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Continuing value in firm valuation by the discounted cash flow model

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Abstract

The discounted cash flow model, like other firm valuation models, proceeds in two periods. For each year in the explicit forecast period, there is an individual forecast of free cash flow. On the other hand, all of the years in the post-horizon period are represented through one single continuing value formula, being the steady-state value of the firm's productive assets at the horizon. Continuing value is typically derived by applying the Gordon formula to a simple extrapolation of free cash flow at the end of the explicit forecast period. This paper examines the components of continuing value, in particular capital expenditures and tax savings due to depreciation of property, plant and equipment (PPE). The estimation of two somewhat elusive parameters related to capital expenditures, equipment economic life and capital intensity, is discussed. A further analysis indicates that a substantial part of continuing value derives from cash flow associated with already acquired equipment. Also, the error resulting from assuming steady-state rather than lumpy capital expenditures is identified. Implementation issues relating to the explicit forecast period are also commented on.

Keywords: Valuation; Free cash flow; Steady state; Discounting; Accounting data

1. Introduction and overview

The discounted cash flow model is one member of a whole family of related models for firm valuation. It values the equity of a firm by discounting free cash flow from the firm's operations to a present value of the productive assets of the firm, and then subtracts the value of the firm's interest-bearing debt to arrive at the value of the equity as a residual.[2] Other members of the same family are the discounted dividend model and the residual income model (and a few other ones). All of these models are equivalent in the sense that they

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[2] It is assumed in this very brief summary, and in what follows, that the firm has zero excess marketable securities, e.g., after netting against interest-bearing debt.

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provide the same equity value, although under very stringent consistency conditions (Fernández, 2002; Penman, 1998; Young et al., 1999). Among the members of this family, the discounted cash flow model has traditionally been a dominant one in practice, although subject to competition in recent years from the residual income model (Lundholm and O'Keefe, 2001, p. 315). The discounted cash flow model has been popularized in an influential book written by an author team associated with the McKinsey consulting company (Copeland et al., 2000).

It is a basic property of firm valuation models that the forecasting of a firm's financial performance is split into two periods. The first period, appropriately labelled the explicit forecast period, consists of perhaps 10–15 individual years. It provides forecasted annual values of the selected valuation attribute (i.e., cash flow from operations in the case of the discounted cash flow model), typically derived from forecasted annual income statements and balance sheets. The second period, often referred to as the post-horizon period, is represented through an infinite discounting operation that provides a continuing value at the beginning of the first post-horizon year. The idea is that the firm is developing in a steady state, as it moves into the post-horizon period. This means that the selected valuation attribute is extrapolated, under an assumption about constant firm growth, starting from the value that is attained in the last year of the explicit forecast period. In other words, the selected valuation attribute is not derived from annual forecasted financial statements in the post-horizon period (except possibly for the first year of that period).[3]

The purpose of this paper is to examine the finer structure of the continuing value formula in the discounted cash flow model. It is clear from the definition of annual free cash flow from operations that several components are included. Two of those components, capital expenditures and tax savings due to depreciation of property, plant and equipment (PPE), are emphasized, since they are somewhat complex. One reason for this complexity is that capital expenditures may be lumpy, so one needs to be clear about the meaning of steady-state development of free cash flow in the post-horizon period. Another reason is a need for precision in relating steady-state capital expenditures and associated tax effects to more basic parameters such as assumed real growth of the firm, inflation, and equipment economic life. The emphasis on capital expenditures and tax savings from depreciation appears to be novel. For instance, this paper is rather different from the discussion of continuing value in the discounted cash flow model by Copeland et al. (2000, Chapter 12).

The outline is as follows. The next section discusses steady-state free cash flow from operations, with particular emphasis on the two components mentioned, capital expenditures and tax savings from depreciation of PPE. It is shown how these two components can be modelled in a reasonably consistent fashion, starting from assumptions about steady-state real growth of the firm's operations, inflation, PPE economic life, PPE life for depreciation for tax purposes, and capital intensity. Two of the parameters just mentioned, PPE economic life and capital intensity, are not necessarily immediately observable from the published financial statements of an individual company.[4] However, estimates can be obtained, if one has access to aggregated financial statements for a large set of companies. This is discussed in Section 3, based on an extensive data set from Statistics Sweden. Section 4 completes the continuing value formula for the firm's operations by adding the discounting part. There is also a brief comparison with a continuing value formula in Copeland et al. (2000, Chapter 12).

The continuing value formula is broken down into further components in Section 5. It is now possible to identify groups of components relating to the continuation of the firm's existing activities, as opposed to components relating to real growth. One can then study the relationship between value derived from the firm's existing PPE and value from replacing existing PPE as it wears out, or value from growth due to capital expenditures beyond what is required to replace existing PPE. It is also possible to identify the error in the continuing value formula that is due to the steady-state assumption. In other words, there is a more exact continuing value formula that takes into account the actual capital expenditures assumed to have taken place in the

[3] The terminology explicit forecast period, post-horizon period, and continuing value is used for instance by Copeland et al. (2000) and by Ohlson and Zhang (1999). Annual forecasts of the valuation attribute should be interpreted as expected values under some scenario.

[4] Published financial statements are also referred to as historical statements in what follows.

explicit forecast period. Those capital expenditures are typically not exactly consistent with a steady-state development of free cash flow in the post-horizon period. Section 6 contains concluding remarks, in particular on implications for the design of the explicit forecast period.

Some of the equations in this paper may look somewhat complex. However, they are applications of nothing more advanced than summations of finite series.

2. Components of free cash flow in a steady state

Suppose that the nominal sales revenue of the firm in the last year of the explicit forecast period is S. Sales revenue represents the activity level of the firm, so all components of free cash flow from operations are related to, or driven by, sales revenue in one way or another. Sales revenue is assumed to increase year by year in the post-horizon period due to real growth g and inflation i. This means that there is a nominal growth of sales revenue of $c = (1 + g)(1 + i) - 1$ in every year of the post-horizon period. In particular, the sales revenue in the first year of that period is $S(1 + c)$.[5]

We now want to relate the steady-state (i.e., in the post-horizon period) development of PPE to the development of sales revenue. Such a relationship is necessary for the following obvious reason: Capital expenditures, which are one component of free cash flow, are equal to this year's net PPE minus last year's net PPE plus this year's depreciation (this is simply an accounting identity; net PPE is equal to gross PPE minus accumulated depreciation). Steady-state development of the firm means that capital expenditures increase by the assumed nominal growth rate c in every year of the post-horizon period. For this to happen, there must be a particular *age distribution* of successive PPE cohorts. More precisely, it must hold that the nominal acquisition value of each PPE cohort increases in line with the assumed nominal growth rate c. In fact, the assumption is that this particular age structure holds already in the last year of the explicit forecast period, i.e., as the firm moves into the post-horizon period.

Proceeding with the steady-state relationship between PPE and sales revenue, there is apparently a need for a specification of how necessary PPE is related to the firm's activity level. It is assumed that *real gross* PPE is related to sales revenue through a capital intensity factor K. In other words, real gross PPE must be equal to sales revenue multiplied by K. Real means expressed in the value of money of the current year in question. Real revenue is equal to nominal revenue for the current year. Real gross PPE means nominal gross PPE adjusted for inflation. Such an adjustment implies revaluing each PPE cohort, through multiplication by a factor that expresses accumulated inflation since that cohort was acquired. The assumption that gross rather than net PPE is related to revenue implies that each piece of PPE is 100% productive until the end of its economic life. At that point in time, it suddenly ceases to function and is retired. This seems like a somewhat more intuitive hypothesis than the alternative, that net PPE is related to revenue, since that would mean that the productivity of each piece of PPE is proportional to its remaining economic life.

We denote by M the steady-state ratio between this year's nominal gross PPE and (nominal) sales revenue. Let H denote steady-state accumulated depreciation as a fraction of nominal gross PPE. $M(1 - H)$ is then the steady-state ratio between nominal net PPE and (nominal) sales revenue. Suppose that a is the acquisition value of the last PPE cohort, which has just been purchased at the end of the last year of the explicit forecast period. That acquisition value is the nominal *and* real one, expressed in current monetary units. Suppose also that the economic life of each PPE cohort is n (integer) years. It is assumed that $n \geqslant 2$. As a cohort becomes n years old, it is retired (and hence also deleted from gross PPE and from accumulated depreciation). Given the steady-state assumption, which implies that the acquisition values of previous cohorts have increased in real

[5] Inflation i is assumed to be non-negative. Assuming negative inflation in the post-horizon period, i.e., over an infinite period, is simply not credible. Cf. Copeland et al. (2000, pp. 242–244), on how to estimate expected inflation from capital market data. Real growth g is also assumed to be non-negative. If one were really considering a scenario with negative real growth in the post-horizon period, then one would want to close down the firm eventually, so a break-up valuation would be more relevant than a going concern valuation (as implied by a continuing value in the post-horizon period). Real growth of the firm in the post-horizon period cannot be higher than the long-term real growth of the surrounding economy as a whole.

terms by the real growth rate g from year to year, the real value of gross PPE is hence $F_g \cdot a$, where (v obviously denotes cohort age)[6]

$$F_g = \sum_{v=0}^{n-1} \left(\frac{1}{1+g}\right)^v = \frac{1+g-(1+g)^{-(n-1)}}{g} \quad \text{if } g > 0; \quad F_g = n \text{ if } g = 0.$$

The physical requirement for gross PPE then implies that

$$F_g \cdot a = K \cdot S.$$

Similarly, the nominal value of gross PPE at the end of the current year, under the steady-state assumption, is $F_c \cdot a$, where

$$F_c = \sum_{v=0}^{n-1} \left(\frac{1}{1+c}\right)^v = \frac{1+c-(1+c)^{-(n-1)}}{c} \quad \text{if } c > 0; \quad F_c = n \text{ if } c = 0.$$

Consequently,

$$F_c \cdot a = M \cdot S.$$

Solving for M,

$$M = (F_c/F_g) \cdot K.$$

Accumulated depreciation as a fraction of gross PPE in a steady state, H, can be written as (using (1) with $Q = n-1$):

$$\begin{aligned} H &= \frac{\sum_{v=0}^{n-1}\left[\left(\frac{1}{1+c}\right)^v \cdot \frac{v}{n}\right]}{F_c} = \frac{\frac{-n(1+c)^{-(n-1)}(1-(1+c)^{-1})+(1-(1+c)^{-n})}{(1-(1+c)^{-1})^2} \cdot \frac{1}{1+c} \cdot \frac{1}{n}}{F_c} = \frac{\frac{1+c-(nc+1)(1+c)^{-(n-1)}}{c^2 n}}{F_c} \\ &= \frac{1}{cn} - \frac{1}{(1+c)^n - 1} \quad \text{if } c > 0; \quad H = \frac{n-1}{2n} \quad \text{if } c = 0. \end{aligned} \tag{2}$$

The steady-state ratio between this year's net PPE and sales revenue is then

$$M(1-H). \tag{3}$$

This steady-state ratio apparently depends on four parameters, the real growth rate g, the inflation rate i (since c depends on g and i), the capital intensity factor K, and the economic life n of the PPE.

Next, we need a ratio for depreciation. Depreciation is applied linearly (that is, by $1/n$ of the acquisition value each year) to last year's gross PPE. The steady-state ratio between depreciation and last year's net PPE is

$$\frac{1}{n} \cdot \frac{1}{1-H}. \tag{4}$$

This steady-state ratio apparently depends on two parameters only, c and n. That is, it does not depend on g and i separately. All that matters for depreciation is nominal growth c, not how that growth comes about due to different combinations of real growth g and expected inflation i.

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[6] For convenience, the reader is reminded of the summation formula that is used for F_g and F_c and also in subsequent formulas:

$$\sum_{v=0}^{Q} x^v = \frac{1-x^{Q+1}}{1-x} \quad (x \neq 1).$$

The following summation formula is also used below

$$\sum_{v=0}^{Q} x^v v = \frac{d}{dx}\left(\sum_{v=0}^{Q} x^v\right) \cdot x = \frac{-(Q+1)x^Q(1-x) + (1-x^{Q+1})}{(1-x)^2} \cdot x \quad (x \neq 1). \tag{1}$$

Values for $c = 0$ and $g = 0$ are obtained using l'Hospital's rule.

Combining Eqs. (3) and (4), we obtain capital expenditures in the first year of the post-horizon period as

$$SM(1-H)\left(c+\frac{1}{n}\cdot\frac{1}{1-H}\right). \tag{5}$$

Capital expenditures in that year are, indeed, seen to be equal to the increase in net PPE between the last explicit forecast year and the first post-horizon year plus depreciation in the first post-horizon year. In subsequent post-horizon years, capital expenditures increase in a steady-state fashion, i.e., by the nominal growth rate c every year. This is the first free cash flow component.

The second component of free cash flow is tax savings due to depreciation of PPE.[7] In the financial reporting to shareholders and other interested parties, depreciation is deducted over the economic life of the PPE, n years. However, it is assumed that linear depreciation over q years is deducted in the tax accounting ($1 \leqslant q \leqslant n$). This means that taxes actually paid in the current year do not agree with computed taxes in the financial reporting. The difference is added to (or subtracted from) *deferred taxes*, an item on the liabilities side of the balance sheet.[8] Deferred taxes are equal to *timing differences* multiplied by the tax rate. For a given piece of PPE that is about to be retired, accumulated depreciation according to the tax accounting and accumulated depreciation over the economic life are both equal to the original acquisition value. Consequently, non-zero timing differences are related to non-retired pieces of PPE only. We need a steady-state ratio between timing differences and this year's net PPE. That ratio is

$$\frac{J}{F_c(1-H)}, \tag{6}$$

where

$$\begin{aligned} J &= \sum_{v=0}^{q-1}\left(\left(\frac{1}{1+c}\right)^v\cdot\frac{v}{q}\right)+\sum_{v=q}^{n-1}\left(\frac{1}{1+c}\right)^v-\sum_{v=0}^{n-1}\left(\left(\frac{1}{1+c}\right)^v\cdot\frac{v}{n}\right) \\ &= \frac{1+c-(qc+1)(1+c)^{-(q-1)}}{c^2q}+\frac{1+c-(1+c)^{-(n-q-1)}}{c}\cdot\frac{1}{(1+c)^q}-\frac{1+c-(nc+1)(1+c)^{-(n-1)}}{c^2n} \end{aligned}$$

if $c > 0$ (see (2) for part of the derivation). The first term in J represents accumulated fiscal depreciation for PPE cohorts that have not yet been written down to zero for tax purposes, the second term accumulated fiscal depreciation for those PPE cohorts that have already been written down to zero for tax purposes but have not yet been retired, and the third term accumulated depreciation over the economic life for PPE cohorts that have not yet been retired. If $c = 0$, then

$$J = 0.5(q-1)+(n-q)-0.5(n-1).$$

The steady-state ratio (6) apparently depends on c, n, and q. Now let τ denote the tax rate. Using (6), one obtains the total tax savings from depreciation of PPE in the first year of the post-horizon period as

$$SM(1-H)\frac{1}{n}\cdot\frac{1}{1-H}\tau+cSM(1-H)\frac{J}{F_c(1-H)}\tau, \tag{7}$$

where the first term is apparently the tax reduction from depreciation over the economic life, and the second term the additional tax reduction from the increase in deferred taxes. Again, this total tax reduction increases by the nominal growth rate c in each subsequent post-horizon year.

It is not difficult to show that (7) can be rewritten as

$$SM\frac{F_c^\tau}{F_c}\cdot\frac{1}{q}\tau,$$

[7] It is assumed that the company is in a tax-paying situation. This is a reasonable assumption. For instance, if NOPLAT is negative, then so is free cash flow when $i \geqslant 0$ and $g \geqslant 0$ (see the definition of NOPLAT at the end of this section). Under such a scenario, the company would be closed down, so a continuing value would not be relevant.

[8] Deferred taxes are not viewed as debt in the discounted cash flow model. Rather, they are viewed as part of equity and are hence not subtracted from the computed value of the firm's operations to obtain the equity value as a residual. Cf. Brealey and Myers, 2003, p. 528.

where

$$F_c^\tau = \sum_{v=0}^{q-1}\left(\frac{1}{1+c}\right)^v = \frac{1+c-(1+c)^{-(q-1)}}{c} \quad \text{if } c>0; \quad F_c^\tau = q \text{ if } c=0.$$

This alternative formula calculates the tax reduction from depreciation of PPE in one single operation (depreciation $1/q$ is directly applied to the sum of nominal acquisition values for PPE cohorts that have not already been written down to zero for tax purposes, $SM[F_c^\tau/F_c]$). The first formula (7) is useful, since in many cases it agrees with published financial statements, for instance in Swedish company groups. That is, deferred taxes are given as a balance sheet item, whereas the sum of nominal acquisition values for PPE cohorts that remain on the books for tax purposes is typically not provided.[9]

A third free cash flow component also needs to be specified. Let steady-state cash costs as a fraction of sales revenue be denoted by z. After-tax sales revenue minus cash costs in the first post-horizon year is hence $S(1+c)(1-z)(1-\tau)$. Also, there is additional investment in working capital.[10] Unlike PPE, working capital turns over very quickly and does not wear out (in other words, is not depreciated). It is hence a reasonable (and standard) assumption that working capital is proportional to sales revenue. If sales revenue increases, there is a need for additional investment in working capital. It is not important in this connection whether sales revenue increases due to real growth or inflation, or a combination of both. Let the steady-state ratio of working capital to sales revenue be denoted by w. With a nominal growth rate of c, the additional investment in working capital in the first post-horizon year is hence cSw. Grouping after-tax sales revenue minus cash costs and additional investment in working capital together, one obtains the third free cash flow component

$$S(1+c)(1-z)(1-\tau) - cSw. \tag{8}$$

Adding up the three components (5), (7), and (8) and rewriting slightly, we obtain free cash flow from operations in the first post-horizon year (more precisely, at the end of that year). For later reference, it is denoted FCF_{T+1}, where T is the index for the last explicit forecast year:

$$\begin{aligned}\mathrm{FCF}_{T+1} = {} & \left(S(1+c)(1-z) - SM(1-H)\frac{1}{n}\cdot\frac{1}{1-H}\right)(1-\tau) + cSM(1-H)\frac{J}{F_c(1-H)}\tau \\ & + SM(1-H)\frac{1}{n}\cdot\frac{1}{1-H} - cSw - SM(1-H)\left(c+\frac{1}{n}\cdot\frac{1}{1-H}\right).\end{aligned} \tag{9}$$

The first term plus the second term in (9) constitute NOPLAT (net operating profit less adjusted taxes). The third term adds back depreciation (which is not a cash cost). The last two terms constitute gross investment, that is, investment in additional working capital plus capital expenditures (cf. Copeland et al., 2000, pp. 163–169).[11] The steady-state assumption implies that (9) increases by c in each subsequent post-horizon year.

[9] In other words, there is a difference between the beginning of the explicit forecast period and the beginning of the post-horizon period. What is available at the start of the explicit forecast period is incomplete information from historical financial statements. At the beginning of the post-horizon period, on the other hand, the complete capital expenditures history for all PPE cohorts that have not yet been retired is known, if the explicit forecast period is at least n years. This follows, since those capital expenditures are generated by the analyst doing the valuation, as part of the scenario under consideration. Hence, the sum of nominal acquisition values for PPE cohorts that remain on the books for tax purposes is also known at the beginning of the first post-horizon year. This difference is further commented on in the concluding Section 6 of this paper.

[10] Working capital is equal to working capital assets minus working capital liabilities. The most important working capital liability is trade credit. Working capital liabilities are considered to pertain to the operations of the firm, not the financing. The reason is that they are paid for not through interest on money borrowed, but rather through higher prices on raw materials purchased (for instance). Working capital liabilities are hence not subtracted from the computed value of the firm's operations in the calculation of the equity value as a residual.

[11] It is seen that (9) can be simplified by eliminating a number of $(1-H)$ factors. This is not done here, for consistency with the explicit forecast period. It is the recommended procedure to forecast net PPE, not gross PPE, as a percentage of sales revenue in the explicit forecast period (see Copeland et al., 2000, pp. 241–242; Jennergren, 2002, pp. 32–36). Also, net PPE is always listed in historical balance sheets for the company that is the object of the valuation exercise, but not always gross PPE. In other words, information about gross PPE may not be easily available at the beginning of the explicit forecast period.

3. Estimates of n and K based on data from statistics Sweden

There are apparently eight parameters that need to be specified for the formula (9) for the free cash flow from operations in the first post-horizon year: The real growth rate g, expected inflation i, cash costs as a fraction of sales revenue z, the tax rate τ, the capital intensity K, PPE economic life n, PPE life for fiscal depreciation q, and the ratio of working capital to sales revenue w. Two parameters that seem somewhat elusive are n and K. A data set from Statistics Sweden (SCB) will now be used to shed light on these two parameters.

Statistics Sweden regularly publishes accounting data for Swedish companies. Prior to 1996, these data are based on information from all companies with at least 50 employees; and from 1996, on information from all companies without size limitation. Aggregated income statements and balance sheets are presented for different industries, classified by SNI numbers. (SNI is the Swedish national industrial classification system.) The data that are used here pertain to the years 1994–1998 and are taken from the publications *Företagen 1994* (Enterprises 1994), *Företagen 1995* (Enterprises 1995), *Ekonomisk redogörelse för företagen 1996* (Financial Accounts for Enterprises 1996), *Ekonomisk redogörelse för företagen 1997* (Structural Business Statistics 1997), and *Ekonomisk redogörelse för företagen 1998* (Structural Business Statistics 1998). These publications also include descriptions (in English as well as in Swedish) of the data and how they were collected.

If one can provide exogenous estimates of g and i, then steady-state estimates of n and K can be computed using the following equations (cf. Eqs. (4) and (3) in Section 2):

$$\text{depreciation}_t = (\text{net PPE})_{t-1} \times \frac{1}{n} \cdot \frac{1}{1-H},$$

$$\text{depreciation}_t = (\text{net PPE})_t \times \frac{1}{1+c} \cdot \frac{1}{n} \cdot \frac{1}{1-H},$$

$$\frac{\text{depreciation}_t}{(\text{net PPE})_t} = \frac{1}{1+c} \cdot \frac{1}{n} \cdot \frac{1}{1 - \left(\frac{1}{cn} - \frac{1}{(1+c)^n - 1}\right)}, \tag{10}$$

$$\frac{(\text{net PPE})_t}{(\text{sales revenue})_t} = M(1-H),$$

$$\frac{(\text{net PPE})_t}{(\text{sales revenue})_t} = \frac{\frac{1+c-(1+c)^{-(n-1)}}{c}}{\frac{1+g-(1+g)^{-(n-1)}}{g}} K\left(1 - \left(\frac{1}{cn} - \frac{1}{(1+c)^n - 1}\right)\right). \tag{11}$$

The left hand side of Eq. (10) can be observed empirically from the Statistics Sweden data, for a given year t and a given industry (according to SNI number).[12] With g and i and hence $c = (1+g)(1+i) - 1$ given, the PPE economic life n can be found as that integer value of n for which the right hand side of (10) is the closest to the left hand side. This estimate of PPE economic life is valid, since the data in the left hand side refer to *the whole industry*, not merely to one company. That is, a single company could, e.g., depreciate over a shorter time period than the economic life, but it is not likely that the whole industry would do that in a given year. With that value of n, one then turns to Eq. (11). The left hand side of (11) can, again, be observed empirically for a given year t and a given industry. K can then be estimated by setting the right hand side of (11) equal to the left hand side. As before, this is a plausible procedure, since the data refer to the totality of companies within one industry: It is more reasonable to imagine that the whole industry is in a steady state than an individual company.[13]

[12] To be more precise: depreciation$_t$ and (net PPE)$_t$ are given in the data base. Depreciation as given in the data base includes depreciation on non-tangible assets (e.g., goodwill). There is a break-down of net PPE that distinguishes such assets. That part of depreciation that refers to non-tangible assets has been eliminated by assuming that non-tangible assets are written off over 12 years.

[13] If $c = 0$, Eqs. (10) and (11) become:

$$\frac{\text{depreciation}_t}{(\text{net PPE})_t} = \frac{1}{n} \cdot \frac{1}{1 - \frac{n-1}{2n}},$$

$$\frac{(\text{net PPE})_t}{\text{revenues}_t} = K\left(1 - \frac{n-1}{2n}\right).$$

Table 1
Estimated n and K using the Statistics Sweden data base

SNI no.	Industry	n						K					
		1994	1995	1996	1997	1998	1994–1998	1994	1995	1996	1997	1998	1994–1998
01–05	Agricultural, forestry, fishing	16	17	16	12	11	14	0.56	0.48	0.67	0.52	0.55	0.55
10–14	Mining and quarrying	12	11	13	13	14	12	1.11	1.08	1.20	1.26	1.45	1.21
15–16	Food, beverages, tobacco	13	14	12	12	13	13	0.36	0.37	0.38	0.38	0.42	0.38
17–19	Textiles, etc.	11	12	12	12	12	12	0.36	0.34	0.36	0.35	0.38	0.36
20	Wood and wood products	13	15	14	15	15	15	0.51	0.55	0.61	0.59	0.60	0.57
21	Pulp and paper products	23	24	24	21	18	22	1.39	1.33	1.71	1.55	1.54	1.50
22	Publishing and printing	10	10	10	8	8	9	0.35	0.36	0.36	0.32	0.31	0.34
23–24	Chemicals, petroleum	14	14	15	14	14	14	0.62	0.64	0.80	0.80	0.78	0.73
25	Rubber and plastic pr.	11	11	11	10	10	11	0.44	0.41	0.47	0.45	0.46	0.45
26	Non-metallic mineral pr.	11	11	11	11	12	11	0.47	0.44	0.47	0.46	0.46	0.46
27	Basic metall industries	14	15	15	16	15	15	0.47	0.44	0.55	0.58	0.65	0.53
28	Fabricated metall pr.	10	11	11	11	12	11	0.34	0.39	0.39	0.42	0.45	0.40
29	Machinery, equipment	9	9	9	9	8	9	0.24	0.23	0.23	0.24	0.23	0.23
30–33	Electrical and optical	8	7	7	6	6	7	0.21	0.18	0.17	0.15	0.14	0.17
34–35	Transport equipment	12	10	9	10	10	10	0.38	0.32	0.36	0.37	0.35	0.36
36–37	Other manufacturing	11	11	12	10	10	11	0.33	0.29	0.30	0.30	0.29	0.30
40–41	Electricity, gas, water	28	28	28	26	28	27	2.63	2.27	2.47	2.59	2.75	2.53
45	Construction	14	14	13	20	20	16	0.32	0.33	0.26	0.45	0.44	0.36
51	Wholesale trade	9	10	10	9	9	9	0.10	0.10	0.11	0.10	0.10	0.10
52	Retail trade	11	11	11	9	10	10	0.13	0.13	0.13	0.11	0.11	0.12
55	Hotels and restaurants	13	14	14	14	13	14	0.44	0.48	0.54	0.58	0.53	0.52
60	Land transportation	9	9	10	12	13	11	0.64	0.64	0.67	0.96	1.02	0.81
61	Sea transportation	21	25	23	24	20	23	1.08	1.30	1.35	1.04	0.94	1.14
62	Air transportation	17	16	18	19	19	18	0.78	0.68	0.87	0.88	0.97	0.84
74	Other (consulting)	7	7	10	8	9	8	0.21	0.20	0.27	0.25	0.23	0.23

Estimated values of n and K are given in Table 1, for a variety of industries. These values are given separately for the years 1994, 1995, 1996, 1997, and 1998, based on data from each one of these years. Estimated n and K are also given for the whole period 1994–1998, i.e., by pooling these five subsequent years and considering them as one observation. Throughout Table 1, the assumed values of g and i are 0.01 and 0.02, respectively.

There are thus six different estimates of n and K for each industry in Table 1. It is noted that there is some instability between years in estimated n values for a few industries, for instance construction. There is also some instability as regards estimated K values in a few cases, e.g., land transportation and sea transportation. However, for many industries estimated n and K values are fairly stable.

Based on experiences from valuation projects undertaken by the author, and by students, some of the K values in Table 1 appear to be on the high side. However, the table provides a clear differentiation between industries that corresponds reasonably well to common-sense considerations about capital intensity. That is, certain industries, like electricity, gas, and water supply, and pulp and paper products, are very capital-intensive, with long-lived PPE. Others, such as retail trade, and electrical and optical, are seen to be light industries when it comes to requirements for gross PPE.

4. Discounting to obtain continuing value, and a comparison with Copeland et al. (2000)

Free cash flow according to (9) is discounted by the firm's nominal weighted average cost of capital (WACC) to obtain the continuing value of operations at the beginning of the first post-horizon year. Let f denote the desired equity weight in WACC and $(1 - f)$ the weight for interest-bearing debt. Nominal WACC is denoted by $r^\star$ and is given by the well-known formula

$$r^{\star} = fr_{\mathrm{E}} + (1-f)r_{\mathrm{D}}(1-\tau), \tag{12}$$

where r_{E} and r_{D} are required nominal rates of return on the firm's equity and debt, respectively. As is also well known, the weight f should be in *market value terms*.[14] It is commonly assumed that the interest-bearing debt is contracted on market conditions, i.e., the borrowing rate is fair. The required rate of return on the debt r_{D} can then be approximated by the borrowing rate. In other words, r_{D} is equal to the firm's borrowing rate, and the market value of the interest-bearing debt, D, is equal to its book value (cf. Lundholm and O'Keefe, 2001, p. 325). The continuing value of the firm's operations, denoted CNTVAL, is

$$\mathrm{CNTVAL} = \frac{\mathrm{FCF}_{T+1}}{r^{\star} - c}, \tag{13}$$

where FCF_{T+1} is given by Eq. (9). This is evidently just the ordinary Gordon formula. Continuing value consists of the three components

$$\frac{S(1+c)(1-z)(1-\tau) - cSw}{r^{\star} - c}, \tag{14}$$

$$\frac{SM(1-H)\left[c + \frac{1}{n}\cdot\frac{1}{1-H}\right]}{r^{\star} - c}, \tag{15}$$

$$\frac{SM(1-H)\frac{1}{n}\cdot\frac{1}{1-H}\tau + cSM(1-H)\frac{J}{F_{\mathrm{c}}(1-H)}\tau}{r^{\star} - c} \tag{16}$$

corresponding to the three free cash flow components (8), (5), and (7). In other words, (13) is equal to (14) minus (15) plus (16).

The value of the firm's equity, E, at the outset of the post-horizon period is the residual

$$E = \mathrm{CNTVAL} - D = \frac{\mathrm{FCF}_{T+1}}{r^{\star} - c} - D. \tag{17}$$

It is assumed in (17) that D is set so that $E/(D+E) = f$ (and $D/(D+E) = (1-f)$). In other words, the equity weight that results from (17) should be equal to f, the desired weight that is used in deriving E as a residual in (17). In fact, it is assumed in (13) that the interest-bearing debt is adjusted in every post-horizon year, in order to maintain the desired capital structure that is expressed by f. This is the *rebalancing assumption* in Brealey and Myers (2003, pp. 535–536) (cf. also Lundholm and O'Keefe, 2001, p. 331).

As for the determination of r_{E} and r_{D}, the discounted cash flow model (as well as other firm valuation models based on accounting data) is fairly silent (cf. Penman, 2001, p. 691). In other words, the discounting parts of firm valuation models are weak. Presumably, r_{E} and r_{D} are not independent of the desired capital structure that is expressed by f. Also, expected inflation i enters as one component of r_{E} and r_{D}. These matters can be brushed aside here, since all that is needed is the nominal WACC $r^{\star}$ that is used for discounting free cash flow from the firm's operations.

Copeland et al. (2000, p. 270) recommend the following value-driver formula for the continuing value of the firm's operations at the beginning of the post-horizon period:

$$\mathrm{CNTVAL} = \frac{\mathrm{NOPLAT}_{T+1}(1 - c/\mathrm{ROIC}_{\mathrm{I}})}{r^{\star} - c}.$$

ROIC means return on invested capital. $\mathrm{ROIC}_{\mathrm{I}}$ according to Copeland et al. is the expected rate of return on new (Incremental) investment and is equal to the increase in NOPLAT between years $T+1$ and $T+2$ divided by the increase in invested capital between years T and $T+1$. Invested capital is the sum of working capital and net PPE. Hence,

[14] WACC in (12) is a weighted average of *nominal* after-tax required rates of return. A slightly higher WACC is obtained by taking a weighted average of *real* after-tax required rates and then adjusting for expected inflation. Cf. Howe (1992).

$$\mathrm{ROIC_I} = \left\{c\left[\left(S(1+c)(1-z) - SM(1-H)\frac{1}{n}\cdot\frac{1}{1-H}\right)(1-\tau) + cSM(1-H)\frac{J}{F_c(1-H)}\tau\right]\right\}$$
$$\div \{c[Sw + SM(1-H)]\}$$
$$= \frac{\mathrm{NOPLAT}_{T+1}}{Sw + SM(1-H)}.$$

In other words, continuing value equals

$$\mathrm{CNTVAL} = \frac{\mathrm{NOPLAT}_{T+1} - c[Sw + SM(1-H)]}{r^\star - c}.$$

Although the starting point of Copeland et al. is rather different, their value-driver formula (as interpreted here) is apparently only a differently written version of the continuing value formula (13) above. For the special case where $\mathrm{ROIC_I} = r^\star$, $\mathrm{CNTVAL} = \mathrm{NOPLAT}_{T+1}/r^\star$, as also mentioned by these authors (Copeland et al., 2000, p. 282).

5. No-real growth and real growth components of continuing value, and an extension to non-steady-state capital expenditures

The continuing value of the firm's operations will now be further decomposed. In fact, (14) will be decomposed into (18), (20), and (23) below. (15) will be decomposed into (21) and (24); and (16) into (19), (22), and (25). Hence, the continuing value is rewritten as the sum of eight components. These eight components can be sorted into groups of components that have to do with production activities associated with already existing PPE cohorts, production activities associated with replacing previously acquired pieces of PPE as they are retired at the end of the economic life (i.e., maintaining the current real level of production), and production activities associated with capital expenditures for real growth.

(i) Present value of after-tax sales revenue minus cash costs, and working capital investment/disinvestment, from already existing PPE cohorts:

$$\sum_{v=0}^{n-2}\left\{SM\frac{1}{F_c}\cdot\frac{1}{(1+g)^v}\cdot\frac{1}{K}\sum_{t=1}^{n-(v+1)}\left[\left((1-z)(1-\tau)-\frac{i}{1+i}w\right)(1+i)^t\frac{1}{(1+r^\star)^t}\right]\right\}$$
$$+\sum_{v=0}^{n-1}\left[SM\frac{1}{F_c}\cdot\frac{1}{(1+g)^v}\cdot\frac{1}{K}w(1+i)^{n-(v+1)}\frac{1}{(1+r^\star)^{n-v}}\right]$$
$$= SM\frac{1}{F_c}\cdot\frac{1}{K}\left((1-z)(1-\tau)-\frac{i}{1+i}w\right)\frac{1+i}{r^\star-i}\left\{A-\left(\frac{1+i}{1+r^\star}\right)^{n-1}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^{n-1}}{1-\frac{1+r^\star}{1+c}}\right\}+SM\frac{1}{F_c}$$
$$\cdot\frac{1}{K}w\frac{1}{1+i}\left(\frac{1+i}{1+r^\star}\right)^n\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^n}{1-\frac{1+r^\star}{1+c}}, \tag{18}$$

where

$$A = \frac{1+g-(1+g)^{-(n-2)}}{g} \quad \text{if } g>0; \quad A = n-1 \text{ if } g = 0.$$

(ii) Present value of tax savings from fiscal depreciation of already existing PPE cohorts:

$$\sum_{v=0}^{q-1}\left[SM\frac{1}{F_c}\cdot\frac{1}{(1+c)^v}\cdot\frac{1}{q}\tau\frac{1-(1+r^\star)^{-(q-v)}}{r^\star}\right] = SM\frac{1}{F_c}\cdot\frac{1}{q}\tau\frac{1}{r^\star}\left(F_c^\tau - \frac{1}{(1+r^\star)^q}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^q}{1-\frac{1+r^\star}{1+c}}\right). \tag{19}$$

(iii) Present value of after-tax sales revenue minus cash costs, and investment in working capital, from maintaining the current real productive capacity (i.e., replacing already existing PPE cohorts over an infinite period):

$$\frac{S(1+i)(1-z)(1-\tau)-iSw}{r^\star-i}-\left[SM\frac{1}{F_\mathrm{c}}\cdot\frac{1}{K}\left((1-z)(1-\tau)-\frac{i}{1+i}w\right)\frac{1+i}{r^\star-i}\right.$$
$$\left.\times\left\{A-\left(\frac{1+i}{1+r^\star}\right)^{n-1}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^{n-1}}{1-\frac{1+r^\star}{1+c}}\right\}+SM\frac{1}{F_\mathrm{c}}\cdot\frac{1}{K}w\frac{1}{1+i}\left(\frac{1+i}{1+r^\star}\right)^{n}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^{n}}{1-\frac{1+r^\star}{1+c}}\right],\tag{20}$$

where

$$A=\frac{1+g-(1+g)^{-(n-2)}}{g}\quad\text{if } g>0;\quad A=n-1\ \text{ if } g=0.$$

(iv) Present value of capital expenditures for maintaining the current real productive capacity:

$$\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^{n}}\sum_{v=0}^{n-1}\left[SM\frac{1}{F_\mathrm{c}}\cdot\frac{1}{(1+g)^{v}}(1+i)^{n-v}\cdot\frac{1}{(1+r^\star)^{n-v}}\right]$$
$$=\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^{n}}\cdot SM\frac{1}{F_\mathrm{c}}\left(\frac{1+i}{1+r^\star}\right)^{n}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^{n}}{1-\frac{1+r^\star}{1+c}}.\tag{21}$$

(v) Present value of tax savings from fiscal depreciation of capital expenditures for maintaining the current real productive capacity:

$$\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^{n}}\cdot SM\frac{1}{F_\mathrm{c}}\left(\frac{1+i}{1+r^\star}\right)^{n}\cdot\frac{1-\left(\frac{1+r^\star}{1+c}\right)^{n}}{1-\frac{1+r^\star}{1+c}}\cdot\frac{1}{q}\tau\frac{1-(1+r^\star)^{-q}}{r^\star}.\tag{22}$$

(vi) Present value of after tax sales revenue minus cash costs, and investment in working capital, from real growth (over an infinite period):

$$\frac{S(1+c)(1-z)(1-\tau)-cSw}{r^\star-c}-\frac{S(1+i)(1-z)(1-\tau)-iSw}{r^\star-i}.\tag{23}$$

(vii) Present value of capital expenditures for real growth:

$$\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^{n}}\cdot\frac{SM\frac{1}{F_\mathrm{c}}F_\mathrm{g}\cdot g(1+i)}{r^\star-c}.\tag{24}$$

(viii) Present value of tax savings from fiscal depreciation of capital expenditures for real growth:

$$\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^{n}}\cdot\frac{SM\frac{1}{F_\mathrm{c}}F_\mathrm{g}\cdot g(1+i)}{r^\star-c}\cdot\frac{1}{q}\tau\frac{1-(1+r^\star)^{-q}}{r^\star}.\tag{25}$$

In the outer summation of the first term in (18), v is the index for the age of not yet retired PPE cohorts. $v=0$ denotes the youngest cohort, acquired at the end of the last explicit forecast year. The economic life of the cohort with age $n-1$ has come to an end, so it has just been retired. Hence, the outer summation is from 0 to $n-2$. $SM(1/F_\mathrm{c})$ is the real (and also nominal) acquisition value of the youngest PPE cohort, acquired at the end of the last explicit forecast year. Real value refers to value of money at the end of the last explicit forecast year. Multiplying by $(1+g)^{-v}$ gives the real acquisition value of the PPE cohort of age v. Further multiplying by $1/K$ gives the real value of the productive capacity (i.e., amount of sales revenue) that is supported by the particular PPE cohort in question. The inner summation in the first term of (18) adds up the after-tax sales revenues minus cash costs, and minus necessary investment in additional working capital, for each remaining year of the economic life of that PPE cohort. For each year in the inner summation, there is an increase in nominal value of after-tax sales revenue minus cash costs, and minus necessary investment in additional working capital, due to inflation. Apparently, everything is discounted to the beginning of the first post-horizon year. The second term in (18) represents the recuperation of working capital. As one PPE cohort is retired, the associated sales revenue that is supported by that cohort ceases, and the working capital that is needed in the last year of the economic life of that PPE cohort can be recuperated in the following year.

In (19), there is a summation over PPE cohorts that have not already, at the end of the last explicit forecast year, been written down to zero for tax purposes. $SM(1/F_c)(1+c)^{-v}$ is the nominal acquisition value of the PPE cohort of age v. The nominal acquisition value is needed, since fiscal depreciation is based on that value. $(1/q)\tau$ is the annual factor for tax saved. $(1-(1+r^{\star})^{-(q-v)})/r^{\star}$ sums all of the present values of subsequent tax savings for the PPE cohort of age v.

Eqs. (20) and (23) are self-explanatory. Eqs. (21) and (24) start with a factor $(1-[(1+i)/(1+r^{\star})]^{n})^{-1}$. This factor derives from the fact that all future capital expenditures, whether for maintaining the current real productive capacity, or for real growth, have to be repeated in cycles of n years (PPE economic life), with nominal prices increasing by $(1+i)^n$ between cycles due to inflation. The summation in (21) represents the first PPE replacement cycle. For each value of the index v, the nominal amount of capital expenditures that is required for replacing the PPE cohort with that age v is calculated and discounted back to the beginning of that cycle (i.e., the beginning of the first post-horizon year). The right hand sides of Eqs. (18), (19), and (21) can be obtained by straightforward calculations.

In Eq. (24), $SM(1/F_c)F_g$ is the sum of real acquisition values of non-retired PPE cohorts at the end of the last explicit forecast year. Multiplying by $g(1+i)$ gives the nominal amount of capital expenditures that is necessary for real growth in the first post-horizon year. That amount increases by c in every subsequent year, as also evidenced by $r^{\star}-c$ in the denominator of the Gordon formula. Eqs. (22) and (25) follow directly from (21) and (24).

Eqs. (18) plus (19) hence represent that part of the continuing value of the firm's operations that derives from already existing PPE. Eqs. (20) minus (21) plus (22) represent that part of the continuing value that is associated with replacing existing PPE only to the extent that is necessary for maintaining the current real productive capacity, i.e., maintaining the real level of sales revenue that is attained in the last explicit forecast year. Eqs. (23) minus (24) plus (25) constitute that part of the continuing value that is associated with real growth of the firm's operations. The last three equations taken together hence constitute a rather precise statement of what is referred to in Brealey and Myers (2003, pp. 71–73), as PVGO (present value of growth opportunities).

One can show that (18) plus (19) plus (20) minus (21) plus (22) plus (23) minus (24) plus (25) are exactly equal to continuing value according to Eq. (13) in Section 4. In particular, (21) plus (24) are equal to (15) in Section 4, i.e., that part of (13) that is associated with capital expenditures. Similarly, (19) plus (22) plus (25) are equal to (16) in Section 4, that part of (13) that is associated with tax savings due to fiscal depreciation. Again, these calculations are straightforward (but tedious).

Three conclusions follow from the preceding decomposition of continuing value. In the first place, the scepticism of continuing value that one finds in some discussions of firm valuation may be misleading. Continuing value is viewed as suspect, because it materializes rather late, being the result of discounting over an infinite period. For instance, Penman (1998, p. 321) writes: "A model is seen as suspect if too much value is placed on the terminal value in the calculation of value." In connection with a highly simplified firm valuation example, Brealey and Myers (2003, p. 77) ask rhetorically: "But doesn't it make you just a little nervous to find that 119 percent of the value of the business rests on the horizon value?" Using this section's decomposition of continuing value into eight components, one can be more precise about how far into the future continuing value takes place. As a matter of fact, that part of continuing value that is associated with already existing PPE cohorts, i.e., Eqs. (18) plus (19), typically constitutes by far the main part of continuing value. One may interpret this to mean that most of continuing value takes place rather soon, in fact no later than n years into the post-horizon period. Hence, continuing value need not be so suspect after all.

In the second place, one can find that value of z (cash costs as a fraction of sales revenue) for which investment in working capital and PPE (for maintaining the current real productive capacity, and for real growth) has zero value. This can be done, for instance, by assuming some positive value of g and then setting (23) minus (24) plus (25) equal to zero and solving for z. If so, the internal rate of return on the investment in working capital and PPE is apparently equal to the discount rate $r^\star$. Incidentally, that value of z for which this equality holds is not the same z for which $\mathrm{ROIC_I}$ as defined by Copeland et al. (see Section 4 above) equals $r^\star$. The reason is that the definition of $\mathrm{ROIC_I}$ by these authors does not agree with the relationship between activity level (sales revenue) and working capital and PPE that is assumed here. In particular, it is assumed here that working capital and PPE are needed in the same year as sales revenue is generated, not one year before as implied by the definition of $\mathrm{ROIC_I}$.[15] One can also let the rate of return vary between working capital and PPE for maintaining the current real productive capacity, and for real growth. This can be done by replacing z in (18) and (20) by z_1 and z in (23) by z_2. In particular, one may let $z_2 > z_1$, so that real growth gets a lower rate of return than maintaining the current real productive capacity. According to Copeland et al., 2000, pp. 277–282, this is a reasonable hypothesis for companies in certain competitive industries.

In the third place, the extent of approximation that is implied by the steady-state assumption can be quantified. That approximation has to do with tax savings from fiscal depreciation of already existing PPE cohorts, and capital expenditures for maintaining the current real productive capacity and associated tax savings from fiscal depreciation. In fact, there are two assumptions: Firstly, the age distribution of different PPE cohorts is such that the nominal acquisition value of each cohort that is not yet retired as the company moves into the post-horizon period is $(1 + c)$ times that of the previous cohort, and the real acquisition value $(1 + g)$ times that of the previous cohort. Secondly, (total) real gross PPE in the last explicit forecast year is K times the sales revenue in that year.[16] The second assumption can be expected to hold, since one may set up the explicit forecast period to satisfy that assumption (cf. the following section). The first assumption, on the other hand, may be more or less violated in a given scenario. Let the actual (i.e., according to the explicit forecast period of the given scenario) nominal acquisition values of PPE cohorts in the last explicit forecast year that have not yet been written down to zero for fiscal purposes be denoted by N_v. Cohort age is denoted by v and goes from 0 (the youngest cohort acquired in the last explicit forecast year) to $q - 1$, so N_v is equal to nominal capital expenditures v years prior to the last explicit forecast year. Similarly, let the actual real acquisition values (in value of money at the end of the last explicit forecast year) of PPE cohorts that have not yet reached the end of the economic life be denoted by R_v. Here, v goes from 0 to $n - 1$, so R_v is equal to real capital expenditures v years before the last explicit forecast year. The inflation index that is involved in calculating the R_v could be based on assumed inflation that is different from i and even different from one explicit forecast year to another. Given that the explicit forecast period comprises at least n years, the N_v and R_v are *known to the analyst*, since it is she/he who generates the scenario.

The present value of the exact tax savings from fiscal depreciation of already existing PPE cohorts can then be written as

$$\sum_{v=0}^{q-1}\left[N_v \cdot \frac{1}{q}\tau\frac{1-(1+r^\star)^{-(q-v)}}{r^\star}\right], \tag{26}$$

which should be compared to (19) above. The present value of the exact capital expenditures for maintaining the current real productive capacity is

$$\frac{1}{1-\left(\frac{1+i}{1+r^\star}\right)^n}\sum_{v=0}^{n-1}\left[R_v(1+i)^{n-v}\cdot\frac{1}{(1+r^\star)^{n-v}}\right], \tag{27}$$

and the present value of tax savings associated with (27) from fiscal depreciation

[15] The same assumption as in this paper, that working capital and PPE are needed in the same year as sales revenue is generated, is actually made elsewhere in Copeland et al., 2000; in particular in the Heineken case study (Chapter 11).

[16] Using the notation R_v that is introduced later in this paragraph, the second assumption is hence $\sum_{v=0}^{n-1}R_v = KS$.

$$\frac{1}{1-\left(\frac{1+i}{1+r^{\star}}\right)^{n}}\sum_{v=0}^{n-1}\left[R_v(1+i)^{n-v}\cdot\frac{1}{(1+r^{\star})^{n-v}}\cdot\frac{1}{q}\tau\frac{1-(1+r^{\star})^{-q}}{r^{\star}}\right]. \tag{28}$$

Eqs. (27) and (28) should be compared to (21) and (22) above.

One can now compute an *exact* continuing value of the firm's operations at the beginning of the post-horizon period as (13) minus (19) plus (21) minus (22) plus (26) minus (27) plus (28). In other words, the difference between (13) and this exact continuing value is the approximation error that is inherent in the steady-state assumption. It seems that this approximation error is often small, at least if the explicit forecast period is implemented as indicated in the following concluding section.

6. Concluding remarks

The discussion in this paper of continuing value of a firm's operations at the outset of the post-horizon period may be briefly summarized as follows. Formula (13) in Section 4 above, with FCF_{T+1} given by (9) in Section 2, may be considered as fairly simple but even so somewhat consistent. More precisely, it does take into account the dependence of capital expenditures (and associated tax savings) on real growth, inflation, and economic life, and also on capital intensity. In so doing, it reflects the fact that PPE is long-lived, with depreciation being applied to original acquisition values but capital expenditures taking place at current (typically higher) equipment prices. However, (13) is not totally consistent. The underlying steady-state assumption disregards the fact that capital expenditures may be lumpy. The further decomposition of continuing value in Section 5 indicates how (13) can be corrected to obtain a continuing value formula that is totally consistent with the scenario that the analyst is investigating. Formula (13) is obviously very simple to implement in a spreadsheet valuation program. The different components of continuing value in Section 5 are a bit more complex, but they, too, are actually reasonably simple to use in such a program.

Two final remarks will be made in closing this paper. In the first place, it was mentioned in Section 5 that continuing value is sometimes viewed as suspect, since it takes place in the distant future. A related criticism is that it is hard to make assumptions about sales, capital expenditures, etc. in the post-horizon period, for the reason that one cannot know very much about things that are supposed to happen far into the future. The upshot is that financial modelling of the post-horizon period is particularly difficult, in any case more difficult than modelling of the explicit forecast period. It follows from the discussion in this paper that such a conclusion is not necessarily correct from a modelling point of view. In fact, the difficult period from a modelling point of view is the explicit forecast period.

The reason why the explicit forecast period is more difficult than the post-horizon period is that a valuation is driven by a scenario. It is the analyst doing the valuation who defines the scenario. As the scenario unfolds, more and more of the relevant information that is used in the calculation of free cash flow in subsequent years comes from previous years covered by the scenario, rather than from published financial statements pertaining to historical years.[17] This is clear from the discussion in Section 5 of the error in the continuing value formula (13). The precise extent of that error can be calculated, given that the number of years in the explicit forecast period is at least n. Nominal and real acquisition values of non-retired PPE cohorts (N_v and R_v) are then known, as the company moves into the post-horizon period of the scenario. The corresponding quantities may not be known, however, in the first year of the explicit forecast period. The information that is available at that point of the scenario, coming from published financial statements of the company being valued, is typically more aggregated, for instance net PPE for a few historical years. This distinction between the explicit forecast period and the post-horizon period has already been alluded to earlier (footnote 9 in Section 2). Also, there may be accounting biases (for instance, "accounting conservatism" in the valuation of PPE) in historical financial statements, and such biases may complicate free cash flow forecasts in the explicit forecast period. However, there is no reason at all for such biases in the post-horizon period, again since it is the analyst who is defining the scenario, including simulated financial statements, for no other purpose than obtaining

[17] Historical years are years before the explicit forecast period. A valuation is typically done as of the beginning of the first explicit forecast year, which is the same as the end of the last historical year.

free cash flow forecasts. In other words, there is no reason for the analyst to complicate her/his own work by incorporating accounting biases into the scenario.

In the second place, the previous discussion leads to a couple of pointers on how to implement the explicit forecast period of the discounted cash flow model. It is clear by now that it is a good idea to include at least n years in that period, so that all relevant information at the outset of the post-horizon period has been generated during the explicit forecast period. Free cash flow apparently needs to be forecasted for every explicit forecast year. To some extent, that is fairly easy. Based on assumptions about real growth and inflation in each explicit forecast year, it is easy (from a technical point of view) to generate annual forecasts of after-tax sales revenue minus cash costs. Also, the annual increase (or decrease) in invested working capital is easy to forecast. The question is then how one should forecast capital expenditures, and tax savings from fiscal depreciation of PPE.

Steady-state ratios between this year's net PPE and sales revenue, between depreciation and last year's net PPE, and between timing differences and this year's net PPE were formulated in Section 2 above. These ratios can be assumed to hold already in the last explicit forecast year (if necessary, by adding one more such year). Corresponding ratios for the first explicit forecast year can be set equal to the same ratios as in the last historical year, as evidenced by historical financial statements.[18] Alternatively, one can set these ratios for the first explicit forecast year equal to averages from several historical years, if several sets of historical financial statements are available. For intermediate years, one can interpolate (in a nonlinear fashion, to satisfy the equality $\sum_{v=0}^{n-1} R_v = KS$ that was mentioned in footnote 16 above) between the first and last years of the explicit forecast period. After setting up these ratios for each year in the explicit forecast period, one obtains each year's net PPE and each year's depreciation, and hence also each year's capital expenditures. One also obtains each year's deferred taxes, and consequently the change in deferred taxes. The year's tax reduction due to fiscal depreciation is then the sum of the tax reduction from economic life depreciation plus the change in deferred taxes. It is hence possible in each explicit forecast year to calculate free cash flow from the firm's operations somewhat like in Eq. (9) in Section 2. However, the three ratios mentioned at the beginning of this paragraph as well as other parameters (such as real growth and inflation) are no longer necessarily constant but can vary from one year to another in the explicit forecast period.

This suggested procedure for the explicit forecast period is simple to implement in a spreadsheet valuation program. It is based on starting information that is typically available in historical financial statements, and it is somewhat consistent with the manner in which free cash flow from operations is modelled in the continuing value formula (9). If the explicit forecast period is implemented in this suggested fashion, then the approximation error that is inherent in the continuing value formula (13) in Section 4 often becomes rather small, as already mentioned at the end of Section 5. Hence, this suggested explicit forecast period procedure fits rather well with continuing value as discussed in this paper.

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[18] For the ratio between depreciation and last year's net PPE, one evidently needs depreciation from the last historical income statement and net PPE from the next-to-last balance sheet. In other words, one needs historical financial statements from the last two historical years. Historical timing differences are computed by dividing historical deferred taxes by the tax rate.

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