Assignment 13
MSO-201: Probability and Statistics

1. Suppose $X_1, \ldots, X_n$ is a random sample from a $N(1, \sigma^2)$. Find the maximum likelihood estimator of σ^2.

2. Suppose $X_1, \ldots, X_n$ is a random sample from a $N(\mu, \sigma^2)$. Find the maximum likelihood estimators of μ and σ^2.

3. Suppose $X_1, \ldots, X_n$ is a random sample from the following distribution with PDF:
$$f(x|\lambda, \mu) = \frac{1}{2\lambda} e^{-\frac{1}{\lambda}|x-\mu|}; \quad -\infty < x < \infty.$$
Find the MLEs of μ and λ.

4. Suppose $X_1, \ldots, X_n$ is a random sample from the following distribution with PDF:
$$f(x|\mu) = e^{-(x-\mu)}; \quad x > \mu,$$
and zero, otherwise. Find the MLE of μ.

5. Suppose $X_1, \ldots, X_n$ is a random sample from the following distribution with PDF:
$$f(x|\theta, \mu) = \frac{1}{\theta} e^{-\frac{1}{\theta}(x-\mu)}; \quad x > \mu,$$
and zero, otherwise. Here $\theta > 0$. Find the MLEs of θ and μ.

6. Suppose $X_1, \ldots, X_n$ is a random sample from the following distribution with PDF:
$$f(x|\theta) = \frac{1}{2\theta} \quad \text{if} \quad -\theta < x < \theta,$$
and zero, otherwise. Here $\theta > 0$. Find the MLEs of θ.

7. Suppose $X_1, \ldots, X_n$ is a random sample from the following Cauchy distribution with PDF:
$$f(x|\theta) = \frac{1}{\pi(1 + (x-\theta)^2)}; \quad -\infty < x < \infty.$$
Find the MLEs of θ.

8. Suppose $X_1, \ldots, X_n$ is a random sample from the following gamma distribution with PDF:
$$f(x|\alpha, \lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}; \quad x > 0,$$
and zero, otherwise. Find the MLEs of λ, in terms of α, assuming α is known.

9. You want to estimate $p = \text{Prob(Head)}$ of a coin. You toss it 6 times and you observe 2 head. What will be the MLE of p. If you know that p can be either 1/3 or 2/3, what will be the MLE of p.

10. Suppose $X_1, \ldots, X_n$ is a random sample from the following distribution with PDF:
$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if} \quad \lambda = 1 \\ \lambda^2 x e^{-\lambda x} & \text{if} \quad \lambda = 2 \end{cases}$$
for $x > 0$, and zero, otherwise. Find the MLE of λ.