Number Systems - Date Representation

Comprehensive Course on Digital Circuits

Bhima Sankar Manthina • Lesson 3 • Oct 22, 2021

Till now :

- Base r Repr
- Conv
- BCD Codes
- r's Comp (r-1)'s Comp

→ BCD Arithmatic

→ Gray Code

→ DATA Repr

BCD Arithmetic :-

Binary Addition :-

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 1 = 2 = 10_2$

$1 + 1 + 1 = 3 = 11_2$

$$\begin{array}{r} 1\,1\,1\,1 \\ 1\,0\,1\,1\,1 \\ +\ 1\,1\,0\,1\,1 \\ \hline 1\,1\,0\,0\,1\,0 \end{array}$$

BCD Addition

$6+6=12 \Rightarrow$
$$\begin{array}{r} 1\,1 \\ 0\,1\,1\,0 \\ +\ 0\,1\,1\,0 \\ \hline 1\,1\,0\,0 \end{array} \rightarrow \text{Binary '12'}$$

BCD $12 \rightarrow 0001\ 0010$

We need to convert Binary Result in to BCD result

$$\left.\begin{matrix} 0-9 \\ + \\ 0-9 \end{matrix}\right\} \quad \overset{\text{Result}}{0 \text{ to } 18}$$

Result Single Digit : 0-9 → i.e., Binary & BCD result are same.

Result > '9' → 1010 (10) - - - - - - 10010 (18) → Binary

+6 → 10000 | +6 → 11000 → BCD

$$\begin{array}{r} 10000 \to 16 \\ -\ 1010 \to 10 \\ \hline 010 \end{array} \qquad \begin{array}{r} 11000 \to 24 \\ -10010 \to 18 \\ \hline 0110 \end{array}$$

BCD Addition:

When we add two BCD numbers using a binary adder to get a valid BCD result we need to perform the following

(a).If the lower order nibble of the result is greater than ‘9’ or Auxiliary carry =1 then add ‘6’ to the lower order nibble of the result

(b).If the Higher order nibble of the result is greater than ‘9’ or Carry from MSB =1 then add ‘6’ to the Higher order nibble of the result .

```
 1          1 1 0 ← AC      1
 65   →     0110          0101
                            =
 29   →   + 0010          1001
                  1      1
 ----     ------------------------
 94       0 1000          1110  → '>9'
 ----     ↑
         Cy
                           110
          ------------------------
            1001          0100
            9             4     → Valid BCD
```

1

64 → 0110 0100

+36 → 0011 0110

100

0 1001 1010 → '79'

110

79 → 1010 0000

110

1 0000 0000

$\underbrace{1}_{1}$ $\underbrace{0000}_{0}$ $\underbrace{0000}_{0}$

```
  1           1 1 1 1 → A.C
 38   →   0 0 1 1     1 0 0 0
 49   →   0 1 0 0     1 0 0 1
----     ---------------------
 87     0 1 0 0 0     0 0 0 1
----     ---------------------
                        1 1 0
         ---------------------
          1 0 0 0     0 1 1 1
             8           7
```

$$\begin{array}{r} 25 \\ +\,23 \\ \hline 48 \end{array} \rightarrow \begin{array}{rr} & \scriptstyle 1\;0 \quad\; 1\,1\,1 \\ & 0010 \;\; 0101 \\ & 0010 \;\; 0011 \\ \hline 0 & 0100 \;\; 1000 \\ & 4 \qquad\; 8 \end{array}$$

$$\begin{array}{r} \scriptstyle 1 \\ 98 \\ +\,98 \\ \hline 196 \end{array} \rightarrow \begin{array}{rr} & \scriptstyle 1\,1 \\ & 1001 \;\; 1000 \\ & 1001 \;\; 1000 \\ \hline 1 & 0011 \;\; 0000 \\ & 110 \;\;\; 110 \\ \hline 1 & 1001 \;\; 0110 \\ 1 & 9 \qquad\; 6 \end{array}$$

Excess-3 Addition:

When we add two Excess-3 numbers using a binary adder to get a valid Excess-3 result we need to perform the following.

If there is a Carry or Aux Carry add ‘3’ to the corresponding nibble otherwise subtract ’3’ from the corresponding nibble.

```
 1
 45        →      1  1 1 1 → AC
                  0111   1000
 29        →      0101   1100
----              -------------
 74            0  1101   0100
               ↑  -------------
               Cy -0011  +0011
                  -------------
                  1010   0111  → Excess-3 result
                  -------------
                   7      4
```

If two numbers in excess - 3 code are added and the result is less than 9, then to get equivalent binary

If we need Excess-3 result

(a) 0011 is subtracted

(b) 0011 is added

(c) 0110 is subtracted

(d) 0110 is added

Code wheel :

6 → 0110

7 → 0111

Binary

5 → 0101 P

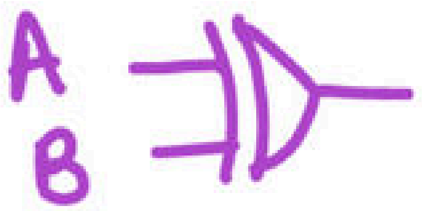
0100 E

Transient

6 → 0110 h

If only one bit varies then no transient

EX-OR :-



$A \oplus B$

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Gray Code :-

It is a unit distance code.

i.e, between any two adjacent numbers only 1-bit difference.

Binary to Gray :

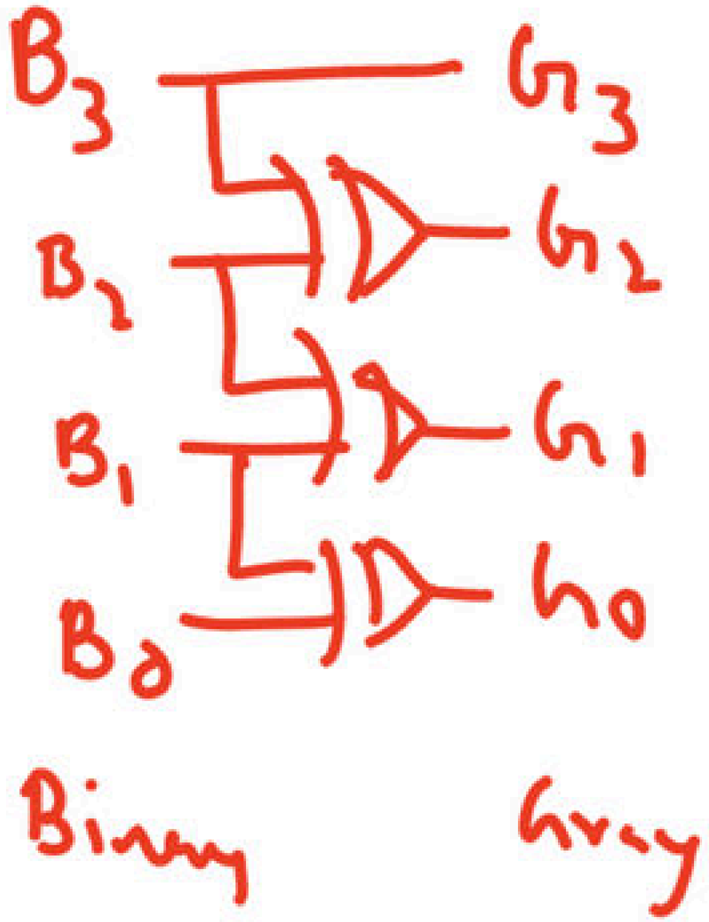
$G_i = B_i$ if (MSB)

$G_i = B_i \oplus B_{i+1}$ (otherwise)

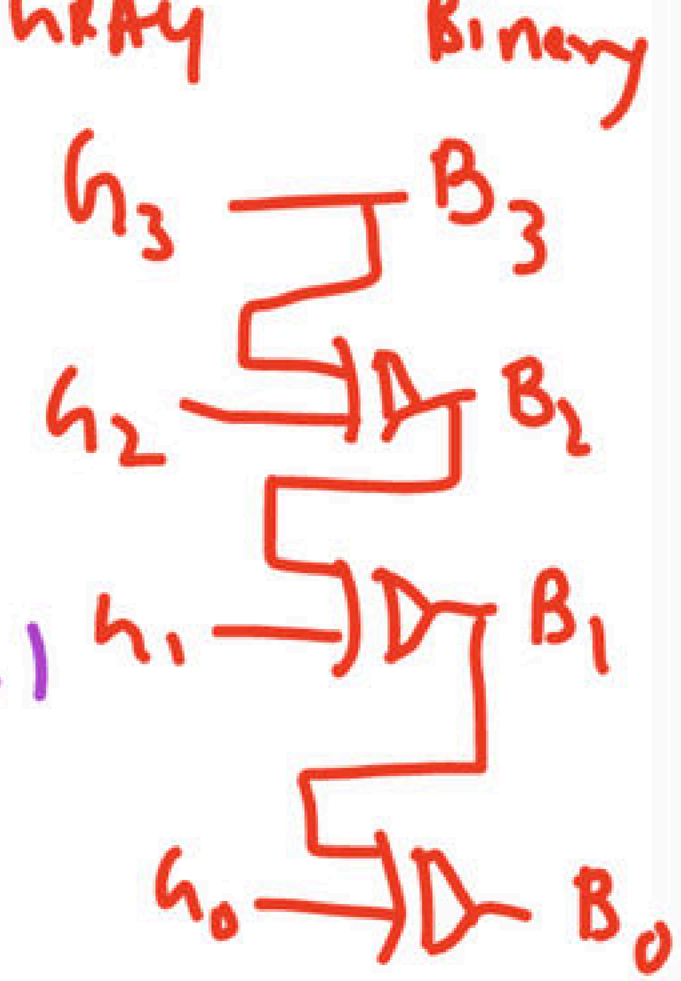
Binary : 1 0 1 1

$\oplus$ $\oplus$ $\oplus$

Gray : 1 1 1 0



Gray to Binary :



$B_i = G_i$ (MSB)

$B_i = B_{i+1} \oplus G_i$ (otherwise)

Gray : 1 1 1 0

$\oplus$ $\oplus$ $\oplus$

Binary : 1 0 1 1

B 0 0 0 | B 0 0 1 | B 0 1 0 | B 0 1 1 | B → 1 0 0 | B → 1 0 1

G 0 0 0 | G 0 0 1 | G → 0 1 1 | G 0 1 0 | G → 1 1 0 | G → 1 1 1

B → 1 1 0 | B → 1 1 1

G → 1 0 1 | G → 1 0 0

Binary	GRAY
000	000
001	001
010	011
011	010
100	110
101	111
110	101
111	100

→ Cyclic Unit Distance Code

(Adjacency Property)

Gray of 0 - 7

→ 0 0 0
→ 0 0 1
→ 0 1 1
→ 0 1 0
→ 1 1 0
→ 1 1 1
1 0 1
1 0 0

In large radar installations, it is required to translate the angular position of a shaft into digital information. This is most generally achieved by employing a code wheel. For unambiguous sensing of the shaft position, one employs a/an

(a) octal code

(b) BCD code

(c) binary Gray code

(d) natural binary code

- DATA Representation.

DATA REPRESENTATION:

- Unsigned Magnitude Representation.
- Signed Magnitude Representation.
- 1’s Compliment Representation.
- 2’s Compliment Representation.

Unsigned Magnitude representation:

Here we represent positive numbers only.

Range: With N bits we can represent $\underline{0}$ to $\underline{2^N\text{-}1}$.

Ex :- N = 4 bits → 0000 to 1111

→ 0 to 15 = 0 to $2^N - 1$ = 0 to $(2^4 - 1)$

Signed Magnitude representation:

Here we can represent both positive and negative numbers.
M.S.B is dedicated for sign , if sign bit is '0' den it's positive num otherwise if sign bit is '1 'its negative number.
Range: With N bits we can represent $-(2^{N-1}-1)$ to $(2^{N-1}-1)$.

	S	Mag	S	Mag	
$-0 \leftarrow$	1	000	0	000	$\rightarrow +0$
$-1 \leftarrow$	1	001	0	001	$\rightarrow +1$
$-2 \leftarrow$	1	010	0	010	$\rightarrow +2$
	⋮		⋮		
$\underline{\underline{-7}} \leftarrow$	1	111	0	111	$\rightarrow \underline{\underline{+7}}$

$0 \to +0 \to 0000$ } Two

$0 \to -0 \to 1000$ } reprns

$n = 4$ bit

Range : -7 to $+7$

$-\left[2^{4-1}-1\right]$ to $\left[2^{4-1}-1\right]$

$-\left(2^{n-1}-1\right)$ to $\left(2^{n-1}-1\right)$

n = 4 bit

+7 = 0111

−7 : 1111

n = 6 bit

+7 = 000111

−7 : 100111

$\underset{S}{1}\,00\,\underbrace{111}_{Mag}$

n = 8-bit

+7 = 0000 0111

$\underset{S}{0}\,000\,0\underbrace{111}_{M}$

−7 : 1000 0111

$\underset{S}{1}\,000\,0\underbrace{111}_{M}$

To Incr no of bits we need to Insert - add 0's between Sign & Magnitude.

Disadvantages:

1. If there is any carry in ~~the addition(end around carry)~~ we need add it back to result.

2.Zero is having two representations.

3.Extending the range is difficult we need to insert 0’s between sign n magnitude

We Can overcome these disadvantages by using Complimentary Representation.

- Signed 1’s Compliment Representation:

In a signed 1’s Compliment representation if number is positive it is represented as it is, if the num is negative it is represented by it’s 1’s compliment form

$-0 \leftarrow 1111$		$0000 \rightarrow +0$
$-1 \leftarrow 1110$		$0001 \rightarrow +1$
$-2 \leftarrow 1101$		$0010 \rightarrow +2$
		⋮
$-7 \leftarrow 1000$		$0111 \rightarrow +7$

$0 \rightarrow +0 : 0000$

$0 \rightarrow -0 : 1111$

Range : -7 to $+7$

$-(2^{n-1}-1)$ to $(2^{n-1}-1)$

n = 4 bits

+7 : 0111

-7 : 1000

n = 6 bits

+7 : 000111

-7 : 111000

n = 8 bits

+7 : 0000 0111

-7 : 1111 1000

To Extend no of bits

Copy MSB

1's Compliment Arithmatic

$A - B = A +$ 1's comp of 'B'

Ex :

2 → 0010

−2 → 1101

5 = 0101 1's comp of 5 = 1010 = '−5'

3 = 0011 1's comp of 3 = 1100 = '−3'

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \qquad \begin{array}{r} 0101 \\ +1100 \\ \hline \text{(1)}\,0001 \\ 1 \\ \hline 0010 \end{array} = 2$$

$$\begin{array}{r} 3 \\ -5 \\ \hline -2 \end{array} \qquad \begin{array}{r} 0011 \\ +1010 \\ \hline 1101 \end{array} \rightarrow \text{'}-2\text{'}$$

no carry ⇒ result = −ve

& it is in 1's comp

[To know mag : 1's comp → $\overline{1101}$ = 0010 → −2]

It's magnitude

In 1's Comp

1 1 0 1 →

$-7 \quad 4 \quad 2 \quad 1$

$-7+4+0+1 = -2$

1 1 1 1 0 1

$-31 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$

$-31+16+8+4+1$

$= -2$

$-(2^{N-1}-1)$

0 0 1 0

$-7 \quad 4 \quad 2 \quad 1$

$0+0+2+0 = 2$

Advantages:

In 1's compliment representation to extend the range is easy we need to copy MSB repeatedly.(sign extension)

Disadvantages:

1. If there is any carry in the addition(end around carry) we need add it back to result.
2.Zero is having two representations.

We can over come by 2's compliment representation.

2's Compliment Representation

Signed 2’s Compliment Representation:

In a signed 2’s Compliment representation if number is positive it is represented as it is, if the num is negative it is represented by it’s 2’s compliment form

$n = 4$ bits

$-1 \leftarrow 1\,111$		$0\,000 \rightarrow 0$
$-2 \leftarrow 1\,110$		$0\,001 \rightarrow +1$
$-3 \leftarrow 1\,101$		$0\,010 \rightarrow +2$
		$\vdots$
$-7 \leftarrow 1\,001$		
$-8 \leftarrow 1\,000$		$0\,111 \rightarrow +7$

$0 \rightarrow +0 :- 0000$

$0 \rightarrow -0 :- 1111$

$+ \quad 1$

$\underline{\;1\;0000\;}$

neglect

Same

Range :- '-8' to '$+7$'

$-(2^{n-1})$ to $(2^{n-1}-1)$

4-bits

+7 : 0111

-7 : 1000
+1
1001

6 bits

+7 : 000111

-7 : 111000
+1
111001

8-bits

+7 : 0000 0111

-7 : 11111000
+1
11111001

To Extend no of bits Copy MSB

$$\boxed{\text{2's Compliment Arithmatic}}$$

A - B = A + 2's comp of 'B'

$2 \rightarrow 0010 \quad -2 \rightarrow \left.\begin{array}{l}1101 \\ +1\end{array}\right\} 1110$

$5 = 0101$

$-5 = $ 2's Comp of $5 = \begin{array}{r}1010 \\ 1 \\ \hline 1011\end{array}$

$3 = 0011$

$-3 = $ 2's Comp of $3 = \begin{array}{r}1100 \\ 1 \\ \hline 1101\end{array}$

$$\begin{array}{rcr} & & 1\ \ 1 \\ 5 & \rightarrow & 0101 \\ -3 & \rightarrow & 1101 \\ \hline 2 & 1 & 0010 \end{array}$$

1 → neglect

0010 → 2

$$\begin{array}{rcr} & & 1\ 1 \\ 3 & \rightarrow & 0011 \\ -5 & \rightarrow & 1011 \\ \hline & & 1110 \rightarrow -'2' \end{array}$$

No carry Ans -ve & in 2's comp

(To know mag :- Find 2's Comp of result

$1110 \xrightarrow{2's} 0001 + 1 = 0010 \rightarrow (-2) \rightarrow$

To know magnitude

$-8\ 4\ 2\ 1$

$1\ 1\ 1\ 0$

$-8+4+2=-2$

$1\ 1\ 1\ 1\ 1\ 1\ 0$

$-64\ 32\ 16\ 8\ 4\ 2\ 1$

$\uparrow$

(-2^{N-1})

Representation	Range
Unsigned Magnitude representation	0 to 2^N-1.
Signed Magnitude representation	$-(2^{N-1}-1)$ to $(2^{N-1}-1)$
Signed 1's Compliment Representation	$-(2^{N-1}-1)$ to $(2^{N-1}-1)$
Signed 2's Compliment Representation	$-(2^{N-1})$ to $(2^{N-1}-1)$

G-2020

Q.No. 38 P, Q, and R are the decimal integers corresponding to the 4-bit binary number 1100 considered in signed magnitude, 1's complement, and 2's complement representations, respectively. The 6-bit 2's complement representation of (P + Q + R) is

(A) 110101 ✓

(B) 110010

(C) 111101

(D) 111001

1100 → −4 (P)

1100 → 1's comp → Q = −7+4+0+0 = −3
−7 4 2 1

1100 → 2's comp → R = −8+4 = −4
−8 4 2 1

P+Q+R = −11

+11 → 001011

−11 → 2's Comp ⇒

110100
+1

110101

$\underbrace{00101}1 \leftarrow$

110101

10010

2's Comp = 01101
+1

01110

$\underbrace{1001}0 \leftarrow$

01110

$\underbrace{1000}1 \leftarrow$

0111 1

10100000

01100000

Dpp → Practice

Image

(1) which of the following is incorrect?

(a) $11100_2 - 10001_2 = 00101_2$

(b) $15E_{16} = 350_{10}$

(c) $81_{10} = 1010001_2$

(d) $37.4_8 = 011\ 111.100$

(2) if 2 NO's in excess -3 code are added and the result is less than 9, then to get equivalent binary

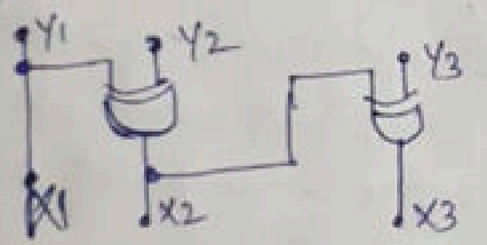
(a) 0011 is subtracted

(b) 0011 is added.

(c) 0110 is subtracted

(d) 0110 is added.

(3) The logic circuit given below converts a gray code $Y_1 Y_2 Y_3$ into



(a) Excess - 3 code (b) Binary code

(c) BCD code (d) Hamming code

(4) The greatest Negative No wch can be stored in a computer that has 8-bit word length and uses 2's complement arithmetic is,

(a) -256 (b) -255 (c) -128 (d) -127

(5) 2's complement of a given 3 or more bit binary No of Non-zero magnitude is the same as the original No if all bits except the

(a) MSB is zero (b) LSB is zero

(c) MSB is one (d) LSB is one

(6) If 73_x (in base x No system) is equal to 54_Y (in base Y No system), the possible values of X and Y are:

(a) 8 and 16 (b) 10 and 12

(c) 9 and 13 (d) 8 and 11

(7) An 8085 micro processor executes the following instructions: 2 No's are represented in signed 2's complement form as

P = 11101101 and Q = 11100110

if Q is subtracted from P, the value obtained in signed 2's complement form is,

(a) 100000111 (b) 00000111

(c) 11111001 (d) 01111001

(8) If $(11X1Y)_8 = (12C9)_{16}$ then the values X and Y are

(a) 5 and 1 (b) 5 and 7
(c) 7 and 5 (d) 1 and 5

(9) The subtraction of a binary No Y from another binary No X, done by adding the 2's complement of Y to X, results in a binary No without overflow. This implies that the result is:

(a) Negative and is in Normal form.
(b) Negative and is in 2's complement form.
(c) Positive and is in Normal form.
(d) Positive and is in 2's complement form.

(10) 2's complement representation of a 16-bit No (one sign bit and 15 magnitude bits) is FFFF. Its Magnitude in decimal representation is

(a) 0 (b) 1 (c) 32,767
(d) 65,535.

(11) A signed integer has been stored in a byte using the 2's complement format. We wish to store the same integer in a 16 bit-word. We should

~~(a) copy the original byte to the more significant byte with zeros.~~
~~(b) copy the original byte to the more significant byte of the word~~

(a) copy the original byte to the less significant byte of the word and fill the more significant byte with zeros.
(b) copy the original byte to the more significant byte of the word and fill the less significant byte with zeros.
(c) copy the original byte to the less significant byte of the word and make eat bit of the more significant bit to the original byte.
(d) copy the original byte to the less significant byte as well as the more significant byte of the word.

(12) An equivalent 2's complement representation of the 2's complement No 1101 is

(a) 110100 (b) 001101

(c) 110111 (d) 111101

(13) The 2's complement representation of -17 is

(a) 101110 (b) 101111

(c) 111110 (d) 110001

(14.) The range of signed decimal No's that can be represented by 6-bit 1's complement No is

(a) -31 to +31 (b) -63 to +63

(c) -64 to +63 (d) -32 to +31

(15.) Decimal 43 in Hexadecimal and BCD no system is respectively.

(a) B2, 0100 0011 (b) 2B, 0100 0011

(c) 2B, 0011 0100 (d) B2, 0100 0100

(16) X = 01110 and Y = 11001 are two 5-bit binary NO's represented in 2's complement format. The Sum of X and Y represented in 2's complement format using 6 bits is

(a) 100111 (b) 001000

(c) 000111 (d) 101001

(17) The No of bytes required to represent the decimal No 1856357 in packed BCD (Binary coded decimal) form is ________

(18) 11001, 1001 and 111001 Correspond to the 2's complement representation of which one of the following sets of No?

(a) 25, 9 and 57 respectively

(b) -6, -6 and -6 respectively

(c) -7, -7 and -7 respectively

(d) -25, -9 and -57 respectively

(19) If X = 111.101 and Y = 101.110 (both in binary) calculate X+Y and X-Y, Y-X by 2's complement method.

20(a) convert 11101.01 to decimal

20(b) perform 11100.101-101.01 using 2's complement method

21. Determine no of 1's in the binary Equivalent of the following
8192 * 6 +
+4096 * 7 + 512 * 4
+ 256 * 2 + 16 * 1

unacademy

▲ 1 • Asked by Arindam

please add me on wp grp and i have text you in telegram

▲ 1 • Asked by Abhishek

Sir plz use white slide...if possible

unacademy

1 • Asked by Abhishek

Sir excees 3 addition once

Plz ...

unacademy

▲ 1 • Asked by Rishav

What was the ambiguity of binary before we discussed the gray

100

011

0 1 0

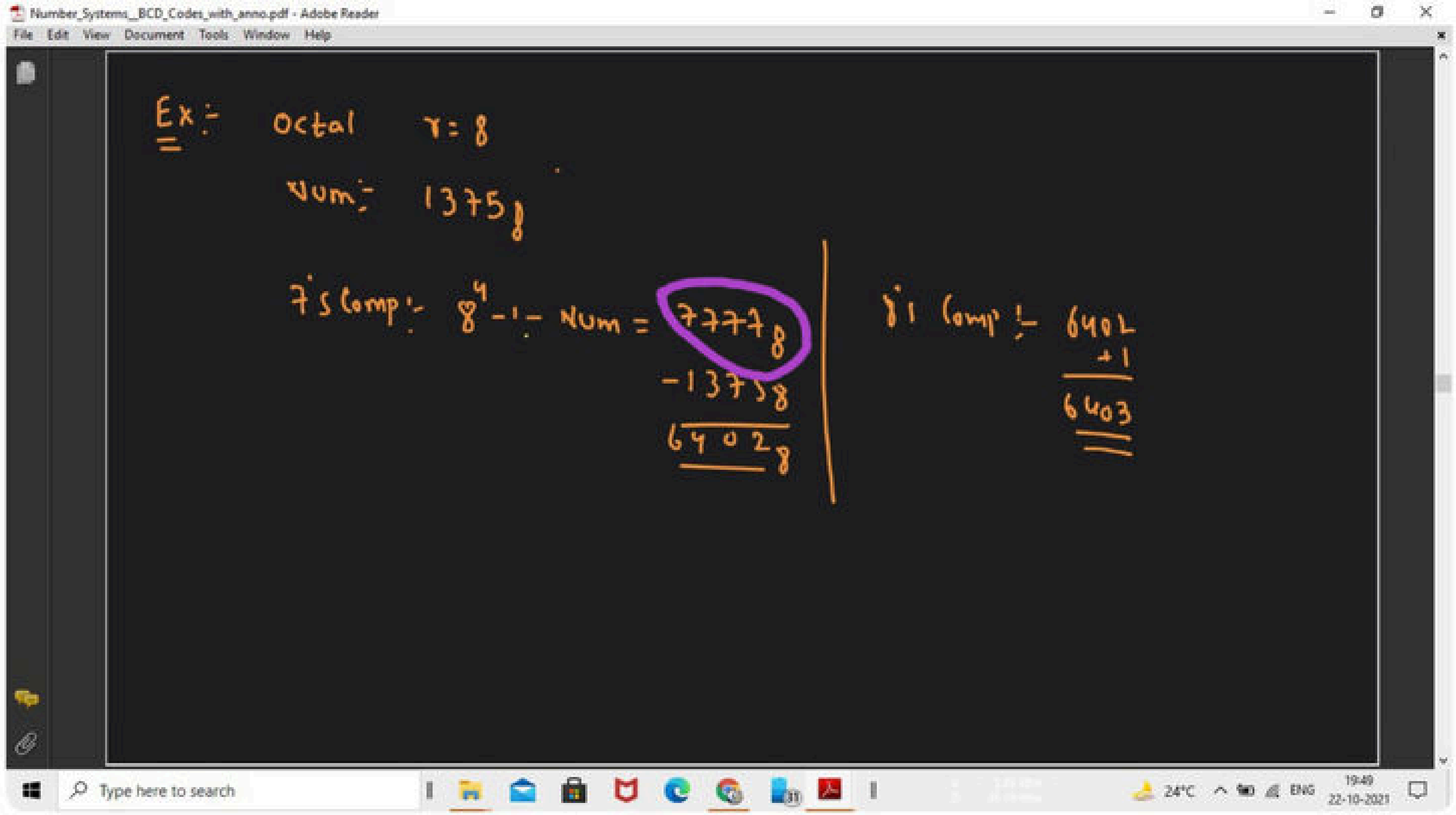
1

0 0 0

100

1 • Asked by Vishal

sir yha pr 7777 kaise aay h R^n-1 -num wale formula se kaise nikala sir ye??



$$8^4 - 1 = 4095_{10}$$

$$\downarrow$$

$$7777_8$$