

Lecture-37

On

INTRODUCTION TO ELECTRICAL ENGINEERING (ESO203)

- Synchronous Machine.

Losses in Synchronous Machine

- Armature winding copper loss
- Field copper loss
- Core loss, (Calculate from the open-circuit core-loss curve at a voltage equal to the internal voltage behind the resistance of the machine).
- Friction and windage loss
- Stray-load losses, (which account for the fact that the effective ac resistance of the armature is greater than the dc resistance because of the skin effect, and for the losses caused by the armature leakage flux.)
- Exciter loss, (only if the exciter is an integral component of the alternator, i.e., shares a common shaft or is permanently coupled)
- **Calculating overall loss in the machine, we can compute efficiency of machine**

Losses in Synchronous Machine (cont...)

Question-1: Consider a 1000-hp, 2300-V, wye-connected, three-phase, 60-Hz, 20-pole cylindrical-rotor synchronous motor having a synchronous reactance of 5.00 ohm/phase. All losses can be neglected.

(a) The motor is operated from an infinite bus supplying rated voltage and rated frequency, and its field excitation is adjusted so that the power factor is unity when the shaft load is such as to require an input of 750 kW. Compute the maximum torque that the motor can deliver, given that the shaft load is increased slowly with the field excitation held constant.

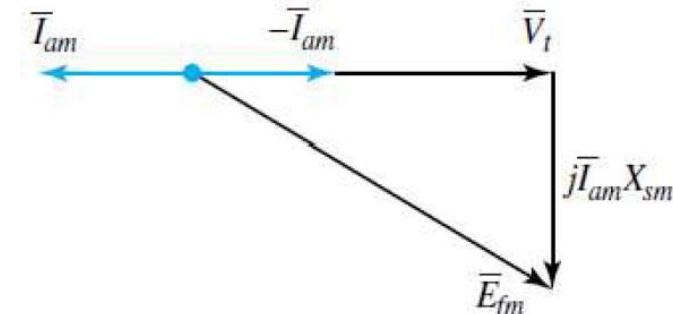
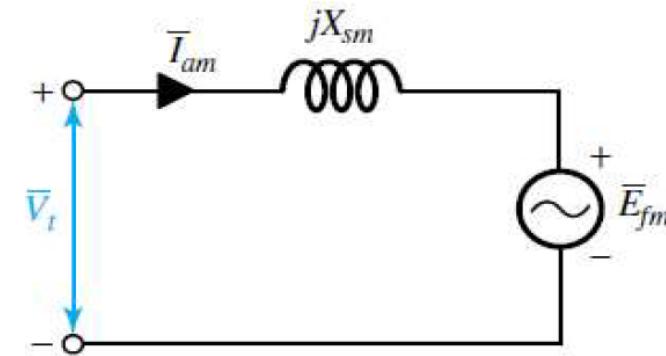
Solution:

$$\text{Rated voltage per phase } V_t = \frac{2300}{\sqrt{3}} = 1328 \text{ V line to neutral voltage}$$

Current per phase

$$I_{am} = \frac{750 \times 10^3}{\sqrt{3} \times 2300 \times 1.0} = 188.3 \text{ A}$$

$$I_{am} X_{sm} = 188.3 \times 5 = 941.5 \text{ V}$$



Losses in Synchronous Machine (cont...)

$$E_{fm} = \sqrt{1328^2 + 941.5^2} = 1628$$

$$P_{max} = \frac{E_{fm}V_t}{X_{sm}} = \frac{1628 \times 1328}{5} = 432.4 \text{ kW per phase, or } 1297.2 \text{ kW for three phases}$$

$$\text{Synchronous Speed} = \frac{120 \times 60}{20} = 360 \text{ r/min, or } 6 \text{ r/s}$$

$$\omega_s = 2\pi \times 60 = 37.7 \text{ rad/s}$$

$$T_{max} = \frac{1297.2 \times 10^3}{37.7} = 34.408 \text{ N.m}$$

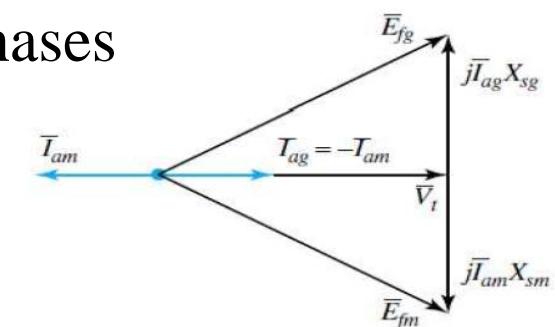
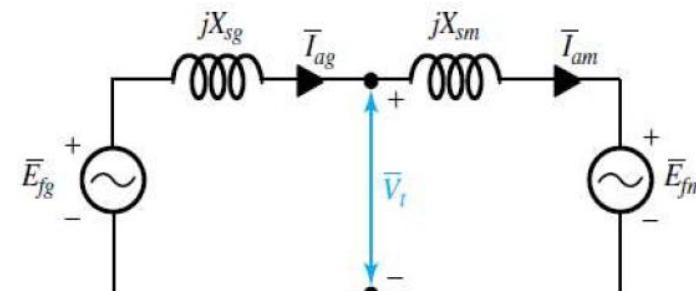
Losses in Synchronous Machine (cont...)

- Instead of an infinite bus as in part (a), let the power to the motor be supplied by a 1000-kVA, 2300-V, wye-connected, three-phase, 60-Hz synchronous generator whose synchronous reactance is also 5.00 ohm/phase. The generator is driven at rated speed, and the field excitations of the generator and motor are adjusted so that the motor absorbs 750kW at unity power factor and rated terminal voltage. If the field excitations of both machines are then held constant, and the mechanical load on the synchronous motor is gradually increased, compute the maximum motor torque under the conditions. Also determine the armature current, terminal voltage, and power factor at the terminals corresponding to this maximum load.

$$E_{fg} = E_{fm} = 1628 \text{ V}$$

$$P_{max} = \frac{E_{fg}E_{fm}}{X_{sg}+X_{sm}} = \frac{1628 \times 1628}{10} = 265 \text{ kW per phase or } 795 \text{ kW for three phases}$$

$$T_{max} = \frac{795 \times 10^3}{37.7} = 21,088 \text{ N.m}$$



Losses in Synchronous Machine (cont...)

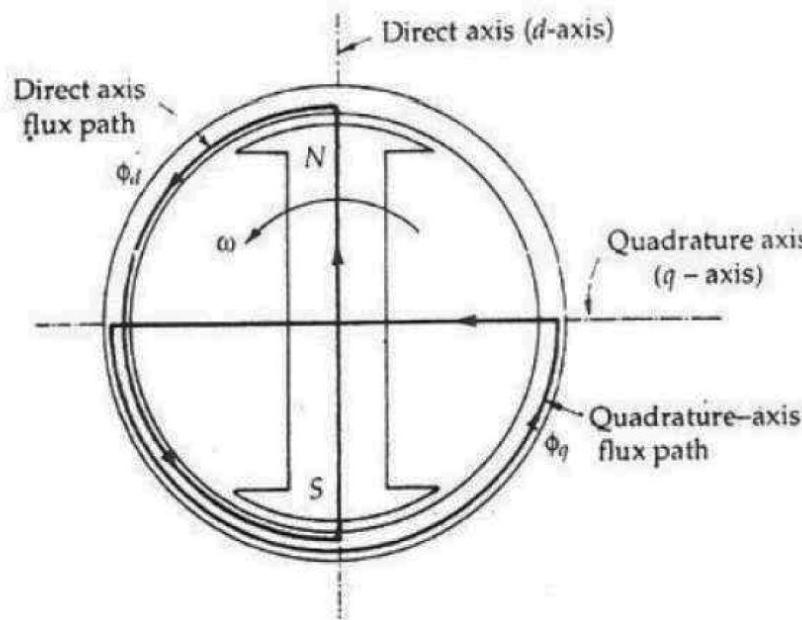
- If a load torque greater than this amount were applied to the motor shaft, synchronism would be lost; the motor would stall, the generator would tend to overspeed, and the circuit would be opened by circuit-breaker action.
- Corresponding to the maximum load, the angle between E_{fg} and E_{fm} is 90° .

$$V_t = \frac{E_{fg}}{\sqrt{2}} = \frac{1628}{\sqrt{2}} = 1151.3 \text{ V line-to-neutral, or } 1994 \text{ V line-to-line}$$

$$I_{ag} = \frac{1151.3}{5} = 230$$

Saliency and Saturation

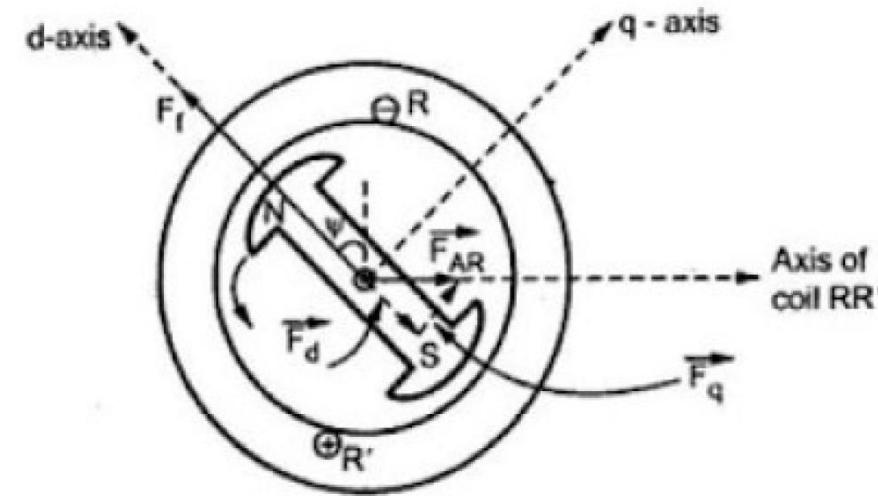
- Because of saliency, the reactance measured at the terminals of a salient pole synchronous machine, as opposed to a cylindrical-rotor machine (with uniform air gap), varies as a function of the rotor position.
- The effects of saliency are taken into account by the *two-reactance theory*.



- The direct axis or **d-axis** component is located along the axis of the rotor of the salient pole. The quadrature axis or **q-axis** component is located perpendicular to the axis of the rotor salient pole.
- The **direct axis flux** path involves two small air gaps and is the path of the **minimum reluctance**. The **q-axis path ϕ** has two large air gaps and is the path of the **maximum reluctance**.

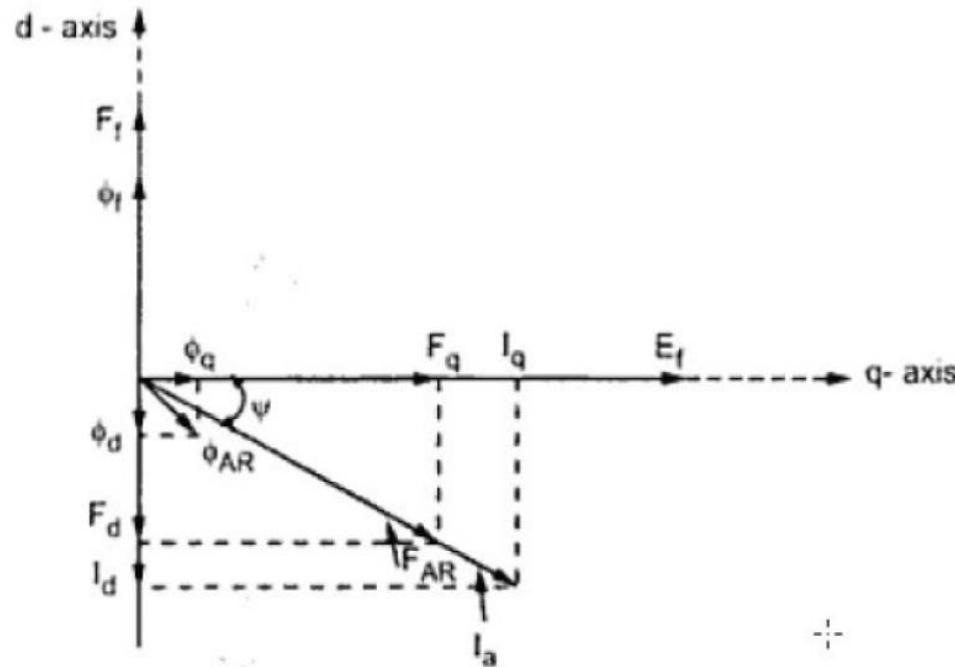
Saliency and Saturation (cont...)

- The instant chosen to show these positions is such that the current in phase R is maximum positive and is lagging E_f by angle Ψ .
- F_f is the m.m.f. wave produced by field winding, and it always acts along the direct axis. This m.m.f. is responsible for an excitation e.m.f. E_f which lags F_f by an angle 90° .
- When armature carries current, it produces its own m.m.f. wave F_{AR} . This can be resolved in two components. Similarly, armature current I_a also can be divided into two components, one along direct axis and along quadrature axis.



Saliency and Saturation (cont...)

- I_a lags E_f by angle Ψ .
- F_d is produced by I_d which is at 90° to E_f while F_q is produced by I_q which is in phase with E_f .



$$\varphi_d = \frac{F_d}{R_d}$$

$$\varphi_q = \frac{F_q}{R_q}$$

- The reluctance offered along the direct axis is less than the reluctance offered along the quadrature axis. Depending upon the reluctances offered along the direct and quadrature axis, the flux Φ_{AR} lags behind I_a .

Saliency and Saturation (cont...)

- E_{ad} is the direct axis component of the armature reaction voltage.
- E_{aq} is the quadrature axis component of the armature reaction voltage.
- Each armature reaction voltage is directly proportional to its stator current and lags behind by 90 degrees angles.

$$E_{ad} = -jX_{ad}I_d$$

$$E_{aq} = -jX_{aq}I_q$$

- X_{ad} is the armature reaction reactance in the direct axis per phase.
- X_{aq} is the armature reaction reactance in the quadrature axis per phase.
- The value of X_{ad} is always greater than X_{aq}

- The total voltage induced in the stator is the sum of EMF induced by the field excitation.

$$E' = E_f + E_{ad} + E_{aq}$$

$$E' = E_f - jX_{ad}I_d - jX_{aq}I_q$$

Saliency and Saturation (cont...)

$$E' = V + R_a I_a + j X_1 I_a \quad I_a = I_d + I_q$$

$$E_f = V + R_a I_a + j X_1 I_a + j X_{ad} I_d + j X_{aq} I_q$$

$$E_f = V + R_a (I_a + I_q) + j X_1 (I_d + I_q) + j X_{ad} I_d + j X_{aq} I_q$$

$$E_f = V + R_a (I_d + I_q) + j (X_1 + X_{ad}) I_d + j (X_{ad} + X_{aq}) I_q$$

Direct-axis synchronous reactance $X_d \triangleq X_1 + X_{ad}$

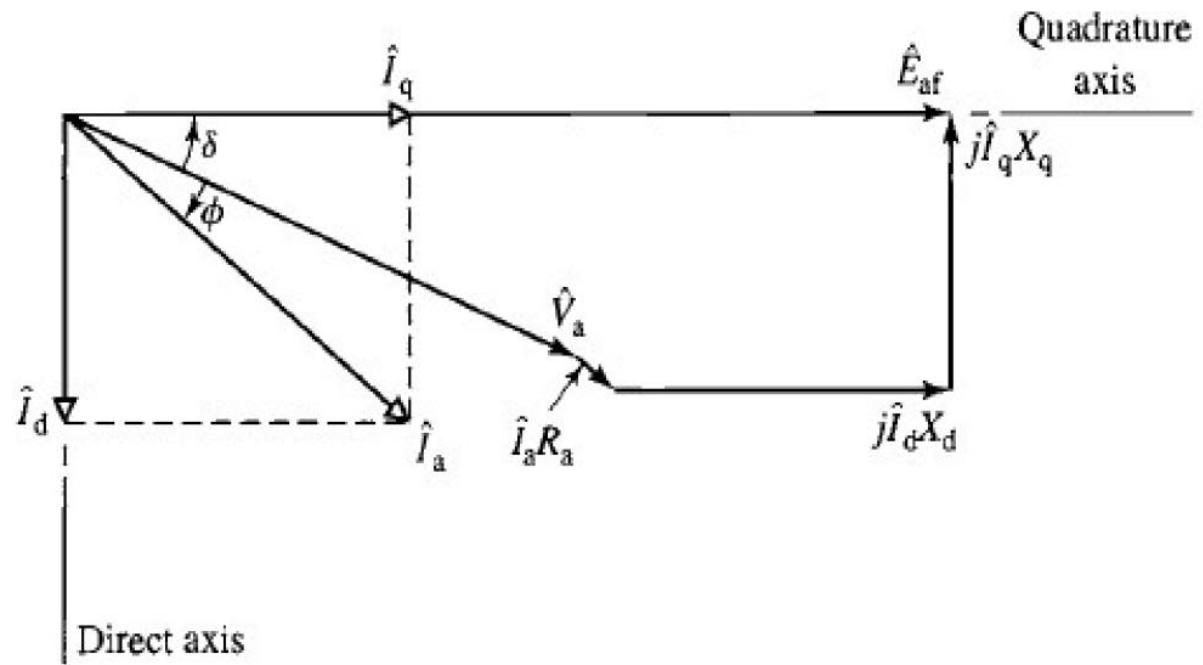
Quadrature-axis synchronous reactance $X_q \triangleq X_1 + X_{aq}$

$$E_f = V + R_a I_d + R_a I_q + j X_d I_d + j X_q I_q$$

$$E_f = V + R_a I_a + j X_d I_d + j X_q I_q$$

Saliency and Saturation (cont...)

$$E_f = V + R_a I_a + j X_d I_d + j X_q I_q$$



Saliency and Saturation (cont...)

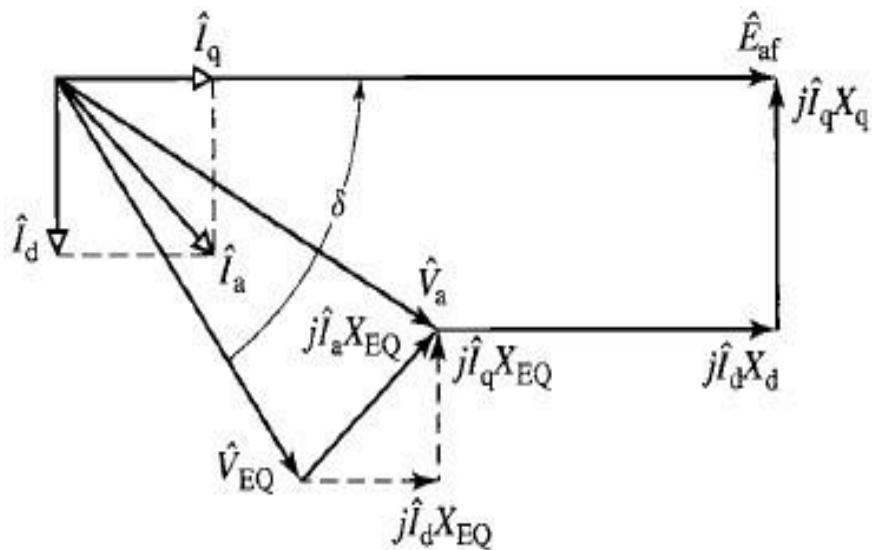
- A salient pole synchronous machine SM is connected to an infinite bus of voltage \hat{V}_{EQ} through a series impedance of reactance X_{EQ} . Resistance is neglected because it is usually small

$$X_{dT} = X_d + X_{EQ} \quad V_d = V_{EQ} \sin \delta$$

$$P = I_d V_d + I_q V_q = I_d V_{ED} \sin \delta + I_q V_{EQ} \cos \delta$$

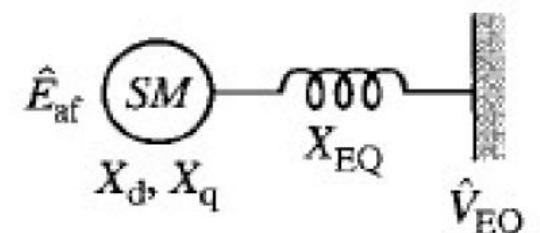
$$X_{qT} = X_q + X_{EQ} \quad V_q = V_{EQ} \cos \delta$$

$$I_d = \frac{E_{af} - V_{EQ} \cos \delta}{X_{dT}}$$

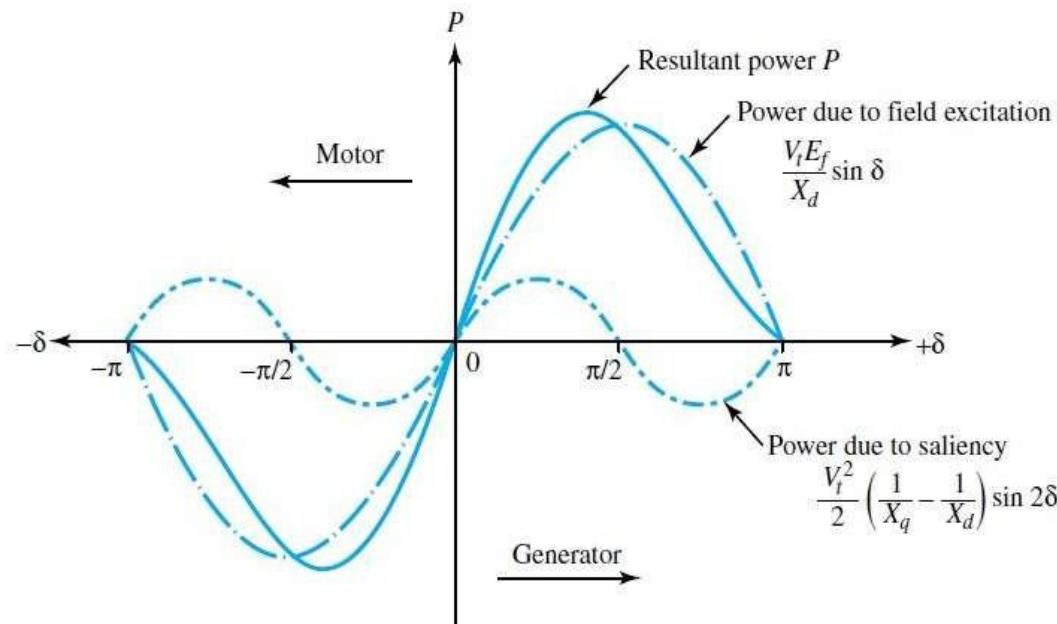


$$I_q = \frac{V_{ED} \sin \delta}{X_{qT}}$$

$$P = \frac{E_{af} V_{EQ}}{X_{dT}} \sin \delta + \frac{V_{EQ}^2 (X_{dT} - X_{qT})}{2 X_{dT} X_{qT}} \sin 2\delta$$



Saliency and Saturation (cont...)



power output occurs at the maximum $\delta < 90^\circ$

- The resulting power has two terms: one due to field excitation and the other due to saliency. The maximum torque that can be developed is somewhat greater because of the contribution due to saliency.

$$P = \frac{|E_f| |V_t|}{X_d} \sin \delta + \frac{V_t^2 (X_d - X_q)}{2X_d X_q} \sin 2\delta$$

The second term is quite small (about 10-20%) compared to the first term and is known as reluctance power.

Parallel Operation

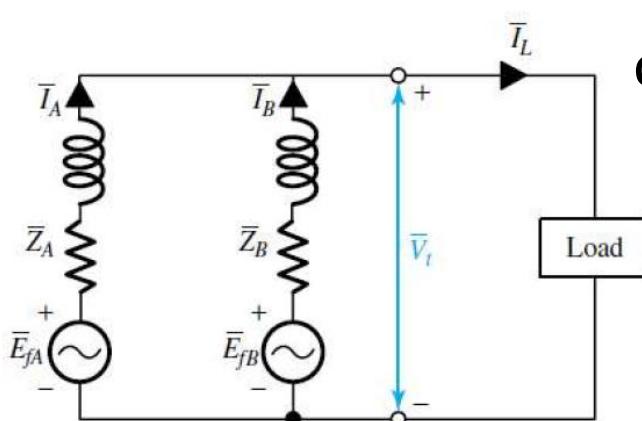
- We need to operate several alternators in parallel, interconnected by various transmission lines, in order to assure continuity of the power supply within prescribed limits of frequency and voltage.
- A generator can be paralleled with an infinite bus (or with another generator running at rated voltage and frequency supplying the load) by driving it at **synchronous speed** corresponding to the system frequency and adjusting its field excitation so that its **terminal voltage** equals that of the bus.
- **Synchronizing** requires the following conditions of incoming machine:
 1. Correct phase sequence.
 2. Phase voltages in phase with those of the system.
 3. Frequency almost exactly equal to that of the system.
 4. Machine terminal voltage approximately equal to the system voltage.
- After the machine has been synchronized and is part of the system, it can be made to take its share of the **active** and **reactive** power by appropriate adjustments.

Parallel Operation (cont...)

Two three-phase, 6.6-kV, wye-connected synchronous generators, operating in parallel, supply a load of 3000 kW at 0.8 power factor lagging. The synchronous impedance per phase of machine A is $0.5+j10$, and of machine B it is $0.4+j12$. The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor, and the governors are set such that the load is shared equally between the two machines. Determine the armature current, power factor, excitation voltage, and power angle of each machine.

$$\bar{I}_L = \frac{3000}{\sqrt{3} \times 6.6 \times 0.8} \angle -\cos^{-1} 0.8 = 328(0.8 - j0.6) = 262.4 - j196.8 \text{ A}$$

For machine A;



$$\cos \phi_A = \frac{1500}{\sqrt{3} \times 6.6 \times 150} = 0.875 \text{ lagging}; \phi_A = 29^\circ; \sin \phi_A = 0.485$$

$$\bar{I}_A = 150(0.874 - j0.485) = 131.1 - j72.75 \text{ A}$$

$$\bar{I}_B = \bar{I}_L - \bar{I}_A = 131.1 - j72.75 - (131.1 - j72.75) = 180.6 \angle -\cos \frac{(131.1)}{(180.6)}$$

Parallel Operation (cont...)

For machine B;

$$\cos \phi_B = \frac{131.3}{180.6} = 0.726 \text{ lagging}$$

$$\bar{E}_{fA} = \bar{V}_t + \bar{I}_A \bar{Z}_A = (6.6/\sqrt{3}) + (131.1 - j72.75)(0.5 + j10) \times 10^{-3} kV \text{ per phase}$$

$$\text{Power angle } \delta_A = \tan^{-1} \left(\frac{1.27}{4.6} \right) = 15.4^\circ$$

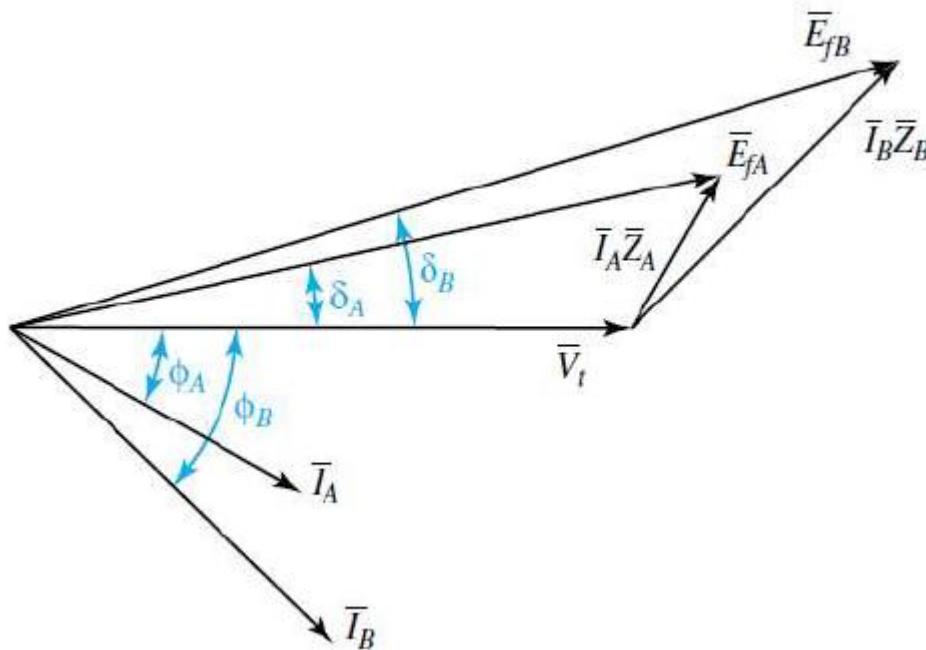
The line-to-line excitation voltage for machine A is $= \sqrt{3} \sqrt{4.6^2 + 127^2} = 8.26 \text{kV}$

$$\begin{aligned} \bar{E}_{fB} &= \bar{V}_t + \bar{I}_B \bar{Z}_B = (6.6/\sqrt{3}) + (131.1 - j124)(0.4 + j12) \times 10^{-3} kV \text{ per phase} \\ &= 5.35 + j1.52 \text{ kV per phase} \end{aligned}$$

Parallel Operation (cont...)

$$\text{Power angle } \delta_B = \tan^{-1} \left(\frac{1.52}{5.35} \right) = 15.9^\circ$$

The line-to-line excitation voltage for machine B is $= \sqrt{3} \sqrt{5.35^2 + 1.52^2} = 9.6\text{kV}$



Synchronous Machine Parameters

- The parameters of the equivalent circuit of a synchronous machine need to be determined to completely describe the machine characteristics and voltage regulation.
- Following tests can be performed to determine the parameters:
 - DC test.
 - Open-circuit test.
 - Short-circuit test.

Synchronous Machine Parameters (Cont...)

□ DC Test:

- DC test is done to determine the armature resistance.
- An adjustable DC source is connected across any 2 stator terminals and a small voltage is applied. The DC resistance is given by the ratio of voltage and current.

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

- To account for the skin effect, DC resistance is usually multiplied by a factor of 1.2.

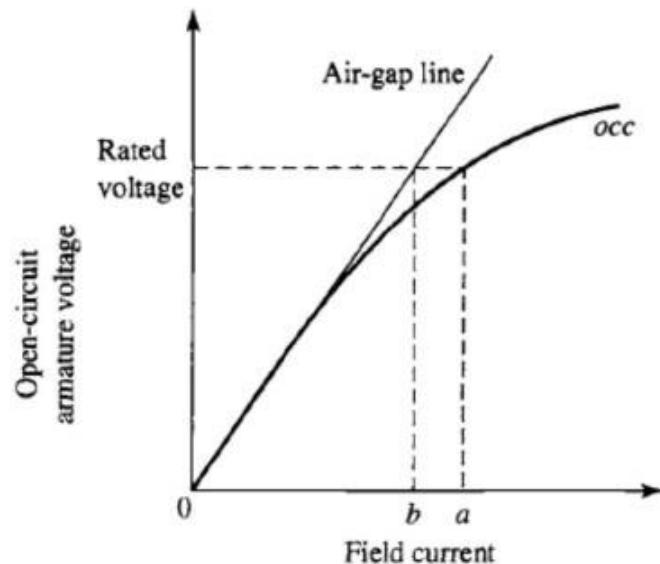
$$R_a = R_{AC} \approx 1.2 \times R_{DC}$$

- Armature resistance can be computed from the above if we know the type of connection (star or delta).

Synchronous Machine Parameters (Cont...)

□ Open Circuit Test:

- For open circuit test, the machine is driven at rated speed by its prime mover, and the field current is adjusted to obtain rated voltage at no-load.
- The quantities measured are: V_{OC} (O.C. voltage) and I_f, oc (field current).
- The open circuit characteristics has the following shape: (note that the actual characteristics is not a straight line). The straight-line tangent to the initial part of the curve is called air-gap line (having constant permeability).



Open-circuit characteristics
of a Synchronous machine

Synchronous Machine Parameters (Cont...)

□ Short Circuit Test:

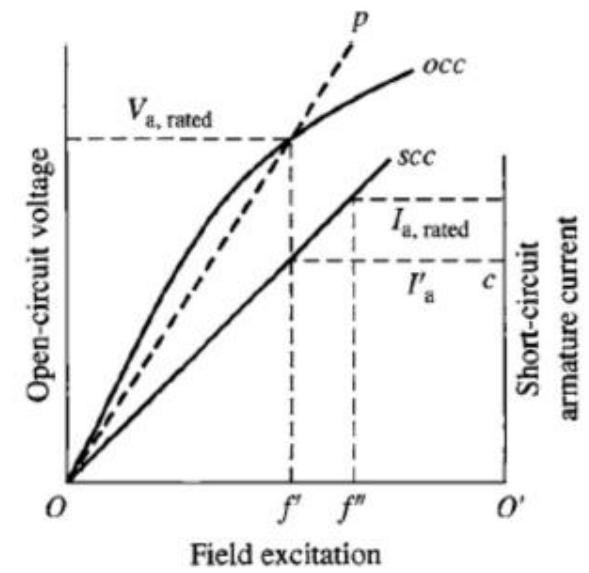
- The three stator leads are short-circuited, machine is driven at rated speed by its prime mover, and the field current is slowly increased to the value it had in the open circuit test.
- Measured quantities: I_{SC} (S.C. armature current) and $I_{f,OC}$ (field current).
- The synchronous impedance is given by (refer to the graph below)

$$Z_s = \frac{V_{OC}}{I_{SC}}$$

- Using armature resistance from the DC test, synchronous reactance can be found from:

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

$V_{a,rated} = V_{OC}$
 $I'_a = I_{SC}$



- If $X_s \gg R_a$

$$X_s = \frac{V_{OC}}{I_{SC}}$$

Finding Synchronous impedance
from OC and SC characteristics

Synchronous Machine Parameters (Cont...)

□ Short Circuit Ratio:

- The short circuit ratio (SCR) of a synchronous generator is the ratio of the field current required to obtain rated voltage at no-load, to the field current required to obtain rated armature current when the armature is short-circuited.

$$SCR = \frac{I_{f,\text{for rated voltage at no load}}}{I_{f,\text{for rated current with armature shorted}}}$$

- The short circuit ratio (SCR) can also be expressed as the reciprocal of the per unit synchronous reactance.

$$SCR = \frac{1}{X_{s,pu}}$$

- SCR affects the machine's sensitivity to loading, or voltage regulation.

Synchronous Machine Parameters (Cont...)

□ Example: Machine Parameters

- The short-circuit, open-circuit, and DC test data for a 3-phase, Y-connected, 50 kVA, 240 V, 60 Hz synchronous machine are:

$$V_{oc,line} = 240V \quad I_{sc,line} = 115.65A \quad V_{DC} = 10.35V \quad I_{DC} = 52.80A$$

Determine (a) Equivalent armature resistance, (b) synchronous reactance, (c) short-circuit ratio.

• Solution:

$$(a) \quad R_{DC} = \frac{V_{DC}}{I_{DC}} = \frac{10.35}{52.80} = 0.1960\Omega$$

Per-phase DC resistances are given by (because of Y-connection),

$$R_Y = \frac{R_{DC}}{2} = \frac{0.1960}{2} = 0.098\Omega$$

Armature Resistance (including skin effect)

$$R_a = 1.2 \times R_Y = 1.2 \times 0.098 = 0.1178\Omega$$

Synchronous Machine Parameters (Cont...)

(b) Synchronous Impedance, $Z_s = \frac{V_{OC,phase}}{I_{SC,phase}} = \frac{240}{\sqrt{3} \times 115.65} = 1.1981\Omega$

Therefore Synchronous Reactance is given by

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{1.1981^2 - 0.1176^2} = 1.1923\Omega$$

(c) Base Phase Voltage, $V_{\phi,base} = \frac{240}{\sqrt{3}} V$

Base Phase Current, $I_{\phi,base} = \frac{50000}{3} \times \frac{1}{\frac{240}{\sqrt{3}}} = \frac{50000}{240\sqrt{3}} A$

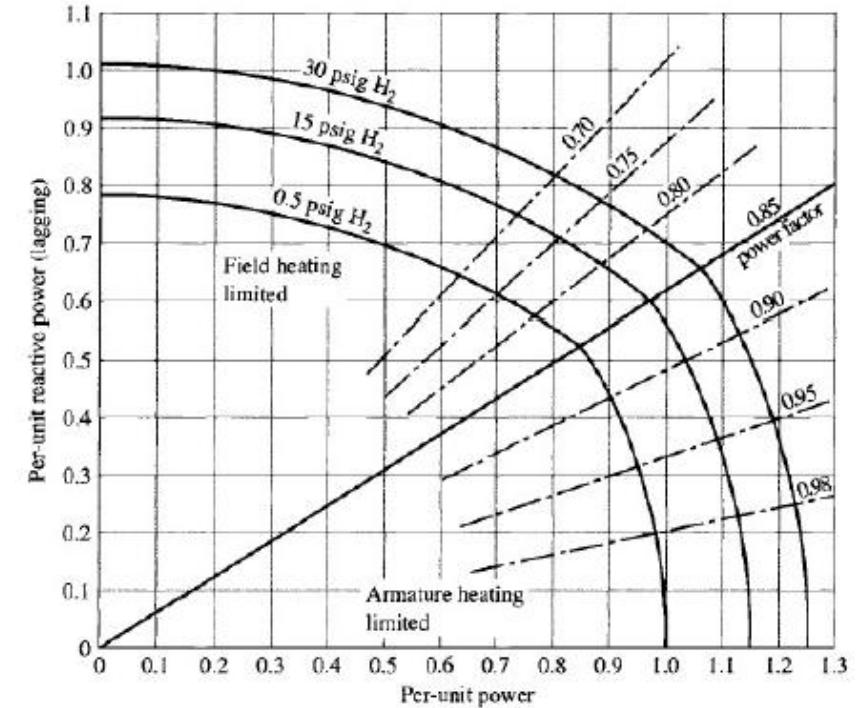
Base Impedance, $Z_{\phi,base} = \frac{V_{\phi,base}}{I_{\phi,base}} = \frac{240}{\sqrt{3}} \times \frac{240\sqrt{3}}{50000} = 1.152\Omega$

Therefore, $SCR = \frac{1}{X_{S,pu}} = \frac{1}{X_s} = \frac{1.1520}{1.1923} = 0.966$

Synchronous Machine Parameters (Cont...)

❑ Capability Curve:

- Synchronous generators are usually rated in terms of the maximum apparent power (kVA or MVA) load at a specific voltage and power factor (often 80, 85, or 90 percent lagging), which they can carry continuously without overheating.
- Capability curves provide a valuable guide both to power system planners and to operators. As system planners consider modifications and additions to power systems, they can see if the various existing or proposed generators can safely supply their required loadings.
- Similarly, power system operators can quickly see whether individual generators can safely respond to changes in system loadings which occur during the normal course of system operation.



Capability curves of an 0.85 power factor,
0.80 short-circuit ratio,
hydrogen-cooled turbine
generator. Base MVA is rated
MVA at 0.5 psig hydrogen.

Synchronous Machine Parameters (Cont...)

□ Drawing Capability Curve:

- Condition of constant terminal voltage and maximum allowable armature current:

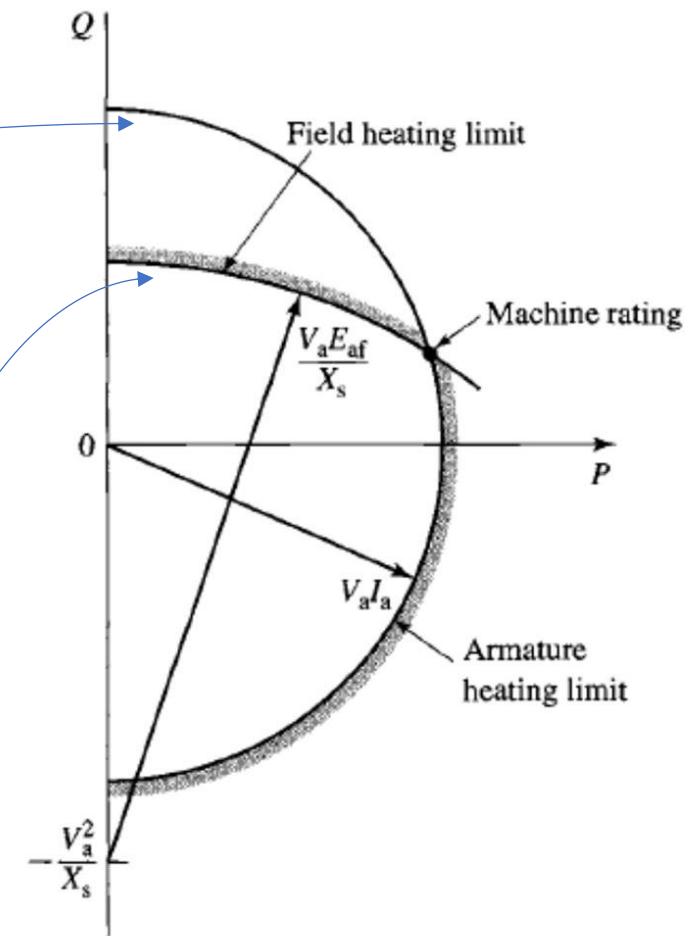
$$P^2 + Q^2 = (V_a I_a)^2$$

- When the field current is limited to its maximum value, the field voltage E_{af} is constant and so is $\frac{V_a E_a}{X_s}$
- From the output equations

$$P = \frac{V_a E_a}{X_s} \sin \delta \quad Q = \frac{V_a E_a}{X_s} \cos \delta - \frac{V_a^2}{X_s}$$

Hence $P^2 + \left(Q + \frac{V_a^2}{X_s}\right)^2 = \left(\frac{V_a E_a}{X_s}\right)^2$

- Machine rating is commonly specified as the intersection of the armature current limiting and field current limiting curves.

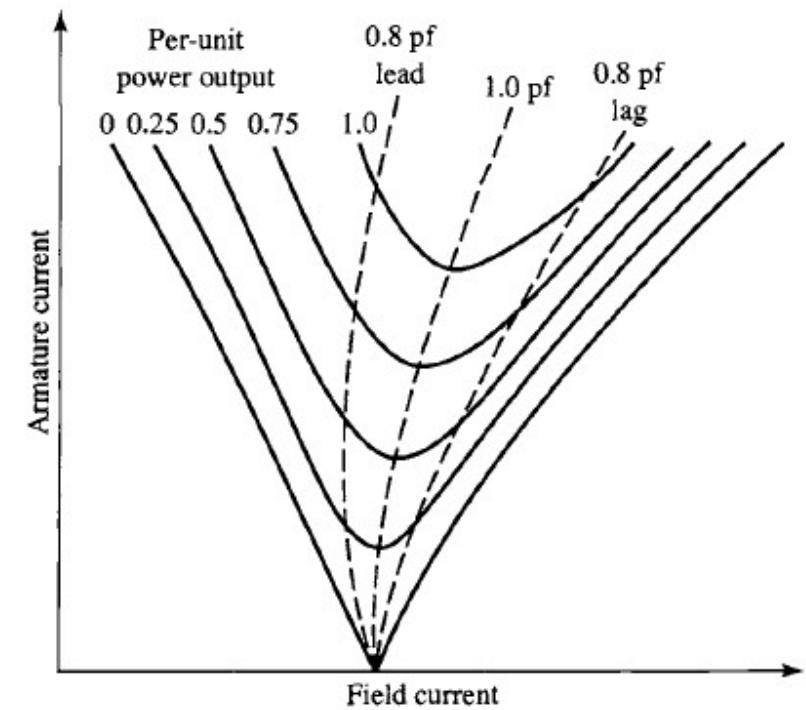


Construction of synchronous generator capability curve

Synchronous Machine Parameters (Cont...)

□ V-Curves:

- For a given real-power loading, the power factor at which a synchronous machine operates, and hence its armature current, can be controlled by adjusting its field excitation.
- The curve showing the relation between armature current and field current at a constant terminal voltage and constant real power is known as a V-curve, because of its characteristic shape.
- For constant power output, the armature current is minimum at unity power factor and increases as the power factor decreases.



Typical form of synchronous generator
V-curve

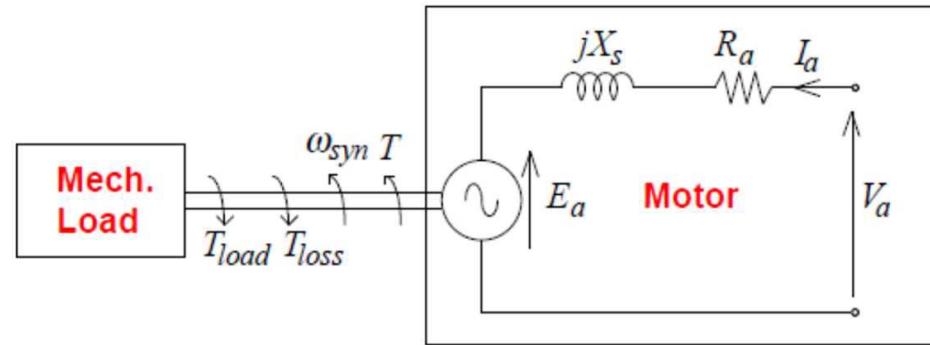
Synchronous Machine Parameters (Cont...)

□ Synchronous Motor:

- The field (rotor) current of the motor produces a magnetic field B_R .
- A 3-phase voltage is applied to the armature (stator), which produces a rotating magnetic field B_S .
- Therefore, there are 2 magnetic fields in the machine, and the rotor field tries to line-up with the stator field. Since the stator field is rotating, the rotor field constantly tries to catch up with it (but never really succeeds!!).
- The induced torque is given by,
$$T = k \mathbf{B}_R \times \mathbf{B}_S$$
- The basic principle of synchronous motor is that the rotor always chases the Rotating magnetic field produced by the stator.

Synchronous Machine Parameters (Cont...)

□ Synchronous Motor Operation:



Motor operation of the synchronous machine

- The rotor field is synchronized (locks in) with the rotating stator field, which drags the rotor along (consequently, rotation occurs).
- Electrical energy is consumed to produce mechanical energy, and the mechanical torque T balances the load torque T_{load} and the loss torque T_{loss} due to friction and windage:

$$T = T_{load} + T_{loss}$$

Synchronous Machine Parameters (Cont...)

□ Output Torque:

- Output power of the motor can be given by (similar to generators as before),

$$P = \frac{3V_a E_a}{X_s} \sin \delta \quad (\text{Considering 3 phases})$$

- The output torque is given by,

$$T = \frac{3V_a E_a}{\omega_m X_s} \sin \delta$$

Where ω_m is the mechanical (Synchronous) speed of the rotor.

- Maximum torque is given by

$$T_{max} = \frac{3V_a E_a}{\omega_m X_s}$$

Synchronous Machine Parameters (Cont...)

□ Example:

- A 480-V, 60 Hz, four-pole synchronous motor draws 50 A from the line at unity power factor and full load. Assuming that the motor is lossless. What is the output torque of this motor?
- Solution:
- If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{in} = \sqrt{3}V_T I_L \cos \theta = \sqrt{3} \times 480 \times 50 \times 1.0 = 41.6kW$$

- Rotational speed in mechanical radians per second is given by,

$$\omega_m = \left(\frac{2}{p}\right) \omega_e = \left(\frac{2}{p}\right) 2\pi f_e = \left(\frac{2}{4}\right) 2\pi \times 60 = 188.4 \text{ mech. rad./s}$$

- The output torque is given by

$$T_{out} = \frac{P_{out}}{\omega_m} = \frac{41.6kW}{188.4 \text{ mech. rad./s}} = 221 \text{ Nm}$$

Synchronous Machine Parameters (Cont...)

