Angle Modulation Indices Measurement in the Presence of Amplitude Modulation

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Abstract. *This paper presents an automated measurement of the phase/frequency indices of angle modulation in the frequency domain on condition that a signal source is simultaneously angle and amplitude modulated by the same phase shifted harmonic signals.*

In the first part of this paper, we propose mathematical model of such modulated signals. Then we investigate the spectrum of signal and discuss some basic properties of such modulated signals, where we restrict ourselves only to a weak amplitude modulation with modulation depth smaller than 100 %. The final section is devoted to the signal parameters derivation techniques based on the power spectrum measurement and non-linear least squares iteration method. The effect of power spectrum measurement errors is also pointed out and a comparison between Bessel null method and proposed technique is presented.

Keywords

Angle modulation, amplitude modulation, FM/PM indices measurement, spectral analysis, power spectrum.

1. Introduction

The frequency modulation is still common and important method of mapping messages into carrier signals in the radio broadcasting. From this reason, the measurement of frequency or phase modulation has received great attention for so many years. Today, two main techniques of frequency/phase indices measurements are used – the well known Bessel null method based on the power spectrum measurement on one hand and the one-box modulation analyzer solution on the other hand. Although it is universally accepted that the second technique is more compact, the power spectrum measurement method has some desirable features: we are not restricted by modulation analyzer limits (bandwidth, sensitivity, peak deviation) and we can use universal measurement instrument – spectrum analyzer – instead of specialized one. In this paper, therefore, we focus on the frequency/phase indices measurement based on the power spectrum measurement.

The Bessel null technique, although conceptually very simple, has two main drawbacks. Firstly, it is the ability to precise measure the indices only for restricted set of indices values (due to the periodic nature of Bessel functions roots) and, secondly, it is the decreasing accuracy of measurement for the signal sources with simultaneous angle and amplitude modulation due to the spectral spreading and leakage. Therefore, we propose a new method to measure angle modulated signals in place of Bessel null method. This method is based on simultaneous angle and amplitude modulated signal model and utilizes all the power components to calculate arbitrary signal parameters.

2. Problem Analysis

In this section, we focus on determination of a mathematical model of simultaneously angle and amplitude modulated signal, its power spectrum derivation and property discussion.

2.1 Mathematical model

We start with the generalized angle modulation mathematical model in the usual fashion

$$x_c(t) = A_c \cos\left(\omega_c t + \varphi(t) + \varphi_0\right) = A_c \cos\left(\Phi(t)\right) \quad (1)$$

where $\Phi(t)$ is the instantaneous phase and $f(t) = \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t)$ is the instantaneous frequency of the modulated carrier. The special cases of angle modulation are phase and frequency modulation, where in phase modulation the carrier time-dependent phase deviation $\varphi(t)$ is varied proportionally to the baseband message signal $x_m(t)$, i.e. $\varphi(t) = k_p x_m(t)$, and in frequency modulation, the phase deviation is varied proportionally to the integral of the message signal, i.e. $\varphi(t) = k_f \int_{t_0}^{t} x_m(\tau)\,\mathrm{d}\tau$. The constants k_p and k_f are usually called phase sensitivity and frequency deviation, respectively.

In the special case of single-tone angle modulation, the message signal $x_m(t)$ is a sinusoid with amplitude A_m and angle modulated carrier is given by

$$x_c(t) = A_c \cos\left(\omega_c t + k \sin(\omega_m t + \theta_m) + \varphi_0\right) \quad (2)$$

where the factor k represents the maximum phase deviation.

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In case of PM, the factor k is simply equal to k_p/A_m. However, in case of FM, the factor k is called the modulation index defined as $k = k_f A_m/\omega_m$ where the product $k_f A_m$ represents the peak frequency deviation of modulated signal.

We find that – unless we know some information about the modulation process – it is not possible to distinguish whether the signal $x_c(t)$ is phase or frequency modulated. In other words, without a priori knowledge of message signal, the mathematical model $\sin(\omega_c t + k\sin(\omega_m t))$ may be phase modulated carrier with modulating signal $x_m(t) = A_m \sin(\omega_m t)$ or frequency modulated carrier with modulating signal $x_m(t) = A_m \cos(\omega_m t)$, where $t_0 = n\pi$, $n \in \mathbb{Z}$, $t_0 < t$.

The $x_c(t)$ is nonlinear function of $x_m(t)$, however, it may be expanded into a series of sinusoids

$$x_c(t) = A_c \cdot \ldots \cdot \sum_{n=-\infty}^{\infty} J_n(k) \cos(\omega_c t + \varphi_0 + n\omega_m t + n\theta_m) \quad (3)$$

whose coefficients $J_n(k)$ are given by the Bessel function of the first kind and nth order.

Now we include sinusoidal amplitude fluctuations around A_c originated from message signal in the model, i.e.

$$x_c(t) = A_c (1 + m\sin(\omega_m + \vartheta_m)) \cdot \ldots \cdot \sum_{n=-\infty}^{\infty} J_n(k) \cos[(\omega_c + n\omega_m)t + \varphi_0 + n\theta_m]. \quad (4)$$

2.2 Power spectrum

After some mathematical manipulations of (4) in order to collect spectral components, one obtains

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} \Big[J_n(k) \cos(\Psi_n^0(t)) + \ldots + \sum_{p=\pm 1} J_{n-p}(k) \frac{pm}{2} \sin(\Psi_n^p(t)) \Big] \quad (5)$$

where

$$\Psi_n^p(t) = (\omega_c + n\omega_m)t + \varphi_0 + (n-p)\theta_m + p\vartheta_m. \quad (6)$$

The power spectrum of $x_c(t)$ can be calculated by means of Fourier transform of (5). Following this way, we get spectrum representation $X_c(t)$ of $x_c(t)$, i.e.

$$X_c(t) = \sum_{n=-\infty}^{\infty} \sum_{r=\pm 1} \delta(f + rf_c + rnf_m) e^{-jr(\varphi_0 + n\theta_m)} \cdot \ldots \cdot \frac{A_c}{2} \left[J_n(k) + \sum_{s=\pm 1} J_{n+s}(k) \frac{rsm}{2j} e^{-jrs(\theta_m - \vartheta_m)} \right] \quad (7)$$

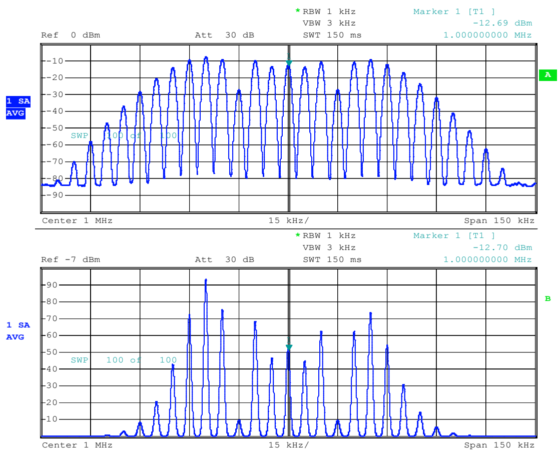

Fig. 1: The power spectrum of simultaneously frequency and amplitude modulated signal; $A_c = 224\,\text{mV}$ ($P_{tot} = 0\,\text{dBm}$), $f_m = 5\,\text{kHz}$, $k = 6.4$ (peak frequency deviation $k_f A_m/2\pi = 32\,\text{kHz}$), $m = 0.2$.

and, finally, the power spectrum $S_x(f) = X_c(f)X_c^*(f)$ can be expressed as

$$S_x(f) = \frac{A_c^2}{4} \sum_{n=-\infty}^{\infty} \sum_{r=\pm 1} \delta(f + rf_c + rnf_m) \cdot \ldots \cdot \Big[J_n^2(k) + \frac{m^2}{4} (J_{n-1}(k) + J_{n+1}(k))^2 + \ldots + mJ_n(k)\sin(\vartheta_m - \theta_m)(J_{n-1}(k) + J_{n+1}(k)) - \ldots - m^2 J_{n-1}(k) J_{n+1}(k) \cos^2(\vartheta_m - \theta_m) \Big]. \quad (8)$$

It is evident that there are components at the infinite set of frequencies. The power distribution is centered at the carrier frequency. The crucial thing resulting from the properties of Bessel function is that the higher parameter k, the more dispersed power distribution. The bandwidth of single-tone angle modulated signal can be estimated by "rules-of-thumb". One such rule is Carson's rule which states that appx. 98 % of the total power is contained in the bandwidth $B = 2(k+1)f_m$. The similar result holds for simultaneous angle and amplitude modulation, therefore, it is possible to approximate the power of spectral component P_n at frequency $f = f_c + nf_m$, $n = 0, \pm 1, \pm 2, \ldots$

$$P_n \approx \frac{A_c^2}{2} \Big[J_n^2(k) + \frac{m}{2} \Big\{ (J_{n-1}(k) + J_{n+1}(k))^2 + \ldots + 2J_n(k)(J_{n-1}(k) + J_{n+1}(k))\sin(\Delta) - \ldots - 2mJ_{n-1}(k)J_{n+1}(k)\cos^2(\Delta) \Big\} \Big] \quad (9)$$

on condition that $B \ll f_c$, where $\Delta = \vartheta_m - \theta_m$.

The first term in square bracket corresponds to pure angle modulation, while the second term is excited by amplitude modulation. This term is responsible for the spectral leakage and power spectrum asymmetry following from $\sin(\Delta)$ term and Bessel function property (see Fig. 1).

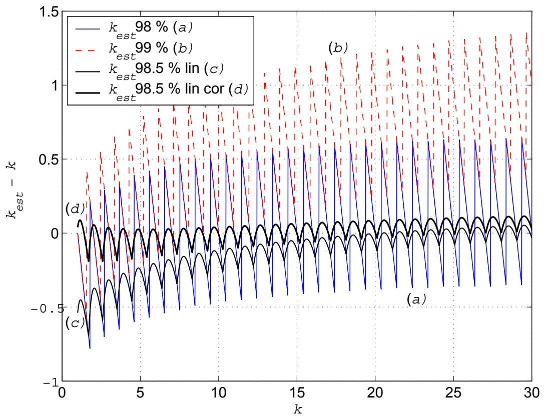

Fig. 2: Four types of k estimation; (a) and (b) straightforward Carson's rule, (c) power change linearization, (d) power change linearization and bias correction.

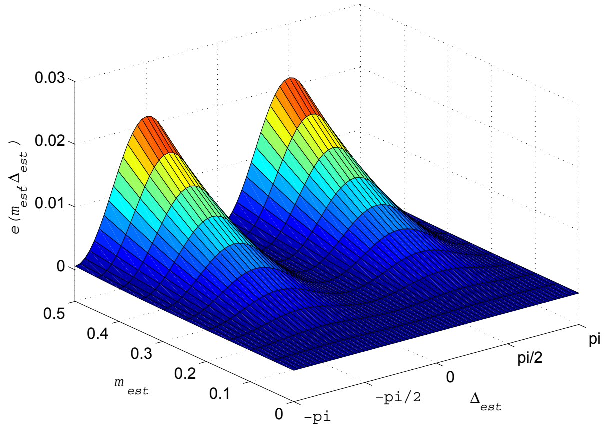

Fig. 3: Sum of the squared deviations between the exact power spectrum ($A_c = \sqrt{2}$, $k = 20$, $m = 0$, $\Delta = 0$) and estimated power spectrum ($A_{c,est} = A_c$, $k_{est} = k$, m_{est} and Δ_{est} variable).

3. Measurement and calculation

This section deals with the description of angle modulation parameters measurement in the frequency domain. The proposed method is based on NLLSQ algorithm (see sec. 3.3) and consists of three main steps: power spectrum measurement and spectral components derivation, estimation of initial values for the parameters, and parameters estimation using weighted NLLSQ method.

3.1 Power spectrum measurement

At first, we have to measure the power spectrum of analyzed signal by spectrum analyzer, see Fig. 1. The proper frequency, span, and resolution filter must be selected to measure all significant spectral components P_n. Note that modern spectrum analyzers can measure thousands of frequency points over one sweep (for example, we used R&S FSP3 [4] with 8001 points), thus we are not limited by screen resolution and the high ratio of SPAN/RBW can be set to measure wide-band angle modulation. In our case, the measurement of power spectrum was controlled by GPIB and the measured results were downloaded to PC for further processing. The derivation of components P_n is a standard task utilizing a peak search algorithm; the average noise floor power N is also calculated.

3.2 Parameters estimation

The second step is to estimate parameters A_c, k, m, and Δ in (9). *Amplitude estimation* is simply calculated from the sum of all measured power components P_n, i.e. $A_{c,est} = \sqrt{2\sum_n P_n}$. We omit slight bias of estimation induced by amplitude modulation. *Estimation of factor* k follows upper mentioned Carson's rule, where the bandwidth B is searched as a carrier frequency centered interval containing defined portion of total measured power. Since the power spectrum has a discrete nature, the straightforward calculation of B generates significant k_{est} ripple (see Fig. 2, lines *a* and *b*), which can cause NLLSQ algorithm to diverge or stop in a local extreme due to the periodic nature of squared Bessel functions. One way to decrease this ripple is to utilize linearization of power change between adjacent spectral components P_n (Fig. 2, line *c*). Finally, a correction must be incorporated because of Carson's rule bias for the lower values of k (Fig. 2, line *d*). The *estimation of* m *and* Δ is slightly complicated. We are not able to recognize whether Δ is in interval $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$ or $\langle -\pi, -\frac{\pi}{2} \rangle \cup \langle \frac{\pi}{2}, \pi \rangle$, thus we focus only on first interval (Fig. 3). For $\Delta > 0$ the power spectrum is skewed to the lower frequency and vice versa. Therefore we start with $\Delta_{est} = \pm\frac{\pi}{4}$ and $m_{est} = 0.1$. These values ensure that the NLLSQ algorithm starts with good initial conditions. For symmetrical power spectrum, m_{est} tends towards zero and Δ_{est} loses influence on $A_{c,est}$ and k_{est} calculation.

3.3 NLLSQ and weighted NLLSQ

To find the parameters A_c, k, m, and Δ we use the iterative process called non-linear least squares method (NLLSQ) based on Matlab's Optimization Toolbox routines lsqcurvefit and lsqnonlin. NLLSQ algorithm finds numerical values for the parameters to minimize the sum of the squared deviations between the measured power spectrum and the functional portion of our model (9). Mathematically, the sum of squares to be minimized is given by

$$e = \sum_n w_n \left[P_n^m - P_n^c(A_{c,est}, k_{est}, m_{est}, \Delta_{est})\right]^2 \qquad (10)$$

where P_n^m are measured spectral components, P_n^c are calculated spectral components, and w_n are weight coefficients.

In the first instance we run NLLSQ algorithm, where all power quantities in (10) are in linear scale and weight coefficients are set to one. The outstanding feature of this step is its insensitivity to the low level noisy spectral components and smoothness of error function (10). Therefore, the NLLSQ algorithm quickly converges to a local minimum of (10). On the other hand, suppression of low level power components results in worse parameter estimation because we move around the flat maxima of Bessel function whereas the sharp minima are strongly damped – see (9).

For this reason, we follow up with weighted NLLSQ algorithm in logarithmic scale. Since the standard deviation of the error term varies over all values of the measured P_n^m, we employ weight coefficients in the form of $w_n = u_n v_n$, where u_n is one for $P_n^m > L_0$ and zero elsewhere, and

$$v_n = \begin{cases} 1, & P_n \geq L_1, \\ 0, & P_n < L_2, \\ \frac{P_n - L_2}{L_1 - L_2} & \text{elsewhere.} \end{cases} \tag{11}$$

The levels L_0, L_1, and L_2 depend on SA parameters and actual measurement results P_n and N so that $L_0 > N$ and $\max(P_n) > L_1 > L_2 > N$. Thus, the coefficients u_n ensure that significantly low level spectral components are suppressed. The simple form of v_n follows the fact that the power level detection in SA is performed by logarithmic amplifier with the ability to measure signals up to 80 dB dynamic range or more, where the typical total accuracy is tenths of decibels and the components P_n^m have increasing uncertainty toward the average noise floor power N.

k [−]		6.4	6.4	20	20
σ_p [dB]		0.3	1.5	0.3	1.5
A_{est}	μ	1.0035	1.0345	1.0038	1.0325
[V]	σ	0.0106	0.0607	0.0065	0.0359
k_{est}	μ	6.2781	6.3105	20.0118	20.0113
[−]	σ	0.0450	0.1492	0.0203	0.1298
A_1	μ	1.0003	1.0283	1.0007	1.0264
[V]	σ	0.0133	0.0728	0.0089	0.0477
k_1	μ	6.4006	6.4166	19.9995	20.0007
[−]	σ	0.0206	0.1157	0.0116	0.0602
m_1	μ	0.1253	0.1998	0.1201	0.1714
[−]	σ	0.0417	0.1226	0.0272	0.0979
$\|\Delta_1\|$	μ	1.1220	1.0109	1.0399	0.9664
[rad]	σ	0.4550	0.6162	0.3416	0.5765
A_2	μ	1.0003	0.9990	1.0002	0.9954
[V]	σ	0.0087	0.0463	0.0051	0.0263
k_2	μ	6.3970	6.3907	19.9997	19.9997
[−]	σ	0.0112	0.0321	0.0043	0.0186
m_2	μ	0.0993	0.0933	0.0993	0.1009
[−]	σ	0.0093	0.0469	0.0077	0.0326
$\|\Delta_2\|$	μ	1.2771	1.3233	1.2595	1.2581
[rad]	σ	0.1502	0.3359	0.1314	0.2362

Tab. 1: The estimates of the mean, μ, and standard deviation, σ, of calculated parameters under normal distribution assumption; $A_{c,est}, \Delta_{est}$ estimated parameters, $A_{c,1} \ldots \Delta_1$ unweighted NLLSQ results, $A_{c,2} \ldots \Delta_2$ weighted NLLSQ results.

4. Results

Tab. 1 summarizes some results obtained from previously mentioned algorithms for two different k and power measurement uncertainties σ_p, where $A_c = 1$ V, $m = 0.1$, and $\Delta = -1.2$. We analyzed 500 spectra samples for each pair of k and σ_p and none of them diverge. Firstly, we can notice small negative bias of k_{est} for $k = 6.4$, which results from estimator ripple (see Fig. 2). As for unweighted NLLSQ algorithm, A_1 and k_1 parameters are estimated quite well whereas m_1 and Δ_1 are rather inaccurate as evident from deviation values. This effect is caused by suppression of the low level power components due to linear power scale. The power of weighted NLLSQ algorithm working in logarithmic scale is apparent. Not only A_2 and k_2, but also m_2 and Δ_2 are estimated very well. Still, the large deviation of the last two parameters remains because of noisy information about m_2 and Δ_2 included in measured power components P_n^m – consult (9). Finally, we have to note that similar conclusions hold for arbitrary parameters A_c, k, m, and Δ.

5. Conclusion

We have demonstrated a new technique to determine angle modulations indices from signal power spectrum analysis. Based on this work it is now possible to measure the frequency/phase indices from signal power spectrum without the restriction on indices values. Moreover, the proposed algorithm takes into account potential simultaneous angle and amplitude modulation process caused by the same phase shifted harmonic signals. To our knowledge the measurement of such signals was previously possible only by strenuous measurement methods supported by complicated mathematical treatments. Besides, we have shown that the proposed technique makes it feasible to make completely automated measurements. Therefore, it is possible to implement it in the form of spectrum analyzer firmware routine or it can be used as a stand-alone computer program processing external data. The data can be, for example, directly downloaded from spectrum analyzer as we present in this paper.

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References

[1] MIDDLETON, D. *An Introduction to Statistical Communication Theory*. New York: McGraw-Hill, 1960.

[2] ABRAMOVITZ, M., STEGUN, I. A. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. 9th printing. New York: Dover 1972.

[3] UEBERHUBER, C. W. *Numerical Computation 2: Methods, Software, and Analysis*. Berlin: Springer-Verlag 1997.

[4] Spectrum Analyzer R&S FSP3 – Operating Manual. Munich: Rohde&Schwarz, 2006.

[5] MATĚJKA, Š. Accurate AM Depth Measurement in the Presence of Angle Modulation. In *Proceedings of 17th International Conference Radioelektronika 2007*. Brno (Czech Republic), 2007, p. 507 - 510.