

ASSIGNMENT 7
MSO-201: PROBABILITY AND STATISTICS

1. Suppose the PDF of a random variable X is $f_X(x) = e^{-x}$, for $0 < x < \infty$, and zero elsewhere. Find the PDF of $\frac{X}{X+1}$.
2. Suppose the PDF of a random variable X is $f_X(x) = 6x(1-x)$, for $0 < x < 1$ and zero elsewhere. Find the mean and variance of X .
3. If μ and σ^2 denote the mean and variance of X find $P(\mu - 2\sigma < X < \mu + 2\sigma)$ for the problem # 2.
4. If the variance of the random variable X exists, show that

$$E(X^2) \geq [E(X)]^2.$$

5. Let a random variable X of the continuous type have a PDF $f_X(x)$ whose graph is symmetric with respect to $x = c$. If the mean value of X exists, show that $E(X) = c$.
6. Find the moment generating function of the random variable X having the pdf $f_X(x) = 1/3$, $-1 < x < 2$, zero elsewhere.
7. Let X be a random variable such that $E[(X-b)^2]$ exists for all real b . Show that $E[(X-b)^2]$ is a minimum when $b = E(X)$.
8. Find the mean and the variance of the distribution that has the CDF

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{8} & \text{if } 0 \leq x < 2 \\ \frac{x^2}{16} & \text{if } 2 \leq x < 4 \\ 1 & \text{if } 4 \leq x. \end{cases}$$

9. Let X have the PDF $f_X(x) = \frac{1}{2}e^{-|x|}$, for $-\infty < x < \infty$. Find the MGF of X .
10. Let X be a random variable of the continuous type with PDF $f_X(x)$, which is positive provided $0 < x < b < \infty$, and is equal to zero elsewhere. Show that

$$E(X) = \int_0^\infty (1 - F_X(x))dx,$$

where $F_X(x)$ is the CDF of X .