PROJECT 3 - EUROPEAN OPTION PRICING

OBJECTIVE:

Estimate the value of an ATM Call Option using the **Black-Scholes -Merton** Option Pricing Model and using **Monte-Carlo Simulation**.

Underlying: Amazon shares

Expiry: 1 year

BLACK-SCHOLES FORMULA

$c = S * \Phi(d1) - K * \ \Phi(d2) * e^{-rt}$

Where $d1 = (\log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right) * T)/(\sigma * sqrt(T))$

$d2 = d1 - T^{\frac{1}{2}}$

[All the alphabets and symbols have their usual meaning.]

We will use the US T-Bond yields as a proxy for the risk-free rate, $r = 0.0425$

VALUATION METHOD	IMPORTANT ASSUMPTIONS	METHODOLOGY	ESTIMATE
1. BLACK-SCHOLES-MERTON	1. Volatility is assumed to be constant.	2. We use the formula mentioned above to estimate the price.	7.603 $
2. MONTE-CARLO SIMULATION	1. Share Price follows a Geometric Brownian Motion. 2. 1 year of data is taken to estimate μ and σ to be used in the Simulation exercise.	1. We use the closed form solution of the SDE: dS(t) = S(t)*[μdt+σdW(t)] S(t) = S * exp{(mu-0.5*sigma^2)*T + Sigma * W} W : Standard Brownian Motion S(t): share price at time t 2. Calculate the payoff at the end of the expiry period and discount it to the present.	5.809 $

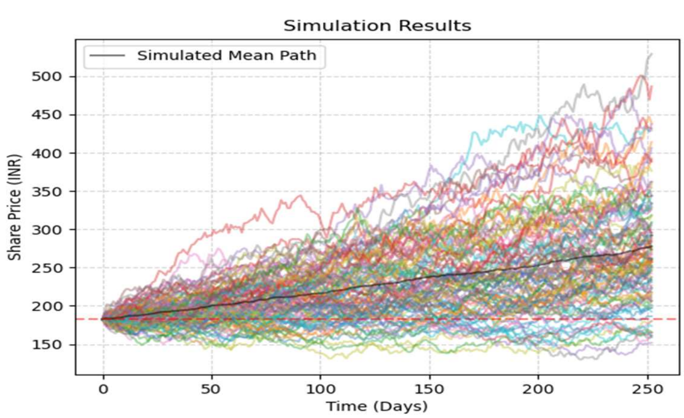

Fig: Simulated Price Paths and the Mean Price Path.

CONCLUSION

The BSM Formula provides an estimation of the Option Price under some unrealistic assumptions, yet it a good approximation/baseline price.

The Monte Carlo Simulation relies on historical estimates of μ and σ of returns to generate the Future paths. Running the simulation ones generated an Option Price that is close to the BSM estimate. However, on running the simulation again may give up an Option Price that is far apart from the BSM estimate.

Some degree of randomness is expected when we are simulating the share price. In this case, probably more because of the positive drift term as evident in the graph above. In the Python code, if we set μ = 0, the Simulated estimate will be close to the BSM estimate.