Transmission Lines - II

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$$Z_0 = \frac{V(z)}{I(z)} = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (\Omega)$$

☝ Characteristic Impedance

- The ratio of the voltage and the current at any z for an infinitely long line is independent of z and it is called the Characteristic Impedance.

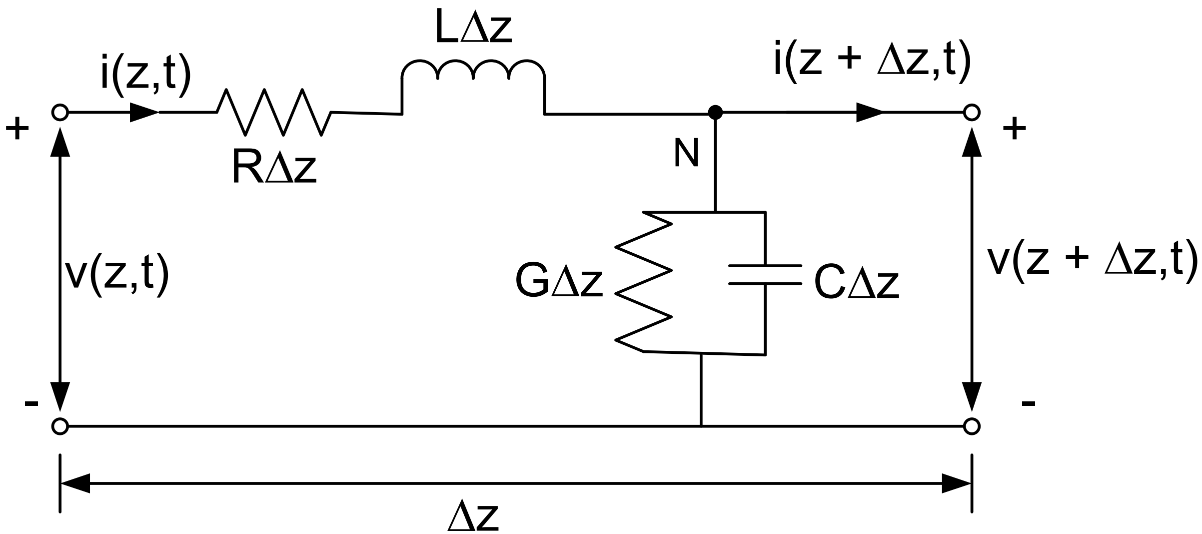
$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

γ: Propagation Constant

α: Attenuation Constant (Np/m)

β: Phase Constant (rad/m)

γ and Z_0 are the characteristics of a transmission line.

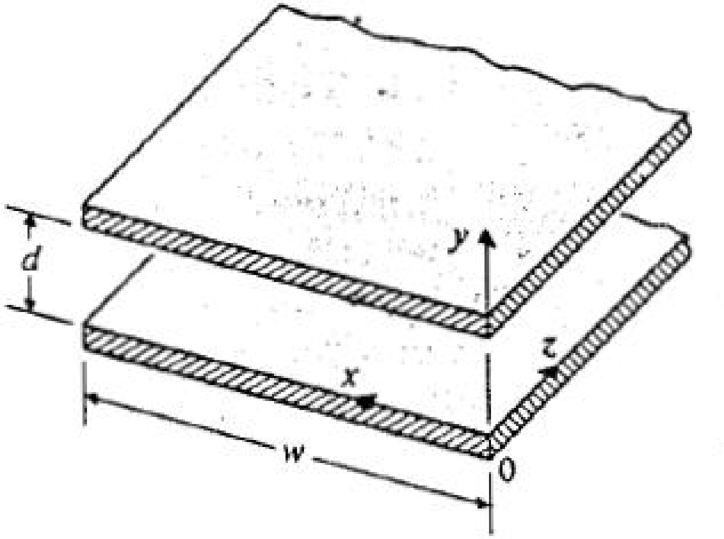


$$\overline{E} = \overline{a_y} E_y = \overline{a_y} E_0 e^{-\gamma z}$$

$$\overline{H} = \overline{a_x} H_x = -\overline{a_x} \frac{E_0}{\eta} e^{-\gamma z}$$

$$\gamma = j\beta = j\omega\sqrt{\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$



Capacitance per unit length of the parallel plate transmission line

$$C \equiv \varepsilon \frac{w}{d} \qquad (\mathrm{F/m})$$

Inductance per unit length of the parallel plate transmission line

$$L \equiv \mu \frac{d}{w} \qquad (\mathrm{H/m})$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad (\Omega)$$

Lossless Line ($R = 0$, $G = 0$)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \equiv \sqrt{\frac{L}{C}} \ (\Omega)$$

$$R_0 = \sqrt{\frac{L}{C}}$$

☞ Characteristic Impedance

- ❑ The characteristic impedance becomes purely real and constant.

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

☞ Phase velocity

- ❑ Phase velocity is independent of the frequency, which basically means that all the frequency components travel with same velocity.

Low Loss Line ($R \ll \omega L, G \ll \omega C$)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\equiv \sqrt{j\omega L\left(1 + \frac{R}{j\omega L}\right) \times j\omega C\left(1 + \frac{G}{j\omega C}\right)}$$

$$\equiv j\omega\sqrt{LC}\left(1 + \frac{R}{j\omega L}\right)^{1/2} \times \left(1 + \frac{G}{j\omega C}\right)^{1/2}$$

$$\cong j\omega\sqrt{LC}\left[1 + \frac{R}{2j\omega L}\right] \times \left[1 + \frac{G}{2j\omega C}\right]$$

$$\cong j\omega\sqrt{LC}\left[1 + \frac{R}{2j\omega L} + \frac{G}{2j\omega C} + \cdots\right]$$

$$\cong j\omega\sqrt{LC} + j\omega\sqrt{LC}\left[\frac{R}{2j\omega L} + \frac{G}{2j\omega C}\right]$$

$$\cong j\omega\sqrt{LC} + \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$

$$\alpha \cong \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$

$$\beta \cong \omega\sqrt{LC}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

☝ Phase velocity

Low Loss Line ($R \ll \omega L, G \ll \omega C$)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

☞ Characteristic Impedance

$$\equiv \sqrt{\frac{L}{C}} \frac{\left(1 + \frac{R}{j\omega L}\right)^{1/2}}{\left(1 + \frac{G}{j\omega C}\right)^{1/2}} \cong \sqrt{\frac{L}{C}} \left[1 + \frac{R}{2j\omega L}\right] \times \left[1 - \frac{G}{2j\omega C}\right]$$

$$\cong \sqrt{\frac{L}{C}} \left[1 + \frac{R}{2j\omega L} - \frac{G}{2j\omega C} \cdots\right] \cong \sqrt{\frac{L}{C}} + \frac{1}{2j\omega} \sqrt{\frac{L}{C}} \left[\frac{R}{L} - \frac{G}{C}\right]$$

$$R_0 \cong \sqrt{\frac{L}{C}} \qquad X_0 \cong -\frac{1}{2\omega} \sqrt{\frac{L}{C}} \left[\frac{R}{L} - \frac{G}{C}\right]$$

- ❑ The characteristic impedance becomes purely real and constant.

Distortion-less Line ($R / L = G / C$)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)\left(\frac{RC}{L} + j\omega C\right)}$$

$$\equiv \sqrt{(R + j\omega L)\frac{C}{L}(R + j\omega L)}$$

$$\equiv (R + j\omega L)\sqrt{\frac{C}{L}}$$

$$\alpha = R\sqrt{\frac{C}{L}}$$

$$\beta = \omega\sqrt{LC}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

- ❑ All the frequency components travel with same velocity and get attenuated with the same factor, therefore shape of wave remains the same for the distortion-less transmission line.

- ✓ The distortion would result for a lossy line when different frequency components of a signal attenuate by different amount even when they travel at the same speed.

Distortion-less Line ($R / L = G / C$)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{RC/L + j\omega C}}$$

$$= \sqrt{\frac{(R + j\omega L)L}{RC + j\omega LC}} \equiv \sqrt{\frac{L}{C}}$$

$$\alpha = R\sqrt{\frac{C}{L}}$$

$$R_0 = \sqrt{\frac{L}{C}}; \qquad X_0 = 0$$

$$\beta = \omega\sqrt{LC} \qquad u_p = \frac{1}{\sqrt{LC}}$$

✓ The characteristic impedance becomes purely real.

- ❑ If different frequency components travel with different velocity, it would lead to dispersion.
- ❑ A general lossy line may be dispersive as the phase constant may not be a linear function of the radial frequency.

Attenuation Constant from Power Relations

$$\alpha = \Re e[\gamma] = \Re\mathrm{e}\left[\sqrt{(R + j\omega L)(G + j\omega C)}\right]$$

- For an infinitely long transmission line, there would not be any reflection, and hence only forward traveling waves would be present.

No reflections

$$V(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-(\alpha + j\beta)z}$$

$$I(z) = I_0^+ e^{-\gamma z} = I_0^+ e^{-(\alpha + j\beta)z} = \frac{V_0^+}{Z_0} e^{-(\alpha + j\beta)z}$$

$$P(z) = \frac{1}{2}\mathrm{Re}[V(z) I^*(z)]$$

$$P(z) = \frac{1}{2}\frac{V_0^2}{|Z_0|^2} R_0 e^{-2\alpha z}$$

The power flow along a lossy transmission line, in the absence of the reflection, may be written using the following expression:

$$P(z) = P_0 e^{-2\alpha z}$$

$$P_0 = \frac{1}{2}\frac{V^2{}_0}{R_0}$$

where P_0 is the power at the z=0 plane and α is the attenuation constant we wish to determine.

Let us define the power loss per unit length along the line as:

$$\mathcal{P}_l(z) \equiv -\frac{\partial P}{\partial z} = 2\alpha P_0 e^{-2\alpha z}$$

$$P(z) = P_0 e^{-2\alpha z}$$

$$= 2\alpha\, P(z)$$

✓ The negative sign on the derivative was introduced so that P_l would be a positive quantity.

From the above expression, the attenuation constant may be derived as:

$$\alpha = \frac{\mathcal{P}_l(z)}{2P(z)} = \frac{\mathcal{P}_l(z=0)}{2P_0} \quad \text{(Np/m)}$$