

Lecture-19

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Hybrid Parameters.
- Transmission Parameters.

Hybrid Parameters

- The z and y parameters of a two-port network do not always exist.
- So there is a need for developing another set of parameters.
- This third set of parameters is based on making V_1 and I_2 the dependent variables.
- They are very useful for describing electronic devices such as transistors.
- It is much easier to measure experimentally the h parameters of such devices than to measure their z or y parameters.
- In fact, we have seen that the ideal transformer does not have z parameters.
- However, the ideal transformer can be described using the hybrid parameters.

Hybrid Parameters (Cont...)

- The relation between the terminal voltage and the terminal current can be established using the following equations:

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- Here the **h** terms are known as the hybrid parameters or the **h** parameters.
- They are called so because they are a hybrid combination of ratios.

Hybrid Parameters (Cont...)

- The values of the parameters can be evaluated by setting $\mathbf{V}_2 = \mathbf{0}$ (output port short-circuited) or $\mathbf{I}_1 = \mathbf{0}$ (input port open-circuited).
- The individual parameters are evaluated as follows:

If $\mathbf{V}_2 = \mathbf{0}$, then

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1}$$

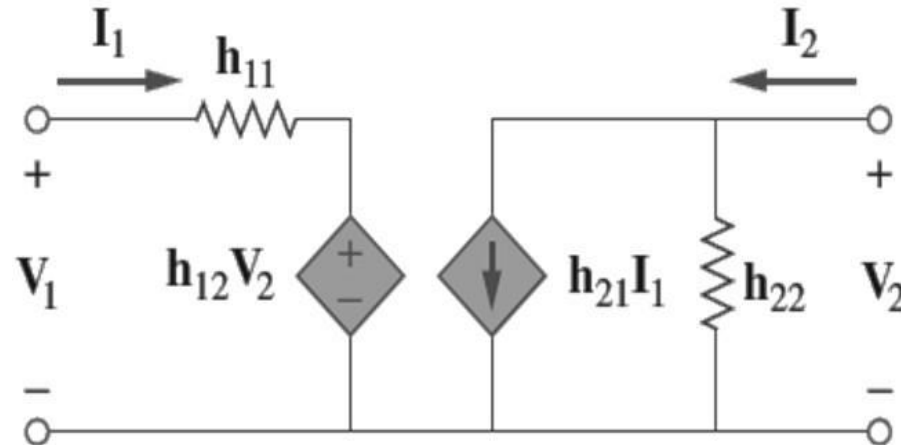
If $\mathbf{I}_1 = \mathbf{0}$, then

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2}, \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$$

- The parameters \mathbf{h}_{11} , \mathbf{h}_{12} , \mathbf{h}_{21} , and \mathbf{h}_{22} represent an impedance, a voltage gain, a current gain, and an admittance, respectively.
- This is why they are called the hybrid parameters.

Hybrid Parameters (Cont...)

- Specifically,
 - \mathbf{h}_{11} = Short-circuit input impedance
 - \mathbf{h}_{12} = Open-circuit reverse voltage gain
 - \mathbf{h}_{21} = Short-circuit forward current gain
 - \mathbf{h}_{22} = Open-circuit output admittance
- The **h** parameters are determined similar to the **z** or **y** parameters.
- For reciprocal circuits, **$\mathbf{h}_{12} = -\mathbf{h}_{21}$** .
- The hybrid model of a two-port network is as shown in the below figure.



Hybrid Parameters (Cont...)

- Another set of parameter closely related to the **h** parameters are the **g** parameters or the **inverse hybrid** parameters.
- They are used to describe terminal currents and voltages as

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{g}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

- Here the **g** terms are known as the **inverse hybrid** parameters or the **g** parameters.

Hybrid Parameters (Cont...)

- The values of the parameters can be evaluated by setting $\mathbf{V}_1 = \mathbf{0}$ (input port short-circuited) or $\mathbf{I}_2 = \mathbf{0}$ (output port open-circuited).
- The individual parameters are evaluated as follows:

If $\mathbf{I}_2 = \mathbf{0}$, then

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1}$$

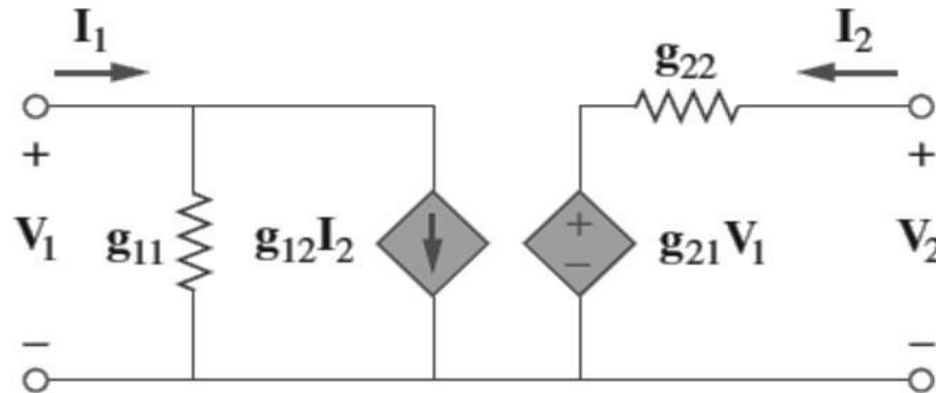
If $\mathbf{V}_1 = \mathbf{0}$, then

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2}, \mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$

- The parameters \mathbf{g}_{11} , \mathbf{g}_{12} , \mathbf{g}_{21} , and \mathbf{g}_{22} represent an admittance, a current gain, a voltage gain, and an impedance, respectively.

Hybrid Parameters (Cont...)

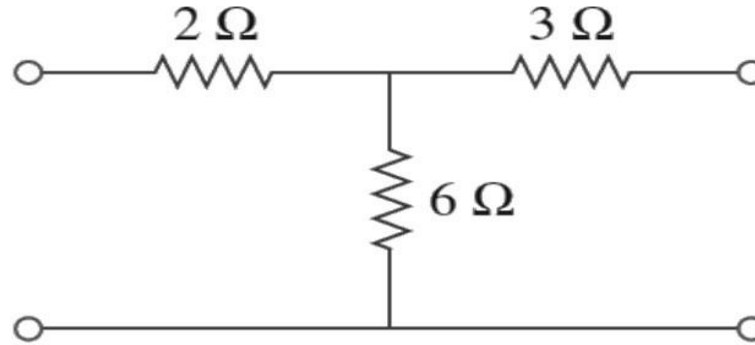
- Specifically,
 - g_{11} = Open-circuit input admittance
 - g_{12} = Short-circuit reverse current gain
 - g_{21} = Open-circuit forward voltage gain
 - g_{22} = Short-circuit output impedance
- The g parameters are determined similar to the z or y parameters.
- The inverse hybrid model of a two-port network is as shown in the below figure.



Hybrid Parameters (Cont...)

□ Example:

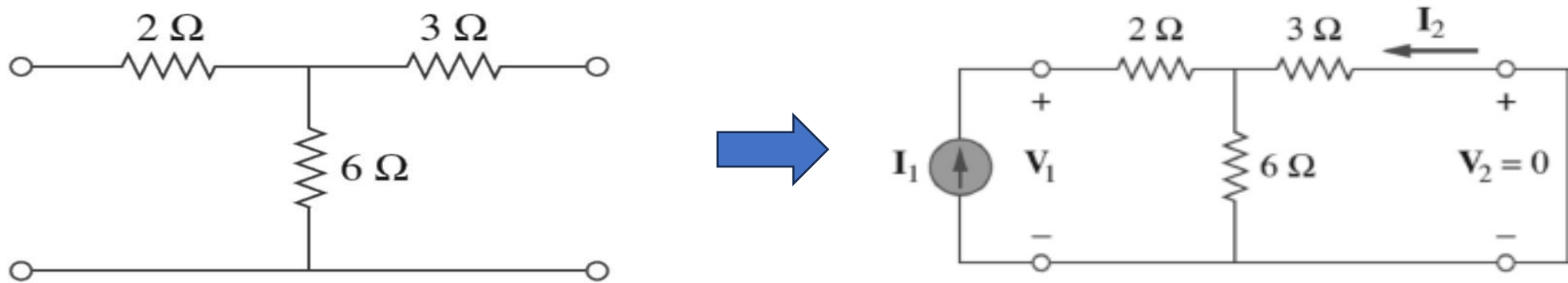
- Determine the **h**-parameters for the below circuit?



Solution: The parameters are determined using the equations discussed earlier in the lecture.

Hybrid Parameters (Cont...)

- To determine \mathbf{h}_{11} and \mathbf{h}_{21} , we connect a current source \mathbf{I}_1 to the input port and short the output port as in the following figure.
- Since the terminal is shorted, 3 and 6 ohm resistors are in parallel.



$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3||6) = 4\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{4\mathbf{I}_1}{\mathbf{I}_1} = 4\Omega$$

If $\mathbf{V}_2 = 0$, then

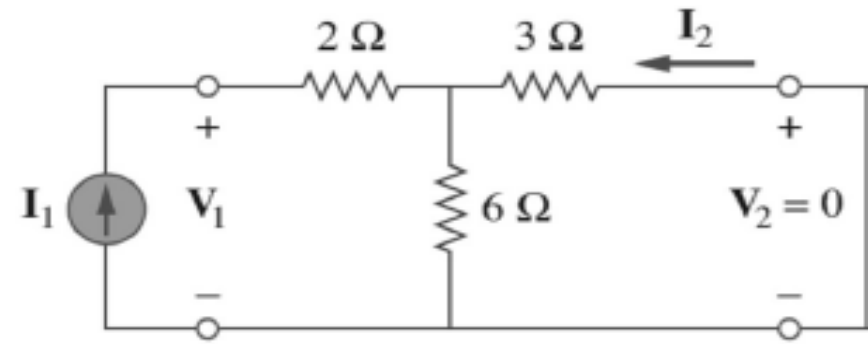
$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1}$$

If $\mathbf{I}_1 = 0$, then

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2}, \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2}$$

Hybrid Parameters (Cont...)

- By current division,



$$-I_2 = \frac{6}{6 + 3} I_1 = \frac{2}{3} I_1$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-\frac{2}{3} I_1}{I_1} = -\frac{2}{3}$$

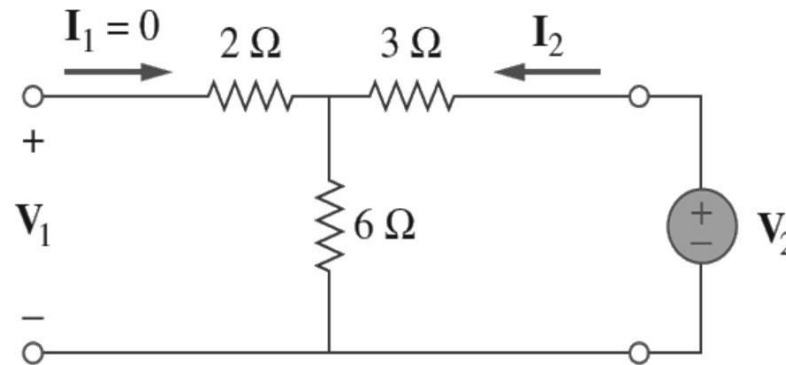
If $V_2 = 0$, then

$$h_{11} = \frac{V_1}{I_1}, h_{21} = \frac{I_2}{I_1}$$

If $I_1 = 0$, then

$$h_{12} = \frac{V_1}{V_2}, h_{22} = \frac{I_2}{V_2}$$

- To determine h_{12} and h_{22} , we connect a voltage source V_2 to the output port and open-circuit the input port as in the following figure.



Hybrid Parameters (Cont...)

- The 2 ohm resistor is open circuited. Hence, by voltage division

$$V_1 = \frac{6}{6 + 3} V_2 = \frac{2}{3} V_2$$

$$h_{12} = \frac{V_1}{V_2} = \frac{\frac{2}{3} V_2}{V_2} = \frac{2}{3}$$

- Also,

$$V_2 = I_2(3 + 6) = 9I_2$$

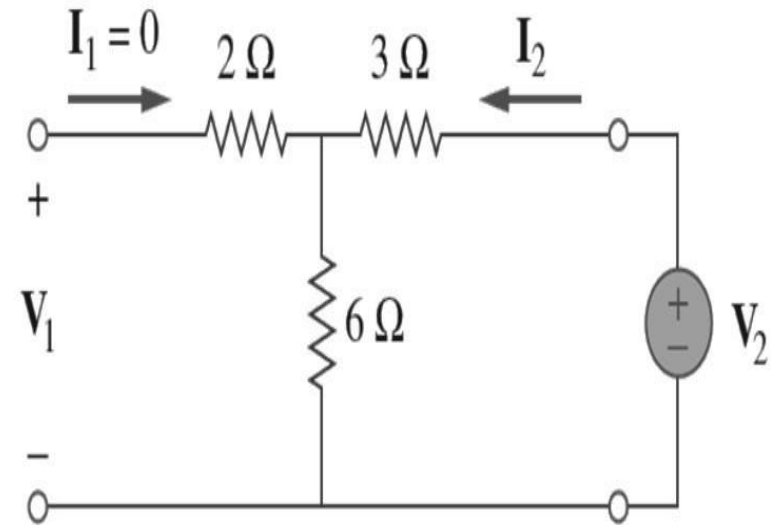
$$h_{22} = \frac{I_2}{V_2} = \frac{I_2}{9I_2} = \frac{1}{9} S$$

If $V_2 = 0$, then

$$h_{11} = \frac{V_1}{I_1}, h_{21} = \frac{I_2}{I_1}$$

If $I_1 = 0$, then

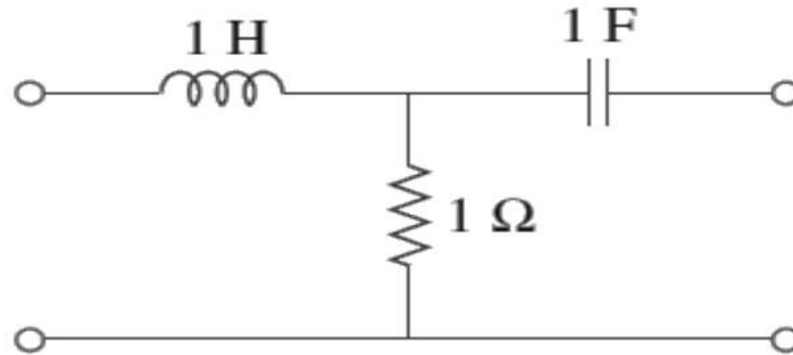
$$h_{12} = \frac{V_1}{V_2}, h_{22} = \frac{I_2}{V_2}$$



Hybrid Parameters (Cont...)

□ Example:

- Determine g -parameters as function of s for the following circuit?

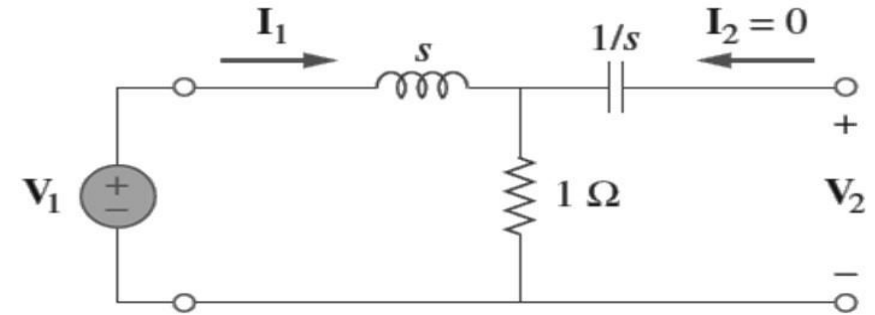
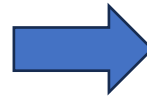
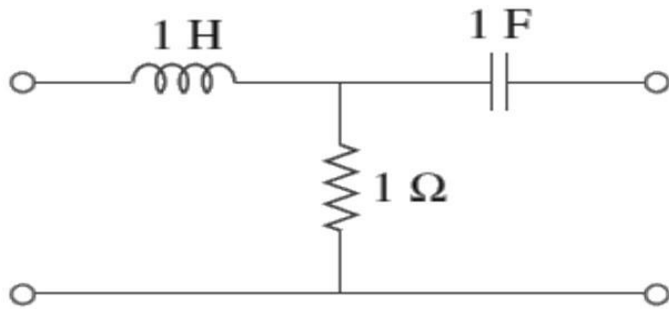


Solution: The above circuit can be solved using the same procedure discussed in the previous example.

First we need to convert the circuit into s -domain.

Hybrid Parameters (Cont...)

- To determine g_{11} and g_{21} , we connect a voltage source V_1 to the input port and open-circuit the output port as in the following figure.
- Since the terminal is open-circuited, the capacitor need not be considered in calculating I_1 .



$$I_1 = \frac{V_1}{s + 1}$$

$$g_{11} = \frac{I_1}{V_1} = \frac{1}{s + 1}$$

If $I_2 = 0$, then

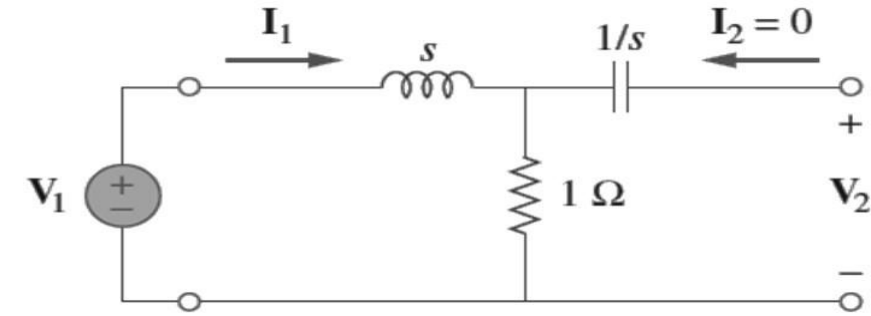
$$g_{11} = \frac{I_1}{V_1}, g_{21} = \frac{V_2}{V_1}$$

If $V_1 = 0$, then

$$g_{12} = \frac{I_1}{I_2}, g_{22} = \frac{V_2}{I_2}$$

Hybrid Parameters (Cont...)

- By voltage division,



$$V_2 = \frac{1}{s+1} V_1$$

$$g_{21} = \frac{V_2}{V_1} = \frac{1}{s+1}$$

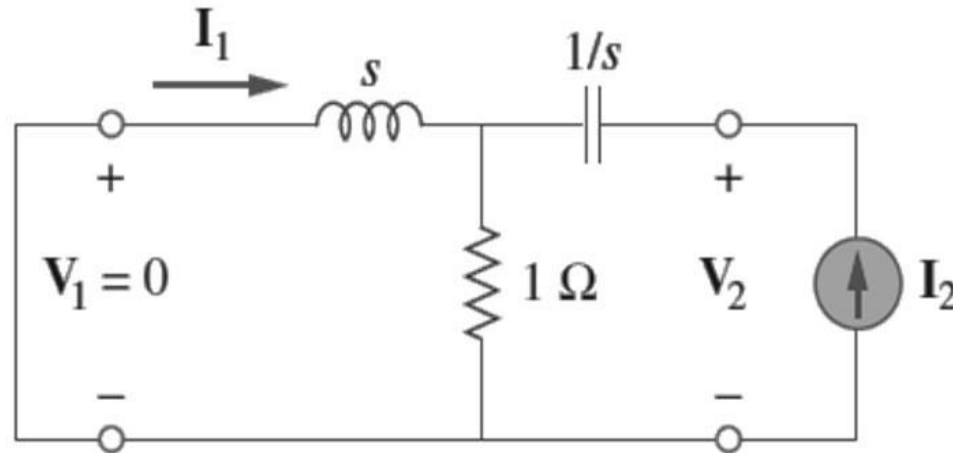
If $I_2 = 0$, then

$$g_{11} = \frac{I_1}{V_1}, g_{21} = \frac{V_2}{V_1}$$

If $V_1 = 0$, then

$$g_{12} = \frac{I_1}{I_2}, g_{22} = \frac{V_2}{I_2}$$

- To determine g_{12} and g_{22} , we connect a current source I_2 to the output port and short-circuit the input port as in the following figure.



Hybrid Parameters (Cont...)

- By current division

$$\mathbf{I}_1 = -\frac{1}{s+1} \mathbf{I}_2$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -\frac{1}{s+1}$$

- Also,

$$\mathbf{V}_2 = \mathbf{I}_2 \left(\frac{1}{s} + s \parallel 1 \right)$$

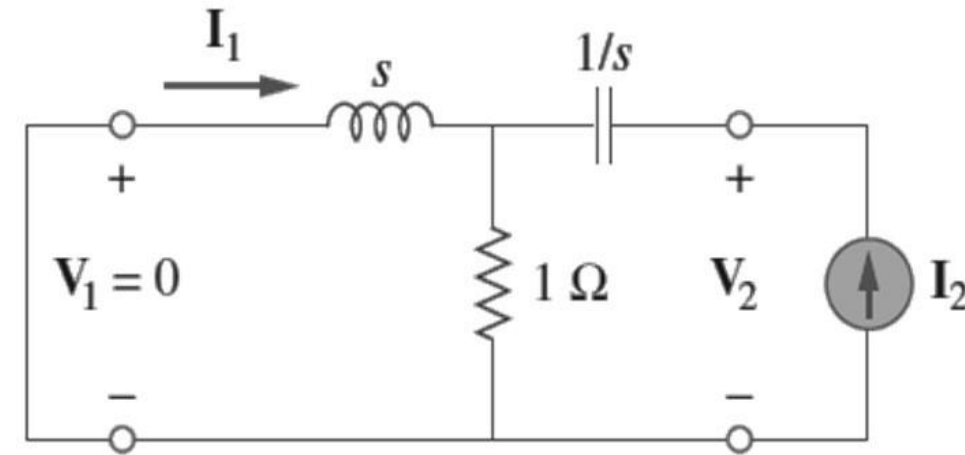
$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} + \frac{s}{s+1} = \frac{(s^2 + s + 1)}{s(s+1)}$$

If $\mathbf{I}_2 = 0$, then

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1}$$

If $\mathbf{V}_1 = 0$, then

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2}, \mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}$$



Transmission Parameters

- As there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we can generate many sets of parameters.
- Today we will discuss another set of parameters that relates the variables at the input port to those at the output port.
- This set of parameters are known as **ABCD** parameters or **transmission** parameters.
- The **ABCD** parameters provide a measure of how a circuit transmits voltage and current from a source to a load.

Transmission Parameters (Cont...)

- The relation between the terminal voltage and the terminal current can be established using the following equations:

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

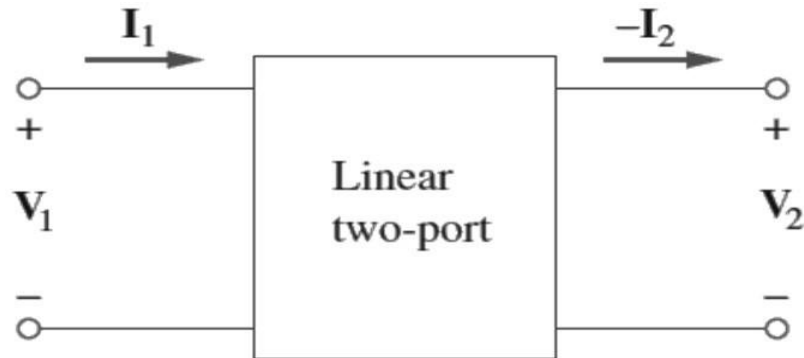
- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

- Here the **T** terms are known as the **ABCD** parameters or the **transmission** parameters.
- They are useful in the analysis of transmission lines because they express sending-end variables in terms of the receiving-end variables.
- For this reason, they are called transmission parameters.

Transmission Parameters (Cont...)

- The above equations relate the input variables ($\mathbf{V_1}$ and $\mathbf{I_1}$) to the output variables ($\mathbf{V_2}$ and $-\mathbf{I_2}$).
- Notice that in computing the transmission parameters, $-\mathbf{I_2}$ is used rather than $\mathbf{I_2}$, because the current is considered to be leaving the network, as shown in below figure.
- It is so, because in the power systems we consider $\mathbf{I_2}$ as leaving the port.



Transmission Parameters (Cont...)

- The values of the parameters can be evaluated by setting $\mathbf{V}_2 = \mathbf{0}$ (output port short-circuited) or $\mathbf{I}_2 = \mathbf{0}$ (output port open-circuited) in the following equations –

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

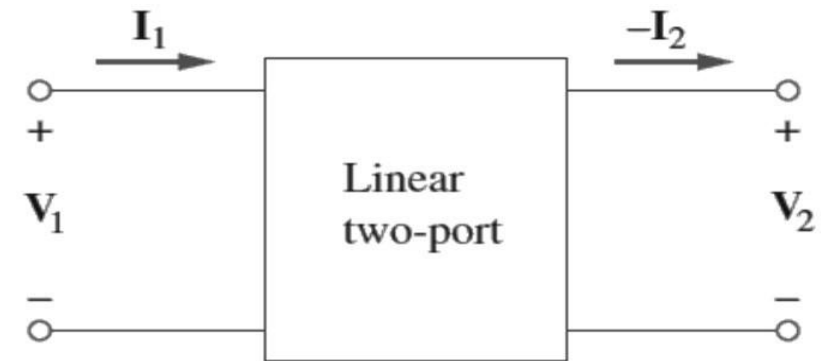
- The individual parameters are evaluated as follows:

If $\mathbf{I}_2 = \mathbf{0}$, then

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2}, \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2}$$

If $\mathbf{V}_2 = \mathbf{0}$, then

$$\mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2}, \mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2}$$



- The parameters \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} represent a **voltage gain**, an **impedance**, an **admittance**, and a **current gain**, respectively.

Transmission Parameters (Cont...)

- Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.
- Specifically,
 - **A** = Open-circuit voltage ratio
 - **B** = Negative short-circuit transfer impedance
 - **C** = Open-circuit transfer admittance
 - **D** = Negative short-circuit current ratio
- The **T** parameters can also be determined similar to the **z** or **y** parameters.
- For reciprocal circuits, **$AD - BC = 1$** .

Transmission Parameters (Cont...)

- The last set of parameter is obtained by expressing the variables at the output port in terms of variables at the input port:

$$\mathbf{V}_2 = \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1$$

$$\mathbf{I}_2 = \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1$$

- This can alternatively be expressed in matrix form as,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix}$$

- Here the parameters are known as the **inverse transmission** or **t** -parameters.
- The values of the parameters can be evaluated by setting **$\mathbf{V}_1 = \mathbf{0}$** (input port short-circuited) or **$\mathbf{I}_1 = \mathbf{0}$** (input port open-circuited).

Transmission Parameters (Cont...)

- The individual parameters are evaluated as follows:

If $\mathbf{I}_1 = 0$, then

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1}, \mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1}$$

If $\mathbf{V}_1 = 0$, then

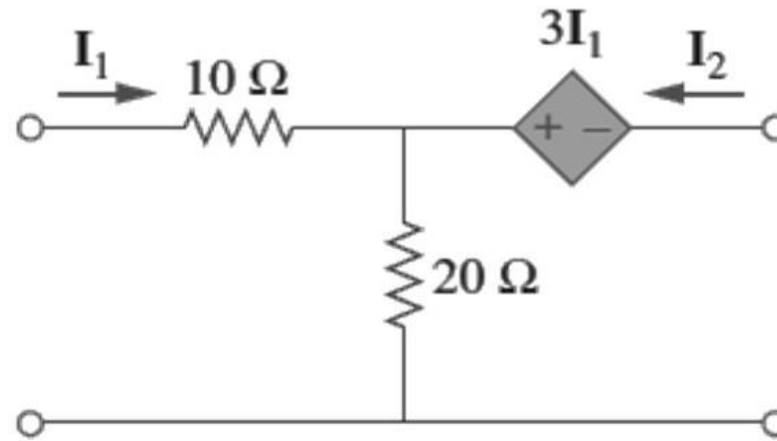
$$\mathbf{b} = -\frac{\mathbf{V}_2}{\mathbf{I}_1}, \mathbf{d} = -\frac{\mathbf{I}_2}{\mathbf{I}_1}$$

- Specifically,
 - \mathbf{a} = Open-circuit voltage ratio
 - \mathbf{b} = Negative short-circuit transfer impedance
 - \mathbf{c} = Open-circuit transfer admittance
 - \mathbf{d} = Negative short-circuit current ratio
- For reciprocal circuits, $\mathbf{ad} - \mathbf{bc} = 1$.

Transmission Parameters (Cont...)

□ Example:

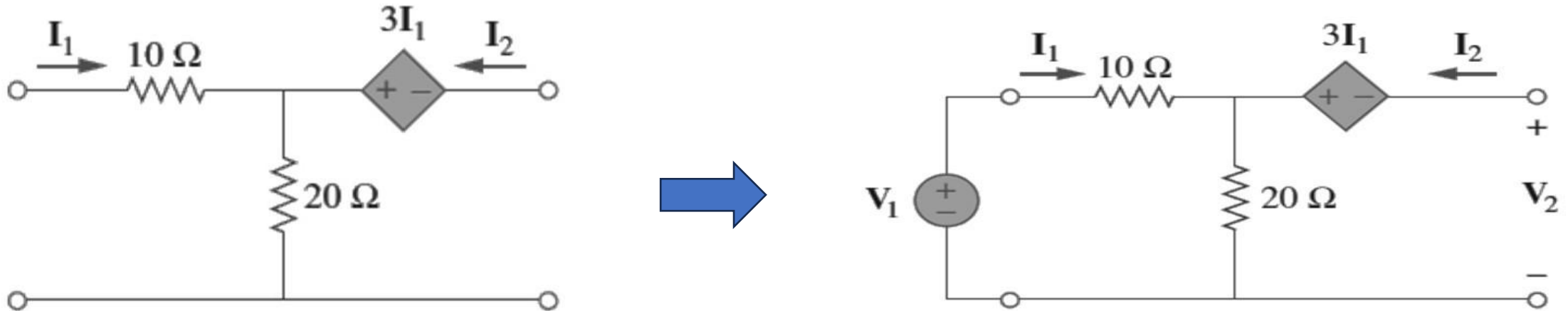
- Determine the transmission parameters for the below circuit?



Solution: The parameters are determined using the equations discussed earlier in the lecture.

Transmission Parameters (Cont...)

- To determine **A** and **C**, we connect a voltage source **V₁** to the input port and open the output port as in the following figure.



$$V_1 = I_1 (10 + 20) = 30I_1$$

$$V_2 = I_1(20 - 3) = 17I_1$$

If **I₂ = 0**, then

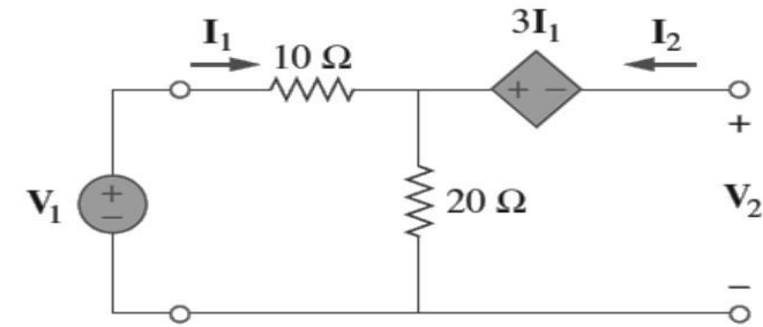
$$A = \frac{V_1}{V_2}, C = \frac{I_1}{V_2}$$

If **V₂ = 0**, then

$$B = -\frac{V_1}{I_2}, D = -\frac{I_1}{I_2}$$

Transmission Parameters (Cont...)

- Thus,



$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588\mathbf{S}$$

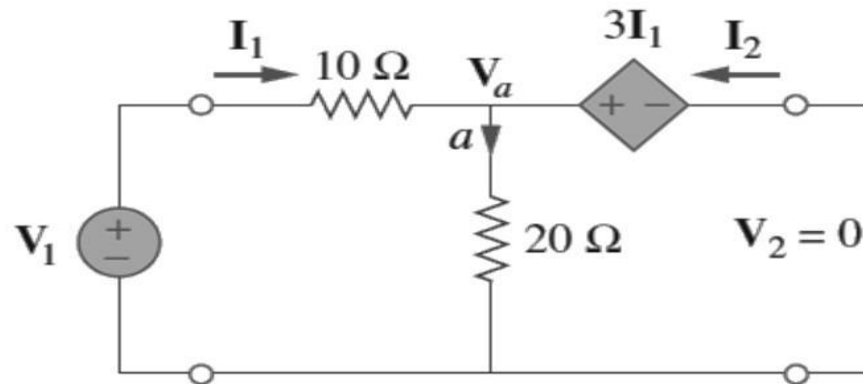
If $\mathbf{I}_2 = 0$, then

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2}, \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2}$$

If $\mathbf{V}_2 = 0$, then

$$\mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2}, \mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2}$$

- To determine \mathbf{B} and \mathbf{D} , we connect a voltage source \mathbf{V}_1 to the input port and short the output port as in the following figure.



Transmission Parameters (Cont...)

- At node a in the circuit, KCL gives,

$$\frac{V_1 - V_a}{10} - \frac{V_a}{20} + I_2 = 0$$

- But $V_a = 3I_1$ and $I_1 = (V_1 - V_a)/10$. Using these we get,

$$V_a = 3I_1 \text{ and } V_1 = 13I_1$$

- Substituting the above values in the KCL,

$$I_1 - \frac{3I_1}{20} + I_2 = 0 \Rightarrow \frac{17}{20} I_1 = -I_2$$

- Therefore,

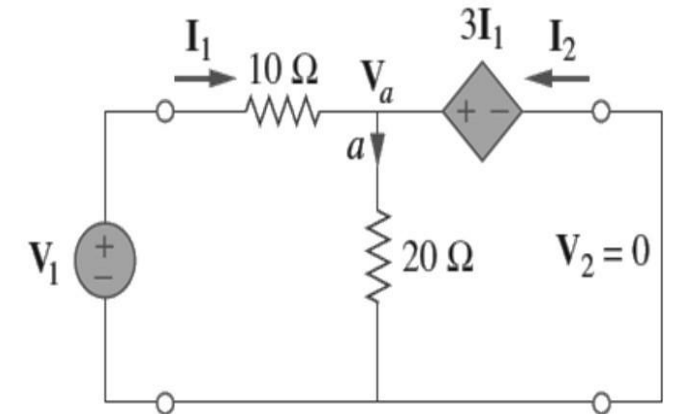
$$\begin{aligned} \mathbf{D} &= -\frac{I_1}{I_2} = \frac{20}{17} = 1.176, & \mathbf{B} &= -\frac{V_1}{I_2} = \frac{-13I_1}{\left(-\frac{17}{20}\right) I_1} \\ &= 15.29\Omega \end{aligned}$$

If $I_2 = 0$, then

$$\mathbf{A} = \frac{V_1}{V_2}, \mathbf{C} = \frac{I_1}{V_2}$$

If $V_2 = 0$, then

$$\mathbf{B} = -\frac{V_1}{I_2}, \mathbf{D} = -\frac{I_1}{I_2}$$

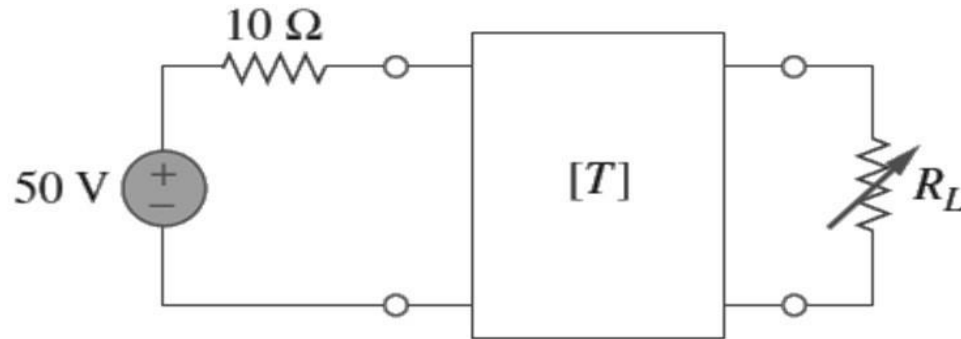


Transmission Parameters (Cont...)

□ Example:

- The transmission parameters for the below circuit are $\begin{bmatrix} 4 & 20\Omega \\ 0.1S & 2 \end{bmatrix}$. The

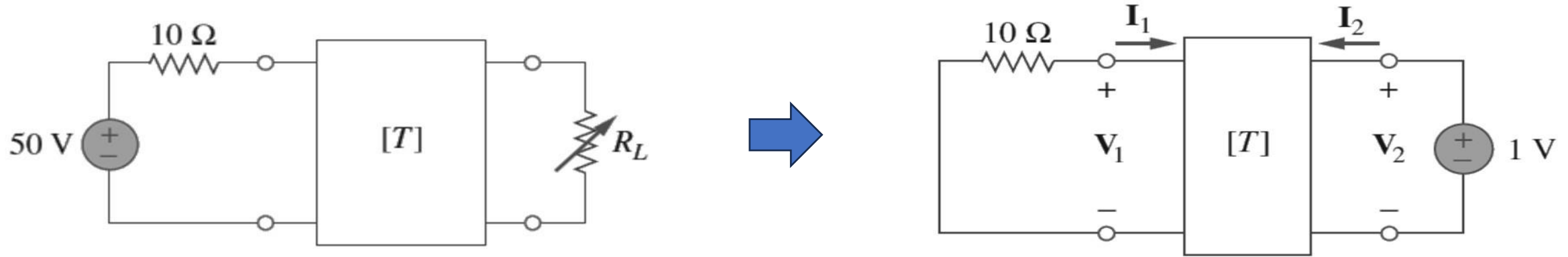
output port is connected to a variable load for maximum power transfer. Find R_L and maximum power transferred?



Solution: The parameters are determined using the equations discussed earlier in the lecture.

Transmission Parameters (Cont...)

- We need to determine the Thevenin equivalent (Z_{Th} and V_{Th}), at the load or output port.
- We find Z_{Th} using the below circuit.



- Our goal here is to set $Z_{Th} = V_2/I_2$
- This can be found out using the **ABCD** parameters.

Transmission Parameters (Cont...)

- Substituting the given parameters we obtain,

$$\mathbf{V_1 = 4V_2 - 20I_2}$$

$$\mathbf{I_1 = 0.1V_2 - 2I_2}$$

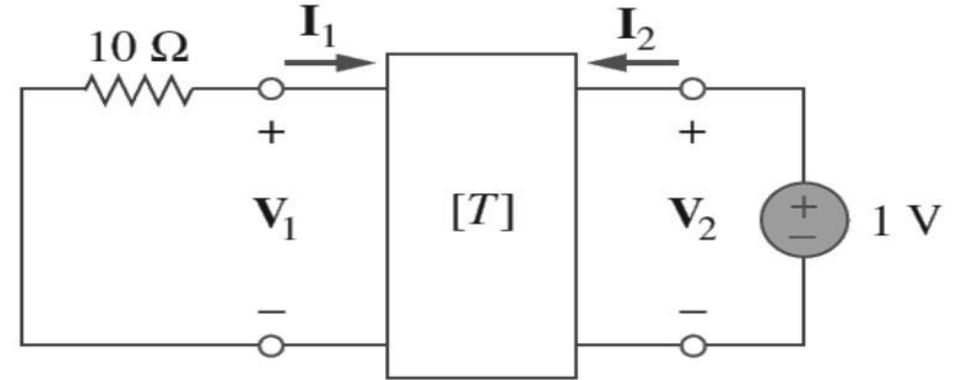
- At the input port $\mathbf{V_1 = -10I_1}$
- Substituting this relation in the first equation, we get,

$$\mathbf{-10I_1 = 4V_2 - 20I_2 \rightarrow I_1 = -0.4V_2 + 2I_2}$$

- Applying the above value to the second equation of the ABCD parameters, we get,

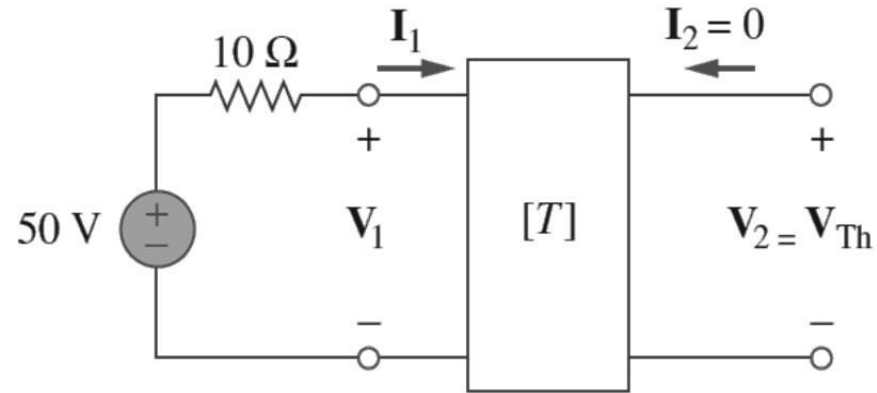
$$\mathbf{-0.4V_2 + 2I_2 = 0.1V_2 - 2I_2 \rightarrow 0.5V_2 = 4I_2}$$

$$\mathbf{Z_{Th} = \frac{V_2}{I_2} = \frac{4}{0.5} = 8\Omega}$$



Transmission Parameters (Cont...)

- To determine V_{Th} , we use the below circuit.



$$V_1 = 4V_2 - 20I_2$$

$$I_1 = 0.1V_2 - 2I_2$$

- At the output $I_2 = 0$ and the input port $V_1 = 50 - 10I_1$
- Applying the above to the **ABCD** parameter equations,

$$50 - 10I_1 = 4V_2$$

$$I_1 = 0.1V_2 \rightarrow V_2 = 10I_1$$

Transmission Parameters (Cont...)

- Therefore,

$$50 - 10\mathbf{I}_1 = 4\mathbf{V}_2 \rightarrow 50 - \mathbf{V}_2 = 4\mathbf{V}_2$$

$$5\mathbf{V}_2 = 50 \rightarrow \mathbf{V}_2 = 10$$

- Thus,

$$V_{\text{Th}} = \mathbf{V}_2 = 10\text{V}$$

- For maximum power transfer,

$$R_L = Z_{\text{Th}} = 8\Omega$$

- The maximum power transfer is therefore,

$$P = I^2 R_L = \left(\frac{V_{Th}}{2R_L} \right)^2 R_L = 3.125\text{W}$$

