EE340: Electromagnetic Theory

Transmission Lines - I

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Free Space ⇔ Wireless Communication

Transmission of Electromagnetic energy through unguided media

Transmission Lines ⇔ Wired Communication

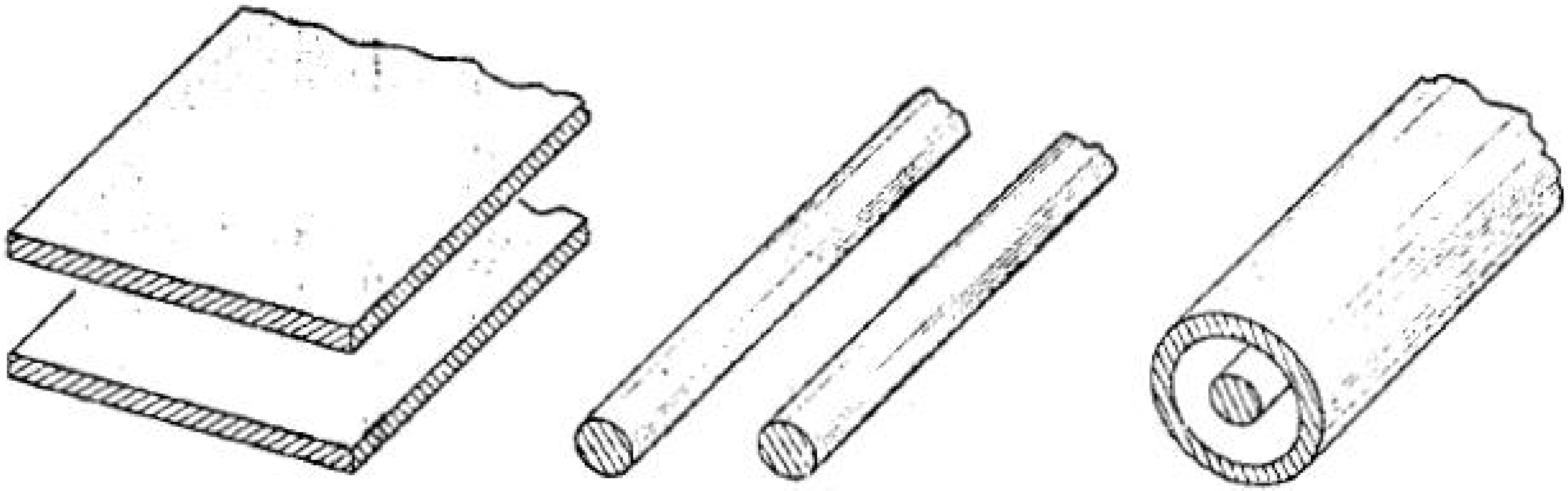
Transmission of Electromagnetic energy through guided media

RF Cables

Commonly used Transmission Lines

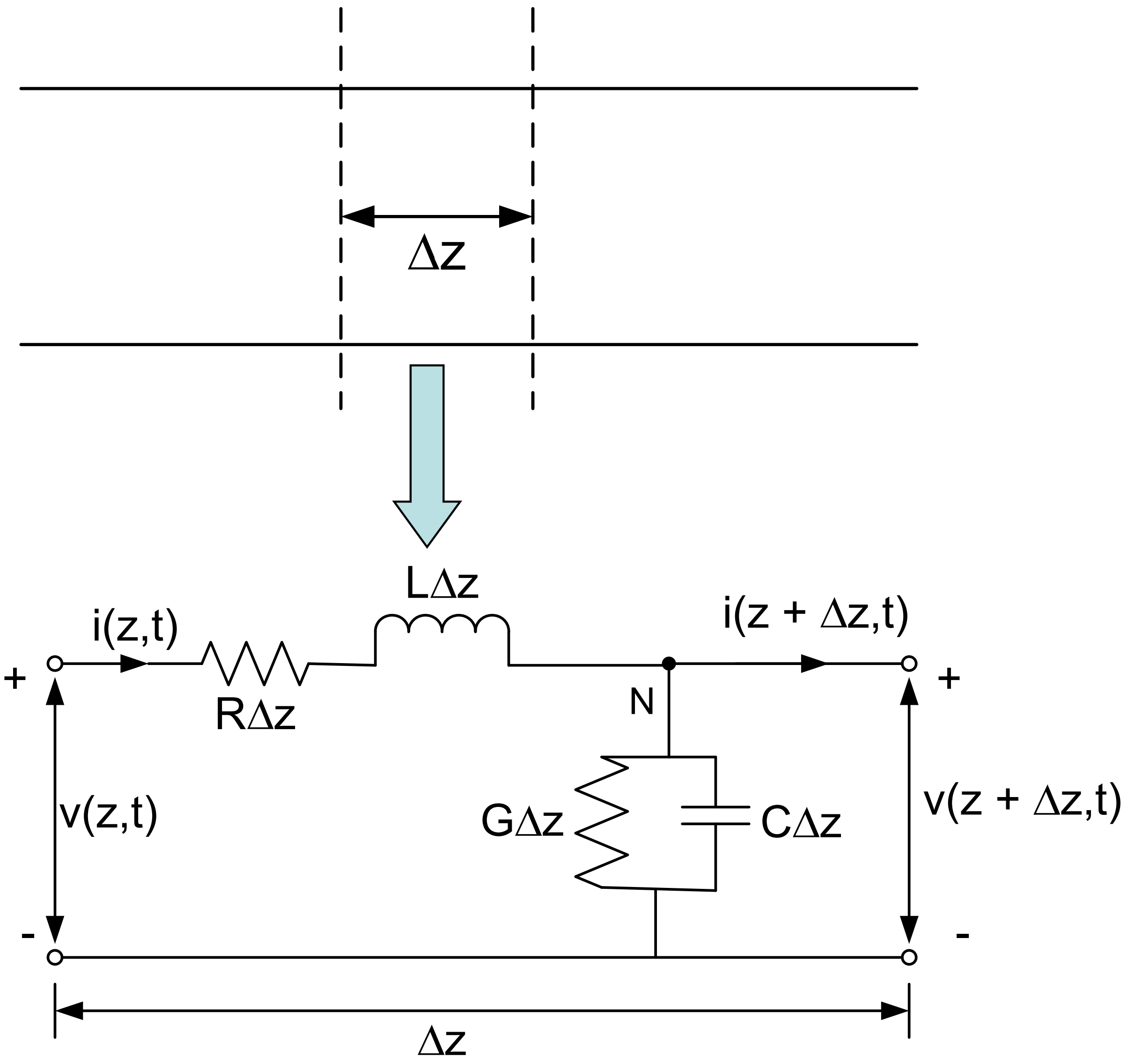
Distributed effects are important

Parallel Plate lines | Two wire lines | Coaxial lines



☝ The elements are distributed along the length of conductors.

General Transmission Line Equations



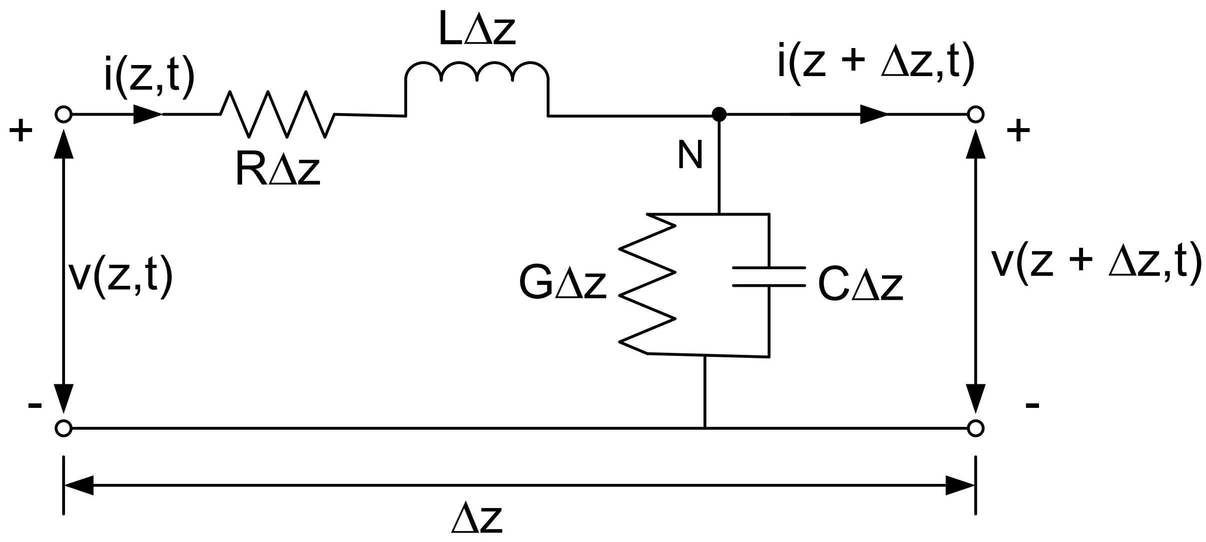
R: (Ω/m)

G: (S/m)

L: (H/m)

C: (F/m)

Equivalent Circuit of a small length z of two wire transmission line



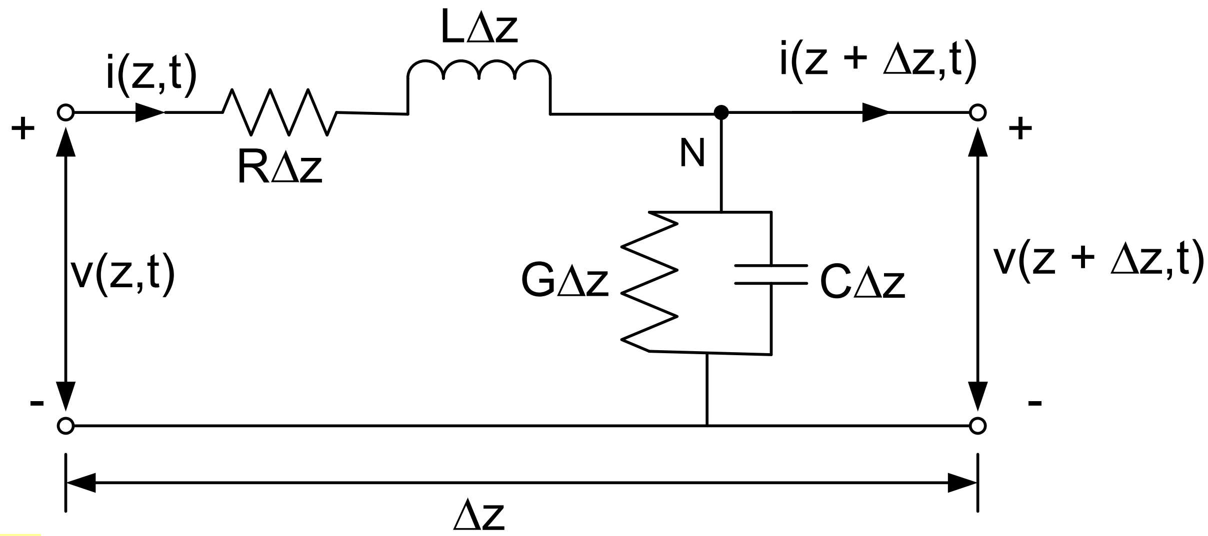
Applying KVL

$$v(z,t) - R\Delta z\ i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = R\ i(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

$\Delta z \to 0$

$$-\frac{\partial v(z,t)}{\partial z} = R\ i(z,t) + L\frac{\partial i(z,t)}{\partial t}$$



Applying KCL at node N

$$i(z,t) - G\Delta z\ v(z+\Delta z,t) - C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

$$-\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = G\ v(z+\Delta z,t) + C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

$\Delta z \to 0$

$$-\frac{\partial i(z,t)}{\partial z} = G\ v(z,t) + C\frac{\partial v(z,t)}{\partial t}$$

$$-\frac{\partial v(z,t)}{\partial z} = R\ i(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

General transmission line equations

$$-\frac{\partial i(z,t)}{\partial z} = G\ v(z,t) + C\frac{\partial v(z,t)}{\partial t}$$

$$v(z,t) = \mathcal{R}e\left[V(z)e^{j\omega t}\right]$$
$$i(z,t) = \mathcal{R}e\left[I(z)\ e^{j\omega t}\right]$$

☞ Phasor form

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$
$$-\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$

☞ Time harmonic transmission line equations

$$-\frac{dV(z)}{\partial z} = (R + j\omega L)I(z)$$
$$-\frac{dI(z)}{\partial z} = (G + j\omega C)V(z)$$

$$-\frac{d^2V(z)}{dz^2} = (R + j\omega L)\frac{dI(z)}{\partial z}$$

$$\frac{d^2V(z)}{dz^2} = (R + j\omega L)(G + j\omega C)\,V(z)$$

$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z)$$

$$\frac{d^2I(z)}{dz^2} = \gamma^2 I(z)$$

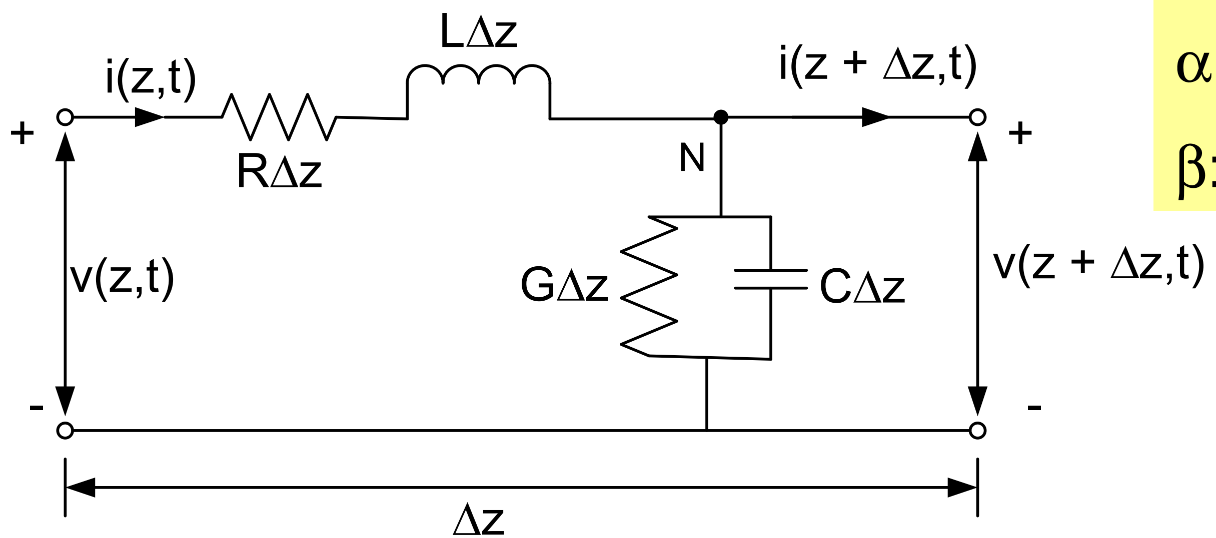
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

☝ Wave characteristic on Transmission Lines

γ: Propagation Constant

α: Attenuation Constant (Np/m)

β: Phase Constant (rad/m)



$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z)$$

$$\frac{d^2I(z)}{dz^2} = \gamma^2 I(z)$$

General Solution

$$V(z) = V^+(z) + V^-(z)$$

$$\equiv V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I^+(z) + I^-(z)$$

$$\equiv I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

V$^+$ and **I**$^+$ waves propagating in + z-direction

V$^-$ and **I**$^-$ waves propagating in - z-direction

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$

$$-\frac{d[V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}]}{dz} = (R + j\omega L)[I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}]$$

$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} \equiv (R + j\omega L)I_0^+ e^{-\gamma z} + (R + j\omega L)I_0^- e^{\gamma z}$$

$$\gamma V_0^+ = (R + j\omega L)I_0^+$$

$$\gamma V_0^- = -(R + j\omega L)I_0^-$$

$$\gamma\, V_0^+ = (R + j\omega L) I_0^+$$

$$\gamma\, V_0^- = -(R + j\omega L) I_0^-$$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

- For an infinite long line transmission line with source at the left end, the term $e^{\gamma z}$ term disappears (because of **no reflection**)

$$V(z) = V^+(z) = V_0^+ e^{-\gamma z}$$
$$I(z) = I^+(z) = I_0^+ e^{-\gamma z}$$

$$Z_0 = \frac{V(z)}{I(z)} = \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma}$$

$$Z_0 = \frac{V(z)}{I(z)} = \frac{V^+}{I^+} = \frac{R + j\omega L}{\gamma}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (\Omega)$$

☝ Characteristic Impedance

❑ The ratio of the voltage and the current at any z for an infinitely long line is independent of z and it is called the Characteristic Impedance.

γ and Z_0 are the characteristics of a transmission line.

☝ **These parameters depend on R, L, G and C, but are independent of the length of transmission line.**

☞ The Characteristic Impedance of a line primarily depends upon the material properties and the line geometry.