Lecture-4

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Series and Parallel resistor, capacitor and inductor.
- Voltage division rule.
- Current division rule.
- Mesh analysis.
- Nodal analysis.

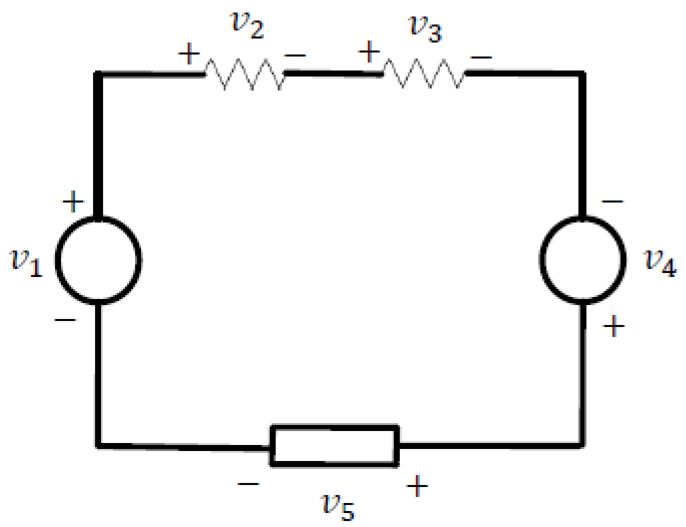
Kirchhoff's Law (cont…)

- Kirchhoff's second law is based on the principle of conservation of energy.
- This law is known as Kirchhoff's voltage law and states that the algebraic sum of all the voltages around a closed loop is zero.

$$\sum_{m=1}^{M} v_m = 0$$

- Here, M is the number of voltages in the loop and v_m is the m^{th} voltage.
- Following the above convention and applying KVL in the loop in the figure,

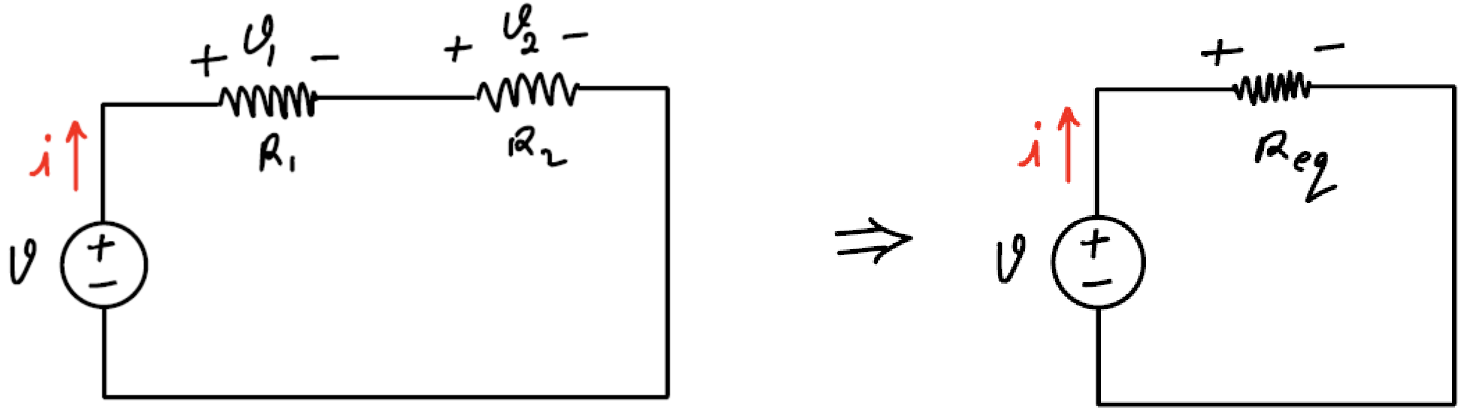
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$
$$\Rightarrow v_1 + v_4 = v_2 + v_3 + v_5$$



- This implies that the sum of voltage drops is equal to the sum of voltage rises.

Series Resistors and Voltage Division

- Consider the circuit below, where two resistors are connected in series as the same current flows through both of them.



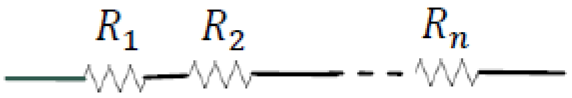
- Applying Ohm's law to each resistor, we have v_1=iR_1 and v_2=iR_2.
- Applying KVL to the loop we get, $v = v_1$+$v_2 = i(R_1$+$R_2)$=iR_{eq}.

Series Resistors and Voltage Division (cont…)

- The equivalent resistance of the circuit is therefore,

$$R_{eq} = R_1 + R_2$$

- The same can be extended for multiple resistors.
- The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.



$$R_{eq} = R_1 + R_2 + \cdots + R_n = \sum_{i=1}^{n} R_i$$

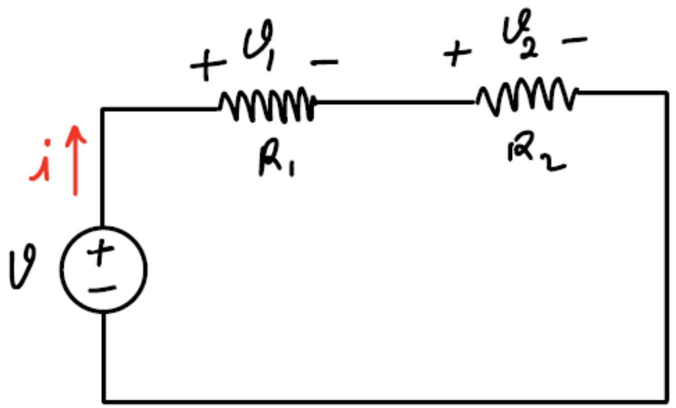
- The voltage is divided among the resistors in direct proportion to their resistances.
- The voltage across resistor n, v_n is given by

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_n} v$$

Series Resistors and Voltage Division (cont…)

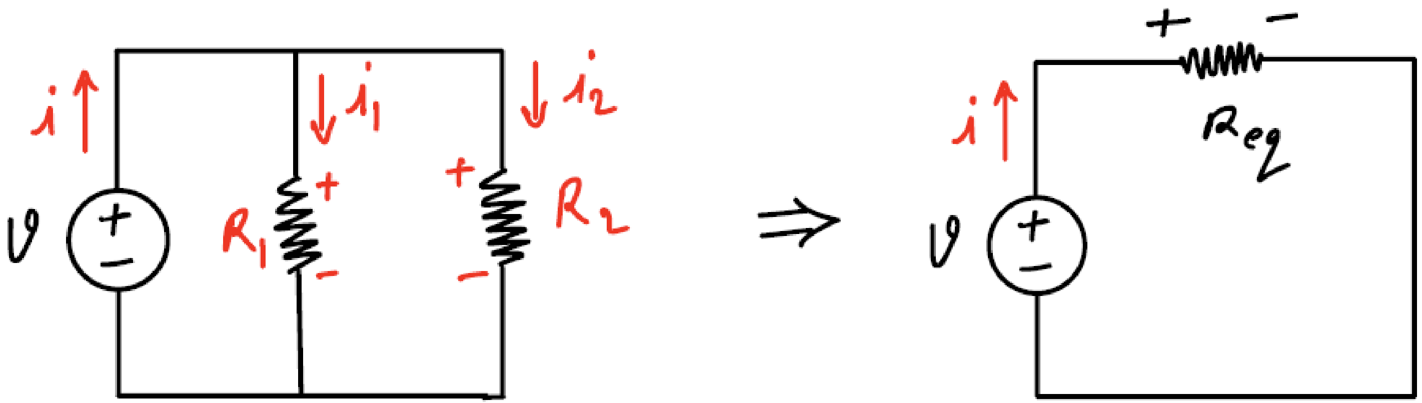
- The voltage across resistor n, v_n is given by

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_n} v$$



Parallel Resistors and Current Division

- Consider the circuit below, where two resistors are connected in parallel as they have the same voltage across them.



- Applying Ohm's law to each resistor, we have $v = i_1 R_1 = i_2 R_2$.
- Applying KCL at the node we get,

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

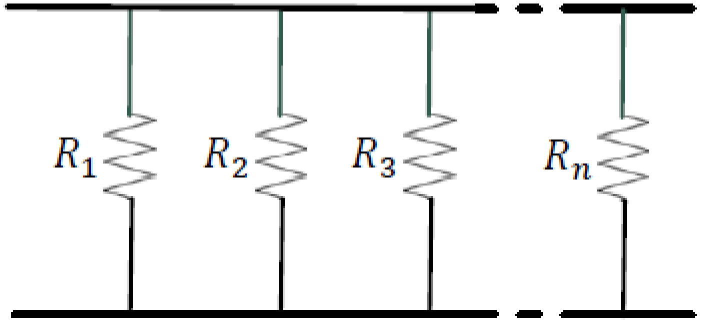
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Parallel Resistors and Current Division (cont…)

- The equivalent resistance of the circuit is therefore,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- The equivalent resistance of any number of resistors connected in parallel is the reciprocal of the sum of the reciprocal of the individual resistances.



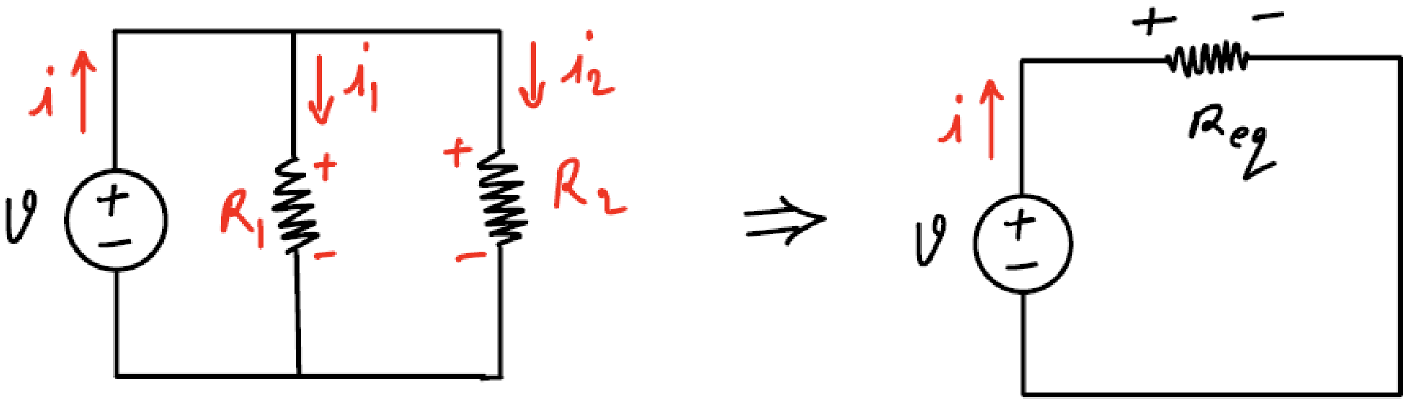
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

- For the circuit given in the previous slide the total current, i is shared by the resistors in inverse proportion to their resistances. The currents through the resistors are expressed as,

$$i_1 = \frac{iR_2}{R_1 + R_2} \;\&\; i_2 = \frac{iR_1}{R_1 + R_2}$$

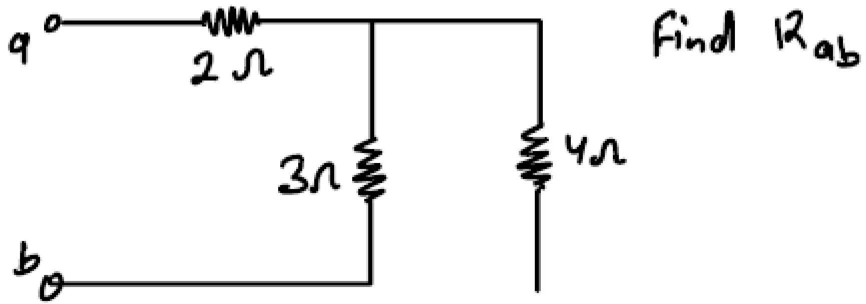
Parallel Resistors and Current Division (cont…)

- For the circuit given in the previous slide the total current, i is shared by the resistors in inverse proportion to their resistances. The currents through the resistors are expressed as,

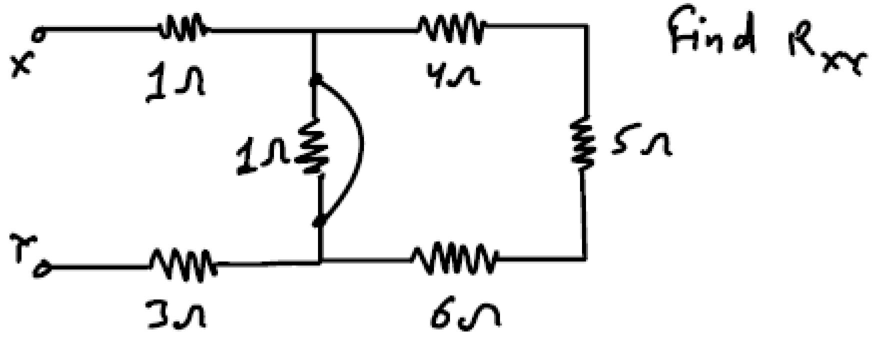


$$i_1 = \frac{iR_2}{R_1 + R_2} \ \& \ i_2 = \frac{iR_1}{R_1 + R_2}$$

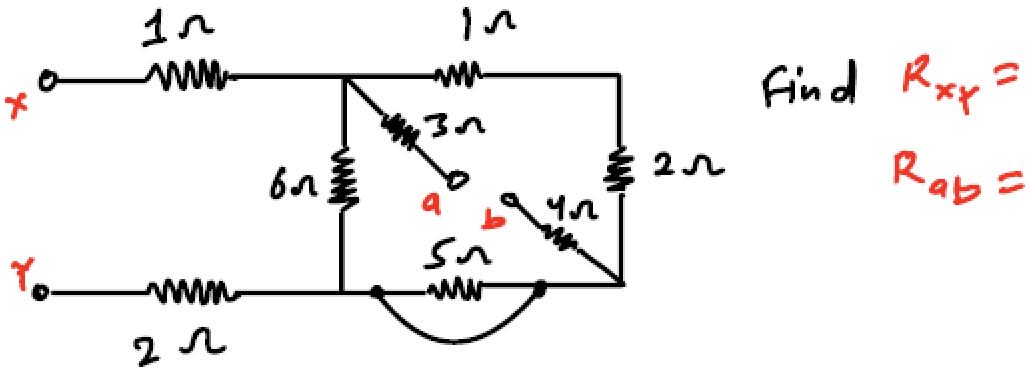
EXAMPLE



$R_{ab} = 5\Omega$



$R_{xy} = 4\Omega$

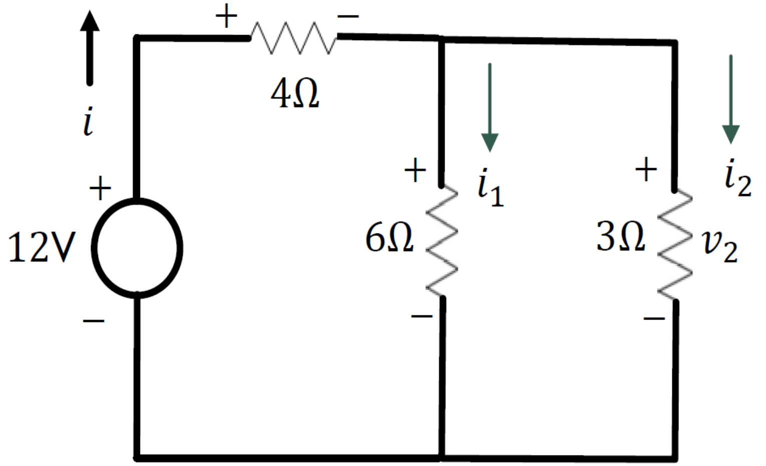


$R_{xy} = 5\Omega$

$R_{ab} = 9\Omega$

EXAMPLE (cont…)

- Find i_2 and v_2 in the circuit shown in the figure? Also, calculate the power dissipated in the 3Ω resistor?



SOLUTION: The equivalent resistance for the circuit is

$$R_{eq}=4+(6||3)=4+\frac{6*3}{6+3}=6\ \Omega$$

The current i is equal to,

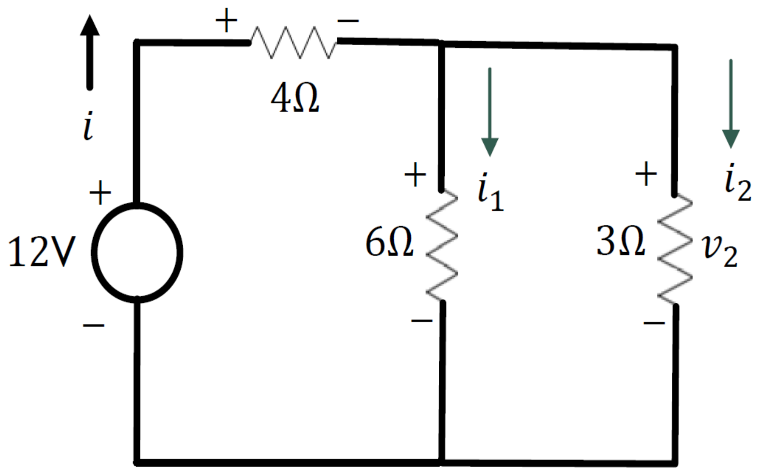
$$i=\frac{v}{R_{eq}}=\frac{12}{6}=2\text{ A}$$

Apply current division rule:

$$i_2=\frac{i\times 6\Omega}{6\Omega+3\Omega}=\frac{2\times 6}{9}=\frac{4}{3}\ A$$

EXAMPLE (cont…)

$$i_2 = \frac{i \times 6\Omega}{6\Omega + 3\Omega} = \frac{2\times6}{9} = \frac{4}{3}\ A$$



Voltage v_2 is calculated as:

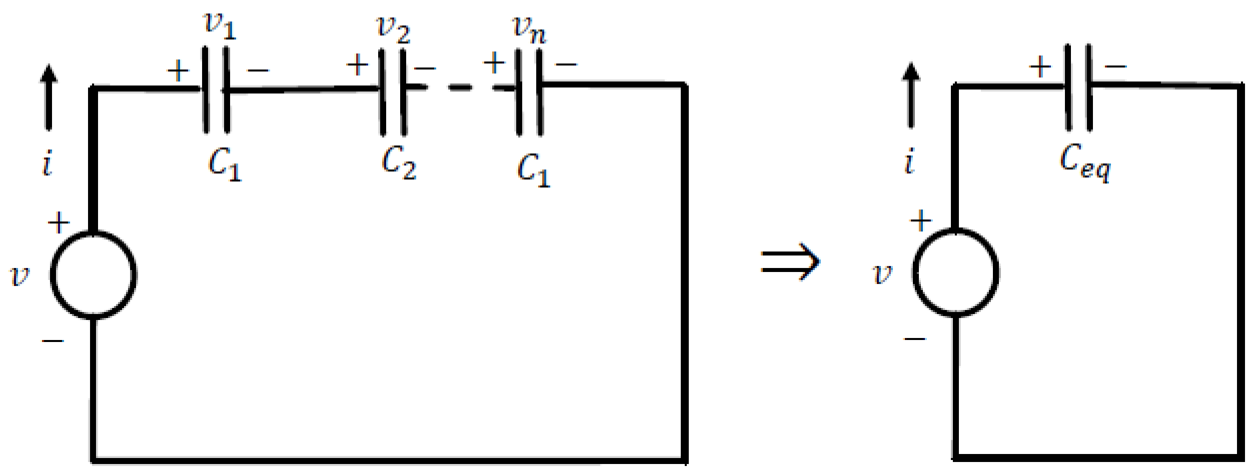
$$v_2 = i_2 * 3\Omega = 4\text{v}$$

The power dissipated is given by,

$$P = v_2 i_2 = 4 * \left(\frac{4}{3}\right) = 5.33W$$

Series Capacitors

- Consider the circuit below, where n capacitors are connected in series and its corresponding equivalent circuit



- The capacitors have the same current flowing through them.
- Applying KVL in the loop we get, $v = v_1 + v_2 + \cdots + v_n$.

Series Capacitors (cont..)

- But we know that $v_{k(t)} = \frac{1}{c_k}\int_0^t idt + v_{k(t_o)}$ .Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} idt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} idt + v_2(t_0) + \cdots + \frac{1}{C_n}\int_{t_0}^{t} idt + v_n(t_0)$$
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)\int_{t_0}^{t} idt + v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0) = \frac{1}{C_{eq}}\int_{t_0}^{t} idt + v(t_0)$$

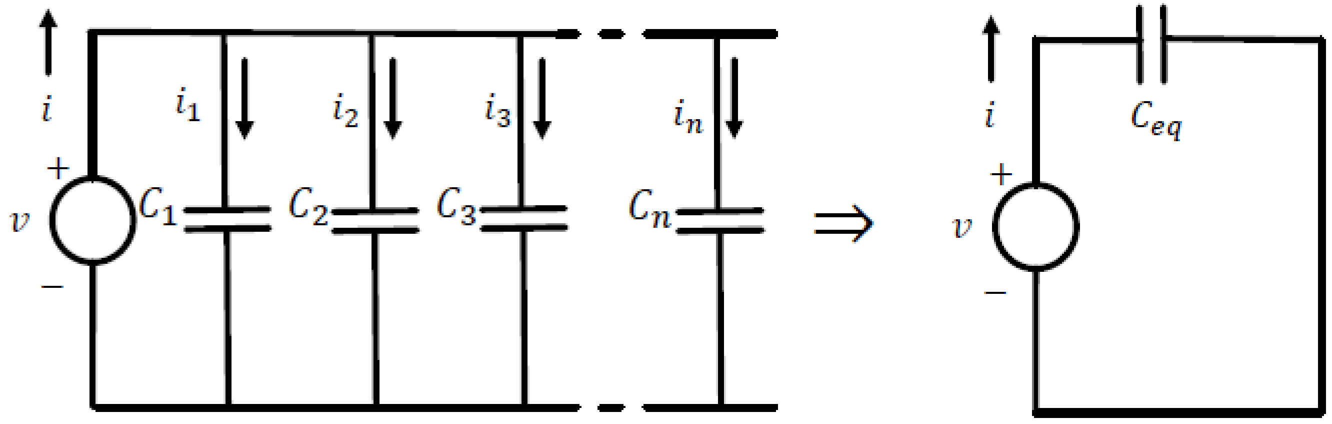
Therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \sum_{i=1}^{n}\frac{1}{C_i}$$

- The equivalent capacitance of **n** series-connected capacitors is the reciprocal of the sum of the reciprocal of the individual capacitances.
- We observe that capacitors in series combine in the same manner as resistors in parallel.

Parallel Capacitors

- Consider the circuit below, where $\boldsymbol{n}$ capacitors are connected in parallel.



- The capacitors have the same voltage across them
- Applying KCL at the node we get, $i = i_1 + i_2 \ldots + i_n$.

Parallel Capacitors (cont…)

But we know that $i_k = c_k \frac{dv}{dt}$. Therefore,

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \cdots + C_n \frac{dv}{dt}$$
$$= \sum_{i=1}^{n} C_i \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

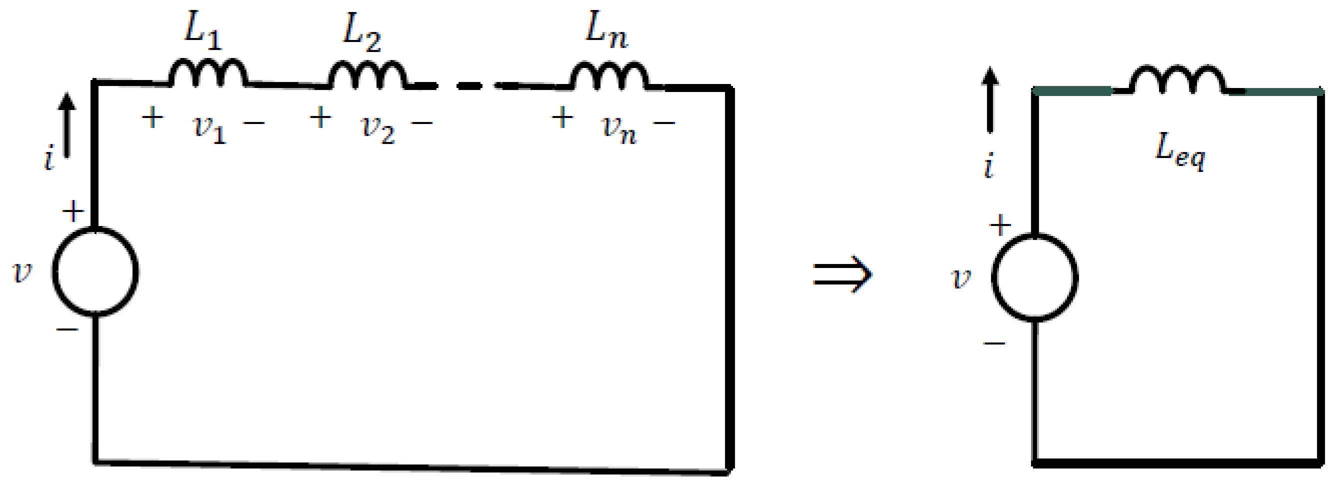
Therefore,

$$C_{eq} = C_1 + C_2 + \cdots + C_n = \sum_{i=1}^{n} C_i$$

- The equivalent capacitance of n parallel-connected capacitors is the sum of the individual capacitances.
- We observe that capacitors in parallel combine in the same manner as resistors in series.

Series Inductors

- Consider the circuit below, where n inductors are connected in series and its corresponding equivalent circuit



- The inductors have the same current flowing through them.
- Applying KVL in the loop we get, $v = v_1 + v_2 + \cdots + v_n$.

Series Inductors (cont…)

- But we know that $v_k = l_k \frac{di}{dt}$. Therefore,

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_n \frac{di}{dt}$$
$$= \sum_{i=1}^{n} L_i \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

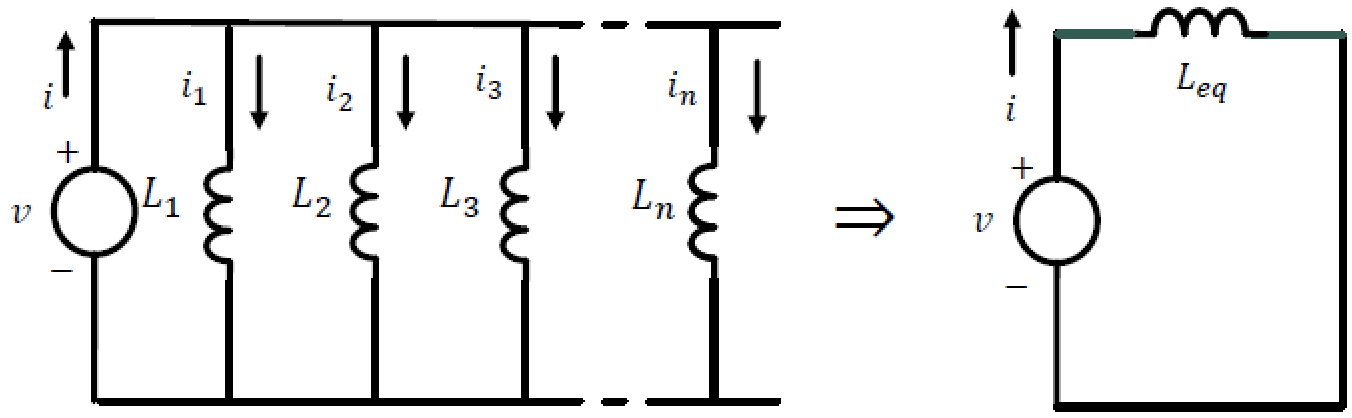
- Therefore,

$$L_{eq} = L_1 + L_2 + \cdots + L_n = \sum_{i=1}^{n} L_i$$

- The equivalent inductance of n series-connected inductors is the sum of the individual inductances.
- We observe that inductors in series combine in the same manner as resistors in series.

Parallel Inductors

- Consider the circuit below, where n inductors are connected in parallel –



- The inductors have the same voltage across them.
- Applying KCL at the node we get, $i = i_1 + i_2 + \cdots + i_n$.

Parallel Inductors (cont…)

- But we know that $i_k = \frac{1}{L_k}\int_{t_o}^{t} v dt + i_k(t_o)$. Therefore,

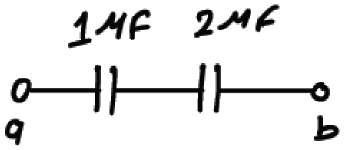
$$i = \frac{1}{L_1}\int_{t_0}^{t} vdt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} vdt + i_2(t_0) + \cdots + \frac{1}{L_n}\int_{t_0}^{t} vdt + i_n(t_0)$$
$$= \left(\frac{1}{l_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}\right)\int_{t_0}^{t} vdt + i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} vdt + i(t_0)$$

- Therefore,

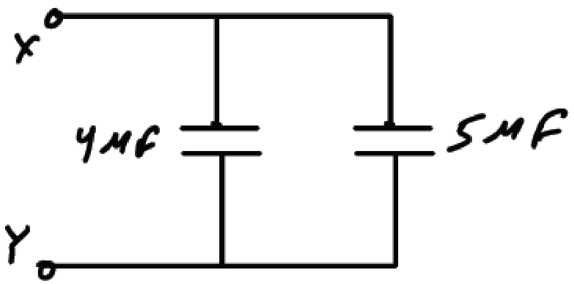
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} = \sum_{i=1}^{n} \frac{1}{L_i}$$

- The equivalent inductance of n parallel-connected inductors is the reciprocal of the sum of the reciprocal of the individual inductances.
- We observe that inductors in parallel combine in the same manner as resistors in parallel.

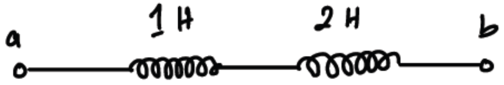
EXAMPLE



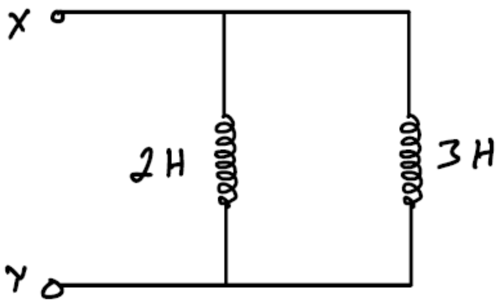
$$\frac{1}{C_{ab}} = \frac{1}{1\times10^{-6}} + \frac{1}{2\times10^{-6}}$$



$$C_{ab} = 4\times10^{-6} + 5\times10^{-6}$$



$$L_{ab} = 1 + 2$$

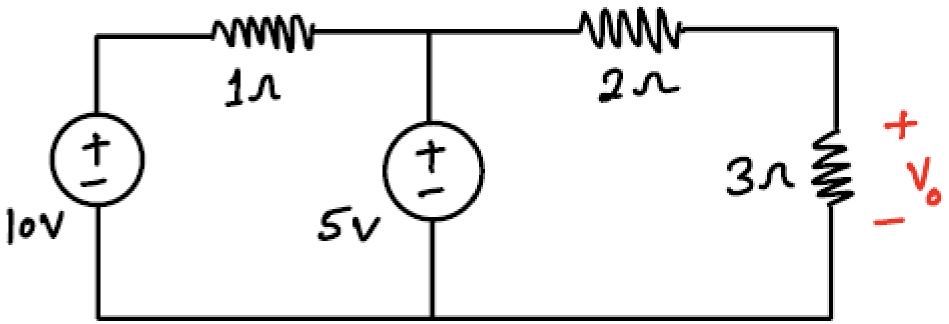


$$\frac{1}{L_{ab}} = \frac{1}{2} + \frac{1}{3}$$

Methods of Analysis

- Mesh Analysis = KVL + OHM's.
- Nodal Analysis = KCL + OHM's.

Q1: Using mesh analysis find the value of v_o?



Mesh-1:

$$-10 + i_1 + 5 = 0$$

$$i_1 = 5\ A$$

Mesh-2:

$$-5 + 2i_2 + 3i_2 = 0$$

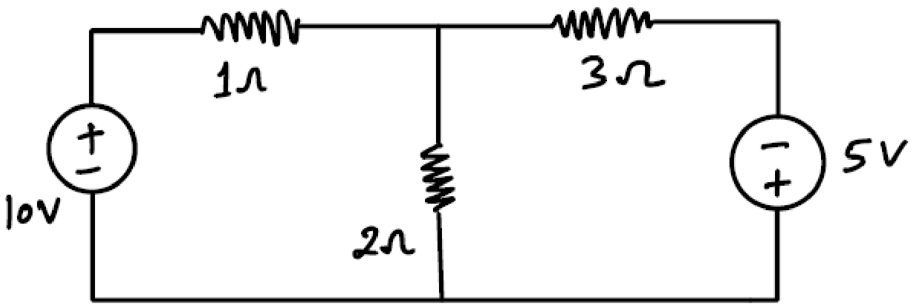
$$i_2 = 1\ A$$

$$v_o = 3i_2 = 3 \times 1 = 3V$$

Methods of Analysis (cont…)

- Mesh Analysis = KVL + OHM's.
- Nodal Analysis = KCL + OHM's.

Q1: Using mesh analysis find the value of v_o?



Mesh-1:

$$-10 + i_1 + 2(i_1 - i_2) = 0$$

$$3i_1 - 2i_2 = 10$$

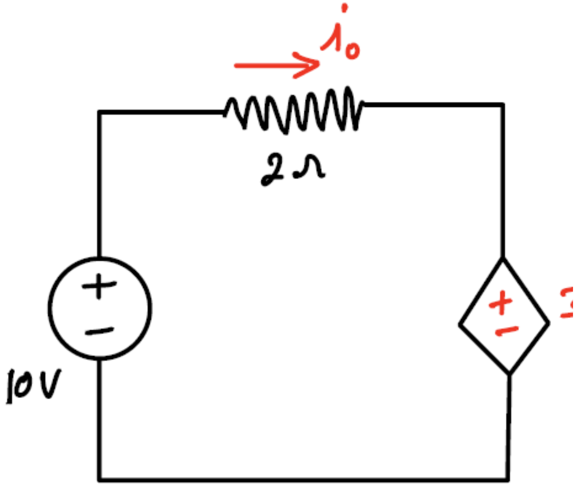
Mesh-2:

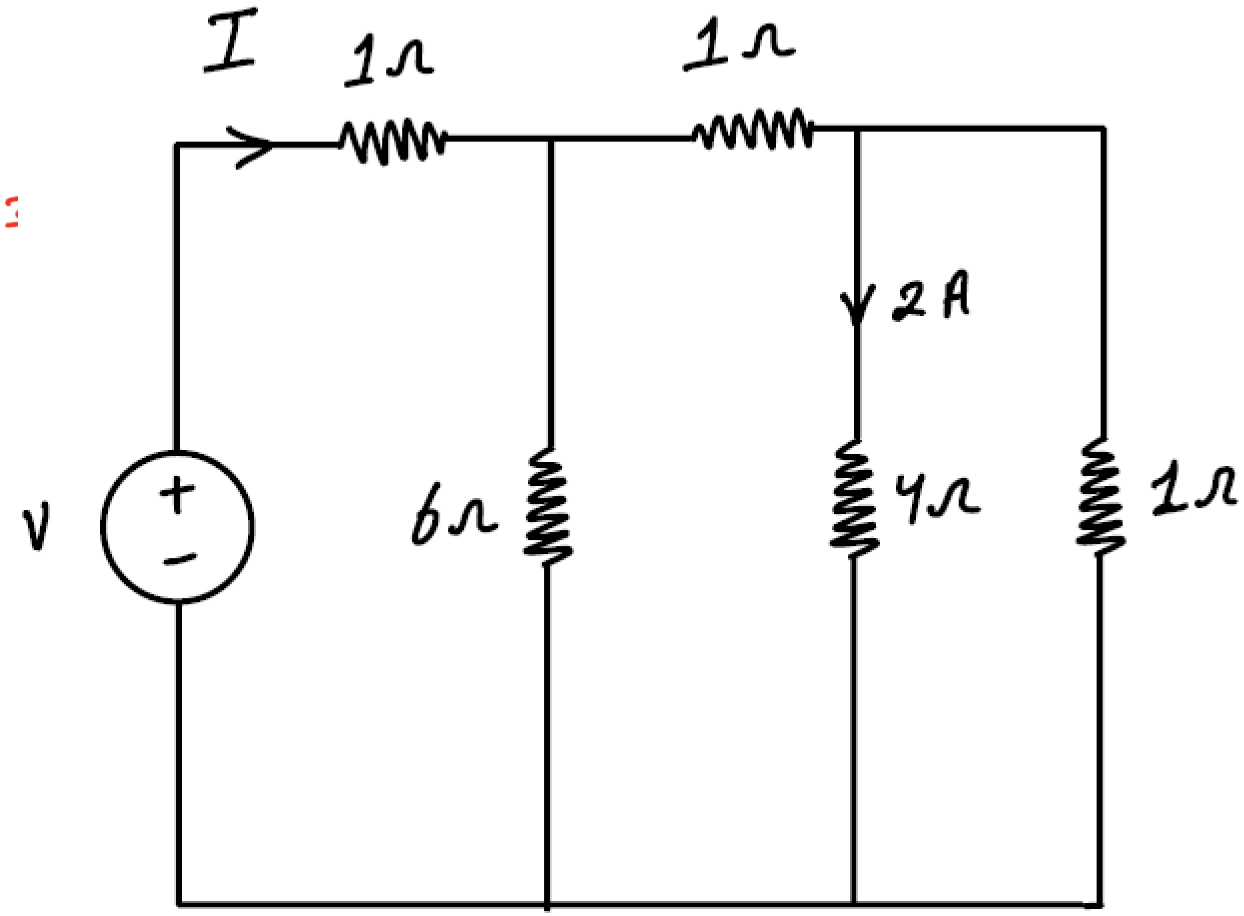
$$2(i_2 - i_1) + 3i_2 - 5 = 0$$

$$-2i_1 + 5i_2 = 5$$

EXAMPLE

- Find power delivered by dependent source?

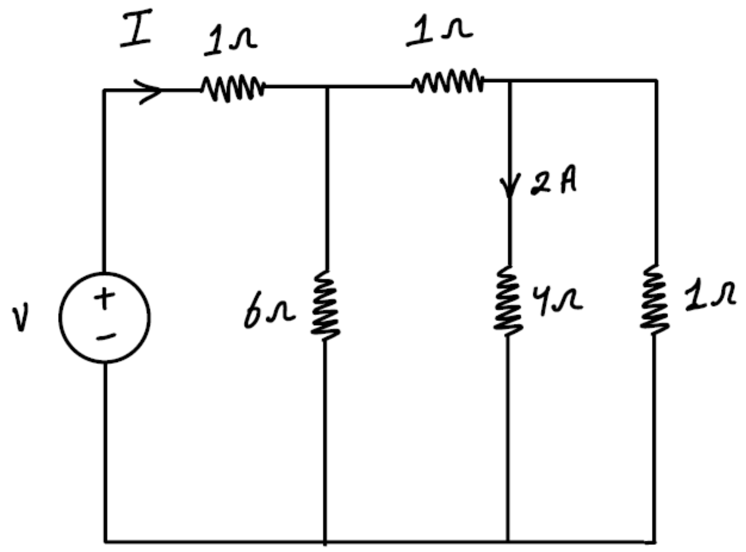




12 W

EXAMPLE

- Find the value of V and I, if current through 4Ω resistor is 2A.




THANKS
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