1. Let the random vector (X, Y) have the following joint PMF

$$P(X = 0, Y = 0) = P(X = 1, Y = 1) = \frac{1}{3}, P(X = 0, Y = 1) = P(X = 1, Y = 0) = \frac{1}{6}.$$

   Find the PMF of X and the PMF of Y. Find $E(X)$, $E(Y)$. Are they independent?

2. Let the random vector (X, Y) have the following joint PMF

$$P(X = i, Y = j) = c \times \frac{e^{-\lambda j}(\lambda j)^i}{i!}; \quad i = 0, 1, 2, \ldots \quad \text{and} \quad j = 1, 2, 3.$$

   Find c. Find $P(Y = j)$ and $P(X = i)$.

3. Let the random vector (X, Y) have the following joint PMF

$$P(X = i, Y = j) = \binom{n}{i}\binom{m}{j}\frac{1}{2^{m+n}}; \quad i = 0, 1, \ldots, n, \quad j = 0, 1, \ldots, m.$$

   Find the PMF of X, the PMF of Y, $\mathrm{Corr}(X, Y)$.

4. Let the random vector (X, Y) have the following joint PMF

$$P(X = m, Y = n) = \frac{e^{-(\lambda+\mu)}\lambda^m \mu^n}{m!n!}; \quad m, n = 0, 1, \ldots,$$

   Find $P(X + Y = k)$, for $k = 0, 1, \ldots$.

5. Let the random vector (X, Y) have the following joint PDF

$$f_{X,Y}(x, y) = c(x + y)e^{-(x+y)}; \quad 0 < x, y < \infty,$$

   and zero, otherwise. Find c. Find the PDF of X, the PDF Y

6. In the above question find $E(X)$, $E(Y)$, $V(X)$, $V(Y)$, $E(XY)$.

7. Suppose X is $N(0, 1)$ and $Y = X^2$. Find $\mathrm{Corr}(X, Y)$.

8. Suppose X has $\mathrm{Gamma}(\alpha, \lambda)$ distribution, $\alpha > 0, \lambda > 0$, i.e. the PDF of X is

$$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}; \quad x > 0,$$

   and zero otherwise. Find the $\mathrm{Corr}(X, Y)$, where $Y = \sqrt{X}$.

9. Let the random vector (X, Y) have the following joint PDF

$$f_{X,Y}(x, y) = \lambda^2 e^{-\lambda(x+y)}; \quad 0 < x, y < \infty,$$

   and zero, otherwise. Find $P(X < Y)$.

10. In the above question suppose $Z = X + Y$, find $P(Z < 1)$. Find the CDF of Z and PDF of Z.