Assignment 5
MSO-201: Probability and Statistics

1. Let a card be selected from an ordinary deck of playing cards. The outcome c is one of these 52 cards. Let $X(c) = 4$ if c is an ace, let $X(c) = 3$ if c is a king, let $X(c) = 2$ if c is a queen, let $X(c) = 1$ if c is a jack, and let $X(c) = 0$ otherwise. Suppose that the probability of drawing any particular card is $1/52$. Find the probability mass function of the random variable X. Find the distribution function X.

2. Let $p_X(x) = x/15$, $x = 1, 2, 3, 4, 5$, zero elsewhere, be the PMF of X. Find the distribution function of X. Find, $P(X = 1 \text{ or } 2)$, $P(1/2 < X < 5/2)$, $P(1 \le X \le 2)$.

3. Let us select five cards at random and without replacement from an ordinary deck of playing cards. (a) Find the pmf of X, the number of hearts in the ve cards. (b) Determine $P(X \le 1)$.

4. Let the probability set function of the random variable X be $P_X(D) = \int_D f(x)dx$, where $f(x) = 2x/9$, for $x \in D = \{x : 0 < x < 3\}$. Let us define the events $D_1 = \{x : 0 < x < 1\}$ and $D_2 = \{x : 1/2 < x < 5/2\}$. Compute $P_X(D_1)$, $P_X(D_2)$, and $P_X(D_1 \cup D_2)$, $P(D_1|D_2)$.

5. Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} cx^{\alpha-1}e^{-\lambda x^{\alpha}}; & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x \le 0. \end{cases}$$

   Here $\alpha > 0$, $\lambda > 0$. Find the constant c, so that $f_X(x)$ is a proper PDF. Find the distribution of X. Find $P(a \le X \le b$, for $0 < a < b < \infty$.

6. Suppose the random variable X has the CDF

$$F_X(x) = \begin{cases} 0 & \text{if} \quad x < -1 \\ \frac{x+2}{4} & \text{if} \quad -1 \le x < 1 \\ 1 & \text{if} \quad x \ge 1. \end{cases}$$

   Sketch the graph of $F_X(x)$. Find $P(1/2 < X \le 1/2)$; (b) $P(X = 0)$; (c) $P(X = 1)$; (d)$P(2 < X \le 3)$.

7. Cast a die two independent times and let X equal the absolute value of the dierence of the two resulting values (the numbers on the up sides). Find the PMF of X.

8. Let a bowl contain 10 chips of the same size and shape. One and only one of these chips is red. Continue to draw chips from the bowl, one at a time and at random and without replacement, until the red chip is drawn. (a) Find the PMF of X, the number of trials needed to draw the red chip. (b) Compute $P(X \le 4)$.

9. Let X have the pmf $p(x) = (1/2)^x; x = 1, 2, 3, \ldots$, and zero elsewhere. Find the PMF of (a) $Y = X^3$, (b) $Y = \dfrac{X}{X+1}$.

10. Divide a line segment $[0, 1]$ into two parts by selecting a point at random. Find the probability that the length of the larger segment is at least three times the length of the shorter segment. Assume a uniform distribution.