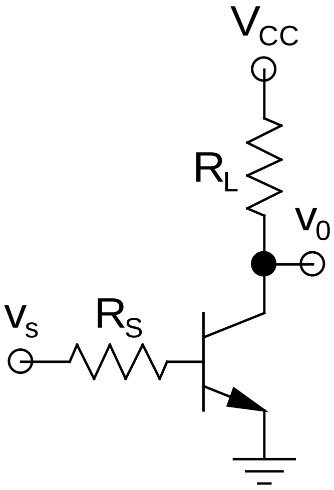
High-Frequency Response

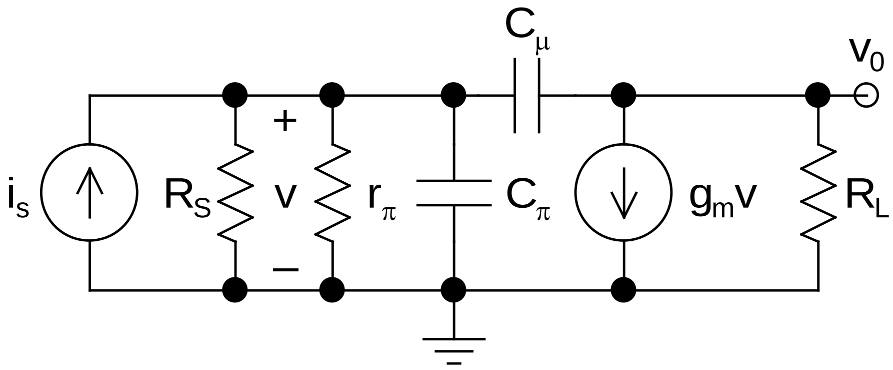
- Will consider ***3 methods***:
  - ***Exact Analysis***:
    - The ***most accurate*** and the ***most rigorous***
    - ***Gives information about all poles and zeros of the system***
  - ***Miller Effect Approximation***:
    - ***One level of approximation***
    - ***Gives information about the Dominant Pole (DP) and one Non-Dominant Pole (NDP)***

- ***Zero-Value Time Constant (ZVTC) Technique***:
  - ***The easiest one***
  - ***Information regarding only the DP***
  - ***Suppresses information about all other poles and zeros of the system***
  - ***Reasonable accuracy***
  - ***Underestimates f_H slightly*** (***better than overestimating and not achieving it***!)
  - ***Based on heuristic***
  - ***Similar to the IVTC technique***, ***based on an algorithm***

- ***Exact Analysis of a CE Stage*:**



ac Schematic



High-Frequency Equivalent

- ***Biasing circuits omitted for simplicity***
- ***Converted input v_s to its Norton equivalent***

- ***KCL at input node*** (using ***Laplace operator*** s = jω and $R = R_S||r_\pi$):

$i_s = v/R + sC_\pi v + sC_\mu(v - v_0)$

$= [1/R + s(C_\pi + C_\mu)]v - sC_\mu v_0$

- ***KCL at output node***:

$sC_\mu(v_0 - v) + g_m v + v_0/R_L = 0$

$$\Rightarrow v = -\frac{1/R_L + sC_\mu}{g_m - sC_\mu} v_0$$

$$\Rightarrow \frac{v_0}{i_s}(s) = -\frac{R_L R\left(g_m - sC_\mu\right)}{1 + s\left(R_L C_\mu + RC_\mu + RC_\pi + g_m R_L RC_\mu\right) + s^2 R_L RC_\pi C_\mu}$$

- Thus, the ***voltage gain***:

$$A_v(s) = \frac{v_0}{v_s} = -\frac{g_m R_L R}{R_S} \frac{\left(1 - sC_\mu / g_m\right)}{1 + sa + s^2 R_L R C_\pi C_\mu} \qquad (1)$$

$$a = R_L C_\mu + R(C_\pi + C_\mu) + g_m R_L R C_\mu$$

- Hence, the circuit has ***one zero*** and ***two poles***

$$\Rightarrow A_v(s) = A_{v0} \frac{\left(1 - s/z_1\right)}{\left(1 - s/p_1\right)\left(1 - s/p_2\right)} \qquad (2)$$

A_{v0} = ***midband gain*** $= - g_m R_L R / R_S$

$= - g_m R_L r_\pi / (r_\pi + R_S)$

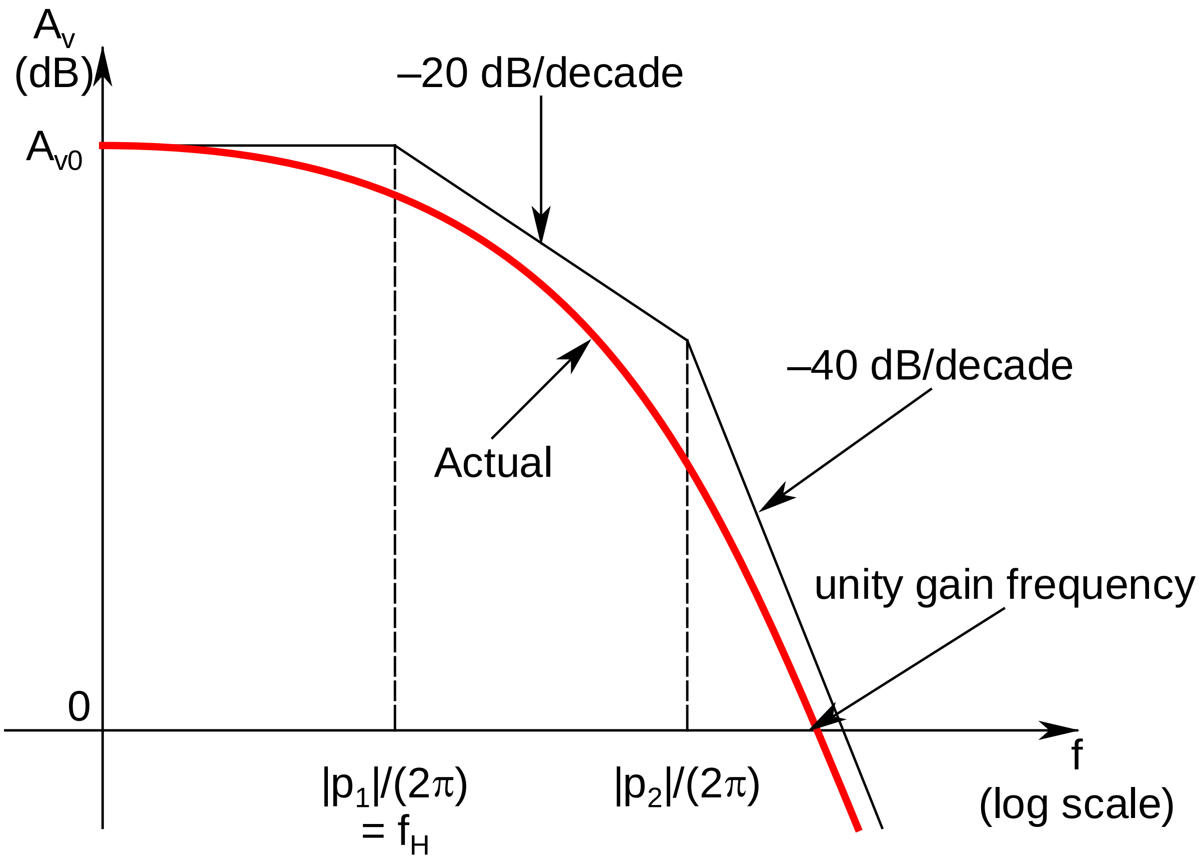
- z_1 (= g_m/C_μ): ***positive real zero***
- The ***frequency*** corresponding to z_1 occurs at $z_1/(2\pi)$, which is ***extremely high***, and generally, is ***not of much consequence***
- ***Computation of the two poles p_1 and p_2 is slightly more tricky***
- From Eqs.(1) and (2), it is obvious that ***both p_1 and p_2 are real and negative***

- To find these, ***write the denominator*** of Eq.(1) as:

$$D(s) = (1 - s/p_1)(1 - s/p_2)$$
$$= 1 - s(1/p_1 + 1/p_2) + s^2/(p_1 p_2) \qquad (3)$$

- ***Matching coefficients*** with Eq.(2), we can get p_1 and p_2, however, the ***resulting algebra*** will become ***extremely tedious***
- Hence, we invoke the ***Dominant Pole Approximation*** (DPA)

- ***DPA***:
  - The ***smallest pole*** [***Dominant Pole*** (DP)] is ***at least 10 times away from its nearest pole***
  - This is an ***excellent approximation for practical analog circuits***
- ***Apply this approximation*** and ***assume p_1 to be the DP*** and ***at least 10 times away from p_2*** [***Non-Dominant Pole*** (NDP)]
- The ***pole frequencies*** are $|p_1|/(2\pi)$ and $|p_2|/(2\pi)$
- ***Note***: $|p_1|/(2\pi)$ is the ***Upper Cutoff Frequency*** (f_H)



Bode Plot of the Frequency Response of a 2-Pole System

- ***2-pole system***
- ***For frequencies till the first pole p_1, gain remains constant at its midband value of $20\log_{10}A_{v0}$***
- ***Beyond this***, the ***gain rolls off at –20 dB/decade till the second pole p_2 is encountered***
- ***After this***, the ***gain rolls off at –40 dB/decade***, and ***eventually crosses zero*** (at ***unity gain frequency)***
- ***Beyond this***, the circuit actually ***attenuates the input signal instead of amplifying it*** (***gain magnitude drops below unity***)

- ***It's assumed that z_1 is >> $|p_2|$***
- ***Task remains to find p_1 and p_2***
- ***Under DPA***, Eq.(2) can be simplified as:

  $$D(s) \approx 1 - s/p_1 + s^2/p_1 p_2 \qquad (4)$$

- ***Comparing*** Eq.(4) with the ***denominator*** of Eq.(1):

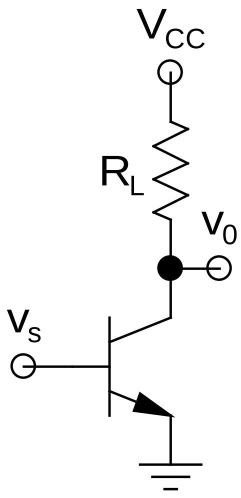
$$p_1 = -\frac{1}{(R_S \| r_\pi) C_\pi + \left[(R_S \| r_\pi) + R_L + g_m (R_S \| r_\pi) R_L \right] C_\mu}$$

$$p_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{(R_S \| r_\pi) C_\pi} + \frac{1}{R_L C_\pi} + \frac{g_m}{C_\pi} \right)$$

- In general, $|p_2| >> |p_1|$
- ***Ex.***: I_C = 1 mA, β = 200, R_S = 1 kΩ, R_L = 2 kΩ, C_π = 10 pF, C_μ = 0.5 pF

  $\Rightarrow$ ***DPF*** = 3.8 MHz, ***NDPF*** = 798.8 MHz, ***ZF*** = 12.3 GHz, and f_H **= *DPF*** = 3.8 MHz
- ***Note***: Even for a ***simple CE circuit***, the ***analysis is so cumbersome***, and the ***results are so complicated***
- ***Definitely not acceptable for routine application***, ***particularly for circuits having more than one active device***

- ***Miller Effect Approximation***:
  - ***Technique by which an input-output coupled circuit can be decoupled by removing the coupling element***
  - This ***removal*** is done by ***splitting*** it into ***two components*** - putting ***one in the input circuit***, and the ***other in the output circuit***
  - We take the ***same example*** as the ***CE circuit*** discussed earlier, but now ***without*** R_S



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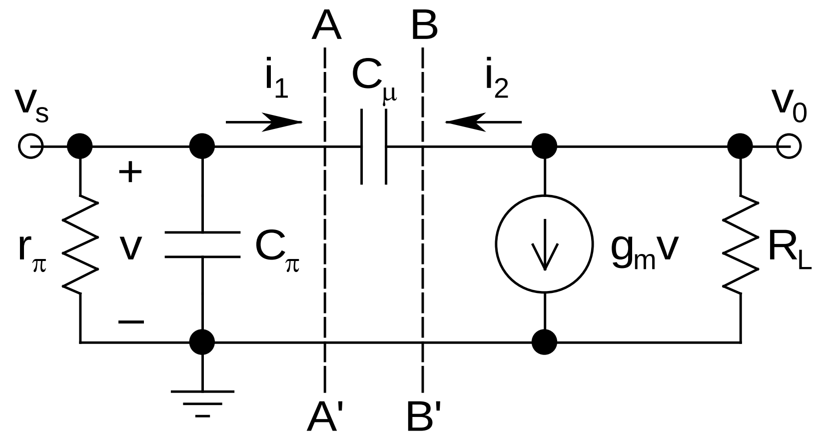

High-Frequency Equivalent

➢ ***Identify C_μ as the input-output coupling element***

➢ After ***application*** of the ***technique***, this ***coupling element*** will be ***removed*** by ***splitting*** it into ***two parts*** - ***one at input***, ***other at output***

- These ***two parts*** can be found by ***evaluating*** the ***impedances*** looking into the ***planes*** AA′ and BB′
- ***KCL at output node***:

$$g_m v + v_0/R_L + sC_\mu(v_0 - v) = 0$$

- Noting that $v = v_s$, the ***voltage gain***:

$$A_v(s) = -\, g_m R_L (1 - sC_\mu/g_m)/(1 + sR_L C_\mu)$$

$\Rightarrow$ ***Midband or low-frequency gain***:

$$A_v(0) = -\, g_m R_L$$

This result can also be written from inspection

- ***Current entering plane AA′***:

$$i_1 = sC_\mu(v - v_0) = sC_\mu[1 - A_v(s)]v$$

- Hence, the ***admittance*** looking into the ***plane*** AA′:

$$y\big|_{AA'} = i_1/v = sC_\mu\left[1 - A_v(s)\right]$$

- This ***admittance*** is ***capacitive*** in nature, and is known as the ***Miller Capacitance*** C_M:

$$C_M = C_\mu[1 - A_v(s)]$$

- Now, since ***$A_v(s)$ is a function of frequency***, ***so would C_M*** $\Rightarrow$ ***Problem***!

- Here, we invoke the ***Miller Effect Approximation*** (MEA)
  - ***$A_v(s)$ is replaced by $A_v(0)$***, i.e., by its ***midband value***, which is a ***constant***
  - Thus, ***C_M becomes a constant*** with a value of

    $$C_M = [1 - A_v(0)]C_\mu = (1 + g_m R_L)C_\mu$$
- Thus, ***$C_M >> C_\mu$***, since, in general, ***$g_m R_L >> 1$***
- This effect is known as the ***Miller Effect Multiplication***
- ***Care***: ***The gain that multiplies C_μ is across its two ends***

- Similarly, ***current entering plane BB′***:

$$i_2 = sC_\mu(v_0 - v) = sC_\mu[1 - 1/A_v(s)]v_0$$

- Hence, the ***admittance*** looking into the ***plane*** BB′:

$$y'\big|_{BB'} = i_2/v_0 = sC_\mu\left[1 - 1/A_v(s)\right]$$

- Again ***replacing*** $A_v(s)$ by $A_v(0)$, we get:

$$C'_M = \left[1 - 1/A_v(0)\right]C_\mu = \left[1 + 1/(g_m R_L)\right]C_\mu$$

- In general, $g_m R_L >> 1 \Rightarrow \; C'_M \simeq C_\mu$