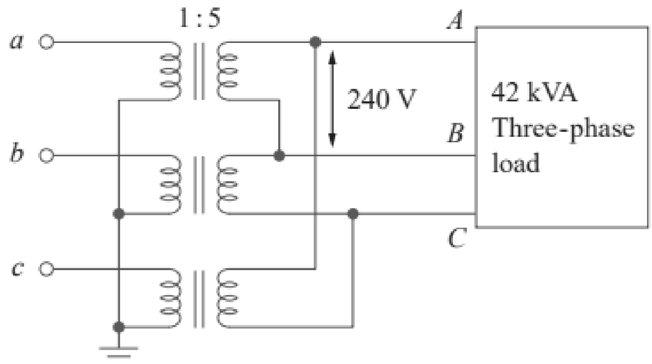
ESO203 Tutorial 6

Question1: The 42-kVA balanced load depicted below is supplied by a three-phase transformer. (a) Determine the type of transformer connections. (b) Find the line voltage and current on the primary side. (c) Determine the kVA rating of each transformer used in the transformer bank. Assume that the transformers are ideal.



Solution:

(a) A careful observation of the figure shows that the primary side is Y-connected, while the secondary side is Δ -connected. Thus, the three-phase transformer is $Y - \Delta$.

(b) Given a load with total apparent power $S_T = 42\ kVA$, the turns ratio $n = 5$ and the secondary line voltage $V_{L_s} = 240\ V$, we can find the secondary line current

$$I_{L_s} = \frac{S_T}{\sqrt{3}V_{L_s}} = \frac{42000}{\sqrt{3}(240)} = 101\ A$$

$$I_{L_p} = \frac{n}{\sqrt{3}} I_{L_s} = \frac{5 \times 101}{\sqrt{3}} = 292\ A$$

$$V_{L_p} = \frac{\sqrt{3}}{n} V_{L_s} = \frac{\sqrt{3} \times 240}{5} = 83.14\ V$$

(c) Because the load is balanced, each transformer equally shares the total load and since there are no losses (assuming ideal transformers), the kVA rating of each transformer is $S = \frac{S_T}{3} = 14\ kVA$ Alternatively, the transformer rating can be determined by the product of the phase current and phase voltage of the primary or secondary side. For the secondary side, for example, we have a delta connection, so that the phase voltage is the same as the line voltage of 240 V, while the phase current is $\frac{I_{Ls}}{\sqrt{3}} = 58.31\ A$. Hence, $S = 240 \times 58.31 \approx 14\ kVA$.

Question2: A 20-kVA, 50-Hz, 2000/200-V distribution transformer has a leakage impedance of 0.42 + j 0.52 in the high-voltage (HV) winding and 0.004 + j 0.05 in the low-voltage (LV) winding. When seen from the LV side, the shunt branch admittance Y_0 is (0.002 – j 0.015) ℧ (at rated voltage and frequency). Draw the equivalent circuit referred to (a) HV side and (b) LV side, indicating all impedances on the circuit.

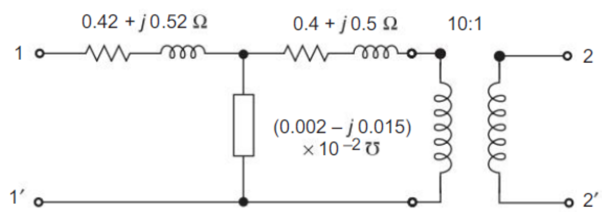
Solution: The HV side will be referred as 1 and LV side as 2.

$$a = \frac{N_1}{N_2} = \frac{2000}{200}$$

(a) Equivalent circuit referred to HV side (side 1)

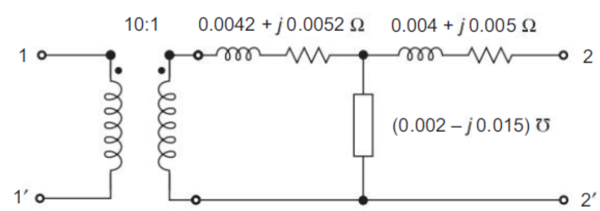
$$\overline{Z_2'} = a^2 \times (0.004 + j0.005) = 100 \times (0.004 + j0.005) = 0.4 + j0.5\Omega$$

$$\overline{Y_0'} = \frac{1}{a^2} \times (0.002 - j0.015) = \frac{1}{100} \times (0.002 - j0.015)$$
$$= (0.002 - j0.015) \times 10^{-2} ℧$$

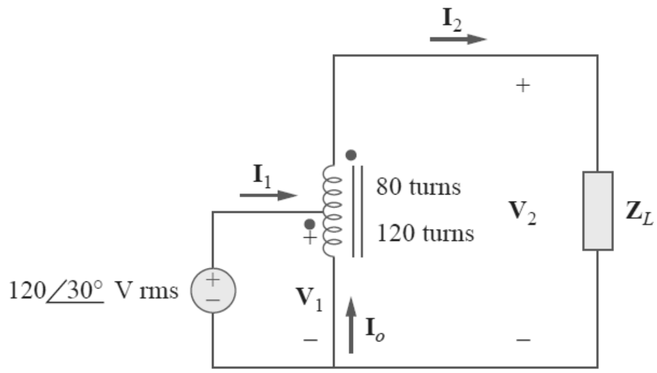


(b) Equivalent circuit referred to LV side (side 2).

$$\overline{Z_1'} = \frac{1}{a^2} \times (0.42 + \mathrm{j}\,0.52) = \frac{1}{100} \times (0.42 + \mathrm{j}\,0.52)$$
$$= (0.42 + \mathrm{j}\,0.52) \times 10^{-2}\Omega$$



Question3: An autotransformer shown in fig. Calculate a) I_1, I_2 and I_0 if Z_L=8+j6Ω and (b) the complex power supplied to load.



Solution:

(a) This is a step-up autotransformer with N_1=80 and N_2=120

$V_1 = 120\angle 30^0 V$ and by relation-

$$\frac{V_1}{V_2} = \frac{N_1}{N_1 + N_2} = \frac{80}{200}$$

So,

$$V_2 = 300\angle 30^0 \mathrm{V}$$

And

$$I2 = \frac{V_2}{Z_L} = \frac{300\angle 30^0}{8 + \mathrm{j}6} = \frac{300\angle 30^0}{10\angle 36.87^0} = 30\angle -6.87^0$$

But,

$$\frac{I_1}{I_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

So,

$$I_1 = 75\angle -6.87^0 \mathrm{A}$$

By applying KCL at node,

$$I_1 + I_0 = I_2$$

$$I_0 = I_2 - I_1 = 45\angle 173.13^0 \mathrm{A}$$

(b) The complex power supplied to the load is-

$$S_2 = V_2 \times I_2^* = Z_L \times |I_2|^2 = 9KVA$$

Question4: A single-phase transformer rated at 5 kVA, 230V/115V, 50 Hz undergoes an **open circuit test** on the **low voltage (LV) side**. The readings obtained are:

- Voltage, $V_{oc} = 115V$
- Current, $I_{oc} = 2A$
- Power, $P_{oc} = 100W$

Determine:

1. The **core loss** of the transformer.
2. The **magnetizing reactance** (X_m) and **core resistance** (R_c).

Solution:

Given Data:

Voc=115V, Ioc=2A, Poc=100W

(a) Core Loss Calculation

The power measured in the open circuit test is mainly due to core loss, given by:
$P_1 = 100\ W$

(b) Core Resistance R_0

$R_0 = \frac{{V_1}^2}{P_1}$

$R_0 = \frac{115^2}{100} = 132.25\Omega$

(c) Magnetizing Reactance X_0

$$X_0 = \frac{V_1}{I_m}$$

where,

$$I_m = \sqrt{{I_0}^2 - {I_w}^2}$$

1. Compute wattful (core loss) current I_w:

$$I_w = \frac{P_1}{V_1} = \frac{100}{115} = 0.8696A$$

2. Compute magnetizing current I_m:

$$I_m = \sqrt{{I_0}^2 - {I_w}^2} = \sqrt{2^2 - 0.8696^2} = 1.8\,A$$

$$X_0 = \frac{115}{1.8} = 63.89\Omega$$

Final Open Circuit Test Values:

Rc=132.25Ω, Xm=63.89Ω

Question5: A **short circuit test** is performed on the **high voltage (HV) side** of the transformer, and the following readings are obtained:

- Voltage, $V_{sc} = 20V$
- Current, $I_{sc} = 21.7A$
- Power, $P_{sc} = 200W$

Determine:

1. The **equivalent resistance** (R_{eq}).
2. The **equivalent reactance** (X_{eq}).

Solution:

Given Data:

Vsc=20V, Isc=21.7A, Psc=200W

(a) Equivalent Resistance Req:

$$\boldsymbol{R_{01} = \frac{P_C}{{I_1}^2} = \frac{200}{21.7^2} = 0.4245\Omega}$$

(b) Equivalent Impedance Zeq

$$\boldsymbol{Z_{01} = \frac{V_{sc}}{I_1} = \frac{20}{21.7} = 0.9212\,\Omega}$$

(c) Equivalent Reactance $\boldsymbol{X_{01}}$

$$\boldsymbol{X_{01} = \sqrt{{Z_{01}}^2 - {R_{01}}^2} = \sqrt{0.9212^2 - 0.4245^2} = 0.8175\,\Omega}$$

Thus, the equivalent parameters from the short circuit test are:

$$\boldsymbol{R_{01} = 0.4245\Omega, X_{01} = 0.8175\Omega}$$

<u>Question6:</u> A current transformer (CT) is designed to measure 10 kA in a 13.8 kV conductor and has $\boldsymbol{N_1 = 1}$ and $\boldsymbol{N_2 = 20}$.The short-circuited current $\boldsymbol{I_2}$ across the secondary winding is

<u>Solution:</u>

For a current transformer

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$I_2 = \frac{N_1}{N_2} * I_1$$

$$I_2 = 10000 * \frac{1}{20} = 500A$$

Question7: A 4-pole dc motor is lap-wound with 400 conductors. The pole shoe is 20 cm long and average flux density over one-pole-pitch is 0.4 T, the armature diameter being 30 cm. Find the torque and gross mechanical power developed when the motor is drawing 25 A and running at 1500 rpm.

Solution:

ϕ=flux per pole, n =speed in rpm, Z = total armature conductors, P = number of poles, A= number of parallel paths

$$\text{Induced emf (} E_a\text{)} = \frac{\phi n Z}{60}(P/A)$$

$$\phi = B_{av}\left(\frac{2\pi r}{P}\right)l$$

$$\text{Flux/pole} = 0.4 \times \frac{\pi \times 30 \times 10^{-2}}{4} \times 20 \times 10^{-2} = 0.0188 \text{ Wb}$$

$$E_a = \frac{0.0188 \times 1500 \times 400}{60} \times \left(\frac{4}{4}\right) = 188 \text{ V}$$

Gross mechanical power developed = induced emf X armature current= $\frac{188 \times 25}{1000} =$ 4.7 kW.

Power = torque X speed of the machine

$$\text{Torque developed} = \frac{4700}{\left(\frac{2\pi \times 1500}{60}\right)} = 29.9 \text{ Nm}$$