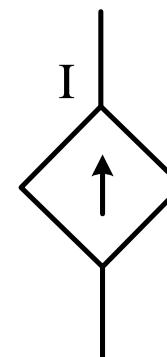
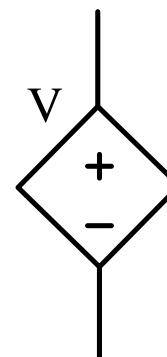
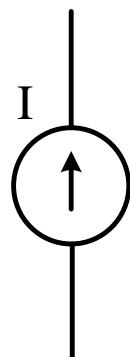
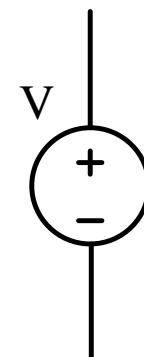
$$\text{Power } p = -4500 \sin(60\pi * 3 * 10^{-3}) \cos(60\pi * 3 * 10^{-3}) = -6.396 \text{ kW}$$

CIRCUIT ELEMENTS

- There are two types of elements found in electric circuits: **passive elements** and **active elements**.
- An active element is capable of generating energy while a passive element is not.
- Examples of passive elements are resistors, capacitors, and inductors.
- Typical active elements include generators, batteries, and operational amplifiers.
- The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.

CIRCUIT ELEMENTS (Cont...)

- There are two kinds of sources: independent and dependent sources.
- An ideal independent source provides a specified voltage or current that is completely independent of other circuit elements.
- An ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage.
- Physical sources such as batteries may be regarded as approximations to ideal voltage sources.
- In an ideal dependent (or controlled) source, source quantity is controlled by another voltage or current.



Independent sources

Dependent sources

CIRCUIT ELEMENTS (Cont...)

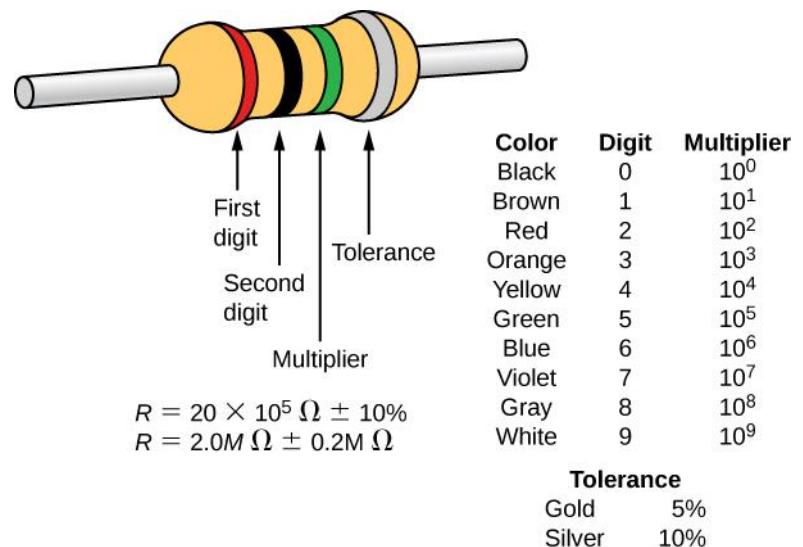
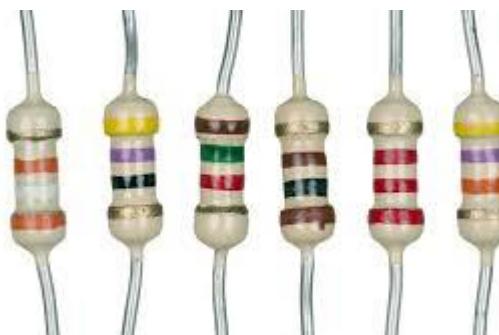
□Resistance:

- A physical property or ability of an element to resist the current, is known as resistance and is represented by the symbol R , and is measured in ohms.
- The resistance of any material with a uniform cross-sectional area A depends on A and its length, l .
- It is expressed mathematically as,

$$R = \rho l/A$$

where, ρ is known as the resistivity of the material in ohm-meters.

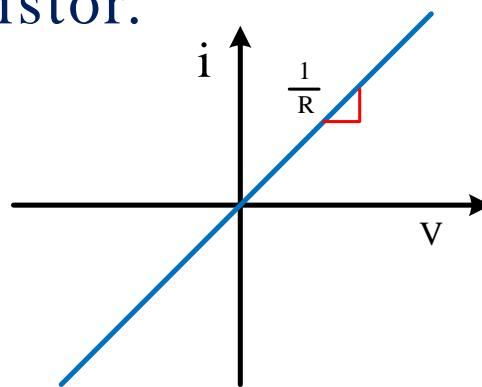
- The common circuit symbol used for a resistor is as shown in the figure below:



CIRCUIT ELEMENTS (Cont...)

□ Resistance (Cont.):

- Georg Simon Ohm (1787–1854), a German physicist, developed the relationship between current and voltage for a resistor.
- This relationship is known as Ohm's law.
- Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.



- Ohm defined the constant of proportionality for a resistor to be the resistance, R .
- From the above equation,

$$R = \frac{v}{i} \quad \Rightarrow \quad 1\Omega = 1 \frac{V}{A}$$

CIRCUIT ELEMENTS (Cont...)

□ Resistance (Cont.):

- The value of R can range from zero to infinity.
- An element with $R = 0$ is called a short circuit.
- For a short circuit,

$$v = iR = 0$$

- The short circuit voltage across an element is zero.
- Similarly, an element with $R = \infty$ is known as open circuit.
- For an open circuit,

$$i = \lim_{R \rightarrow \infty} \frac{v}{R} = 0$$

- Thus, for an open circuit the current through the element is zero.

CIRCUIT ELEMENTS (Cont...)

□ Conductance:

- Another useful element in circuit analysis is the reciprocal of resistance R , known as conductance G .
- It is defined as a measure of how well an element can conduct electric current.
- The unit of conductance is siemens (S) or mho (Ω).
- It can be expressed mathematically as,

$$G = \frac{1}{R} = \frac{i}{v}$$

- Thus, the unit of conductance can be expressed as,

$$1\text{S} = 1\Omega = 1\text{A/V}$$

- The same resistance can be expressed in terms of ohms or siemens. For example, $10\ \Omega$ is same as $0.1\ \Omega$.

