

Discounted Cash Flow

A Theory of the Valuation of Firms

Lutz Kruschwitz and Andreas Löffler



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List of Symbols

| | |
|-----------------------|------------------------------------|
| $E[\cdot]$ | (conditional) expectation |
| $ \mathcal{F}_t$ | information in t |
| \widetilde{CF}_t | cash flow in t |
| \widetilde{GCF}_t | gross cash flow before tax in t |
| \widetilde{FCF}_t | free cash flow in t |
| \widetilde{FCF}_t^u | free cash flow of unlevered firm |
| \widetilde{FCF}_t^l | free cash flow of levered firm |
| Div | dividend |
| \widetilde{Tax} | tax payment |
| \widetilde{Tax}^u | tax payment of unlevered firm |
| \widetilde{Tax}^l | tax payment of levered firm |
| \widetilde{Inv} | investments |
| \widetilde{Accr} | accruals |
| \widetilde{EBIT} | earnings before interest and taxes |
| \widetilde{I} | interest |
| \widetilde{l} | leverage ratio to book values |
| \widetilde{l} | leverage ratio to market values |
| \widetilde{L} | debt–equity ratio to book value |
| \widetilde{L} | debt–equity ratio to market value |
| \widetilde{V}^u | market value of unlevered firm |
| \widetilde{V} | book value of unlevered firm |
| \widetilde{V}^l | market value of levered firm |
| \widetilde{E} | book value of equity |
| \widetilde{E} | market value of equity |
| \widetilde{D} | book value of debt |
| \widetilde{R} | debt repayment |
| \widetilde{D} | market value of debt |

| | |
|--------------------|--|
| $\tilde{\epsilon}$ | increase in subscribed capital |
| τ | tax rate |
| k | cost of capital |
| d^u | dividend–price ratio of unlevered firm |
| k^ϕ | WACC, type 1 |
| $WACC$ | WACC, type 2 |
| k^D | cost of debt |
| $k^{E,l}$ | cost of equity of levered firm |
| $k^{E,u}$ | cost of equity of unlevered firm |
| k | cost of capital of untaxed firm |
| r_f | riskless interest rate |

List of Definitions, Theorems, etc.

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