Lecture-12

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Maximum Power Transfer.
- Reciprocity Theorem.
- Compensation Theorem.

Maximum Power Transfer (Cont…)

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

- Differentiating

$$\frac{dp}{dR_L} = \frac{V_{Th}^2 [(R_{Th} + R_L) - 2R_L]}{(R_{Th} + R_L)^3}$$

$$= \frac{V_{Th}^2 [(R_{Th} + R_L) - 2R_L]}{(R_{Th} + R_L)^3} = 0$$

- Equating the numerator to 0

$$(R_{Th} + R_L) - 2R_L = 0 = R_{Th} - R_L$$

- Therefore,

$$R_{Th} = R_L$$

Maximum Power Transfer (Cont…)

- This shows that the maximum power transfer takes place when the load resistance is equal to Thevenin resistance.
- This can be confirmed by double differentiating the power with respect to R_L and proving that this is less than 0.

$$\frac{dp}{dR_L} = \frac{V_{Th}^2[(R_{Th} + R_L) - 2R_L]}{(R_{Th} + R_L)^3} = \frac{V_{Th}^2[R_{Th} - R_L]}{(R_{Th} + R_L)^3}$$

$$\frac{d^2p}{dR_L^2} = \frac{V_{Th}^2[-3(R_{Th} - R_L) - 1]}{(R_{Th} + R_L)^4}$$

$$\frac{d^2p}{dR_L^2} < 0$$

Maximum Power Transfer (Cont…)

- Therefore, the maximum power transferred is given by substituting $R_L = R_{Th}$, to obtain,

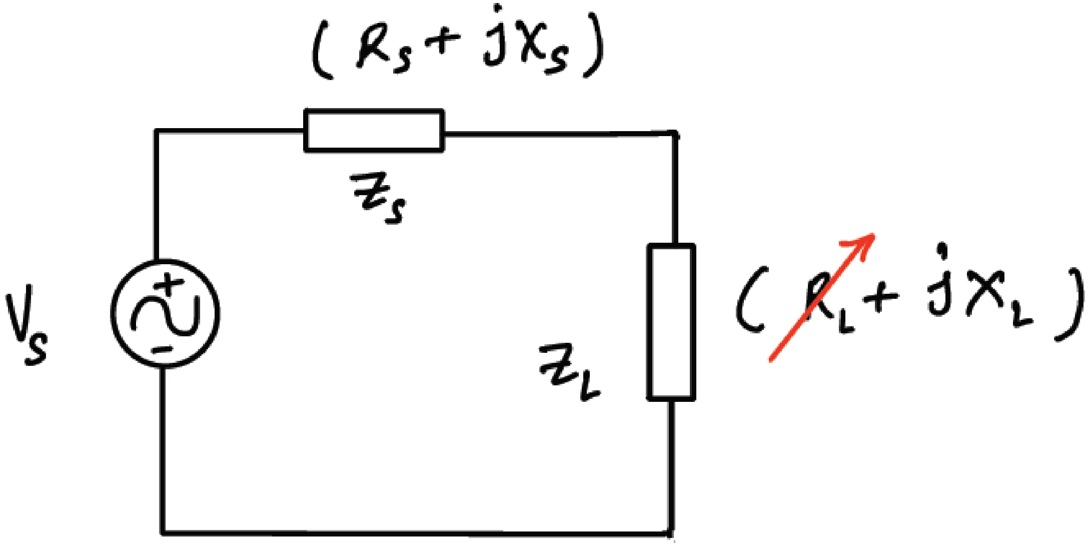
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\text{So, } p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

- This is applicable only when $R_L = R_{Th}$.
- When they are not equal, power delivered is calculated using as $i^2 R_L$.

Maximum Power Transfer (Cont…)

❑ Case-1

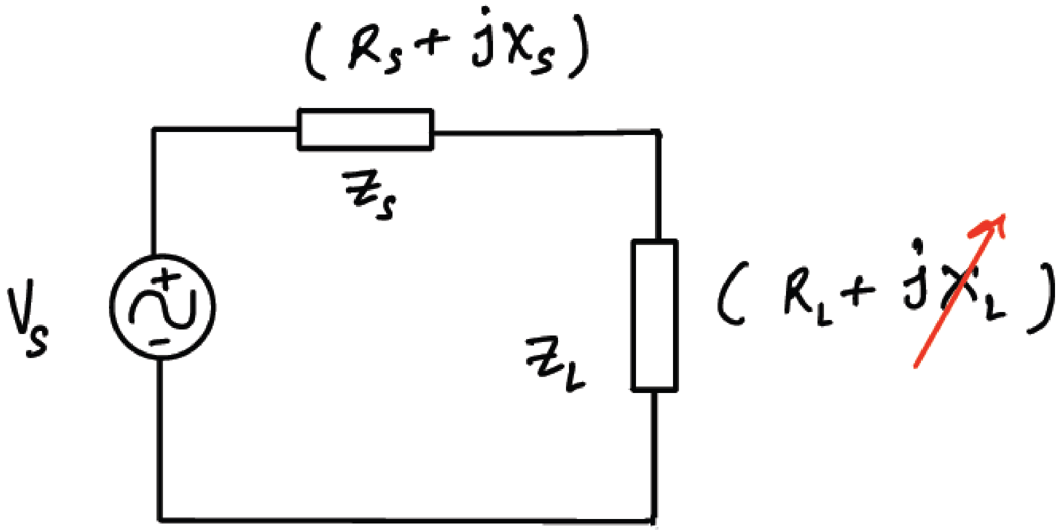


- Maximum power transfer occurs when,

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

Maximum Power Transfer (Cont…)

- Case-2



- Maximum power transfer occurs when,

$$\boxed{X_L + X_s = 0}$$

or

$$\boxed{X_L = -X_s}$$

Maximum Power Transfer (Cont…)

❑ Some Key Points:

- There is a distinct difference between drawing maximum power from a source and delivering maximum power to a load.
- If the load is sized such that its resistance is equal to the Thevenin's resistance of the network, to which it is connected, it will receive maximum power from that network.
- Any change to the load resistance will reduce the power delivered to the load.
- On the other hand, we draw the maximum possible power from the voltage source by drawing the maximum possible current which is achieved by shorting the network terminals.

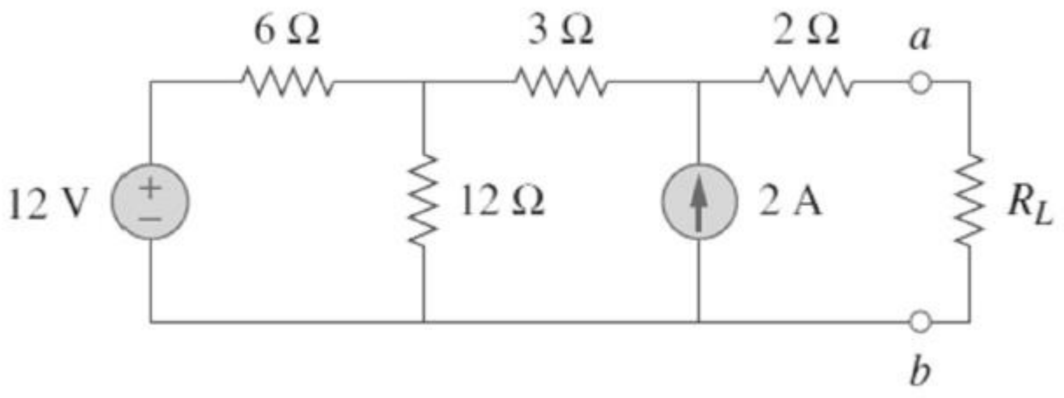
Maximum Power Transfer (Cont…)

❑ Some Key Points…

- However, in this extreme example when source is short circuited, we deliver zero power to the load as $p = i^2R$ and we just set $R = 0$ by shorting the network terminals.
- It is also not uncommon for the maximum power theorem to be misinterpreted.
- It is designed to help us select an optimum load in order to maximize power absorption.
- However, if the load resistance is already specified, the maximum power theorem is of no assistance.
- Practically, if for some reason we can affect the size of the Thévenin equivalent resistance of the network connected to our load, setting it equal to the load does not guarantee maximum power transfer to our, predetermined load.
- A quick consideration of the power lost in the Thevenin resistance will clarify this point.

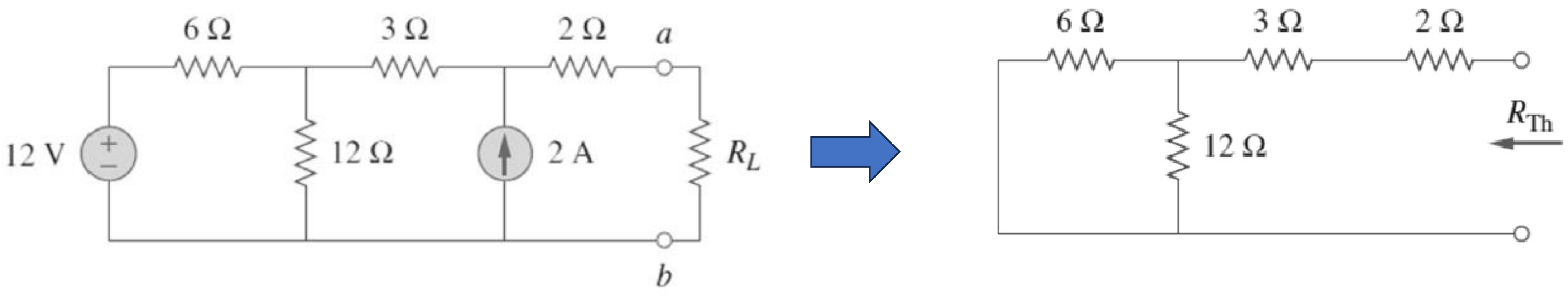
❑ Example:

Find the value of R_L for maximum power transfer in the below circuit?



Solution: We find the value of Thevenin equivalent resistance and the Thevenin voltage across the terminals a and b .

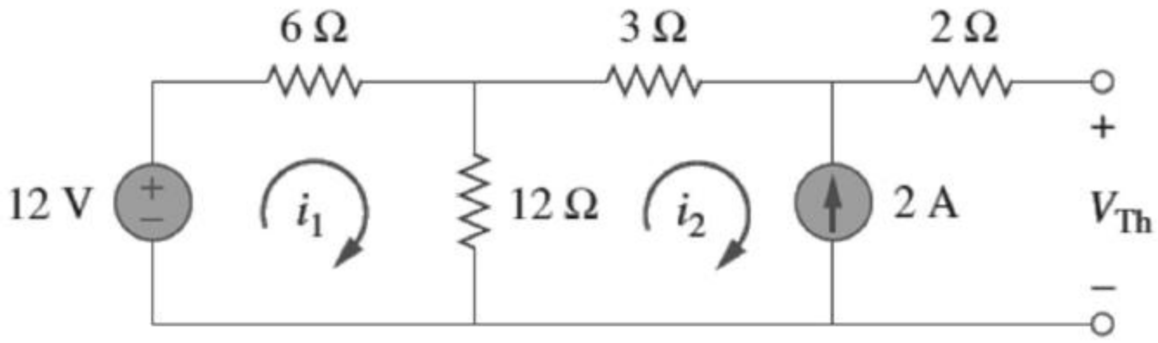
Set the independent sources to zero, to obtain the following circuit,



From the above circuit,

$$R_{Th} = 2 + 3 + 6||12 = 5 + \frac{6 * 12}{18} = 9\Omega$$

- To find V_{Th} we open the terminals a and b.



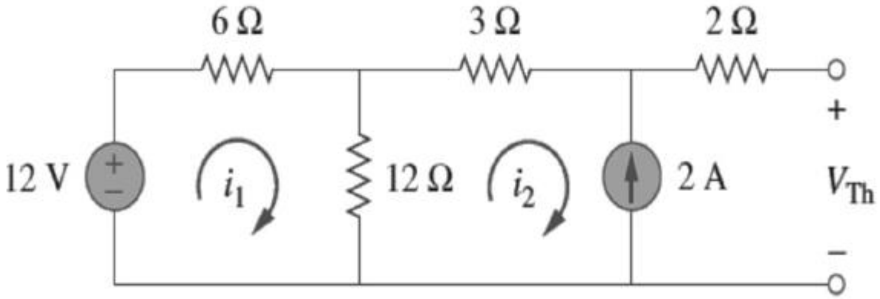
- Applying mesh analysis, we get,

$$-12 + 18i_1 - 12i_2 = 0$$

$$i_2 = -2A$$

- Solving for i_1, we get

$$i_1 = -\frac{2}{3}A$$



- Applying KVL to the outer loop to evaluate V_{Th},

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$V_{Th} = 22V$$

- For maximum power transfer, $R_L = R_{Th} = 9\Omega$

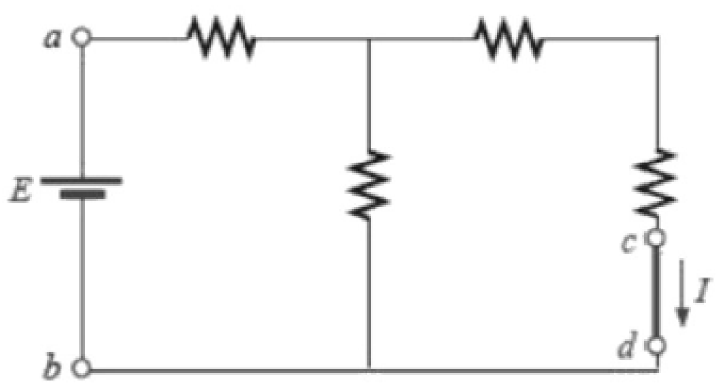
- The maximum power is then given by, $p_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{22^2}{4*9} = 13.44W$

Reciprocity Theorem

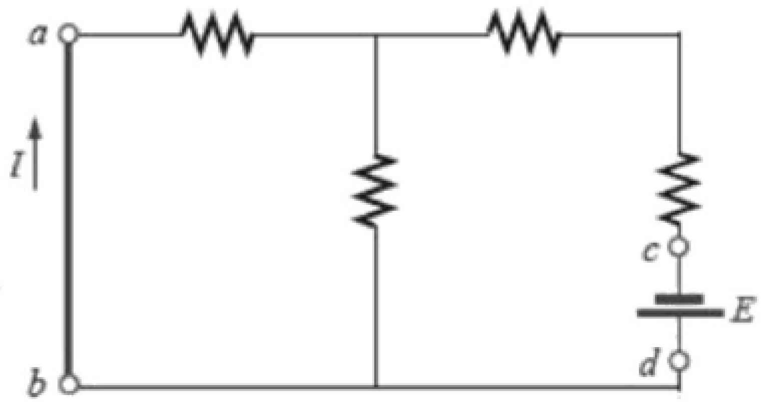
- An important aspect of reciprocity theorem is that it does not hold for all networks.
- Therefore, its restrictions should be remembered.
- Using reciprocity wrongly may lead to seriously erroneous results.
- This lecture will explain what the reciprocity theorem is, and give examples to make its meaning clear.
- Reciprocity theorem states that if an emf E in one branch of a reciprocal network produces a current I in another branch, then if the emf E is moved from the first to the second branch, it will cause the same current in the first branch, where the emf has been replaced by a short circuit.

Reciprocity Theorem (Cont…)

- The theorem can be explained with the help of the circuit diagram shown below.



(a)



(b)

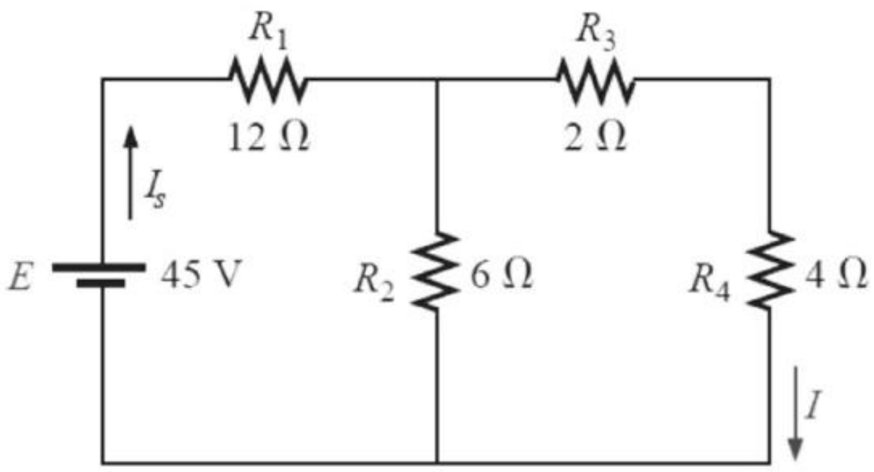
- The network where reciprocity theorem is applied should be linear.
- The circuit should not have any time-varying elements.

Reciprocity Theorem (Cont…)

- In other words, the reciprocity theorem can be stated as – when the places of voltage source and current in any network are interchanged the amount or magnitude of voltage and current flowing in the circuit remains the same.
- However, the polarity of the voltage source should be identical with the direction of the branch current in each position.
- The reciprocity theorem is applicable only to single source networks.
- It is therefore, not a theorem applied to solve multi-source networks.

❑ Example:

Prove reciprocity theorem for the circuit shown below.

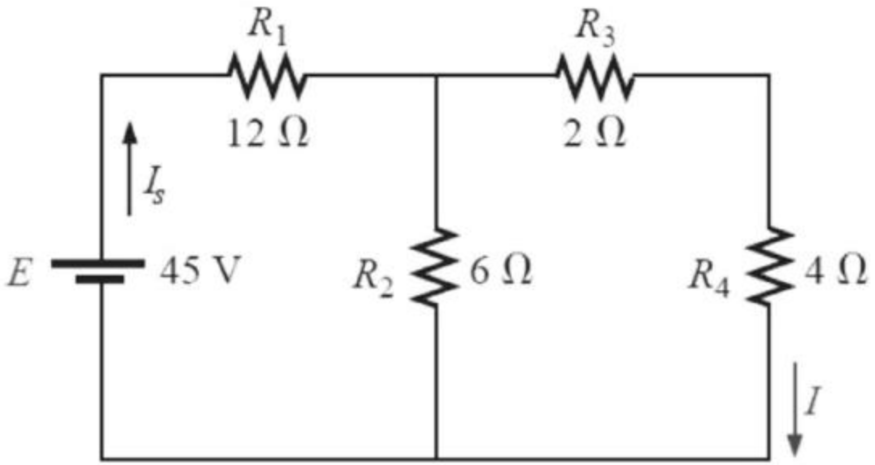


Solution: We find the value of equivalent resistance and the source current for the circuit.

To find the resistance, R_{eq} = ((2 + 4) || 6)+ 12 = 15Ω

- The source current I_s is then evaluated as,

$$I_s = \frac{45}{15} = 3A$$

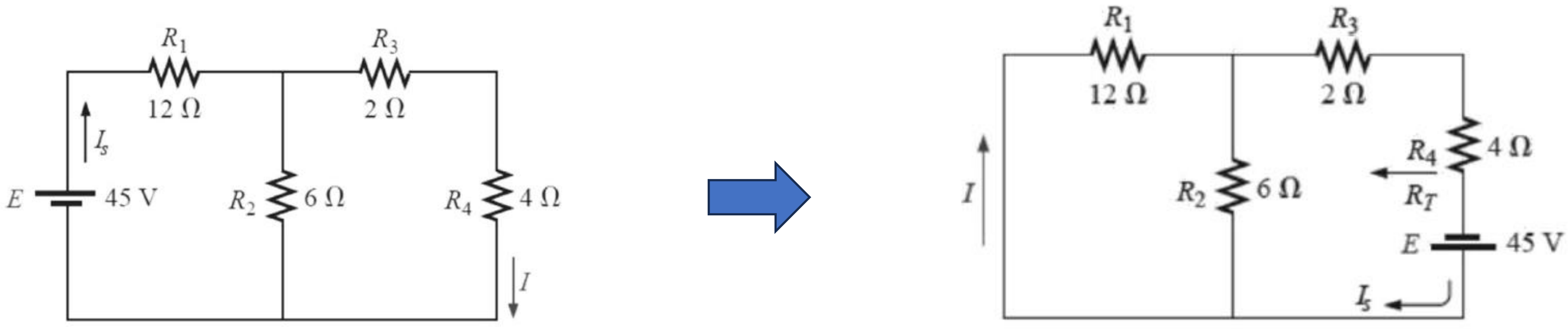


- By using current division rule,

$$I = \frac{3 * 6}{12} = 1.5A$$

- To prove reciprocity theorem, we interchange E and I as shown in the next figure.

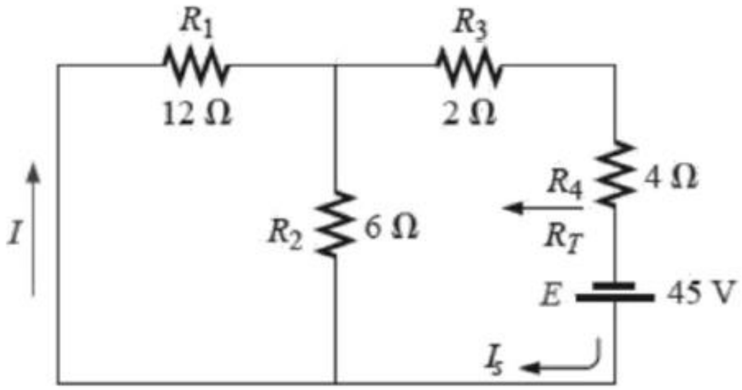
- Since I is pointing downwards in the given problem, the positive terminal is placed downward when we interchange them.



- To find the resistance, $R_{eq} = (12||\,6) + 2 + 4 = 10\Omega$

- The source current I_s is then evaluated as,

$$I_s = \frac{45}{10} = 4.5A$$



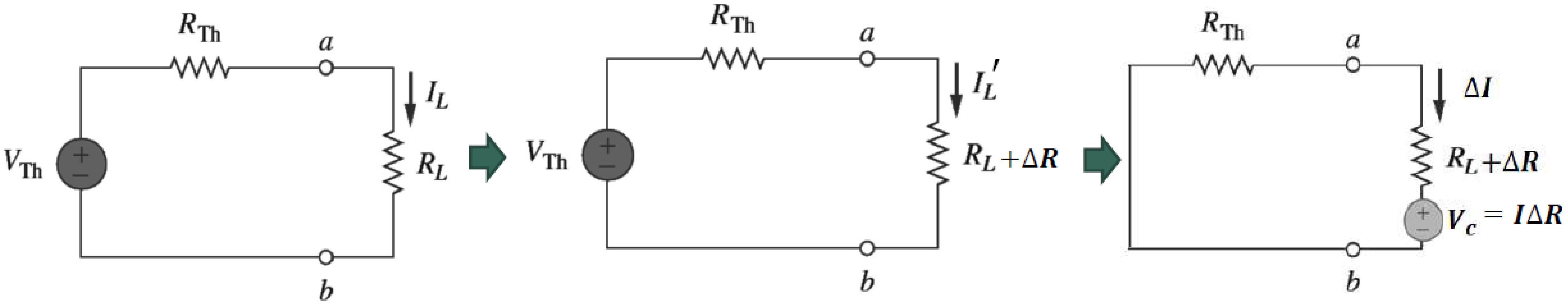
- By using current division rule,

$$I = \frac{4.5 * 6}{12 + 6} = 1.5A$$

- Since in both cases I is equal, therefore, reciprocity theorem is proved.

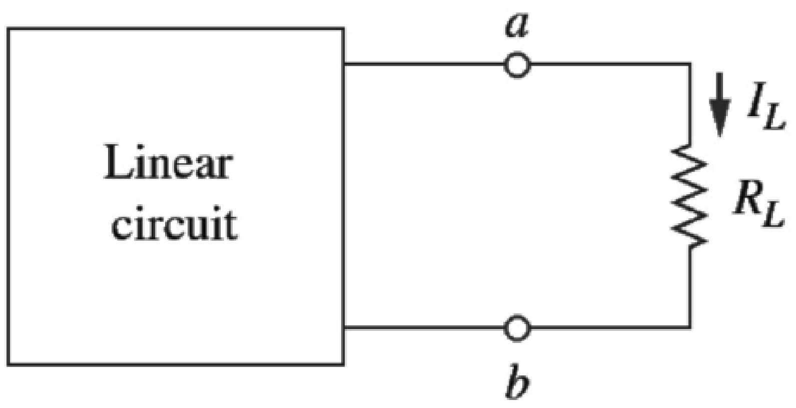
Compensation Theorem

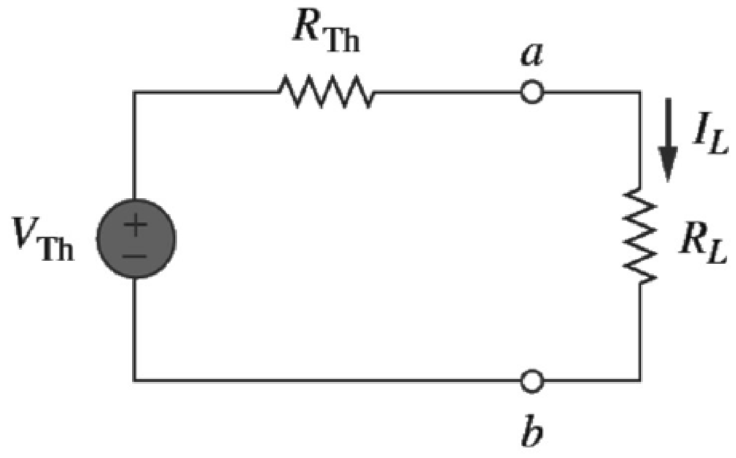
- Compensation theorem is also one of the important theorems in network analysis, which finds it's application mostly in calculating the sensitivity of electrical circuits.
- Compensation Theorem: The calculation of current change ΔI, in a circuit, may be carried out by replacing the added resistance ΔR by the series combination of a voltage source $I(\Delta R)$ and resistance ΔR, while at the same time replacing all sources in a circuit by their equivalent impedances.



Compensation Theorem (Cont…)

- In compensation theorem, the source voltage $I(\Delta R)$ opposes the original current.
- In simple words compensation theorem can be stated as – the resistance of any network can be replaced by a voltage source, having the same voltage as the voltage drop across the resistance which is replaced.
- This can be further explained using the below example.
- Let us assume a load R_L be connected to a DC source network.



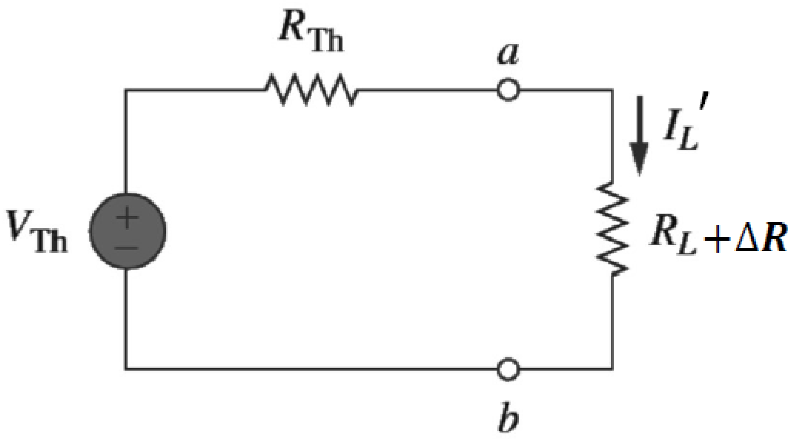


Compensation Theorem (Cont…)

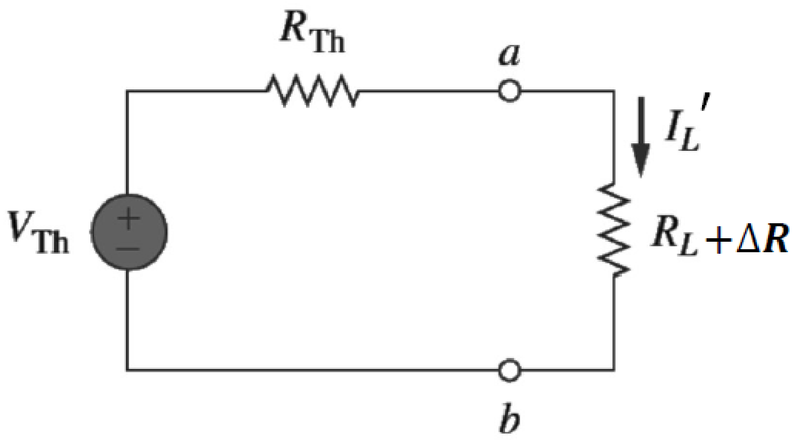
- Thevenin's equivalent gives V_{Th} as the Thevenin's voltage and R_{Th} as the Thevenin's resistance as shown in the figure.
- Here,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

- Let the load resistance R_L be now changed to $R_L + \Delta R$.
- Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the following figure.



Compensation Theorem (Cont…)

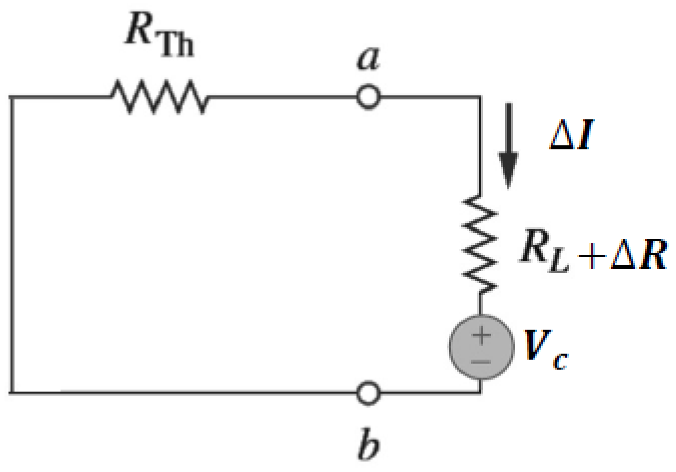


- Here,

$$I_L' = \frac{V_{Th}}{R_{Th} + (R_L + \Delta R)}$$

- The above circuit can be redrawn by replacing the voltage source with its internal resistance as shown in the next figure.

Compensation Theorem (Cont…)



- Let the change of current be ΔI.
- Then, $\Delta I = I' - I$.
- Substituting the value of I' and I in the above equation we get,

$$\Delta I = \frac{V_{Th}}{R_{Th} + (R_L + \Delta R)} - \frac{V_{Th}}{R_{Th} + R_L}$$

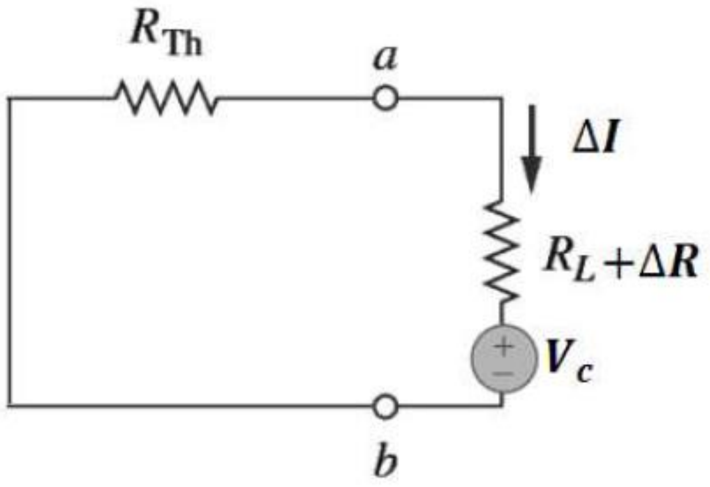
$$= \frac{V_{Th}(R_{Th}+R_L - (R_{Th}+R_L + \Delta R))}{(R_{Th}+(R_L + \Delta R))(R_{Th}+R_L)}$$

Compensation Theorem (Cont…)

- The above equation can then be rewritten as,

$$\Delta I = -\frac{V_{Th}}{R_{Th} + R_L} * \frac{\Delta R}{R_{Th} + R_L + \Delta R}$$

$$= -I * \frac{\Delta R}{R_{Th} + R_L + \Delta R}$$



- But $V_c = -I\Delta R$
- Hence,

$$\Delta I = \frac{V_c}{R_{Th} + R_L + \Delta R}$$

- Hence, compensation theorem tells that with the change of branch resistance, branch currents change, and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

First Order Circuits

- In first order circuits, we will discuss about two types of simple circuits: a circuit comprising a resistor and a capacitor; and a circuit comprising a resistor and an inductor.
- These are called R-C and R-L circuits, respectively.
- Analysis of R-C and R-L circuits is done by applying Kirchhoff's laws, as is done for resistive circuits.
- Applying Kirchhoff's laws to purely resistive circuits results in algebraic equations, while applying the laws to R-C and R-L circuits produces differential equations, more difficult to solve than algebraic equations.
- The differential equations resulting from analyzing R-C and R-L circuits are of the first order. Hence, the circuits are collectively known as first-order circuits.
- A first-order circuit is characterized by a first-order differential equation.

First Order Circuits (Cont…)

Two ways to excite the first-order circuits:

❖ <u>First way:</u>

- To excite the circuit by using initial conditions of the storage elements in the circuits.
- These types of circuits are called as source-free circuits.
- Here, it is assumed that energy is initially stored in the capacitive or inductive element.
- The energy causes current to flow in the circuit and is gradually dissipated in the resistors.
- Although source free circuits are free of independent sources, they may have dependent sources.

❖ <u>Second way:</u>

- Exciting first-order circuits by independent sources.

The Source-Free R-C Circuit

- A source-free R-C circuit occurs when its dc source is suddenly disconnected.
- The energy already stored in the capacitor is released to the resistors.

- Consider a series combination of a resistor and an initially charged capacitor, as shown in Fig. 1 (The resistor and capacitor may be the equivalent resistance and equivalent capacitance of combinations of resistors and capacitors). The objective is to determine the circuit response.

A circuit response is the manner in which the circuit reacts

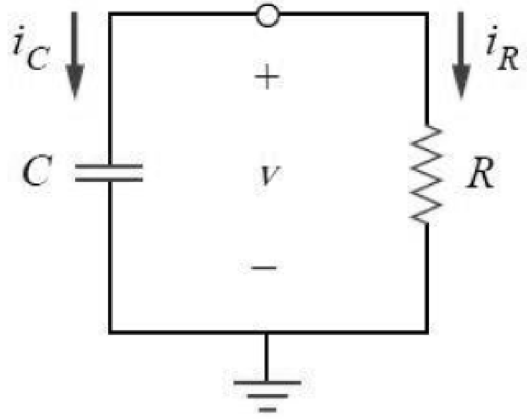


Fig. 1

The Source-Free R-C Circuit (Cont…)

- Assume a voltage v(t) across capacitor. Since the capacitor is initially charged, we can assume that at time t = 0, the initial voltage is

$$v(0) = V_0 \qquad (1)$$

with the corresponding value of the energy stored as

$$w(0) = \frac{1}{2}CV^2 \qquad (2)$$

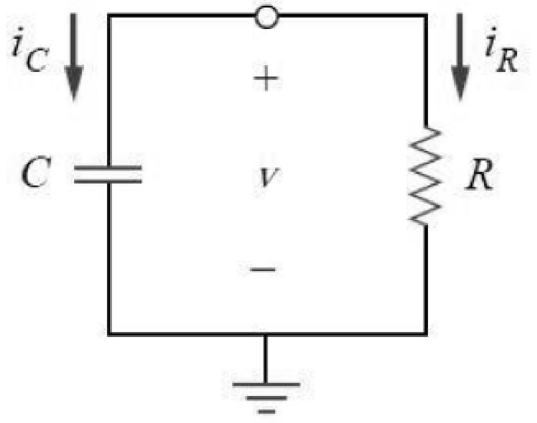


Fig. 1

- Applying KCL at the top node of the circuit in Fig. 1,

$$i_C + i_R = 0 \qquad (3)$$

- By definition, $i_C = C\frac{dv}{dt}$ and $i_R = \frac{v}{R}$, Thus,

$$C\frac{dv}{dt} + \frac{v}{R} = 0 \qquad (4)$$

The Source-Free R-C Circuit (Cont…)

- This is a first-order differential equation, since only the first derivative of v is involved. To solve it, we rearrange the terms as

$$\frac{dv}{v} = -\frac{1}{RC}dt \qquad (5)$$

- Integrating both sides, we get

$$lnv = -\frac{t}{RC} + lnA$$

- where $ln\ A$ is the integration constant. Thus

$$ln\frac{v}{A} = -\frac{t}{RC} \qquad (6)$$

The Source-Free R-C Circuit (Cont…)

- Taking powers of e on both sides produces

$$v(t) = Ae^{-t/RC}$$

- But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0 e^{-t/RC} \tag{7}$$

- This shows that the voltage response of the R-C circuit is an exponential decay of the initial voltage.
- Since the response is due to the initial energy stored and the physical characteristics of the circuit; and not due to some external voltage or current source, it is called the natural response of the circuit.

The natural response of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.

The Source-Free R-C Circuit (Cont…)

- The natural response is illustrated graphically in the figure below. Note that, value at $t = 0$ is the initial condition.
- As t increases, the voltage decreases toward zero. The rapidity with which the voltage decreases is expressed in terms of the time constant, denoted by the lower-case Greek letter tau, τ .
- The value of τ is RC for the R-C circuit

The natural response depends on the nature of the circuit alone, with no external sources. In fact, the circuit has a response only because of the energy initially stored in the capacitor.

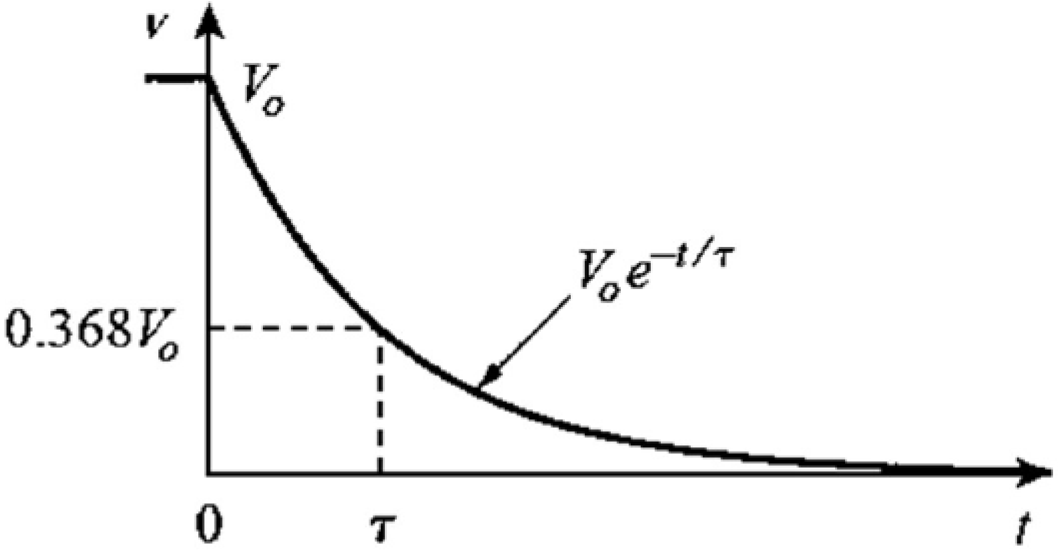


Fig. 2


THANKS
FOR YOUR
ATTENTION