

Lecture-3

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Capacitor.
- Inductor.
- Branch, Node, Loop and Mesh in electrical network.
- Kirchhoff's Law (KCL and KVL).

CIRCUIT ELEMENTS (Cont...)

□ Power dissipated in a resistor:

- The power dissipated in resistor can be expressed in terms of the resistance, R .
- We know the equation for power as $p = vi$.
- From Ohm's law we know that $v = iR$.
- Combining the above two equations, we obtain

$$p = vi = i^2R = \frac{v^2}{R}$$

- The power dissipated can also be expressed in terms of conductance as,

$$p = vi = v^2G = \frac{i^2}{G}$$

CIRCUIT ELEMENTS (Cont...)

□ Example:

- A 30 V voltage is applied across a resistor of resistance 5 k Ω. Calculate the current, the conductance and the power absorbed by the resistor?

□ Solution: The voltage across the resistor $v=30$ V.

$$\text{Current } i = \frac{v}{R} = \frac{30}{5*10^3} = 6 \text{ mA}$$

The conductance is given by,

$$G = \frac{1}{R} = \frac{1}{5*10^3} = 0.2 \text{ mS}$$

The power dissipated or absorbed by the resistor can be evaluated using any of the formulas discussed in the lecture, as follows:

$$p = vi = 30 * (6 * 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2 R = (6 * 10^{-3})^2 * 5 * 10^3 = 180 \text{ mW}$$

or

$$p = v^2 G = (30)^2 * 0.2 * 10^{-3} = 180 \text{ mW}$$

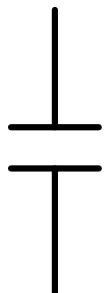
CIRCUIT ELEMENTS (Cont...)

□ Capacitor:

- A capacitor is a passive element designed to store charge in its electric field.
- A capacitor typically is constructed using two conducting plates separated by an insulator/dielectric.
- When a voltage, v is connected to the capacitor a charge q is deposited on one plate and a charge $-q$ is deposited on the other plate.
- Capacitor is therefore said to store charge.
- The amount of charge stored is directly proportional to v and is expressed as,

$$q = Cv$$

- C is the constant of proportionality and is known as the **capacitance** of the **capacitor**.
- In electric circuits a capacitor is represented as



CIRCUIT ELEMENTS (Cont...)

□ Capacitor (cont.):

- Capacitance is therefore defined as the ratio of the **charge on one plate** of a capacitor to the **voltage difference** between the two plates, and is measured in farads (F).
- Therefore, 1 farad = 1 coulomb/volt.
- Capacitance of a capacitor also depends on the physical dimensions of a capacitor and is expressed as,

$$C = \frac{\epsilon A}{d}$$

where, **A** is the surface area of each plate, **d** is the distance between the plates, and **ϵ** is the permittivity of the dielectric between the plates, measured in F/m.

- The permittivity of free space is 8.85×10^{-12} F/m.

CIRCUIT ELEMENTS (Cont...)

□ Capacitor (cont..):

- To obtain the current-voltage relationship in a capacitor, we use the equations $i = dq/dt$ and $q = Cv$, to obtain.

$$i = C \frac{dv}{dt}$$

- The instantaneous power delivered to the capacitor is

$$p = vi = Cv \frac{dv}{dt}$$

- The energy stored in a capacitor is therefore,

$$w = \int_{-\infty}^t p(\tau) d\tau = C \int_{-\infty}^t v \frac{dv}{d\tau} d\tau = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} Cv^2 \Big|_{v(-\infty)}^{v(t)}$$

CIRCUIT ELEMENTS (Cont...)

□ Capacitor (cont..):

- Since the capacitor was uncharged at $t = -\infty$, $v(-\infty) = 0$,

$$w = \frac{1}{2} C v^2 = \frac{q^2}{2C}$$

- The above equation represents the energy stored in the electric field that exists between the plates of the capacitor.
- This energy can be retrieved, since an ideal capacitor cannot dissipate energy.
- In fact, the word **capacitor** is derived from this element's capacity to store energy in an electric field.
- When the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero.
- A capacitor, therefore, acts like an open circuit to DC.

CIRCUIT ELEMENTS (Cont...)

□ Example:

Calculate the charge stored on a 3 pF capacitor with 20 V applied across it? Also, find the energy stored in the capacitor?

□ Solution:

Since

$$q = Cv$$

$$q = 3 * 10^{-12} * 20 = 60 \text{ pC}$$

The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} * 3 * 10^{-12} * 20^2 = 600 \text{ pJ}$$

CIRCUIT ELEMENTS (Cont...)

□ Example:

The voltage across a $5 \mu\text{F}$ capacitor is $v(t) = 10\cos 6000t$ V? Calculate the current through it?

□ Solution:

The current through a capacitor is given by,

$$\begin{aligned} i(t) &= C \frac{dv}{dt} = 5 * 10^{-6} * \frac{d}{dt}(10 \cos 6000t) = \\ &= -5 * 10^{-6} * 6000 * 10 \sin 6000t = -0.3 \sin 6000t \text{ A} \end{aligned}$$

CIRCUIT ELEMENTS (Cont...)

□ Example:

Determine the voltage across a $2 \mu\text{F}$ capacitor if the current through it is $i(t)=6e^{-3000t}$ mA? Assume the initial capacitor voltage to be zero.

□ Solution:

The voltage across a capacitor is expressed as,

$$v(t) = \frac{1}{C} \int_0^t i dt + v(0) \text{ and } v(0) = 0$$

$$v = \frac{1}{2 \cdot 10^{-6}} * \int_0^t 6e^{-3000t} dt * 10^{-3} = 1 - e^{-3000t} \text{ V}$$

CIRCUIT ELEMENTS (Cont...)

□ Inductor :

- An inductor is another passive element designed to store energy in its magnetic field.
- An inductor consists of a coil of conducting wire.
- If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.
- The above relation can be mathematically expressed as,

$$v = L \frac{di}{dt}$$

- **L** is the constant of proportionality and is known as the inductance of the inductor.
- In electric circuits an inductor is represented as,



CIRCUIT ELEMENTS (Cont...)

□ Inductor (cont...):

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).
- Therefore, 1 henry = 1 volt second/ampere.
- Inductance of an inductor depends on its physical dimensions and its construction.
- For a solenoid shaped inductor,

$$L = \frac{N^2 \mu A}{l}$$

where, **N** is the number of turns, **l** is the length, **A** is the cross-sectional area, and **μ** is the permeability of the core.

CIRCUIT ELEMENTS (Cont...)

□ Inductor (cont...):

- The current-voltage relationship of an inductor, is expressed as,

$$di = \frac{1}{L} v dt$$

- The instantaneous power delivered to the inductor is

$$p = vi = L \frac{di}{dt} i$$

- The energy stored in a capacitor is therefore,

$$w = \int_{-\infty}^t p(\tau) d\tau = L \int_{-\infty}^t i \frac{di}{d\tau} d\tau = L \int_{i(-\infty)}^{i(t)} i di = \frac{1}{2} Li^2 \Big|_{i(-\infty)}^{i(t)}$$

CIRCUIT ELEMENTS (Cont...)

□ Inductor (cont...):

- Since the current at $t = -\infty$ is zero, $i(-\infty) = 0$,

$$w = \frac{1}{2} Li^2$$

- The above equation represents the energy stored in the magnetic field of an inductor.
- The voltage across an inductor is zero when the current through the inductor is constant.
- An inductor, therefore, acts like a short circuit to DC.
- An important property of the inductor is its opposition to the change in current flowing through it.
- The current through an inductor cannot change abruptly.
- An ideal inductor, like an ideal capacitor cannot dissipate energy and the stored energy can be retrieved at a later time.

CIRCUIT ELEMENTS (Cont...)

□ Example:

The current through a 0.1 H inductor is $i(t) = 10te^{-5t}$ A? Find the voltage across it and the energy stored in the inductor?

□ Solution:

Since,

$$v = L \frac{di}{dt} \text{ and } L = 0.1 \text{ H}$$

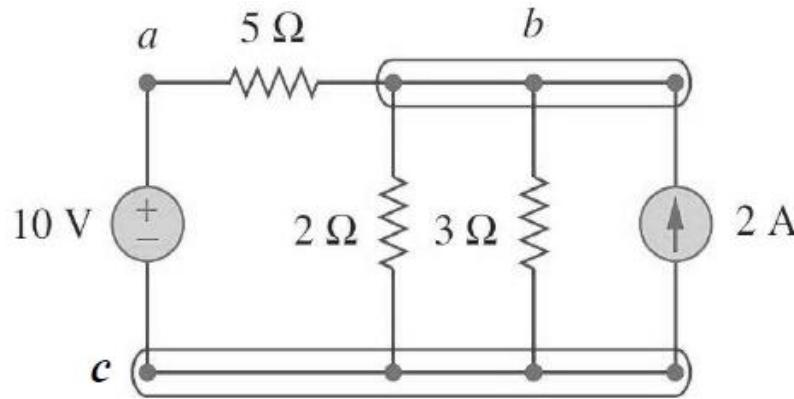
$$\begin{aligned} v &= 0.1 * \frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} \\ &= e^{-5t}(1 - 5t) \text{ V} \end{aligned}$$

The energy stored in the inductor is given by,

$$w = \frac{1}{2}Li^2 = \frac{1}{2} * 0.1 * 100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

Branch

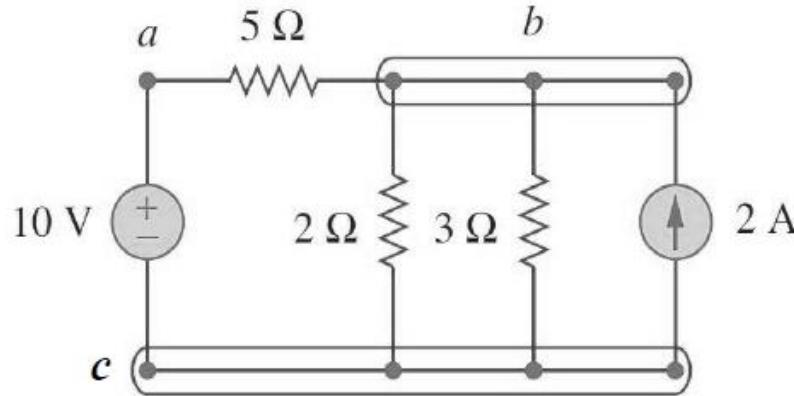
- A branch represents a single element such as a voltage source or a resistor.



- A branch represents any two-terminal element.
- The circuit in above figure has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

Node

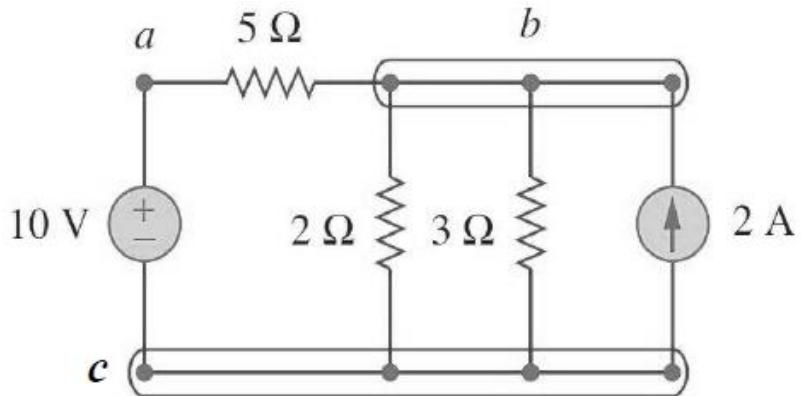
- A node is the point of connection between two or more branches.



- The circuit in above figure has three nodes **a**, **b**, and **c**.
- A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two, the two nodes constitute a single node.
- The three points that form node **b** are connected by perfectly conducting wires and therefore constitute a single point.
- The same is true of the four points forming node **c**.

Loop

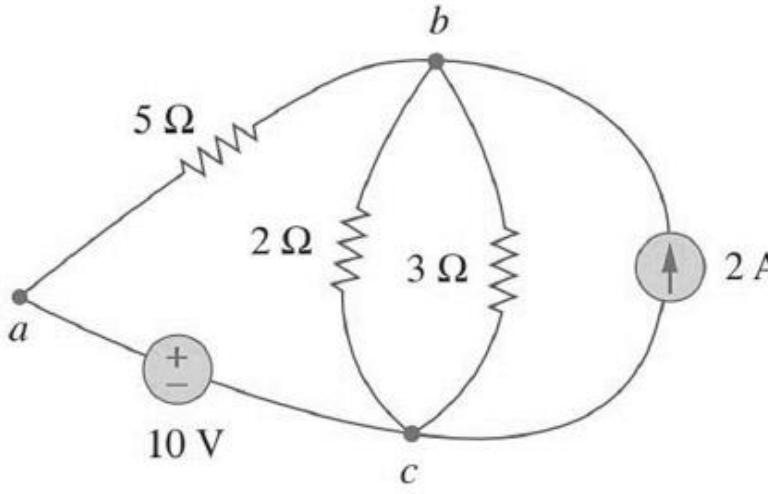
- A loop is any closed path in a circuit.



- It is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- It is said to be independent if it contains at least one branch which is not a part of any other independent loop.

Loop (cont...)

- loop is said to be independent if it contains at least one branch which is not a part of any other independent loop.



- In above figure, **abca** with the 2Ω resistor is independent.
- A second loop with the 3Ω resistor and the current source is independent.
- The third loop could be the one with the 2Ω resistor in parallel with the 3Ω resistor. This does form an independent set of loops.
- **Mesh (m):** A mesh is a closed path of a circuit or network which should not have further closed path in it.
- **Loop:** A loop is a closed path of a circuit or network which may have further closed path in it.

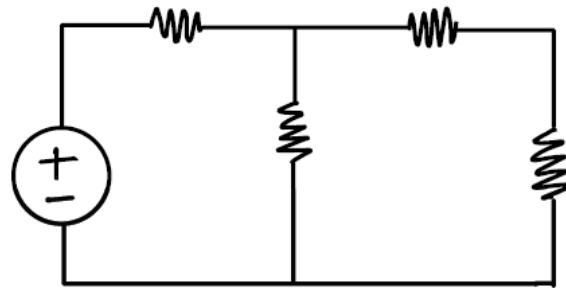
Nodes, Branches, and Loops/Mesh

- A network with **b** branches, **n** nodes, and **m** mesh will satisfy the fundamental theorem of network topology:

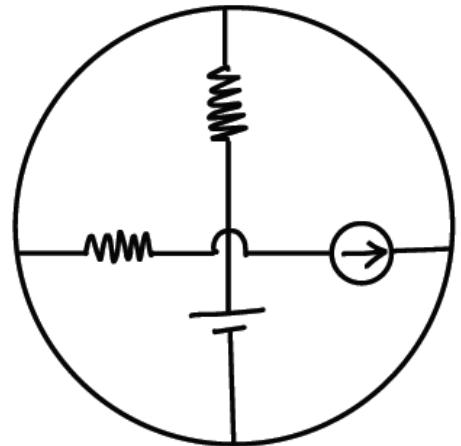
$$m = b-n+1$$

- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently, have the same voltage across them.

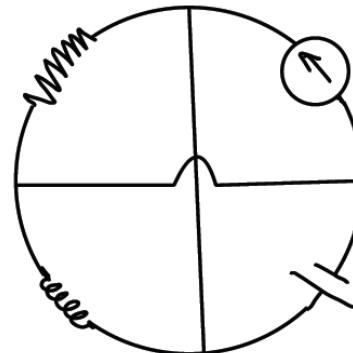
Nodes, Branches, and Loops/Mesh



$n=4, b=5, m=2$



$n=3, b=4, m=2$



$n=2, b=4, m=2$

Kirchhoff's Law

- Ohm's law by itself is not sufficient to analyze circuits.
- However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.
- These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).
- Kirchhoff's first law, i.e. KCL, is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

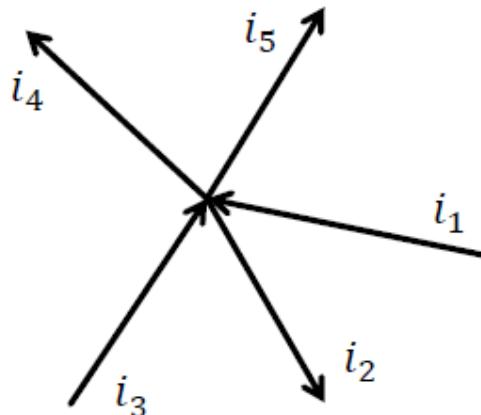
Kirchhoff's Law (cont...)

- Mathematically KCL implies that,

$$\sum_{n=1}^N i_n = 0$$

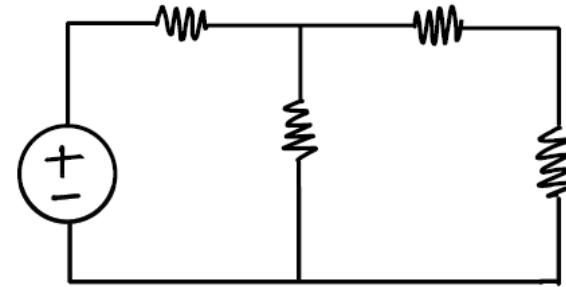
- Here, N is the number of branches connected to the node and i_n is the n^{th} current entering or leaving the node.
- By this law, the current entering a terminal are regarded positive and the currents leaving the node are taken to be negative.
- Following the above convention and applying KCL at the node in the figure,

$$i_1 + (-i_2) + i_3 + (-i_4) + (-i_5) = 0 \\ \Rightarrow i_1 + i_3 = i_2 + i_4 + i_5$$



- The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Kirchhoff's Law (cont...)



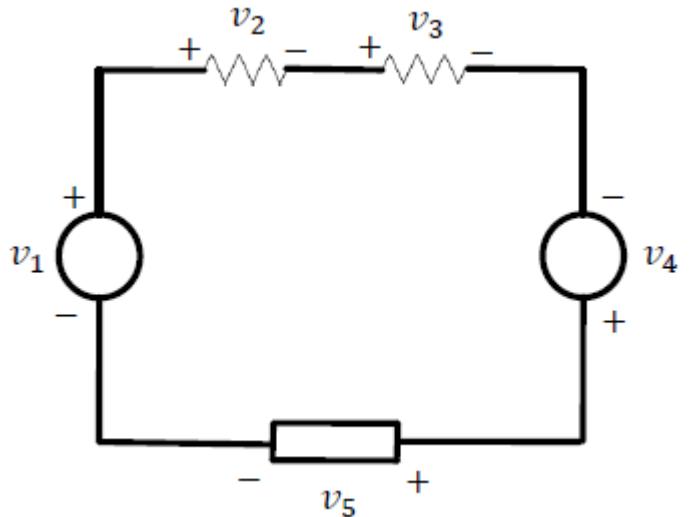
Kirchhoff's Law (cont...)

- Kirchhoff's second law is based on the principle of conservation of energy.
- This law is known as Kirchhoff's voltage law and states that the algebraic sum of all the voltages around a closed loop is zero.

$$\sum_{m=1}^M v_m = 0$$

- Here, M is the number of voltages in the loop and v_m is the m^{th} voltage.
- Following the above convention and applying KVL in the loop in the figure,

$$\begin{aligned}-v_1 + v_2 + v_3 - v_4 + v_5 &= 0 \\ \Rightarrow v_1 + v_4 &= v_2 + v_3 + v_5\end{aligned}$$



- This implies that the sum of voltage drops is equal to the sum of voltage rises.

Kirchhoff's Law (cont...)

