Lecture-22

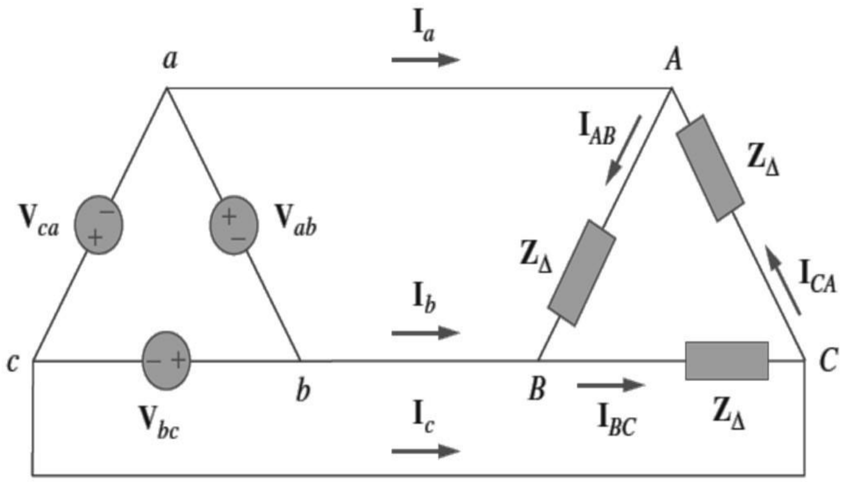
On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Δ-Δ system.
- Δ-Y system.
- Power in balance system.
- Power in unbalance system.
- Three Phase Power Measurement.

Balanced Delta-Delta Connection

- A balanced Δ-Δ system consists of a balanced Δ-connected source feeding a balanced Δ-connected load.
- The balanced Δ-Δ system is shown in the below figure.
- There will be no neutral connection from source to load for this case.



Balanced Delta-Delta Connection (Cont…)

- Assuming the positive sequence, the phase voltage for a Δ-connected source are,

$$\mathbf{V}_{\mathrm{ab}} = V_{\mathrm{p}}\angle 0°$$

$$\mathbf{V}_{\mathrm{bc}} = V_{\mathrm{p}}\angle -120°$$

$$\mathbf{V}_{\mathrm{ca}} = V_{\mathrm{p}}\angle 120°$$

- The line voltages are same as the phase voltages.
- Assuming there is no line impedance, the phase voltages of the delta-connected source are equal to the voltage across the impedance,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Balanced Delta-Delta Connection (Cont…)

- From these voltages, we can obtain the phase currents as,

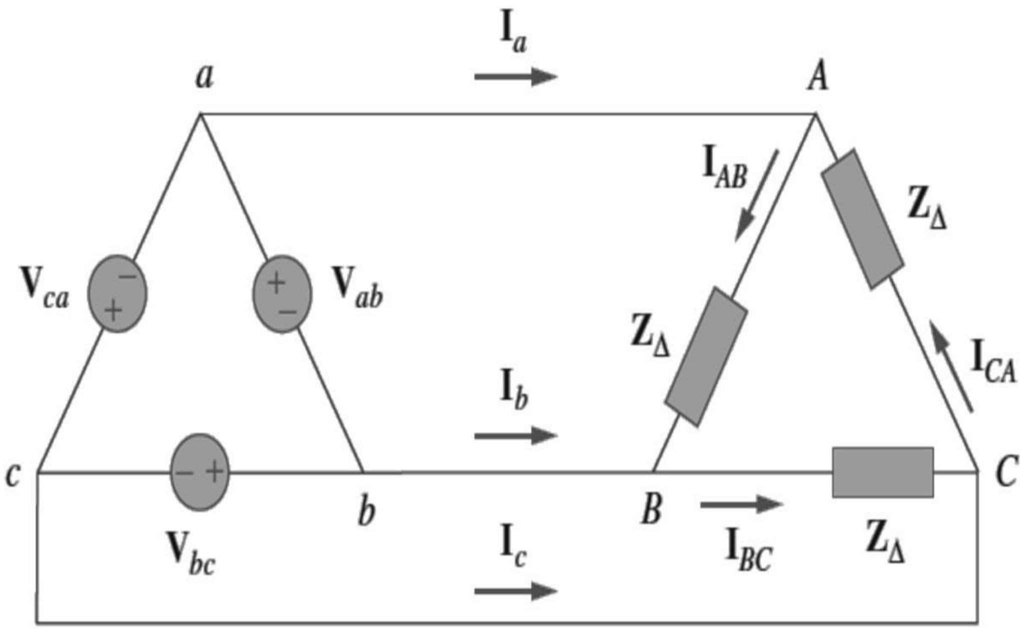
$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{\Delta}}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{\Delta}} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{\Delta}}$$

- These currents have the same magnitude but are out of phase with each other by 120°.

Balanced Delta-Delta Connection (Cont…)



- The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C.

Balanced Delta-Delta Connection (Cont…)

- Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

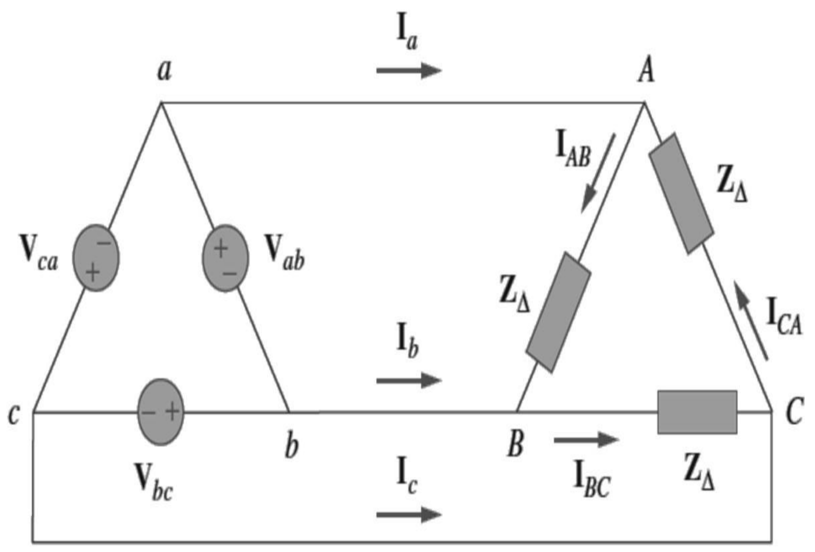
- Since,

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240°$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240°)$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866)$$

$$= \mathbf{I}_{AB}\sqrt{3}\angle -30°$$



- Similarly,

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = \mathbf{I}_{BC}\sqrt{3}\angle -30°$$

$$\mathrm{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = \mathbf{I}_{CA}\sqrt{3}\angle -30°$$

- This shows that the magnitude of the line current is $\sqrt{3}$ times the magnitude of the phase current, i.e.,

$$\mathbf{I}_\mathrm{L} = \sqrt{3}\,\mathbf{I}_\mathrm{p}$$

Balanced Delta-Delta Connection (Cont…)

- An alternative way of analyzing the circuit is to transform the Δ-Δ connected circuit to an equivalent Y-Y connected circuit.
- The load can be transformed using,

$$\mathbf{z}_Y = \frac{\mathbf{z}_\Delta}{3}$$

Example:

- A balanced Δ-connected having an impedance of $(20 - j15)\Omega$ per phase is connected to a Δ- connected positive sequence generator having $\mathbf{V}_{ab} = 330\angle 0°$ V. Calculate the phase currents of the load and the line currents?

Solution: The load impedance is,

$$\mathbf{Z}_{\Delta} = (20 - j15)\ \Omega = 25\angle -36.87°\Omega$$

$$\mathbf{V}_{ab} = 330\angle 0°\ \text{V} = \mathbf{V}_{AB}$$

Balanced Delta-Delta Connection (Cont…)

- The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{330\angle 0^\circ}{25\angle -36.87^\circ} = 13.2\angle 36.87^\circ\,\mathbf{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 13.2\angle -83.13^\circ\,\mathbf{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120^\circ = 13.2\angle 156.87^\circ\,\mathbf{A}$$

- The line currents are

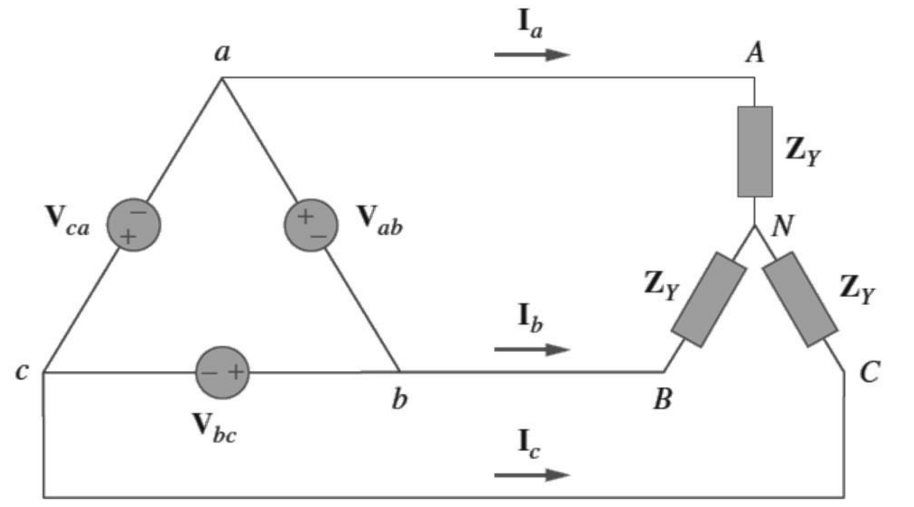
$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(13.2)\angle(36.87^\circ - 30^\circ) = 22.86\angle 6.87^\circ\,\mathbf{A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle -120^\circ = 22.86\angle -113.13^\circ\,\mathbf{A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120^\circ = 22.86\angle 126.87^\circ\,\mathbf{A}$$

Balanced Delta-Wye Connection

- A balanced delta-star system consists of a balanced Δ-connected source feeding a balanced Y-connected load.
- The balanced Δ-Y system is shown in the below figure, where the source is Δ-connected and the load is Y-connected.



Balanced Delta-Wye Connection (Cont…)

- Assuming the positive sequence, the phase voltages of a Δ connected source are,

$$\mathbf{V}_{\mathrm{ab}} = V_{\mathrm{p}}\angle 0°$$

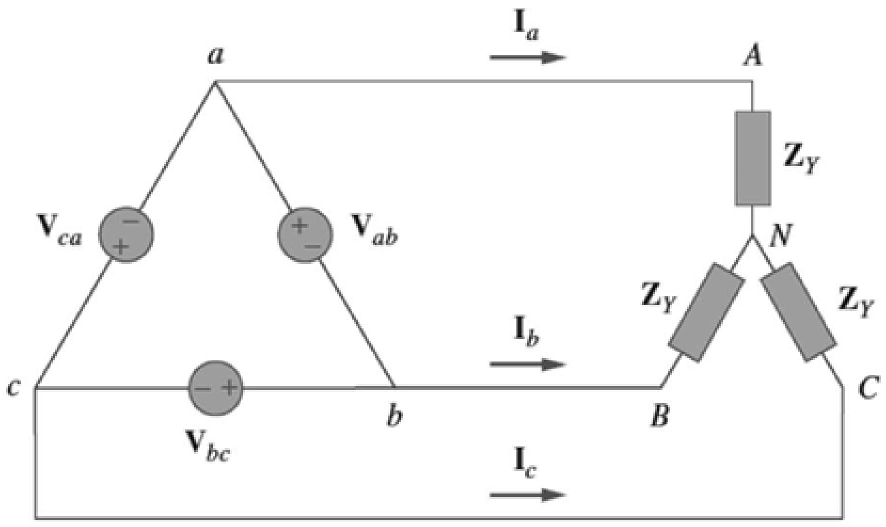
$$\mathbf{V}_{\mathrm{bc}} = V_{\mathrm{p}}\angle -120°$$

$$\mathbf{V}_{\mathrm{ca}} = V_{\mathrm{p}}\angle 120°$$

- These are also the line voltages as well as the phase voltages.

- The line currents are evaluated using KVL as detailed in the next slide.

Balanced Delta-Wye Connection (Cont…)



- Applying KVL around the around the loop $aANBba$ gives,

$$-\mathbf{V}_{ab} + \mathbf{Z}_F\mathbf{I}_a - \mathbf{Z}_F\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_F(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = \mathbf{V}_p\angle 0°$$

- Therefore,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{\mathbf{V}_p\angle 0°}{\mathbf{Z}_Y}$$

Balanced Delta-Wye Connection (Cont…)

- But $\mathbf{I}_b$ lags $\mathbf{I}_a$ by 120°, i.e.,

$$\mathbf{I}_\mathrm{b} = \mathbf{I}_\mathrm{a}\angle -120°$$

- Hence,

$$\mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = \mathbf{I}_\mathrm{a}(1 - 1\angle -120°)$$

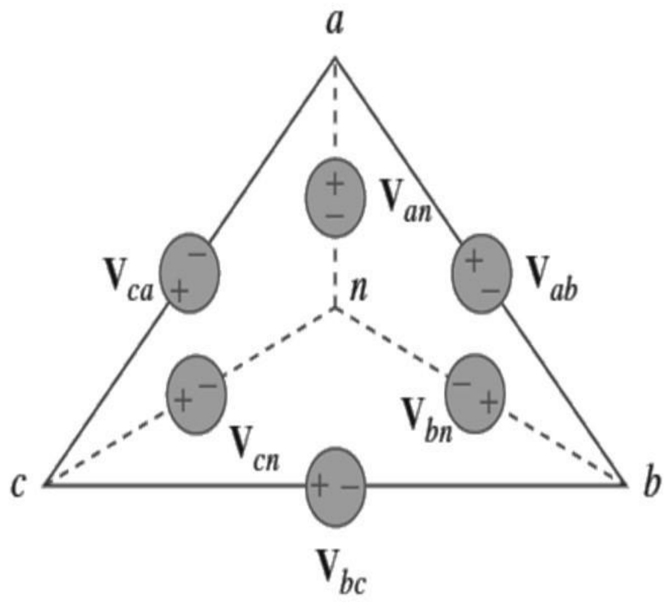
$$= \mathbf{I}_a\left(1 + \frac{1}{2} + \frac{j\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\angle 30°$$

- Therefore,

$$\mathbf{I}_a = \frac{\frac{\mathbf{V}_p}{\sqrt{3}}\angle -30°}{\mathbf{Z}_Y}$$

Balanced Delta-Wye Connection (Cont…)

- Another way to obtain the line currents is to replace the delta connected source with its equivalent Y connected source as shown below.



- We had already discussed that the line-line voltages of a wye-connected source leads their corresponding phase voltages by 30 degree and is $\sqrt{3}$ times in magnitude as well.

Balanced Delta-Wye Connection (Cont…)

- Therefore, the equivalent Y connected source has the phase voltages,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_p}{\sqrt{3}} \angle -30°$$

$$\mathbf{V}_{bn} = \frac{\mathbf{V}_p}{\sqrt{3}} \angle -150°$$

$$\mathbf{V}_{cn} = \frac{\mathbf{V}_p}{\sqrt{3}} \angle 90°$$

- If the delta connected source has a source impedance of $\mathbf{Z}_S$ per phase the wye-connected source will have a source impedance of $\mathbf{Z}_s/3$ per phase.
- Once the source is transformed to Y the circuit becomes a Y-Y network and can be solved using the single phase analysis as discussed previously.

Balanced Delta-Wye Connection (Cont…)

Example:

- A balanced Y-connected load having an impedance of $(40 + j25)\Omega$ per phase is connected to a Δ- connected positive sequence source having $\mathbf{V}_{ab} = 210\angle 0°$ V. Calculate the phase currents of the load and the line currents?

Solution: The load impedance is,

$$\mathbf{Z}_F = (40 + j25)\ \Omega = 47.17\angle 32°\Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0°\ \text{V}$$

Balanced Delta-Wye Connection (Cont…)

- When the delta connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \angle -30^\circ = 121.2\angle -30^\circ$$

- The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.17\angle 32^\circ} = 2.57\angle -62^\circ \mathbf{A}$$

$$\mathbf{I}_\mathrm{b} = \mathbf{I}_\mathrm{a}\angle -120^\circ = 2.57\angle 178^\circ\ \mathrm{A}$$

$$\mathbf{I}_\mathrm{c} = \mathbf{I}_\mathrm{a}\angle 120^\circ = 2.57\angle 58^\circ\ \mathrm{A}$$

Power in a Balanced System

- We now consider the power in a balanced three-phase system.

- To find the instantaneous power absorbed by the load, the analysis needs to be done in the time domain.

- For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos\omega\, t$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega\, t + 120°)$$

- where the factor $\sqrt{2}$ is necessary because V_{p} has been defined as the rms value of the phase voltage.

Power in a Balanced System (Cont…)

- If $\mathbf{Z}_{\mathrm{Y}} = Z\angle\theta$, the phase currents lag behind their corresponding phase voltages by θ.

- Thus,

$$i_{\mathrm{a}} = \sqrt{2}\, I_{\mathrm{p}} \cos(\omega t - \theta)$$

$$i_{\mathrm{b}} = \sqrt{2}\, I_{\mathrm{p}} \cos(\omega t - \theta - 120°)$$

$$i_{\mathrm{c}} = \sqrt{2}\, I_{\mathrm{p}} \cos(\omega t - \theta + 120°)$$

where I_{p} is the rms value of the phase current.

Power in a Balanced System (Cont…)

- The total instantaneous power in the load is the sum of the instantaneous power in the three phases, i.e.,

$$p = p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c$$

$$= 2V_p I_p [\cos\omega t \cos(\omega t - \theta) +$$
$$\cos(\omega t - 120°)\cos(\omega t - \theta - 120°) +$$
$$\cos(\omega t + 120°)\cos(\omega t - \theta + 120°)]$$

- Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

Power in a Balanced System (Cont…)

- The above identity transforms the instantaneous power equation as follows,

$$p = V_p I_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240°) + \cos(2\omega t - \theta + 240°)]$$

- Let $\alpha = 2\omega t - \theta$

$$p = V_p I_p[3\cos\theta + \cos\alpha + \cos\alpha \cos 240° + \sin\alpha \sin 240° + \cos\alpha \cos 240° - \sin\alpha \sin 240°]$$

$$= V_p I_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_p I_p \cos\theta$$

- Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time while the instantaneous power of each phase does change.

Power in a Balanced System (Cont…)

$$V_p = 3V_p I_p \cos\theta$$

- Since the total instantaneous power is independent of time, the average power per phase for the delta or wye connected load is $p/3$

$$P_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}} \cos\theta$$

- The reactive power per phase can be given by,

$$Q_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}} \sin\theta$$

- The apparent power per phase is,

$$S_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}}$$

- The complex power per phase is, therefore,

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

where $\mathbf{V}_{\mathrm{p}}$ and $\mathbf{I}_{\mathrm{p}}$ are the phase voltages and phase currents with magnitude V_{p} and I_{p}, respectively.

Power in a Balanced System (Cont…)

- The total average power is the sum of the average powers in the three phases,

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

- For a Y connected load $I_L = I_p$ and $V_L = \sqrt{3}\, V_p$, whereas for a delta connected load $I_L = \sqrt{3}\, I_p$ and $V_L = V_p$.

- Thus the above power equation is applicable for both wye and delta connected loads.
- Similarly, the total reactive power is,

$$Q = 3V_p I_p \sin\theta = 3Q_p = \sqrt{3}\, V_L I_L \sin\theta$$

- Therefore, the total complex power is given by,

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3\mathbf{I}_p^2 \mathbf{Z}_p = \frac{3\mathbf{V}_p^2}{\mathbf{Z}_p^*}$$

Power in a Balanced System (Cont…)

- Here, $\mathbf{Z}_p = Z_p\angle\theta$ is the load impedance per phase.
- Alternatively,

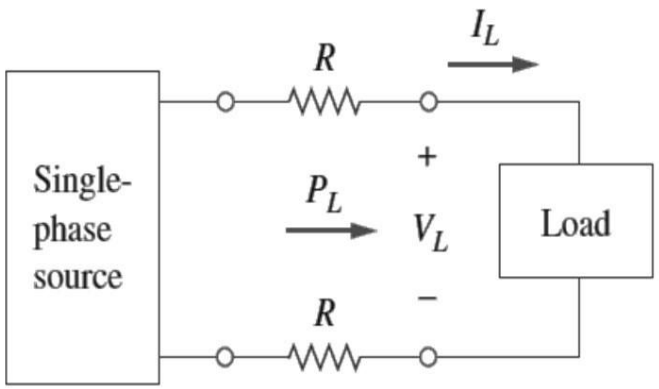
$$\mathbf{S} = P + jQ = \sqrt{3}V_L I_L \angle\theta$$

- Remember that V_L, I_L, V_p, and I_p are all rms values and θ is the angle of the load impedance or the angle between the phase voltage and the phase current.
- A major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage V_L and the same absorbed power P_L.
- We will compare these cases and assume in both that the wires are of the same material (e.g., copper with same resistivity), of the same length, l, and that the loads are resistive (i.e., unity power factor).

Power in a Balanced System (Cont…)

- For the two wire single phase system, shown in the figure below,

$$I_L = \frac{P_L}{V_L}$$



- The power loss in the two wires is,

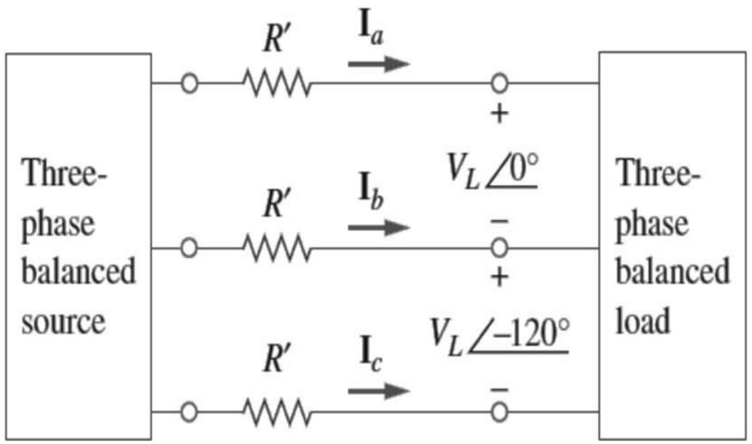
$$P_{loss} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2}$$

Power in a Balanced System (Cont…)

$$V_p = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta$$

- For the three wire three phase system, shown in the figure below,

$$I_L' = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = \frac{P_L}{\sqrt{3}V_L}$$



- The power loss in the three wires is,

$$P_{loss}' = 3(I_L')^2R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$

Power in a Balanced System (Cont…)

- From the power loss equations, for the same total power delivered P_L and the same line voltage V_L-

$$2 - wire\ sys.,\ P_{loss} = 2R\frac{P_L^2}{V_L^2}$$

$$3 - wire\ sys.,\ P'_{loss} = R'\frac{P_L^2}{V_L^2}$$

$$\frac{P_{loss}}{P'_{loss}} = \frac{2R}{R'}$$

- But resistance $R = \rho l/\pi r^2$ and $R' = \rho l/\pi r'^2$, where r and r' are the radius of the wires.
- Thus,

$$\frac{P_{loss}}{P'_{loss}} = \frac{2r'^2}{r^2}$$

- If the same power loss is tolerated in both the systems, then $r^2 = 2r'^2$.
- The ratio of the material required is determined by the number of wires and their volumes.

Power in a Balanced System (Cont…)

- This can be expressed mathematically as,

$$\frac{Material\ for\ \ 1 - phase}{Material\ \ for\ \ 3 - phase} = \frac{2(\pi r^2 l)}{3(\pi r'^2 l)} = \frac{2r^2}{3r'^2}$$

- But $r^2 = 2r'^2$,
- Therefore the above equation reduces to,

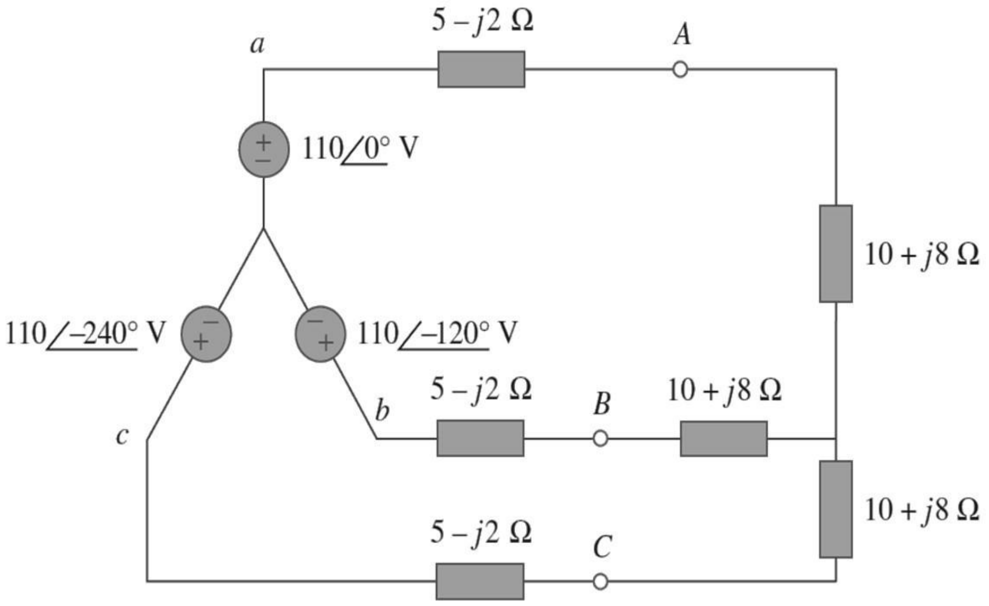
$$\frac{2r^2}{3r'^2} = \frac{2}{3}(2) = 1.33$$

- This shows that the single-phase system uses 33 percent more material than the three-phase system, alternatively, three-phase system uses only 75 percent of the material used in the equivalent single-phase system.
- In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

Power in a Balanced System (Cont…)

Example:

- For the circuit given below determine the total average power, reactive power, and complex power at the source and at the load?



Solution: It is sufficient to consider one phase as the system is balanced.

- Considering phase-a

$$\mathbf{V}_\mathrm{p} = 110\angle 0°,\ \ \mathbf{I}_\mathrm{p} = 6.81\angle -21.8° \text{ (yesterday lecture-21)}$$

- Thus at the source the complex power absorbed is,

$$\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* = -3(110\angle 0°)(6.81\angle 21.8°) = -2247\angle 21.8° = -(2087 + j834.6)\text{VA}$$

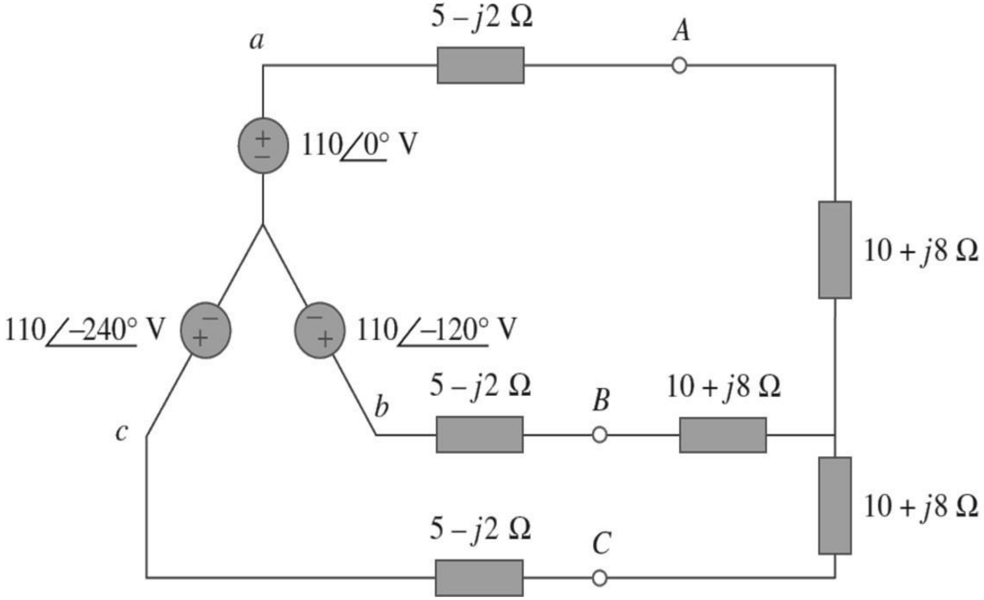
Power in a Balanced System (Cont…)

$$\mathbf{S}_s = -(2087 + j834.6)\text{VA}$$

- Therefore, the real or average power absorbed at the source is -2087 W and the reactive power absorbed is -834.6 VAR.
- At the load end $\mathbf{Z}_\text{p} = 10 + j8 = 12.81\angle 38.66°$ and $\mathbf{I}_\text{p} = \mathbf{I}_\text{a} = 6.81\angle -21.8°$
- Hence, the complex power absorbed is

$$\mathbf{S}_L = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p = 3(6.81)^2 * 12.81\angle 38.66° = 1782\angle 38.66° = 1392 + j1113\ \text{VA}$$

- The real power absorbed is 1392 W and the reactive power absorbed is 1113 VAR.



Power in a Balanced System (Cont…)

- The difference between the two complex powers is absorbed by the line impedance $(5 - j2)\ \Omega$

- Hence, the complex power absorbed by the line is

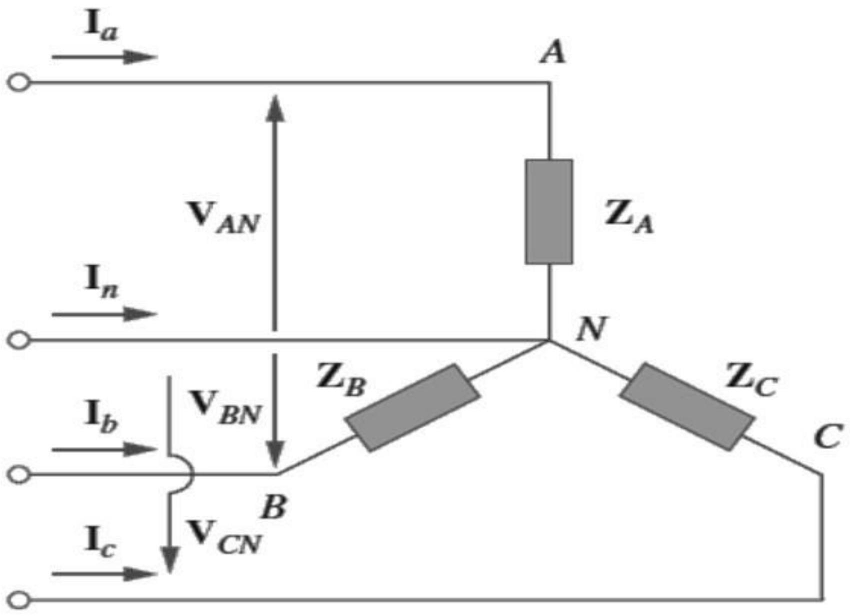
$$\mathbf{S}_l = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_l = 3(6.81)^2 * (5 - j2) = 695.6 - j278.3\text{VA}$$

- This is the difference between $\mathbf{S}_s$ and $\mathbf{S}_L$.
- Therefore,

$$\mathbf{S}_s + \mathbf{S}_L + \mathbf{S}_l = 0$$

Power in an Unbalanced System

- An unbalanced system is caused by two possible situations:
  - The source voltages are not equal in magnitude and/or differ in phase by angles that are unequal
  - Load impedances are unequal.
- Thus, an unbalanced system is due to unbalanced voltage sources or an unbalanced load.
- To simplify analysis, we will assume balanced source voltages, but an unbalanced load.

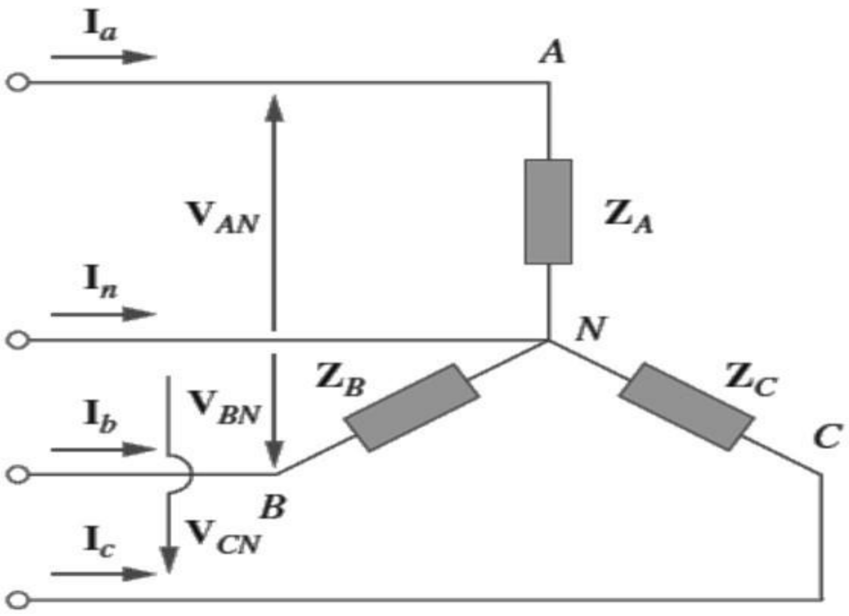


- Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis.
- The below figure shows an example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the figure) and an unbalanced Y-connected load (shown in the figure).
- Since the load is unbalanced, $\mathbf{Z}_A$, $\mathbf{Z}_B$, and $\mathbf{Z}_C$ are not equal.
- The line currents are determined by Ohm's law as

$$\mathrm{I_a} = \frac{V_{AN}}{Z_A}$$

$$I_b = \frac{V_{BN}}{Z_B}$$

$$I_c = \frac{V_{CN}}{Z_C}$$



Power in an Unbalanced System (Cont…)

- This set of unbalanced line currents produces current in the neutral line, which is not zero as in a balanced system.
- Applying KCL at node *N* gives the neutral line current as
- The line currents are determined by Ohm's law as

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$

- In a three-wire system where the neutral line is absent, we can still find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ using mesh analysis.
- At node *N*, KCL must be satisfied so that $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$ in this case.
- The same could be done for an unbalanced Δ-Y, Y-Δ, or Δ-Δ three-wire system.
- As mentioned earlier, in long distance power transmission, conductors in multiples of three (multiple three-wire systems) are used, with the earth itself acting as the neutral conductor.

Power in an Unbalanced System (Cont…)

- To calculate power in an unbalanced three-phase system requires that we find the power in each phase using the following equations.

$$P_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}} \cos\theta$$

$$Q_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}} \sin\theta$$

$$S_{\mathrm{p}} = V_{\mathrm{p}} I_{\mathrm{p}}$$

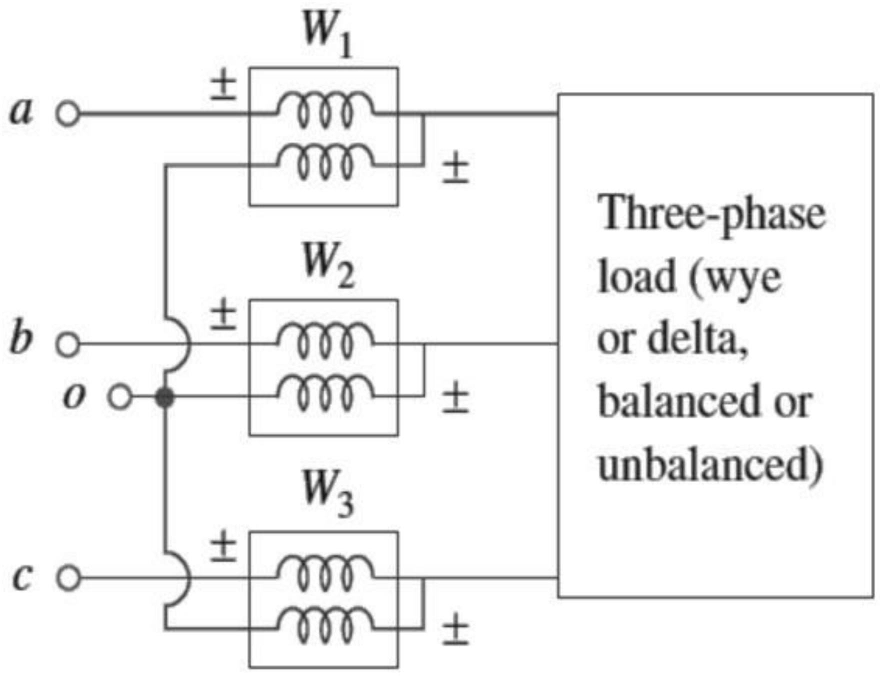
$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

- The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

$$P = P_{\mathrm{a}} + P_{\mathrm{b}} + P_{\mathrm{c}}$$

Three Phase Power Measurement

- A single wattmeter can also measure the average power in a three-phase system that is balanced, so that $P_1 = P_2 = P_3$: the total power is three times the reading of that one wattmeter.
- However, two or three single-phase wattmeters are necessary to measure power if the system is unbalanced.
- The three wattmeter method of power measurement will work regardless of whether the load is balanced or unbalanced, wye or delta-connected.

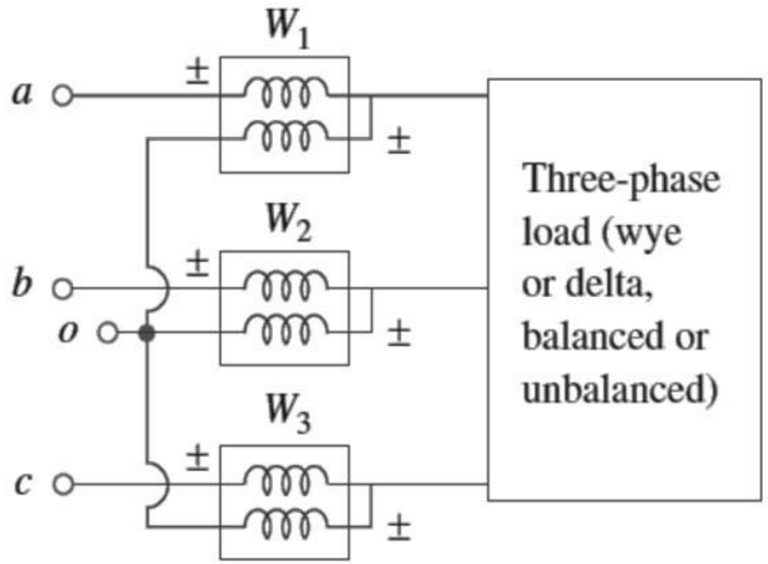


Three Phase Power Measurement (Cont…)

- The three-wattmeter method is well suited for power measurement in a three-phase system where the power factor is constantly changing.
- The total average power is the algebraic sum of the three wattmeter readings,

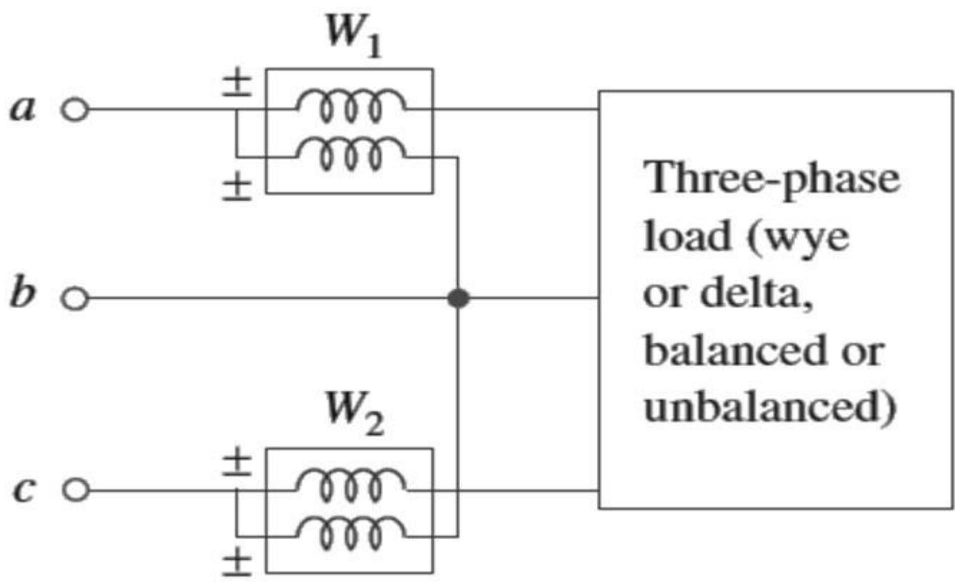
$$P_T = P_1 + P_2 + P_3$$

where, P_1, P_2, and P_3 correspond to readings of wattmeter W_1, W_2, and W_3.



- Notice that the common or reference point o is selected arbitrarily.
- If the load is wye-connected, point o can be connected to the neutral point n.
- For a delta-connected load, point o can be connected to any point.
- If point o is connected to point b, for example, the voltage coil in wattmeter W_2 reads zero and $P_2 = 0$ indicating that wattmeter W_2 is not necessary.
- Thus, two wattmeters are sufficient to measure the total power.

Three Phase Power Measurement (Cont…)



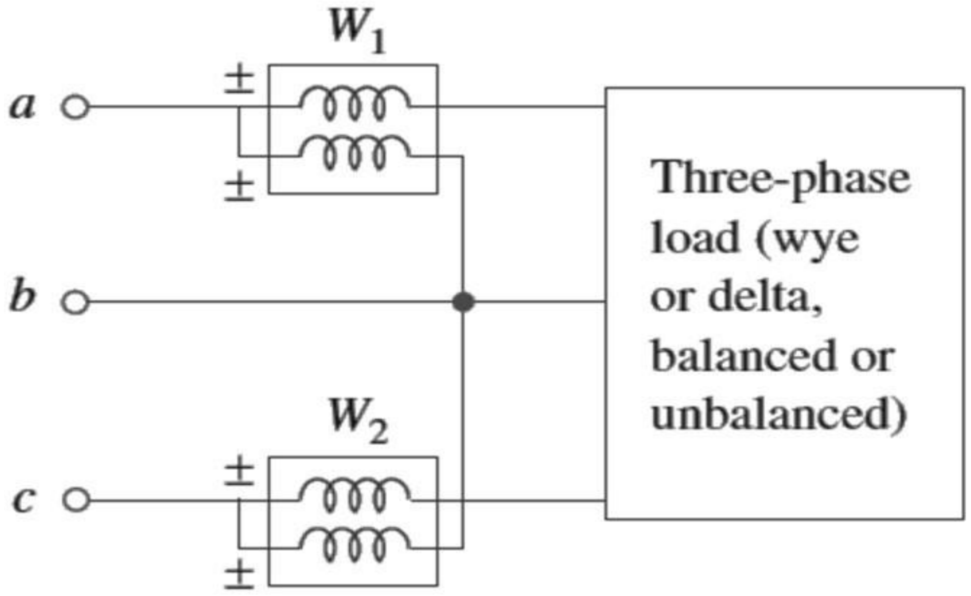
- The *two-wattmeter method* is the most commonly used method for three-phase power measurement.
- The two wattmeters must be properly connected to any two phases, as shown in the figure.
- Notice that the current coil of each wattmeter measures the line current, while the respective voltage coil is connected between the line and the third line and measures the line voltage.

Three Phase Power Measurement (Cont…)

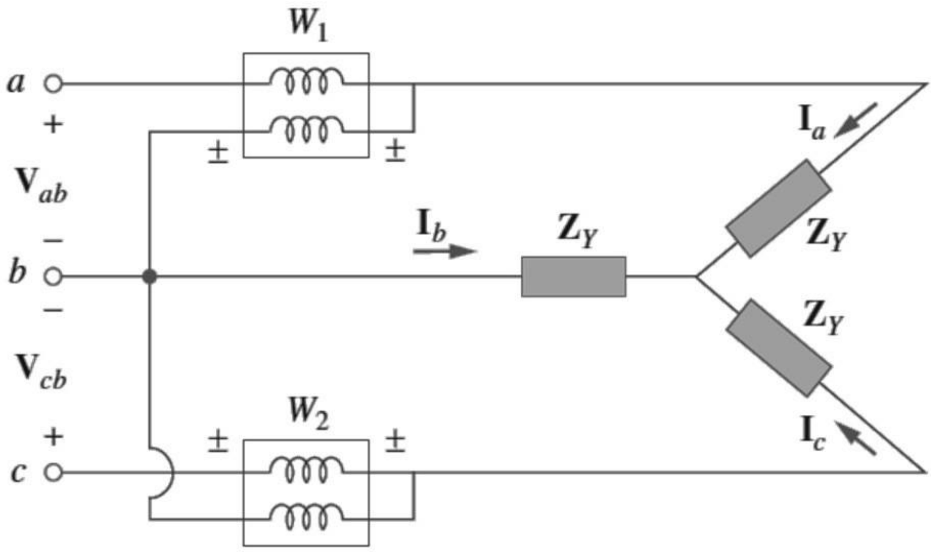
- Also notice that the ± terminal of the voltage coil is connected to the line to which the corresponding current coil is connected.
- Although the individual wattmeters no longer read the power taken by any particular phase, the algebraic sum of the two wattmeter readings equals the total average power absorbed by the load, regardless of whether it is wye- or delta-connected, balanced or unbalanced.
- The total real power is equal to the algebraic sum of the two wattmeter readings,

$$P_T = P_1 + P_2$$

- We will study the two-wattmeter method for a balanced three-phase system.



Three Phase Power Measurement (Cont…)



- Consider the balanced, wye-connected load in the above figure.
- Our objective is to apply the two-wattmeter method to find the average power absorbed by the load.
- Assume the source is in the *abc* sequence and the load impedance $\mathbf{Z}_Y = Z_Y\angle\theta$.
- Due to the load impedance, each voltage coil leads its current coil by θ so that the power factor is $\cos\theta$.
- We recall that each line voltage leads the corresponding phase voltage by 30°.

Three Phase Power Measurement (Cont…)

- Thus, the total phase difference between the phase current $\mathbf{I}_a$ and line voltage $\mathbf{V}_{ab}$ is $\theta + 30°$, and the average power read by wattmeter W_1 is

$$P_1 = \mathrm{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab}I_a\cos(\theta + 30°) = V_L I_L \cos(\theta + 30°)$$

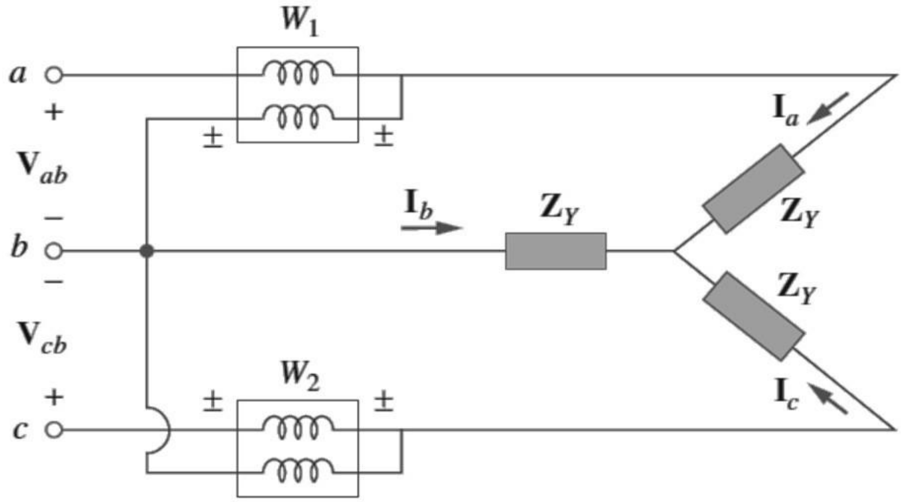
- Similarly, we can show that the average power read by wattmeter 2 is,

$$P_2 = \mathrm{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c\cos(\theta - 30°) = V_L I_L \cos(\theta - 30°)$$

- We use the trigonometric identities,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

to find the sum and difference of two wattmeter readings.



Three Phase Power Measurement (Cont…)

- Therefore,

$$P_1 + P_2 = V_L I_L[\cos(\theta + 30°) + \cos(\theta - 30°)]$$

$$= V_L I_L[\cos\theta\ \cos 30° - \sin\theta\ \sin 30° + \cos\theta\ \cos 30° + \sin\theta\ \sin 30°]$$

$$= V_L I_L 2\cos\theta\ \cos 30° = \sqrt{3} V_L I_L \cos\theta$$

- From the above it can be observed that the sum of the wattmeter readings in the two wattmeter method gives the total average power, i.e.,

$$P_T = P_1 + P_2$$

- Similarly,

$$P_1 - P_2 = V_L I_L[\cos(\theta + 30°) - \cos(\theta - 30°)]$$

$$= V_L I_L[\cos\theta \cos 30° - \sin\theta \sin 30° - \cos\theta \cos 30° - \sin\theta \sin 30°]$$

$$= -V_L I_L 2\sin\theta \sin 30° = -V_L I_L \sin\theta$$

- Thus the total reactive power can be evaluated using,

$$Q_T = \sqrt{3}(P_2 - P_1)$$

Three Phase Power Measurement (Cont…)

- Therefore, the total apparent power can be obtained as,

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

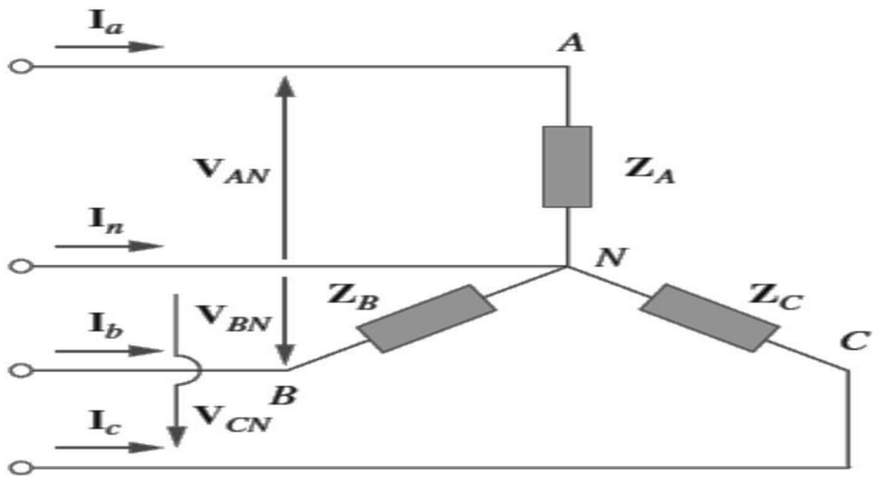
- Power factor is evaluated using,

$$\tan\theta = \frac{Q_T}{P_T} = \frac{P_2 - P_1}{P_1 + P_2}$$

- From this we can obtain the power factor as $pf = \cos\theta$.
- Thus, the two-wattmeter method not only provides the total real and reactive powers, it can also be used to compute the power factor.
- From the above equations it can be observed that,
  - If $P_2 = P_1$, the load is resistive.
  - If $P_2 > P_1$, the load is inductive.
  - If $P_2 < P_1$, the load is capacitive.

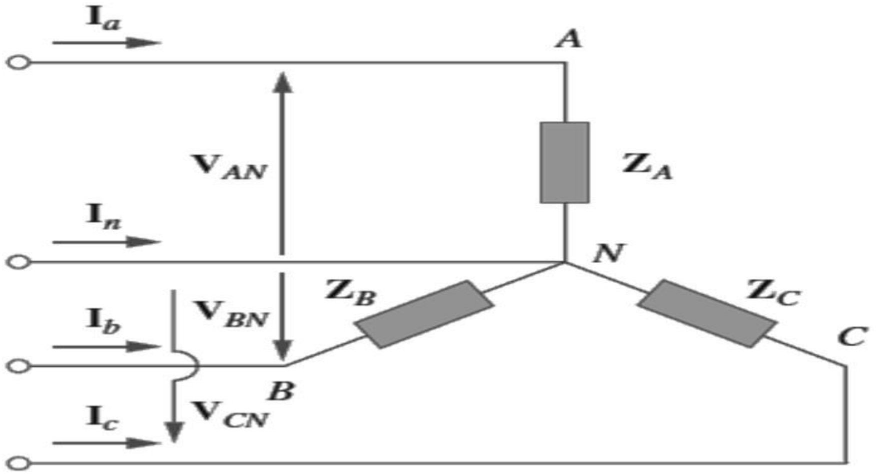
Example:

- The unbalanced Y load in the figure given below has balanced voltages of 100 V and acb sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15\Omega$, $\mathbf{Z}_B = 10 + j5\Omega$, $\mathbf{Z}_C = 6 - j8\Omega$.



Solution: Each line current has to be solved separately since the system is unbalanced.

Three Phase Power Measurement (Cont…)



- The line currents are evaluated as,

$$\mathbf{I}_a = \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ A$$

$$\mathbf{I}_b = \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \mathbf{A}$$

$$\mathbf{I}_c = \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ A$$

- The current in the neutral line is evaluated as,

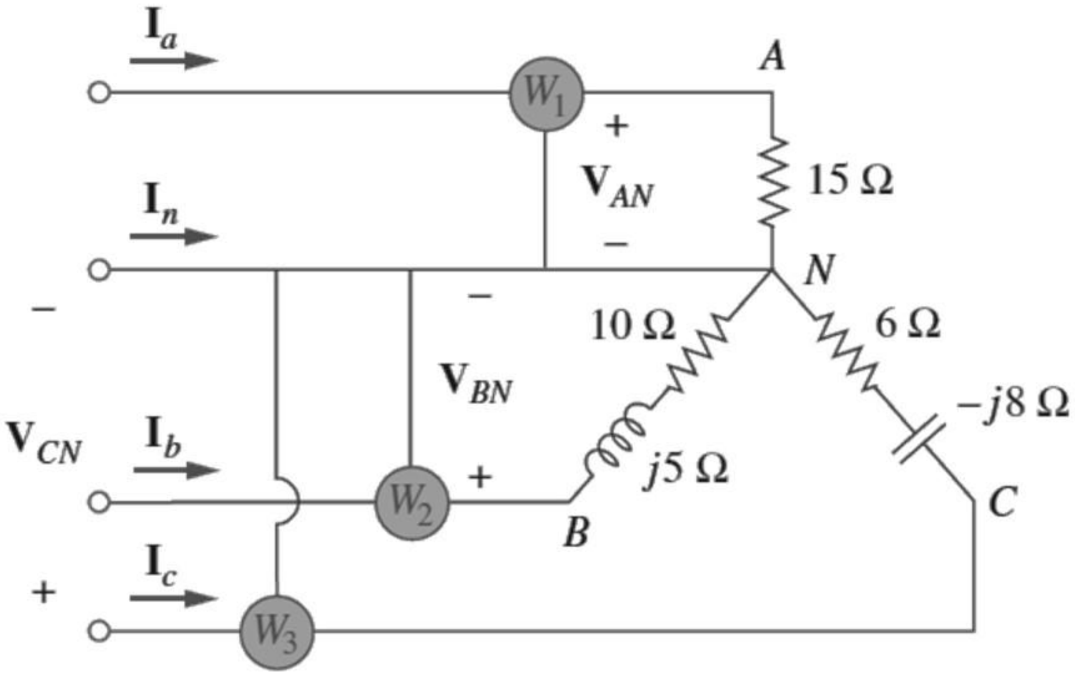
$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 10.06\angle 178.4^\circ \mathbf{A}$$

Three Phase Power Measurement (Cont…)

Example:

- For the previous problem three wattmeters are connected to the individual phases to measure the total power absorbed by the unbalanced Y-connected load. Predict the wattmeter readings and the total power absorbed.

- Solution: The wattmeters are connected as shown in the figure below.



Three Phase Power Measurement (Cont…)

- From the previous problem we know that,

$$\mathbf{V}_{\mathrm{AN}} = 100\angle 0°, \quad \mathbf{V}_{\mathrm{BN}} = 100\angle 120°, \quad \mathbf{V}_{\mathrm{CN}} = 100\angle -120°$$

While

$$\mathbf{I}_{\mathrm{a}} = 6.67\angle 0° \text{ A}, \ \mathbf{I}_{\mathrm{b}} = 8.94\angle 93.44° \text{A}, \ \mathbf{I}_{\mathrm{c}} = 10\angle -66.87° \text{A}$$

- The wattmeter readings are evaluated as,

$$P_1 = \mathrm{Re}(\mathbf{V}_{AN}\mathbf{I}_a^*) = V_{AN}I_a\cos\left(\theta_{\mathbf{V}_{AN}} - \theta_{\mathbf{I}_a}\right) = 100 * 6.67 * \cos(0° - 0°) = 667\text{W}$$

$$P_2 = \mathrm{Re}(\mathbf{V}_{BN}\mathbf{I}_b^*) = V_{BN}I_b\cos\left(\theta_{\mathbf{V}_{BN}} - \theta_{\mathbf{I}_b}\right) = 100 * 8.94 * \cos(120° - 93.44°) = 800\text{W}$$

$$P_3 = \mathrm{Re}(\mathbf{V}_{CN}\mathbf{I}_c^*) = V_{CN}I_c\cos\left(\theta_{\mathbf{V}_{CN}} - \theta_{\mathbf{I}_c}\right) = 100 * 10 * \cos(-120° + 66.87°) = 600\text{W}$$

- The total power absorbed is therefore,

$$P_{\mathrm{T}} = P_1 + P_2 + P_3 = 2067\text{W}$$

- The power absorbed by the individual resistors can be found as

$$P_T = |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) = 2067W$$

which is exactly same as the wattmeter readings.


THANKS
FOR YOUR
ATTENTION