Duality Theorem :-

In Boolean algebra, Theorem States that for any valid Boolean equation, a dual equation Can be derived by swapping 'AND' and 'OR' operators and exchanging 0s & 1's.

[importance:- Simplifying complex Boolean expressions.]
- Circuit design optimization.

OR $\xleftrightarrow{\text{interchange}}$ AND means '+' $\longleftrightarrow$ '·'

0 $\longleftrightarrow$ 1

NOT $\xleftrightarrow{\text{same}}$ NOT

XOR $\longleftrightarrow$ XNOR

{But variable does not change. like:- $A, \bar{A}$ → as it is.}

Ex: EXOR → $\bar{A}\cdot B + A\cdot\bar{B}$

$\Rightarrow (\bar{A}+B)\cdot(A+\bar{B})$

$\Rightarrow \cancel{A\bar{A}} + \bar{A}\bar{B} + BA + \cancel{B\bar{B}} \rightarrow \bar{A}\bar{B} + AB$

Some examples of Dual:-

(i) $AB\bar{C} + \bar{A}BC + ABC$

$= (A+B+\bar{C})\cdot(\bar{A}+B+C)\cdot(A+B+C)$

(ii) $XYZ + \bar{X}Y\bar{Z} + \bar{Y}Z = 1$ interchange to 0

$(X+Y+Z)\cdot(\bar{X}+Y+\bar{Z})\cdot(\bar{Y}+Z) = 0$

$= A\cdot A + AC + AB + BC$

$= A(A+C) + B(A+C) \Rightarrow \underline{(A+C)(B+A)}$ H.P.

Demorgan's Theorems:-

(i) Complement of a product is equal to the sum of the complements. If variables are A & B.

$$\boxed{\overline{AB} = \bar{A} + \bar{B}}$$

(ii) Complement of a sum is equal to the product of the complements.

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

Minimization (Simplification) of Boolean Expression using algebric method:-

(1) $AB + BC + \bar{B}C = AB + C$ $\quad \because B+\bar{B}=1$

L.H.S

$AB + BC + \bar{B}C \Rightarrow AB + C(B+\bar{B})$

$\Rightarrow AB + C \cdot 1$

$\Rightarrow AB + C$

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(2) $\bar{A} \cdot B + A \cdot B + \bar{A} \cdot \bar{B}$ — Simplify this expression.

$\Rightarrow (\bar{A}+A) \cdot B + \bar{A} \cdot \bar{B}$ $\quad \because (A+\bar{A}B = A+B)$

$\Rightarrow 1 \cdot B + \bar{A}\bar{B}$

$\Rightarrow B + \bar{A}$

③ Complement the expression $\bar{A}B + C\bar{D}$. (30

An

$$\overline{\bar{A}B + C\bar{D}} \Rightarrow (\overline{\bar{A}B}) \cdot (\overline{C\bar{D}})$$

$$\Rightarrow (\bar{\bar{A}} + \bar{B}) \cdot (\bar{C} + \bar{\bar{D}})$$

$$\Rightarrow (A + \bar{B}) \cdot (\bar{C} + D)$$

④ $AB + \overline{AC} + A\bar{B}C(AB + C)$

$$\Rightarrow AB + \overline{AC} + \cancel{A\bar{B}C \cdot AB}^{\,0} + A\bar{B}\underline{C \cdot C} \to C$$

$$\Rightarrow AB + \overline{AC} + A\bar{B}C$$

$\because B \cdot \bar{B} = 0$

$\because C \cdot C = C$

$$\Rightarrow AB + \bar{A} + \bar{C} + A\bar{B}C$$

$\because A + \bar{A}B = A + B$

$$\Rightarrow \bar{A} + AB + \bar{C} + C\bar{B}$$

$B + \bar{B} = 1$

$$\Rightarrow \bar{A} + B + \bar{C} + \bar{B}$$

$$\Rightarrow \bar{A} + \bar{C} + 1$$

$$\Rightarrow 1$$

Sum of Products & Product of Sums :-

1 SOP :- The logical sum of two or more logical product terms, is called a sum of product expression.

- It is OR operation of AND operated variable.

(i) $Y = AB + BC + AC$

(ii) $Y = AB + \bar{A}C + BC$

minterms :- A product term containing all the K-variables of the function in either complemented or uncomplemented form is called a minterm.

→ Two variable function has four possible combinations, $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$ & AB. These are minterms.

→ For 3-binary input, there are 8-minterms

→ In minterms, a variable appears either in uncomplemented form, if it possesses a value of 1 in the corresponding combination, or in complemented form, if it contains the value 0.

minterm Table

A	B	C	minterm
0	0	0	$\bar{A}\bar{B}\bar{C}$
0	0	1	$\bar{A}\bar{B}C$
0	1	0	$\bar{A}B\bar{C}$
0	1	1	$\bar{A}BC$
1	0	0	$A\bar{B}\bar{C}$
1	0	1	$A\bar{B}C$
1	1	0	$AB\bar{C}$
1	1	1	ABC

Canonical SOP expression:- SOP need not contain all variables but in Canonical form, each product term contains all the variables either in complemented or uncomplemented form.

How to express ^Canonical SOP :-

$$Y = \sum_m (0, 5, 6)$$

$$= m_0 + m_5 + m_6$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

- Check for variable that are missing in each product which is not a minterm. Multiply the product by $(X+\bar{X})$, for each variable X that is missing
- Multiply all products & omit the reduntant terms.

Ex: Obtain the Canonical sum of product form of the function $Y(A \cdot B) = A + B$ → A missing → B missing

Sol

$$\Rightarrow A+B = A \cdot 1 + B \cdot 1 \qquad \because B+\bar{B} \rightarrow 1$$

$$= A(B+\bar{B}) + B(A+\bar{A})$$

$$= AB + A\bar{B} + BA + B\bar{A} \qquad A+A \rightarrow A$$

$$= \underset{11}{AB} + \underset{10}{A\bar{B}} + \underset{10}{B\bar{A}}$$

$$Y = \sum m(3, 2, 12) \Rightarrow m_2 + m_3 + m_{12}$$

Ex: $Y(A,B,C) = A + BC$

$$Y = A + BC = A(B+\bar{B})(C+\bar{C}) + BC(A+\bar{A})$$

$$= (AB + A\bar{B})(C+\bar{C}) + BCA + BC\bar{A}$$

$$= \underline{ABC} + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \underline{BCA} + BC\bar{A}$$

$$= \underset{111}{ABC} + \underset{110}{AB\bar{C}} + \underset{101}{A\bar{B}C} + \underset{100}{A\bar{B}\bar{C}} + \underset{011}{\bar{A}BC}$$

$(\because A + A \rightarrow A$

$$Y = \sum m(7, 6, 5, 4, 3)$$

$$= m_3 + m_4 + m_5 + m_6 + m_7$$

Ex: $Y = AB + ACD$

$$Y = AB(C+\bar{C})(D+\bar{D}) + A(B+\bar{B})CD$$

$$= (ABC + AB\bar{C})(D+\bar{D}) + ABCD + A\bar{B}CD$$

$$= \underline{ABCD} + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + \underline{ABCD}$$

$$+ A\bar{B}CD$$

$\because A + A \rightarrow A$

$$= \underset{1111}{ABCD} + \underset{1110}{ABC\bar{D}} + \underset{1101}{AB\bar{C}D} + \underset{1100}{AB\bar{C}\bar{D}} + \underset{1011}{A\bar{B}CD}$$

$$Y = \sum m(15, 14, 13, 12, 11)$$

$$Y = m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$$

Ex: Canonical SOP form function $Y = \sum m(2, 3)$.

A 2 variable has four possible combination

	A	B	
0	0	0	$\rightarrow \bar{A}\bar{B}$
1	0	1	$\rightarrow \bar{A}B$
2	1	0	$\rightarrow A\bar{B}$
3	1	1	$\rightarrow AB$

- According to question, function has two two variable then expression will be

$$Y = A\bar{B} + AB$$

$$Y = A(B + \bar{B})$$

$\because B + \bar{B} = 1$

$$\boxed{Y = A}$$

Ans

POS : (Product of Sum) ⟶ A product of sums expression is a logical product of two or more logical sum terms. It is basically an AND operation of OR operated variables, such as:-

(i) $Y = (A+B)(B+C)(C+\bar{A})$

(ii) $Y = (A+B+C)(A+\bar{C})$

- **Maxterm:-** A sum term containing all the K-variables of the function in either complemented or uncomplemented form is called a Maxterm.

→ 2 variable function has 4-possible combinations

→ 3-binary i/p variable function has 8-maxterms like -

$\{\Pi m\}$; $0 \to A$, $1 \to \bar{A}$

A	B	C	Maxterm	Represented By
0	0	0	$A+B+C$	M_0
0	0	1	$A+B+\bar{C}$	M_1
0	1	0	$A+\bar{B}+C$	M_2
0	1	1	$A+\bar{B}+\bar{C}$	M_3
1	0	0	$\bar{A}+B+C$	M_4
1	0	1	$\bar{A}+B+\bar{C}$	M_5
1	1	0	$\bar{A}+\bar{B}+C$	M_6
1	1	1	$\bar{A}+\bar{B}+\bar{C}$	M_7

- In the maxterm, a variable appears either in uncomplemented form if it possesses the value '0' in the corresponding combination or in complemented form if it contains the value 1.

- **Canonical POS:-**

POS need not contain all variables/literals but in Canonical form, each sum terms contain all variables either in complemented or uncomplemented form.

- Check for variables that are missing in each sum, which is not a maxterm. Add $(X\bar{X})$ to the sum term, for each variable X that are missing.
- Expand expression using distributive property & eliminate redundant terms.

Ex: Obtain the canonical product of sum expression of
$$Y(ABC) = (A+\bar{B})(B+C)(A+\bar{C})$$

Sol
$$Y(ABC) = (A+\bar{B}+0)(B+C+0)(A+\bar{C}+0)$$
$$= (A+\bar{B}+C\bar{C})(B+C+A\bar{A})(A+\bar{C}+B\bar{B})$$

$\because$ use distributive property, ($\because A+BC=(A+B)(A+C)$)

$$= (A+\bar{B}+C)\underline{(A+\bar{B}+\bar{C})}(B+C+A)(B+C+\bar{A})(A+\bar{C}+B)\underline{(A+\bar{C}+\bar{B})}$$

$$Y(ABC) = \underset{010}{(A+\bar{B}+C)}\ \underset{011}{(A+\bar{B}+\bar{C})}\ \underset{000}{(A+B+C)}\ \underset{100}{(\bar{A}+B+C)}\ \underset{001}{(A+B+\bar{C})}$$

$\hookrightarrow$ This is maxterm canonical form or canonical POS expression. ($\because A\cdot A \to A$)

$$Y = M_2\,M_3\,M_0\,M_4\,M_1$$
$$Y = \Pi_m(0,1,2,3,4)$$

Ex2:- Express the function $Y = A+\bar{B}C$ in
(a) Canonical SOP (b) Canonical POS.

(a) **Canonical SOP:-**
$$Y = A+\bar{B}C$$
$$= A(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$$
$$= (AB+A\bar{B})(C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$$
$$= \underset{111}{ABC} + \underset{110}{AB\bar{C}} + \underset{101}{A\bar{B}C} + \underset{100}{A\bar{B}\bar{C}} + A\bar{B}C + \underset{001}{\bar{A}\bar{B}C} \qquad (A+A \to A)$$

$$Y = m_7 + m_6 + m_5 + m_4 + m_1$$

$$Y = \Sigma_m (1, 4, 5, 6, 7)$$

(b) Canonical POS:-

$$Y = A + \bar{B}C$$

$$Y = (A+\bar{B})(A+C)$$

{ ∵ Use distributive property $A+BC = (A+B)(A+C)$

$$Y = (A+\bar{B}+C\bar{C})(A+B\bar{B}+C)$$

} again use distributive property

$$Y = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(A+\bar{B}+C)$$

{ ∵ $A \cdot A \rightarrow A$ }

$$Y = \underset{0\ 1\ 0}{(A+\bar{B}+C)}\ \underset{0\ 1\ 1}{(A+\bar{B}+\bar{C})}\ \underset{0\ 0\ 0}{(A+B+C)}$$

$$Y = M_2 M_3 M_0$$

$$Y = \Pi_m (0, 2, 3)$$

Q3: $Y = (A+\bar{B})(\bar{C}+\bar{D})(\bar{B}+\bar{C})$ ∵ Distributive property

$$Y = (A+\bar{B}+C\bar{C}+D\bar{D})(\bar{C}+\bar{D}+A\bar{A}+B\bar{B})(\bar{B}+\bar{C}+D\bar{D}+A\bar{A})$$

$$Y = (A+\bar{B}+C)(A+\bar{B}+\bar{C})+(D\bar{D}) \cdot (\bar{C}+\bar{D}+A)(\bar{C}+\bar{D}+\bar{A})+(B\bar{B})$$
$$\cdot (\bar{B}+\bar{C}+D)(\bar{B}+\bar{C}+\bar{D})+(A\bar{A})$$

$$Y = (A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$$
$$(\bar{C}+\bar{D}+A+B)(\bar{C}+\bar{D}+A+\bar{B})(\bar{C}+\bar{B}+\bar{A}+B)(\bar{C}+\bar{D}+\bar{A}+\bar{B})$$
$$(\bar{B}+\bar{C}+D+A)(\bar{B}+\bar{C}+\bar{A}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$
$$(\bar{A}+B+\bar{C}+\bar{D})$$

✱ Deriving SOP expression from a T.T. :-

- SOP expression for Boolean function can be derived from its T. T. by summing (OR operation) the product terms that correspond to the combinations containing a function value 1.

- 3-i/p function Y consider T.T. :- Assume $Y = (2, 3, 5, 7)$

I/P A	B	C	Y- O/P (assume)	Product term (SOP)	Sum term (POS)
0	0	0	0		$A+B+C$
0	0	1	0		$A+B+\bar{C}$
0	1	0	1	$\bar{A}B\bar{C}$	
0	1	1	1	$\bar{A}BC$	
1	0	0	0		$\bar{A}+B+C$
1	0	1	1	$A\bar{B}C$	
1	1	0	0		$\bar{A}+\bar{B}+C$
1	1	1	1	ABC	

- Now, the final SOP expression for the O/P Y is obtained by summing (OR operation of) the four product terms as follows.

$$Y = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

- Give a product term for each I/P combination in the table, containing an O/P value of 1.
- Each product term contain its I/P variables in either complemented or uncomplemented form.
- If I/P variable $\rightarrow 0$, it appears in complemented form.

  If I/P variable $\rightarrow 1$, $\rightarrow$ it appears in uncomplemented form
- All the product terms are OR operated together in order to produce the final SOP expression of the O/P.

Deriving POS expression from a Truth Table:-

- POS expression for a Boolean (Switching) function can also be obtained from a truth table by AND operation of the sum terms corresponding to the combinations for which the function assumes the value 0.

  0 value appear $\rightarrow$ then it is represented by (A) uncomplemented form

  1 value appear $\rightarrow$ O/P value represented by complemented form $\bar{A}$

Now POS expression for o/p Y is obtained by the AND operation of the four sum terms.

$$Y = (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

- The POS expression for a Boolean function can also be obtained from its SOP expression using $\bar{\bar{Y}} = Y$

Consider a function

$$Y = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

$$Y = \bar{\bar{Y}} = \overline{\overline{\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC}}$$

The complement $\bar{Y}$ can be obtained by the OR operation of the minterms which are not available in Y. Therefore –

$$\bar{Y} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$$

$$Y = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}}$$

$$= (\overline{\bar{A}\bar{B}\bar{C}})(\overline{\bar{A}\bar{B}C})(\overline{A\bar{B}\bar{C}})(\overline{AB\bar{C}})$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

Karnaugh Map (K-map) :- K-map is a graphical method which is used to simplify a logic equation or to convert a truth table to its corresponding logic circuit. It is modified form of a Truth table.

- Karnaugh map is a minimization techniques to simplify the boolean equations with up to four input variables.
- In an n-variable K-map, there are 2^n cells.

- **2-variable K-map :-**

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	00 m_0	01 m_1
A 1	10 m_2	11 m_3

- **3-variable K-Map :-**

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	000 m_0	001 m_1	011 m_3	010 m_2
A 1	100 m_4	101 m_5	111 m_7	110 m_6

- **4 variable K-Map :-**

CD \ AB	00	01	11	10
00	0000 m_0	0001 m_1	0011 m_3	0010 m_2
01	0100 m_4	0101 m_5	0111 m_7	0110 m_6
11	1100 m_{12}	1101 m_{13}	1111 m_{15}	1110 m_{14}
10	1000 m_8	1001 m_9	1011 m_{11}	1010 m_{10}

Q1: Simplify the following expression using the K-map for the 4-variables A, B, C and D.

$$Y = m_1 + m_3 + m_5 + m_7 + m_8 + m_9 + m_{12} + m_{13}$$

Ans:- These are minterms, so enter 1 in the cells at the place of given minterms & 0 in other cells.

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	1	1	0	0
$A\bar{B}$ 10	1	1	0	0

unique essential (prime implicants)

① first quad: minimized term $= \bar{A}D$

② Second quad: minimized term $= A\bar{C}$

prime implicants (common/covered by other group so, don't include in result).

$$\boxed{Y = \bar{A}D + A\bar{C}}$$

Q2:- Plot logical expression $ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C + AB$ on a 4-variable K-map. obtained simplified from the map.

Ans:- To enter into K-map, a logic expression must be either in the canonical SOP form or in the canonical POS form. The canonical SOP form of the given expression can be obtained as follows.

$$Y = ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C + AB$$

Ans:-

$$Y = ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C(D+\bar{D}) + AB(C+\bar{C})(D+\bar{D})$$

$$Y = ABCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + (ABC + AB\bar{C})(D+\bar{D})$$

$$Y = \underset{1111}{\boxed{ABCD}} + \underset{1000}{A\bar{B}\bar{C}\bar{D}} + \underset{1011}{A\bar{B}CD} + \underset{1010}{A\bar{B}C\bar{D}} + \boxed{ABCD} + \underset{1110}{ABC\bar{D}} + \underset{1101}{AB\bar{C}D} + \underset{1100}{AB\bar{C}\bar{D}}$$

$\therefore A + A \to A$

$$Y = \Sigma m(8, 10, 11, 12, 13, 14, 15)$$

$$Y = m_8 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14} + m_{15}$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	0 (1)	0 (3)	0 (2)
$\bar{A}B$ 01	0 (4)	0 (5)	0 (7)	0 (6)
AB 11	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$ 10	1 (8)	0 (9)	1 (11)	1 (10)

3^{rd} quad $\to A\bar{D}$

2^{nd} quad $\to AB$

1^{st} quad $\to AC$

$$\boxed{Y = AC + AB + A\bar{D}}$$

Q3: Simplify the expression $Y = \Sigma m(7, 9, 10, 11, 12, 13, 14, 15)$, using the K-map method.

Ans

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	1	1	1

4^{th} group $\to BCD$

2^{nd} quad $\to AB$

1^{st} quad $\to AC$

3^{rd} quad $\to AD$

$$Y = A(B + C + D) + BCD$$

Q4:- Simplify the expression

$$Y = \Pi(0,1,4,5,6,8,9,12,13,14) \text{ using K-map.}$$

Ans:- This is POS form.

$$Y = (A+B+C+D)(A+B+C+\bar{D})(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$$
$$(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})(\bar{A}+\bar{B}+C+D)$$
$$(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

uncomplemented
$\because 0 \rightarrow A$
$1 \rightarrow \bar{A}$
complement

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	1

1st quad $\rightarrow (\bar{B}+D)$

one octet $\rightarrow C$

$$\boxed{Y = C\cdot(\bar{B}+D)}$$

Q5: Obtain (a) minimal Sum of Product & (b) minimal product of sum expressions for the function given below.

$$F = (A,B,C,D) = \Sigma_m(0,1,2,5,8,9,10)$$

Ans Here, Cells with 1 are grouped to obtain the minimal sum of product; cells with 0 are grouped to obtain minimal product of sum.

$\rightarrow$ Design K-map

Ans (a) SOP $Y = \bar{B}\bar{D} + \bar{A}\bar{C}D + A\bar{B}\bar{C}$

(b) POS $Y = (\bar{A}+\bar{B})(\bar{C}+\bar{D})(\bar{B}+D)$

SOP

Ans:

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

SOP

one quad with four Corners.

2 pairs. (SOP)

1 quad in (POS)

2^{nd} quad in (POS) → $(\bar{C}+\bar{D}$

SOP

$$Y = \bar{B}\bar{D} + \bar{A}\bar{C}D + A\bar{B}\bar{C}$$

POS

$$Y = (\bar{C}+\bar{D})(\bar{B}+D)(\bar{A}+\bar{B})$$

Q4:- Simplify the expression $Y = \Pi(0,1,4,5,6,8,9,12,13,14)$ using the K-map method.

Ans:- <u>POS form</u>

$$Y = (A+B+C+D)(A+B+C+\bar{D})(A+\bar{B}+C+D)(A+\bar{B}+C+\bar{D})$$
$$(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})(\bar{A}+\bar{B}+C+D)$$
$$(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

AB \ CD	CD 00	$C\bar{D}$ 01	$\bar{C}\bar{D}$ 11	$\bar{C}D$ 10
AB 00	1	1	0	0
$A\bar{B}$ 01	1	1	0	1
$\bar{A}\bar{B}$ 11	1	1	0	1
$\bar{A}B$ 10	1	1	0	0

→ 1 octet → C

1st quad ↓ $(\bar{B}+D)$

$$\boxed{Y = C\cdot(\bar{B}+D)}$$

- <u>prime implicants</u> → you have to cover all one's.
- <u>Essential Prime implicants</u> → is a prime implicant that covers at least one minterm that is not covered by any other prime implicant. (at least one '1', do not participate in other group (uniqueness)).

Don't Care (d/X/Φ) Conditions in K-map:-

- In K-maps, "don't care" conditions indicate input combinations where the output value is irrelevant to the circuit's functionality. These conditions can be treated as either 0 or 1 during minimization.

- This condition during minimization, allows more flexibility in grouping and simplifying the boolean expression.

- They arise from invalid input combinations or when the output value for a specific input is not important.

- when an incompletely specified function, i.e., a function with don't care combinations, is simplified to obtain minimal SOP expression, the value '1' or 'd' can be assigned to select don't care combinations.

- This is done in order to increase the no. of 1's in the selected groups, for further simplification is possible.

- Also, a don't care combination need not to be used in grouping if it does not cover a large no. one's (1's).

⇒ Questions related Don't Care Conditions (d) in K-map →

Note:- Consider that 'd', which is used to make big group with one (1), cover only all 1's. After this any 'd' not cover in grouping then ignore it. (Covers only minterms)

Q6: Simplify the Boolean function.

$$F(A,B,C,D) = \sum_m (1,3,7,11,15) + \sum_d (0,2,5)$$

Ans:- The 1's & d's are combined in order to enclose the maximum no. of adjacent cells with 1. By combining the 1's & d's, two quads can be obtained.

Now, expression in SOP form is:-

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	d	1	1	d
$\bar{A}B$ 01	0	d	1	0
AB 11	0	0	1	0
$A\bar{B}$ 10	0	0	1	0

$$\boxed{Y = \bar{A}\bar{B} + CD}$$

Q7: Using K-map, simplify the Boolen function and obtain (i) minimal SOP &

$$Y = \sum_m (0,2,3,6,7) + \sum_d (8,10,11,15)$$

Ans

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	1	1
$\bar{A}B$ 01	0	0	1	1
AB 11	0	0	d	0
$A\bar{B}$ 10	d	0	d	d

SOP expression:-

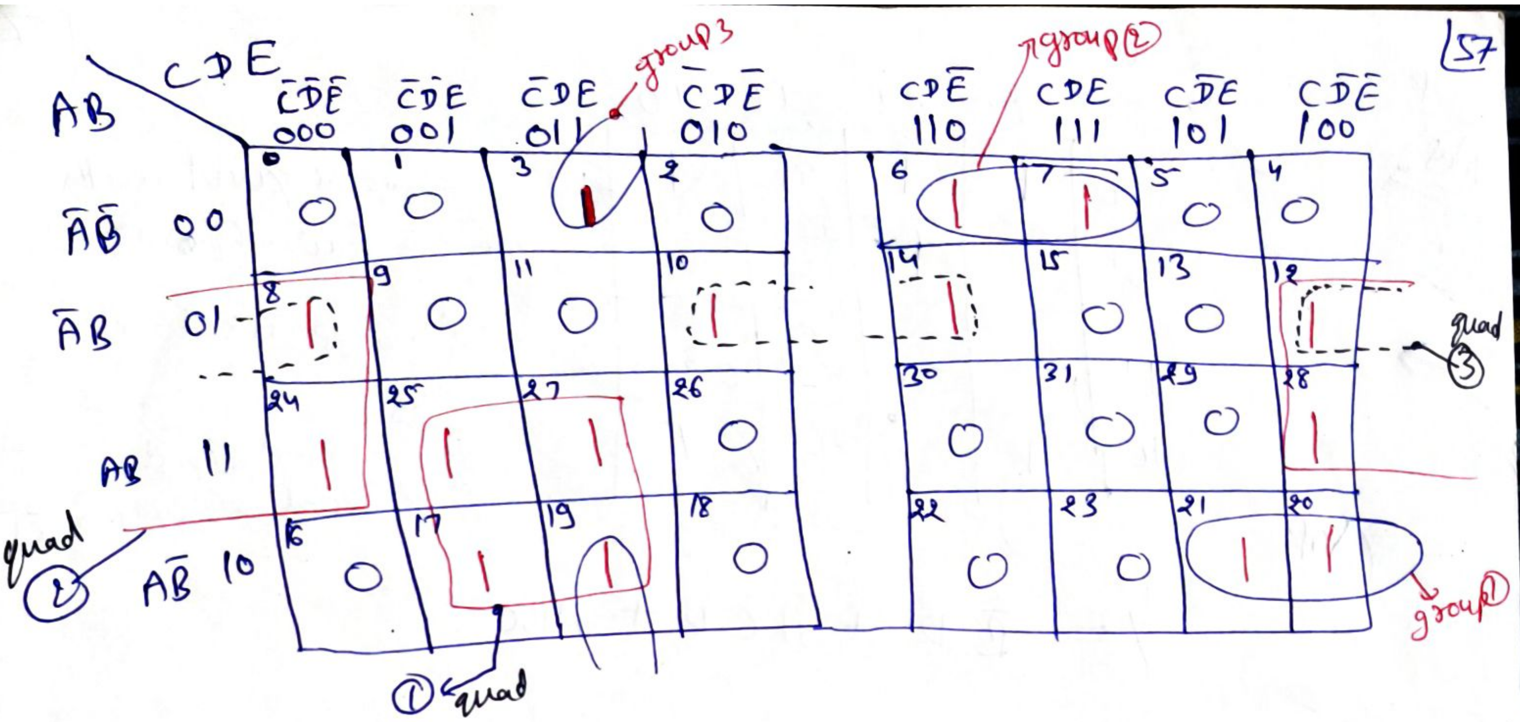
$$\boxed{Y = C\bar{A} + \bar{B}\bar{D}}$$

- Combine 1's & d's to make big group.
- Group only that d's, which is used to group all 1's. Otherwise ignor remaing d's.

Five - Variable K-Map :-

- A Five variable K-map contains ($2^5 \rightarrow 32$) cells which is used to simplify any 5-variable logic expression.

AB \ CDE	000	001	011	010	110	111	101	100
00	00000 0	00001 1	00011 3	00010 2	00110 6	00111 7	00101 5	00100 4
01	01000 8	01001 9	01011 11	01010 10	01110 14	01111 15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20



Q1 Simplify $Y = \Sigma m(3,6,7,8,10,12,14,17,19,20,21,24,25,27,28)$

Ans $A\bar{C}E + B\bar{D}\bar{E} + \bar{A}B\bar{E} + \bar{B}\bar{C}DE + \bar{A}\bar{B}CD + A\bar{B}C\bar{D}$

12, 8, 14, 10
12, 8, 24, 28
25, 27, 17, 19

Q2 Obtain the minimal SOP expression for the function.

$$Y = \Sigma m(1,5,7,13,14,15,17,18,21,22,25,29) + \Sigma d(6,9,19,23,30)$$

Ans:- $Y = \bar{D}E + \bar{A}CD + A\bar{B}D$

By Combining the 1's & d's, one octet & two quads.

practice it →

Quine - Mc Cluskey OR Tabular Method :-

- K-map is a very effective tool for minimization of logic functions with 4 or less variables.
- For logic expression with more than 4-variables, the visualization of adjacent cells and the drawing of the K-map become more difficult.
- QC/tabular method can be employed in such cases to minimize switching functions.
- This method employs a systematic, step-by-step procedure to produce a simplified standard form of expression for a function with any no. of variables.
- The tabular method, is a systematic, algorithmic approach that uses minterm combination to find prime implicants, making it suitable for a large no. of variables (more than 4-6) where K-maps are difficult.
- The primary advantage of tabular method is its ability to handle a large number of input variables and its suitability for computer programming.

Q1: Find the minimal SOP for the boolean expressions $f = \Sigma(1,2,3,7,8,9,10,11,14,15)$ using Quine-McCluskey method. $\boxed{59}$

Ans:- Step I:- Represent the minterms in binary form.

Table: 1

minterms	A	B	C	D
m_1	0	0	0	1
m_2	0	0	1	0
m_3	0	0	1	1
m_7	0	1	1	1
m_8	1	0	0	0
m_9	1	0	0	1
m_{10}	1	0	1	0
m_{11}	1	0	1	1
m_{14}	1	1	1	0
m_{15}	1	1	1	1

(Columns A–D under the heading "Equivalent Binary Representation")

Step II: Grouping based on number of 1's, present in their binary equivalents.

Table 2:

No. of 1's	minterms	A	B	C	D
1	m_1	0	0	0	1
	m_2	0	0	1	0
	m_8	1	0	0	0
2	m_3	0	0	1	1
	m_9	1	0	0	1
	m_{10}	1	0	1	0
3	m_7	0	1	1	1
	m_{11}	1	0	1	1
	m_{14}	1	1	1	0
4	m_{15}	1	1	1	1

(Columns A–D under the heading "Binary Representation of variables")

Step 3:- Any two numbers in these groups which differ from each other by only one variable can be chosen and combined to get two-cell combinations.

Table 3:- 2-Cell Combinations (possible merging of minterms from adjacent group.)

Group Name	minterm Combinations	A	B	C	D
G1	(1,3)	0	0	—	1
	(1,9)	—	0	0	1
	(2,3)	0	0	1	—
	(2,10)	—	0	1	0
	(8,9)	1	0	0	—
	(8,10)	1	0	—	0
G2	(3,7)	0	—	1	1
	(3,11)	—	0	1	1
	(9,11)	1	0	—	1
	(10,11)	1	0	1	—
	(10,14)	1	—	1	0
G3	(7,15)	—	1	1	1
	(11,15)	1	—	1	1
	(14,15)	1	1	1	—

Note:- if bit change, place dash (—) at that place, other remain same.

Step 4:- From the two cell combinations, one variable and a dash in the same position can be combined to form 4-cell combinations.

Table 4:- 4 cell Combinations

Combination	A	B	C	D
(1,3,9,11)	—	0	—	1
(2,3,10,11)	—	0	1	—
(8,9,10,11)	1	0	—	—
(3,7,11,15)	—	—	1	1
(10,11,14,15)	1	—	1	—

- (1,3) (9,11) & (1,9) (3,11) are same.

Step 5:- write all minterms in prime implicant table.

Prime implicants	1	2	3	7	8	9	10	11	14	15
* (1,3,9,11)	X		X			X		X		
* (2,3,10,11)		X	X				X	X		
* (8,9,10,11)					X	X	X	X		
* (3,7,11,15)			X	X				X		X
* (10,11,14,15)							X	X	X	X
	✓	✓		✓	✓				✓	

- Columns having only one cross mark correspond to essential prime implicants.
- A tick (✓) mark is put against every column which has only one cross mark.
- A star (*) mark is placed against every essential primary implicant.
- The sum of the prime implicants gives the function in its minimal SOP form. Since all prime implicants are essential prime implicants.

$$\boxed{f = \bar{B}\cdot D + \bar{B}\cdot C + A\bar{B} + C\cdot D + A\cdot C}$$

Ans

Q3: $f(W,X,Y,Z) = \Sigma(1,3,4,5,9,10,11) + \Sigma d\,(6,8)$

using Quine - McCluskey Method. (Find the minimal SOP for boolean Expression).

Ans:- Firstly represent in binary form.

Step1: Table:1 Binary representation of minterms

Minterms	Variables			
	W	X	Y	Z
1	0	0	0	1
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1

Step2 :- The above binary representation is grouped into a no. of sections in terms of a no. of 1's.

Table.2: Group of minterms for different no. of 1's.

No. of 1's	Minterms	Variable			
		W	X	Y	Z
1	1 ✓	0	0	0	1
	4 ✓	0	1	0	0
	8 ✓	1	0	0	0
2	3 ✓	0	0	1	1
	5 ✓	0	1	0	1
	6 ✓	0	1	1	0
	9 ✓	1	0	0	1
	10 ✓	1	0	1	0
3	11 ✓	1	0	1	1

Step 3: Any two no. in these groups which differs from each other by only one variable can be chosen and combined to get two cell combinations

Step3: Table 3: 2 Cell Combinations

Combination	w	x	y	z
(1,3) ✓	0	0	–	1
(1,5)	0	–	0	1
(1,9) ✓	–	0	0	1
(4,5)	0	1	0	–
(4,6)	0	1	–	0
(8,9) ✓	1	0	0	–
(8,10) ✓	1	0	–	0
(3,11) ✓	–	0	1	1
(9,11) ✓	1	0	–	1
(10,11) ✓	1	0	1	–

Step4:- From the cell Combinations, one variable and a dash in the same position can be Combined to form 4-cell Combinations.

Table 4:- 4 cell Combinations

Combination	w	x	y	z
(1,3,9,11)	–	0	–	1
(8,9,10,11)	1	0	–	–

- cell (1,3) & (9,11) same as cell (1,9) (3,11).

- Don't care minterms cannot be listed as column headings in the chart because they do not have to covered by a minimal expression.

Step 5:-

Table 5: Prime implicants table

Prime implicants	Minterms 1	3	4	5	9	10	11
(1, 5)	X			X			
(4, 5)			X	X			
(4, 6)			X				
(1, 3, 9, 11)*	X	X			X		X
(8, 9, 10, 11)*					X	X	X
		✓				✓	

- The Column having only one cross mark Correspond to the essential prime implicants. A tick mark is put aginst every column which has only one cross mark.
- A star mark is put against every essential primary implicant. The prime implicant which cover the minterm (1, 3, 9, 11) is the essential prime implicant
- So, in order to cover the remaining minterms, the reduced prime implicant Chart is formed.

Table 6:- Reduced prime implicants

Prime Implicants	Minterms 1	3	4	5	9	10	11
(1, 5)	X			X			
✓ (4, 5)*			X	X			
(4, 6)			X				

- To Cover minterms (4, 5), the prime implicants (4, 5) select
- The minimal expression:-

$$\boxed{f(w, x, y, z) = \bar{x}z + \bar{w}x\bar{y} + w\bar{x}}$$

Q:3: $y = (A,B,C,D) = \sum m(1,2,3,5,9,12,14,15) + \sum d(4,8,11)$ using Quine-Mc Cluskey method.

Ans:- StepI: Represent the minterms in binary form.

Table1:-

minterms	Equivalent Binary Representation A	B	C	D
m_1	0	0	0	1
m_2	0	0	1	0
m_3	0	0	1	1
m_5	0	1	0	1
m_9	1	0	0	1
m_{12}	1	1	0	0
m_{14}	1	1	1	0
m_{15}	1	1	1	1
m_4	0	1	0	0
m_8	1	0	0	0
m_{11}	1	0	1	1

StepII: Grouping based on number of 1's, present in their binary equivalents.

Table 2:

No. of 1's	Minterms	Binary Representation of Variables A	B	C	D
1	1	0	0	0	1 ✓
	2	0	0	1	0 ✓
	4	0	1	0	0 ✓
	8	1	0	0	0
2	3	0	0	1	1 ✓
	5	0	1	0	1 ✓
	9	1	0	0	1 ✓
	12	1	1	0	0 ✓
3	11	1	0	1	1 ✓
	14	1	1	1	0 ✓
4	15	1	1	1	1 ✓

Step3:- Any two no in these groups which differs from each other by only one variable can be chosen and combined to get two cell combinations.

Table 3:- 2-cell Combinations

Combination	A	B	C	D
(1,3)	0	0	—	1 ✓
(1,5)	0	—	0	1
(1,9)	—	0	0	1 ✓
(3,11)	—	0	1	1 ✓
(9,11)	1	0	—	1 ✓
(12,14)	1	1	—	0
(11,15)	1	—	1	1
(14,15)	1	1	1	—
(2,3)	0	0	1	—
(4,5)	0	1	0	—
(8,9)	1	0	0	—
(4,12)	—	1	0	0
(8,12)	1	—	0	0

These Term have atleast '1' minterm (without don't care) So include in prime implicant table.

Step4:- 4 cell Combinations

Combination	A	B	C	D
(1, 3, 9, 11)	—	0	—	1

- Here only one 4-cell Combination, in which one variable and one dash at same position. Combine that one to form 4-cell Combination.
- Do not care minterms cannot be listed in prime implicant listed table 5.

Step 6:- Reduced prime implicants table → To cover remaining minterms, So, form this table

Prime implicants	minterms							
	1	2	3	5	9	12	14	15
(1, 3, 9, 11) *	X		X		X			
(1, 5) *	X			X				
(2, 3) *		X	X					
(4, 5)				X				
(4, 12)						X		
(8, 9)					X			
(8, 12)						X		
(11, 15)								X
(12, 14) *						X	X	
(14, 15) *							X	X

(Don't count don't care term, Cover only remaining minterms)

Table 5

Prime implicants	All Minterms							
	1	2	3	5	9	12	14	15
(1, 3, 9, 11) *	X		X		X			
	✓		✓		✓			

→ Now write minimize boolean expression using tabular method include only star mark term. (from 2-Cell Combination, write expression. (for (1, 3, 9, 11), write answer from 4-Cell Combination.

$$Y = \bar{B}D + \bar{A}\bar{C}D + \bar{A}\bar{B}C + AB\bar{D} + ABC$$