

## Lecture-36

On

# INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Synchronous Machine.

## 3-Phase Synchronous Machine

- Each phase carries an alternating current that varies sinusoidally with time.
- Current in each phase are displaced from each other by 120 degrees due to the geometry of the stator winding.

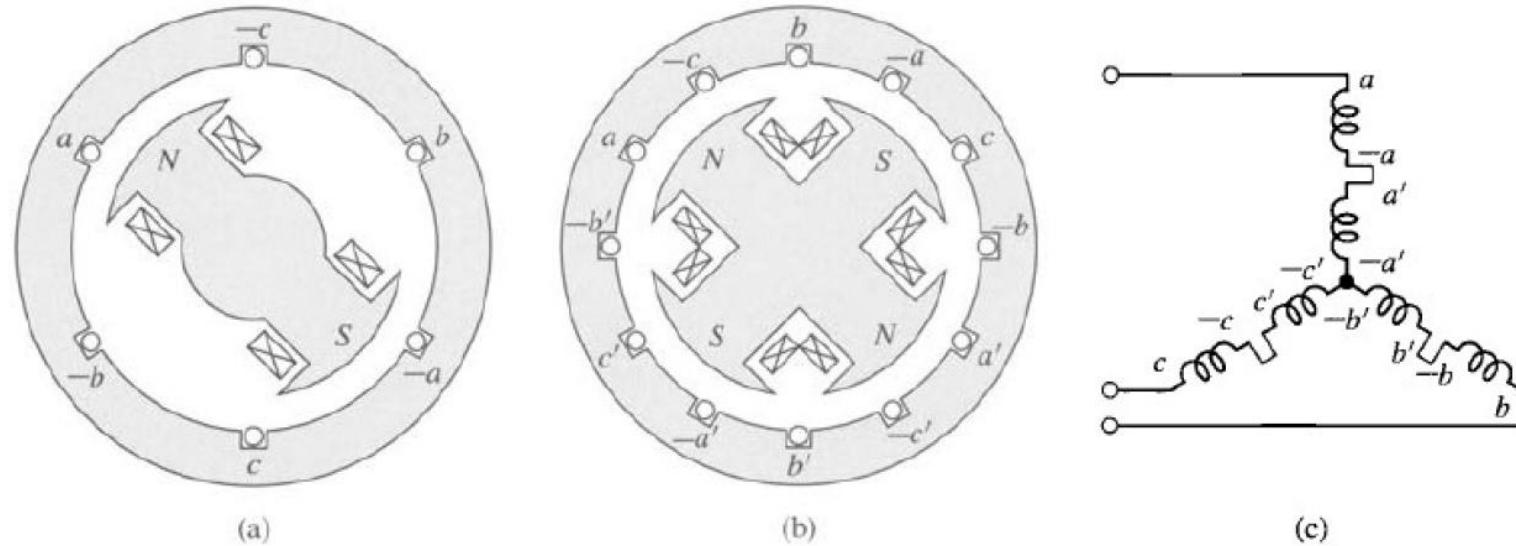


Fig.6: Three-phase generators : (a) 2-pole, (b) 4-pole,(c) Y-connected stator winding

## 3-Phase Synchronous Machine (cont...)

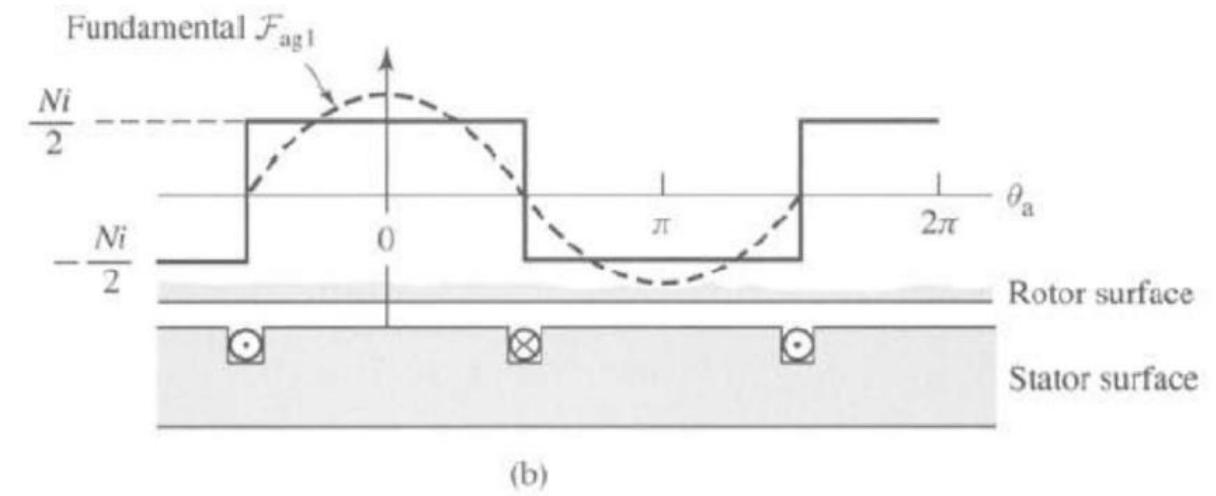
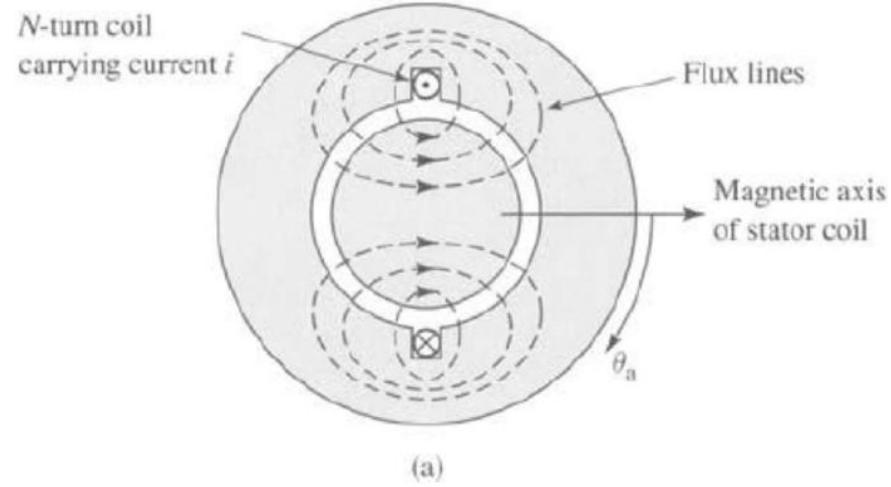


Fig.7: (a) Flux produced by a single coil winding in the stator (b) air-gap mmf produced by this stator winding

# MMF by Distributed Stator Winding

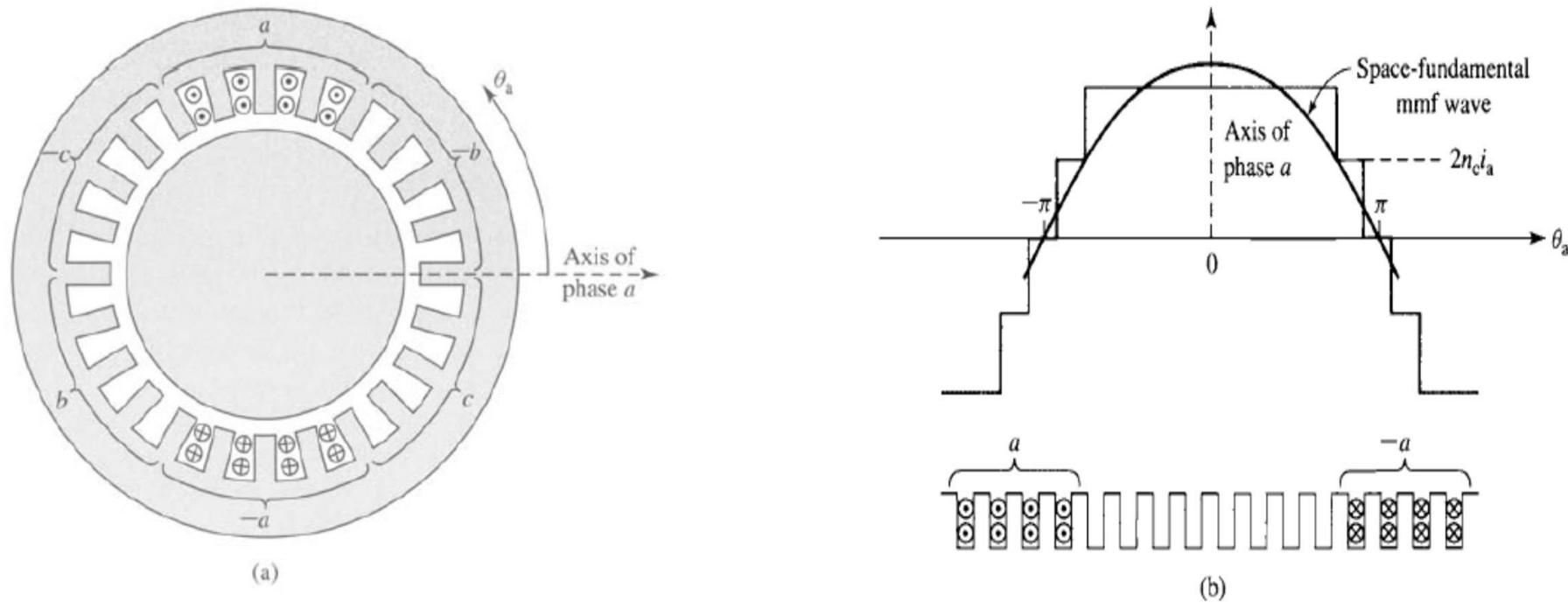


Fig.8: (a) Distributed 3-phase stator winding (b) mmf produced by the phase 'a' of the distributed winding

## Combined MMF

- Under balanced 3-phase condition, the instantaneous currents are:

$$i_a = I_m \cos(\omega_e t)$$

$$i_b = I_m \cos(\omega_e t - 120^\circ)$$

$$i_c = I_m \cos(\omega_e t + 120^\circ)$$

Since mmf is proportional to the current (recall  $Hl=Ni$ ),

$$F_a = F_{max} \cos(\omega_e t)$$

$$F_b = F_{max} \cos(\omega_e t - 120^\circ)$$

$$F_c = F_{max} \cos(\omega_e t + 120^\circ)$$

At time  $t$ , all three phases contribute to the air-gap mmf at a point P (whose spatial angle is  $\theta$ ). The resultant MMF is then given by,

$$F = F_a \cos(\theta) + F_b \cos(\theta - 120^\circ) + F_c \cos(\theta + 120^\circ) = \frac{3}{2} F_{max} \cos(\omega_e t - \theta)$$

### Points to note:

- The resultant mmf wave rotates at constant angular speed  $\omega$ .
- Its value depends on the spatial position  $\theta$  as well as time.

## Combined MMF (cont...)

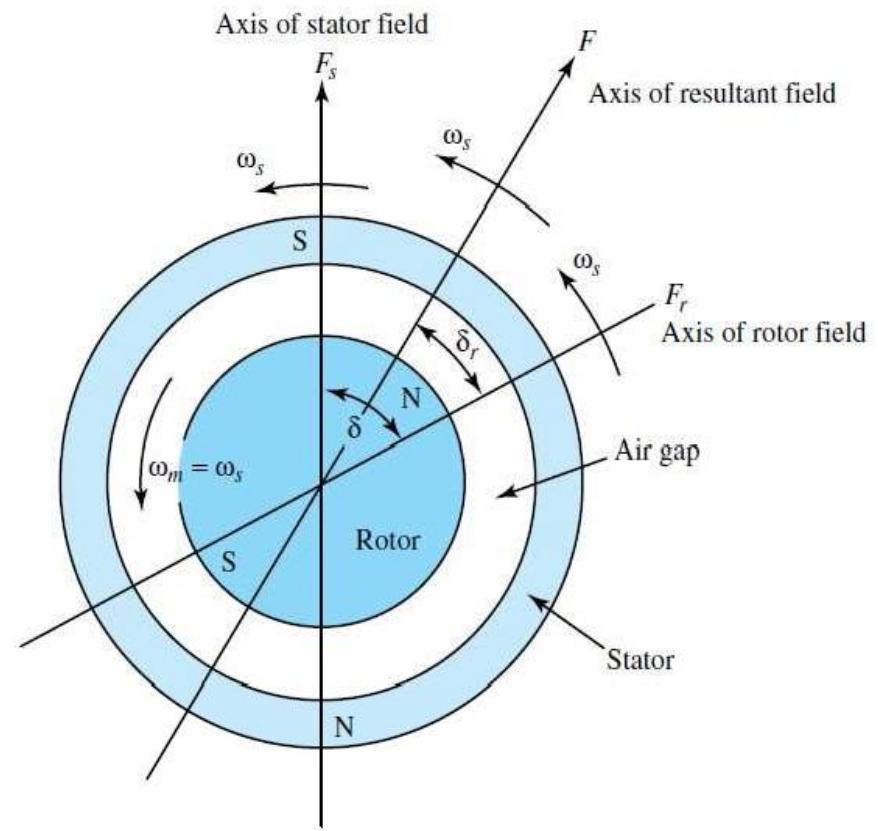
$$\text{Synchronous Speed } N = \frac{120f_s}{P} \text{ r/min}$$

$$\omega_m = \omega_s = \frac{2\pi f}{P/2}$$

$$T_e = K\Phi F_r \sin \delta_r$$

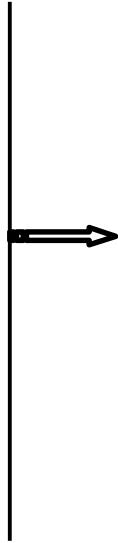
P is the number of poles

$\Phi$ : the resultant flux per pole produced by the combined effect of the stator and rotor mmfs



## The Synchronous Speed

- $\theta_{ea} = \frac{P}{2} \theta_a$
- Where,
- $\theta_{ea}$ =electrical angle
- $\theta_a$ =mechanical angle
- P= number of poles



$$\text{Now, } \omega_e = \frac{d\theta_{ea}}{dt}$$

$$\text{and } \omega_m = \frac{d\theta_a}{dt}$$

(Rotational speed in mechanical radians/second)

Rotational speed of the resultant mmf produced by the stator is therefore,

$$\omega_e = \frac{P}{2} \omega_m$$

(synchronous speed in electrical radians/second)

The electrical frequency  $f_e$  of the voltage generated in a synchronous machine is therefore,  
 $f_e = \frac{P}{2} s$  (synchronous frequency in Hz)

Where, S is the mechanical speed of the rotor in rotations per second.

## Synchronous Speed: Example

**Example 1:** List the four highest possible synchronous speeds (in mechanical radians per second) for a 60 Hz synchronous generator.

**Solution:**

We have,

$$f_e = \frac{P}{2} \times s = 60 \quad \text{Or,} \quad s = \frac{2f_e}{P} \text{ r.p.s} = \frac{120f_e}{P} \text{ r.p.m.}$$

The highest speed is determined by the number of poles. The smallest number of poles is 2.

Therefore, the highest speed is given by,  $s = \frac{120 \times 60}{2} = 3600$  r.p.m. The next three highest speeds are 1800-, 1200- and 900-rpm for  $p = 4, 6$  and 8 respectively.

## Internal Generated Voltage

- Voltage induced is dependent upon flux and speed of rotation, hence from what we have learnt so far, the induced voltage can be found as follows:

$$E_A = \sqrt{2} N \phi \pi f$$

Above can also be written as,

$$E_A = k \phi \omega$$

where,

$$k = \frac{N}{\sqrt{2}}, \text{ if } \omega \text{ is in electrical radians}$$

$$k = \frac{NP}{\sqrt{2}}, \text{ if } \omega \text{ is in mechanical radians}$$

## Armature Reaction

- The voltage  $E_A$  is the internal generated voltage produced in one phase of a synchronous generator. If the machine is not connected to a load (no armature current flowing), the terminal voltage will be equivalent to the voltage induced at the stator coils. This is due to the fact that there are no current flow in the stator coils hence no losses. When there is a load connected to the generator, there will be differences between  $E_A$  and terminal voltage  $V$ . These differences are due to :
  - a) Distortion of the air gap magnetic field by the current flowing in the stator called armature reaction.
  - b) Self inductance of the armature coil.
  - c) Resistance of the armature coils.

## Armature Reaction (Cont...)

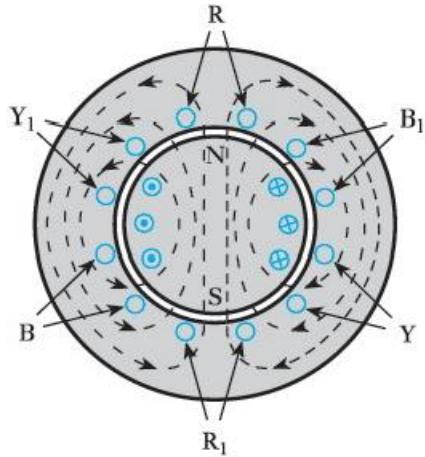
- When the rotor is spun, a voltage  $E_A$  is induced in the stator windings. If a load is attached to the terminals of the generator, a current flows.
- But a 3-phase stator current flow will produce a magnetic field of its own. This stator magnetic field will distort the original rotor magnetic field, changing the resulting phase voltage.
- This effect is called armature reaction because the armature (stator) current affects the magnetic field, which produced it in the first place.
- The voltage produced in the stator due to this field is proportional to the armature current, and can be given by,

$$E_A = -jXI_A$$

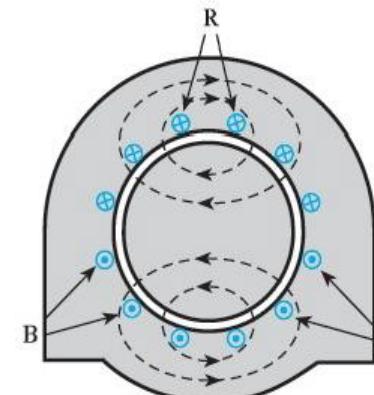
Where  $X$  is the reactance that accounts for the armature reaction,  $I_A$  is the armature current.

## Armature Reaction (Cont...)

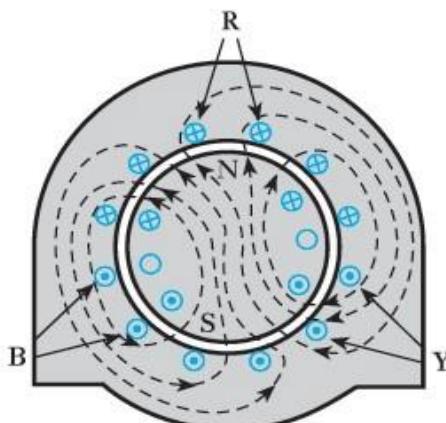
- ‘Armature reaction’ is the influence of the stator m.m.f. upon the value and distribution of the magnetic flux in the air gaps between poles and the stator core’.



If the machine is on open circuit there is no stator current, and the magnetic flux due to the rotor current is distributed symmetrically.

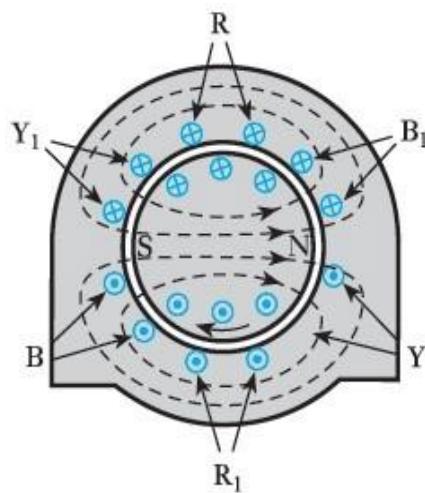
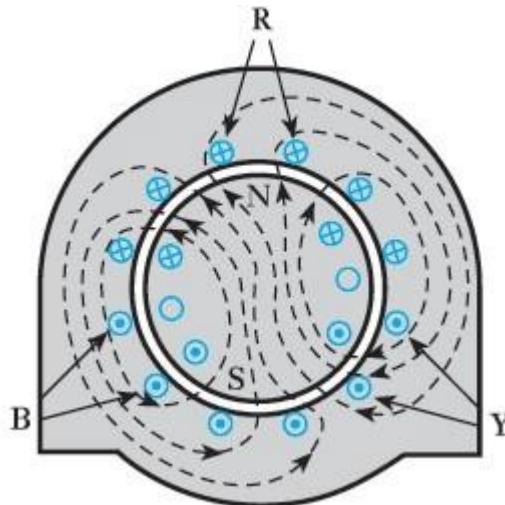


Flux due to the stator currents alone at the instant when the current in phase R is at its maximum positive value.

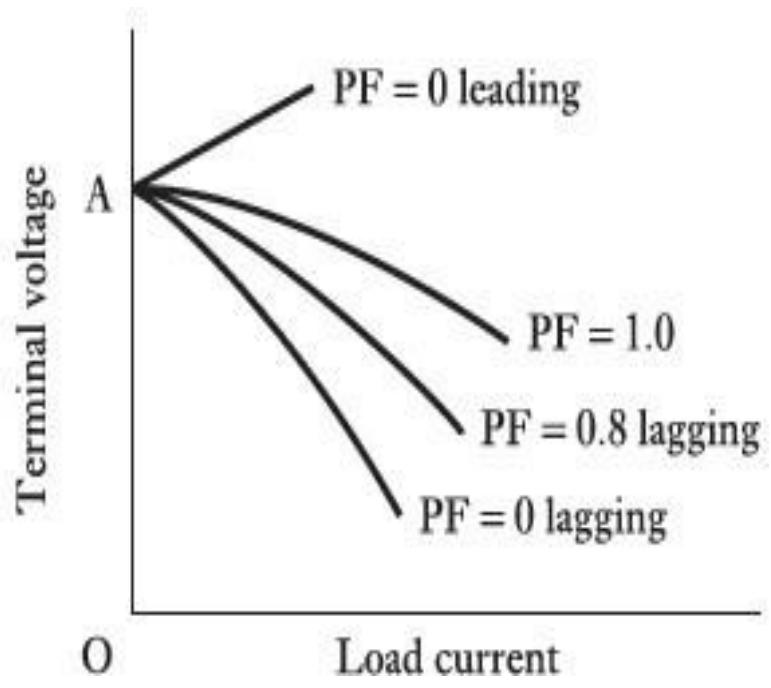


The direction of most of the lines of flux in the air gaps has been skewed and thereby lengthened.

## Armature Reaction (Cont...)

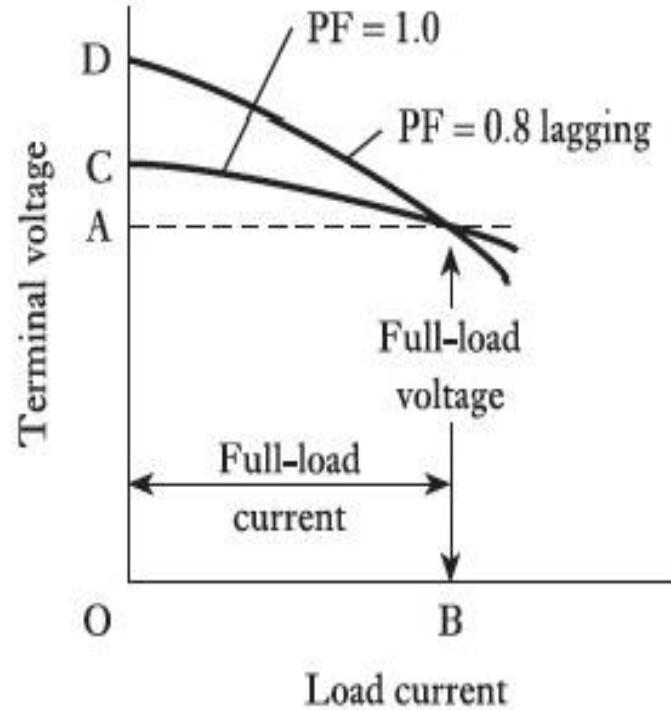


- Armature reaction exerts a backward pull on the rotor.
- To overcome the tangential component of this pull, the engine driving the generator has to exert a larger torque than that required on no load.
- Since magnetic flux due to the stator currents rotates synchronously with the rotor, the flux distortion remains the same for all positions of the rotor.
- When the current lags the generated e.m.f. by a quarter of a cycle, by the time the current in phase R reaches its maximum value, the poles will have moved forward through half a pole pitch. The stator m.m.f. would have a flux in direct opposition to the flux produced by the rotor m.m.f.
- When the current leads the e.m.f. by a quarter of a cycle, the flux due to the stator m.m.f strengthens the rotor flux.



- The field current is maintained constant at a value giving an e.m.f. OA on open circuit condition.
- When the power factor of the load is unity, the fall in voltage with increase of load is comparatively small.
- With an inductive load, the demagnetizing effect of armature reaction causes the terminal voltage to fall much more rapidly.
- With a capacitive load, the magnetizing effect of armature reaction causes the terminal voltage to increase with increase of load.

## VOLTAGE REGULATION



- If the field current is adjusted to give the terminal voltage OA when the generator is supplying current OB at unity power factor, then when the load is removed but with the field current and speed kept unaltered, the terminal voltage rises to OC.

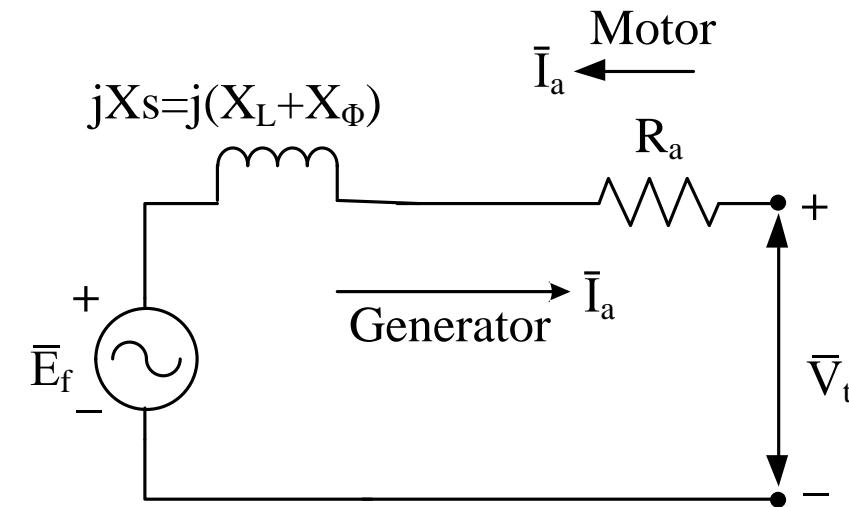
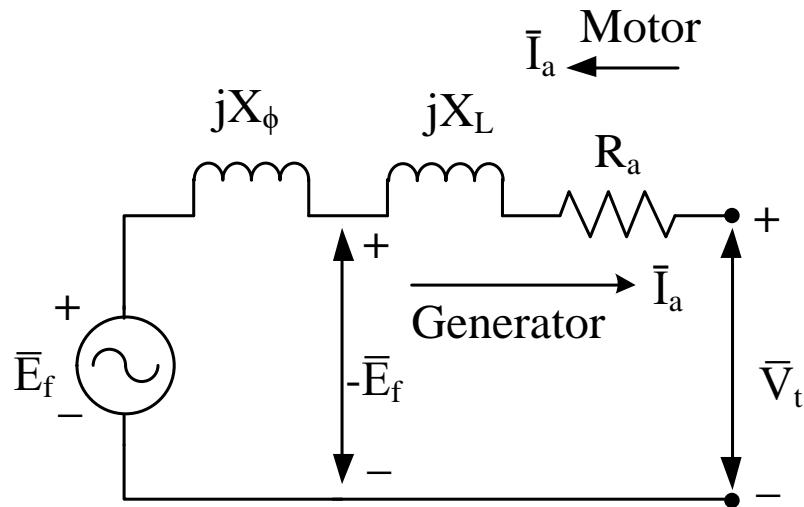
Per unit voltage regulation =  $\frac{\text{change of terminal voltage when full load is removed}}{\text{full-load terminal voltage}}$

## Equivalent Circuit

- Also, we have to include the self-inductance and resistance of the armature coils. If the stator self-inductance is called  $L_l$  (reactance is  $X_l$ ) while the stator resistance is called  $R_a$ , then we can write (for generator):

$$V = E_f - jX_\phi I_a - jX_l I_a - R_a I_a = E_f - jX_s I_a - R_a I_a$$

Where,  $X_s = X_\phi + X_l$  (also known as the synchronous reactance)

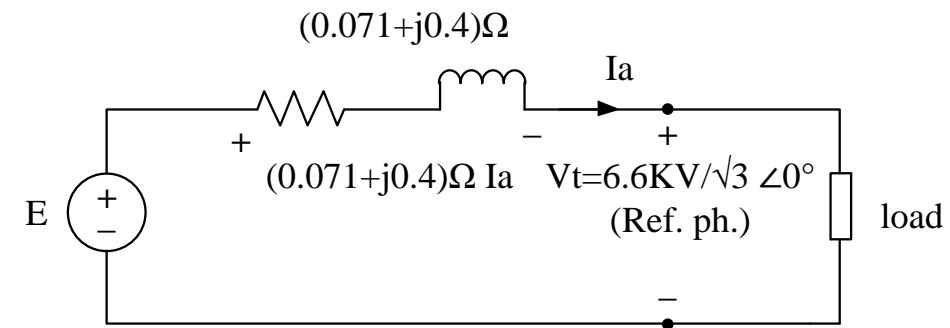


## Synchronous Generator: Example

**Example 2:** Calculate the % voltage regulation for a three-phase wye-connected 2.5 MVA, 6.6 KV synchronous generator operating at rated KVA load, 0.8 lagging power factor. The synchronous impedance is  $Z_s = R_a + jX_s = (0.071 + j10.4) \Omega$  (N.B.:  $6.6 \text{ KV}/\sqrt{3} \approx 3811 \text{ V}$ ).

**Solution:** Equivalent circuit:

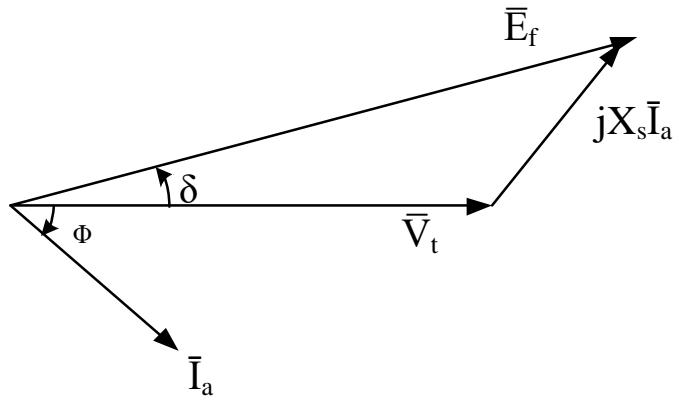
$$\text{Now, } I_a = |I_a| \angle \theta$$



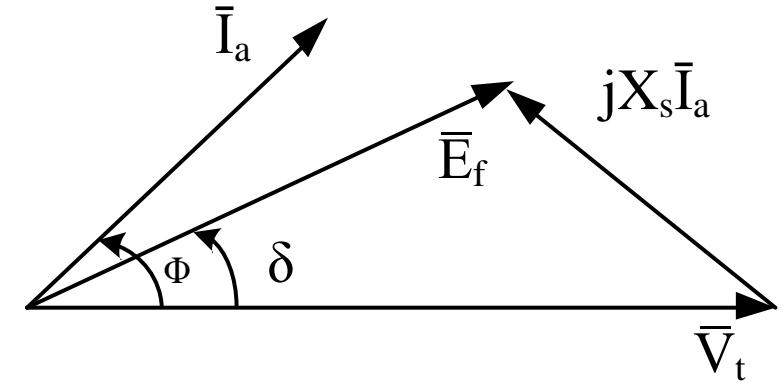
$$\text{Where } |I_a| = \frac{2.5 \text{ MVA}}{3} \times \frac{1}{6.6 \text{ KV}/\sqrt{3}} = 218.7 \text{ A}, \text{ And } \theta = \cos^{-1} 0.8 \approx -36.87^\circ$$

$$\text{Therefore, } E = V_t + Z_s I_a = 3811 + 10.4 \angle 89.67^\circ \times 218.7 \angle -36.87^\circ \approx 5495 \angle 19.2^\circ \text{ V}$$

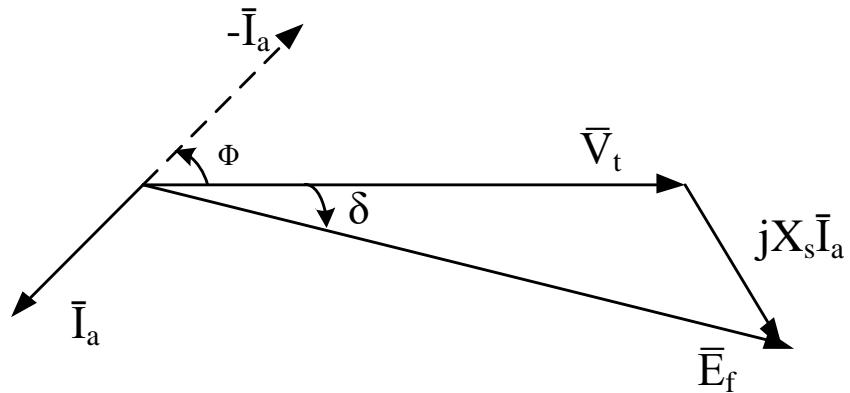
$$\text{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100 = \frac{|E| - |V_t|}{|V_t|} \times 100 = \frac{5495 - 3811}{3811} \times 100 = 44.2\%$$



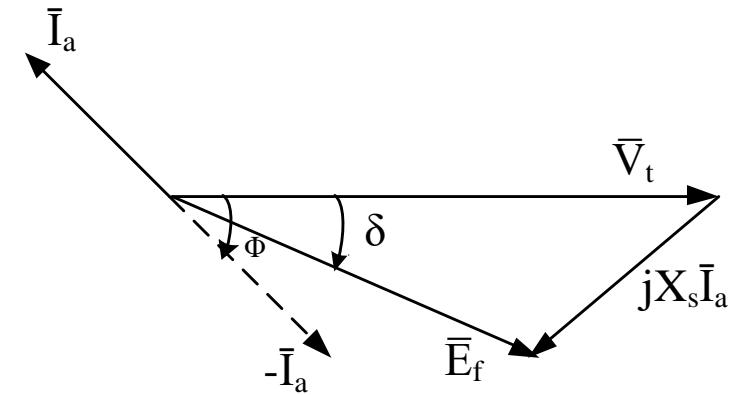
Overexcited generator (power factor lagging),  
 $P > 0, Q > 0, \delta > 0.$



Under excited generator (power factor leading),  
 $P > 0, Q < 0, \delta > 0.$



Overexcited motor (power factor leading),  
 $P < 0, Q > 0, \delta < 0.$



Under excited motor (power factor lagging),  
 $P < 0, Q < 0, \delta < 0$

## POWER ANGLE

- It is convenient to adopt a convention such that the real power  $P$  and the reactive power  $Q$  delivered by an overexcited generator is positive.
- The generator action corresponds to positive values of  $\delta$ , whereas the motor action corresponds to negative values of  $\delta$ .
- It follows that  **$P > 0$  for generator operation**, whereas  **$P < 0$  for motor operation**.
- Positive  $Q$  means delivering inductive VARs for a generator action or receiving inductive VARs for a motor action; negative  $Q$  means delivering capacitive VARs for a generator action or receiving capacitive VARs for a motor action.
- Incidentally, the power factor is lagging when  $P$  and  $Q$  have the same sign and leading when  $P$  and  $Q$  have opposite signs.

$$\bar{S} = \mathbf{P} + j\mathbf{Q} = \bar{V}_t \bar{I}_a^*$$

$$\bar{E}_f = E_f (\cos \delta + j \sin \delta)$$

$$\bar{V}_t = V_t + j0$$

$$\bar{I}_a = \frac{\bar{E}_f - \bar{V}_t}{jX_s} = \frac{E_f \cos \delta - V_t + jE_f \sin \delta}{jX_s}$$

$X_s$  is the synchronous reactance per phase

$$\bar{I}_a^* = \frac{E_f \cos \delta - V_t - jE_f \sin \delta}{-jX_s} = \frac{E_f \sin \delta}{X_s} + j \frac{E_f \cos \delta - V_t}{X_s}$$

$$P = \frac{V_t E_f \sin \delta}{X_s}$$
$$Q = \frac{V_t E_f \cos \delta - V_t^2}{X_s}$$

for a cylindrical-rotor synchronous generator with negligible armature resistance.

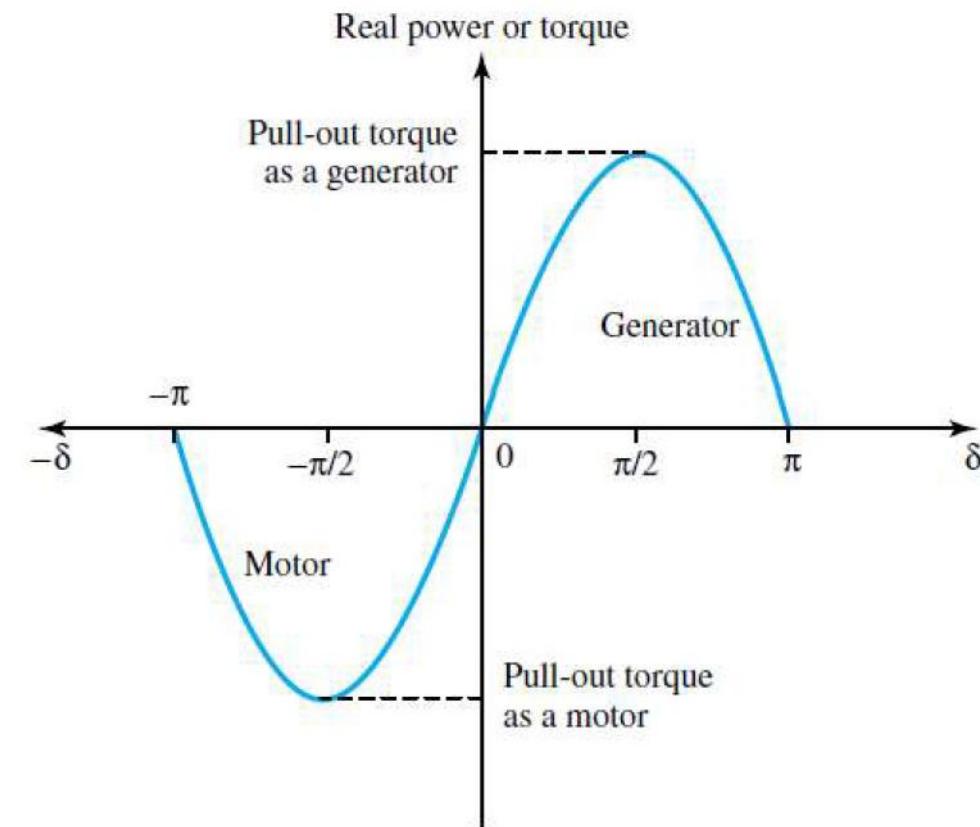
- To obtain the total power for a three-phase generator, multiply by 3 when the voltages are line to neutral. If the line-to-line magnitudes are used for the voltages, however, these equations give the total three-phase power.

## POWER ANGLE/TORQUE-ANGLE CHARACTERISTICS

- The maximum real power output per phase of the generator for a given terminal voltage and a given excitation voltage.

$$P_{max} = \frac{V_t E_f}{X_s}$$

- Any further increase in the prime-mover input to the generator causes the real power output to decrease. The excess power goes into accelerating the generator, thereby increasing its speed and causing it to pull out of synchronism.
- The steady-state stability limit is reached when  $\delta = \frac{\pi}{2}$
- For normal steady operating conditions, the power angle or torque angle is well below  $90^\circ$ .



- The maximum torque or **pull-out torque** per phase that a round-rotor synchronous motor can develop for a gradually applied load is-

$$T_{max} = \frac{P_{max}}{\omega_m} = \frac{P_{max}}{2\pi n_s/60} \quad n_s \text{ is the synchronous speed in r/min}$$

- In normal steady-state operation, **the electromechanical torque balances the mechanical torque** applied to the shaft. In a generator, the prime-mover torque acts in the direction of rotation of the rotor, pushing the rotor mmf wave ahead of the resultant air-gap flux. The electromechanical torque then **opposes** rotation.
- The opposite situation exists in a synchronous motor, where the electromechanical torque is in the direction of rotation, in opposition to the retarding torque of the mechanical load on the shaft.

**Note:**

- To address hunting of machine at a new operating position, damper windings are provided.

